Numerical Developments in Relativistic Spin Hydrodynamics

Sushant Kumar Singh
(University of Florence)

in collaboration with

R. Ryblewski (IFJ PAN, Kraków) and W. Florkowski (UJ, Kraków)

ECT* workshop on Spin and quantum features of QCD plasma
September 16-20, 2024

Finanziato dall'Unione europea NextGenerationEU

Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

ICSC Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

NATIONAL SCIENCE CENTRE POLAND
SONATA BIS 8 Grant No. 2018/30/E/ST2/00432

Outline

- Motivation
- Equations
- Numerical details
- Results
  - Code test
  - Spin polarization[1]
- Summary

[1] Presented results are preliminary and may subject to updates.

Motivation

- **Global polarization** of Λ hyperons is described by **standard viscous hydrodynamics** in terms of **thermal vorticity of fluid flow** evaluated at freeze-out.
- Description of the **local polarization** data involves introduction of **thermal shear** contribution. However, the forms of shear contribution differ for **different derivation methods** [F. Becattini, M. Buzzegoli, and A. Palermo, PLB 820, 136519 (2021) and S. Y. F. Liu and Y. Yin, JHEP 07 (2021) 188] and matching the data requires **additional assumptions**.
- **Spin hydrodynamics** may capture some **nontrivial spin dynamics before freeze-out** which above methods may omit.
- Kinetic-theory based extension of hydrodynamic theory was recently developed [W. Florkowski *et al.*, Prog. Part. Nucl. Phys. 108 (2019) 103709 and N. Weickgenannt *et al.*, PRL 127, 052301 (2021)].
- No numerical implementations available so far.

Relativistic Spin Hydrodynamics

- A state-of-the-art viscous hydrodynamic equations are [PRD 85, 114047 (2012), PRC 94, 024907 (2016), Comput. Phys. Commun. 185 (2014), 3016]

$$D_\mu T^{\mu\nu} = 0 \quad , \quad D_\mu J^\mu = 0$$

$$\dot{\Pi} = \frac{\Pi_{NS} - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\Pi\theta + \frac{\lambda_{\Pi\pi}}{\tau_\Pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} = \frac{\pi^{\mu\nu}_{NS} - \pi^{\mu\nu}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\mu\nu}\theta + \frac{\phi_7}{\tau_\pi}\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\Pi\sigma^{\mu\nu}$$

where

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \quad , \quad J^\mu = nu^\mu$$

- The conservation of total angular momentum implies the conservation of spin current when energy-momentum tensor is symmetric [W. Florkowski *et al.*, Prog. Part. Nucl. Phys. 108 (2019) 103709]

$$D_\mu J^{\mu\lambda\nu} = 0 \quad \rightarrow \quad D_\mu S^{\mu\lambda\nu} = T^{\nu\lambda} - T^{\lambda\nu} \quad \rightarrow \quad D_\mu S^{\mu,\beta\gamma} = 0$$

Spin equations in Milne coordinates

In arbitrary coordinate system

$$\partial_\alpha S^{\alpha,\beta\gamma} + \Gamma^{\alpha}_{\rho\alpha} S^{\rho,\beta\gamma} + \Gamma^{\beta}_{\delta\alpha} S^{\alpha,\delta\gamma} + \Gamma^{\gamma}_{\omega\alpha} S^{\alpha,\beta\omega} = 0$$

Choosing Milne coordinates, the only non-vanishing symbols are

$$\Gamma^{\eta}{}_{\tau\eta} = \Gamma^{\eta}{}_{\eta\tau} = \frac{1}{\tau} \qquad , \qquad \Gamma^{\tau}{}_{\eta\eta} = \tau.$$

The spin equations become

$$\begin{aligned}
\partial_\tau(\tau S^{\tau,\tau x}) + \partial_x(\tau S^{x,\tau x}) + \partial_y(\tau S^{y,\tau x}) + \partial_\eta(\tau S^{\eta,\tau x}) + \tau^2 S^{\eta,\eta x} &= 0 \\
\partial_\tau(\tau S^{\tau,\tau y}) + \partial_x(\tau S^{x,\tau y}) + \partial_y(\tau S^{y,\tau y}) + \partial_\eta(\tau S^{\eta,\tau y}) + \tau^2 S^{\eta,\eta y} &= 0 \\
\partial_\tau(\tau S^{\tau,\tau \eta}) + \partial_x(\tau S^{x,\tau \eta}) + \partial_y(\tau S^{y,\tau \eta}) + \partial_\eta(\tau S^{\eta,\tau \eta}) + S^{\tau,\tau\eta} &= 0 \\
\partial_\tau(\tau S^{\tau,xy}) + \partial_x(\tau S^{x,xy}) + \partial_y(\tau S^{y,xy}) + \partial_\eta(\tau S^{\eta,xy}) &= 0 \\
\partial_\tau(\tau S^{\tau,x\eta}) + \partial_x(\tau S^{x,x\eta}) + \partial_y(\tau S^{y,x\eta}) + \partial_\eta(\tau S^{\eta,x\eta}) + S^{\tau,x\eta} + S^{\eta,x\tau} &= 0 \\
\partial_\tau(\tau S^{\tau,y\eta}) + \partial_x(\tau S^{x,y\eta}) + \partial_y(\tau S^{y,y\eta}) + \partial_\eta(\tau S^{\eta,y\eta}) + S^{\tau,y\eta} + S^{\eta,y\tau} &= 0
\end{aligned}$$

Spin equations for numerical solution

Define $\tilde{\partial}_\nu$ and $\tilde{S}$ as

$$\tilde{\partial}_\nu = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{1}{\tau}\frac{\partial}{\partial \eta} \right\},$$

$$\widetilde{S}^{\alpha,\beta\gamma} = S^{\alpha,\beta\gamma} \qquad \text{for} \qquad \alpha, \beta, \gamma \neq \eta$$

$$\widetilde{S}^{\alpha,\beta\gamma} = \tau S^{\alpha,\beta\gamma} \qquad \text{if any one of } \alpha, \beta, \gamma \text{ is } \eta$$

$$\widetilde{S}^{\alpha,\beta\gamma} = \tau^2 S^{\alpha,\beta\gamma} \qquad \text{if any two of } \alpha, \beta, \gamma \text{ is } \eta$$

Spin equations become

$$\widetilde{\partial}_\tau(\tau \widetilde{S}^{\tau,\tau x}) + \widetilde{\partial}_x(\tau \widetilde{S}^{x,\tau x}) + \widetilde{\partial}_y(\tau \widetilde{S}^{y,\tau x}) + \widetilde{\partial}_\eta(\tau \widetilde{S}^{\eta,\tau x}) + \widetilde{S}^{\eta,\eta x} = 0$$

$$\widetilde{\partial}_\tau(\tau \widetilde{S}^{\tau,\tau y}) + \widetilde{\partial}_x(\tau \widetilde{S}^{x,\tau y}) + \widetilde{\partial}_y(\tau \widetilde{S}^{y,\tau y}) + \widetilde{\partial}_\eta(\tau \widetilde{S}^{\eta,\tau y}) + \widetilde{S}^{\eta,\eta y} = 0$$

$$\widetilde{\partial}_\tau(\tau \widetilde{S}^{\tau,\tau \eta}) + \widetilde{\partial}_x(\tau \widetilde{S}^{x,\tau \eta}) + \widetilde{\partial}_y(\tau \widetilde{S}^{y,\tau \eta}) + \widetilde{\partial}_\eta(\tau \widetilde{S}^{\eta,\tau \eta}) = 0$$

$$\widetilde{\partial}_\tau(\tau \widetilde{S}^{\tau,xy}) + \widetilde{\partial}_x(\tau \widetilde{S}^{x,xy}) + \widetilde{\partial}_y(\tau \widetilde{S}^{y,xy}) + \widetilde{\partial}_\eta(\tau \widetilde{S}^{\eta,xy}) = 0$$

$$\widetilde{\partial}_\tau(\tau \widetilde{S}^{\tau,x\eta}) + \widetilde{\partial}_x(\tau \widetilde{S}^{x,x\eta}) + \widetilde{\partial}_y(\tau \widetilde{S}^{y,x\eta}) + \widetilde{\partial}_\eta(\tau \widetilde{S}^{\eta,x\eta}) + \widetilde{S}^{\eta,x\tau} = 0$$

$$\widetilde{\partial}_\tau(\tau \widetilde{S}^{\tau,y\eta}) + \widetilde{\partial}_x(\tau \widetilde{S}^{x,y\eta}) + \widetilde{\partial}_y(\tau \widetilde{S}^{y,y\eta}) + \widetilde{\partial}_\eta(\tau \widetilde{S}^{\eta,y\eta}) + \widetilde{S}^{\eta,y\tau} = 0$$

Assumptions

- We consider the GLW form for the spin current [W. Florkowski *et al.*, Prog. Part. Nucl. Phys. 108 (2019) 103709]

$$S^{\alpha,\beta\gamma}_{\rm GLW} = A_1 u^\alpha \varpi^{\beta\gamma} + A_2 u^\alpha u^{[\beta} k^{\gamma]} + A_3 \left(u^{[\beta} \varpi^{\gamma]\alpha} + g^{\alpha[\beta} k^{\gamma]} \right)$$

  where $k_\mu = \varpi_{\mu\alpha} u^\alpha$, and A_1, A_2, A_3 are given by

$$A_1 = \mathcal{C} \frac{T^3}{\pi^2} \left[\left(4 + \frac{m^2}{2T^2} \right) K_2\left(\frac{m}{T}\right) + \frac{m}{T} K_1\left(\frac{m}{T}\right) \right]$$

$$A_2 = 2\mathcal{C} \frac{T^3}{\pi^2} \left[\left(12 + \frac{m^2}{2T^2} \right) K_2\left(\frac{m}{T}\right) + 3\frac{m}{T} K_1\left(\frac{m}{T}\right) \right]$$

$$A_3 = -\mathcal{C} \frac{T^3}{\pi^2} \left[4 K_2\left(\frac{m}{T}\right) + \frac{m}{T} K_1\left(\frac{m}{T}\right) \right]$$

  where $\mathcal{C} = \cosh(\mu/T)$ and m is mass of Λ i.e. $m = m_\Lambda = 1.115$ GeV.
- **Unless stated otherwise ϖ denotes the spin polarization tensor. Thermal vorticity will be denoted by $\varpi_{\rm th}$**

Assumptions

- We further assume that the spin polarization is small which implies that the energy-momentum conservation equations do not receive feedback from spin conservation, hence they decouple from spin dynamics.
- As a result we may first solve **equations of motion for $T^{\mu\nu}$ and N^{μ} (we call it "background hydrodynamics")** and subsequently we **evolve $S^{\lambda\mu\nu}$ (we call it "spin hydrodynamics")**.

Numerical Method

- Finite volume method. Reconstruction at cell boundaries using minmod slope limiter.
- Approximate solution to local Riemann problems is obtained using relativistic HLLE (Harten-Lax-van Leer-Einfeldt) prescription. [Nucl. Phys. A 595 (1995) 346-382 and Comput. Phys. Commun. 185 (2014), 3016]
- We extend this method to additional 6 equations for spin current.
- Due to the choice of the spin current, we have

$$S = \mathbb{M}(u^\mu, T)\ \varpi$$

For example,

$$S = \{\widetilde{S}^{\tau,\tau x}, \widetilde{S}^{\tau,\tau y}, \widetilde{S}^{\tau,\tau\eta}, \widetilde{S}^{\tau,xy}, \widetilde{S}^{\tau,x\eta}, \widetilde{S}^{\tau,y\eta}\}^T$$

$$\varpi = \{\widetilde{\varpi}^{\tau x}, \widetilde{\varpi}^{\tau y}, \widetilde{\varpi}^{\tau\eta}, \widetilde{\varpi}^{xy}, \widetilde{\varpi}^{x\eta}, \widetilde{\varpi}^{y\eta}\}^T$$

Results

Testing the code

Assuming boost invariance

$$\tau S_1 \dot{\tilde{\varpi}}^{\tau x} + \left(S_1 + \tau S_1' \dot{T} + \frac{A_3}{2} \right) \tilde{\varpi}^{\tau x} = 0$$

$$\tau S_1 \dot{\tilde{\varpi}}^{\tau y} + \left(S_1 + \tau S_1' \dot{T} + \frac{A_3}{2} \right) \tilde{\varpi}^{\tau y} = 0$$

$$S_1 \dot{\tilde{\varpi}}^{\tau \eta} + \left(S_1 + \tau S_1' \dot{T} \right) \tilde{\varpi}^{\tau \eta} = 0$$

$$\tau A_1 \dot{\tilde{\varpi}}^{xy} + \left(A_1 + \tau A_1' \dot{T} \right) \tilde{\varpi}^{xy} = 0$$

$$\tau A_1 \dot{\tilde{\varpi}}^{x\eta} + \left(A_1 + \tau A_1' \dot{T} - \frac{A_3}{2} \right) \tilde{\varpi}^{x\eta} = 0$$

$$\tau A_1 \dot{\tilde{\varpi}}^{y\eta} + \left(A_1 + \tau A_1' \dot{T} - \frac{A_3}{2} \right) \tilde{\varpi}^{y\eta} = 0$$

where $S_1 = A_1 - \frac{A_2}{2} - A_3$. Bjorken hydrodynamics gives $\dot{T} = -\frac{T}{3\tau}$.

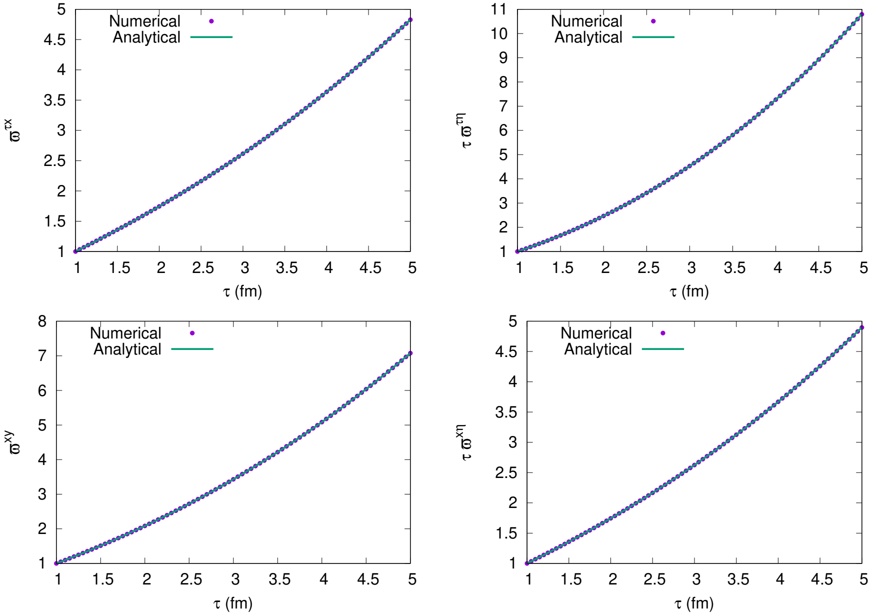


Figure: Comparison of spinhydro code with Mathematica solution.

Other inputs for polarization studies

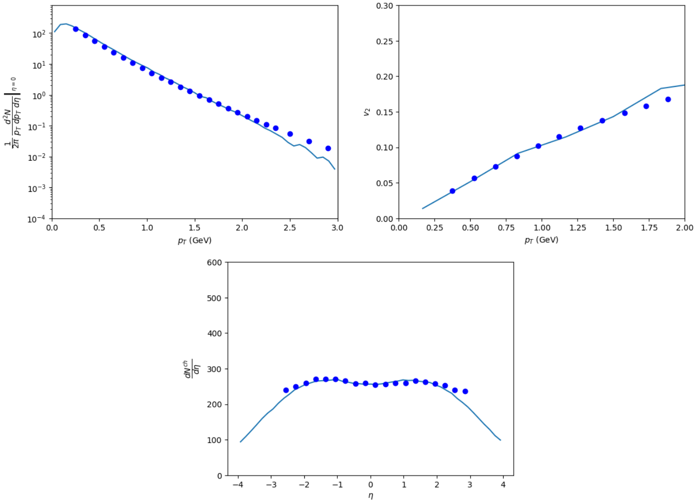
- The **background hydrodynamics** is initialized using "smooth" variant of superMC initial condition. (PRC 102, 014909 (2020)and PRC 106, 014905 (2022))

- The parameters of initial condition model and switching temperature are determined by fitting model results to experimentally mesured bulk observables. For this purpose on the switching hypersurface we generate 2000 events with SMASH afterburner.

- The initial condition for **spin hydrodynamics** is not a priori known. We initialize spin polarization tensor ϖ with thermal vorticity $\varpi_{\rm th}$. For this purpose background hydrodynamics is evolved for **one** timestep to determine flow gradients at the initial time.

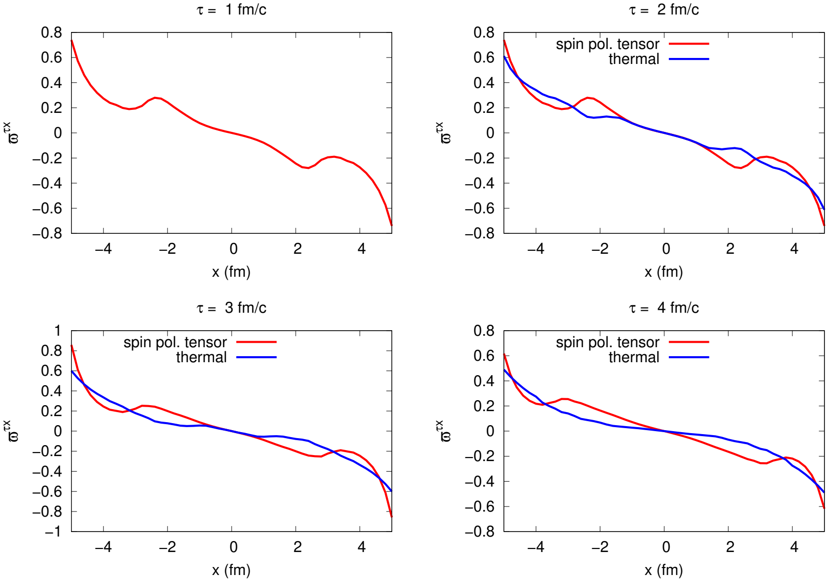
- Subsequently we evolve spin hydrodynamics using the hydrodynamic fields (T, μ, u^μ) determined with background hydrodynamics.

Bulk observables

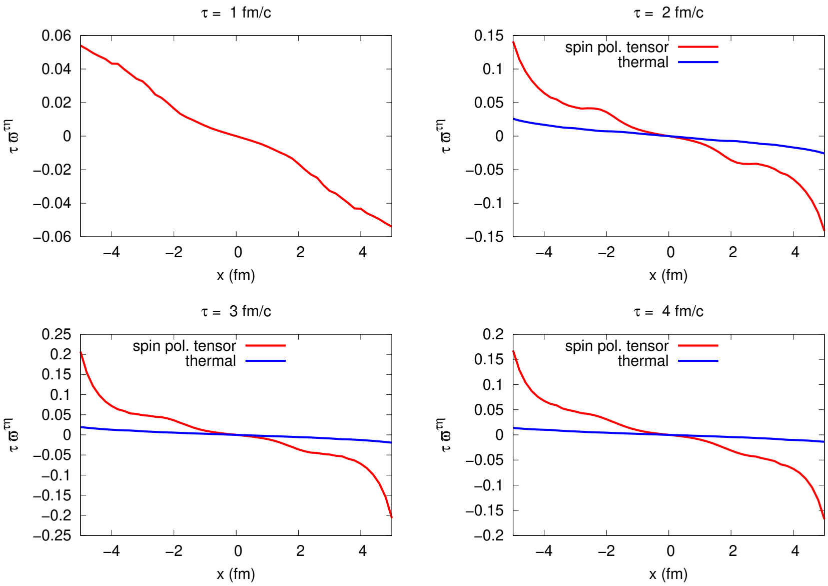


Comparison of numerics with experimental data in the 20-30% centrality for Au+Au collisions@200 GeV

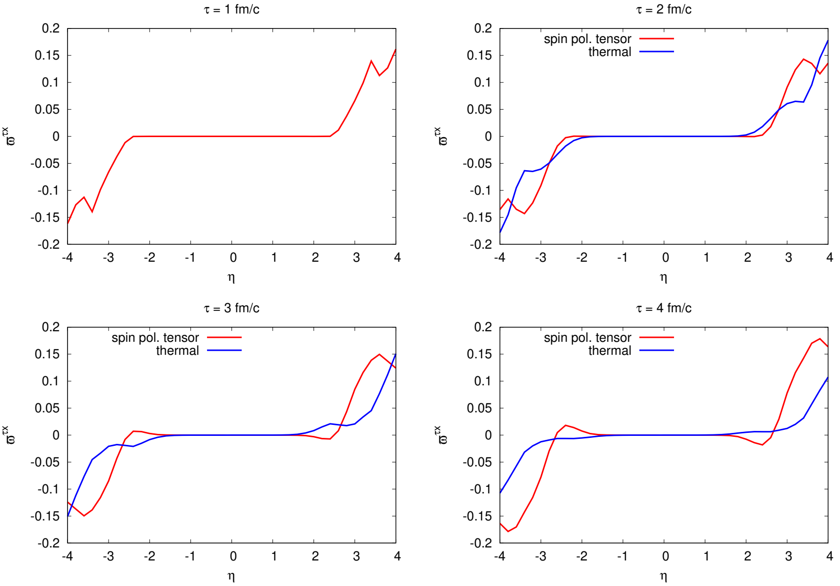
Time evolution ($y = \eta_s = 0$)



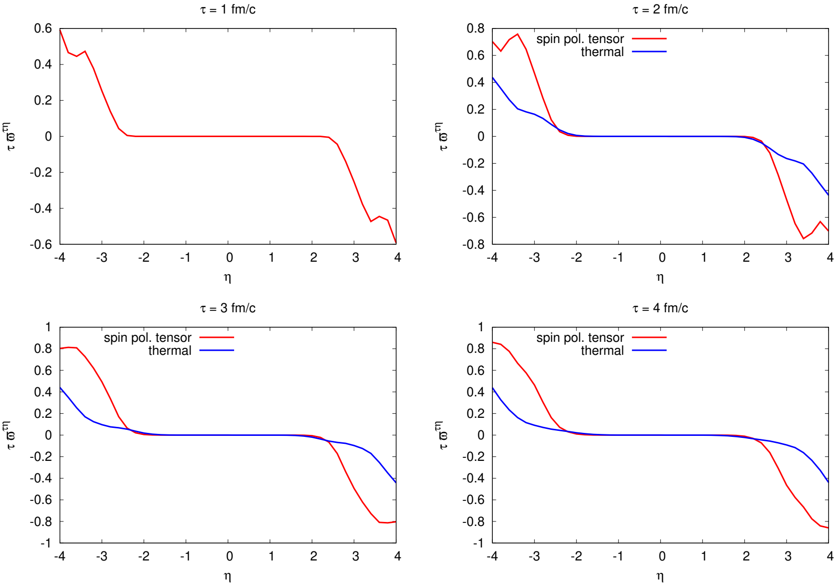
Time evolution ($y = \eta_s = 0$)



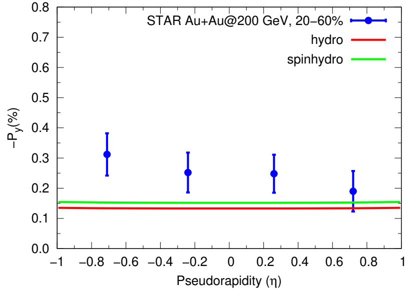
Time evolution ($x = y = 0$)

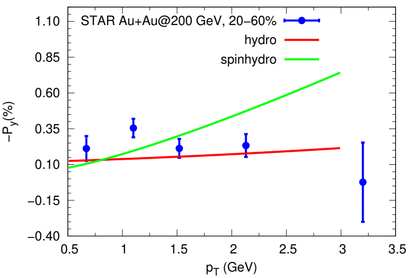


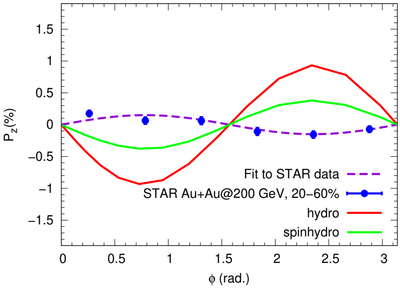
Time evolution ($x = y = 0$)



Spin Polarization







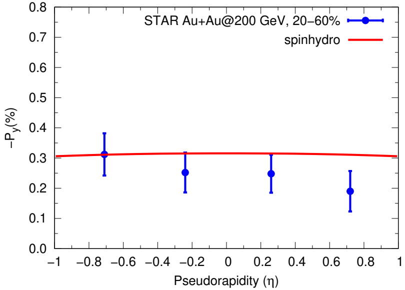
$$S^\mu(x,p) \sim -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}$$

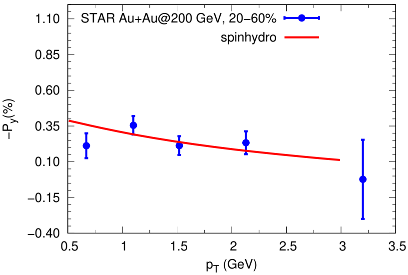
A comment !

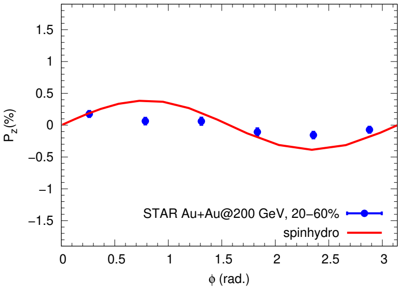
$$S^\mu \sim (\cdots)\varpi + (\cdots)(\xi + \varpi - \varpi_{\rm th})$$

X.L.-Sheng et al., arXiv:2407.12130

Spin Polarization from different initial condition







$$\varpi^{\tau x}(\tau=\tau_0) = -\varpi^{\tau x}_{\rm th}$$

$$\varpi^{\tau y}(\tau=\tau_0) = \varpi^{\tau y}_{\rm th}$$

$$\varpi^{\tau \eta}(\tau=\tau_0) = -\varpi^{\tau \eta}_{\rm th}$$

$$\varpi^{xy}(\tau=\tau_0) = \varpi^{xy}_{\rm th}$$

$$\varpi^{x\eta}(\tau=\tau_0) = \varpi^{x\eta}_{\rm th}$$

$$\varpi^{y\eta}(\tau=\tau_0) = \varpi^{y\eta}_{\rm th}$$

Summary

- For the first time we developed a (3+1)-dimensional relativistic spin hydrodynamics code with standard viscous hydrodynamic background.
- The model results obtained with spin polarization tensor initialized by thermal vorticity do not agree with experimental data on spin polarization of Λ – the predicted longitudinal polarization is closer to the data than that of the "spin-thermal" prescription.
- Exploiting the freedom of choosing IC we test the impact of flipping the sign of τx, τy and $\tau\eta$ components of the initial condition for spin polarization tensor – we obtain correct sign of longitudinal spin polarization.

Acknowledgements

1. INFN-Firenze Computing and Networking Service (PC-FARM)
2. This work was supported in part by the Polish National Science Centre Grants No. 2022/47/B/ST2/01372 (W.F.) and No. 2018/30/E/ST2/00432 (R.R., S.K.S).

Thank You !!!