# Simulating spin polarization in high-energy heavy ion collisions

*Based on Eur. Phys. J. C 84 (2024) 9, 920*

In collaboration with E. Grossi, I. Karpenko and F. Becattini

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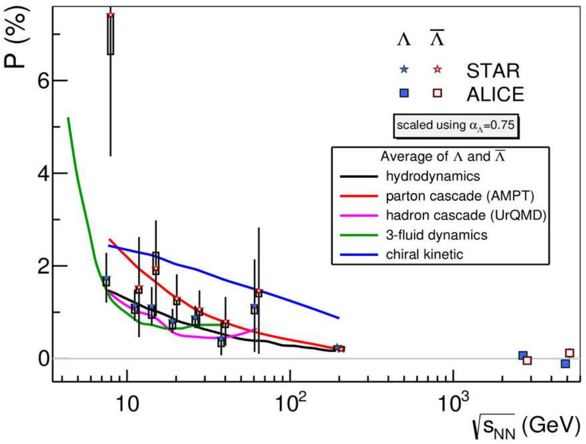
# Motivations

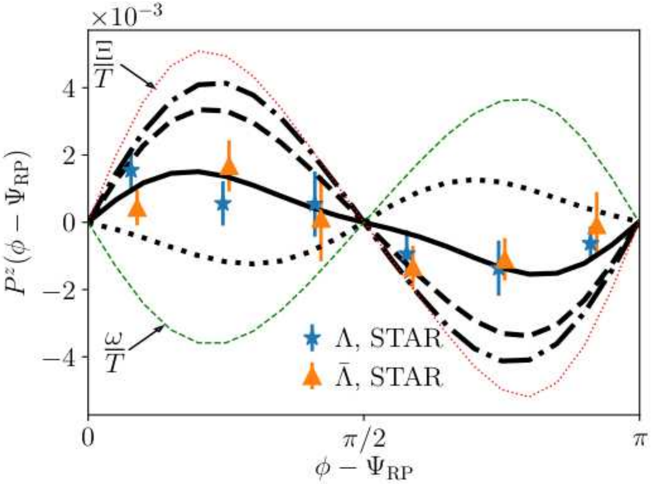
The theoretical understanding of the $\Lambda$ polarization has improved, and high energy data can be reproduced with isothermal equilibration

$$\beta^\mu = \frac{u^\mu}{T} \qquad \omega_{\mu\nu} = \frac{1}{2}\left(\partial_\nu u_\mu - \partial_\mu u_\nu\right), \qquad \Xi_{\mu\nu} = \frac{1}{2}\left(\partial_\nu u_\mu + \partial_\mu u_\nu\right)$$

$$S^\mu(p) = -\frac{S(S+1)}{3}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_{FB}(1+(-1)^{2S}n_{FB})\left[\omega_{\rho\sigma} + 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon}\Xi_{\lambda\sigma}\right]}{2mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_{FB}}$$




**Global polarization**



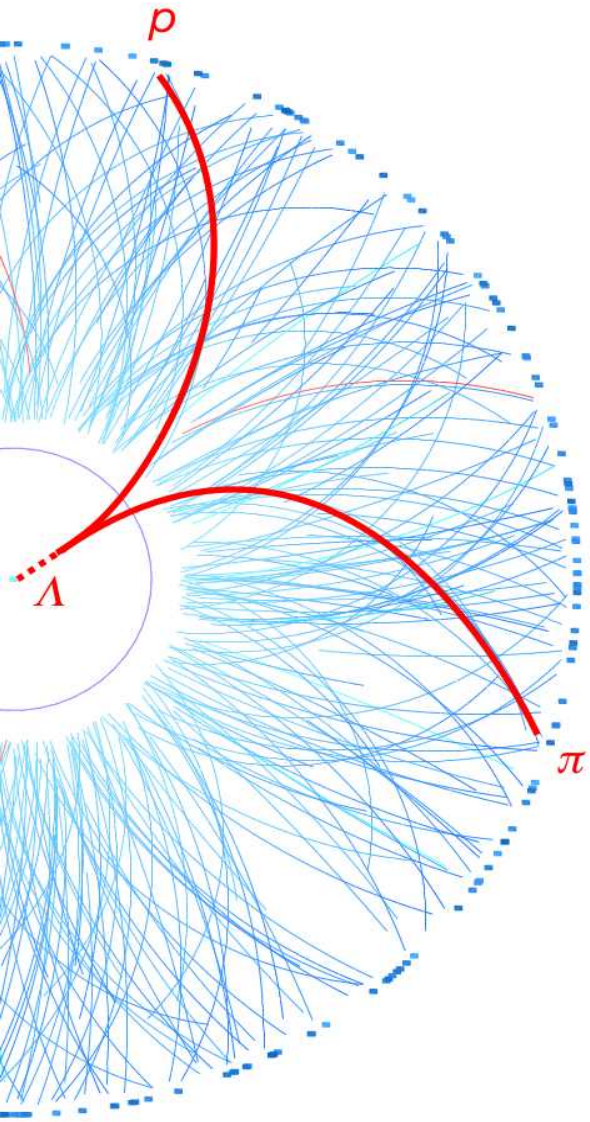

**Local polarization**

*F.Becattini, M.Buzzegoli, AP, I.Karpenko, G.Inghirami Phys.Rev.Lett.127 (2021)*

# Results

Each of the stage of the simulation has an effect on polarization

- **Initial conditions** – Transverse polarization probes the initial momentum flow
- **Hydrodynamic stage**
  - ➔ **Viscosity** – Longitudinal polarization is very sensitive to bulk viscosity
- **Afterburning**
  - ➔ **Feed down** – reduces polarization by $10\%$ at most




# Outline

- **Short theoretical summary of $\Lambda$ polarization**
- **Simulations**
- **Effects of initial conditions, viscosity, and feed-down on polarization**

# Density operator

Given a space-time foliation in hypersurfaces $\Sigma(\tau)$, the entropy $S = -\mathrm{Tr}\left(\widehat{\rho}\log\widehat{\rho}\right)$ is maximized constraining the mean values of the operators to be the actual ones *(van Weert, Annals of Physics 1982)*.

$$n_\mu \langle \widehat{T}_B^{\mu\nu} \rangle = n_\mu \mathrm{Tr}\left(\widehat{\rho}\widehat{T}_B^{\mu\nu}\right) = n_\mu T_B^{\mu\nu}{}_{\mathrm{true}}$$

$$n_\mu \langle \widehat{j}^\mu \rangle = n_\mu \mathrm{Tr}\left(\widehat{\rho}\widehat{j}^\mu\right) = n_\mu j^\mu_{\mathrm{true}}$$

"$\mathrm{n}$" is the normal vector to the hypersurface $\Sigma(\tau)$.
The density operator reads:

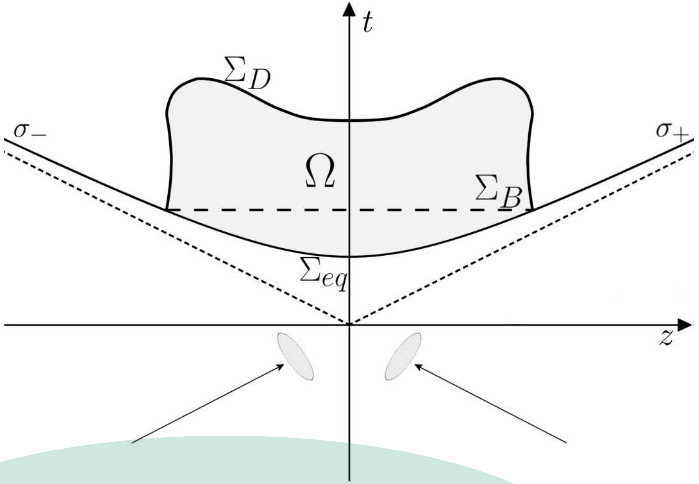
$$\widehat{\rho} = \frac{1}{Z}\exp\left(-\int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu}\beta_\nu - n_\mu \widehat{j}^\mu \zeta\right)$$

$$Z = \mathrm{Tr}\left[\exp\left(-\int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu}\beta_\nu - n_\mu \widehat{j}^\mu \zeta\right)\right]$$

β and ζ are Lagrange multipliers: $\beta^\mu = \frac{u^\mu}{T}$ $\quad \zeta = \frac{\mu}{T}$

**Warning:** The true density operator should be constant in the Heisenberg representation!

$$\widehat{\rho}_{true} = \frac{1}{Z} \exp\left( - \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu} \beta_\nu - n_\mu \widehat{j}^\mu \zeta \right)$$




We use Gauss theorem to connect with present time.

$$\widehat{\rho}_{true} = \frac{1}{Z} \exp\left[ - \int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \left( \widehat{T}^{\mu\nu} \beta_\nu - n_\mu \widehat{j}^\mu \zeta \right) + \int_\Omega \mathrm{d}\Omega \left( \widehat{T}^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right]$$

**Local equilibrium** **Dissipation**

# Theory: local-equilibrium density operator

Ideal fluid at local equilibrium $\beta^\mu = \frac{u^\mu}{T}$ $\widehat{\rho}_{LE} = \frac{1}{Z_{LE}} \exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}\beta_\nu\right]$

**Hydrodynamic approximation:** gradients are small, **linear response theory**

$$S^\mu(k) = \frac{1}{2}\frac{\int_{\Sigma_D} \mathrm{d}\Sigma\cdot k\ \mathrm{tr}\left(\gamma^\mu\gamma_5 W_+(x,k)\right)}{\int_{\Sigma_D} \mathrm{d}\Sigma\cdot k\ \mathrm{tr}\left(W_+(x,k)\right)}$$

$$W(x,k)_{ab} = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y e^{-ik\cdot y}\langle : \bar{\Psi}_b(x+y/2)\Psi_a(x-y/2) :\rangle$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta(x)\cdot\widehat{P} + \partial_\nu\beta_\mu(x)\int \mathrm{d}\Sigma_\alpha(y)\widehat{T}^{\alpha\mu}(y-x)^\nu\right]$$

Corrections to the spin operator (Pauli-Lubanski vector): $\langle \widehat{O} \rangle_\beta = \frac{1}{Z} \mathrm{Tr} \left( e^{-\beta(x)\cdot \widehat{P}} \widehat{O} \right)$

$$\langle \widehat{S}^\mu(p) \rangle_{LE} = \langle \widehat{S}^\mu(p) \rangle_\beta + \partial_\nu \beta_\mu(x) \int \mathrm{d}\Sigma_\alpha(y) \; (y-x)^\nu \langle \widehat{S}^\mu(p) \widehat{T}^{\alpha\nu}(y) \rangle_\beta$$

The gradients of the four-temperature contribute to polarization:

$$S^\mu(p) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int \mathrm{d}\Sigma \cdot p \; n_F (1-n_F) [\varpi_{\nu\rho} + 2 \hat{t}_\nu \xi_{\lambda\rho} \frac{p^\lambda}{\varepsilon}]}{\int \mathrm{d}\Sigma \cdot p \; n_F}$$

$$\varpi_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \qquad \xi_{\mu\nu} = \frac{1}{2} (\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$$

Vector $\hat{t}^\mu = (1, \mathbf{0})$ in lab frame. Origin: the thermal-shear couples to a **non-conserved operator!**

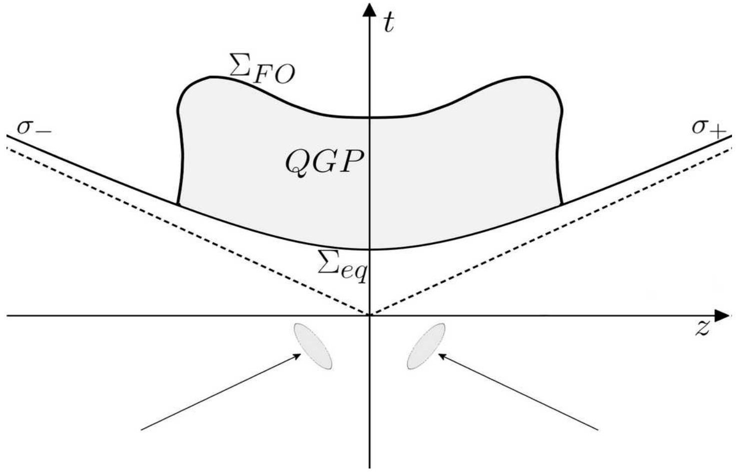
# Polarization: Spin-shear coupling

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left(-\beta(x)\cdot\widehat{P} + \varpi_{\tau\nu}\widehat{J}^{\tau\nu}_x - \xi_{\tau\nu}\widehat{Q}^{\tau\nu}_x\right)$$

$$\widehat{J}^{\tau\nu}_x = \int \mathrm{d}\Sigma_\mu[\widehat{T}^{\mu\tau}(x-y)^\nu - \widehat{T}^{\mu\nu}(x-y)^\tau] = \int \mathrm{d}\Sigma_\mu \widehat{J}^{\mu,\tau\nu}$$

$$\widehat{Q}^{\tau\nu}_x = \int \mathrm{d}\Sigma_\mu[\widehat{T}^{\mu\tau}(x-y)^\nu + \widehat{T}^{\mu\nu}(x-y)^\tau] = \int \mathrm{d}\Sigma_\mu \widehat{Q}^{\mu,\tau\nu}$$

The Q operator depends on the hypersurface!




$$\int_{\Sigma_D} \mathrm{d}\Sigma n_\mu v^\mu = \int_{\Sigma_B} \mathrm{d}^3x t_\mu v^\mu + \int_\Omega \mathrm{d}\Omega \partial_\mu v^\mu$$

$$\partial_\mu \widehat{J}^{\mu,\nu\rho} = 0 \qquad \partial_\mu \widehat{Q}^{\mu,\nu\rho} \neq 0$$

# Isothermal freeze-out

In **high-energy** heavy-ion collisions, the best approximation for the local density operator at high energy involves an isothermal decoupling hypersurface.

F.*Becattini, M.Buzzegoli, A.P. , I.Karpenko, G.Inghirami Phys.Rev.Lett. 127 (2021)*

$$\widehat{\rho}_{LE} = \frac{1}{Z_{LE}} \exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}\beta_\nu\right] = \frac{1}{Z_{LE}} \exp\left[-\frac{1}{T_{\mathrm{dec}}}\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}u_\nu\right]$$

The final formula now depends only on gradients of the four velocity

$$\omega_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu u_\nu - \partial_\nu u_\mu\right) \qquad \Xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu u_\nu + \partial_\nu u_\mu\right)$$

$$S^\mu_{ILE}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F (1-n_F)\left[\omega_{\rho\sigma} + 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon}\Xi_{\lambda\sigma}\right]}{8mT_{\mathrm{dec}}\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

# And in lower energy collisions?
# Iso-energy gradient expansion (preliminary)

More commonly, the freeze out hypersurface is at constant energy density

$$\widehat{\rho}_{LE} = \frac{1}{Z} \exp\left( - \int_{\Sigma_D} \mathrm{d}\Sigma_\mu(y) \widehat{T}^{\mu\nu}(y) \beta_\nu(y) - \frac{\mu(y)}{T(y)} j^\mu(y) \right)$$

$$S^\mu(k) = \frac{1}{2} \frac{\int_{\Sigma_D} \mathrm{d}\Sigma \cdot k \ \mathrm{tr}\left(\gamma^\mu \gamma_5 W_+(x,k)\right)}{\int_{\Sigma_D} \mathrm{d}\Sigma \cdot k \ \mathrm{tr}\left(W_+(x,k)\right)}$$

Both x and y lie on the freeze-out hypersurface, so we can use geometry to try to improve our approximation.

The four-velocity follows hydrodynamics and it is independent of the freeze out. The freeze-out condition is a relation between T and

$$e(T, \mu) = e_{FO}$$

This provide us with a thermodynamic relation that we can use to compute the variations at constant energy density:

$$\delta T = -\mathcal{C}\delta\mu \qquad\qquad \mathcal{C} = \frac{\partial e/\partial\mu|_T}{\partial e/\partial T|_\mu}$$

But the variations in cartesian coordinate will not obey the above equation.

The value of $\mathcal{C}$ is used to establish which variation should be expressed in terms of the cartesian variation of the other.

If $|\mathcal{C}|<1$

$$S^\mu_A = -\epsilon^{\mu\rho\sigma\tau}\frac{p_\tau}{8m}\frac{1}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\frac{1}{T}\left[\omega_{\rho\sigma}+\mathcal{C}\beta_{[\rho}\partial_{\sigma]}\zeta\right]$$

$$S^\mu_S = -\epsilon^{\mu\rho\sigma\tau}\frac{p_\tau\,\hat{t}_\rho p^\lambda}{4m\varepsilon}\frac{1}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\frac{1}{T}\left[\Xi_{\lambda\sigma}+\mathcal{C}\beta_{(\lambda}\partial_{\sigma)}\zeta\right]$$

$$S^\mu_\zeta = +\epsilon^{\mu\rho\sigma\tau}\frac{p_\tau\,\hat{t}_\rho}{16m\varepsilon}\frac{1}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\frac{\partial_\sigma\mu}{T}\left(1+\zeta\mathcal{C}\right)$$

If $|\mathcal{C}|>1$

$$S^\mu_A = -\epsilon^{\mu\rho\sigma\tau}\frac{p_\tau}{8m}\frac{1}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}$$

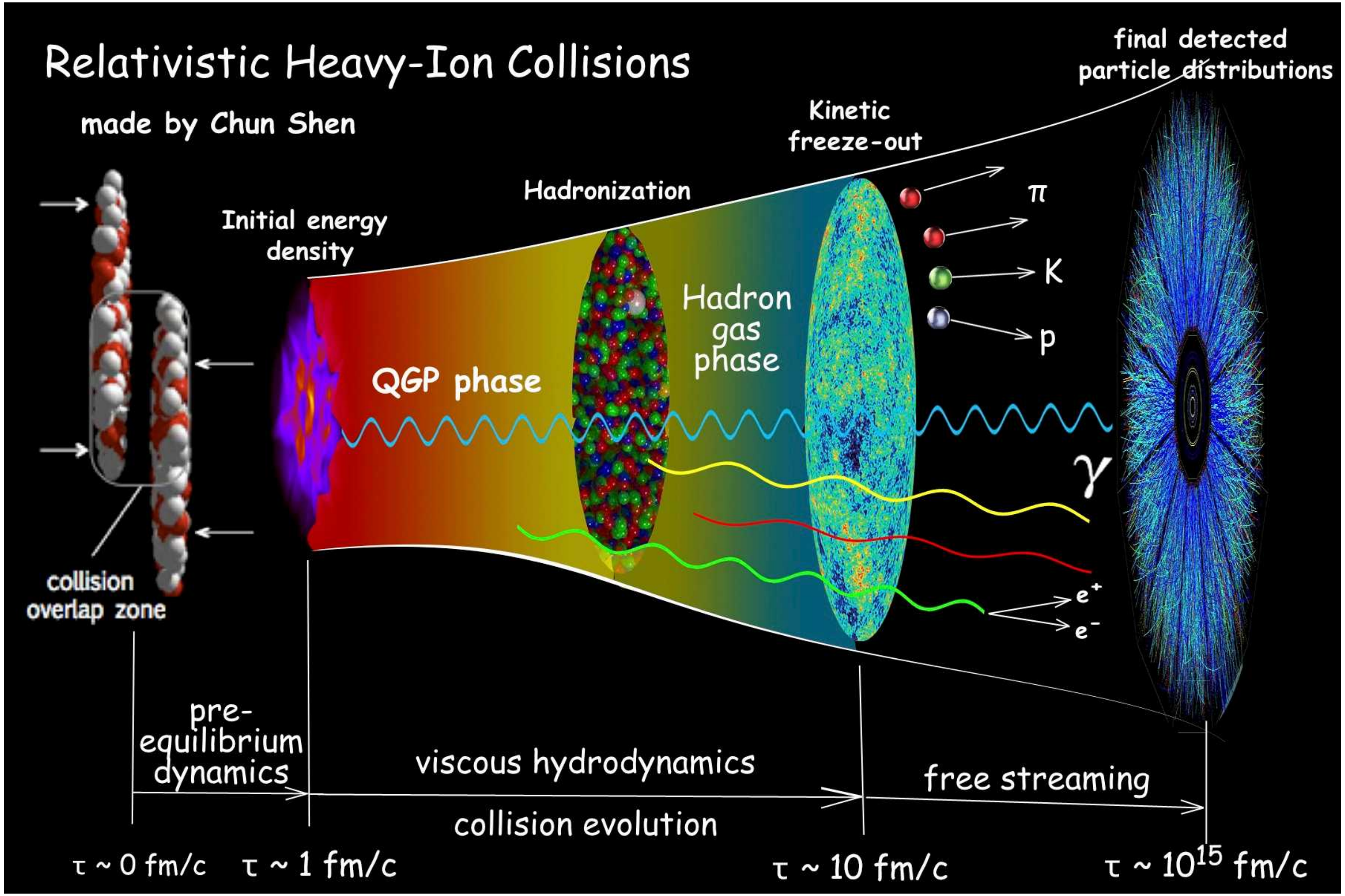
$$S^\mu_S = -\epsilon^{\mu\rho\sigma\tau}\frac{p_\tau\,\hat{t}_\rho p^\lambda}{4m\varepsilon}\frac{1}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\xi_{\lambda\sigma}$$

$$S^\mu_\zeta = -\epsilon^{\mu\rho\sigma\tau}\frac{p_\tau\,\hat{t}_\rho}{16m\varepsilon}\frac{1}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\frac{\partial_\sigma T}{T}\left(\frac{1}{\mathcal{C}}+\zeta\right)$$

Hopefully, improved description of data at lower energies. Stay tuned!

# Simulating Heavy Ion Collisions

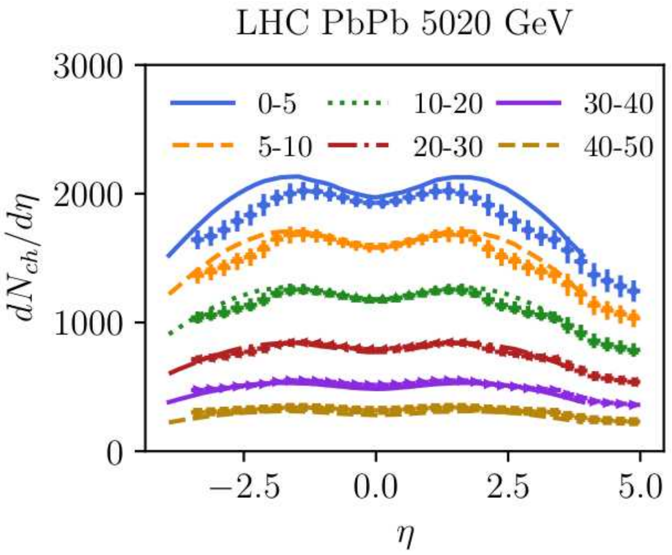
- Initial condition model
- Hydrodynamic evolution, equation of state
- Freeze-Out
- Afterburner
  - Scattering
  - Decays



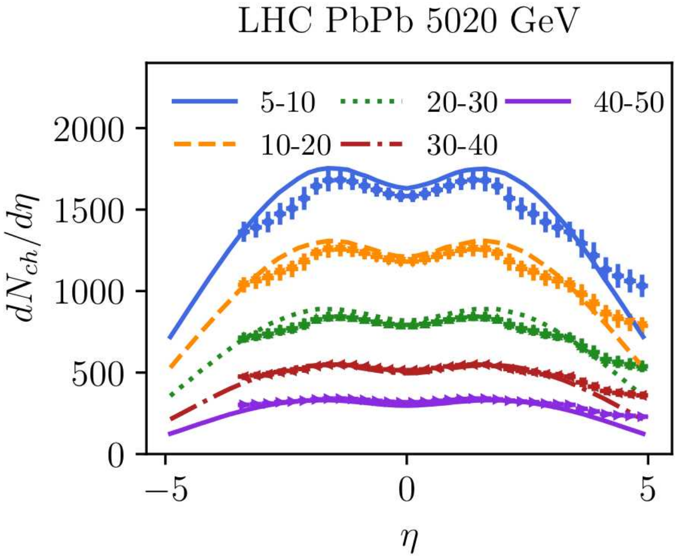
# Numerical setup

An average initial state is generated with superMC and GLISSANDO, hydrodynamics is handled with vHLLE and afterburning with SMASH.

We use a constant $\eta/s$ and a temperature dependent $\zeta/s$. Decoupling happens at $e_{crit}=0.4$ GeV/fm$^3$, that is $T\cong160$ MeV




SuperMC




GLISSANDO

# Polarization is computed at freeze-out

Is based explicitly only on the information at freeze-out $\mathrm{d}\Sigma \quad \hat{n}_\mu \quad u_\mu \quad T \quad \partial_\nu u_\mu$

$$\omega_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu u_\nu - \partial_\nu u_\mu\right) \qquad \Xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu u_\nu + \partial_\nu u_\mu\right)$$

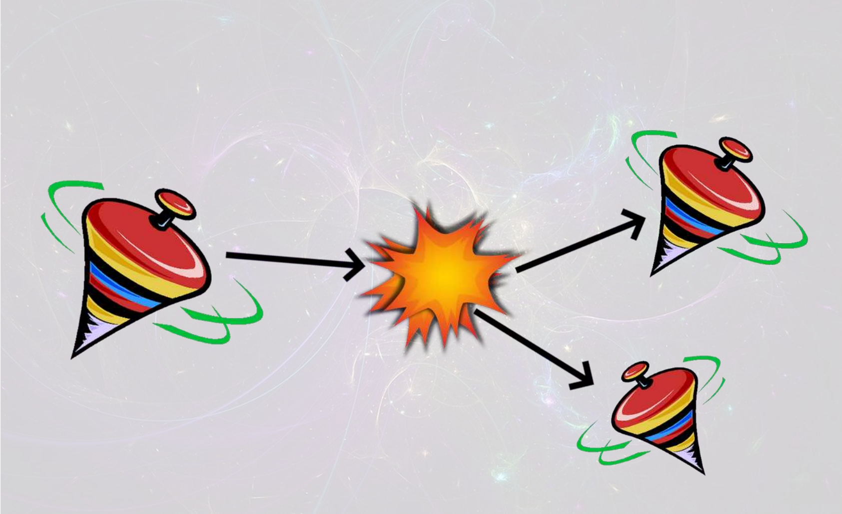
$$S^\mu_{ILE}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F (1 - n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma}\right]}{8 m T_{\mathrm{dec}} \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

The afterburning stage doesn’t include spin (work in progress in Frankfurt…).

# Feed-down corrections

Most $\Lambda$ particles do not come from the QGP but from decays.

$$\mathbf{S}_*^{(M)}(\mathbf{p}) = \frac{\int \mathrm{d}\Omega_* n(\mathbf{P}) F(\mathbf{p}, \Omega_*) \mathbf{S}_{M\to\Lambda}(\mathbf{P}, \mathbf{p})}{\int \mathrm{d}\Omega_* n(\mathbf{P}) F(\mathbf{p}, \Omega_*)}$$



We consider $\Sigma^* \to \Lambda\pi$ and $\Sigma_0 \to \Lambda\gamma$.

The total polarization is the sum of the polarizations from each channel, rescaled by the multiplicity.

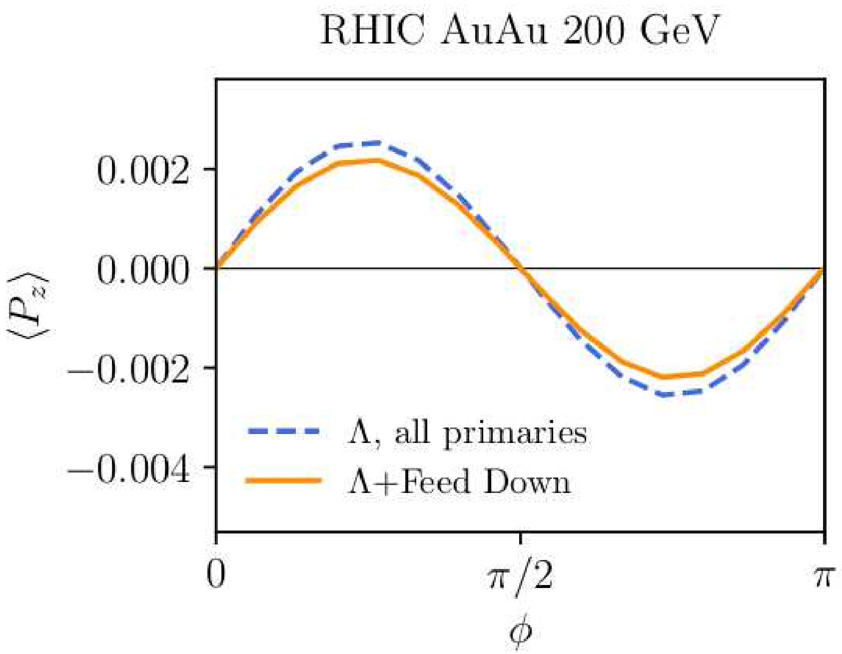
$$\boldsymbol{S}_{\Lambda,tot}(p) = \frac{n_\Lambda^{(FO)} \boldsymbol{S}_\Lambda^{(FO)}(p) + n_\Lambda^{(\Sigma^*)} \boldsymbol{S}^{(\Sigma^*)}(p) + n_\Lambda^{(\Sigma_0)} \boldsymbol{S}^{(\Sigma_0)}(p)}{n_\Lambda^{(FO)} + n_\Lambda^{(\Sigma^*)} + n_\Lambda^{(\Sigma_0)}}$$

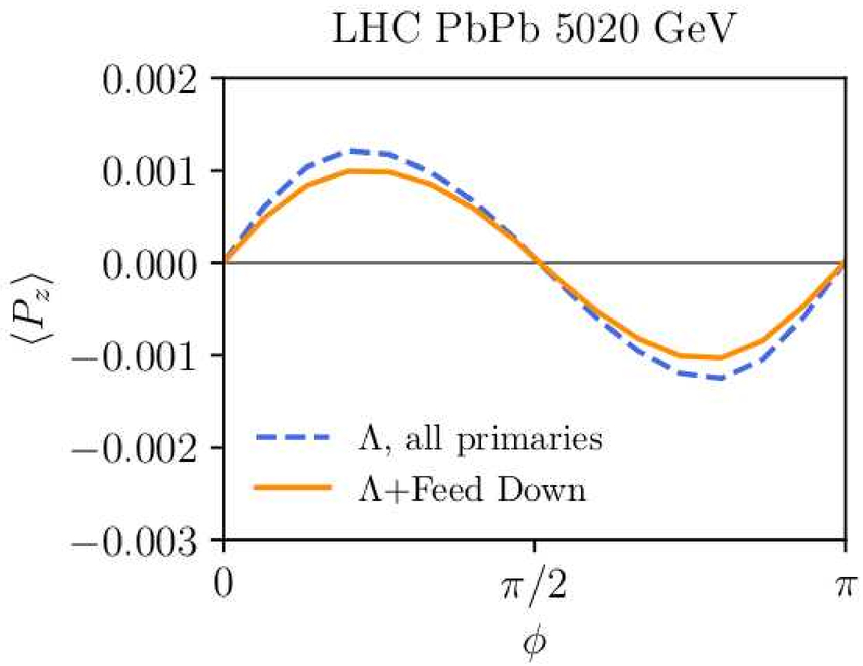
X.Xia, H.Li, X.Huang, X.Huang Phys.Rev.C 100 (2019); F.Becattini, G.Cao, E.Speranza Eur.Phys.J.C 79 (2019)

# Feed-down corrections

Including decays reduces longitudinal polarization by about 10%.

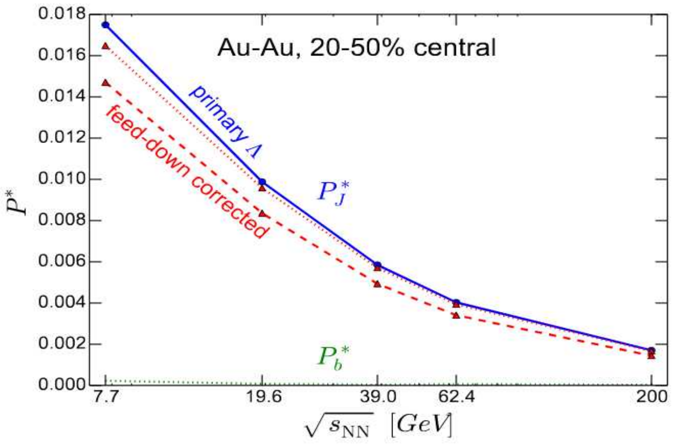
Transverse polarization changes only by 3%.

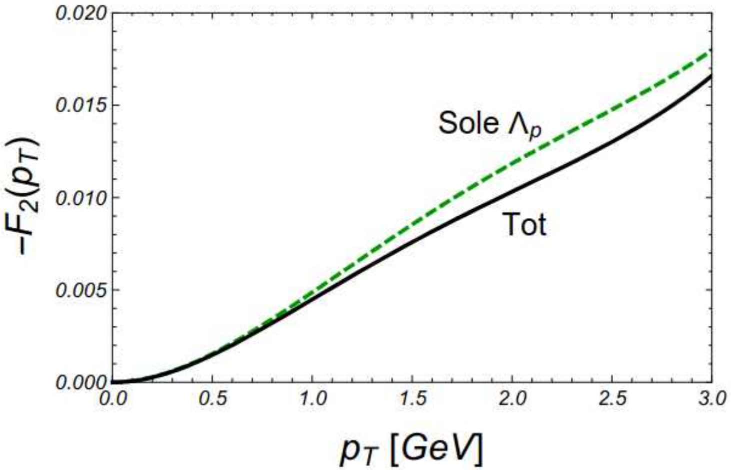


# Confirms previous expectations

Karpenko, Becattini, Eur.Phys.J.C 77 (2017) 4, 213



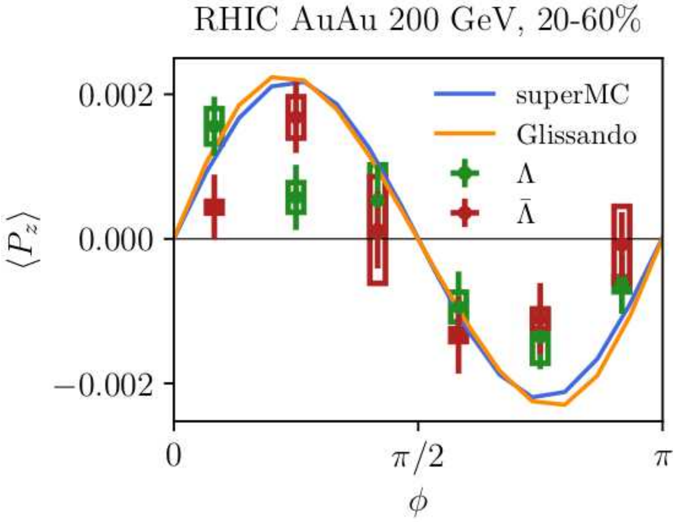



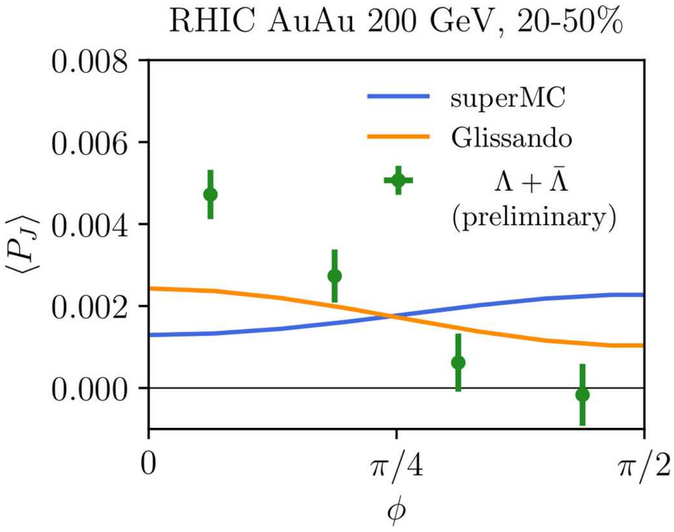

Becattini, Cao, Speranza Eur.Phys.J.C 79 (2019) 9, 741

We include shear induced polarization for mother particles.

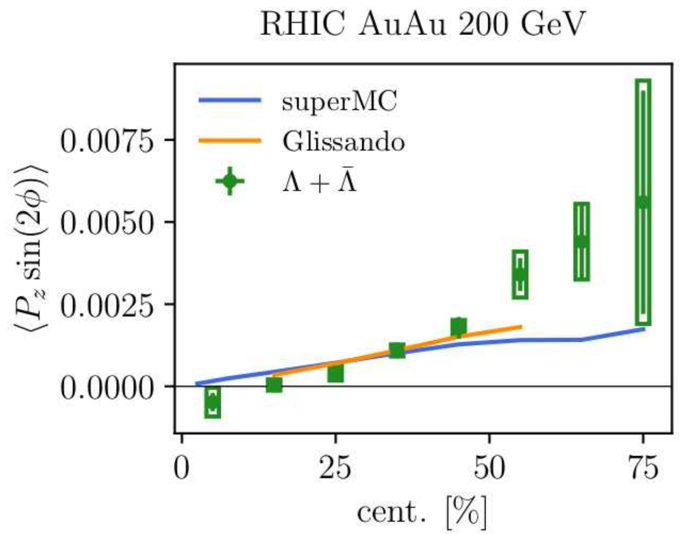
Calculation of feed down corrections to local polarization based on a realistic Freeze-Out surface.

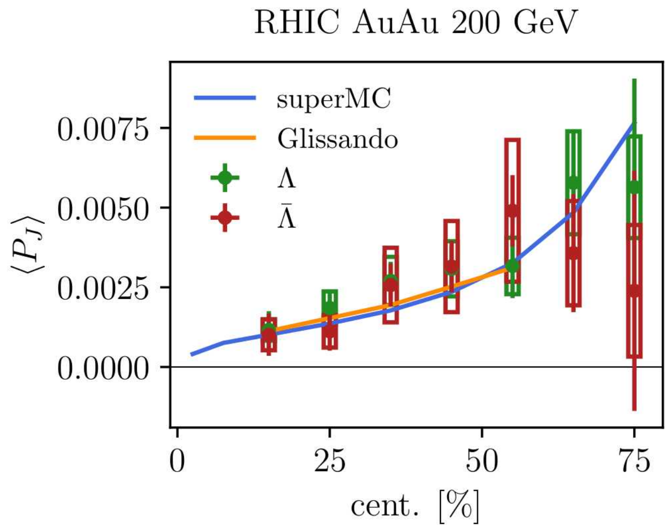
We obtain a good agreement in the longitudinal sector.




The two different initial states produce a **significantly different transverse polarization.**



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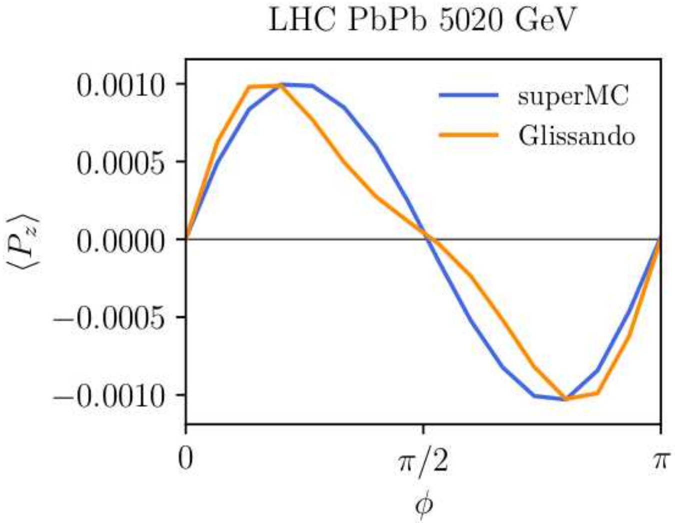


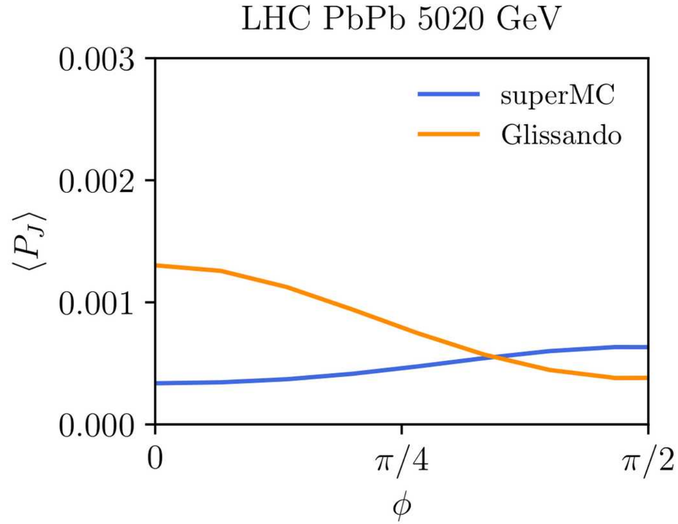


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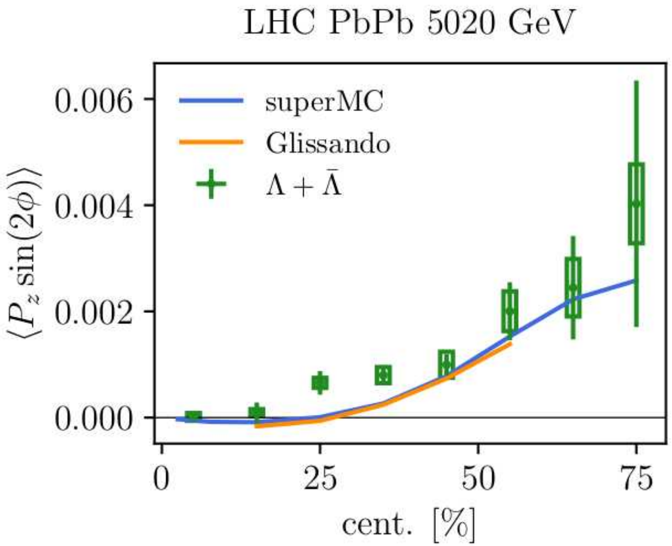
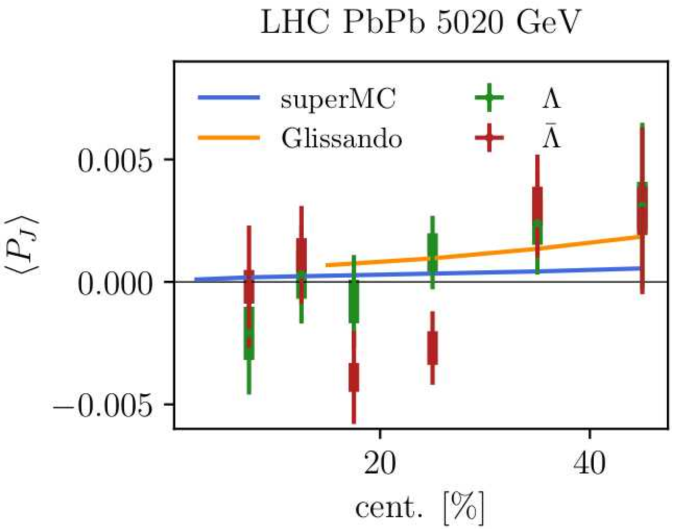


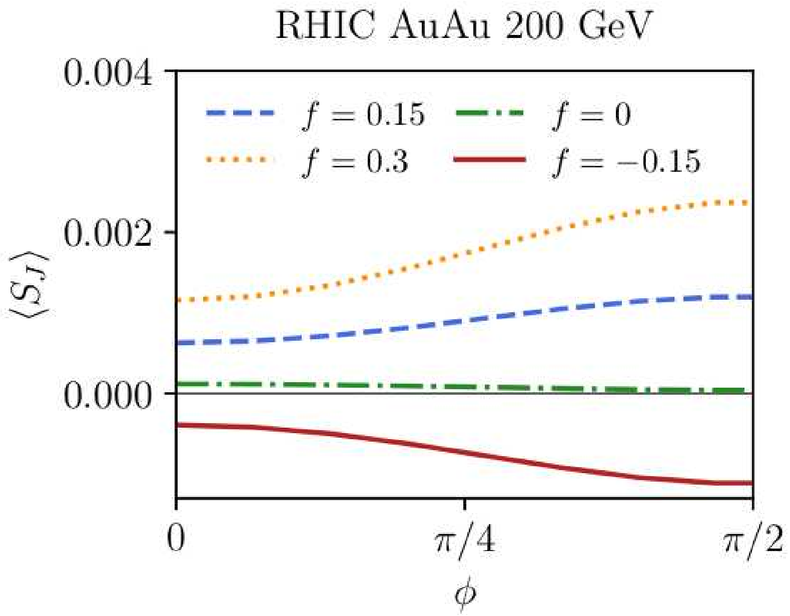



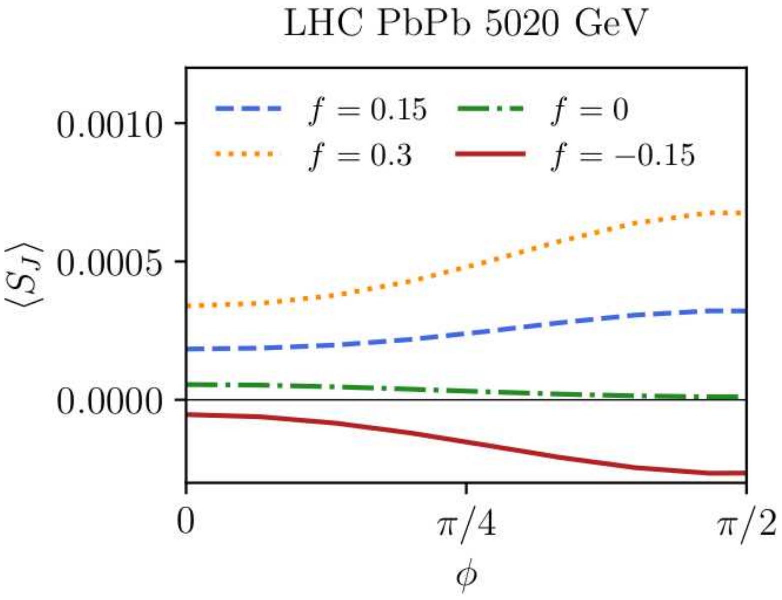
# Initial longitudinal flow

The initial energy momentum tensor (superMC initial state):

$$T^{\tau\tau} = \rho \cosh(f\, y_{CM}), \qquad T^{\tau\eta} = \frac{\rho}{\tau} \sinh(f\, y_{CM})$$

Transverse polarization depends strongly on the initial longitudinal momentum flow
(similar conclusions Z.Jiang, X.Wu, S.Cao, B.Zhang Phys.Rev.C 108 (2023) 6, 064904)



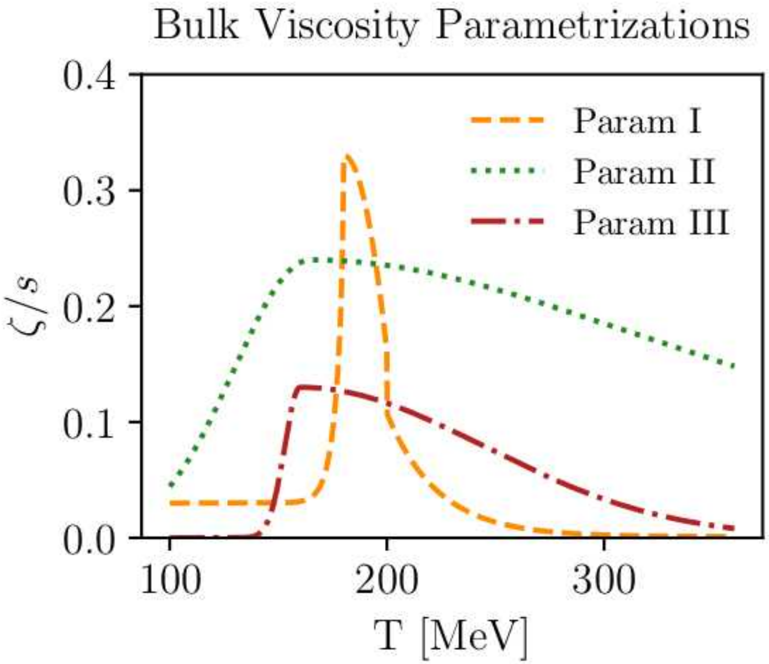
# Viscosity

We study for the first time the dependence of polarization on bulk viscosity

**Param I:** S.Ryu, J-F.Paquet, C.Shen, G.Denicol, B.Schenke, S.Jeon, C.Gale, Phys.Rev.C 97 (2018) 3, 034910

**Param II:** B.Schenke, C.Shen, P.Tribedy, Phys.Rev.C 99 (2019) 4, 044908

**Param III:** B.Schenke, C.Shen, P.Tribedy Phys.Rev.C 102 (2020) 4, 044905



# "Direct" vs "Indirect" dissipative corrections

Viscosity accounts for dissipative corrections to spin in an "indirect" way.

The true "direct" ones would come from the dissipative part of the density operator

$$\widehat{\rho}_{true} = \frac{1}{Z} \exp\left[ -\widehat{P} \cdot \beta + \int_{\Omega} \mathrm{d}\Omega \, \widehat{T}^{\mu\nu} \nabla_{\mu} \beta_{\nu} \right]$$

$$\delta S^{\mu}(p) \approx (S^{\mu}, T^{\rho\sigma}) \partial_{\rho} \beta_{\sigma}$$

→ Kubo Formula, transport coefficients

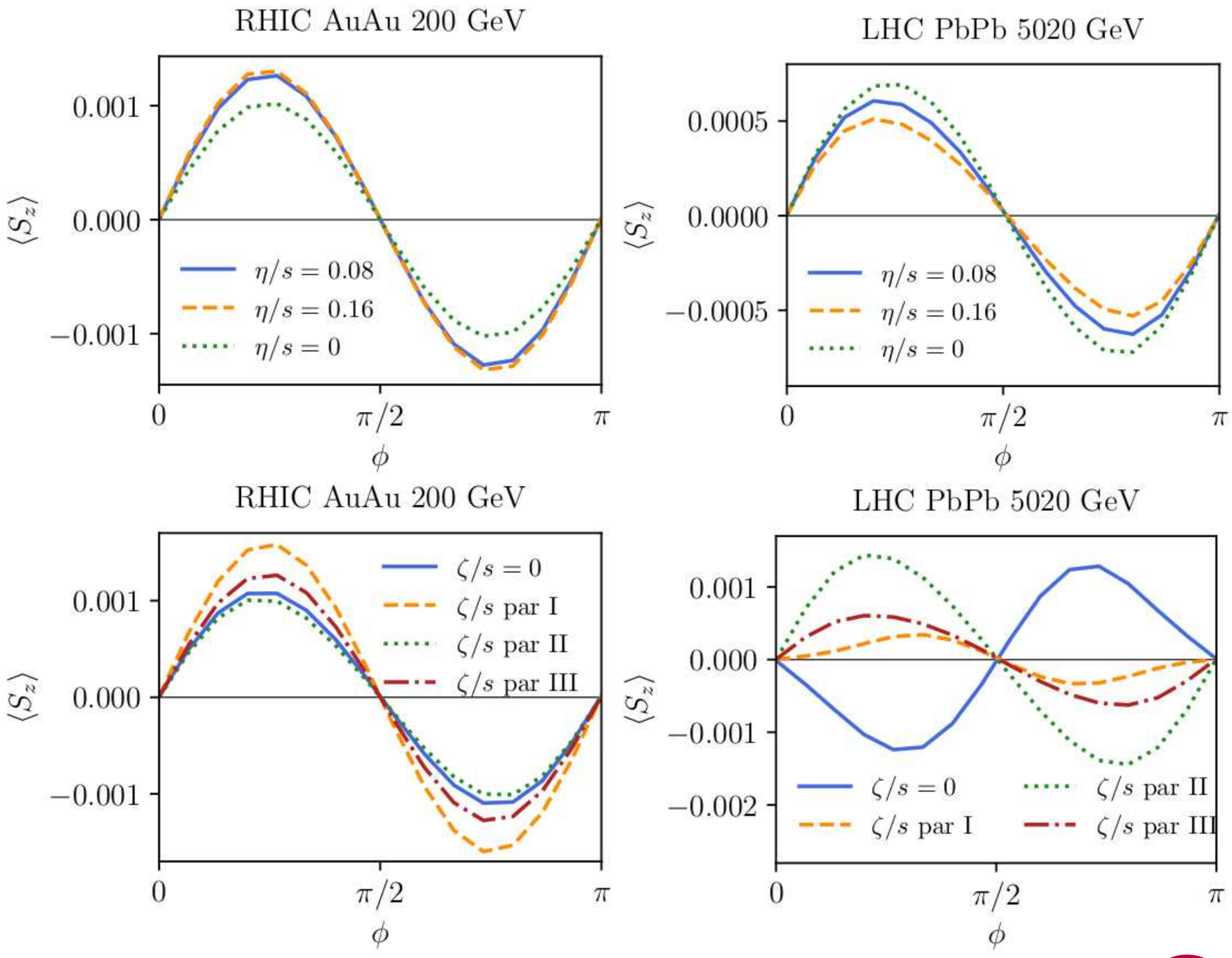
Another direct way to include dissipation is using spin hydrodynamics.

*Banerjee, Bhadury, Florkowski, Jaiswal, Ryblewski 2405.05089*

The shear viscosity has a minor effect on longitudinal and transverse polarization.

**Bulk viscosity has a significant effect**, which becomes more important in higher energy collisions.

Bulk viscosity **can change the sign of longitudinal polarization!** Transverse polarization is also affected, but to minor extent.



We can study how fluid components vary by decomposing the kinematic vorticity and shear:

$$\omega_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\omega^\rho u^\sigma + \frac{1}{2}\left(A_\mu u_\nu - A_\nu u_\mu\right), \quad \Xi_{\mu\nu} = \frac{1}{2}\left(A_\mu u_\nu + A_\nu u_\mu\right) + \sigma_{\mu\nu} + \frac{1}{3}\theta\Delta_{\mu\nu}$$

And we can identify the components of polarization coming from rotation, acceleration, shear and expansion:

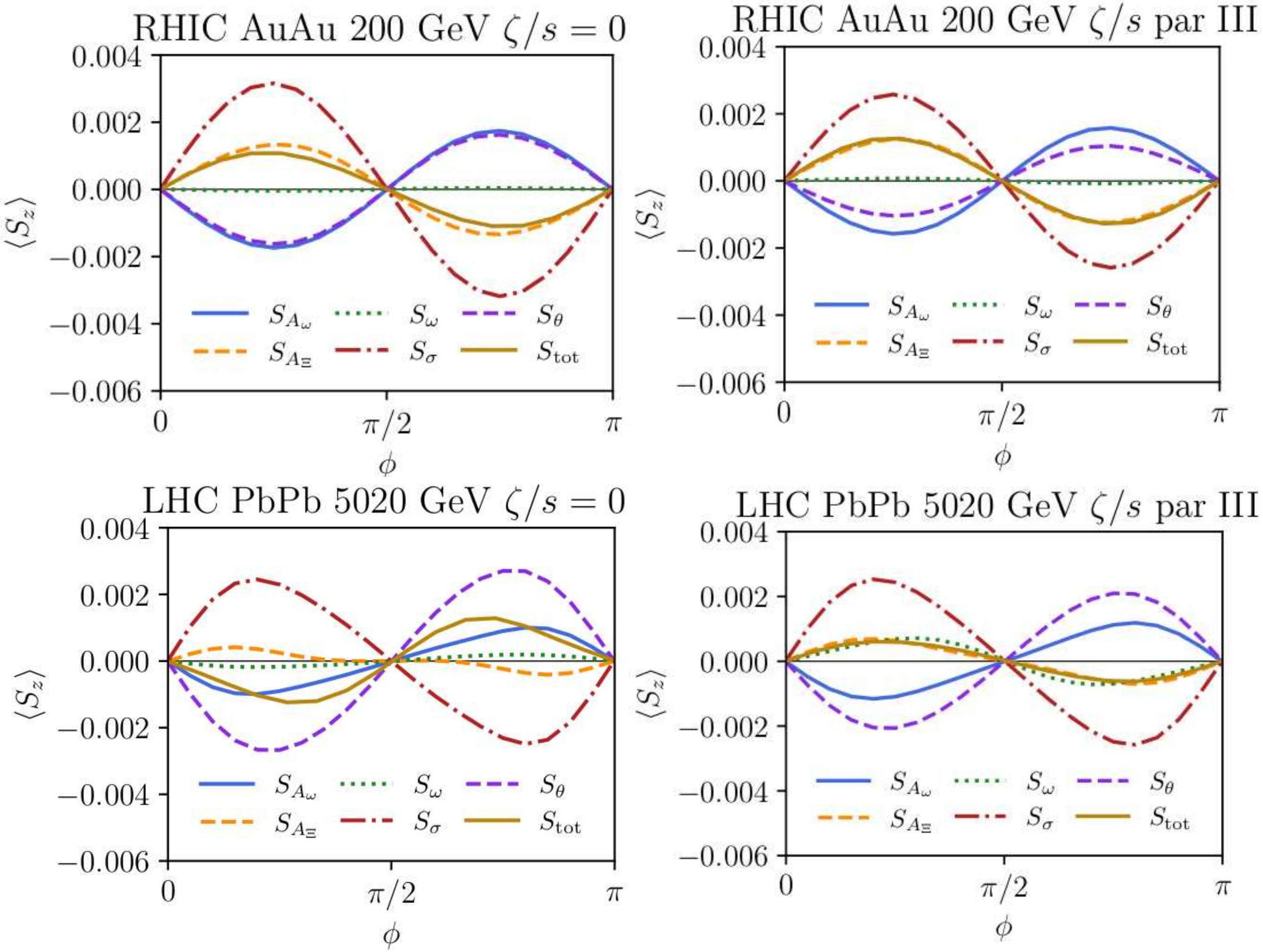
$$S^\mu_{A_\omega} = -\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\; A_\nu u_\rho}{8mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}, \qquad S^\mu_\omega = \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\; \left[\omega^\mu u \cdot p - u^\nu \omega \cdot p\right]}{4mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F},$$

$$S^\mu_\sigma = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho p_\tau \frac{p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\; n_F(1-n_F)\sigma_{\lambda\sigma}}{4mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\; n_F}, \qquad S^\mu_\theta = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho p_\tau \frac{p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\; n_F(1-n_F)\theta\Delta_{\lambda\sigma}}{12mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\; n_F},$$

$$S^\mu_{A_\Xi} = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho \frac{p_\tau}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\; n_F(1-n_F)\left[u_\sigma A \cdot p + A_\sigma u \cdot p\right]}{8mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\; n_F}.$$

For √s=200 GeV the most affected components are $S_\theta$ and $S_\sigma$, but the variations compensate.

For √s=5020 GeV also $S_{A\Xi}$ and $S_\omega$ change significantly.



# Conclusions

Polarization is a paramount probe of the quark-gluon plasma.

- Transverse polarization is **very sensitive** to the initial longitudinal flow

- **Bulk viscosity** has a strong impact on longitudinal polarization at 5.02 TeV

Polarization can be used to **constrain initial conditions and transport coefficients.**

**THANK YOU FOR THE ATTENTION!**