Hyperon spin polarization and phenomenology

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Spin and quantum features of QCD plasma, ECT*
2024.09.16

Outline

- **Global polarization**
- **Local polarization**
- **Spin polarization at pA system**
- **Other related topics and discussion**
- **Summary**

Global polarization

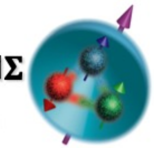
Spin in high energy physics

Striking spin effects have been observed in high energy reactions since 1970s

"Proton spin crisis" 质子自旋危机

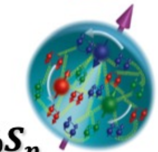
夸克模型：

夸克自旋之和Σ
$=$质子自旋S_p

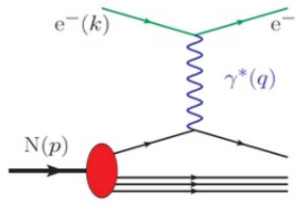


DIS实验：

89年: $\Sigma \sim 0$
目前: $\Sigma \sim 20\% S_p$



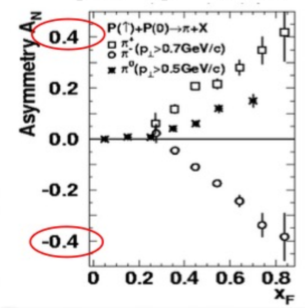
$$e + p \to e + X$$

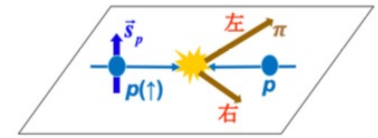


EMC, PLB 206.364 (1988)

"Single spin left-right asymmetry (SSA)"

$$p(\uparrow) + p \to \pi + X$$





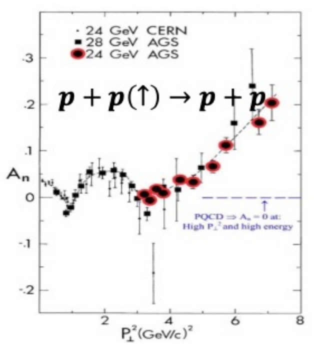
$$A_N \equiv \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

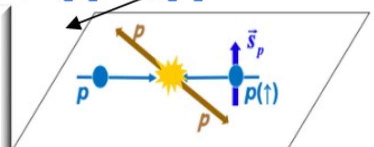
e.g. FNAL E704,
PLB264, 462 (1991)

Predictions of pQCD $\sim$ 0

"Spin analyzing power in $pp \to pp$"

$$p + p(\uparrow) \to p + p$$



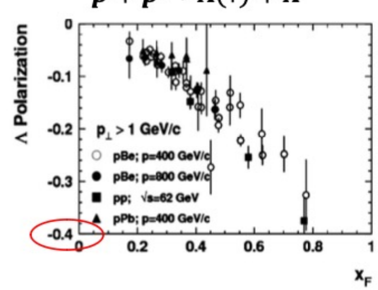


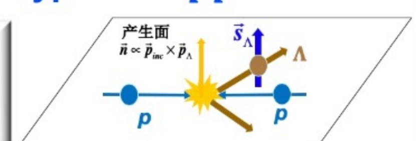
$$A_N \equiv -\frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

e.g. D. Grab *et al.*,
PRL41, 1257 (1978)

"Transverse polarization of hyperon in $pp \to \Lambda X$"

$$p + p \to \Lambda(\uparrow) + X$$



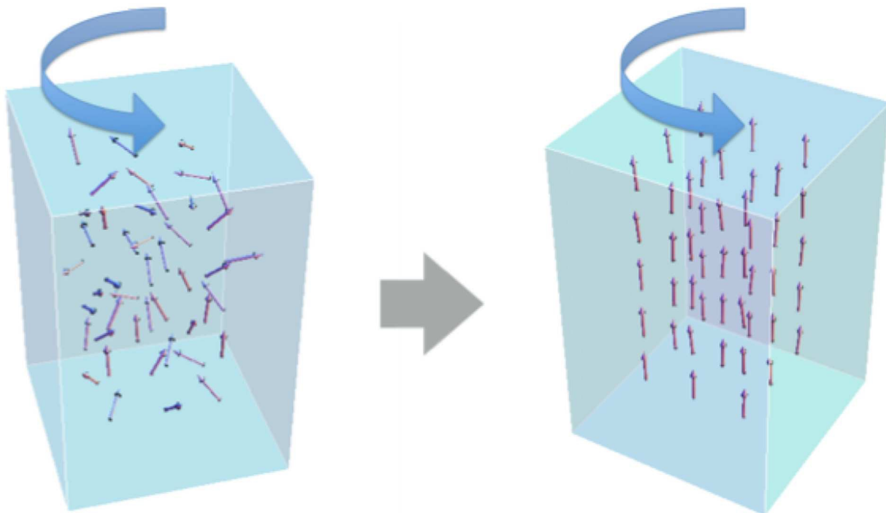


$$P_\Lambda \equiv \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$

e.g. S.A. Gourlay *et al.*,
PRL56, 2244 (1986)

Slides copy from Prof. Zuo-tang Liang's review talk

Barnet and Einstein-de Hass effects



Barnett effect:

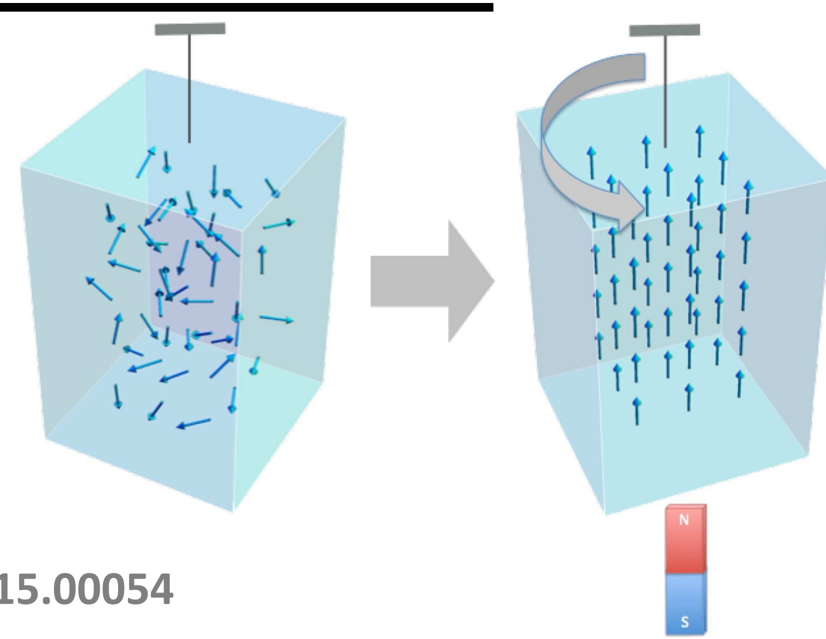
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

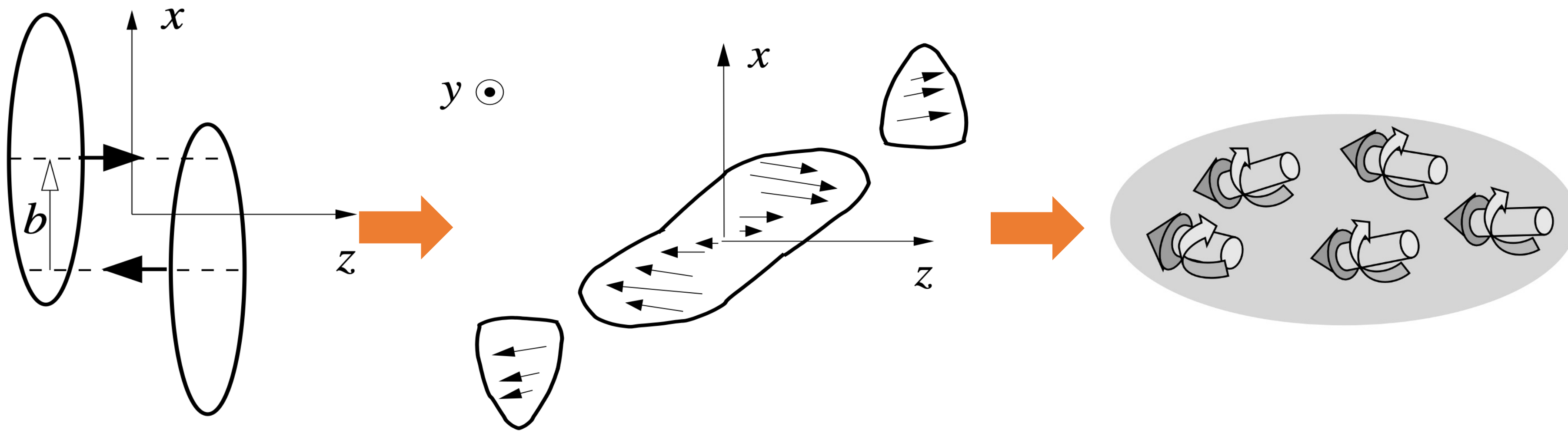
Magnetization $\Longrightarrow$ Rotation

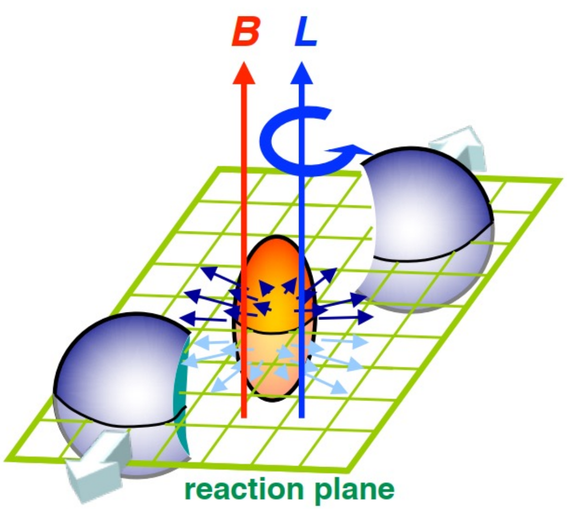
Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.



Figures: copy from paper doi: 10.3389/fphy.2015.00054

OAM to spin polarization in HIC



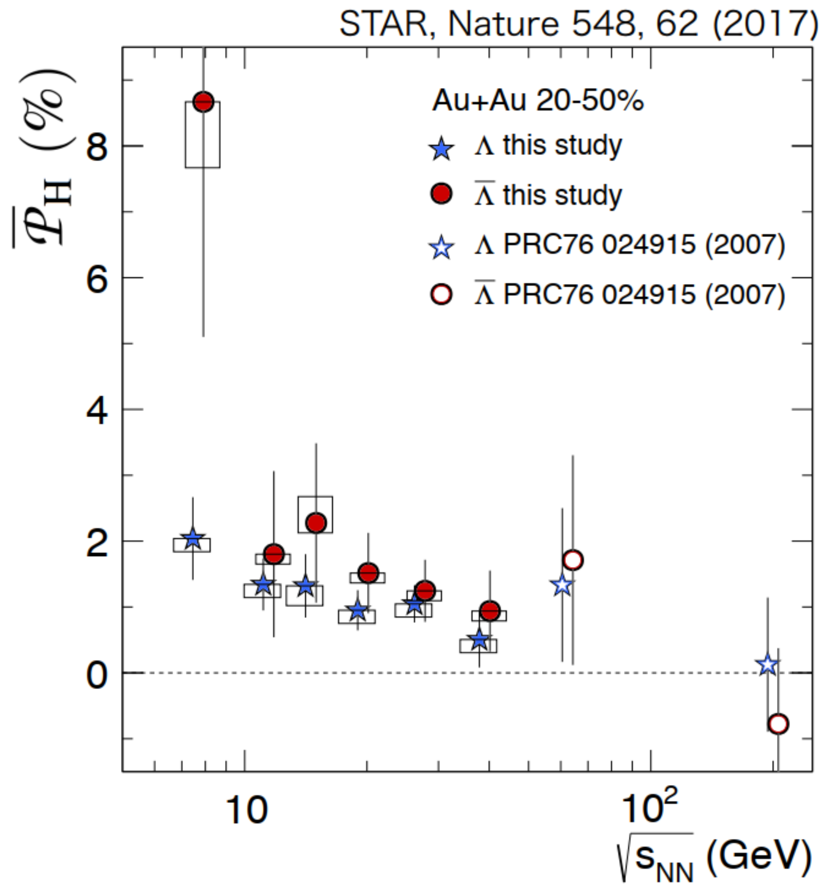


- **Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.**

- **Global orbital angular momentum leads to the polarizations of Λ hyperons and spin alignment of vector mesons through spin-orbital coupling.**

Liang, Wang, PRL (2005); PLB (2005);
Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization for Λ and $\bar{\Lambda}$ hyperons

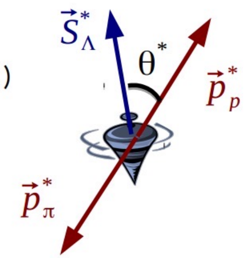


parity-violating decay of hyperons

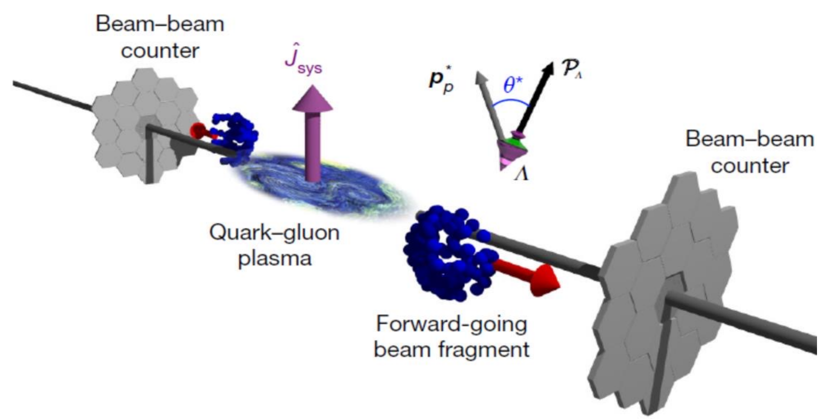
In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$$\Lambda \to p + \pi^+$$
(BR: 63.9%, cτ~7.9 cm)



Most vortical fluid

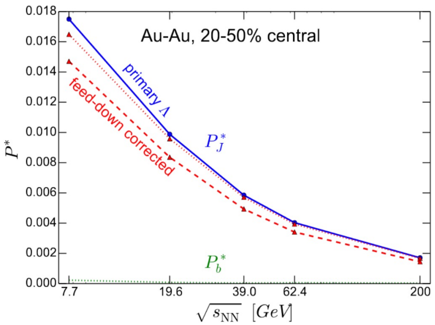
- Estimation given by Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC95, 054902 (2017)

$$
\begin{aligned}
\mathbf{P}_\Lambda &\simeq \frac{\boldsymbol{\omega}}{2T} + \frac{\mu_\Lambda \mathbf{B}}{T} \\
\mathbf{P}_{\overline{\Lambda}} &\simeq \frac{\boldsymbol{\omega}}{2T} - \frac{\mu_\Lambda \mathbf{B}}{T}
\end{aligned}
$$

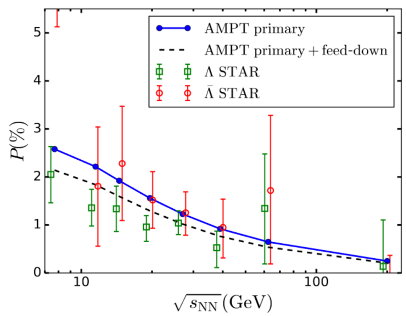
- $\omega = (9 \pm 1)\mathrm{x}10^{21}$/s, greater than previously observed in any system.
- QGP is most vortical fluid so far.

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)
…

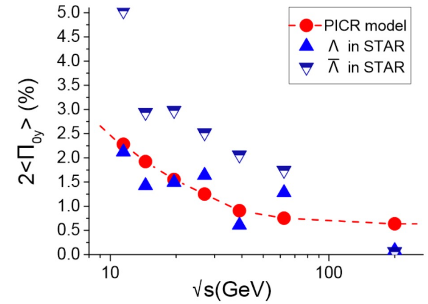
Phenomenological models for global polarization



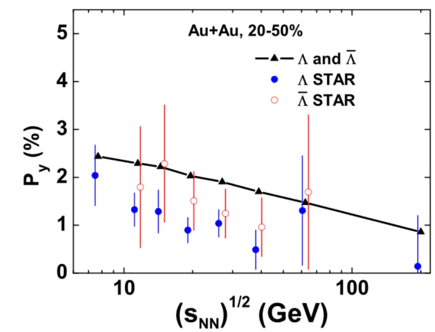
Karpenko, Becattini, EPJC(2017)



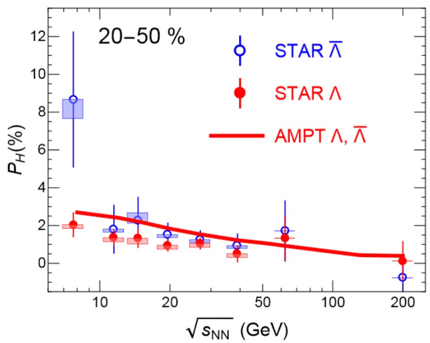
Li, Pang, Wang, Xia PRC(2017)



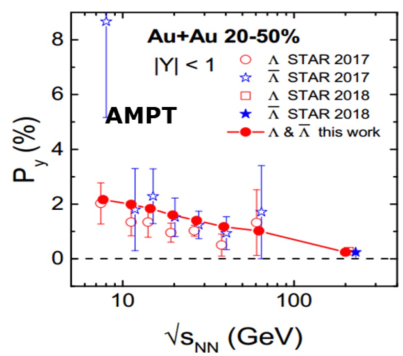
Xie, Wang, Csernai, PRC(2017)



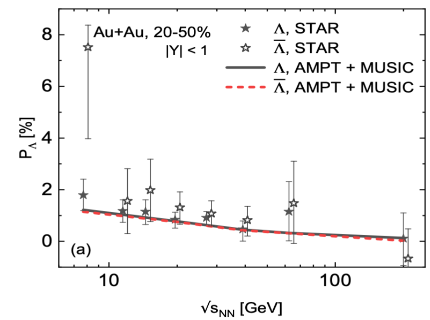
Sun, Ko, PRC(2017)



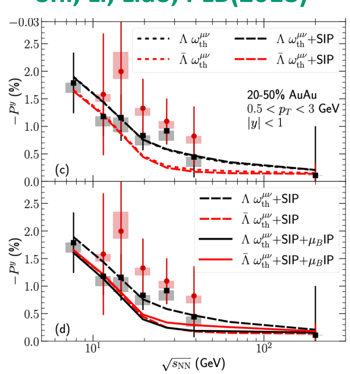
Shi, Li, Liao, PLB(2018)



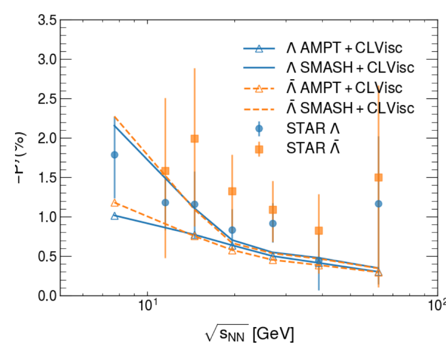
Wei, Deng, Huang, PRC(2019)



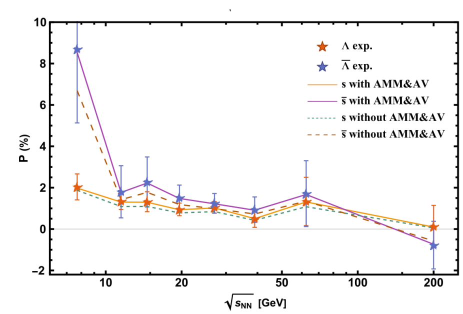
Fu, Xu, Huang, Song, PRC (2021)



S. Ryu, V. Jupic, C. Shen, PRC (2021)



Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

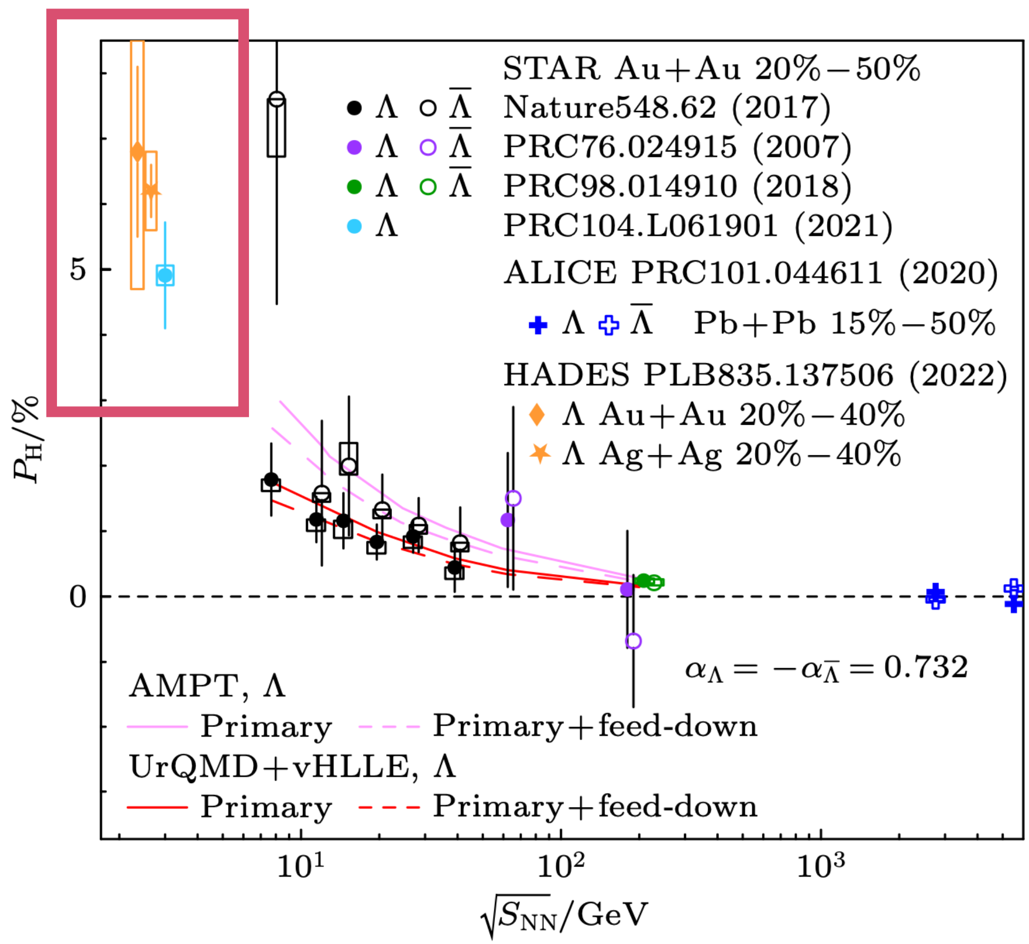


Xu, Lin, Huang, Huang, PRDL (2022)

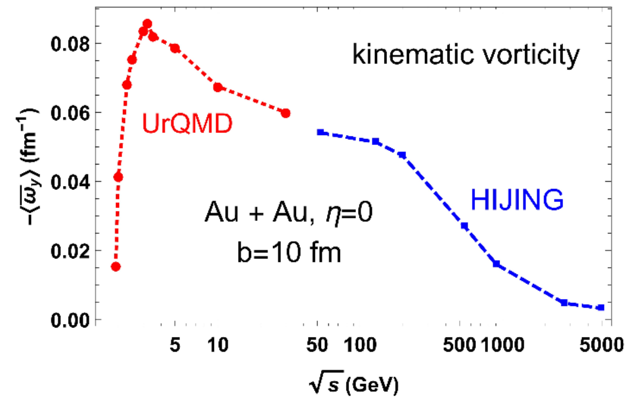
Polarization at low energies

Will the polarization of Lambda be nonzero when $\sqrt{s_{NN}} \to 0$?

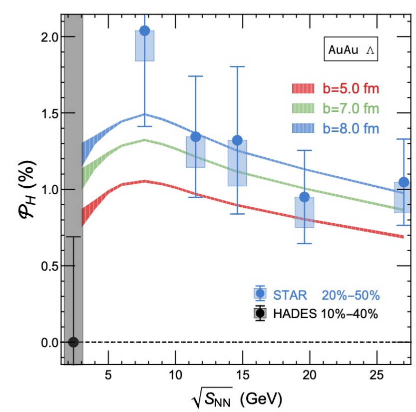
If not, how large the "critical $\sqrt{s_{NN}}$" will be?

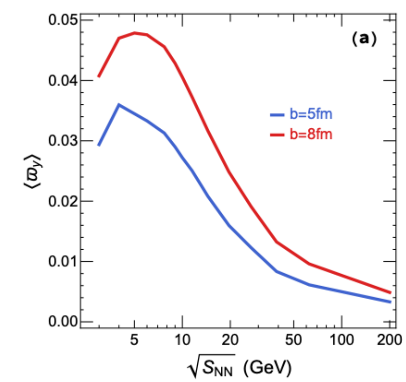


Sun Xu et al., Acta Phys. Sin. 72(7), 072401 (2023)



Deng, Huang 2016; Deng, Huang, et. al 2020



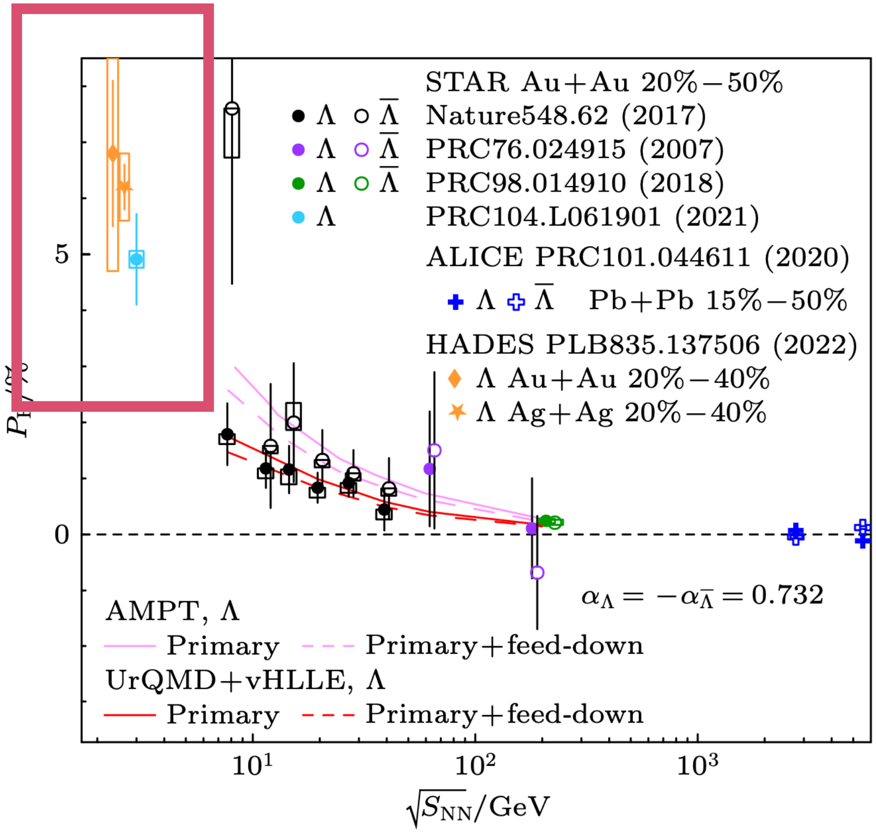


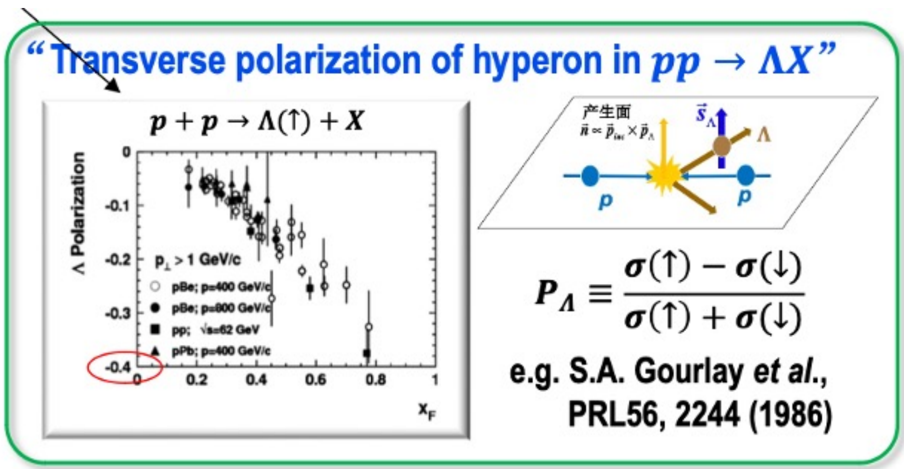
Guo, Liao, et. al, PRC 2021

Polarization at low energies

Will the polarization of Lambda be nonzero when $\sqrt{s_{NN}} \to 0$?

If yes, is the nonvanishing polarization at $\sqrt{s_{NN}} \to 0$ related to the spin puzzle in pp collisions?

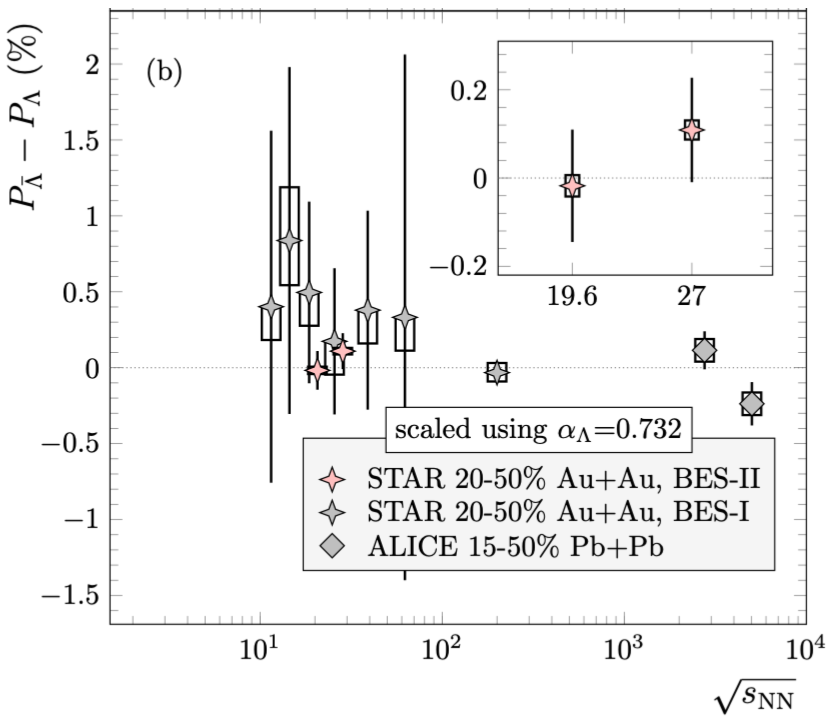




Figures copy from Prof. Zuo-tang Liang's review talk

Sun Xu et al., Acta Phys. Sin. 72(7), 072401 (2023)

Global polarization splitting



- **No splitting between the global polarization of Λ and $\overline{\Lambda}$ hyperons.**
- **The splitting may come from the B fields.**

$$\mathbf{P}_\Lambda \simeq \frac{\boldsymbol{\omega}}{2T} + \frac{\mu_\Lambda \mathbf{B}}{T}$$

$$\mathbf{P}_{\overline{\Lambda}} \simeq \frac{\boldsymbol{\omega}}{2T} - \frac{\mu_\Lambda \mathbf{B}}{T}$$

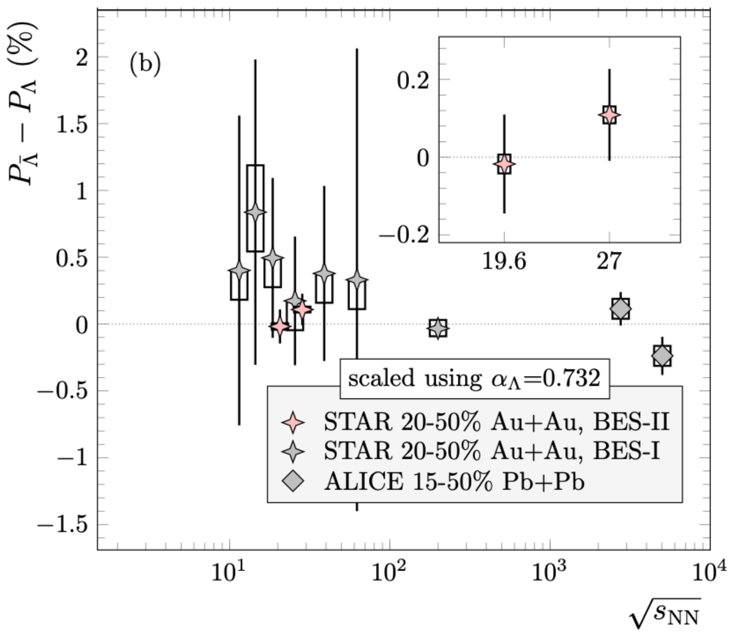
$$|B| \approx \frac{T_s |P_{\bar{\Lambda}} - P_\Lambda|}{2|\mu_\Lambda|}$$

Estimation by STAR data:

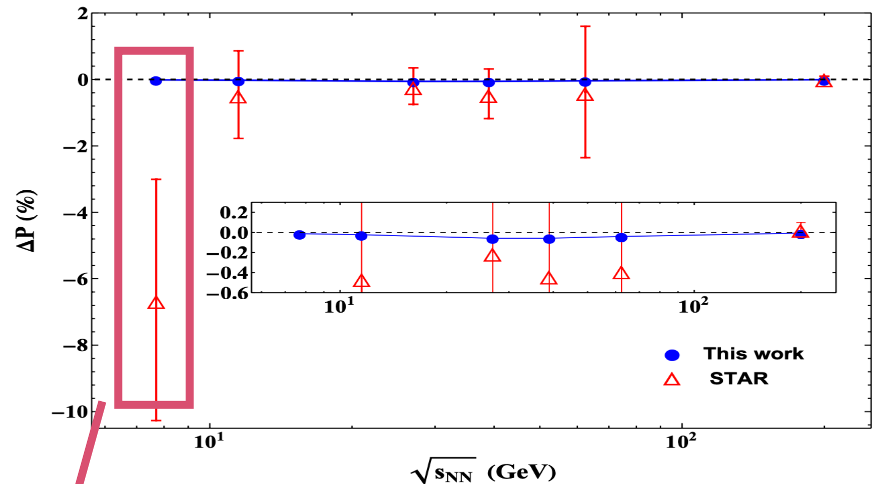
B < 9.4×10^{12} T ($\sqrt{s_{NN}}$ = 19.6 GeV)
B < 1.4×10^{13} T ($\sqrt{s_{NN}}$ = 27 GeV)

STAR, PRC 108 (2023) 1, 014910

Global polarization splitting



STAR, PRC 108 (2023) 1, 014910

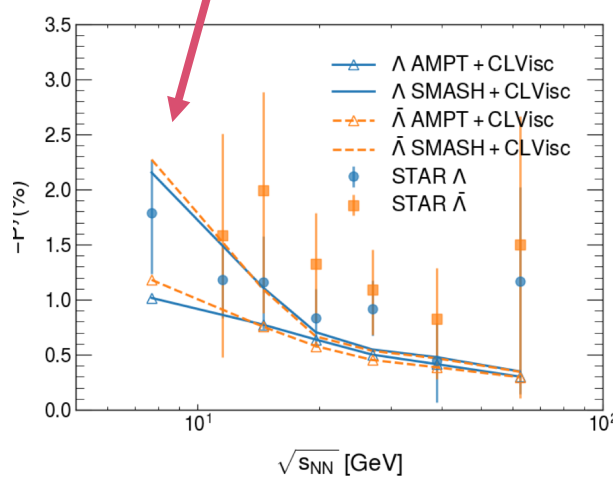


Estimation by Peng, Wu, Wang, She, Pu, PRD(2023)

UrQMD initial B fields (Sheng, Wang, PRC 2021)

+ ideal Magnetohydrodynamics

+ Modified Cooper-Frye formula



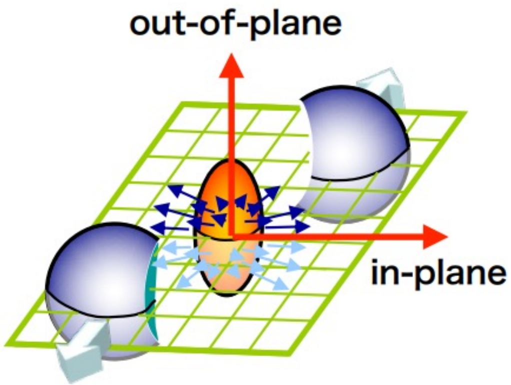
The baryon diffusion plays an important role to the global polarization.

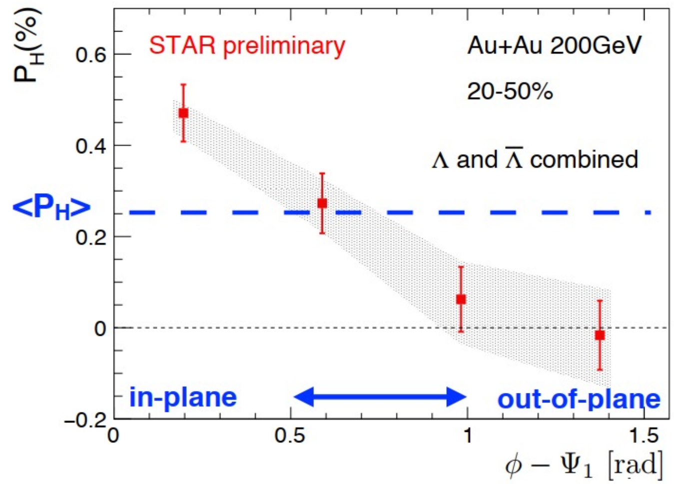
Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

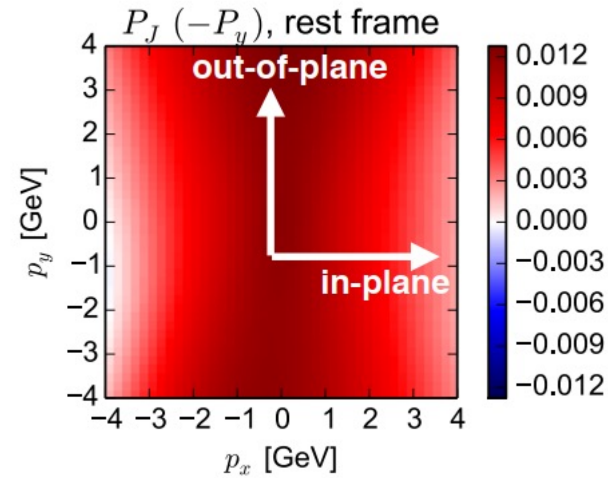
CLVis hydro + SMAHS

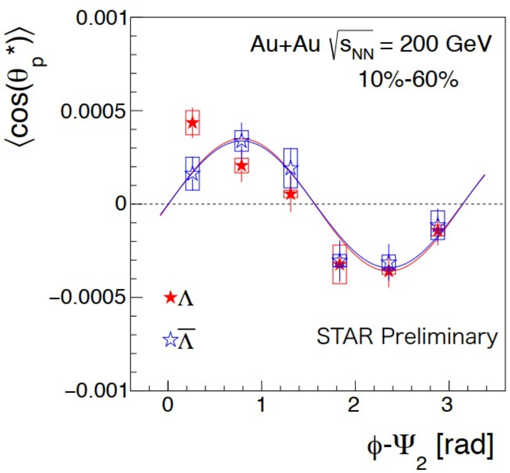
Local polarization

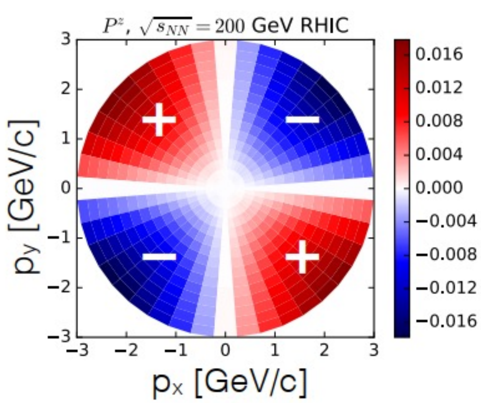
Local polarization and sign problem

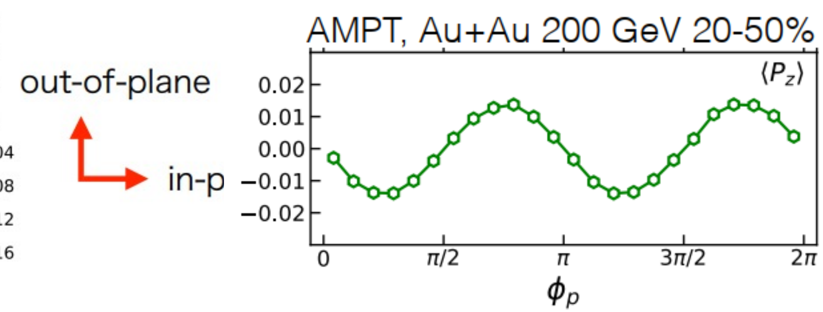












- **UrQMD :**
  Becattini, Karpenko, PRL (2018)
- **AMPT:**
  Xia, Li, Tang, Wang, PRC (2018)

Theoretical developments

- **Spin hydrodynamics (macroscopic approach)**

  Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
  Montenegro, Tinti, Torrieri (2017-2019);
  Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
  Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022); …
  S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
  D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060
  Weickgenannt, Wanger, Speranze, Rischke, PRD 2022; PRD 2022; Weickgennatt, Wanger, Speranza, PRD 2022; arXiv:2306.05936;
  …

- **Quantum kinetic theory with collisions (microscopic approach)**

  Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD 100, 056018 (2019)
  Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.
  Liu, Mameda, Huang, arXiv:2002.03753.
  Gao, Liang, PRD 2019
  Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019)； Z.Y. Wang, arXiv:2205.09334;
  Li ,Yee, PRD100, 056022 (2019)
  Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184; Lin, Wang, arXiv:2206.12573
  Fang, SP, Yang, PRD (2022)

- **Other approaches:**

  Side-jump effect Liu, Sun, Ko PRL(2020)
  Mesonic mean-field Csernai, Kapusta, Welle, PRC(2019)
  Using different vorticity Wu, Pang, Huang, Wang, PRR (2019)

- **Recent reviews:**

  Gao, Ma, SP, Wang, NST (2020)
  Gao, Liang, Wang, IJMPA (2021)
  Hidaka, SP, Yang, Wang, PPNP (2022)
  F. Becattini, M. Buzzegoli, T. Niida, SP, A.H. Tang, Q. Wang, arXiv: 2402.04540

Polarization and axial current

- **The polarization tensor is connected to the axial current in phase space by modified Cooper-Frye formula**
  Karpenko, Becattini, EPJC. (2017); Fang, Pang, QW, Wang, PRC (2016)

$$\mathcal{S}^{\mu}(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^{\mu}(p, X)}{2m_{\Lambda} \int d\Sigma \cdot \mathcal{N}(p, X)},$$

Polarization pseudo-vector~ Spin tensor in phase space

- **For massless fermions, the left and right handed currents can be derived by quantum kinetic theory,**

$$\mathcal{J}_5^{\mu} = \mathcal{J}^{\mu}_{\text{thermal}} + \mathcal{J}^{\mu}_{\text{shear}} + \mathcal{J}^{\mu}_{\text{accT}} + \mathcal{J}^{\mu}_{\text{chemical}} + \mathcal{J}^{\mu}_{\text{EB}},$$

Y. Hidaka, SP, and D.L. Yang, Phys. Rev. D97, 016004 (2018)

Polarization induced by different sources

$$\mathcal{S}^{\mu}(\mathbf{p}) = \mathcal{S}^{\mu}_{\text{thermal}} + \mathcal{S}^{\mu}_{\text{shear}} + \mathcal{S}^{\mu}_{\text{accT}} + \mathcal{S}^{\mu}_{\text{chemical}} + \mathcal{S}^{\mu}_{\text{EB}}$$

Y. Hidaka, SP, D.L. Yang, PRD97, 016004 (2018); C. Yi, SP, D.L. Yang, PRC 2021

Thermal vorticity 热涡旋

$$\mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) = \frac{\hbar}{8m_{\Lambda}N}\int d\Sigma^{\sigma}p_{\sigma}f_V^{(0)}(1-f_V^{(0)})\boxed{\epsilon^{\mu\nu\alpha\beta}p_{\nu}\partial_{\alpha}\frac{u_{\beta}}{T}},$$

Shear viscous tensor 剪切粘滞张量

$$\mathcal{S}^{\mu}_{\text{shear}}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_V^{(0)}(1-f_V^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}}{(u\cdot p)T}\frac{1}{2}\{p^{\sigma}\boxed{(\partial_{\sigma}u_{\nu}+\partial_{\nu}u_{\sigma})-Du_{\nu}}\}$$

Fluid acceleration 流体加速

$$\mathcal{S}^{\mu}_{\text{accT}}(\mathbf{p}) = -\frac{\hbar}{8m_{\Lambda}N}\int d\Sigma\cdot pf_V^{(0)}(1-f_V^{(0)})\frac{1}{T}\epsilon^{\mu\nu\alpha\beta}p_{\nu}u_{\alpha}\boxed{\left(Du_{\beta}-\frac{1}{T}\partial_{\beta}T\right)},$$

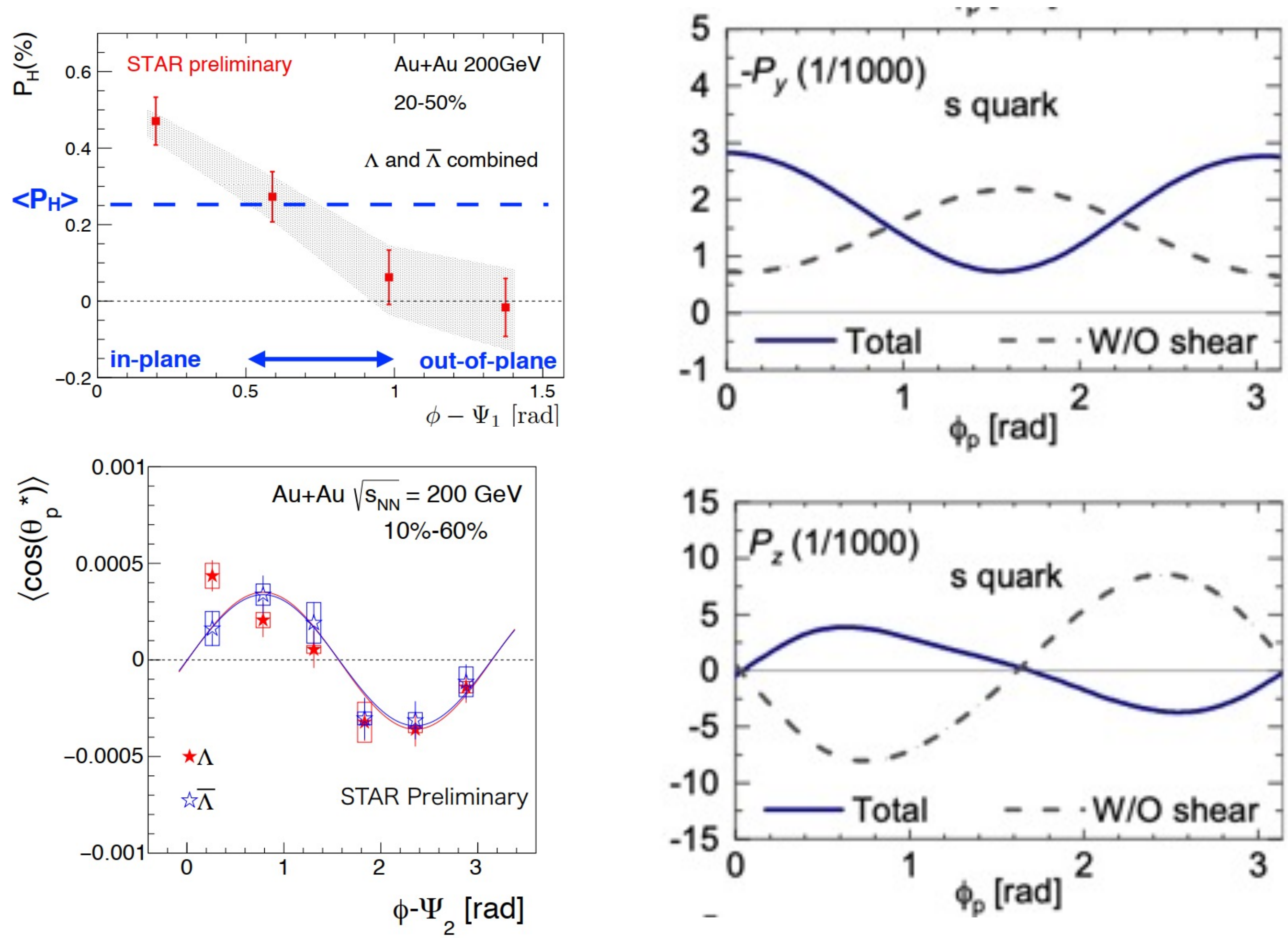
Gradient of chemical potential 化学势/温度梯度

$$\mathcal{S}^{\mu}_{\text{chemical}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_V^{(0)}(1-f_V^{(0)})\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\boxed{\partial_{\nu}\frac{\mu}{T}},$$

Electromagnetic fields 电磁场

$$\mathcal{S}^{\mu}_{\text{EB}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_V^{(0)}(1-f_V^{(0)})\left(\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\boxed{E_{\nu}+\frac{B^{\mu}}{T}}\right)$$

Shear induced polarization: s quark scenario



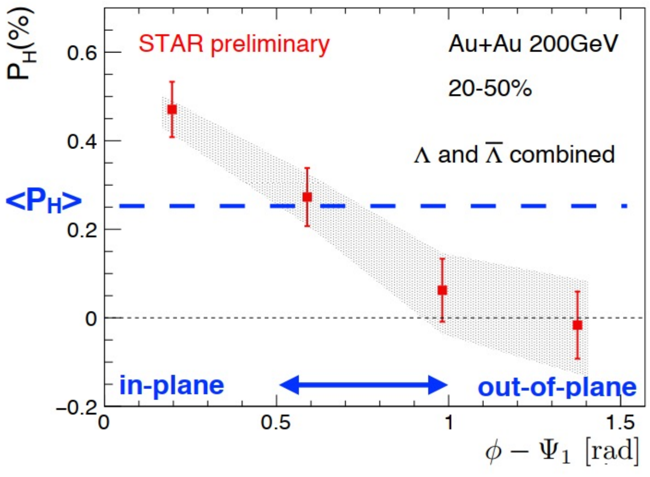
s quark scenarios (Thermal vorticity + shear)

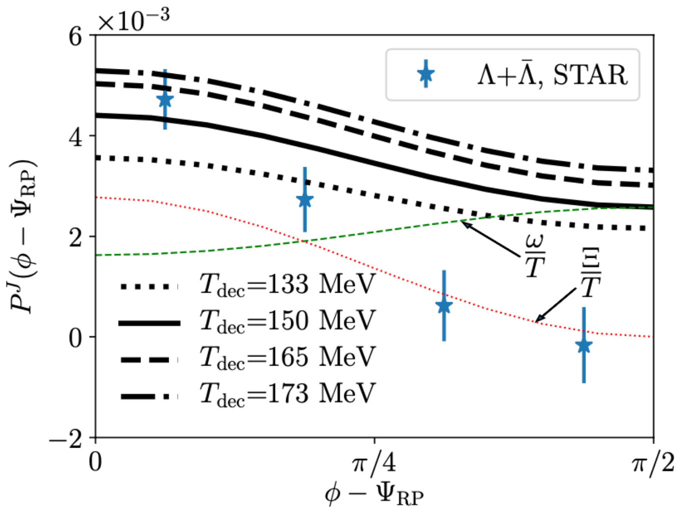
Fu, Liu, Pang, Song, Yin, PRL 2021

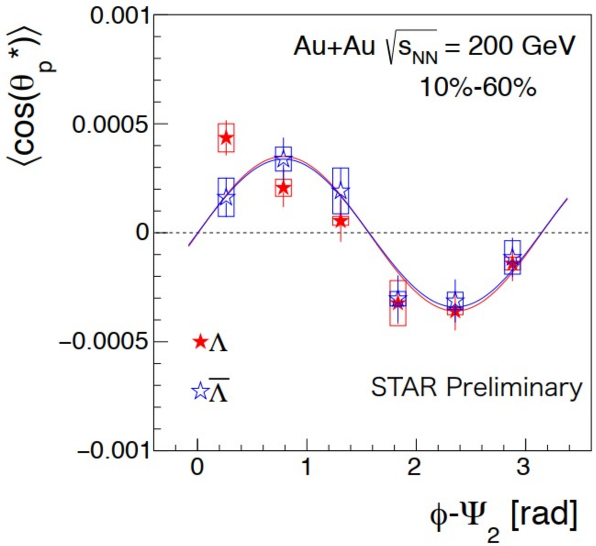
Also see:

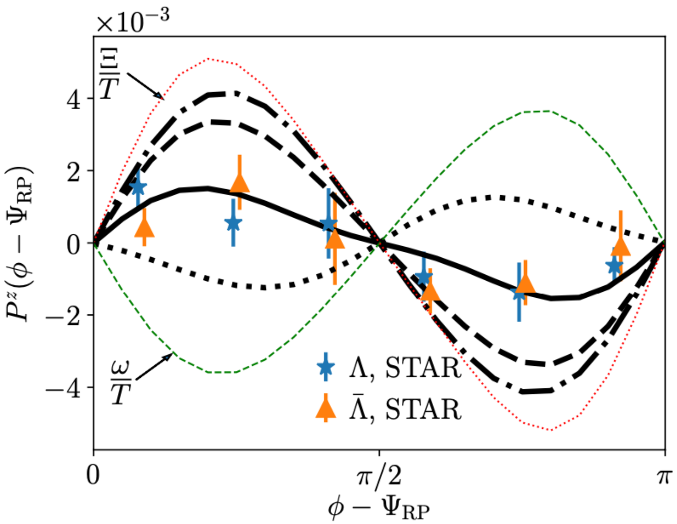
Yi, Pu, Yang, PRC (2021); Yi, Wu, Qin, Pu, PRC (2022); Ryu, Jupic, Shen, PRC (2021)

Shear induced polarization: isothermal equilibrium









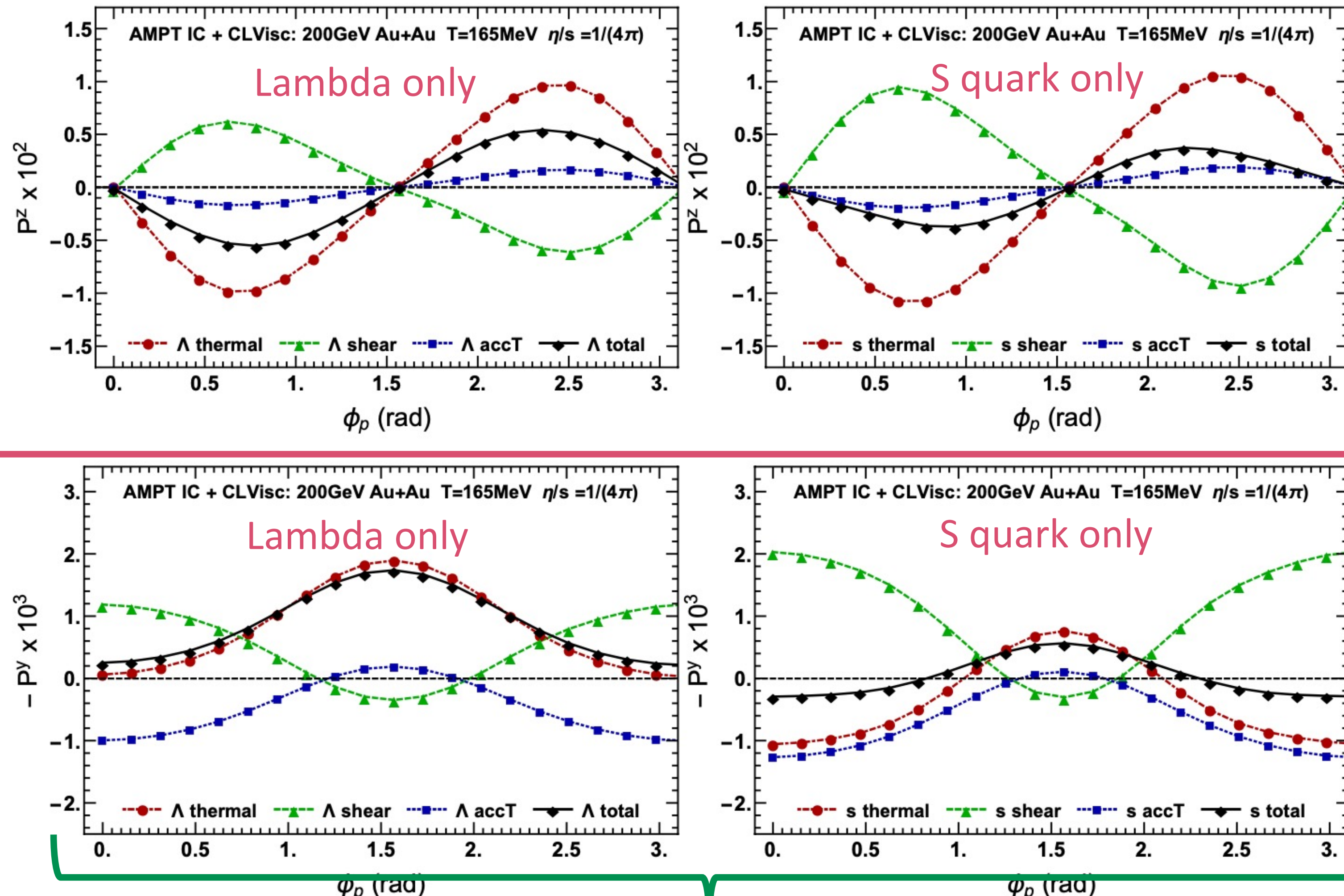
Isothermal equilibrium (Thermal vorticity + shear)

Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021

Local spin polarization induced by shear tensor

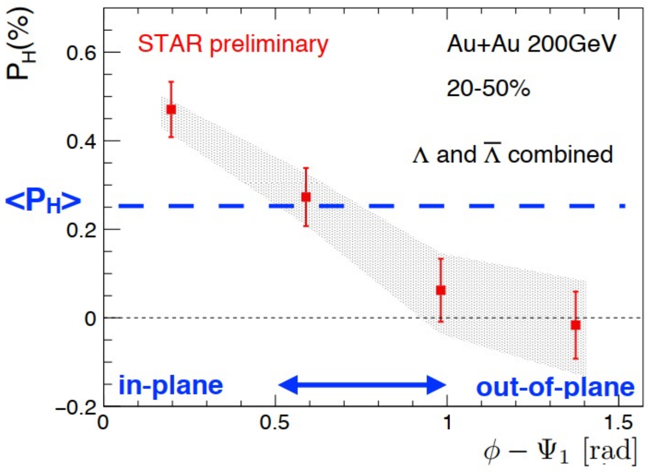
Polarization along beam direction

C. Yi, SP, D.L. Yang, PRC 2021

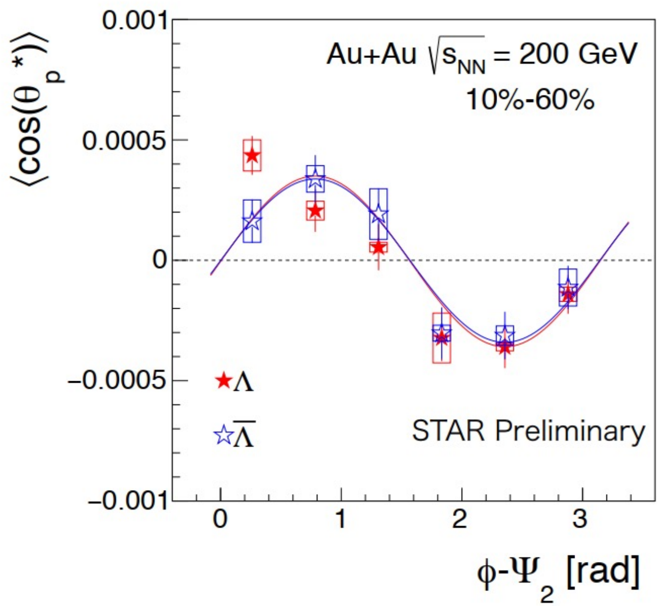


Polarization along out-of-plane direction

$P_{2,y}$ and $P_{2,z}$ across BES



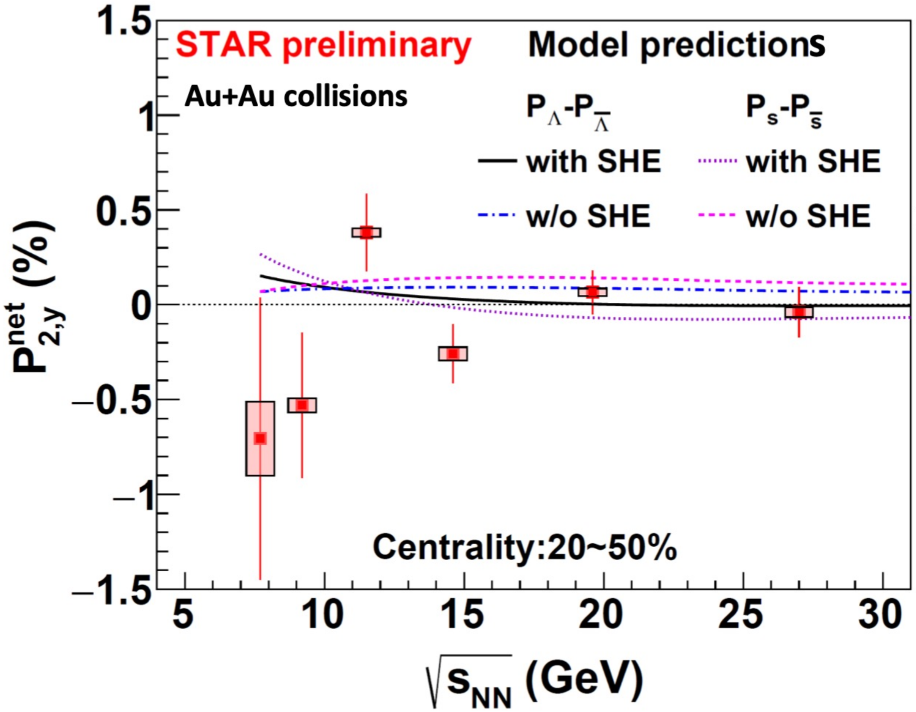
$$P_{2,y} \equiv \langle P_y \cos[2(\phi_\Lambda - \Psi_2)] \rangle$$

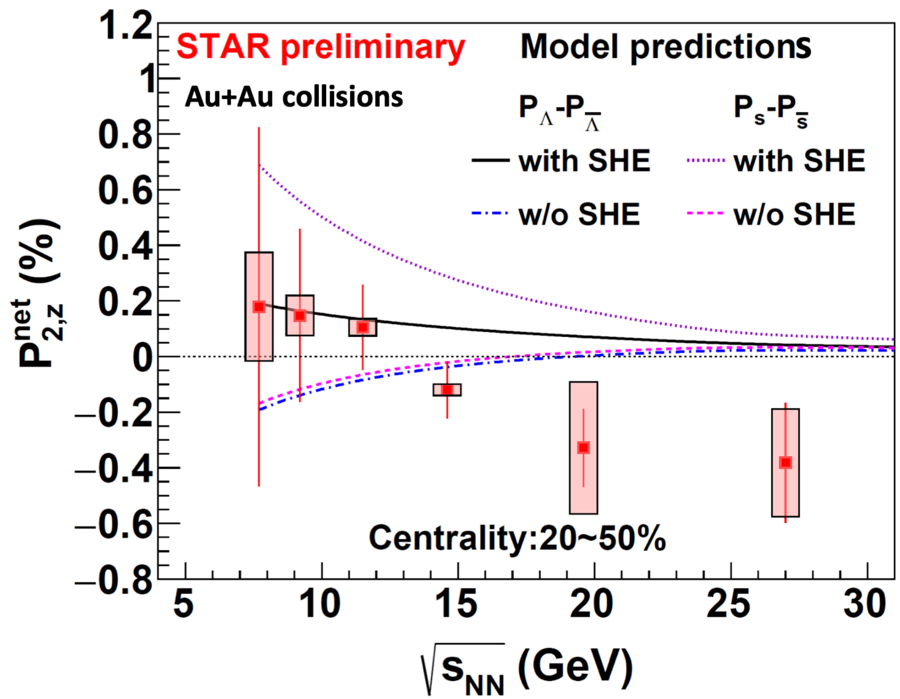


$$P_{2,z} \equiv \langle P_z \sin[2(\phi_\Lambda - \Psi_2)] \rangle$$

Local polarization and spin Hall effect

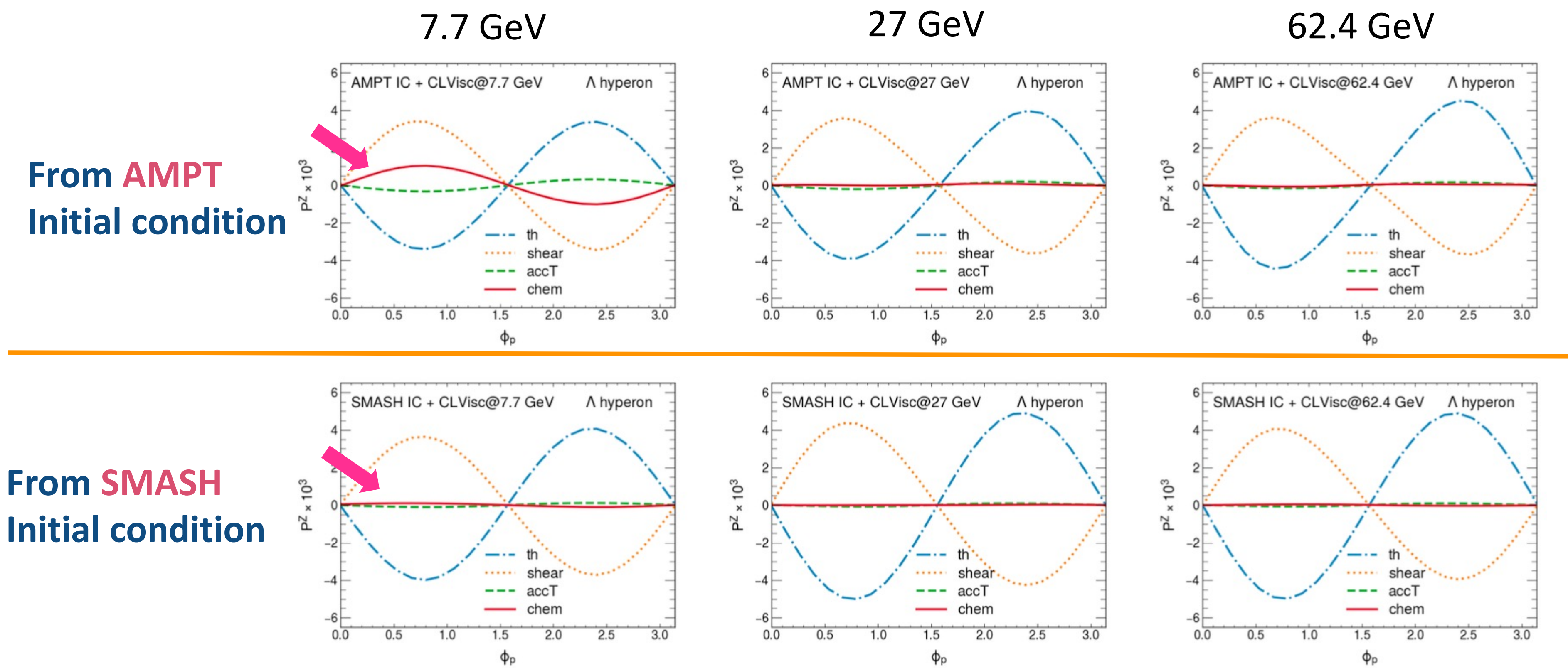
$$\mathcal{S}^{\mu}_{\text{chemical}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\boxed{\partial_{\nu}\frac{\mu}{T}},$$





Prediction: Fu, Pang, Song, Yin, 2208.00430

Local polarization and spin Hall effect: SMASH VS AMPT



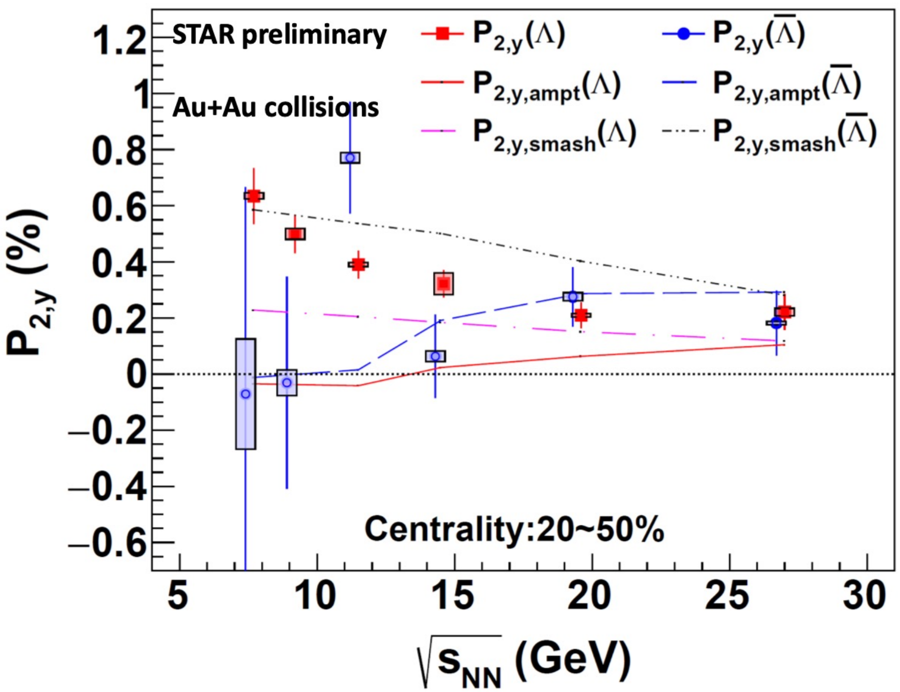
Red lines: contributions from spin Hall effect

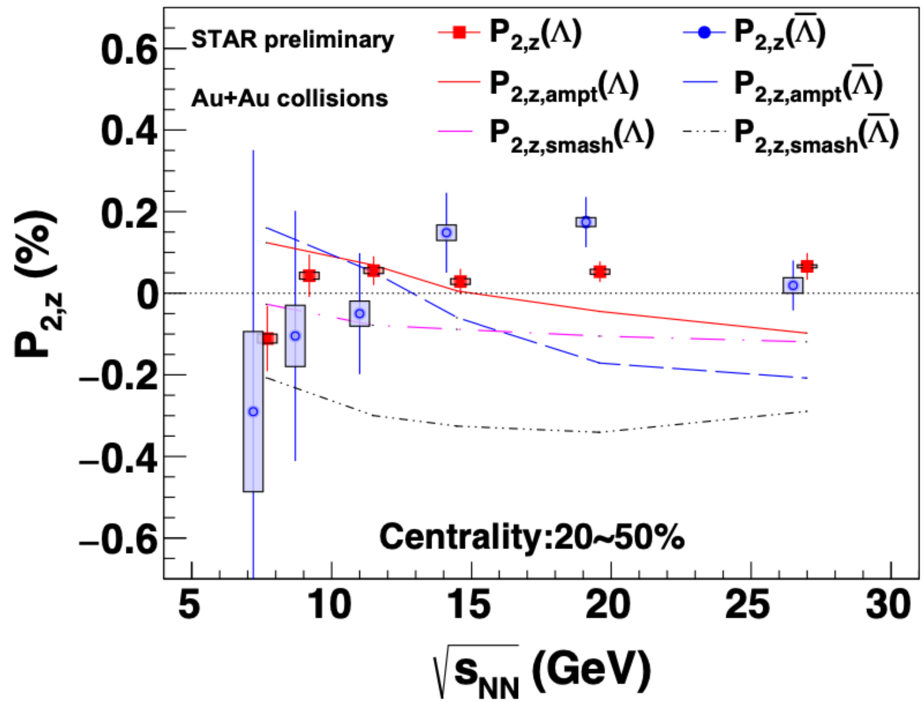
Polarization induced by SHE is almost zero at 27, 62.4GeV and it depends on the initial conditions at 7.7 GeV. For SMASH, Pz is still almost vanishing at 7.7 GeV.

X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

Local polarization splitting

How can we understand the data in low energy collisions?

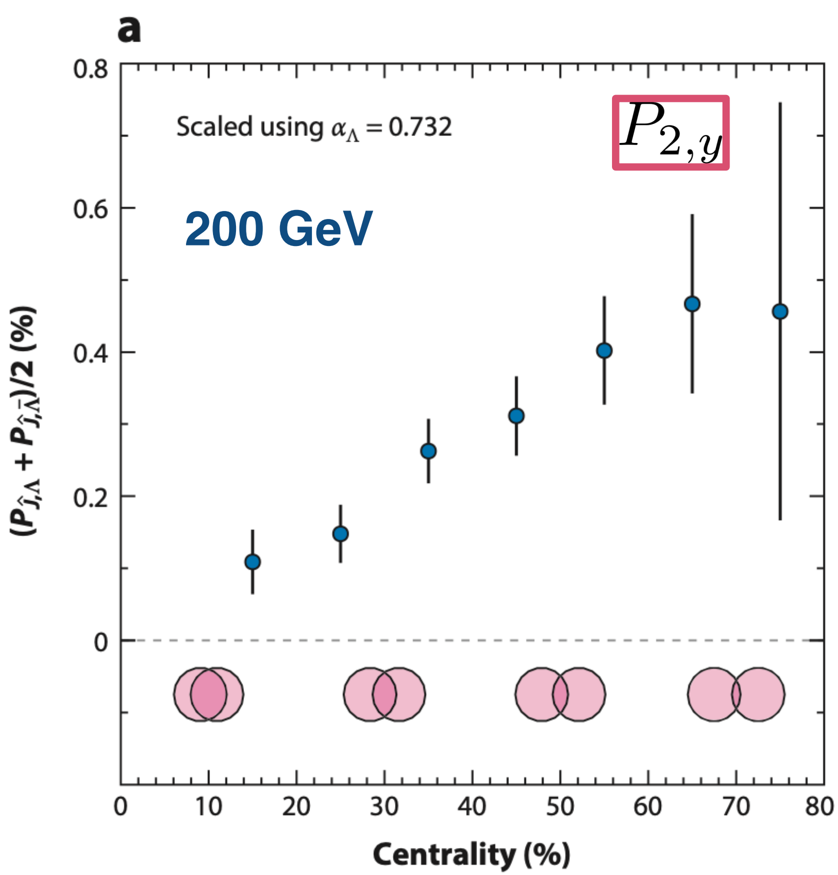


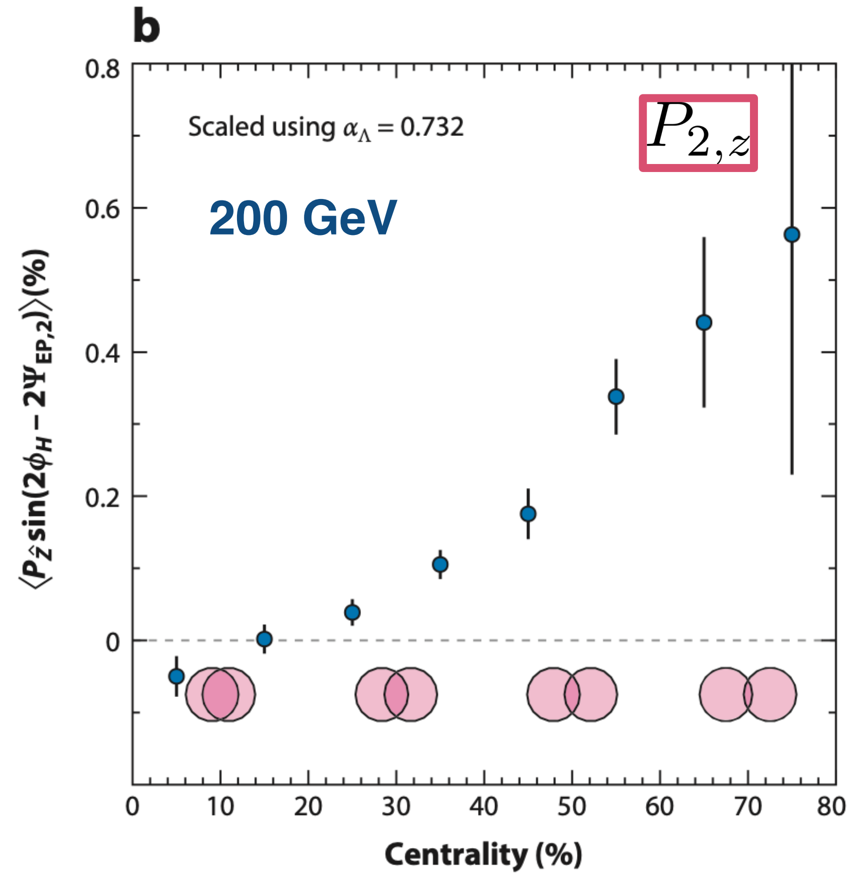


Model Predition: **X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)**

Local polarization VS centrality

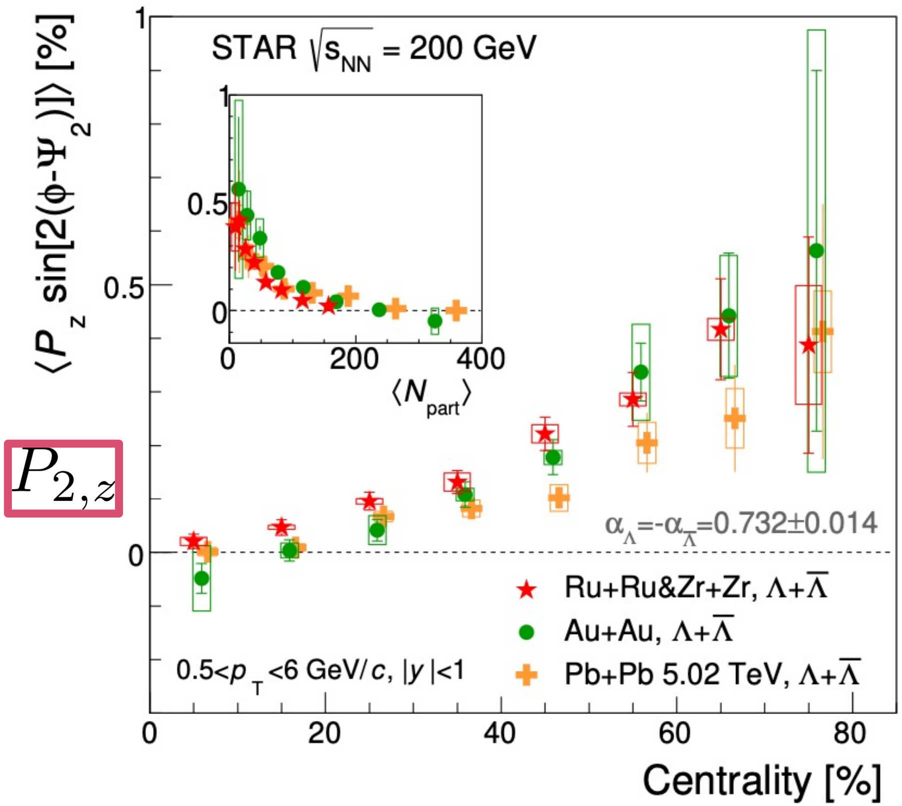
Local polarization increases with growthing centrality.
What happens in ultra-pherial heavy ion collisions?



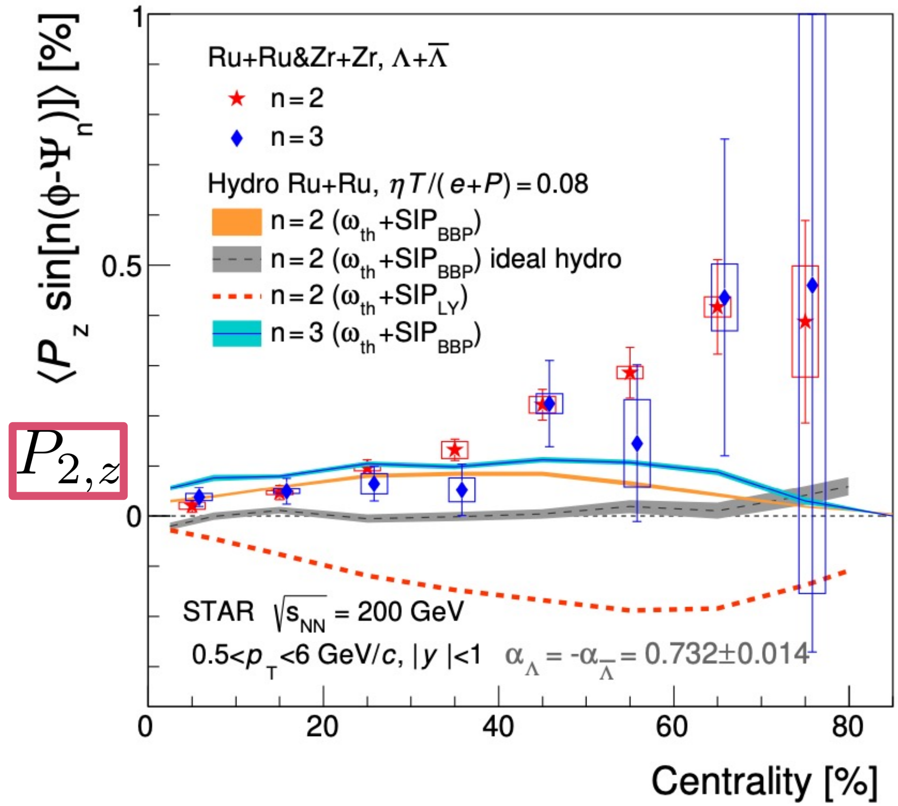


Becattini, Lisa, Annu. Rev. Nucl. Part. Sci. 2020. 70:395–423

Local polarization VS centrality



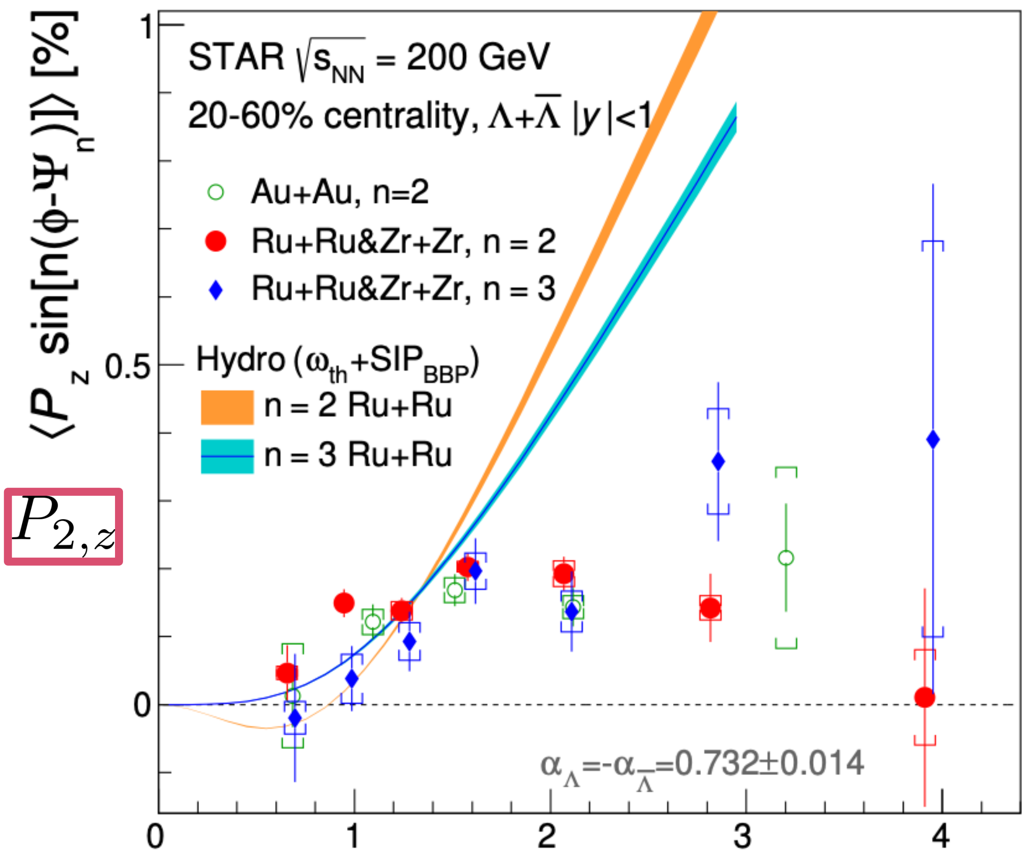
STAR, PRL 123, 132301 (2019)
ALICE, PRL 128, 172005 (2022)



STAR, PRL 131 (2023) 20, 202301

Model Calculation:
BBP: isothermal equilibrium; LY: s quark equilibrium
Simulation: Alzhrani, Ryu, Shen, PRC 106, 014905 (2022)

Local polarization VS p_T



STAR, PRL 131 (2023) 20, 202301

Similar question arises:

How large will the local polarization become when p_T is infinte?

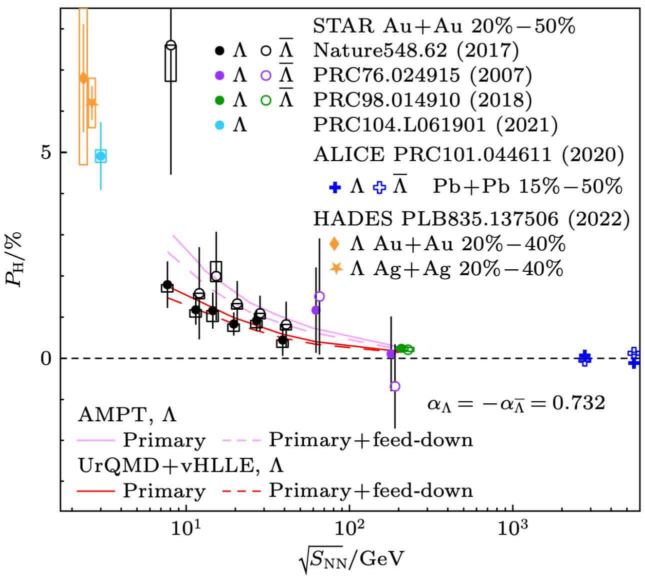
$$\mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) = \frac{\hbar}{8m_\Lambda N}\int d\Sigma^\sigma p_\sigma f_V^{(0)}(1-f_V^{(0)})\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T},$$

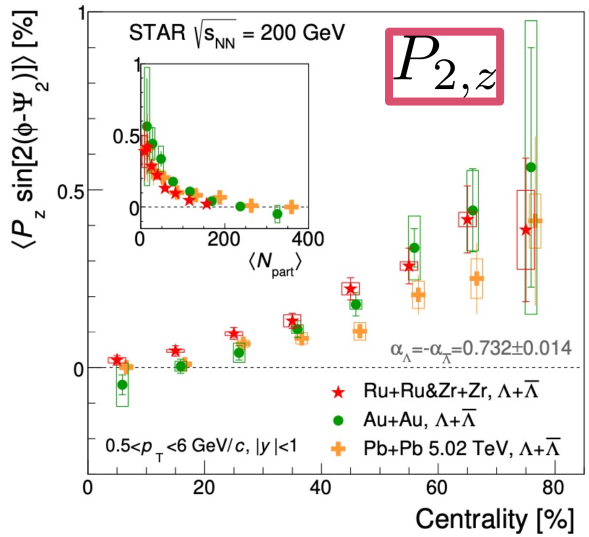
Reason:
For a given thermal vorticity, there is no suppressed factor proportional to p_T in denominator.

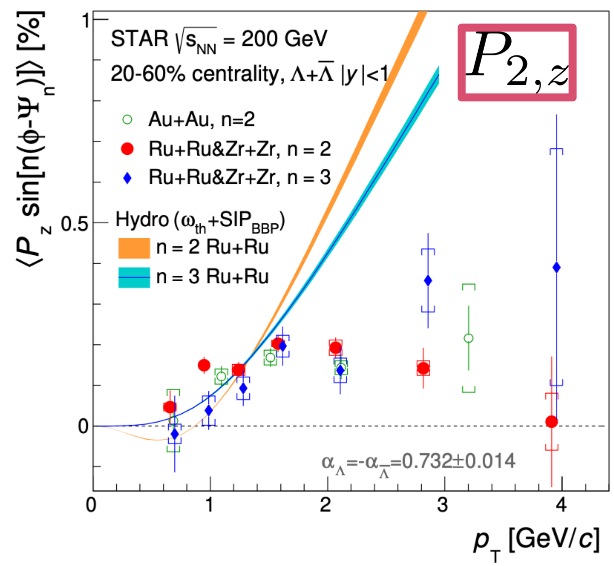
Model Calculation:
BBP: isothermal equilibrium; LY: s quark equilibrium
Simulation: Alzhrani, Ryu, Shen, PRC 106, 014905 (2022)

Brief summary for polarization in AA systems

Question: when will the polarization stop growing?







Spin polarization at pA system

C. Yi, X.Y. Wu, J. Zhu, SP, G.Y. Qin, arXiv: 2408.04296

Setup (I)

- **We follow the modified Cooper-Frye formula to compute the polarization pseudo-vector including the contribution from thermal vorticity and thermal shear tensor.**

$$
\begin{aligned}
\mathcal{S}^{\mu}(\mathbf{p}) &= \mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) + \mathcal{S}^{\mu}_{\text{th-shear}}(\mathbf{p}) \\
\mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) &= \hbar \int d\Sigma \cdot \mathcal{N}_p \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \varpi_{\alpha\beta}, \\
\mathcal{S}^{\mu}_{\text{th-shear}}(\mathbf{p}) &= \hbar \int d\Sigma \cdot \mathcal{N}_p \frac{\epsilon^{\mu\nu\alpha\beta} p_\nu n_\beta}{(n \cdot p)} p^\sigma \xi_{\sigma\alpha}
\end{aligned}
$$

thermal vorticity $\varpi_{\alpha\beta} = \frac{1}{2}\left[\partial_\alpha\left(\frac{u_\beta}{T}\right) - \partial_\beta\left(\frac{u_\alpha}{T}\right)\right],$

thermal shear tensor $\xi_{\alpha\beta} = \frac{1}{2}\left[\partial_\alpha\left(\frac{u_\beta}{T}\right) + \partial_\beta\left(\frac{u_\alpha}{T}\right)\right]$

Setup (II)

- **We consider three different scenarios:**
  - **Λ equilibrium:**

    It is assumed that Λ hyperons reach the local (thermal) equilibrium at the freeze-out hyper-surface.

  - **s quark equilibrium:**

    The spin of Λ hyperons is assumed to be carried by the constituent s quark. We take the s quark's mass instead of Λ's mass in the simulation.

  - **Isothermal equilibrium:**

    The temperature of the system at the freeze-out hyper-surface is assumed to be constant. The time unit vector is taken as fluid velocity for simplicity.

Setup (III)

- **We implement the 3+1D CLVisc hydrodynamics model**

  Pang, Wang, Wang, PRC (2012)

  Wu, Qin, Pang, Wang, PRC (2022)

- **Initial condition: TRENTo-3D model**

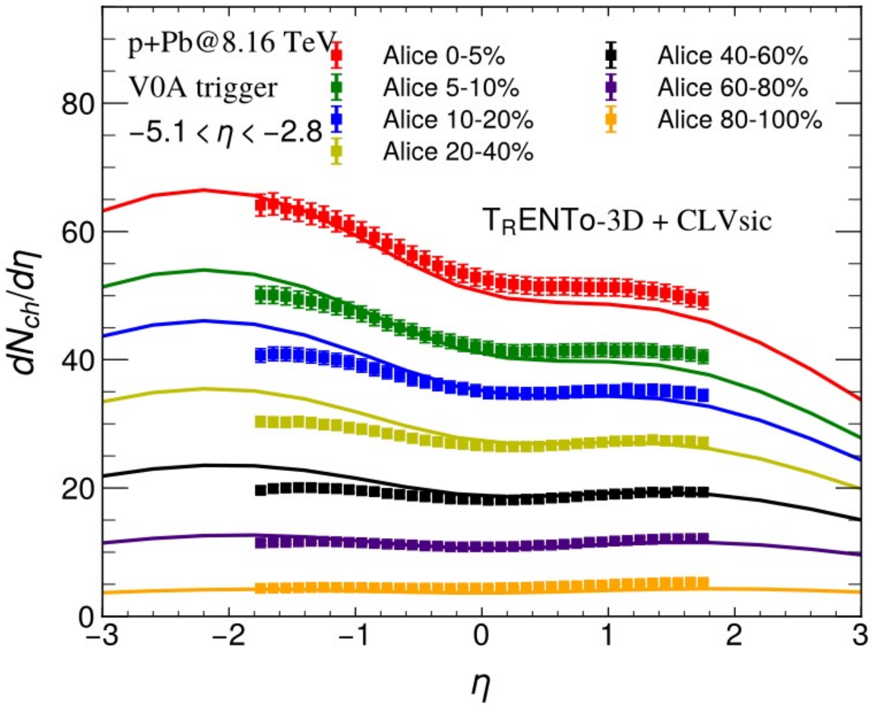
  Soeder, Ke, Paquet, Bass, 2306.08665

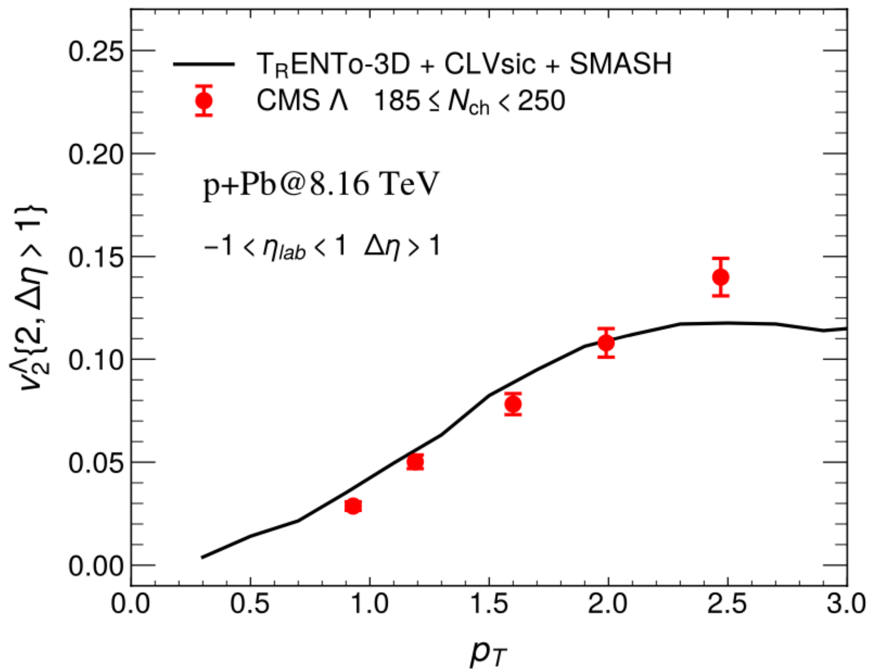
  Moreland, Bernhard, Bass, PRC (2015); PRC (2020)

  Ke, Moreland, Bernhard, Bass, PRC (2017)

- **p+Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV**

Fit parameters and test v2 of Λ

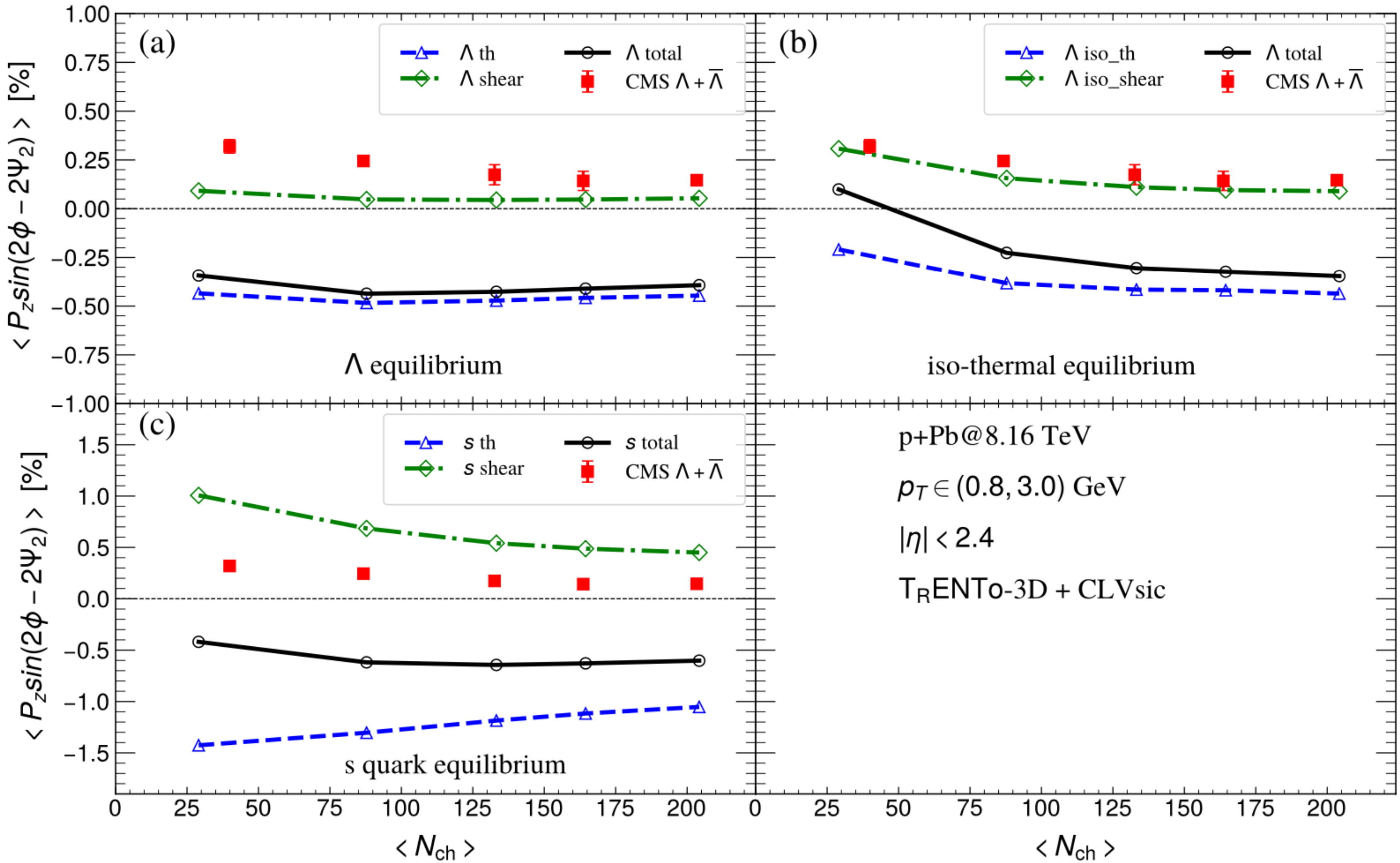




Multiplicity intervals	$\langle N_{\rm ch}\rangle_{\rm exp}$	$\langle N_{\rm ch}\rangle_{\rm CLVisc}$
[185,250)	203.3	204.2
[150,185)	163.6	164.5
[120,150)	132.7	133.57
[60,120)	86.7	87.7
[3,60)	40	29.3

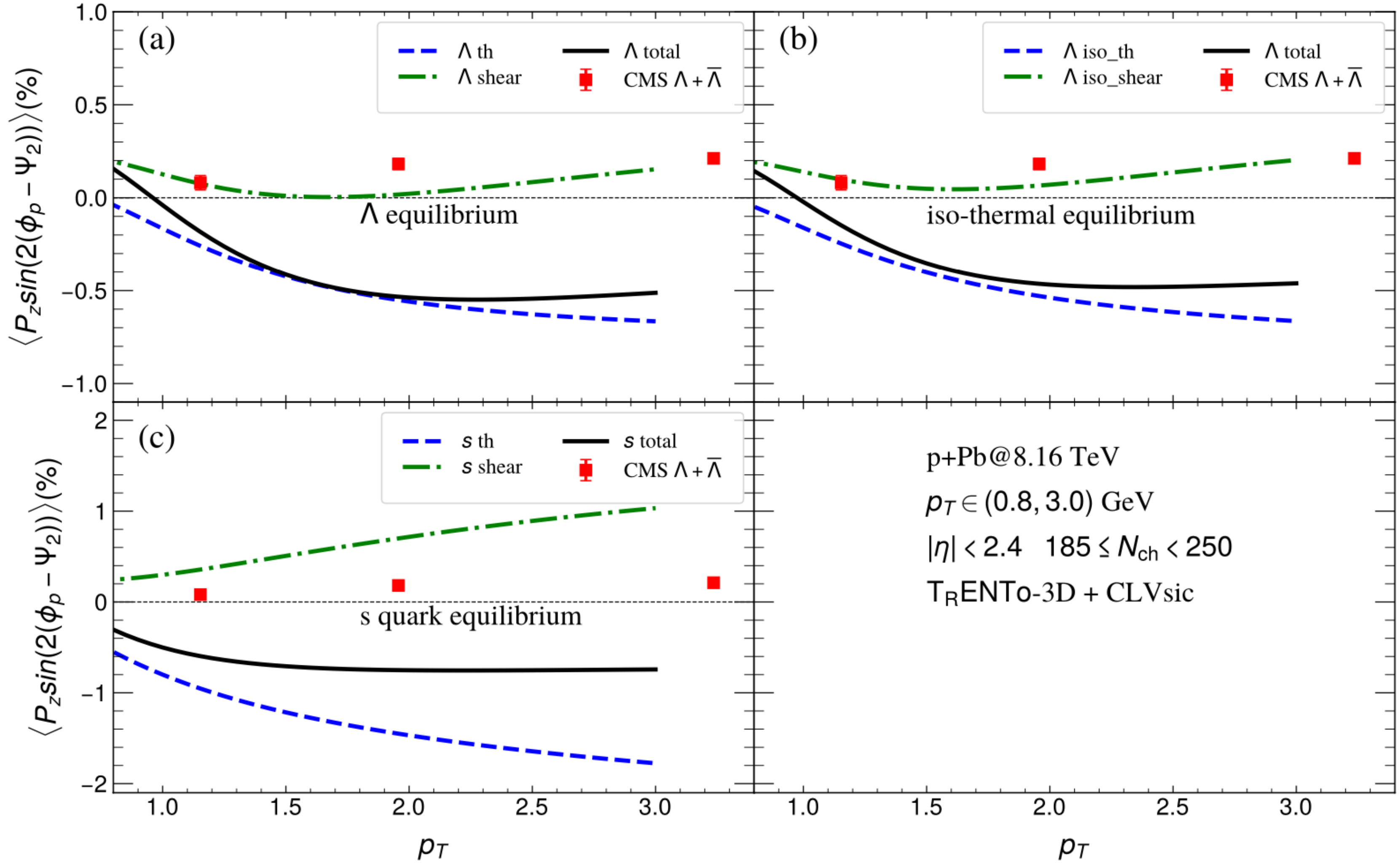
We have run 10^5 minimum bias events to divide the centrality. The centrality-dependent pseudo-rapidity distributions of charged hadrons and elliptic flow for Λ hyperons computed by our model are consistent with the experimental measurements.

Multiplicity (centrality) dependence

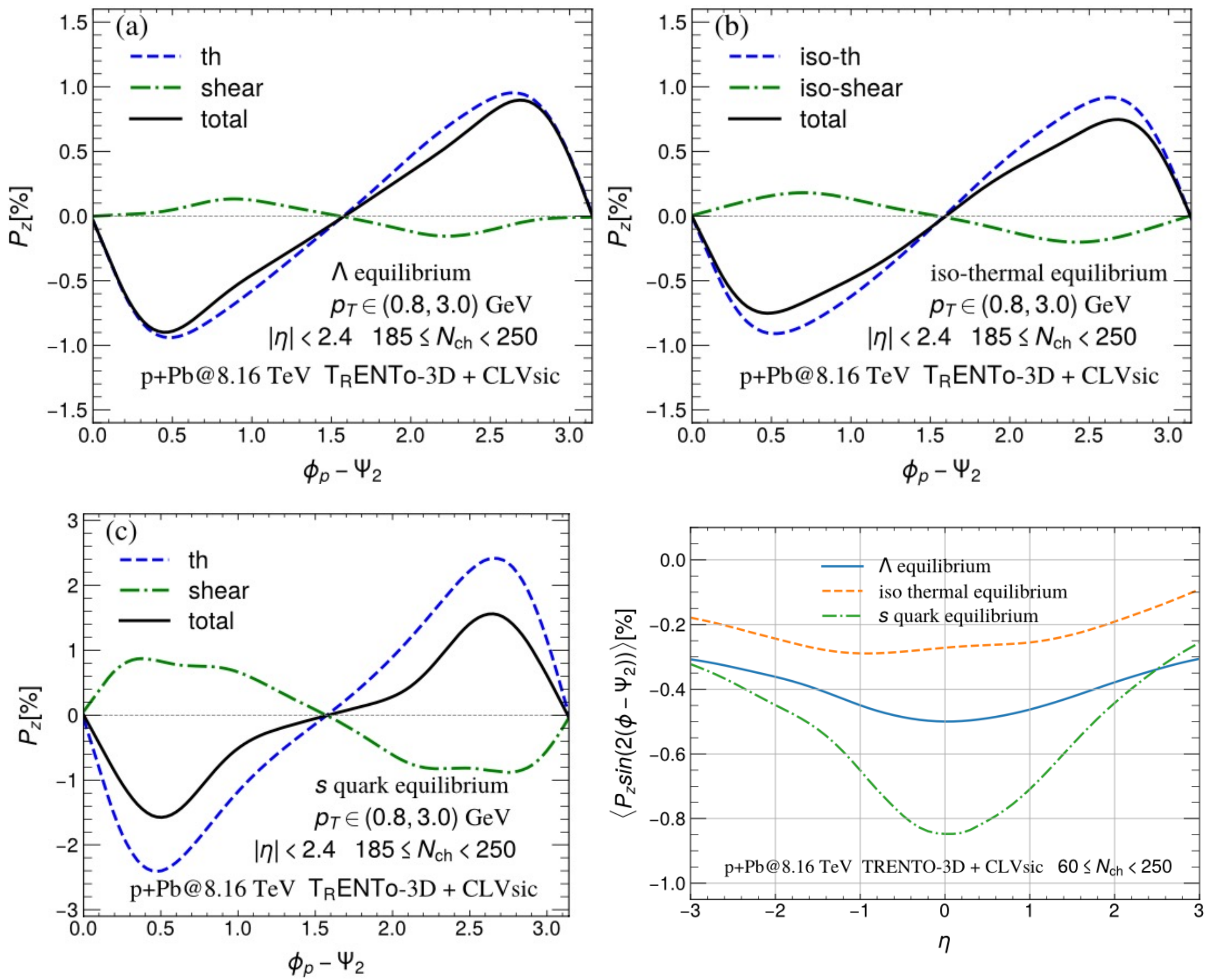


Multiplicity interval

p_T dependence



Azimuthal angle and pesudo-rapidity dependence



Why?

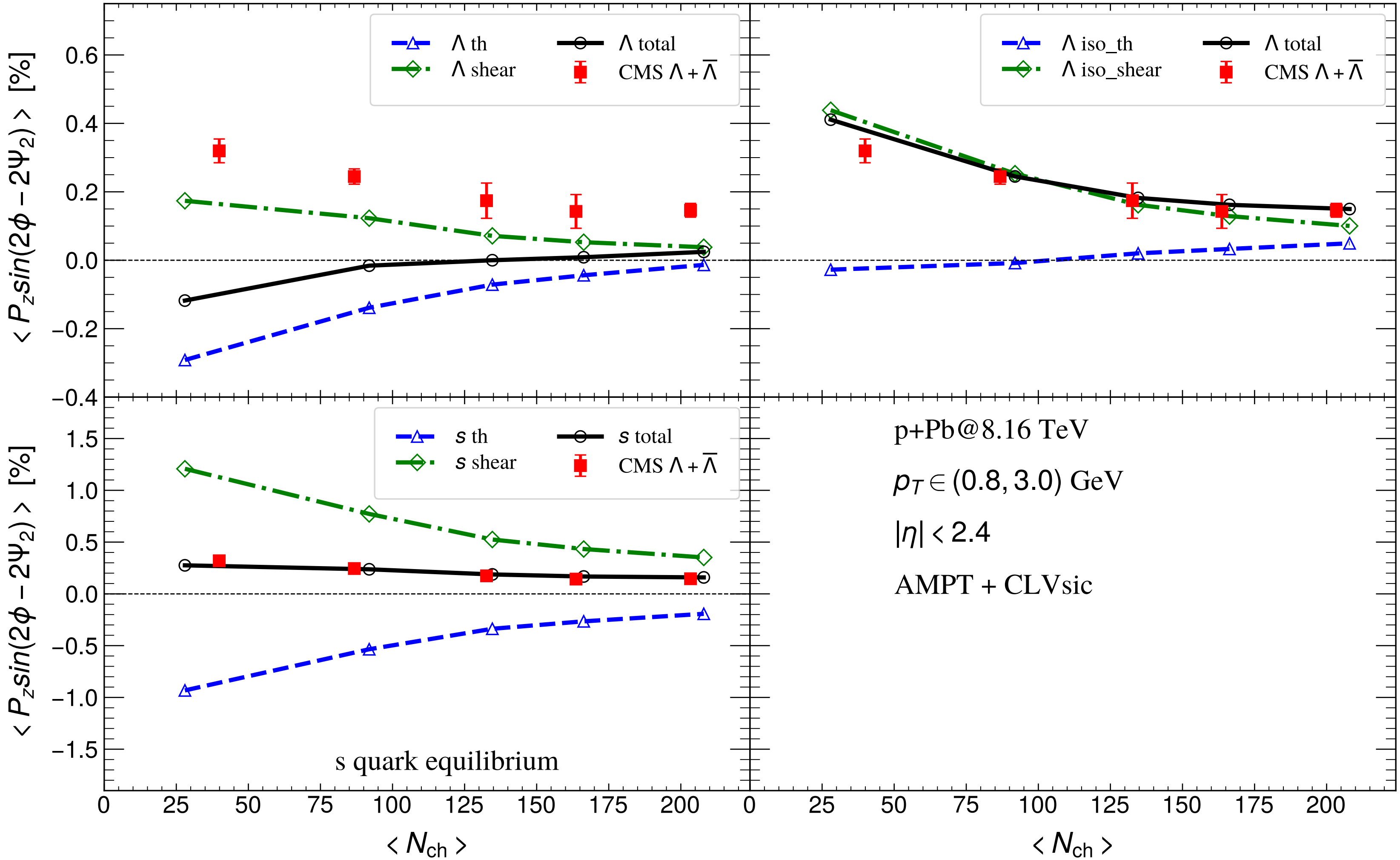
- **We implement the 3+1D CLVisc hydrodynamics model**
  Pang, Wang, Wang, PRC (2012)
  Wu, Qin, Pang, Wang, PRC (2022)

- **Initial condition: TRENTo-3D model**
  Soeder, Ke, Paquet, Bass, 2306.08665
  Moreland, Bernhard, Bass, PRC (2015); PRC (2020)
  Ke, Moreland, Bernhard, Bass, PRC (2017)

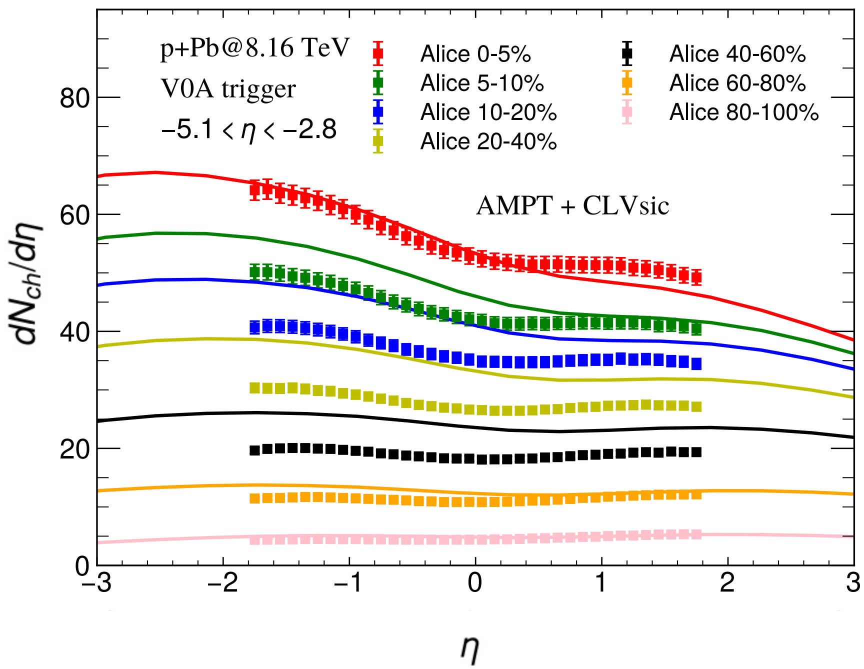
- **p+Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV**

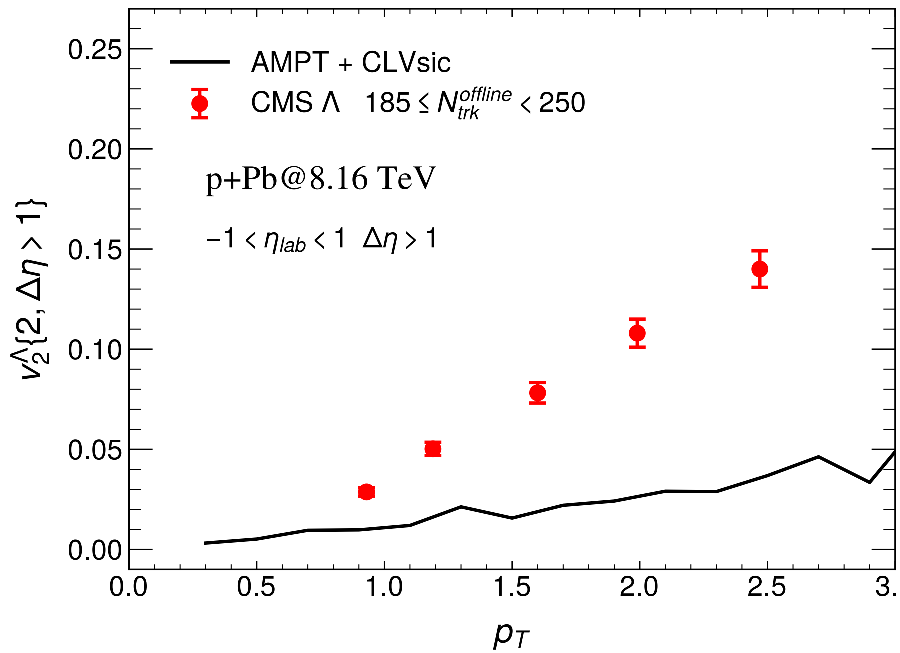
Test for AMPT initial condtions

It describes data well in s quark and isothermal equilbirum scenarios?



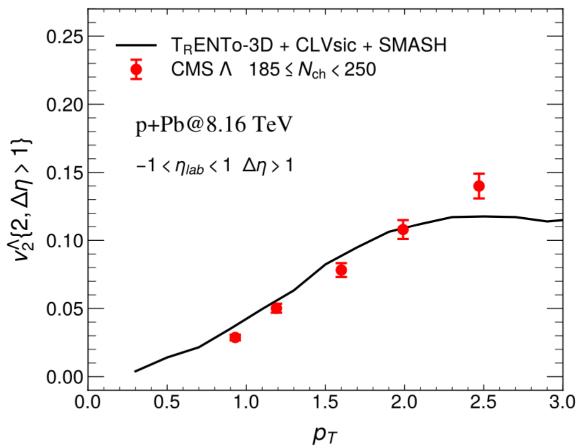
Test for AMPT initial conditions

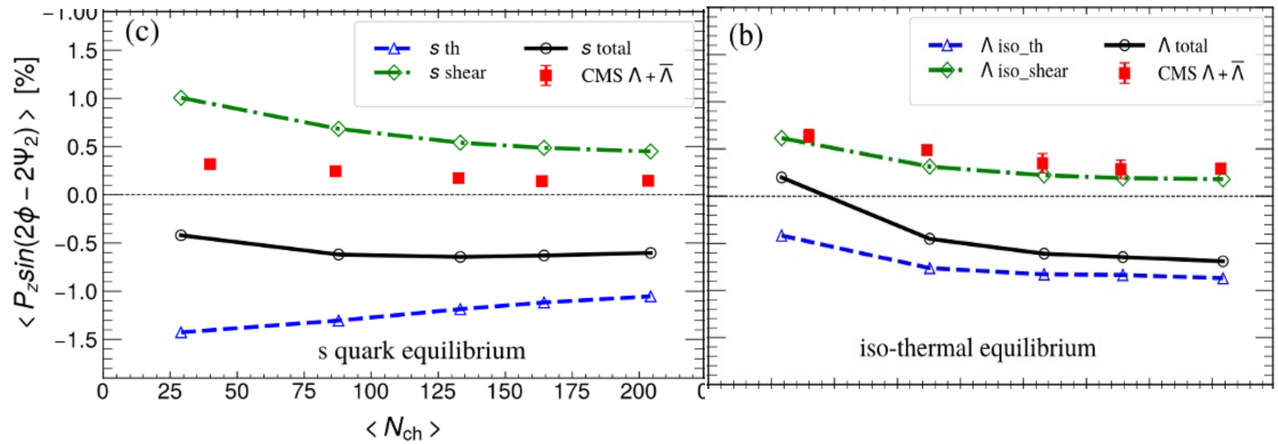




We fix the parameters in 3+1D CLVisc hydrodynamic model with AMPT initial conditions by the spectrum of charged hadrons. But, it cannot describe v2 well.

However ...

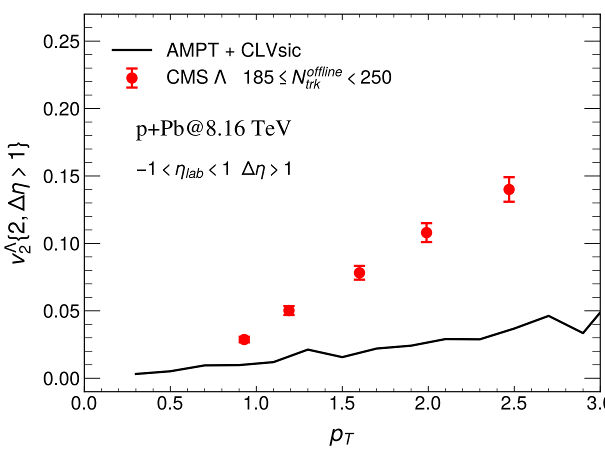


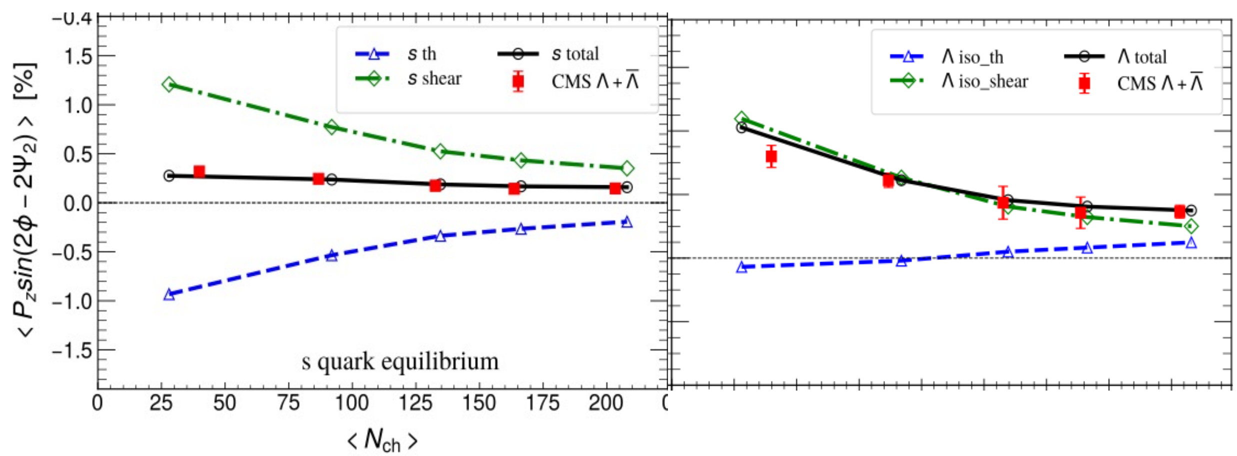


Smaller v2 gives a larger polarization along beam direction ?

Smaller v2, larger shear induced polarization, smaller thermal vorticial induced polarization

Sensitive to initial conditions?





Connection between P_z and v_2

- **Assuming we consider a Bjorken-like flow**

$$\mathcal{S}^z_{\mathrm{thermal}} = -\frac{1}{4m_\Lambda N}\frac{1}{T}\frac{dT}{d\tau}\Big|_\Sigma \partial_\phi \int d\Sigma_\alpha p^\alpha f_V^{(0)} \cosh\eta$$

since

$$\int d\Sigma_\lambda p^\lambda f_V^{(0)} = \frac{dN}{2\pi E_p p_T dp_T dY}\left[1+\sum_{n=1}^{\infty} 2v_n(p_T,Y)\cos n\phi\right]$$

one can get

$$\mathcal{S}^z_{\mathrm{thermal}} \approx \frac{1}{m_\Lambda}\frac{1}{T}\frac{dT}{d\tau}\Big|_\Sigma v_2(p_T,0)\sin 2\phi$$

Becattini, Karpenko, PRL (2018); C. Yi, SP, J.H. Gao, D.L. Yang, PRC (2022)

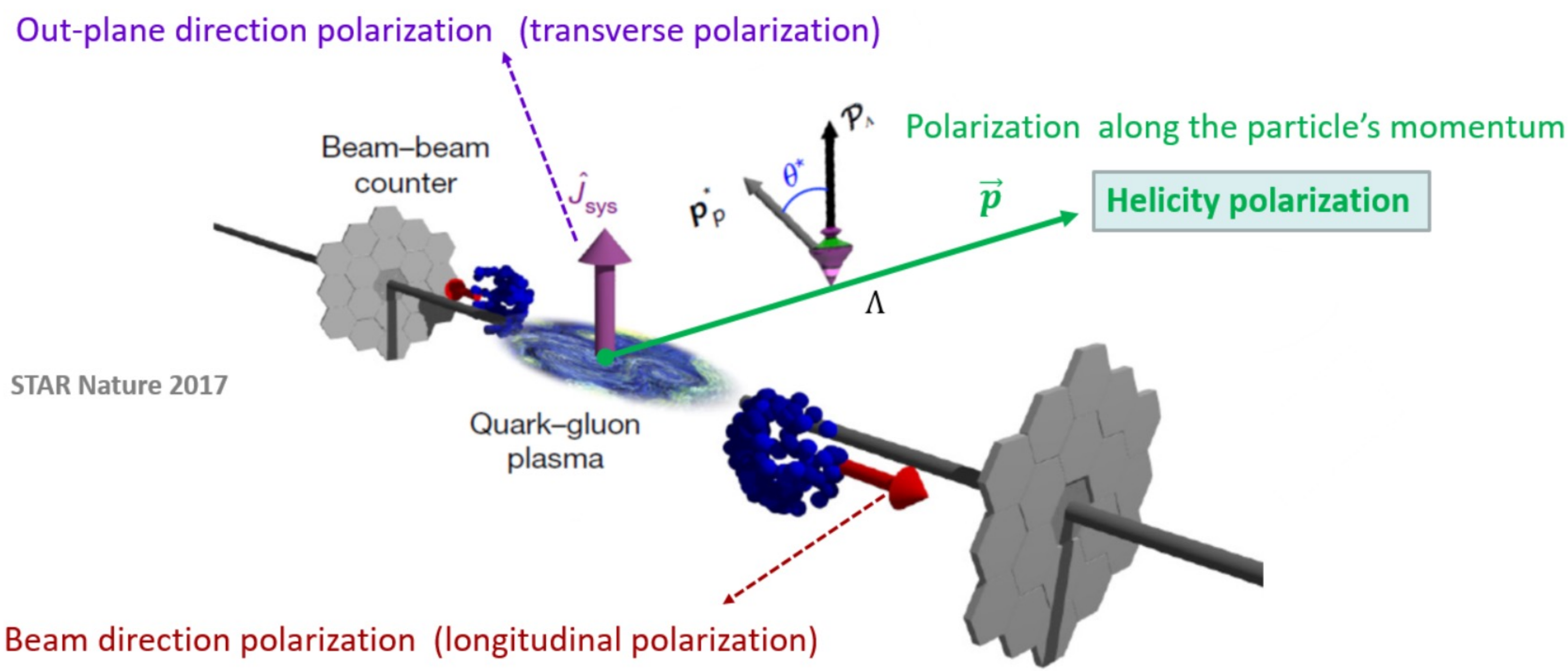
Non-flow effects play a crucial role in the polarization at pA collisions.

Other related topics and discussion

(a) Helicity polarization

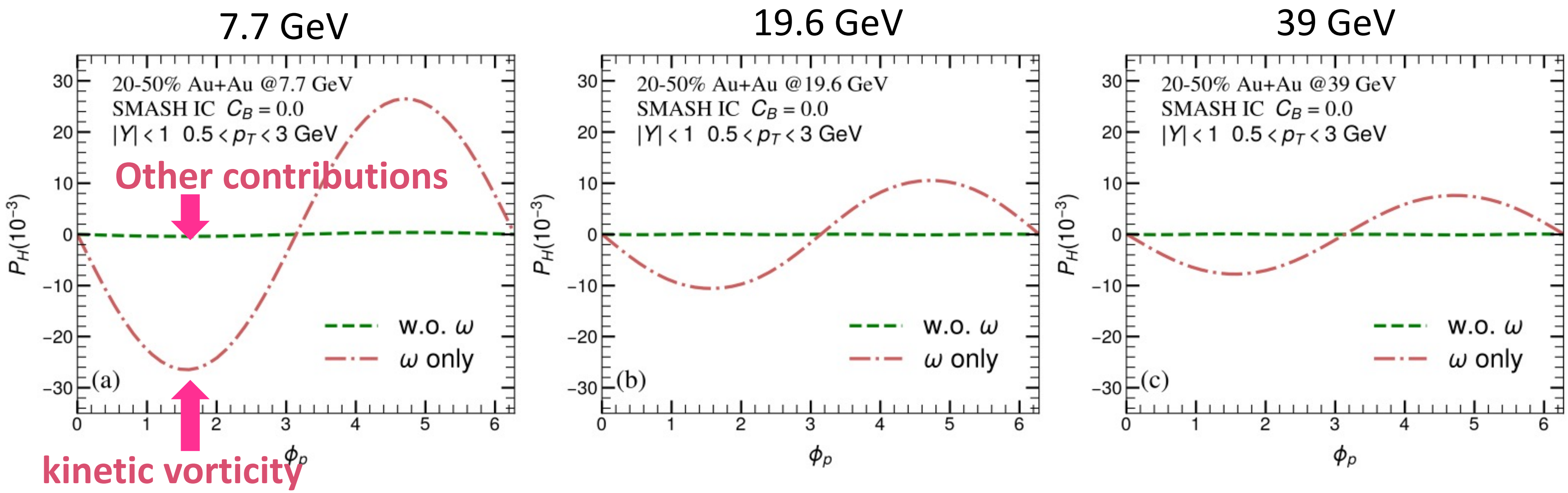
- The original idea for helicity polarization is proposed by Becattini, Buzzegoli, Palermo, Prokhorov, PLB(2021) and Gao, PRD(2021); Yi, Pu, Gao, Yang, PRC (2022) to probe the initial chiral chemical potential.
- Helicity instead of spin is widely-used in high energy spin physics.

$$S^h = \widehat{\mathbf{p}}\cdot \boldsymbol{S}(\mathbf{p}) = \widehat{p}^x \mathcal{S}^x + \widehat{p}^y \mathcal{S}^y + \widehat{p}^z \mathcal{S}^z,$$



Probe fine structure of vorticity by helicity polarization

$$S^h(\mathbf{p}) = S^h_{\mathrm{thermal}} + S^h_{\mathrm{shear}} + S^h_{\mathrm{accT}} + S^h_{\mathrm{chemical}} + S^h_{\mathrm{EB}}$$



- Helicity polarization induced by kinetic vorticity dominates at low energy collisions.

- A possible way to probe the fine structure of kinetic vorticity by mapping the simulations of helicity polarization to the future measurements?
  Yi, Pu, Gao, Yang, PRC (2022); Yi, Wu, Yang, Gao, SP, Qin, PRC(Lett) (2023)

(b) Collisional corrections

- **Collision term with quantum corrections**

  Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD (2019); PRL (2021)

  Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.

  Liu, Mameda, Huang, arXiv:2002.03753.

  Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019); Wang, Guo, Zhuang, EPJC (2021);Wang, Zhuang, arXiv:2105.00915

  Li ,Yee, PRD100, 056022 (2019)

  Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184

  Fang, SP, Yang, PRD (2022)

  Z.Y. Wang, arXiv:2205.09334; Lin, Wang, arXiv:2206.12573

  …

Recent reviews:

  Gao, Ma, SP, Wang, NST 31 (2020) 9, 90

  Gao, Liang, Wang, IJMPA 36 (2021), 2130001

  Hidaka, SP, Yang, Wang, arXiv:2201.07644

(b1) Corrections from self-energies
(b2) Corrections from scattering

(b1) Master equation for Wigner functions

$$\left[\frac{i\hbar}{2}\gamma^{\mu}\nabla_{\mu}+\gamma^{\mu}\Pi_{\mu}-m+\underline{\overline{\Sigma}_{\mathrm{g}}\star}\right]S^{<}(q,X) = -\frac{i\hbar}{2}(\Sigma_{\mathrm{g}}^{>}\star S^{<}-\Sigma_{\mathrm{g}}^{<}\star S^{>}),$$

$$S^{<}\left(-\frac{i\hbar}{2}\gamma^{\mu}\overleftarrow{\nabla}_{\mu}+\gamma^{\mu}\overleftarrow{\Pi}_{\mu}-m\right)+S^{<}\underline{\star\overline{\Sigma}_{\mathrm{g}}} = -\frac{i\hbar}{2}(S^{>}\star\Sigma_{\mathrm{g}}^{<}-S^{<}\star\Sigma_{\mathrm{g}}^{>}),$$

$\star$ denotes the Moyal product

$S^{<}$: Wigner function

$$\overline{\Sigma}_{\mathrm{g}}(q,X) = \Sigma^{\delta}(X)+\mathrm{Re}\Sigma_{\mathrm{g}}^{\mathrm{r}}$$

For a long time, we always neglect the self-energy terms for simplicity. Now, we consider the contributions from them carefully.

Applications to spin polarization

- We consider effects from the thermal QCD background. After a heavy calculation, we get the corrections to polarization vectors from self-energies:

Corrections to polarization induced by:

$$\delta\mathcal{P}^{\mu}_{\text{therm}}(t,\mathbf{q}) = -\frac{\hbar^2}{2mN}\int_{\Sigma} q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{m_f^2 T}{E_q^3}\epsilon^{\mu\nu\alpha\beta}q_\nu \underline{\partial_\alpha\left(\frac{u_\beta}{T}\right)},$$

Thermal vorticity

$$\delta\mathcal{P}^{\mu}_{\text{shear}}(t,\mathbf{q}) = -\frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot d\sigma G_{\omega_1}(E_q,\mathbf{q})\frac{m_f^2}{E_q^3}\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{E_q} \underline{q^\gamma \sigma_{\nu\gamma}},$$

Shear tensor

$$\delta\mathcal{P}^{\mu}_{\text{chem}}(t,\mathbf{q}) = -\frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{C_{\mathrm{F}} g^2 \mu T}{4\pi^2 E_q^2}\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{E_q}\underline{\nabla_\nu\left(\frac{\mu}{T}\right)},$$

Gradient of chemical Potential over temperature

$$\delta\mathcal{P}^{\mu}_{\text{acc}}(t,\mathbf{q}) = \frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{3m_f^2}{E_q^3}\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma \underline{D u_\nu},$$

Fluid acceleration

$$\begin{aligned}\delta\mathcal{P}^{\mu}_{\text{vor}}(t,\mathbf{q}) = \frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot d\sigma \frac{m_f^2}{E_q^2}\Bigg[&\underline{\omega^\mu}\left(4G_{\mathrm{T}}(E_q,\mathbf{q}) - \frac{|q_\perp|^2}{E_q^2}G_{\omega_1}(E_q,\mathbf{q}) + 2G_{\omega_2}(E_q,\mathbf{q})\right) \\ &-\frac{\underline{(\omega\cdot q)}}{E_q}\left(6u^\mu G_{\mathrm{T}}(E_q,\mathbf{q}) + \frac{q_\perp^\mu}{E_q}G_{\omega_1}(E_q,\mathbf{q})\right)\Bigg],\end{aligned}$$

Kinetic vorticity

Shuo Fang, Shi Pu, Di-Lun Yang, PRD (2024), arXiv: 2311.15197

Estimation

It can contribute 30% to the original polarization vector in the case of low momentum.

	$\lvert q_\perp\rvert = 0.5$ GeV	$\lvert q_\perp\rvert = 1.0$ GeV	$\lvert q_\perp\rvert = 2.0$ GeV
$\lvert\delta\mathcal{J}^{5,\mu}_{\mathrm{therm}}/\mathcal{J}^{5,\mu}_{\mathrm{therm,leq}}\rvert$	$\boxed{0.325}$	0.098	0.024
$\lvert\delta\mathcal{J}^{5,\mu}_{\mathrm{shear}}/\mathcal{J}^{5,\mu}_{\mathrm{shear,leq}}\rvert$	0.081	0.028	0.007
$\lvert\delta\mathcal{J}^{5,\mu}_{\mathrm{vor}}/\mathcal{J}^{5,\mu}_{\mathrm{therm,leq}}\rvert$	0.177	0.103	0.030

We have chosen temperature T = 0.165 GeV, chemical potential μ = 0.01 GeV and constituent s quark mass m = 0.3 GeV.

(b2) Spin Boltzmann equations (II)

- **We derive the spin Boltzmann equation incorporating Møller scattering process using hard thermal loop approximations.**

$$p^\mu \partial_\mu f_{\mathrm{A}}^{<}(p) + \hbar \partial_\mu S^{(u),\mu\alpha}(p) \partial_\alpha f_{\mathrm{V}}^{<}(p) \quad = \quad \mathcal{C}_{\mathrm{A}} + \hbar \partial_\mu \left(S_{(u)}^{\mu\alpha} C_{\mathrm{V},\alpha}[f_{\mathrm{V}}^{<}] \right)$$

S. Fang, SP, D.L. Yang, PRD (2022); S. Fang, SP, arXiv:2408.09877

- **Scenario (I): particle distribution function is off-equilibirum**

$$\partial \sim \lambda^{-1} \mathrm{Kn} \ll \lambda^{-1}$$

Kn: Knudsen number
λ: mean free path

- **Scenario (II): particle distribution function is at local equilibrium**

  Similar to standard kinetic theory, e.g. AMY

New corrections to shear induced polarization

- **Scenario (I):**

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I})}(\mathbf{p}) = \delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{shear}} + \delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{chem}} + \mathcal{O}(\hbar^2\partial^2)$$

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{shear}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 g_2(E_{\mathbf{p}})\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \sigma_{\nu\alpha}p^\alpha,$$

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{chem}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 g_1(E_{\mathbf{p}})\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \nabla_\nu \alpha_0.$$

They come from scatterings but do not depend on coupling constant explicitly. They correspond to anomalous spin Hall conductivity in condensed matter.

Also see the similar findings: S. Lin and Z. Wang, arXiv:2406.10003.

New corrections from scattering

Let us start from the kinetic theory for massless fermions.

$$p \cdot \partial f_0 = C_{pp' \to kk'}[\delta f],$$

We consider the system close to the global equilibrium,

$$f = f_0 + \delta f,$$

We can estimate

$$\delta f \sim A p_\mu p_\nu \pi^{\mu\nu}, \quad A \sim 1/C_{pp' \to kk'}[f] \sim 1/e^4,$$

Recalling the current in phase space,

$$j^\mu(p) = p^\mu f + S^{\mu\nu}\partial_\nu f + \int_{p',k,k'} C_{pp' \to kk'}[f]\Delta^\mu,$$

$$j^\mu \sim \int_{p',k,k'} C_{pp' \to kk'}[\delta f]\Delta^\mu \sim \int_{p',k,k'} C_{pp' \to kk'}[A p_\mu p_\nu \pi^{\mu\nu}]\Delta^\mu$$

$$\sim \cancel{e}^4 \frac{1}{\cancel{e}^4} \pi^{\mu\nu} p_\mu p_\nu .$$

It can also be derived by Kubo formula.

Other second order corrections

- **Scenario (II):**

$$\begin{aligned}\delta\mathcal{P}^{\mu}_{(\mathrm{II})}(\mathbf{p}) \quad = \quad & \mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\nabla T}+\mathcal{P}^{\mu}_{(\mathrm{II}),\nabla\omega}+\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\mathrm{chem}}+\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\mathrm{shear}}\\ & +\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{chem}-\nabla\mathrm{T}}+\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{shear}-\nabla\mathrm{T}}+\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{shear}-\mathrm{chem}}+\mathcal{O}(\hbar^{2}\partial^{3})\end{aligned}$$

$$\begin{aligned}
\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\nabla T} &= -\hbar^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot p a^{\mu}_{(\mathrm{II})}\left[d_{2}\left(E_{\mathbf{p}}-d_{1}\frac{1}{\beta_{0}}\right)\omega^{\alpha}\nabla_{\alpha}\beta_{0}-d_{6}\beta_{0}p_{\langle\alpha}p_{\rho\rangle}\omega^{\alpha}\nabla^{\rho}\beta_{0}\right],\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\nabla\omega} &= \hbar^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot p a^{\mu}_{(\mathrm{II})}\left[\left(E_{\mathbf{p}}-d_{1}\frac{1}{\beta_{0}}\right)d_{2}\beta_{0}\nabla^{\alpha}\omega_{\alpha}+d_{6}\frac{1}{2}\beta_{0}^{2}\nabla^{\alpha}\omega^{\rho}p_{\langle\alpha}p_{\rho\rangle}\right],\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\mathrm{chem}} &= \hbar^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot p a^{\mu}_{(\mathrm{II})}\left[\left(E_{\mathbf{p}}-d_{1}\frac{1}{\beta_{0}}\right)d_{3}\beta_{0}\omega^{\alpha}\nabla_{\alpha}\alpha_{0}-d_{8}\beta_{0}^{2}\omega^{\alpha}\nabla^{\rho}\alpha_{0}p_{\langle\alpha}p_{\rho\rangle}\right],\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\omega-\mathrm{shear}} &= -\hbar^{2}\beta_{0}\int_{\Sigma}\mathrm{d}\Sigma\cdot p a^{\mu}_{(\mathrm{II})}\left[-d_{4}\omega^{\rho}\sigma^{\alpha}_{\rho}p_{\langle\alpha\rangle}+d_{9}\beta_{0}^{2}\omega^{\beta}\sigma^{\alpha\lambda}p_{\langle\beta}p_{\alpha}p_{\lambda\rangle}\right]\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{chem}-\nabla\mathrm{T}} &= \hbar^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot p a^{\mu}_{(\mathrm{II})}d_{5}\epsilon^{\rho\nu\alpha\beta}u_{\beta}\nabla_{\nu}\alpha_{0}\nabla_{\rho}\beta_{0}p_{\langle\alpha\rangle},\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{shear}-\nabla\mathrm{T}} &= \hbar^{2}\beta_{0}\int_{\Sigma}\mathrm{d}\Sigma\cdot p a^{\mu}_{(\mathrm{II})}d_{6}\epsilon^{\beta\nu\sigma\rho}\sigma^{\alpha}_{\beta}u_{\sigma}\nabla_{\nu}\beta_{0}p_{\langle\alpha}p_{\rho\rangle},\\
\mathcal{P}^{\mu}_{(\mathrm{II}),\mathrm{shear}-\mathrm{chem}} &= -\hbar^{2}\beta_{0}^{2}\int_{\Sigma}\mathrm{d}\Sigma\cdot p a^{\mu}_{(\mathrm{II})}d_{7}\epsilon^{\mu\nu\sigma\rho}\sigma^{\alpha}_{\mu}u_{\sigma}\nabla_{\nu}\alpha_{0}p_{\langle\alpha}p_{\rho\rangle},
\end{aligned}$$

Corrections from space-time dependent EM fields

- **We derived the corrections to Wigner function and polarization from varying EM fields.**

$$\mathcal{S}^{\mu}_{(2)} = \frac{1}{8mN}\sum_{m=1,2,3}\int d\Sigma^{\sigma}p_{\sigma}X^{\mu}_{(m)}f_5^{(m)},$$

$$\begin{aligned}
X^{\mu}_{(0)} &= \frac{1}{p_u^3}\left(u^{\mu}u^{\nu}u_{\lambda}-\frac{1}{p_u}p^{\mu}u^{\nu}u_{\lambda}-\frac{2}{p_u}u^{\mu}u^{\nu}p_{\lambda}-\frac{1}{2p_u^2}u^{\mu}p^{\nu}p_{\lambda}+\frac{1}{p_u^2}p^{\mu}u^{\nu}p_{\lambda}+u_{\lambda}g^{\mu\nu}\right)F_{\nu\rho}F^{\lambda\rho},\\
X^{\mu}_{(1)} &= \frac{1}{p_u^2}\left(\frac{1}{2}p^{\mu}u^{\nu}u_{\lambda}+u^{\mu}u^{\nu}p_{\lambda}-\frac{1}{4p_u}u^{\mu}p^{\nu}p_{\lambda}-\frac{1}{2p_u}p^{\mu}u^{\nu}p_{\lambda}-p_u g^{\mu\nu}u_{\lambda}\right)(F_{\nu\gamma}\Omega^{\lambda\gamma}+\Omega_{\nu\gamma}F^{\lambda\gamma})\\
&\quad+\frac{\beta}{p_u^3}\left(p^{\mu}u^{\nu}u_{\lambda}+2u^{\mu}u^{\nu}p_{\lambda}-\frac{1}{p_u}u^{\mu}p_{\mu}p_{\lambda}-\frac{2}{p_u}p^{\mu}u^{\nu}p_{\lambda}+\frac{1}{2p_u^2}p^{\mu}p^{\nu}p_{\lambda}-2p_u g^{\mu\nu}u_{\lambda}+g^{\mu\nu}p_{\lambda}\right)F_{\nu\rho}F^{\lambda\rho}
\end{aligned}$$

Corrections for 2nd order constant EM fields

$$\begin{aligned}
X^{\mu}_{(2)} &= \frac{1}{p_u}\left(\frac{1}{2}u^{\mu}p^{\nu}p_{\lambda}+p^{\mu}u^{\nu}p_{\lambda}-p_u g^{\mu\nu}p_{\lambda}\right)\Omega_{\nu\rho}\Omega^{\lambda\rho}\\
&\quad+\frac{\beta}{p_u^2}\left(p^{\mu}u^{\nu}p_{\lambda}+\frac{1}{2}u^{\mu}p^{\nu}p_{\lambda}-\frac{1}{4p_u}p^{\mu}p^{\nu}p_{\lambda}-p_u g^{\mu\nu}p_{\lambda}\right)(F_{\nu\gamma}\Omega^{\lambda\gamma}+\Omega_{\nu\gamma}F^{\lambda\gamma})\\
&\quad+\frac{\beta^2}{2p_u^3}\left(u^{\mu}p_{\mu}p_{\lambda}+2p^{\mu}u^{\nu}p_{\lambda}-\frac{1}{p_u}p^{\mu}p_{\mu}p_{\lambda}-2p_u g^{\mu\nu}p_{\lambda}\right)F_{\nu\rho}F^{\lambda\rho},\\
X^{\mu}_{(3)} &= \frac{\beta}{2p_u}p^{\mu}p^{\nu}p_{\lambda}\Omega_{\nu\rho}\Omega^{\lambda\rho}+\frac{\beta^2}{4p_u^2}p^{\mu}p^{\nu}p_{\lambda}(F_{\nu\gamma}\Omega^{\lambda\gamma}+\Omega_{\nu\gamma}F^{\lambda\gamma})+\frac{\beta^3}{3p_u^3}p^{\mu}p^{\nu}p_{\lambda}F_{\nu\rho}F^{\lambda\rho}.
\end{aligned}$$

$$\mathcal{S}^{\mu}_{\partial,\mathrm{EM}} = \frac{1}{8mN}\sum_{m=0,1,2,3}\int d\Sigma^{\sigma}p_{\sigma}Y^{\mu}_{(m)}f_5^{(m)},$$

Corrections for varying EM fields

$$\begin{aligned}
Y^{\mu}_{(0)} &= -\frac{2}{3p_u^2}\left(u_{\lambda}u_{\nu}-\frac{1}{2p_u}u_{\lambda}p_{\nu}-\frac{1}{2p_u}p_{\lambda}u_{\nu}\right)\partial^{\lambda}F^{\mu\nu}\\
&\quad+\frac{1}{3p_u^2}\left(2u^{\mu}u^{\nu}-\frac{1}{p_u}p^{\mu}u^{\nu}-\frac{1}{p_u}u^{\mu}p^{\nu}\right)\partial^{\lambda}F_{\lambda\nu},\\
Y^{\mu}_{(1)} &= +\frac{2\beta}{3p_u}\left(u_{\lambda}u_{\nu}-\frac{1}{p_u}u_{\lambda}p_{\nu}-\frac{1}{p_u}p_{\lambda}u_{\nu}+\frac{1}{2p_u^2}p_{\lambda}p_{\nu}\right)\partial^{\lambda}F^{\mu\nu}\\
&\quad+\frac{\beta}{6p_u}\left(u^{\mu}u^{\nu}+\frac{4}{p_u}p^{\mu}u^{\nu}+\frac{4}{p_u}u^{\mu}p^{\nu}-\frac{2}{p_u^2}p^{\mu}p^{\nu}\right)\partial^{\lambda}F_{\lambda\nu}\\
&\quad-\frac{\beta}{3p_u^2}\left(u^{\mu}u^{\lambda}p^{\nu}-\frac{1}{2p_u}p^{\mu}u^{\lambda}p^{\nu}-\frac{1}{2p_u}u^{\mu}p^{\lambda}p^{\nu}\right)u^{\rho}\partial_{\lambda}F_{\nu\rho}\\
&\quad\frac{4\beta}{3p_u}\partial_{\lambda}F^{\mu\lambda}+\frac{\beta}{3p_u}u^{\nu}u^{\rho}\partial^{\mu}F_{\nu\rho},
\end{aligned}$$

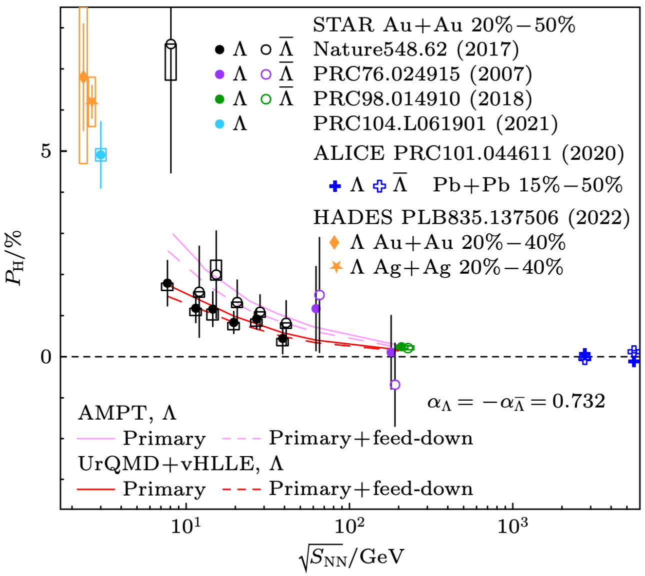
$$\begin{aligned}
Y^{\mu}_{(2)} &= -\frac{\beta^2}{3}\left(u_{\lambda}u_{\nu}-\frac{2}{p_u}u_{\lambda}p_{\nu}+\frac{1}{p_u^2}p_{\lambda}p_{\nu}\right)\partial^{\lambda}F^{\mu\nu}\\
&\quad+\frac{\beta^2}{6}\left(\frac{1}{p_u^2}p^{\mu}u^{\nu}+\frac{2}{p_u^2}p^{\mu}p^{\nu}\right)\partial^{\lambda}F_{\lambda\nu}+\frac{\beta^2}{3p_u}p^{\nu}u^{\rho}\partial^{\mu}F_{\nu\rho}\\
&\quad+\frac{\beta^2}{3p_u}\left(u^{\mu}u^{\lambda}p^{\nu}-\frac{1}{p_u}p^{\mu}u^{\lambda}p^{\nu}-\frac{1}{p_u}u^{\mu}p^{\lambda}p^{\nu}+\frac{1}{2p_u^2}p^{\mu}p^{\lambda}p^{\nu}\right)u^{\rho}\partial_{\lambda}F_{\nu\rho},\\
Y^{\mu}_{(3)} &= \frac{\beta^3}{3p_u}\left(p^{\mu}u^{\lambda}p^{\nu}-\frac{1}{2p_u}p^{\mu}p^{\lambda}p^{\nu}\right)u^{\rho}\partial_{\lambda}F_{\nu\rho}.
\end{aligned}$$

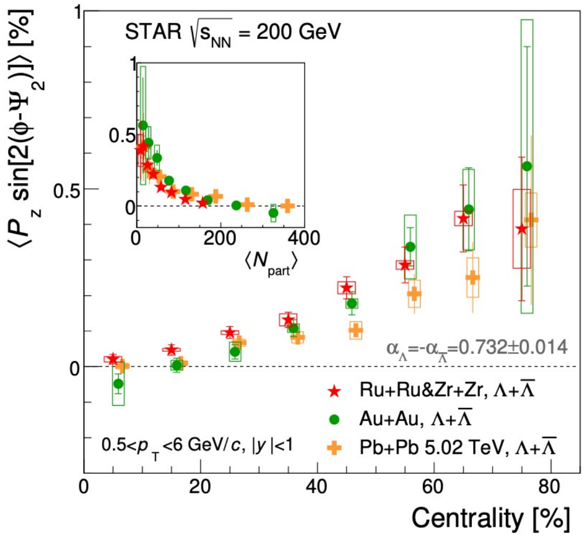
S. Z. Yang, J.H. Gao, SP, arXiv: 2409.00456

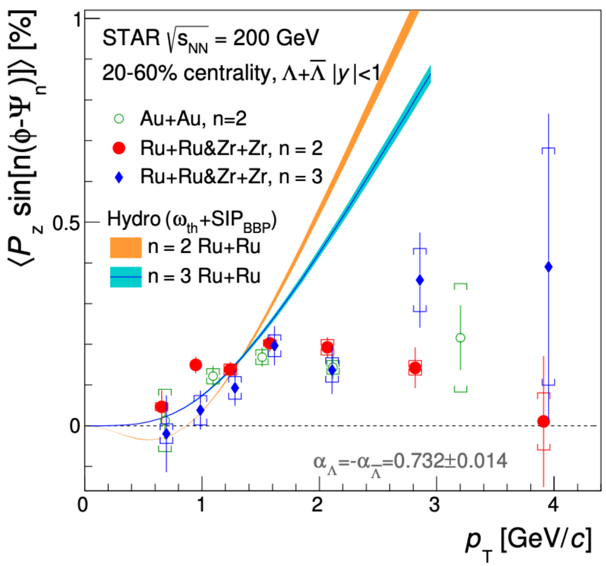
Summary and outlook

Summary (I)

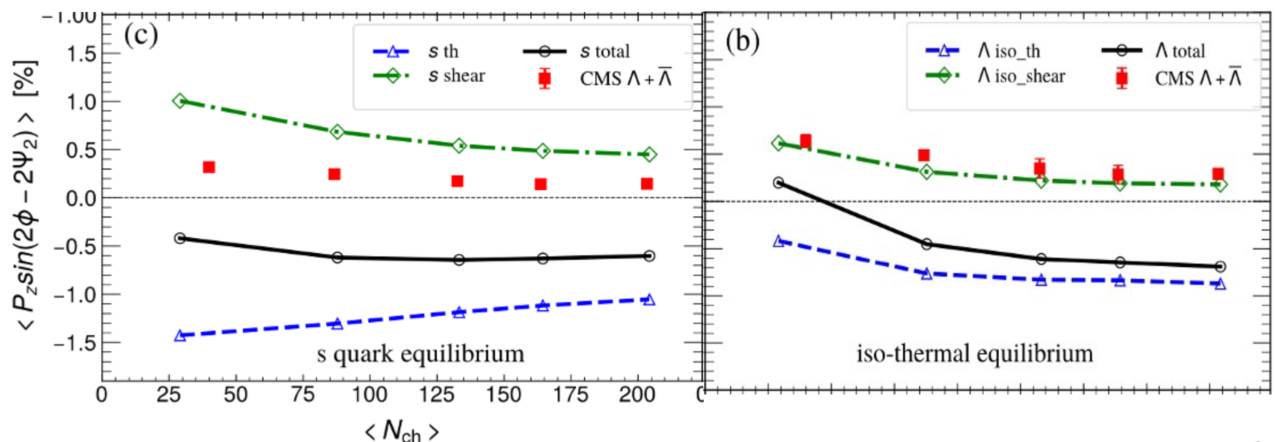
When will the polarization stop growing?







Smaller v2 gives a larger polarization along beam direction ?
Sensitive to initial conditions?



Summary (II)

New corrections to shear induced polarization come from scatterings but do not depend on coupling constant explicitly.

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I})}(\mathbf{p}) = \delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{shear}} + \delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{chem}} + \mathcal{O}(\hbar^2\partial^2)$$

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{shear}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 g_2(E_{\mathbf{p}})\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \sigma_{\nu\alpha}p^\alpha,$$

$$\delta\mathcal{P}^{\mu}_{(\mathrm{I}),\mathrm{chem}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 g_1(E_{\mathbf{p}})\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \nabla_\nu \alpha_0.$$

Thank you for your time!

Any comments and suggestions are welcome!

Backup

Puzzle : T-odd/T-even VS dissipative/non-dissipative

Are shear induced polarization or spin alignment non-dissipative?

Q1: If the coefficient is T-even, it is non-dissipative.

CME $\mathbf{j} \sim C\mathbf{B}$ $\xrightarrow{\text{Taking T transformation}}$ $\mathbf{j} \to -\mathbf{j}$, $\mathbf{B} \to -\mathbf{B}$ C: T-even ✓

Spin polarization vector $\mathcal{S}^i \sim C^{ijk}(\partial_j u_k + \partial_k u_j)$ $\longrightarrow$ $\mathcal{S}^i \to -\mathcal{S}^i$, $\partial_j u_k \to -\partial_j u_k$ C: T-even Non-dissipative?

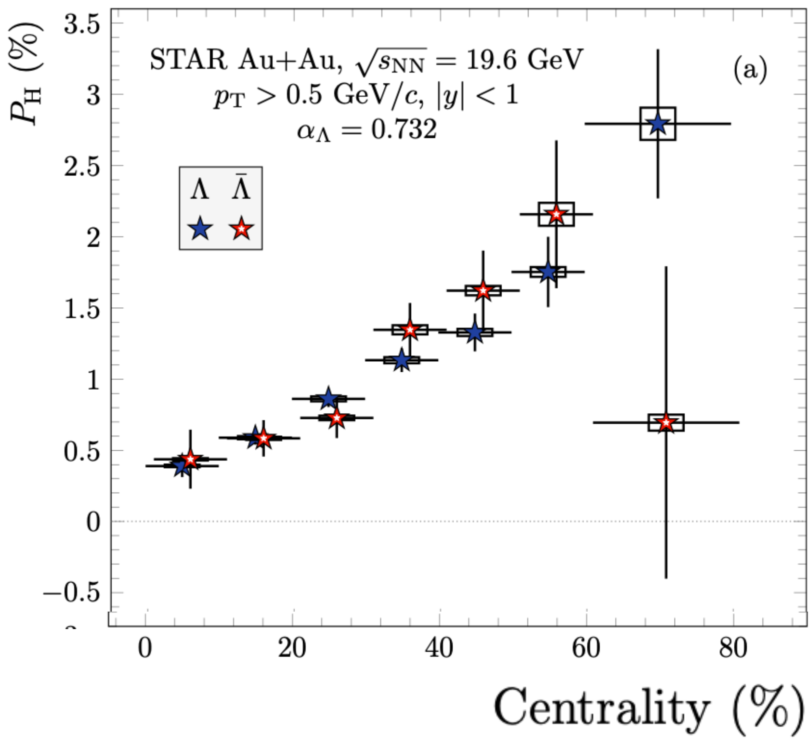
Spin alignment $\epsilon^i(\lambda)\epsilon^{*j}(\lambda')\rho_{\lambda\lambda'} \sim a\pi^{ij}$ $\longrightarrow$

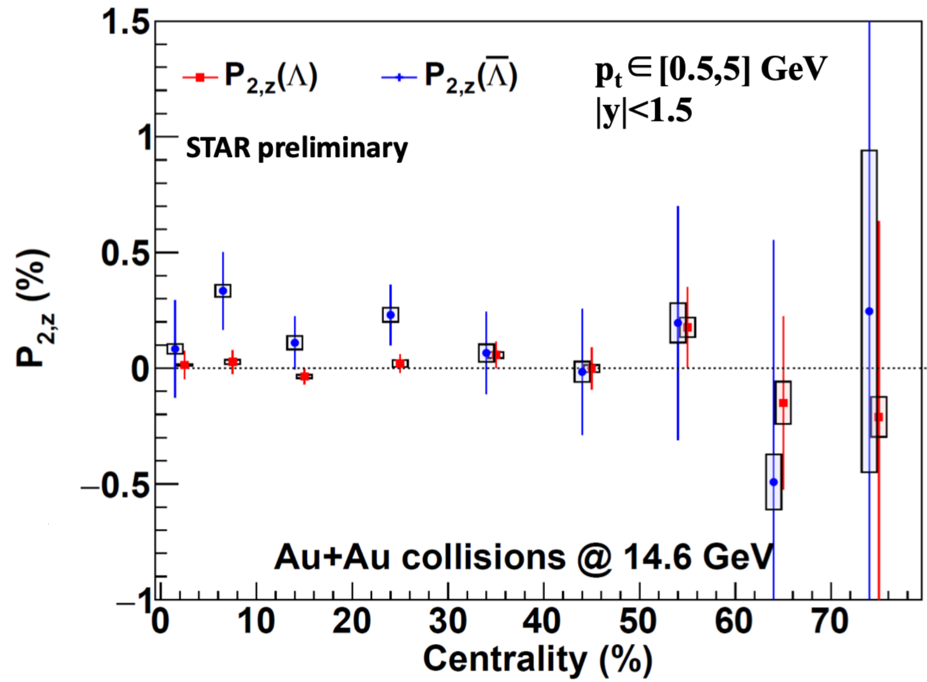
$$\rho_{\lambda\lambda'} \to (-1)^{\lambda+\lambda'}\rho_{\lambda\lambda'}$$
$$\epsilon_\mu^{s*}(\mathbf{p}) \to -(-1)^s \tilde{\delta}_\mu^\alpha \epsilon_\alpha^{-s*}(-\mathbf{p})$$
$$\pi^{ij} \to -\pi^{ij}$$

For ρ_{00}, the coefficient “a” is T-odd. Dissipative?

In Zubarev approach, the non-dissipative means the results does NOT depend on hypersurface. But, shear tensor comes from local equilibrium operators and should always depends on hypersurface. So, shear induced something is always dissipative?

Local polarization VS centrality





STAR, Phys.Rev.C 108 (2023) 1, 014910

Hu's talk at SQM 2024