IN-MEDIUM EFFECTS FROM S-MATRIX

POK MAN LO (盧博文)

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ECT* TRENTO WORKSHOP
13.09.2024

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969).

R. Venugopalan and M. Prakash,
Nucl. Phys. A546, 718 (1992).

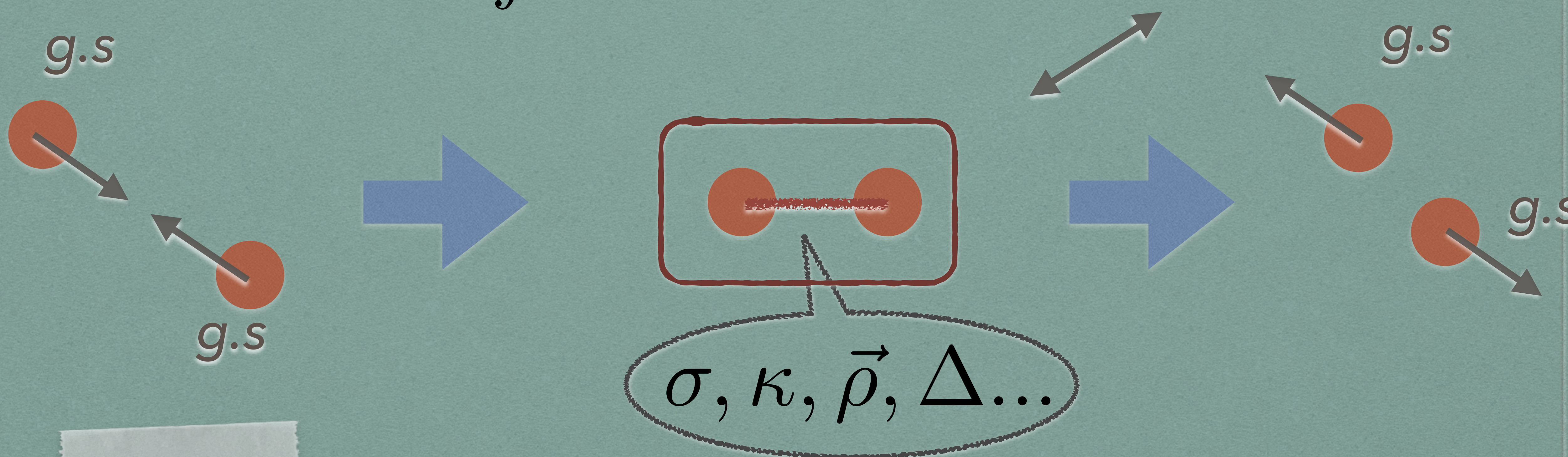
(study notes)

PML, EPJC **77** no.8 533 (2017)

PML PRD **102**, 034038 (2020)

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE \, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr} \, (\delta_E) \, .$$



+ repulsions

PWA

X

S-matrix thermo.

$$\delta \longrightarrow \mathcal{Q}(M) \equiv \frac{1}{2} \mathrm{Im} \, (\mathrm{tr} \ln \mathrm{S})$$

Thermodynamics & Scattering

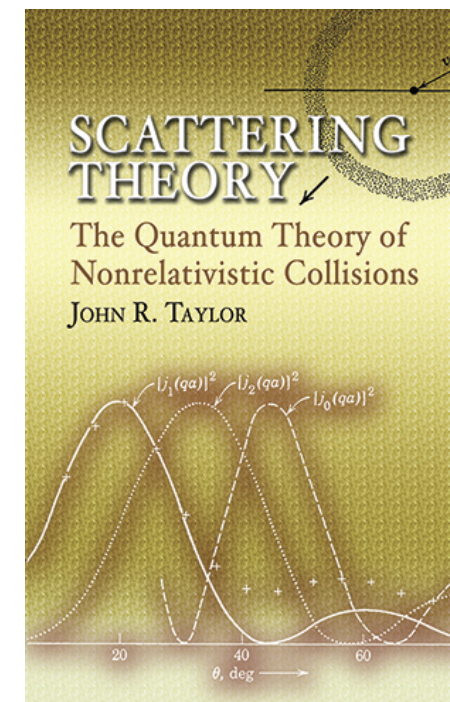
$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

lousy derivation

$$\operatorname{tr} e^{-\beta \hat{H}} \to \int \frac{dE}{2\pi} e^{-\beta E} \operatorname{tr} \underline{2\pi \delta(E - \hat{H})}$$

$$-2\,\mathrm{Im} \frac{1}{E - \hat{H} + i\delta} = \frac{1}{i} \frac{\partial}{\partial E} \ln\left(G^{*\,-1} G \right)$$

subtract the free part $\quad - \frac{1}{i} \frac{\partial}{\partial E} \ln\left(G_0^{*\,-1} G_0 \right)$



$$S_E = G_0^* G^{*\,-1} G G_0^{-1} \to I - 2\pi i\, \delta(E - \hat{H}_0) \times T_E$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr}\left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

QM *N-body* *QFT*

$$\mathrm{tr}\{...\} \iff \int d^3q \, \langle q | \ldots | q \rangle \longrightarrow \int (d\,k)\, \langle k_1 k_2 | \ \ldots \ | k_1 k_2 \rangle$$

Fock Space Expansion

$$\int (dk)\,(\cdots) \to \int \frac{d^3p_1}{(2\pi)^3} \frac{1}{2E_1} \frac{d^3p_2}{(2\pi)^3} \frac{1}{2E_2} (\cdots)$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

$$b_{\pi\pi}\xi_\pi^2 + b_{\pi K}\xi_\pi\xi_K + b_{\pi N}\xi_\pi\xi_N + b_{\pi\eta}\xi_\pi\xi_\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \ldots$$

$$b_{\pi N} = 2 \times b_{\pi N}^{I=1/2} + 4 \times b_{\pi N}^{I=3/2}$$

orbital L:
S, P, D, F, etc..

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

$a_S = 20\, fm$

$r \approx 0.0727$ *LHC*

$r \approx 0.36$ *HADES*

$r \approx 1.92$

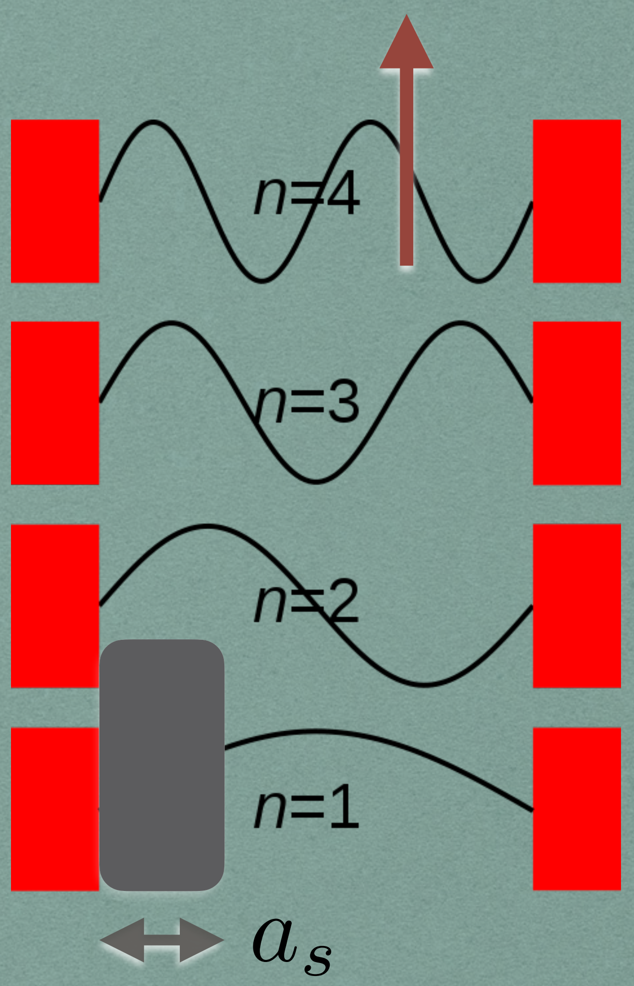
$T = 60\, MeV$

$\mu_B = 700, 800\, MeV$

REPULSION AND RESONANCES

PHASE SHIFT AND DENSITY OF STATES

particle in a box



$$\psi \sim \sin(k^{(0)}x) \qquad k^{(0)} = \frac{n\pi}{L}$$

in the presence of a scattering potential

$$\psi \sim \sin(kx + \delta(k))$$

density of states

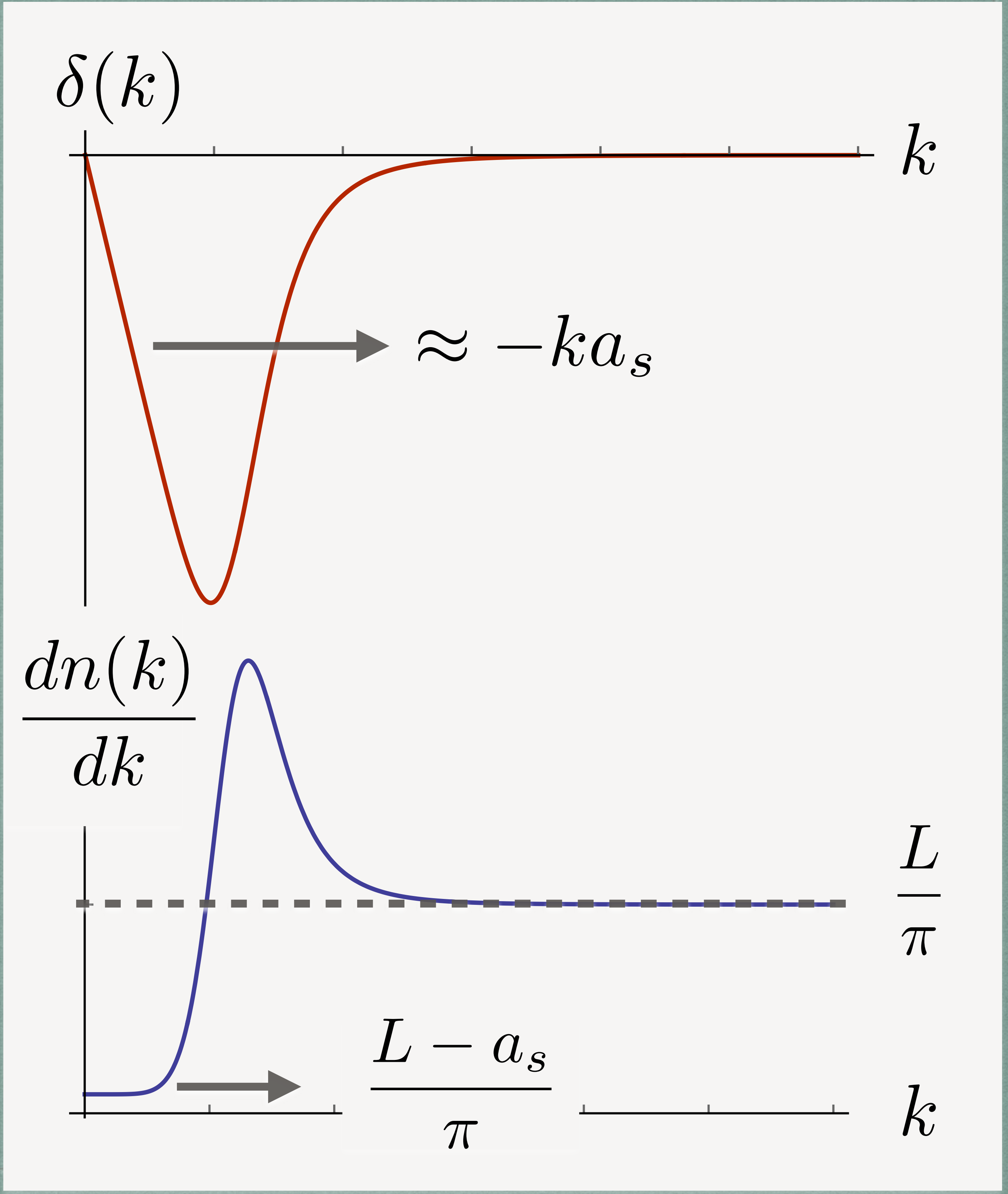
$$kL + \delta(k) = n\pi \quad \Rightarrow \quad \frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

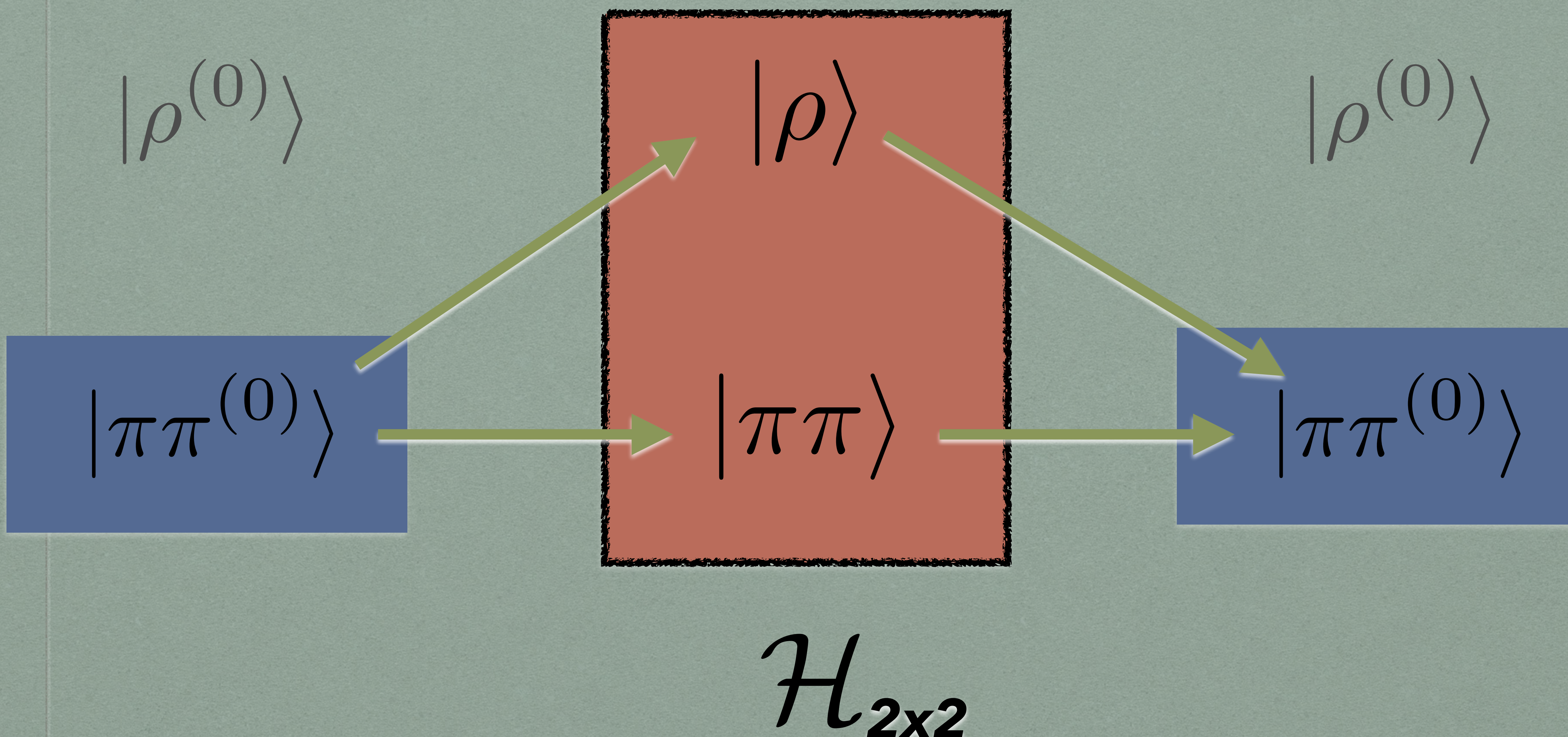
change in d.o.s. due to int.

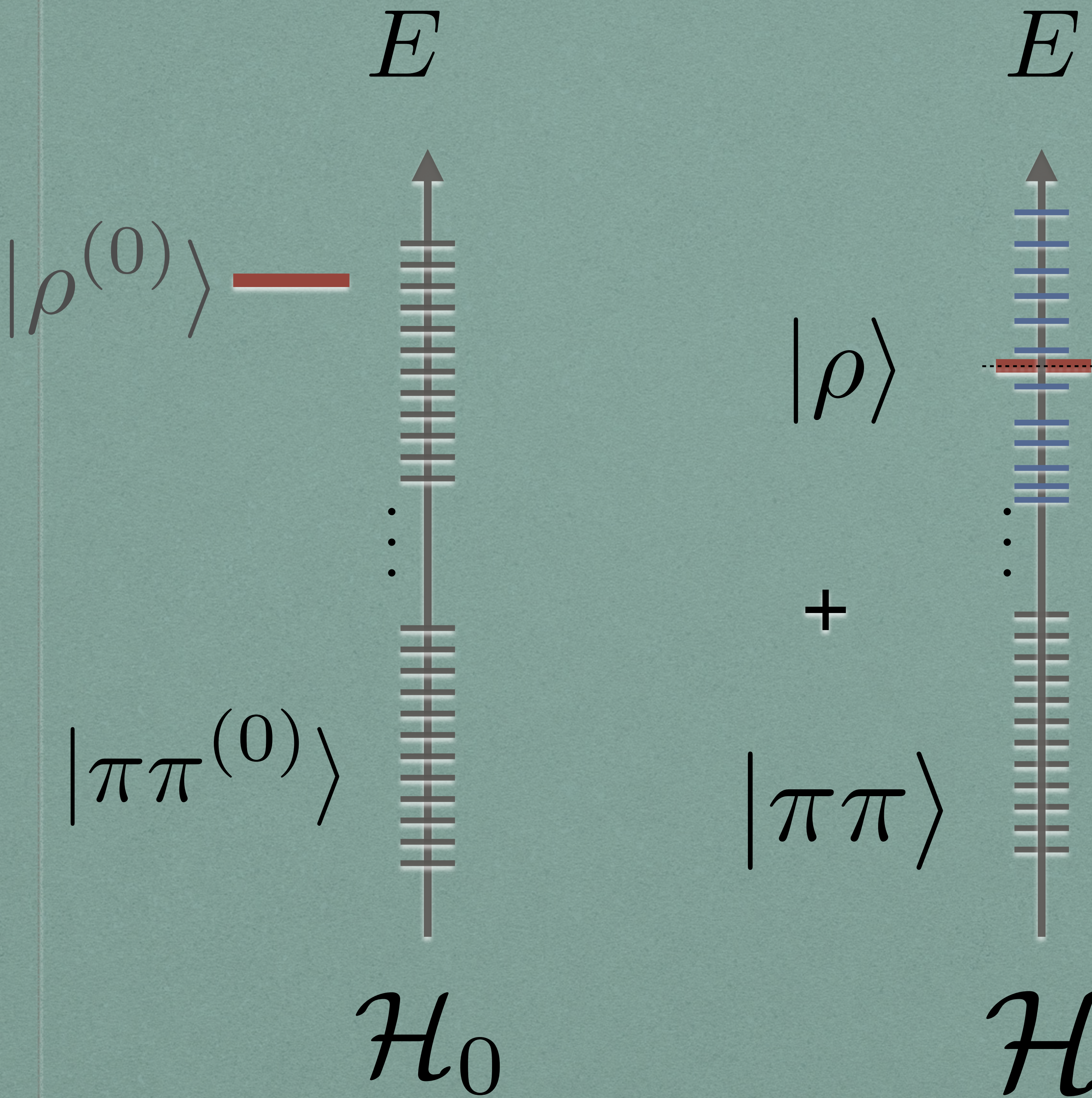
Effect of repulsive interaction: pushing states from low k to high k

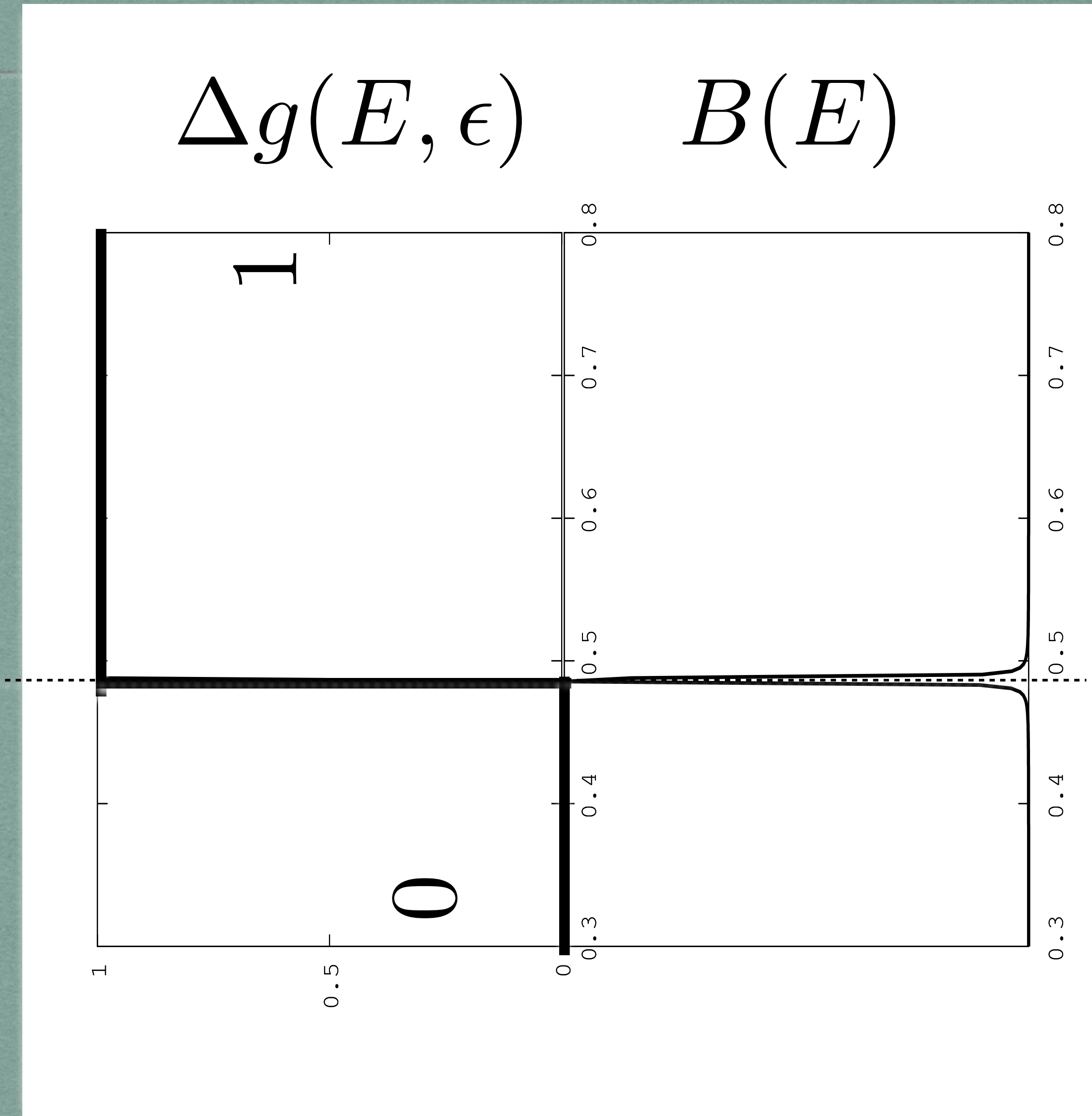


phase shift and d.o.s. (schematics)

SCATTERING THEORY VS HAMILTONIAN (LEE MODEL)



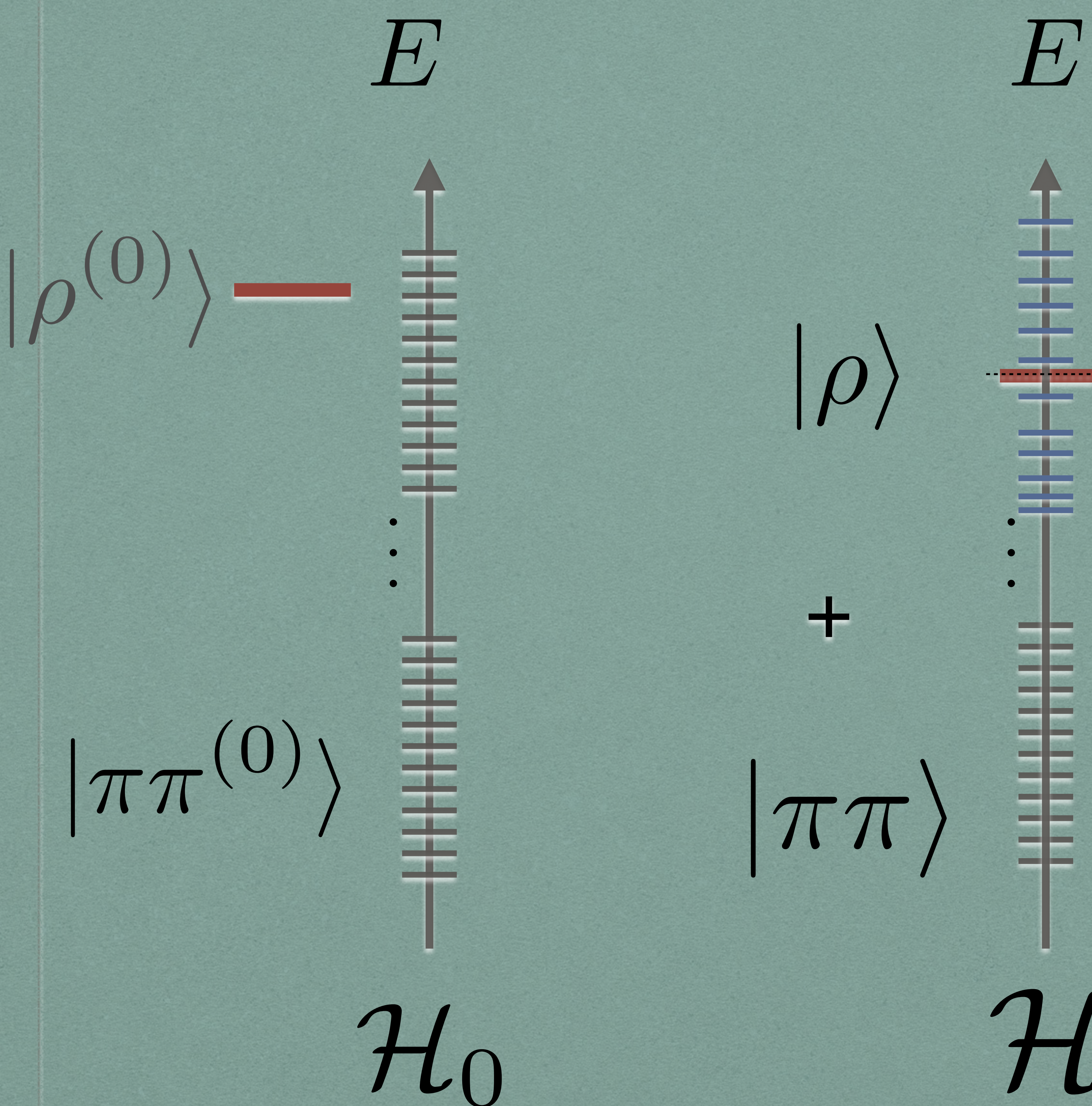


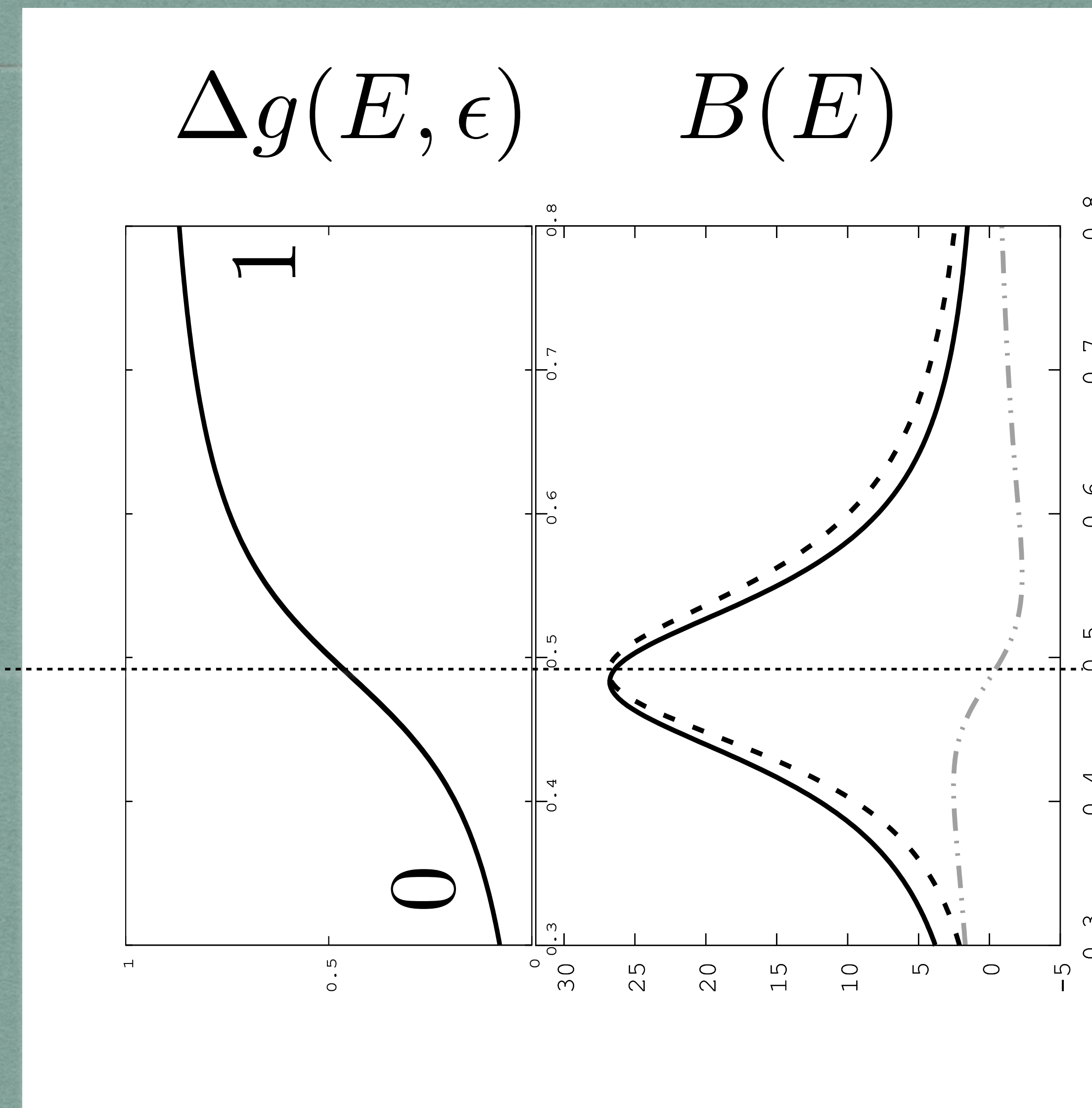


$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE}\Delta g(E,\epsilon)$$

PML, Francesco Giocosa EPJC **79** (2019)





$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E,\epsilon)$$

$$= A_\rho + \Delta A_{\pi\pi}$$

$\mathrm{Tr}\, e^{-\beta \mathcal{H}_0}$ *vs* $\mathrm{Tr}\, e^{-\beta \mathcal{H}}$

PML, Francesco Giocosa EPJC **79** (2019)

W. Weinhold & B. Friman, Phys. Lett. B 433, 236 (1998).

PHYSICS OF B

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

physical interpretation:

$$B = 2\,\frac{\partial}{\partial E}\delta$$

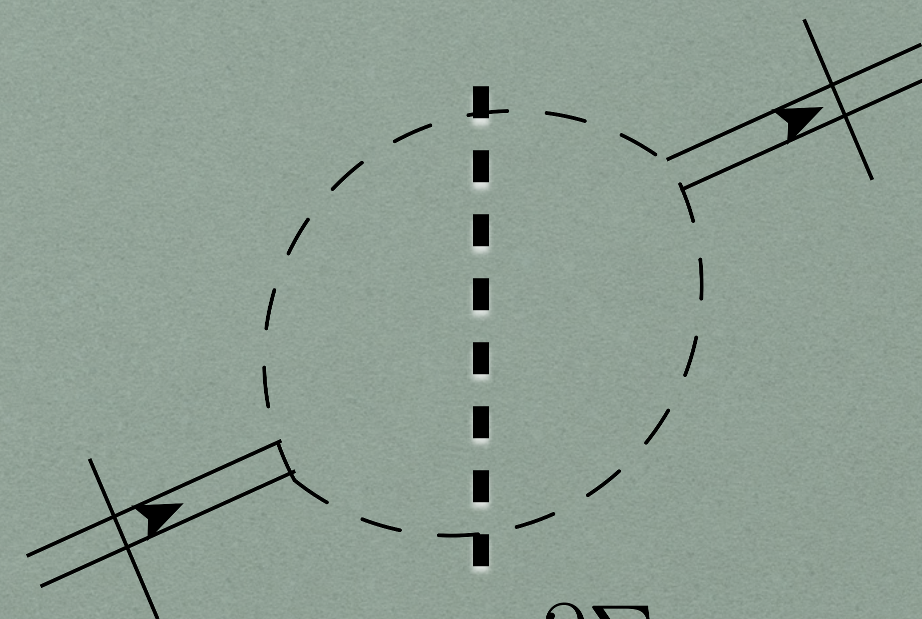
$$= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1}$$

$$= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E}G_\rho]$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

$$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$$ pipi -> pipi

contribution from correlated pi pi pair



$$\frac{\partial \Sigma_\rho}{\partial E}$$

PHYSICS OF B

to rho or not to rho?
that's out of the question!

resonance's picture:

$$B(E) = A_\rho(E) + \Delta A_{\pi\pi}$$

rho

scattering picture:

$$B_1 = \frac{\partial}{\partial E} \mathrm{Tr}\, \hat{t}_{\mathrm{re}}$$

$$B_2 = \frac{1}{2} \mathrm{Im}\, \mathrm{Tr}\, \hat{t}^\dagger \overleftrightarrow{\partial_E}\, \hat{t}$$

pipi -> pipi

$$\delta = -\mathrm{Im}\, \mathrm{T}$$

$$B = 2\, \frac{\partial}{\partial E} \delta$$

$$= -2\, \mathrm{Im} \frac{\partial}{\partial E}$$

$$= -2\, \mathrm{Im}[G$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

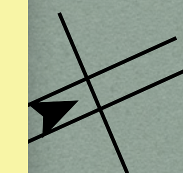
$$-\frac{\partial}{\partial E} \int d\phi_E\, T_{\mathrm{re}}$$

pipi -> pipi

on:

m

air



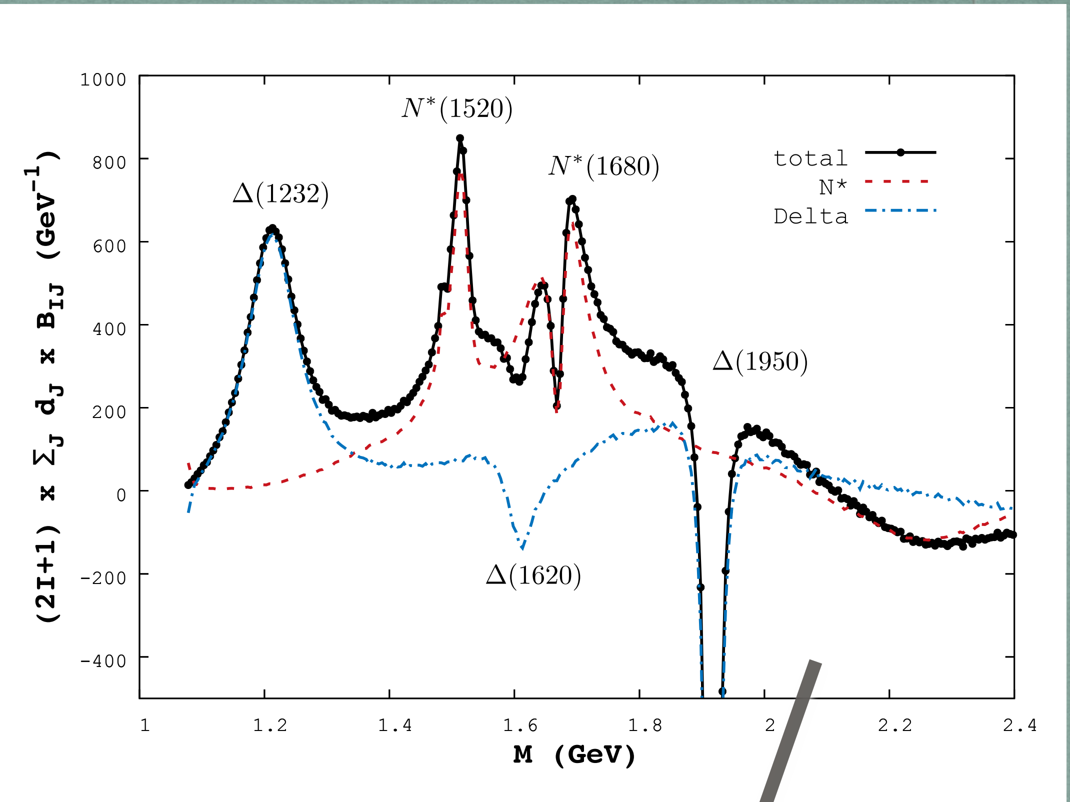
$$\frac{\partial \Sigma_\rho}{\partial E}$$

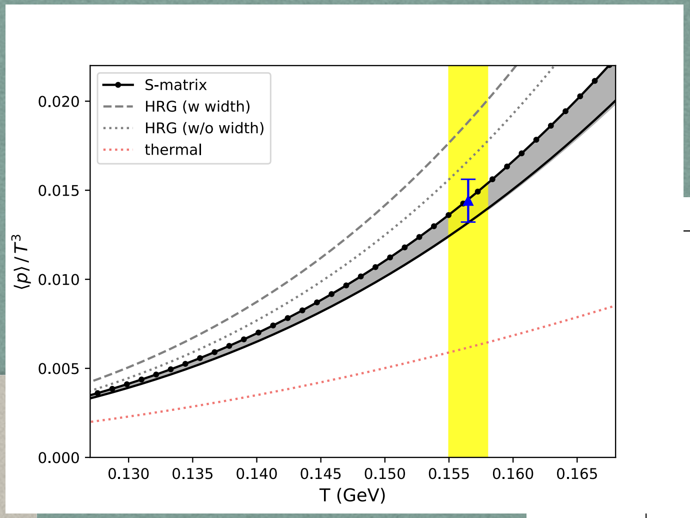
THE S-MATRIX PROGRAM

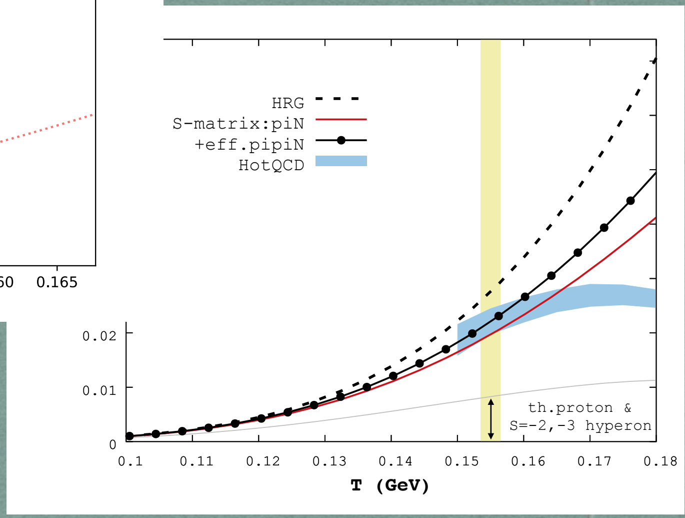
step 1: build a model for S-matrix

step 2: adjust model parameters to match scattering experiments

board resonances & thresholds
coupled channel effects
energy dependent branchings







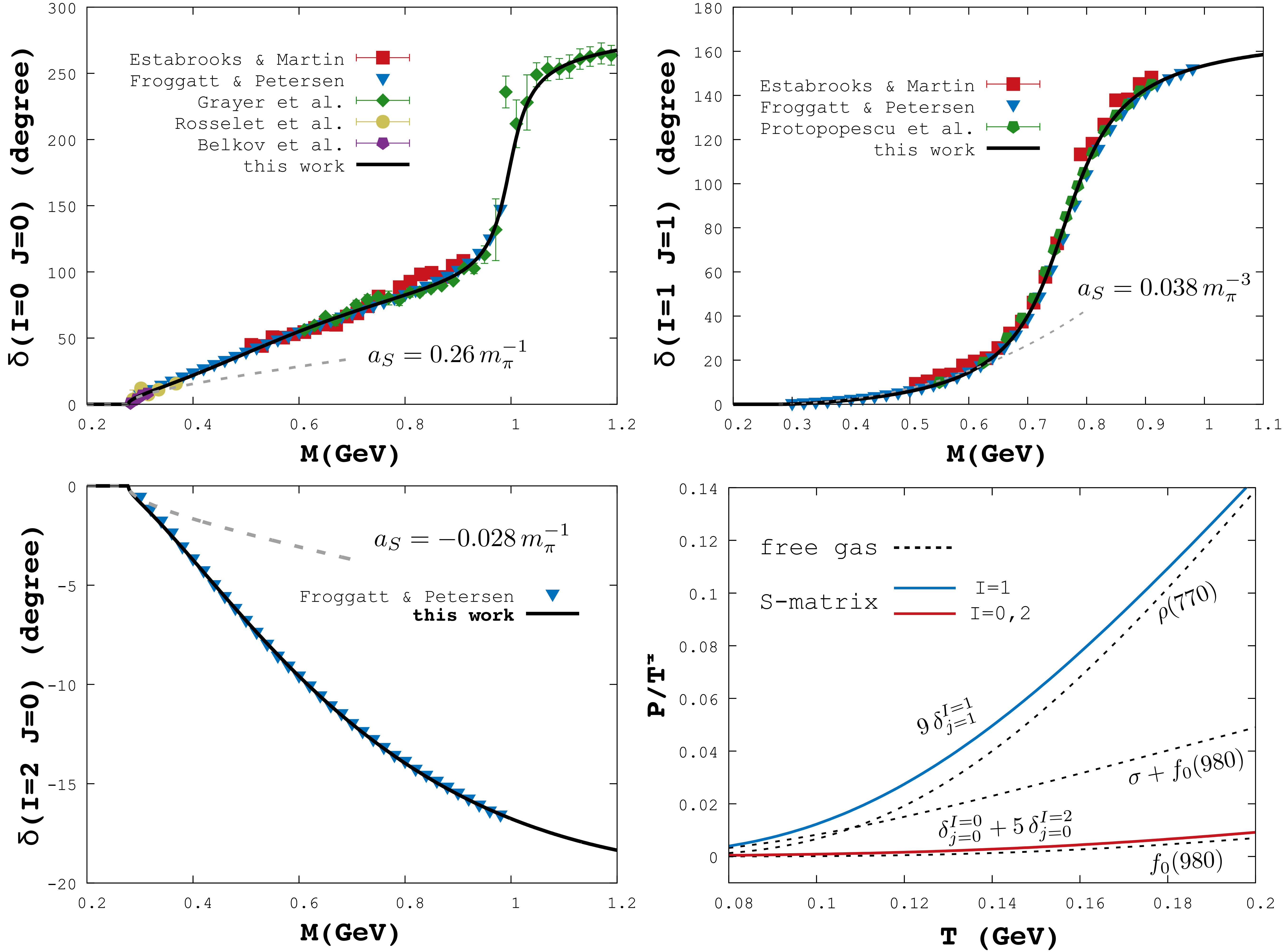
step 3: compute thermodynamics

HICs & LQCD

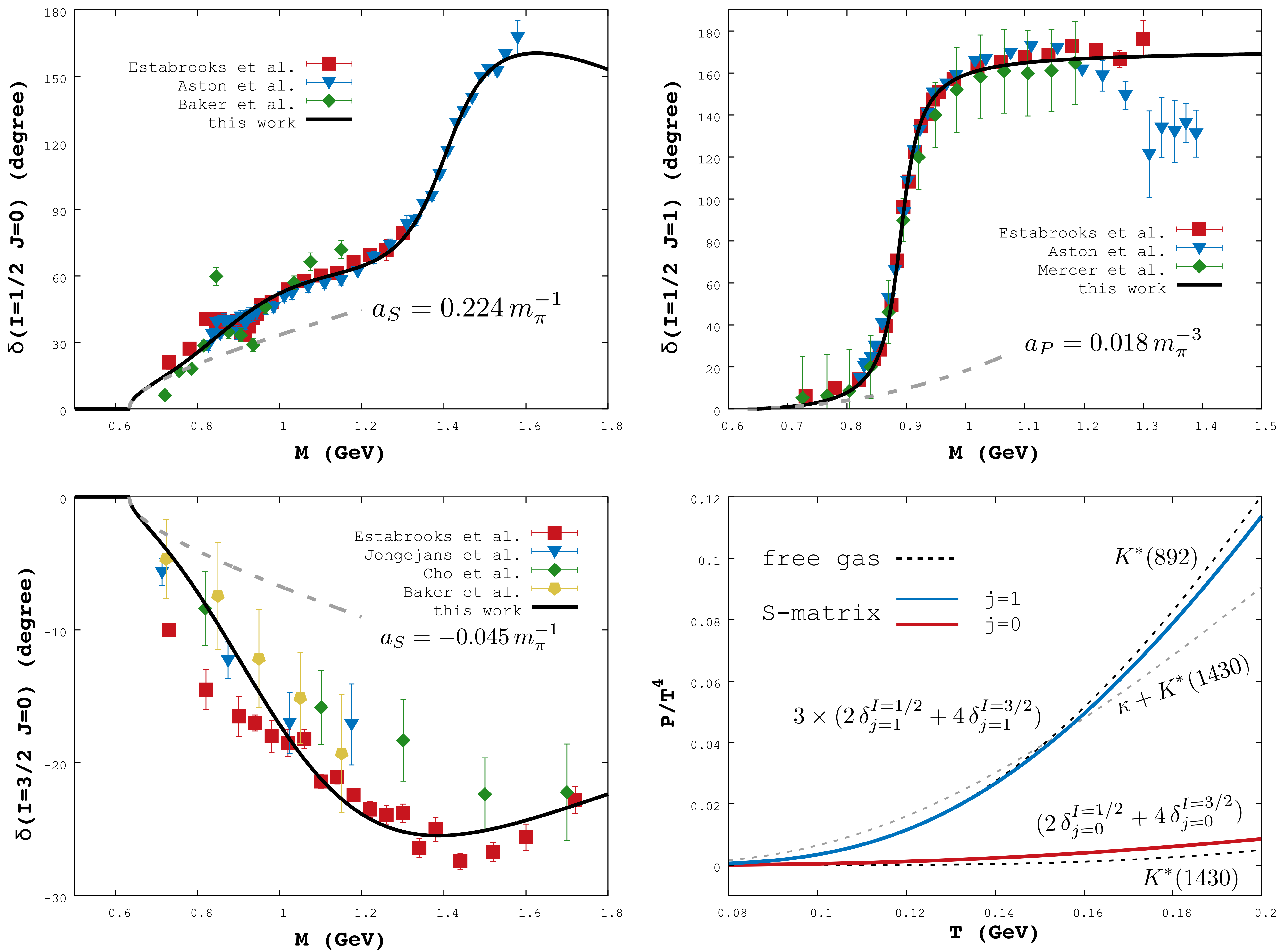
THEORETICAL ISSUES

muB = 0 @T = 155 MeV

- LHC conditions = pion rich: p = pbar; $\langle \pi \rangle / \langle p \rangle \approx 15$
  Need to Take Pions Seriously!
  NN is heavily (Boltzmann) suppressed compared to pi N
- Issues of Missing States
- In-medium Effects from S-matrix



δ(I=0 J=0) (degree)
Estabrooks & Martin
Froggatt & Petersen
Grayer et al.
Rosselet et al.
Belkov et al.
this work
$a_S = 0.26\, m_\pi^{-1}$
M(GeV)
δ(I=1 J=1) (degree)
Estabrooks & Martin
Froggatt & Petersen
Protopopescu et al.
this work
$a_S = 0.038\, m_\pi^{-3}$
M(GeV)
δ(I=2 J=0) (degree)
$a_S = -0.028\, m_\pi^{-1}$
Froggatt & Petersen
this work
M(GeV)
P/T⁴
free gas
S-matrix
I=1
I=0,2
$\rho(770)$
$9\,\delta_{j=1}^{I=1}$
$\sigma + f_0(980)$
$\delta_{j=0}^{I=0} + 5\,\delta_{j=0}^{I=2}$
$f_0(980)$
T (GeV)



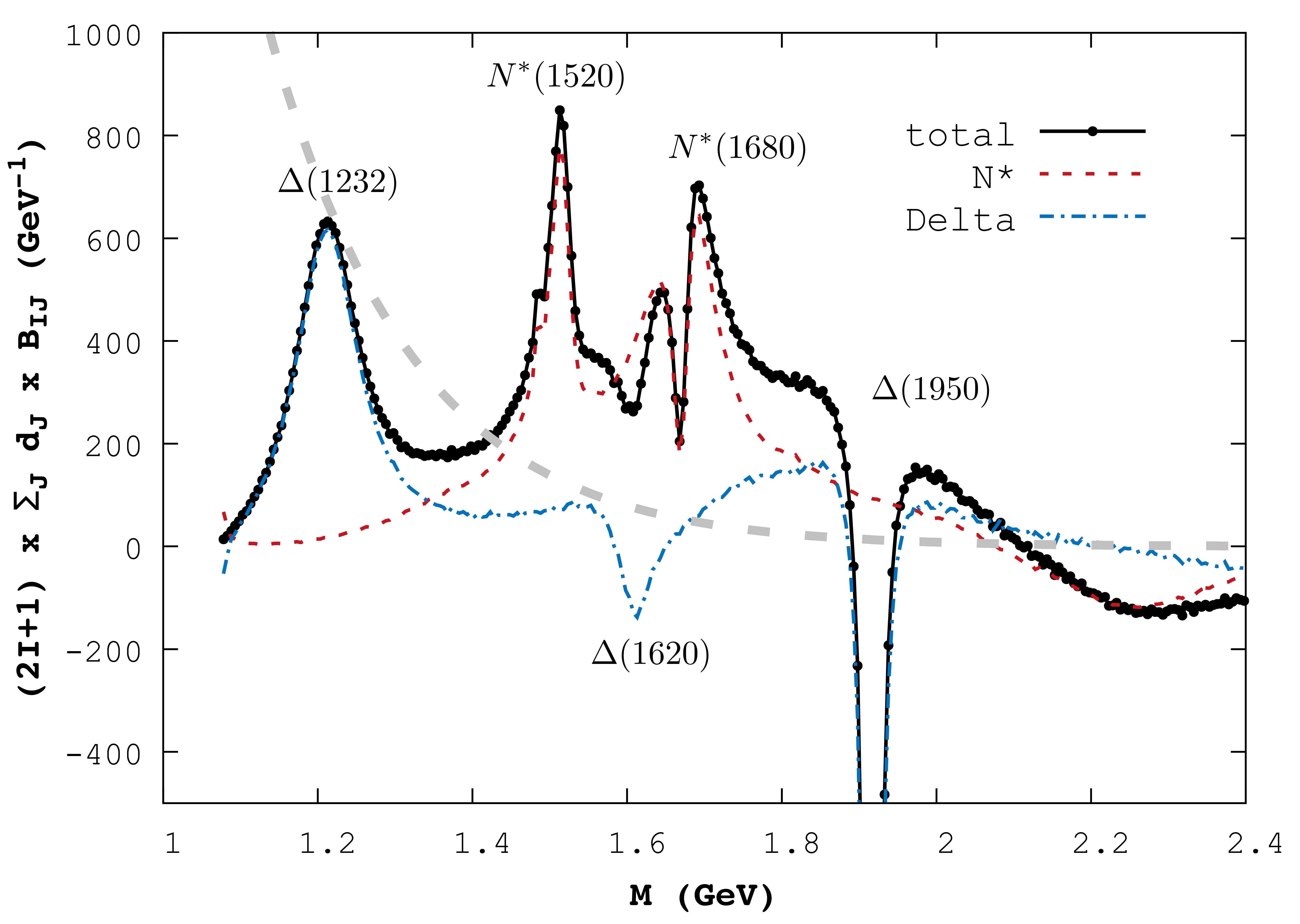
Estabrooks et al.
Aston et al.
Baker et al.
this work
$a_S = 0.224\, m_\pi^{-1}$
δ(I=1/2 J=0) (degree)
M (GeV)
Estabrooks et al.
Aston et al.
Mercer et al.
this work
$a_P = 0.018\, m_\pi^{-3}$
δ(I=1/2 J=1) (degree)
M (GeV)
Estabrooks et al.
Jongejans et al.
Cho et al.
Baker et al.
this work
$a_S = -0.045\, m_\pi^{-1}$
δ(I=3/2 J=0) (degree)
M (GeV)
free gas
S-matrix
j=1
j=0
$K^*(892)$
$\kappa + K^*(1430)$
$3 \times (2\,\delta_{j=1}^{I=1/2} + 4\,\delta_{j=1}^{I=3/2})$
$(2\,\delta_{j=0}^{I=1/2} + 4\,\delta_{j=0}^{I=3/2})$
$K^*(1430)$
P/T⁴
T (GeV)

S = -1 HYPERONS COUPLED CHANNEL SYSTEM

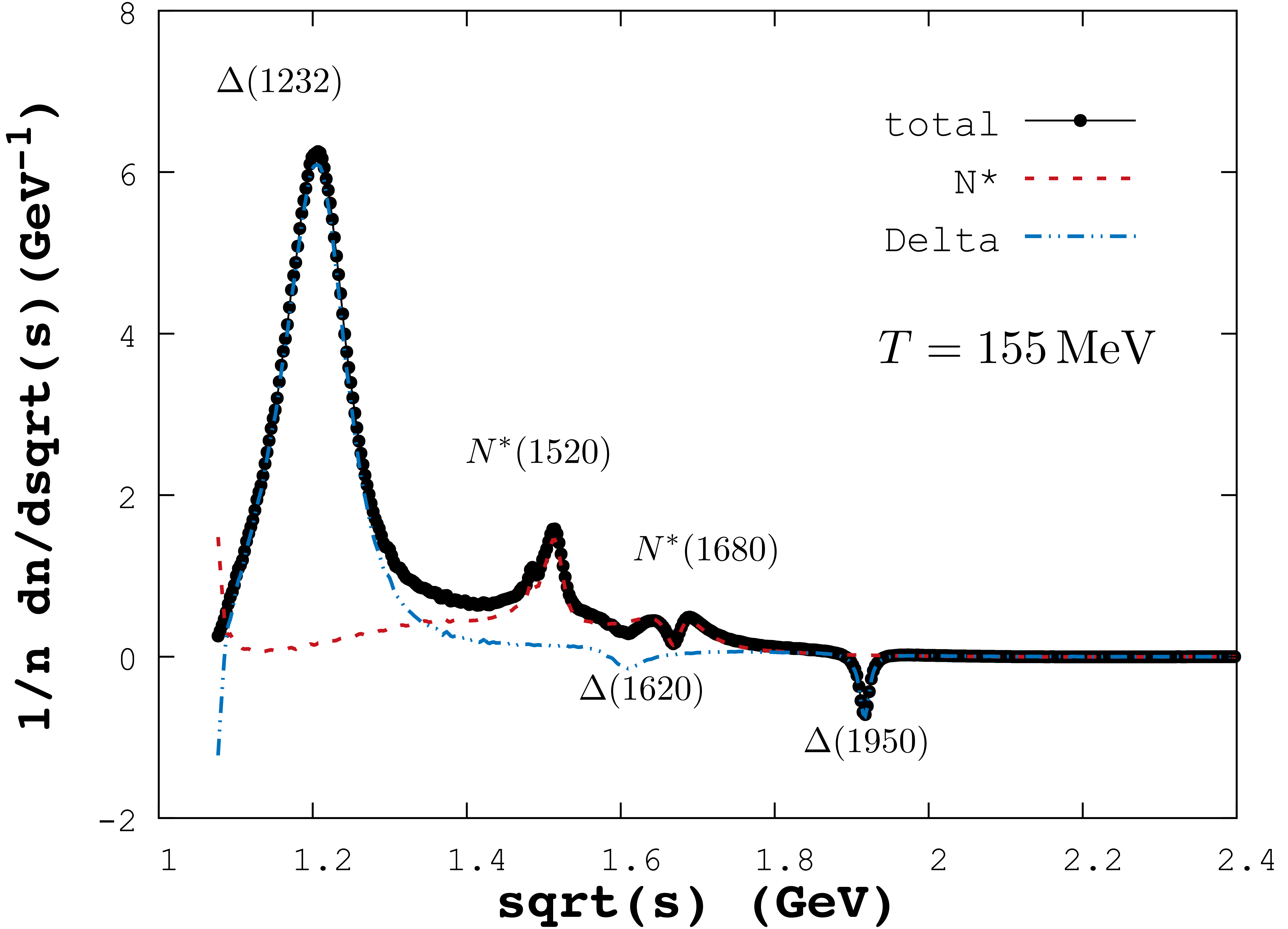
JPAC, PRD **93**, 034029 (2016)

C. Fernandez-Ramirez, PML, and P. Petreczky, PRC **98**, 044910 (2018)

J. Cleymans, PML, K. Redlich, and N. Sharma PRC **103**, 014904 (2021)



(2I+1) x $\sum_J$ d_J x B_{IJ} (GeV^{-1})
M (GeV)
total
N*
Delta
$\Delta(1232)$
$N^*(1520)$
$N^*(1680)$
$\Delta(1620)$
$\Delta(1950)$
1000
800
600
400
200
0
-200
-400
1
1.2
1.4
1.6
1.8
2
2.2
2.4



Δ(1232)
N*(1520)
N*(1680)
Δ(1620)
Δ(1950)
total
N*
Delta
T = 155 MeV
1/n dn/dsqrt(s) (GeV-1)
sqrt(s) (GeV)
8
6
4
2
0
-2
1
1.2
1.4
1.6
1.8
2
2.2
2.4

JPAC, PRD **93**, 034029 (2016)

PHASE SHIFT FROM PWA

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Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

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Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)
Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.

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```

channel	elastic	channel	quasi-elastic	channel	unitarity
1	$\bar{K}N$	6	$\bar{K}_1^* N$	15	$\pi\pi\Lambda$
2	$\pi\Sigma$	7	$[\bar{K}_3^* N]_-$	16	$\pi\pi\Sigma$
3	$\pi\Lambda$	8	$[\bar{K}_3^* N]_+$		
4	$\eta\Lambda$	9	$[\pi\Sigma(1385)]_-$		
5	$\eta\Sigma$	10	$[\pi\Sigma(1385)]_+$		
		11	$[\bar{K}\Delta(1232)]_-$		
		12	$[\bar{K}\Delta(1232)]_+$		
		13	$[\pi\Lambda(1520)]_-$		
		14	$[\pi\Lambda(1520)]_+$		

elastic scatterings (elementary)

quasi elastic scatterings

unitarity background

STRANGENESS CONTENT IN A HADRON GAS

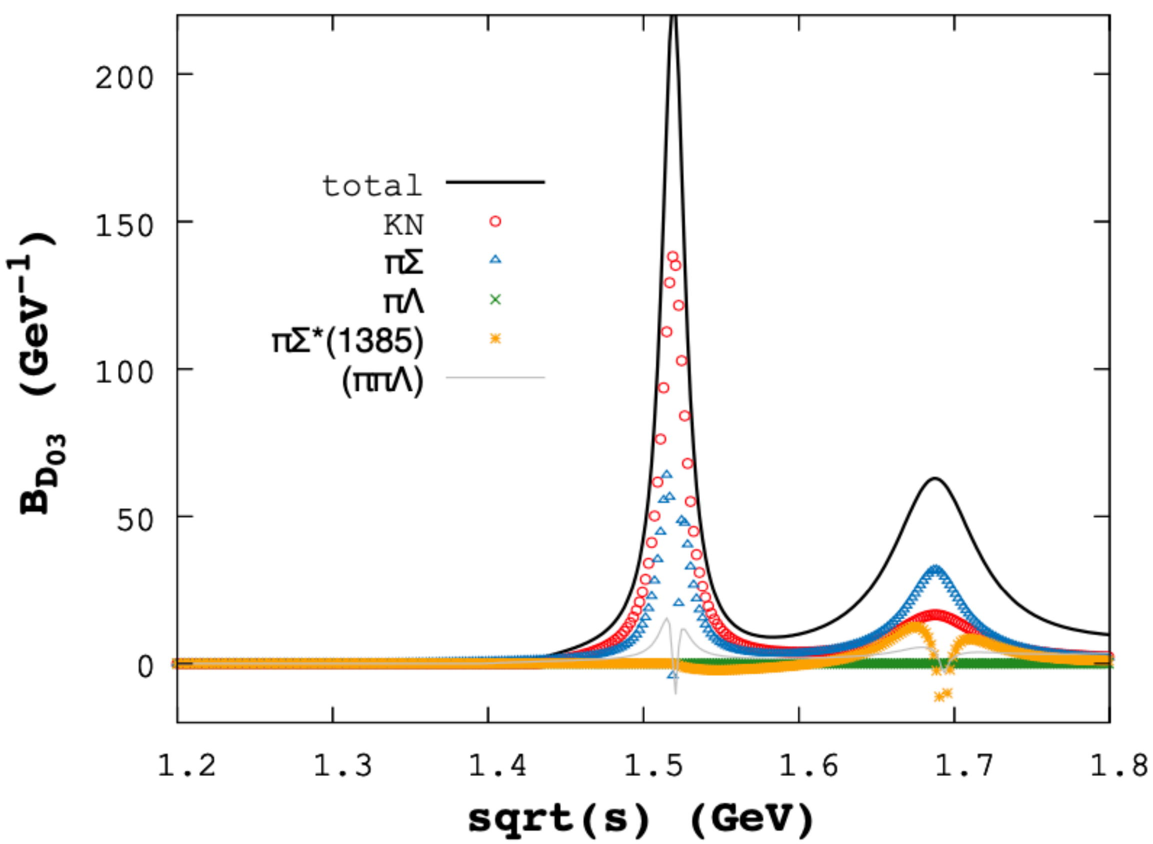
- K-N system requires a coupled channel analysis

$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \ldots$ *16 basis states*

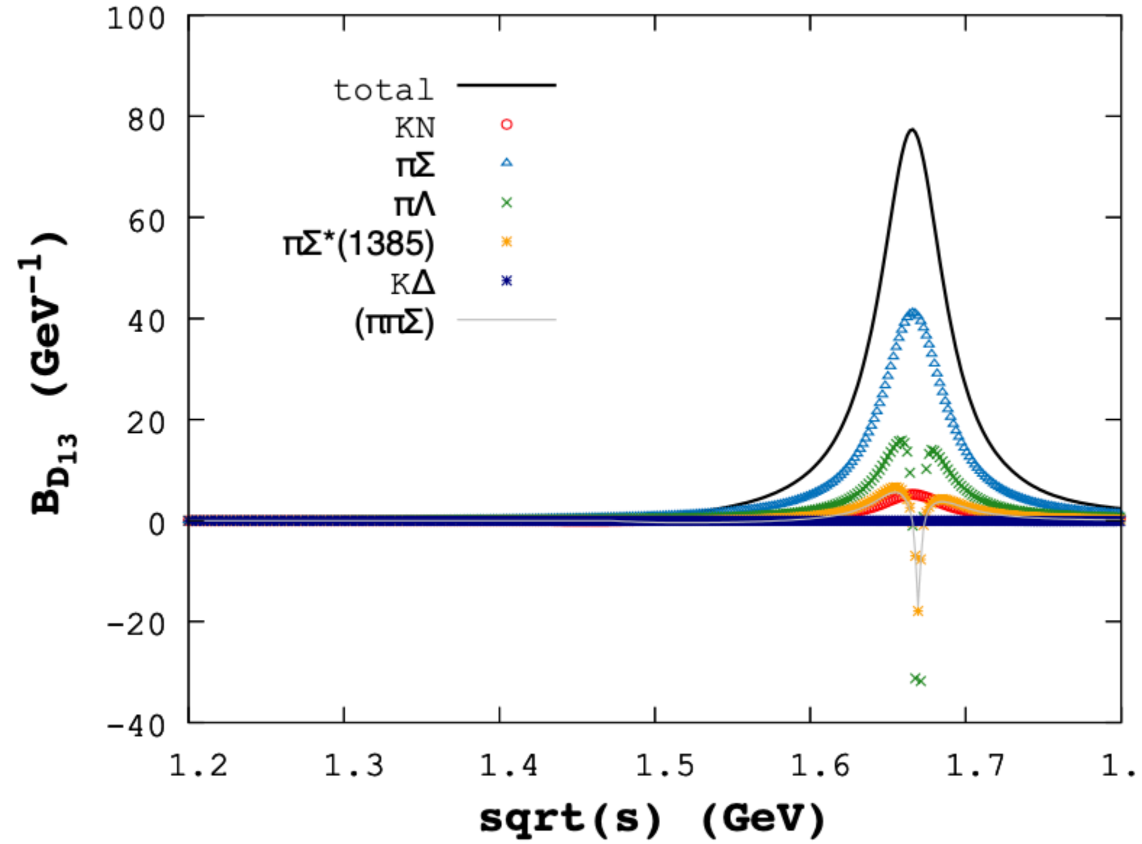
$$\mathcal{Q}(M) \equiv \frac{1}{2}\,\mathrm{Im}\left(\mathrm{tr}\ln\ \mathrm{S}\right)$$
$$= \frac{1}{2}\,\mathrm{Im}\left(\ln\ \det\left[\mathrm{S}\right]\right)$$
$$= \delta_{\bar{K}N} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \ldots$$

Compute det S for each sqrt(s) for each channel isospin conserving

1520, 1690



1670



$\Lambda(1520)\ 3/2^-$ $\qquad I(J^P) = 0(\frac{3}{2}^-)$

Mass $m = 1519.5 \pm 1.0$ MeV [d]
Full width $\Gamma = 15.6 \pm 1.0$ MeV [d]
$p_{\text{beam}} = 0.39$ GeV/c $\qquad 4\pi\lambda\!\!\!^{-2} = 82.8$ mb

Λ(1520) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	45 ± 1%	243
$\Sigma\pi$	42 ± 1%	268
$\Lambda\pi\pi$	10 ± 1%	259
$\Sigma\pi\pi$	0.9 ± 0.1%	169
$\Lambda\gamma$	0.85 ± 0.15%	350

$\Lambda(1690)\ 3/2^-$ $\qquad I(J^P) = 0(\frac{3}{2}^-)$

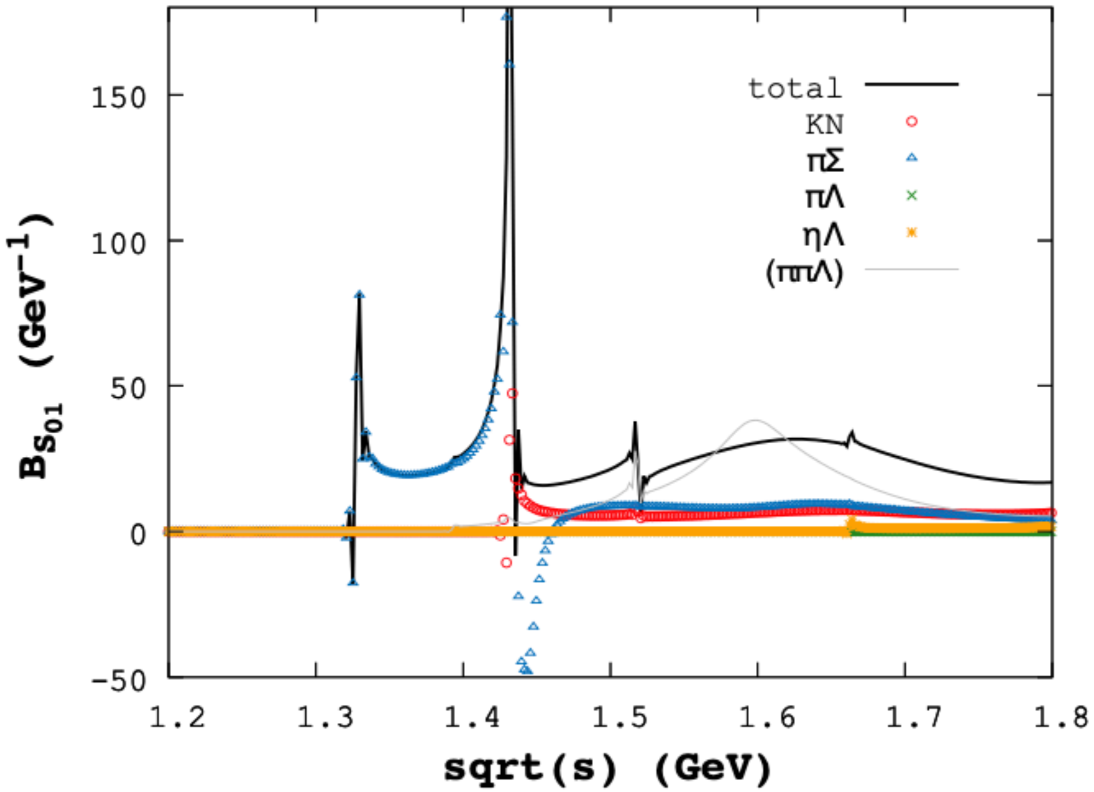
Mass $m = 1685$ to 1695 (≈ 1690) MeV
Full width $\Gamma = 50$ to 70 (≈ 60) MeV
$p_{\text{beam}} = 0.78$ GeV/c $\qquad 4\pi\lambda\!\!\!^{-2} = 26.1$ mb

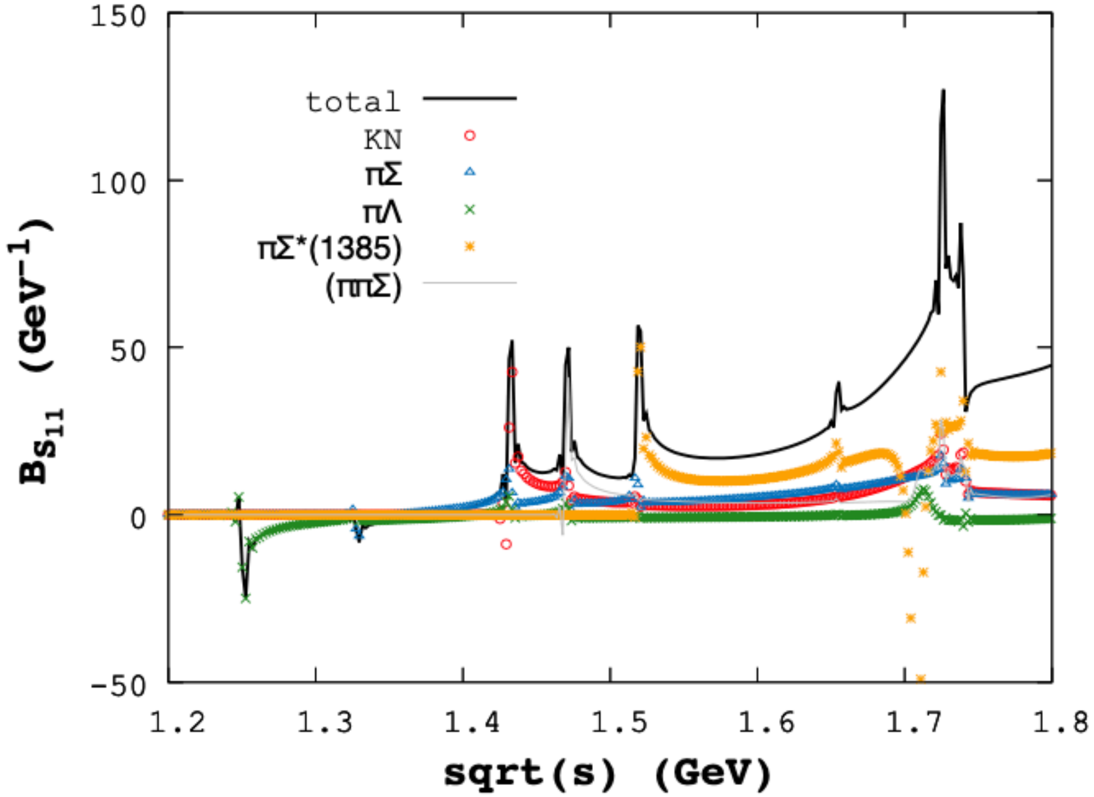
Λ(1690) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	20–30 %	433
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	~ 25 %	419
$\Sigma\pi\pi$	~ 20 %	358

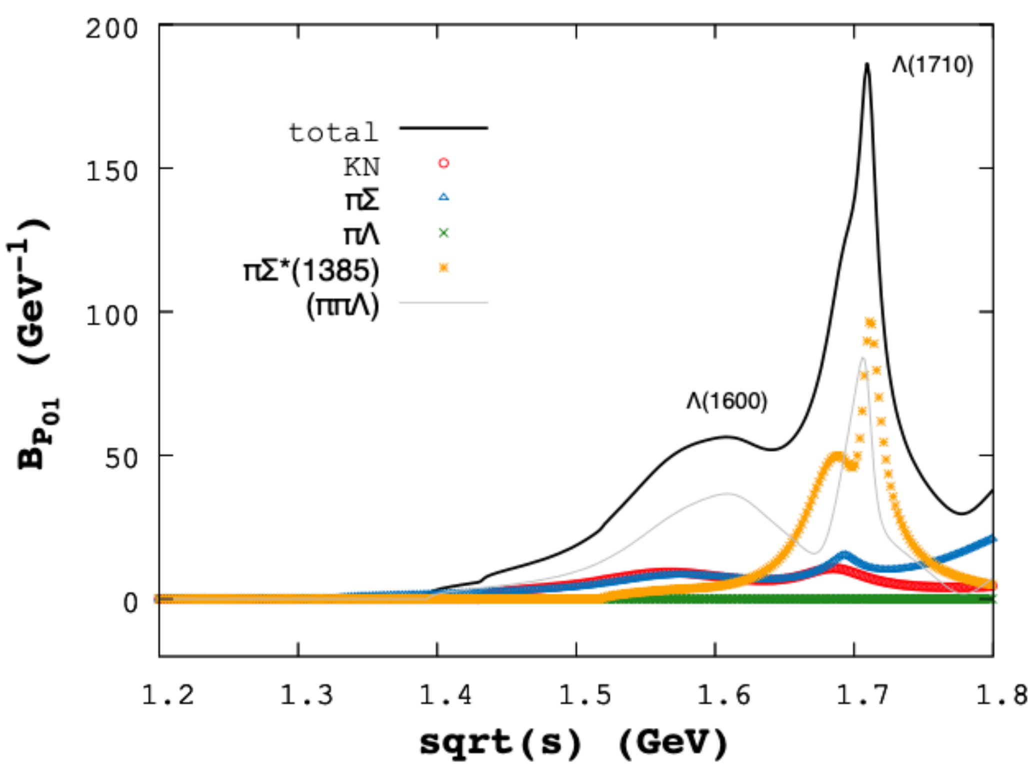
$\Sigma(1670)\ 3/2^-$ $\qquad I(J^P) = 1(\frac{3}{2}^-)$

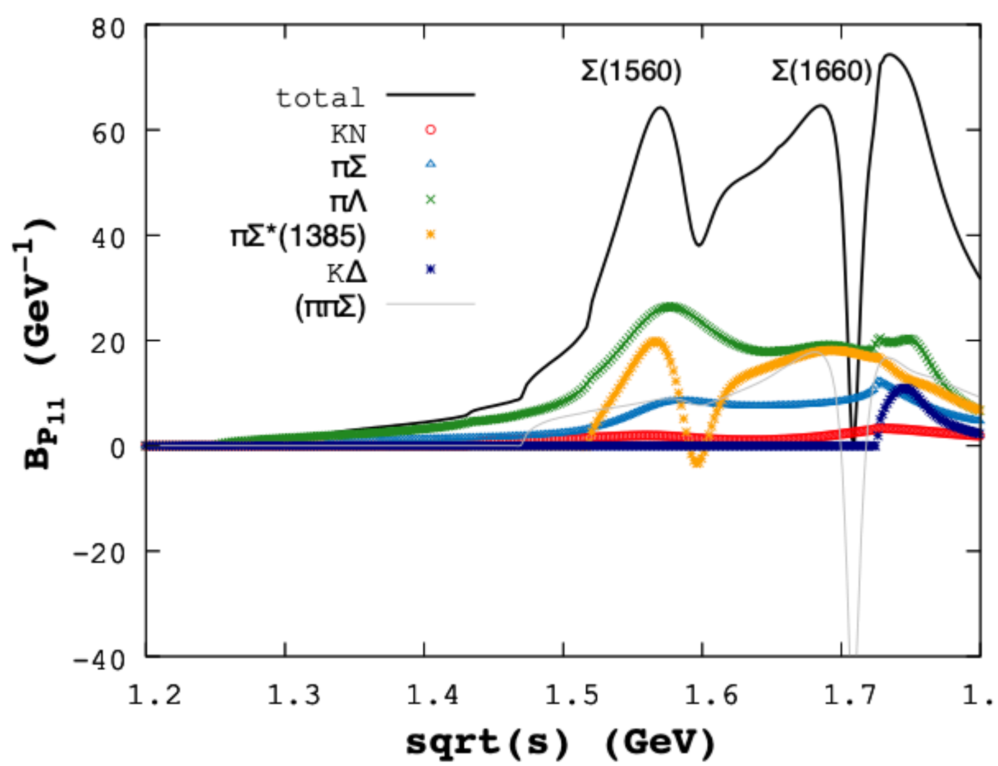
Mass $m = 1665$ to 1685 (≈ 1670) MeV
Full width $\Gamma = 40$ to 80 (≈ 60) MeV
$p_{\text{beam}} = 0.74$ GeV/c $\qquad 4\pi\lambda\!\!\!^{-2} = 28.5$ mb

Σ(1670) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394

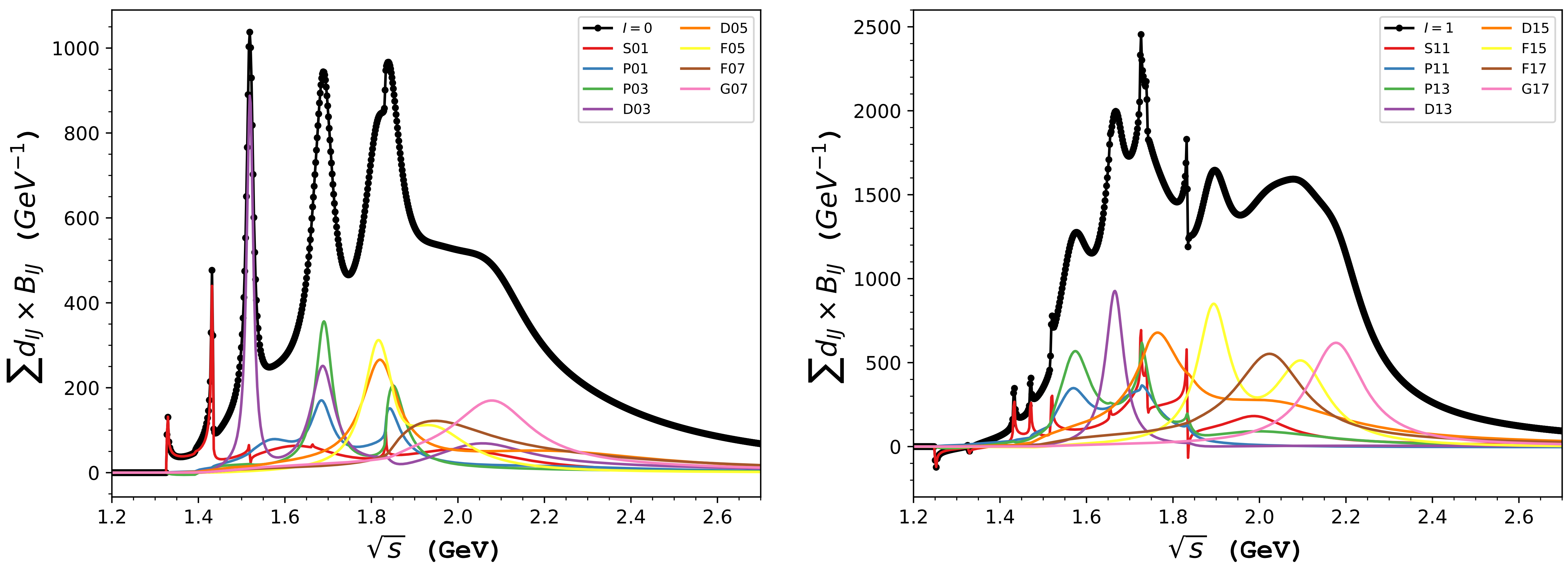








branching ratio?



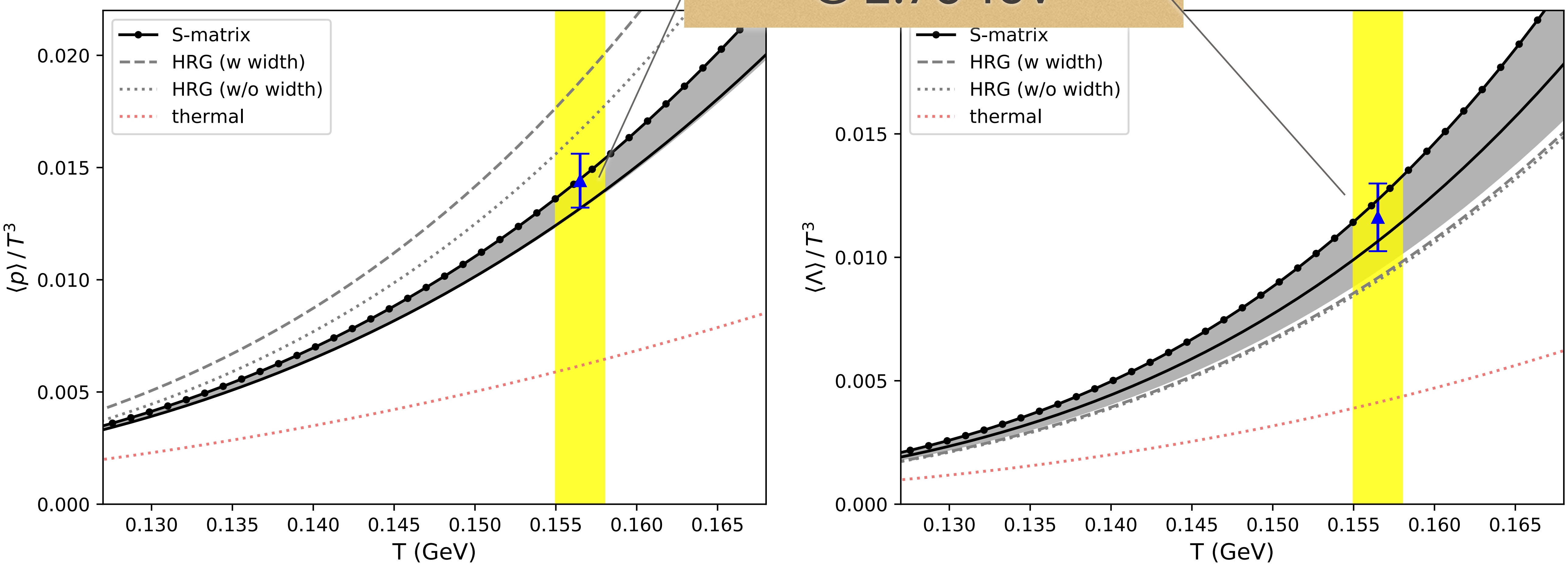
$I=0$
S01
P01
P03
D03
D05
F05
F07
G07
$\sum d_{IJ} \times B_{IJ}$ (GeV^{-1})
$\sqrt{S}$ (GeV)
1000
800
600
400
200
0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
$I=1$
S11
P11
P13
D13
D15
F15
F17
G17
2500
2000
1500
1000
500
0

S-matrix VS HRG

Will still go up!

Andronic et. al. NPA1010 (2021) 122176

ALICE proton yield
@ 2.76 TeV



GW INS

Jefferson Lab
Thomas Jefferson National Accelerator Facility

```
piN phase shifts
pipiN BGs
hyperons
```

Coupled-Channel model:

$$\bar{k}N, \pi\Lambda, \pi\Sigma, \ldots$$

extra hyperon states

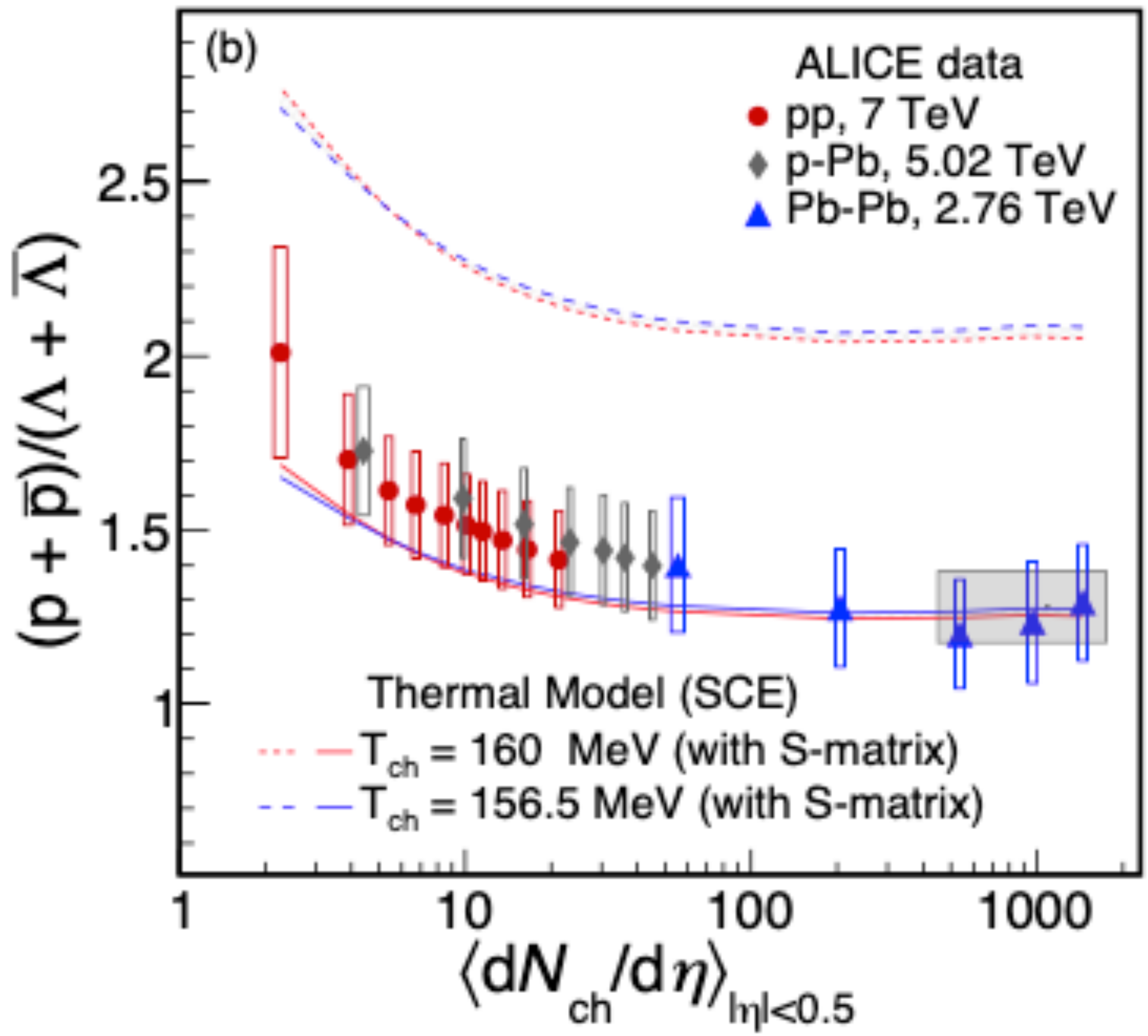
beyond PDG

unitarity BGs

consistent treatment of res and non-res. int.

less protons

more lambdas

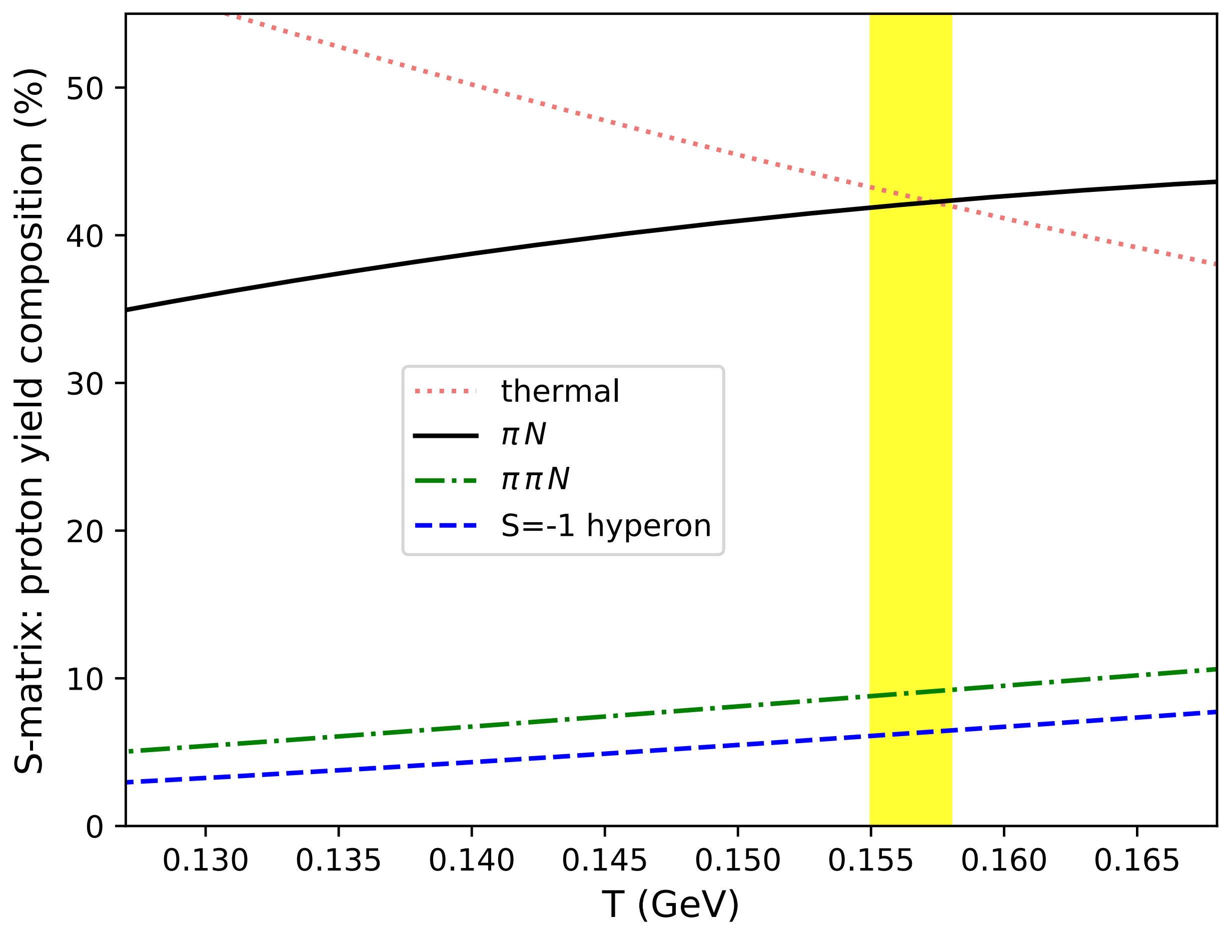


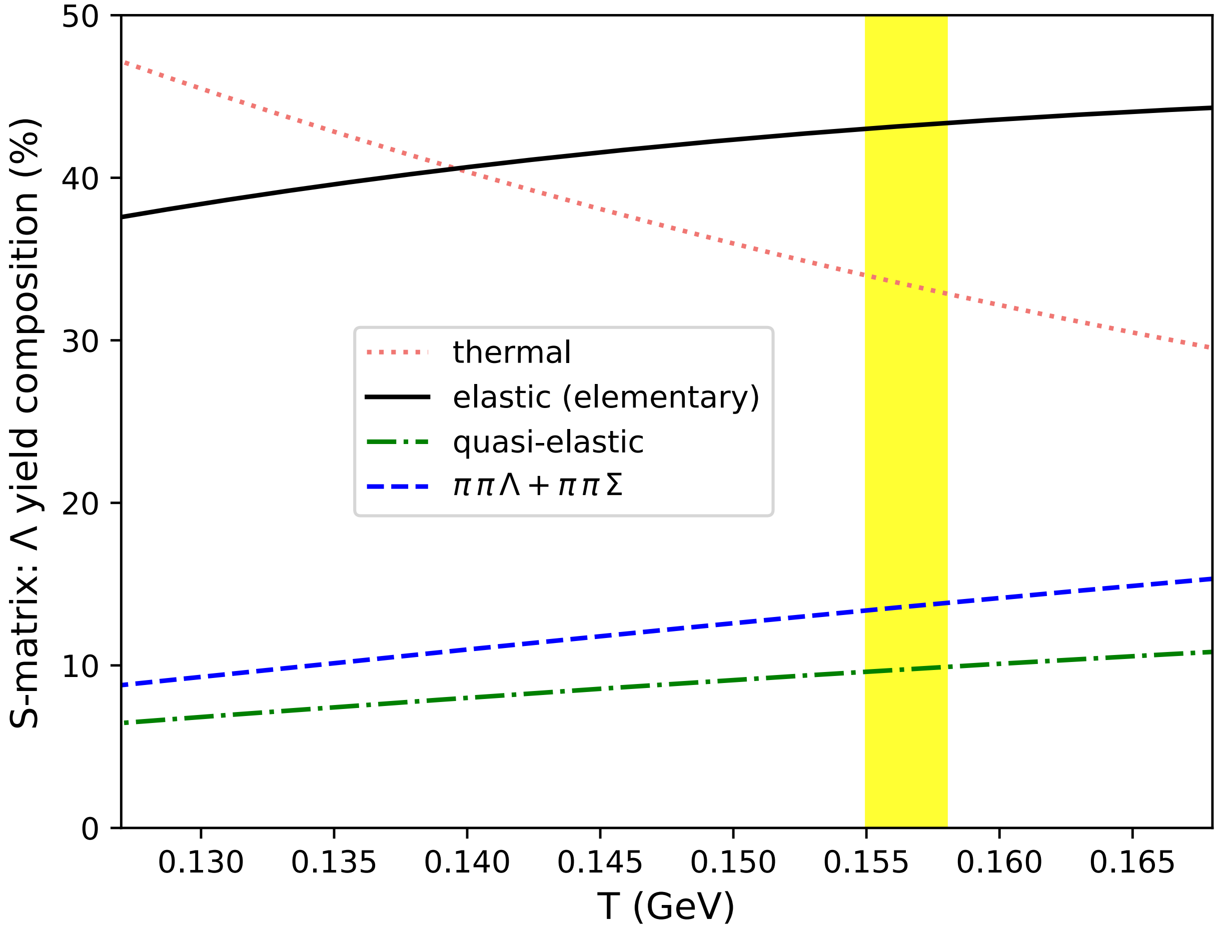
hrg

smat

Phys. Rev. C 103, 014904 (2021).

Phys. Lett. B 792, 304 (2019).





SAID GWU

JPAC

piN phase shifts
pipiN BGs
hyperons

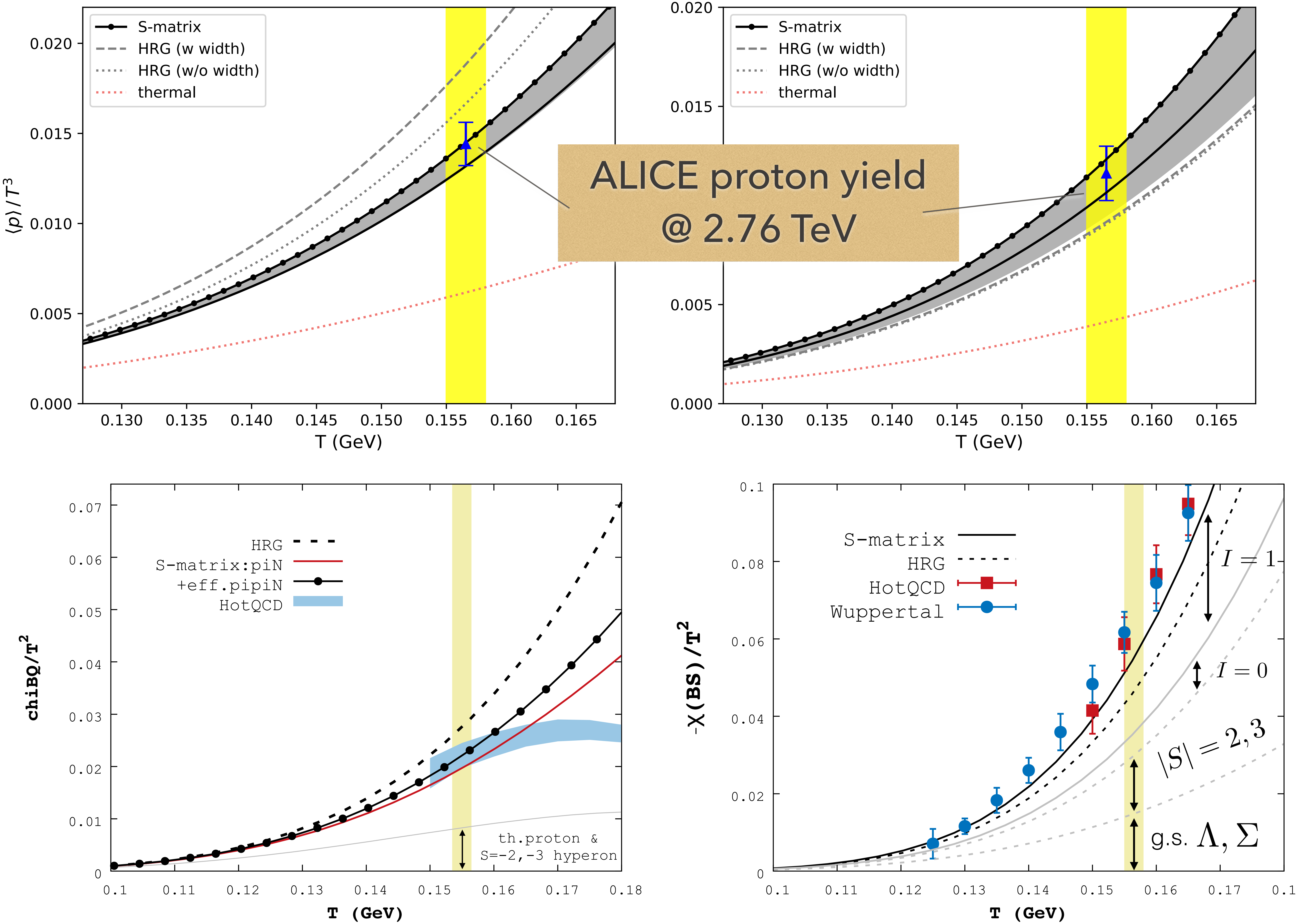
Coupled-Channel system:

$$\bar{k}N, \pi\Lambda, \pi\Sigma, ...$$

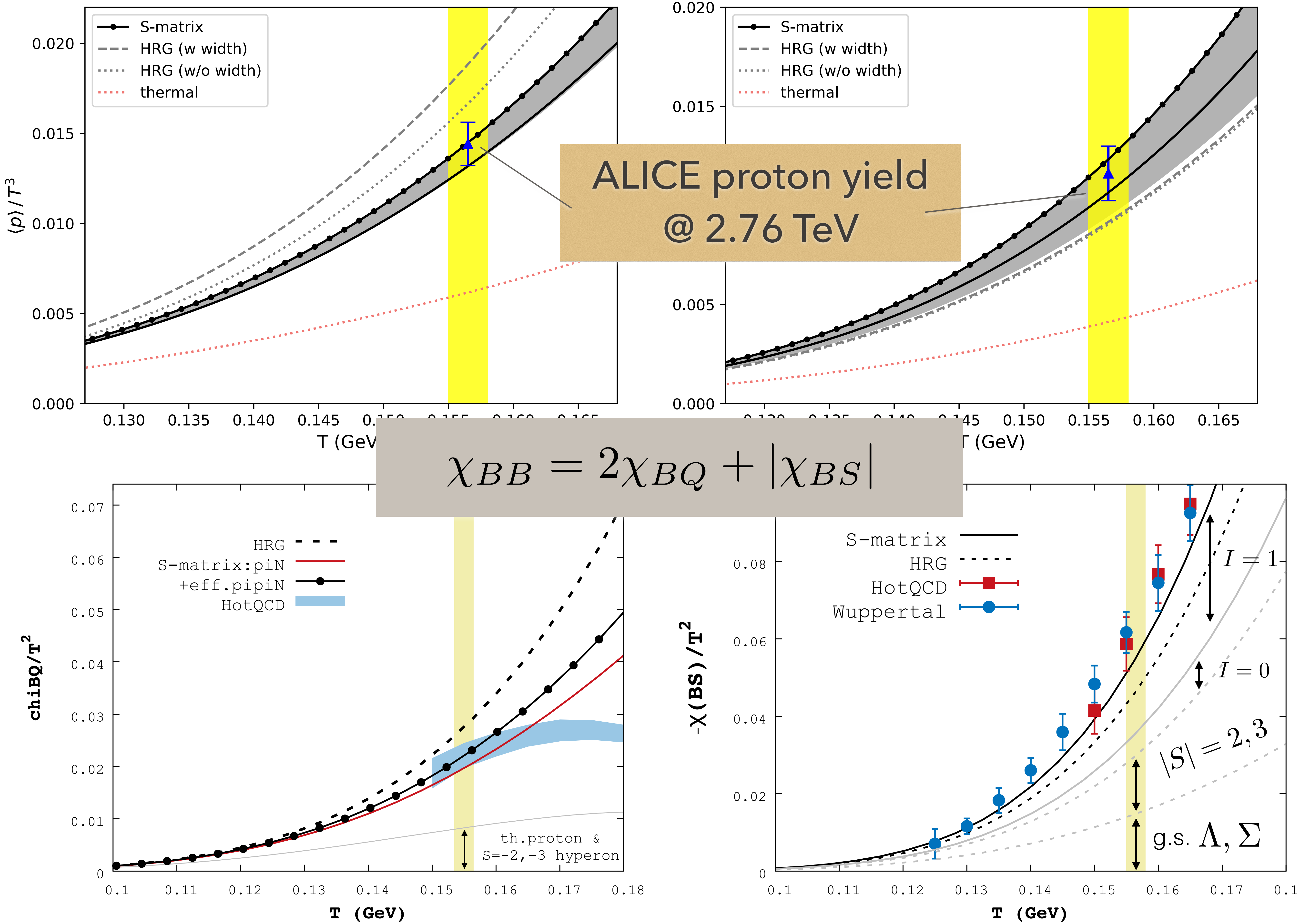
extra hyperon states beyond PDG

unitarity BGs

consistent treatment of res and non-res. int.



S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩ / T^3
T (GeV)
ALICE proton yield
@ 2.76 TeV
HRG
S-matrix:piN
+eff.pipiN
HotQCD
chiBQ/T^2
th.proton &
S=-2,-3 hyperon
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
-χ(BS)/T^2
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ
T (GeV)



$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$

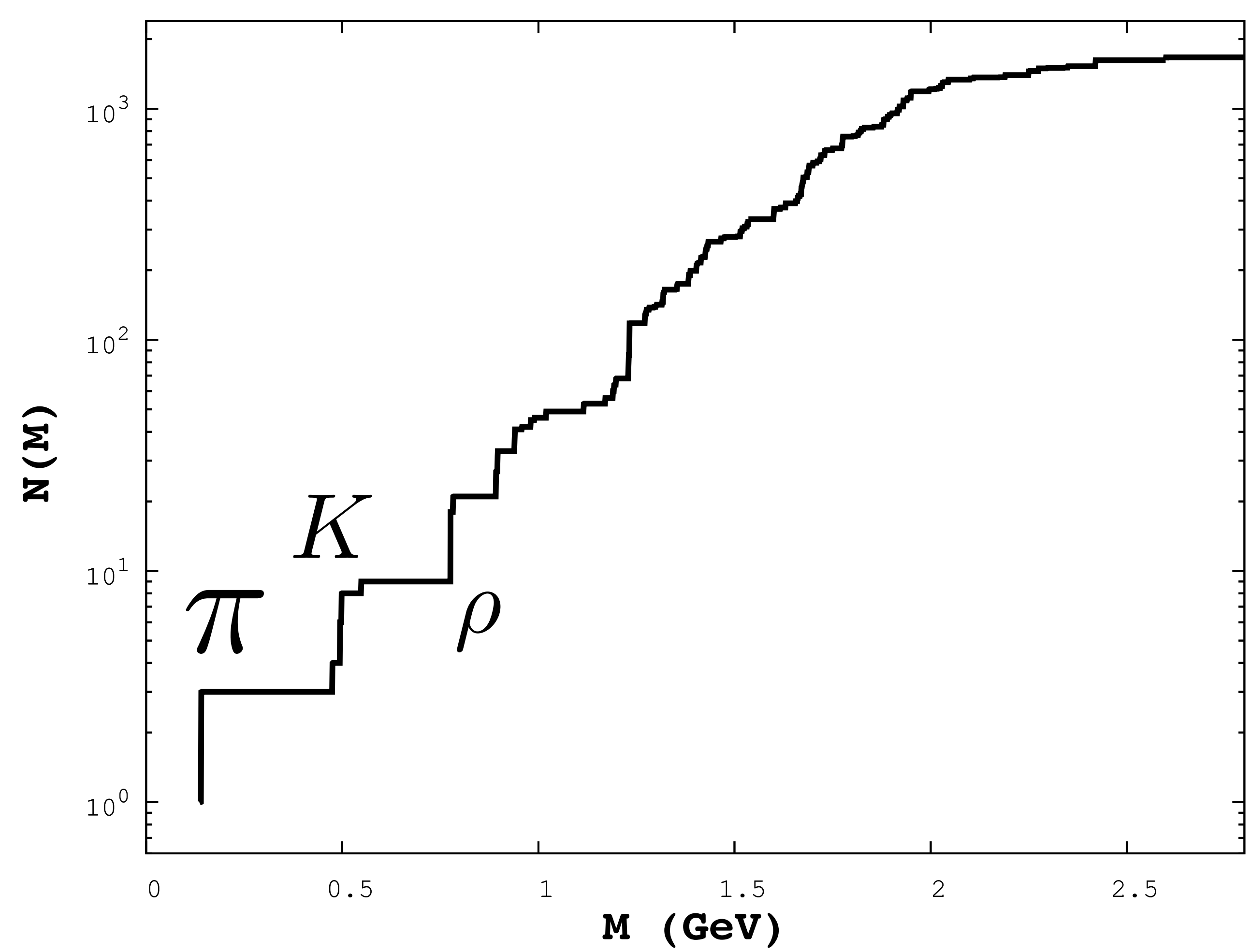
HOW TO PROPERLY ADD STATES

PML PRD **102**, 034038 (2020)

HRG AS AN S-MATRIX SCHEME

$$\det S(E) = \prod_{\{\text{res}\}} \frac{z^{\star}_{\text{res}} - E}{z_{\text{res}} - E}, \qquad z_{\text{res}} \approx m_{\text{res}} - i\,0^{+}.$$

$$\mathcal{Q}_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi\,\theta(E - m_{\text{res}}),$$

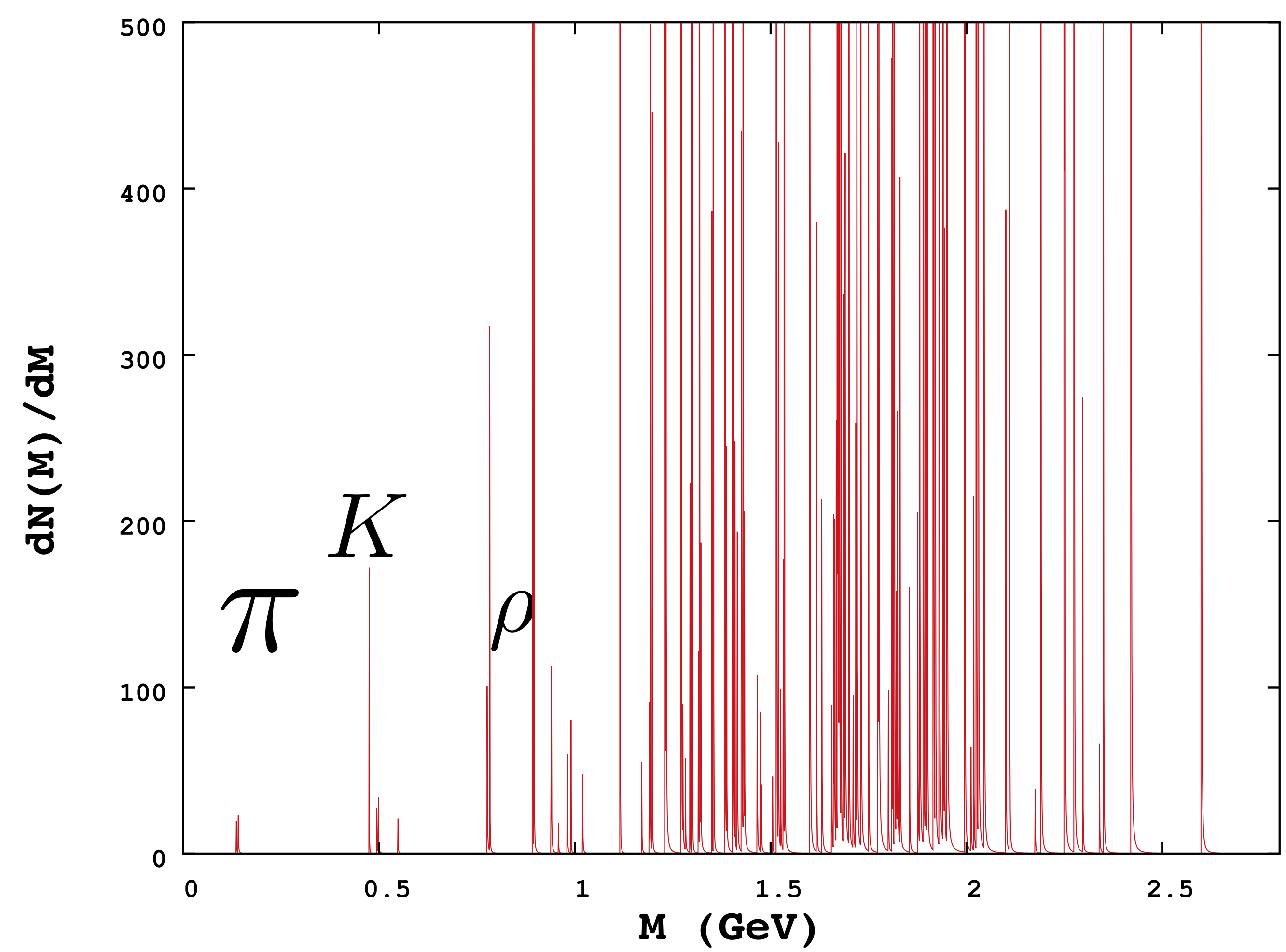


HRG AS AN S-MATRIX SCHEME

$$\det S(E) = \prod_{\{\text{res}\}} \frac{z^{\star}_{\text{res}} - E}{z_{\text{res}} - E}, \qquad z_{\text{res}} \approx m_{\text{res}} - i\,0^{+}.$$

$$\mathcal{Q}_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi\,\theta(E - m_{\text{res}}),$$

$$\frac{\partial}{\partial E}$$



DYNAMICAL GENERATION OF BS / RESONANCES

- dynamical generation of bound states / resonances:
  f(980) close to K Kbar threshold
  f(500) dynamically generated
- coupling of open channels: pipi, kkbar with a $|q\bar{q}\rangle$ state

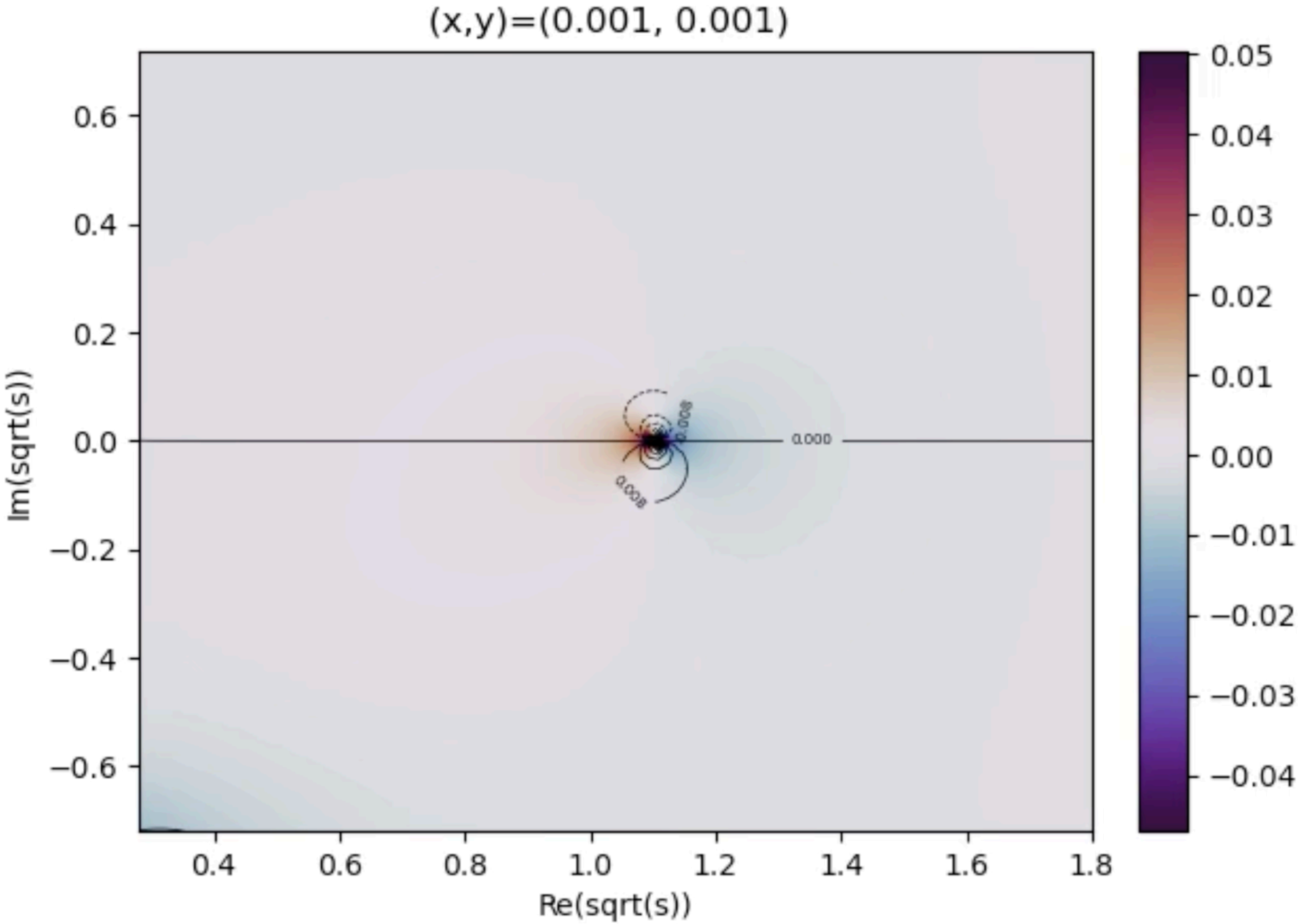
Locher, Markushin, Zheng, EPJC 4 (1998)
Kaminski, Lesniak, Loiseau, EPJC 9 (1999)

what you give $\neq$ *what you get*   *1 in 5 out!*

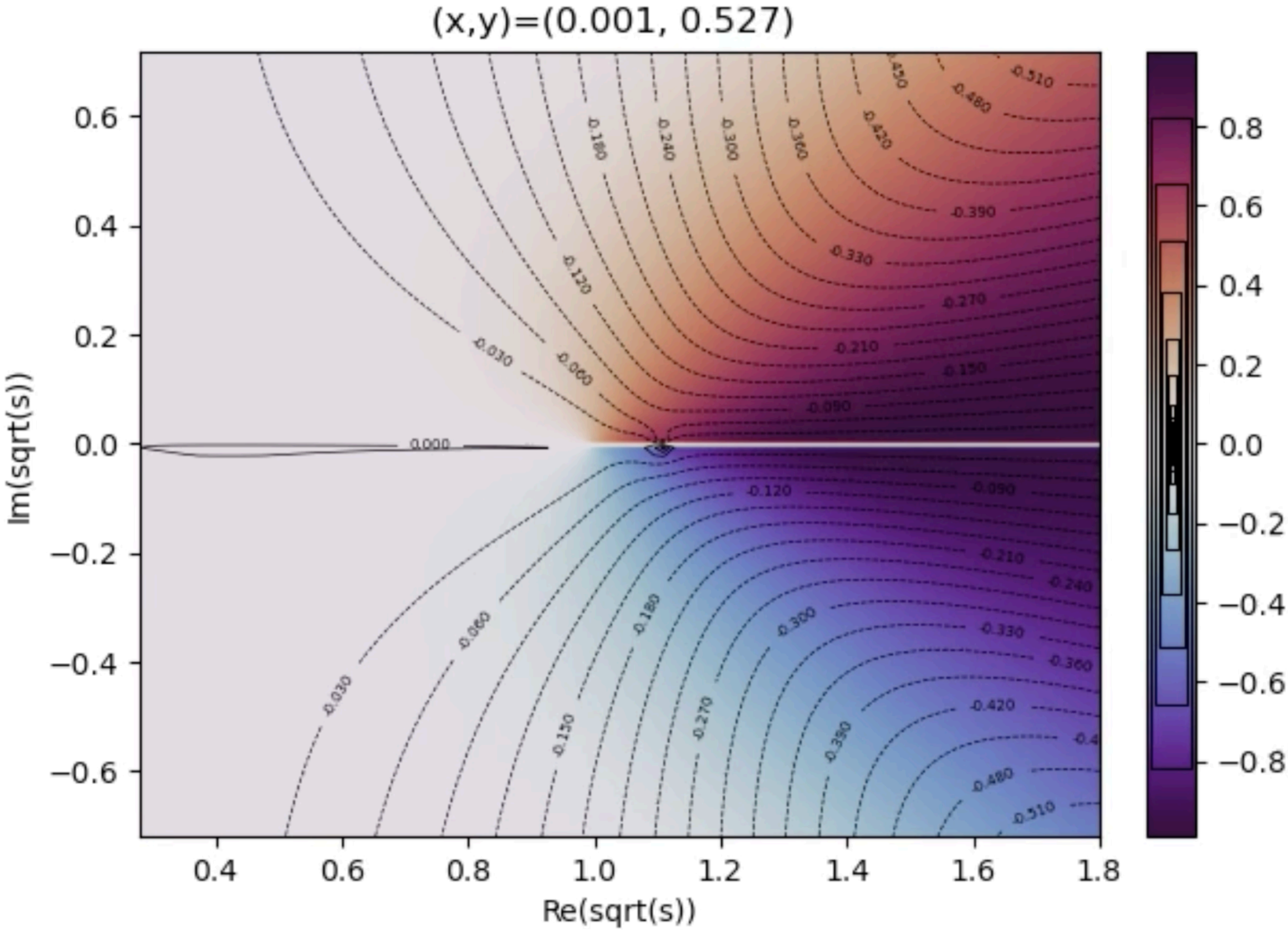
$$\frac{1}{E-\mathcal{H}_0} = \begin{matrix} |\pi\pi\rangle \\ |K\bar{K}\rangle \\ |R^0\rangle \ (|q\bar{q}\rangle) \end{matrix} \begin{bmatrix} \Pi_{\pi\pi}(E) & & \\ & \Pi_{K\bar{K}}(E) & \\ & & \frac{1}{E-m^0_{res}} \end{bmatrix}$$

$$V_{int} = \begin{bmatrix} g_{\pi\pi} & g_{\pi K} & g_{\pi R} \\ g_{\pi K} & g_{KK} & g_{KR} \\ g_{\pi R} & g_{KR} & \end{bmatrix}$$

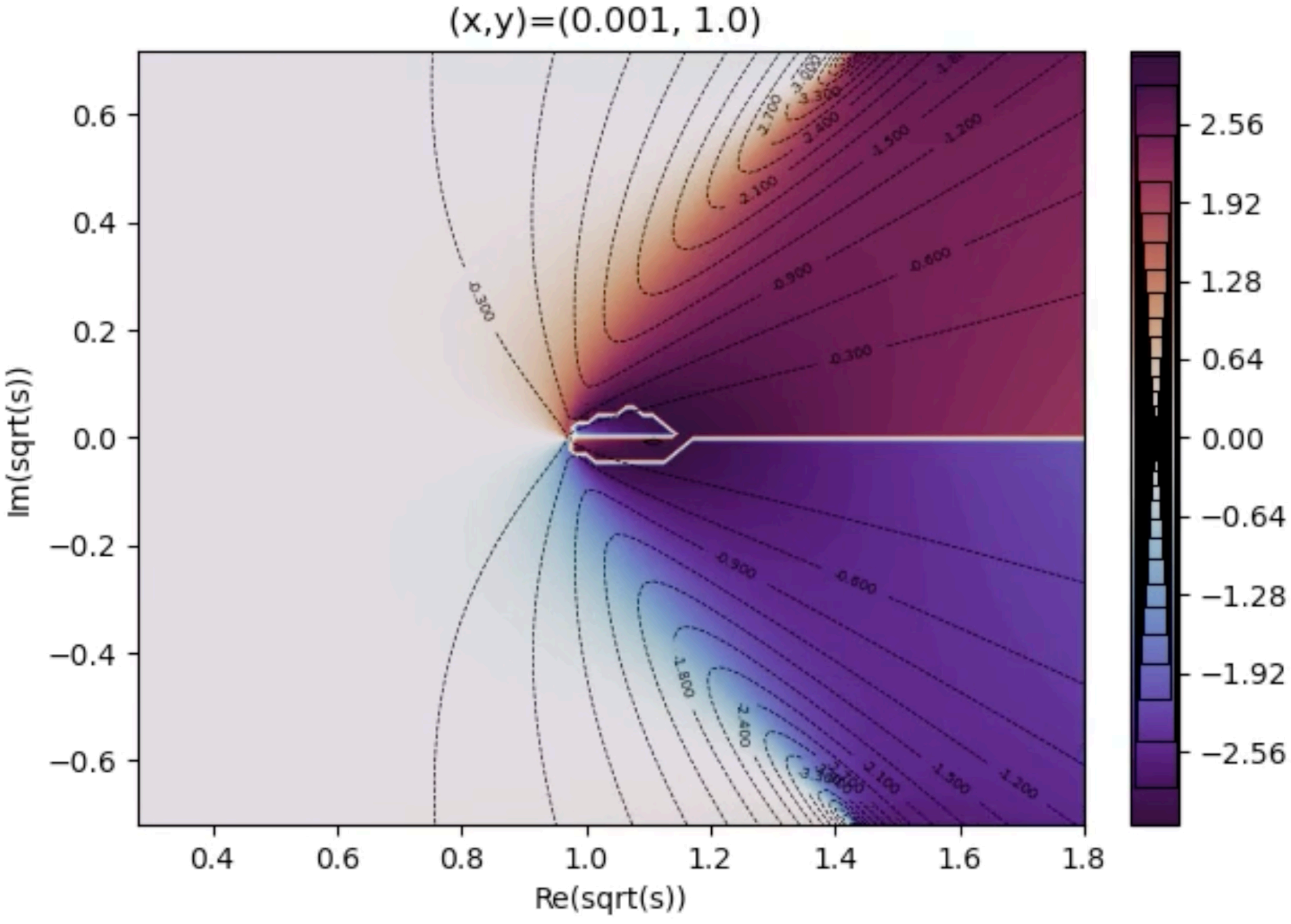
$$G = G_0 + G_0 V_{int} G$$



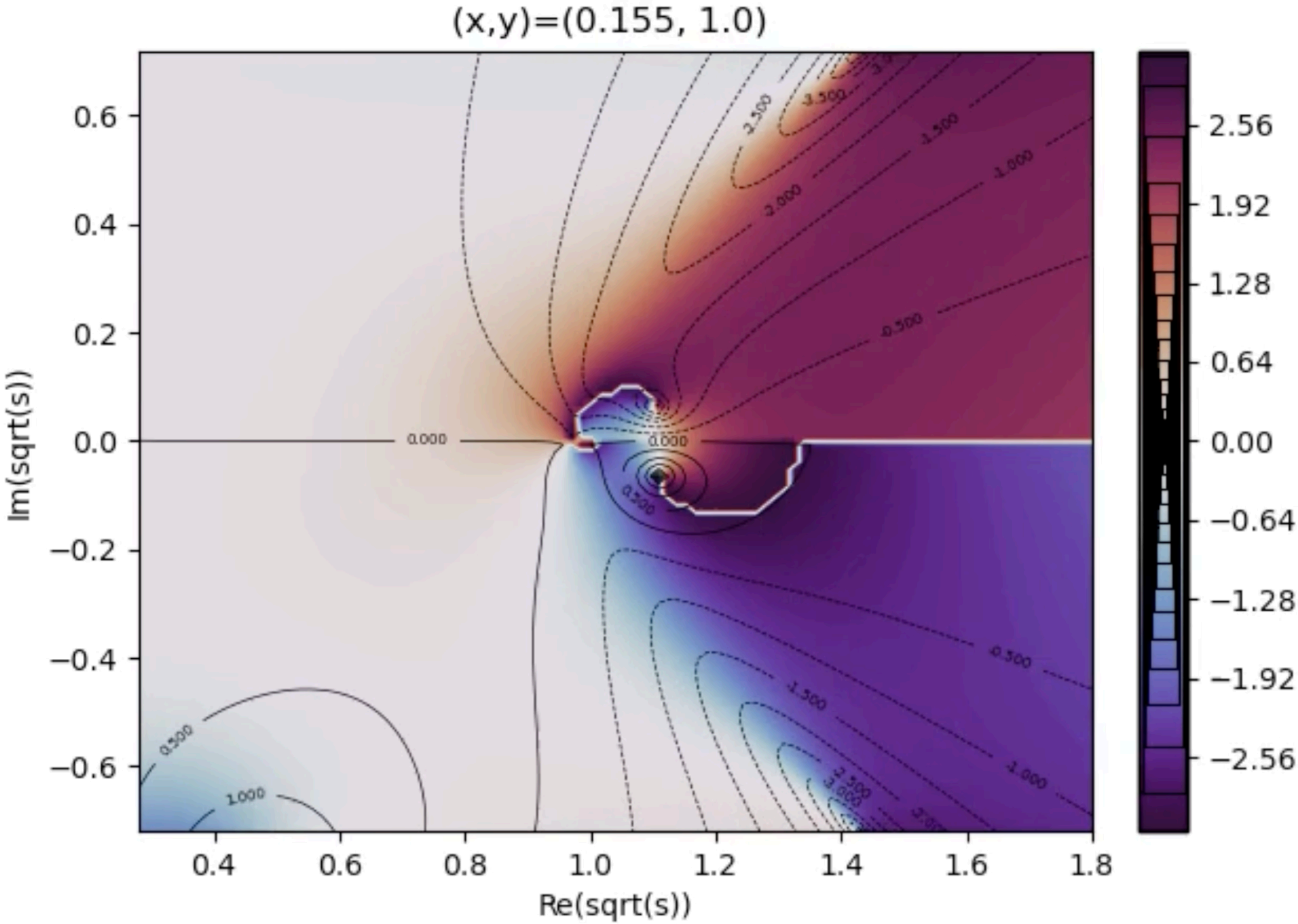
(x,y)=(0.001, 0.001)
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Im(sqrt(s))
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Re(sqrt(s))
0.000
0.008
0.008
0.05
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04



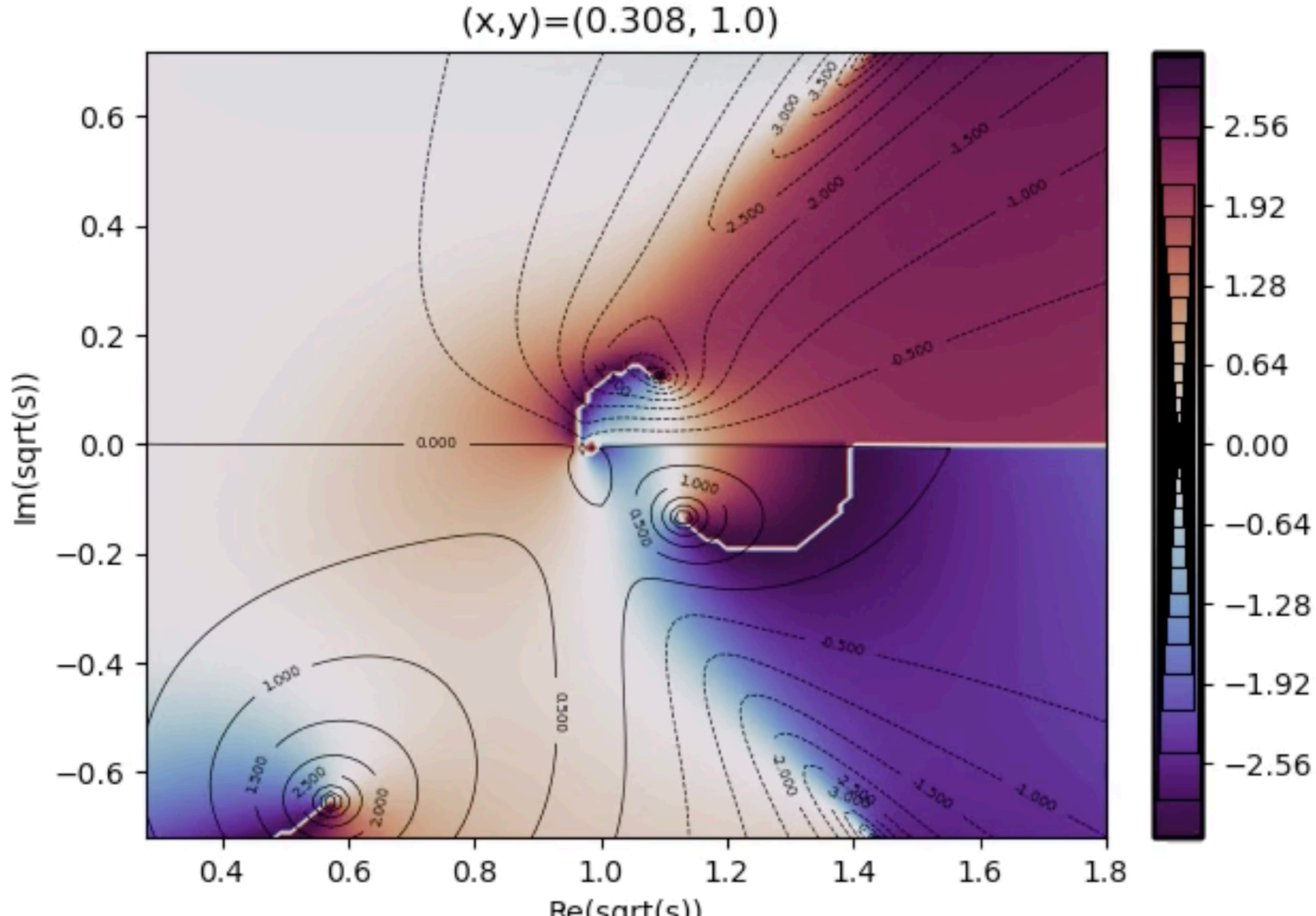
(x,y)=(0.001, 0.527)
Im(sqrt(s))
Re(sqrt(s))
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8



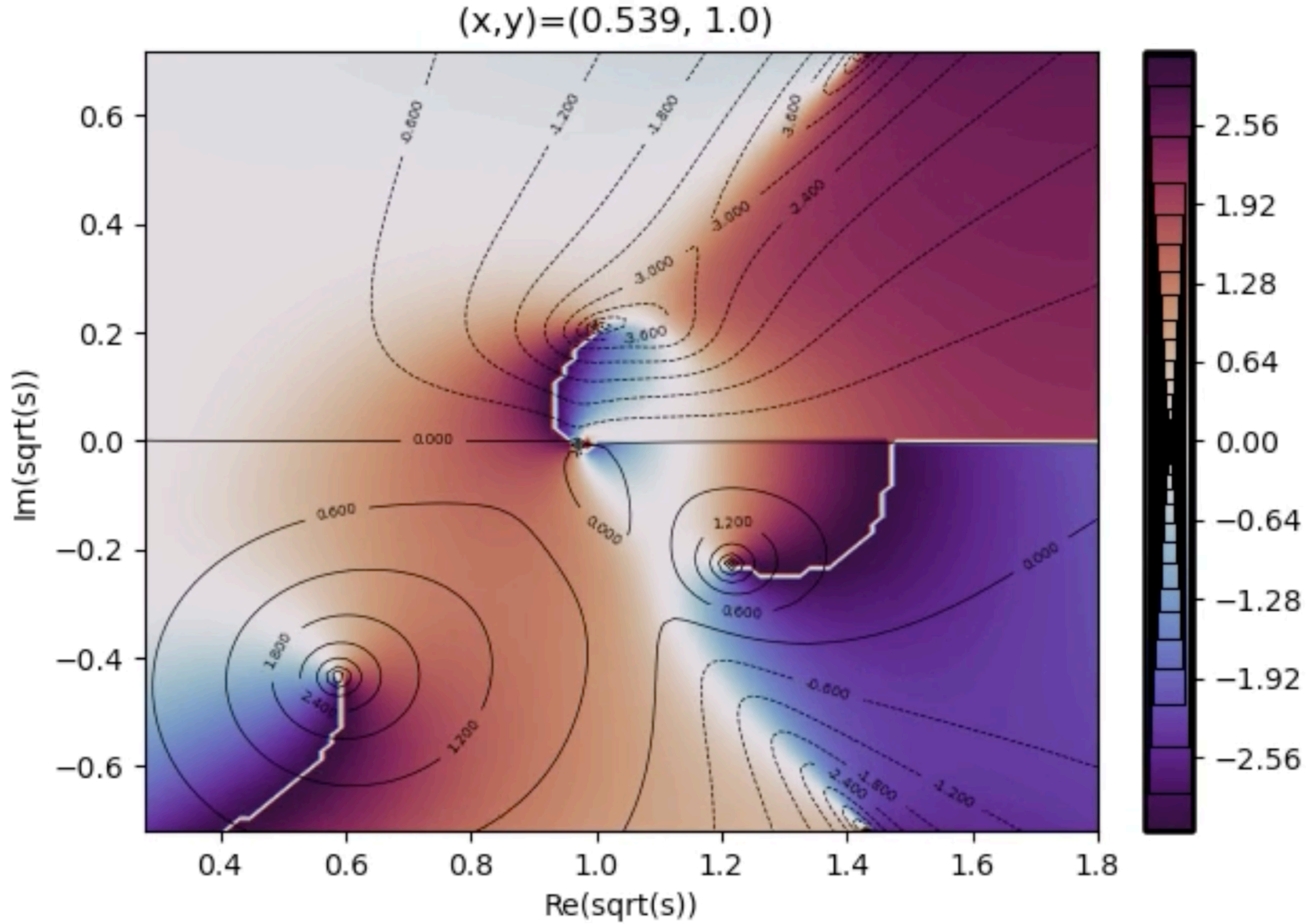
(x,y)=(0.001, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56



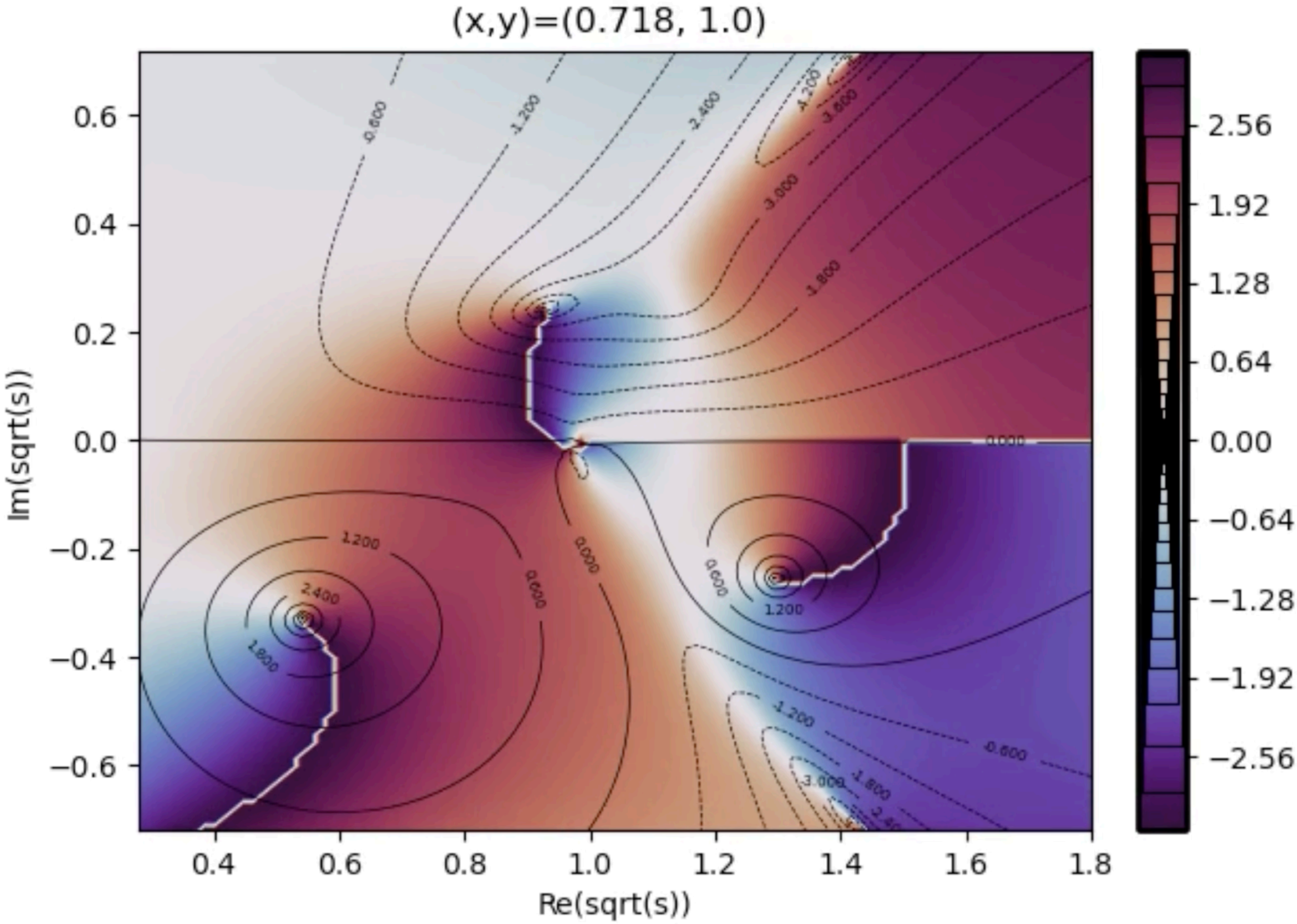
(x,y)=(0.155, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56
0.000
0.500
1.000
-0.500
-1.000
-1.500
-2.000
-2.500
-3.500



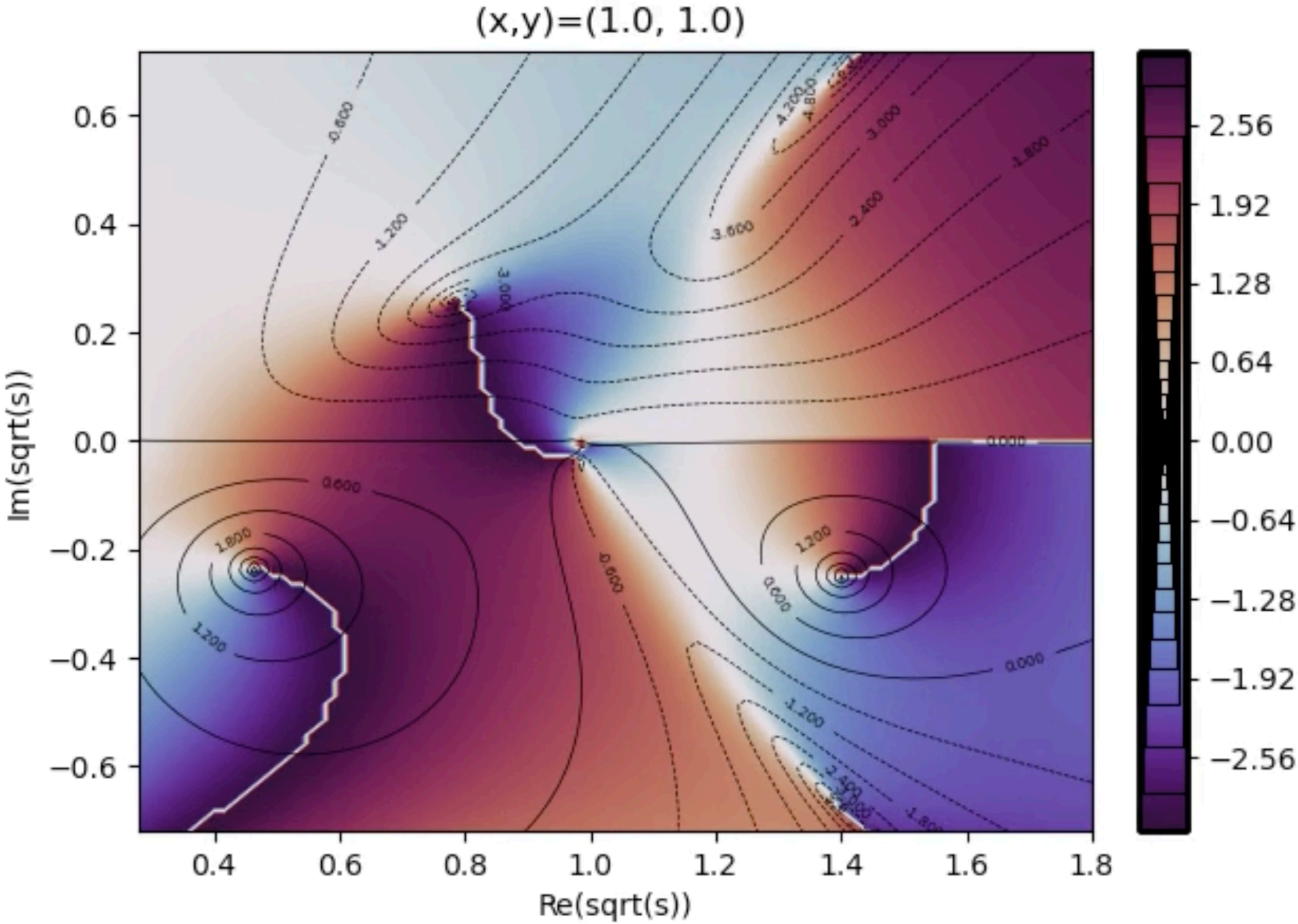
(x,y)=(0.308, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56

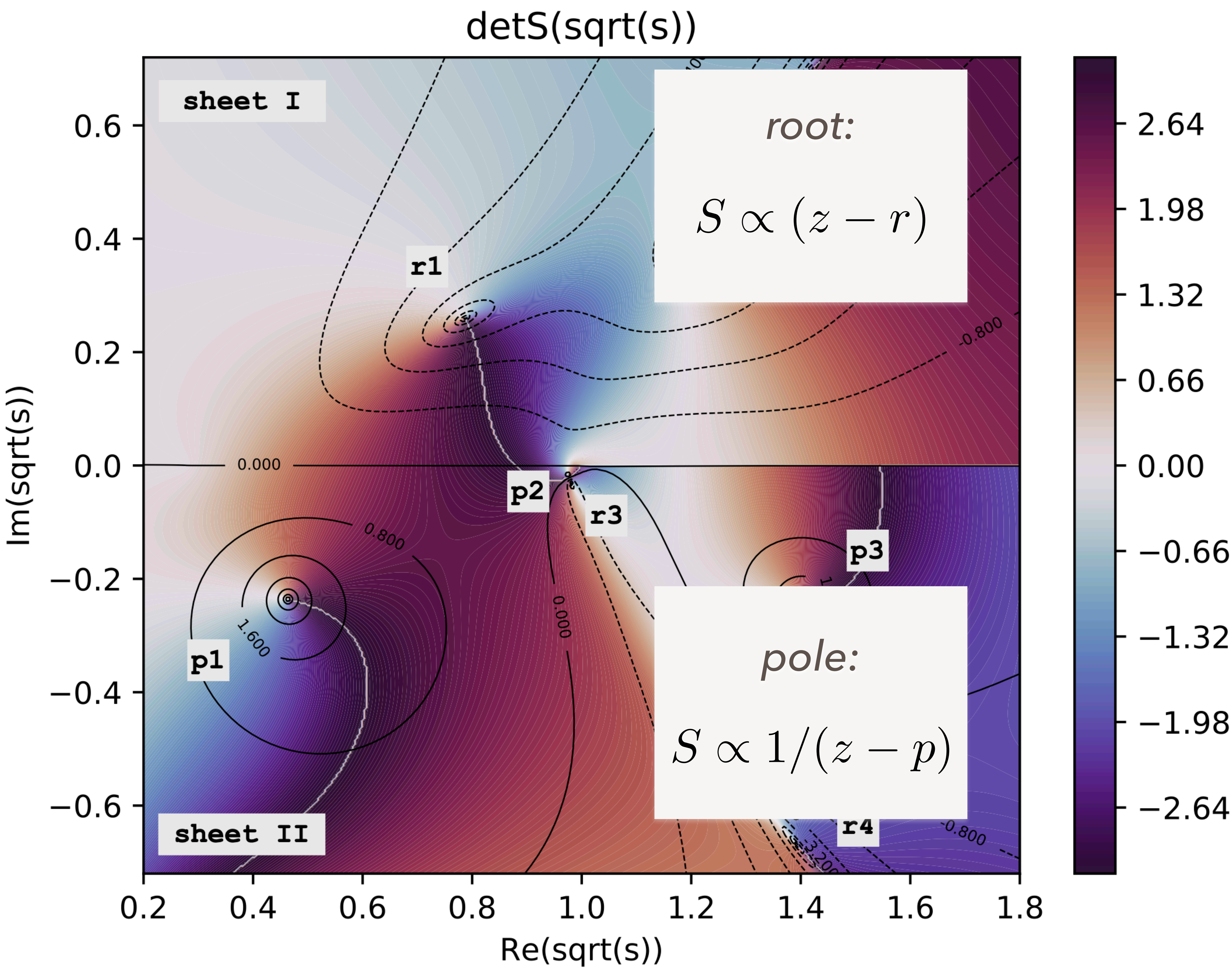


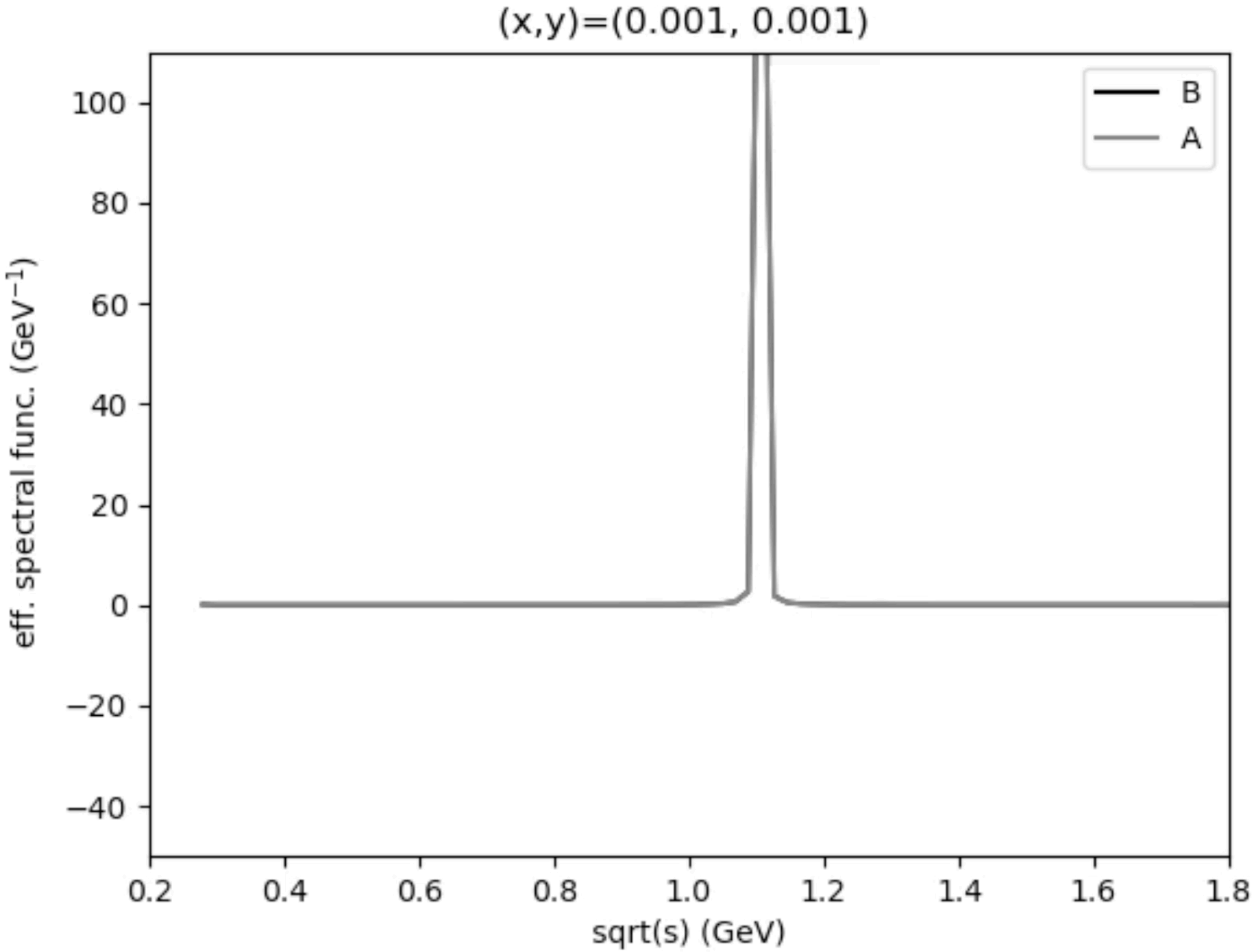
(x,y)=(0.539, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56



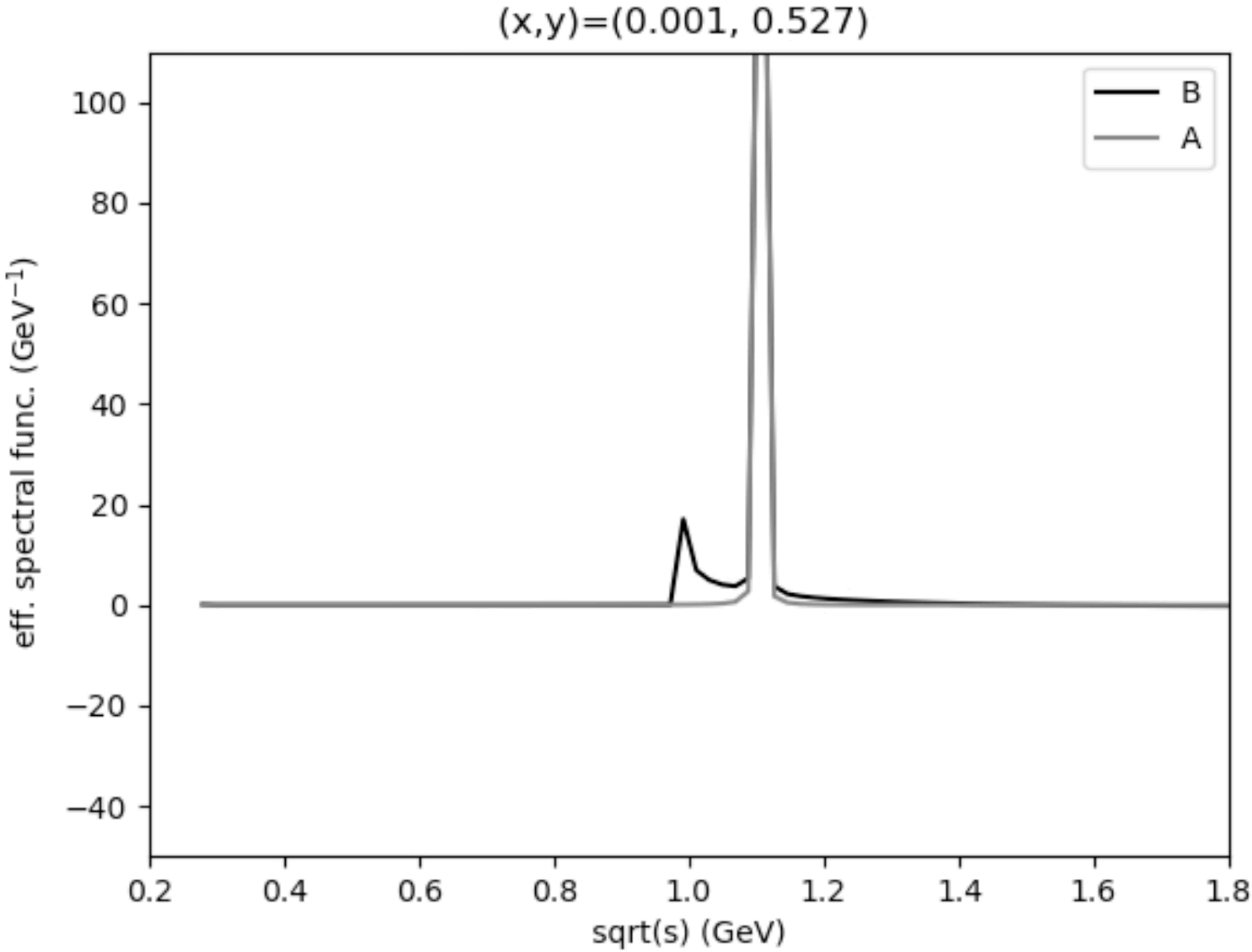
(x,y)=(0.718, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56


(x,y)=(1.0, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56

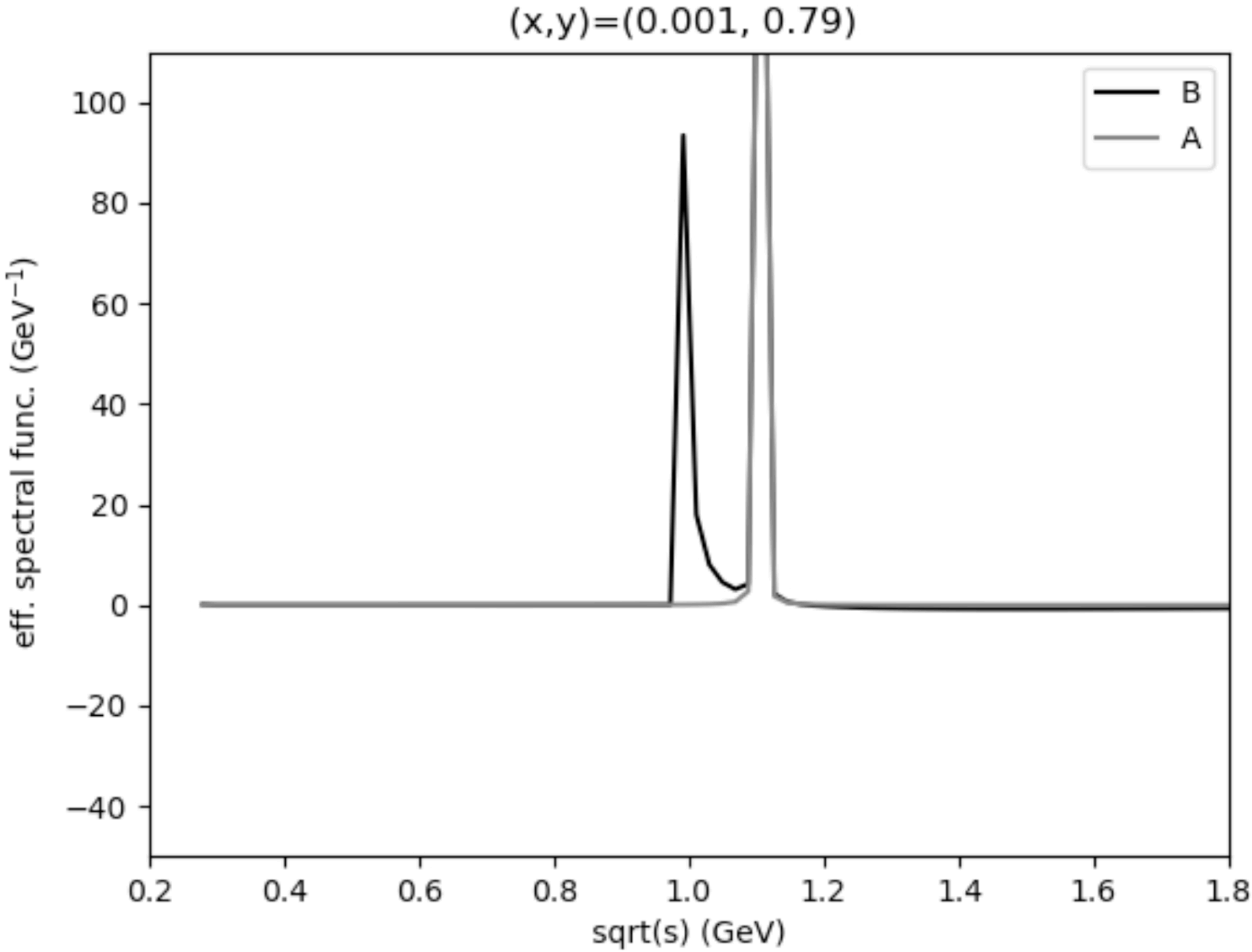

detS(sqrt(s))
sheet I
sheet II
root:
$S \propto (z - r)$
pole:
$S \propto 1/(z - p)$
r1
r3
r4
p1
p2
p3
0.000
0.800
1.600
-0.800
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.64
1.98
1.32
0.66
0.00
−0.66
−1.32
−1.98
−2.64

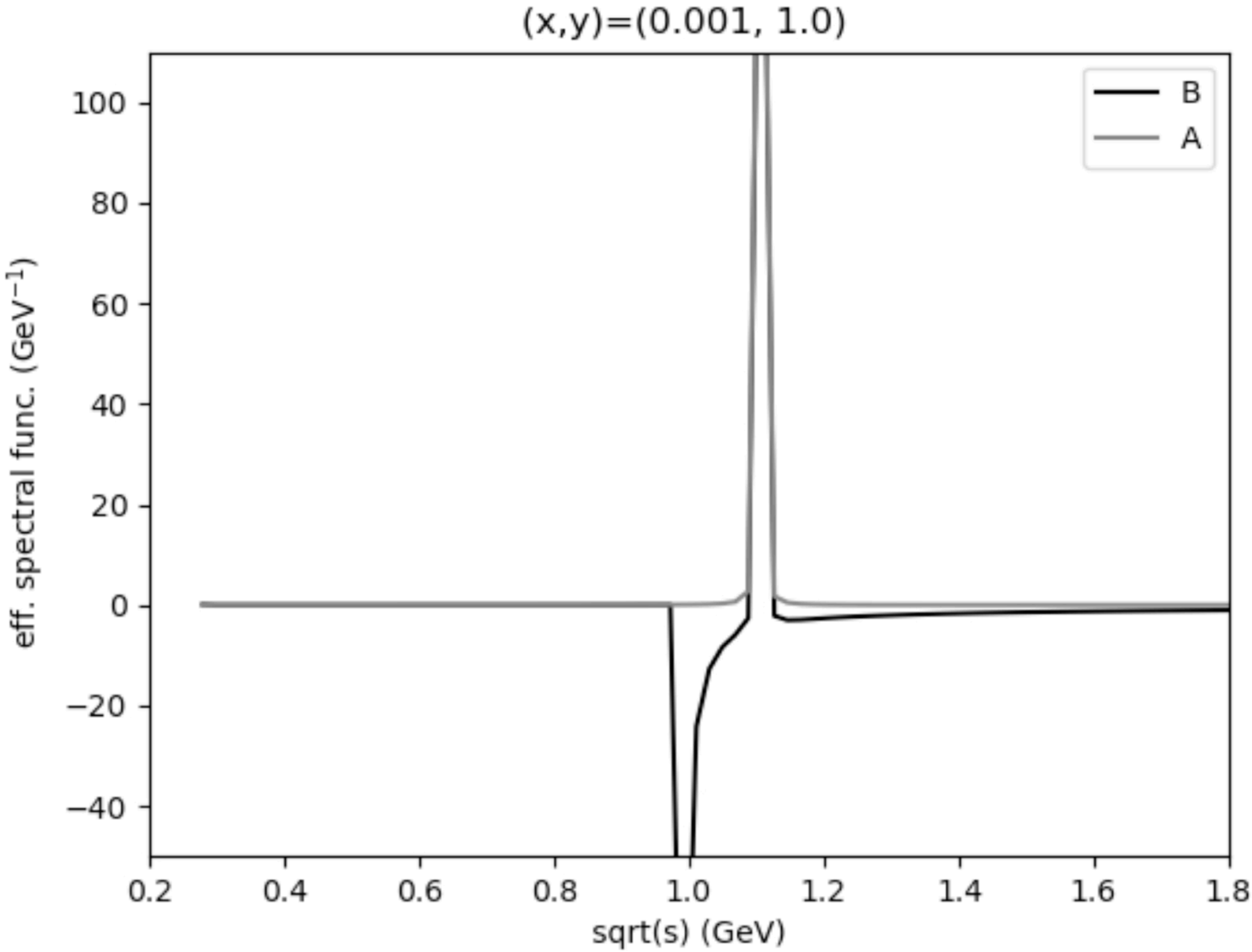


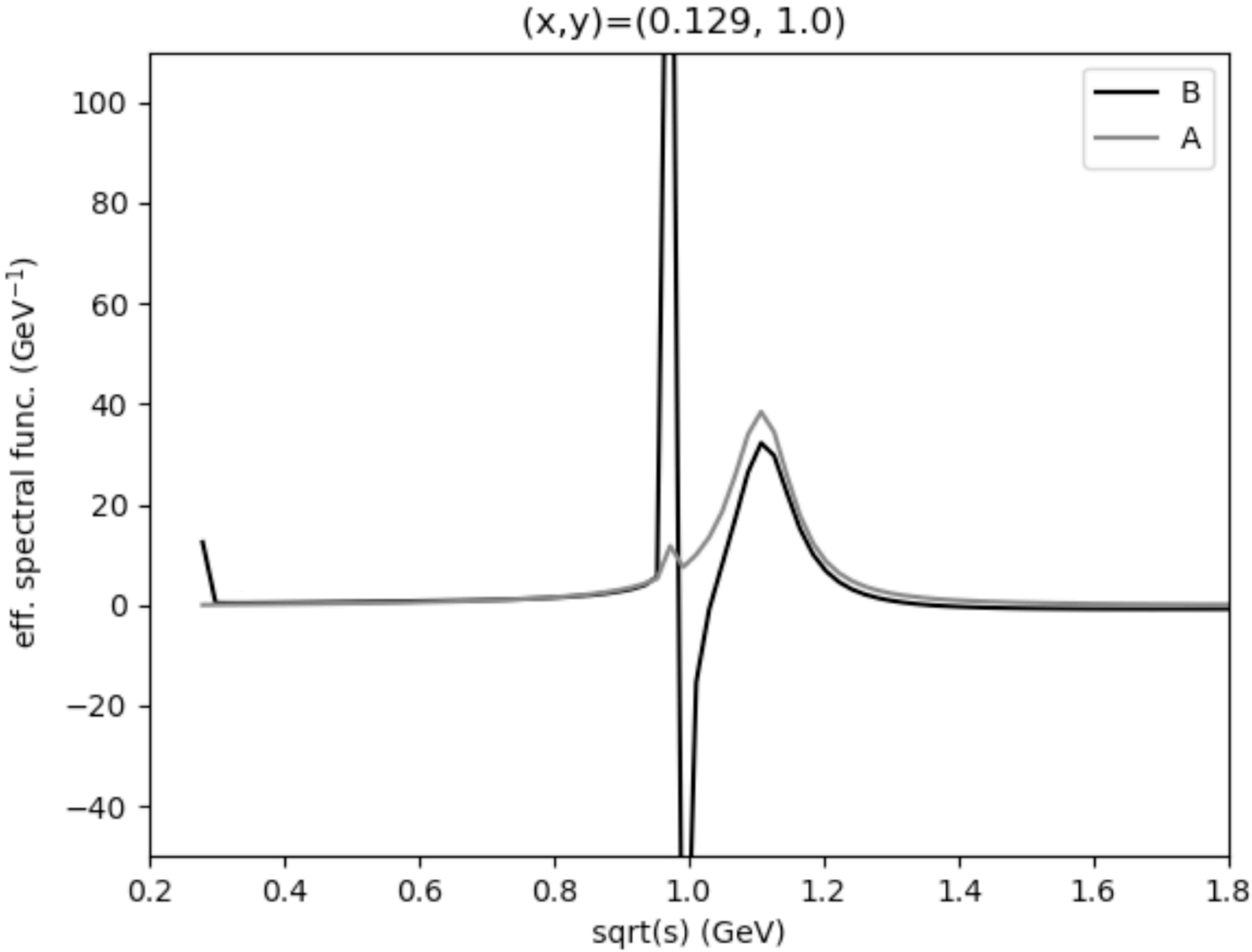
(x,y)=(0.001, 0.001)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)



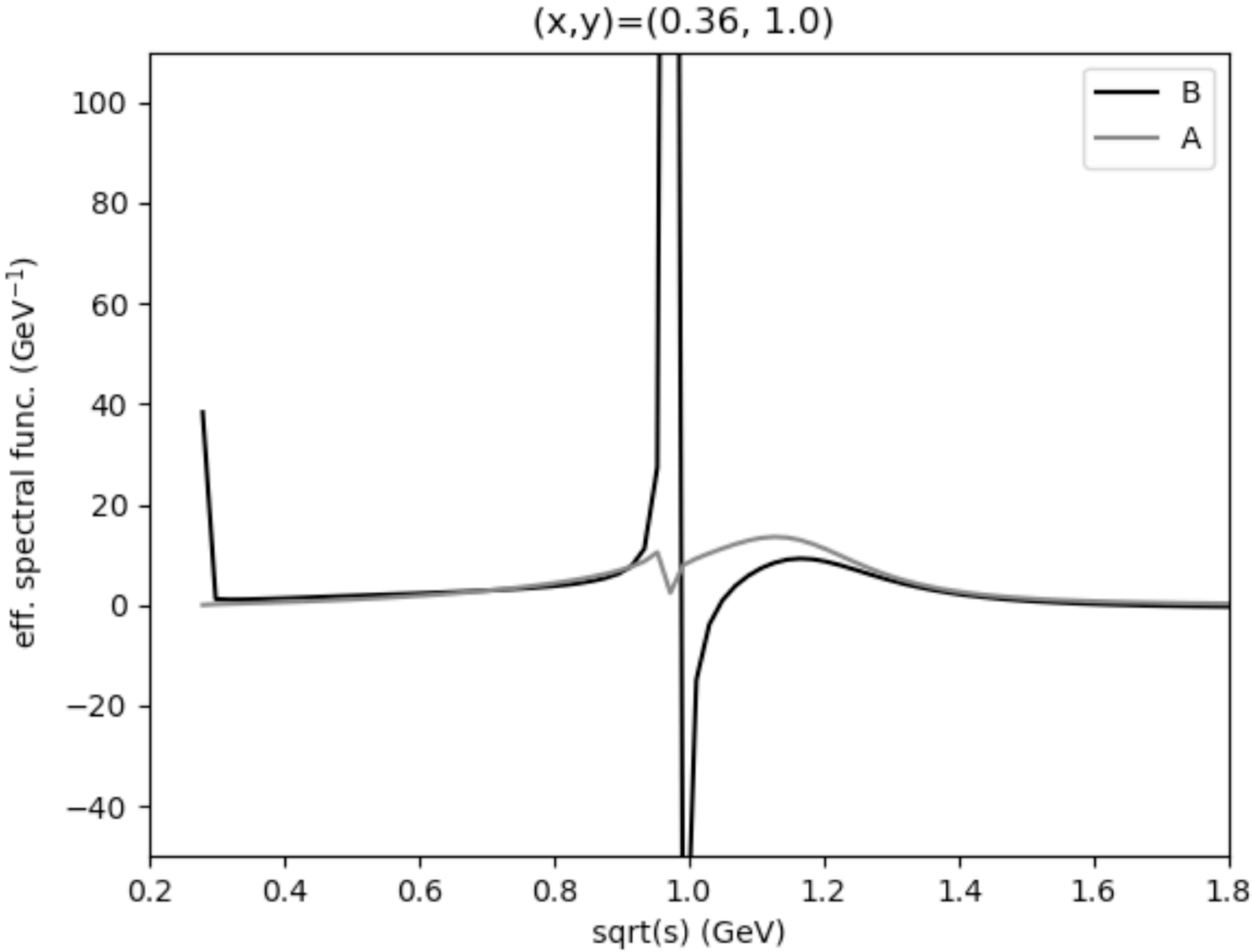
(x,y)=(0.001, 0.527)
B
A
eff. spectral func. (GeV^{-1})
sqrt(s) (GeV)
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

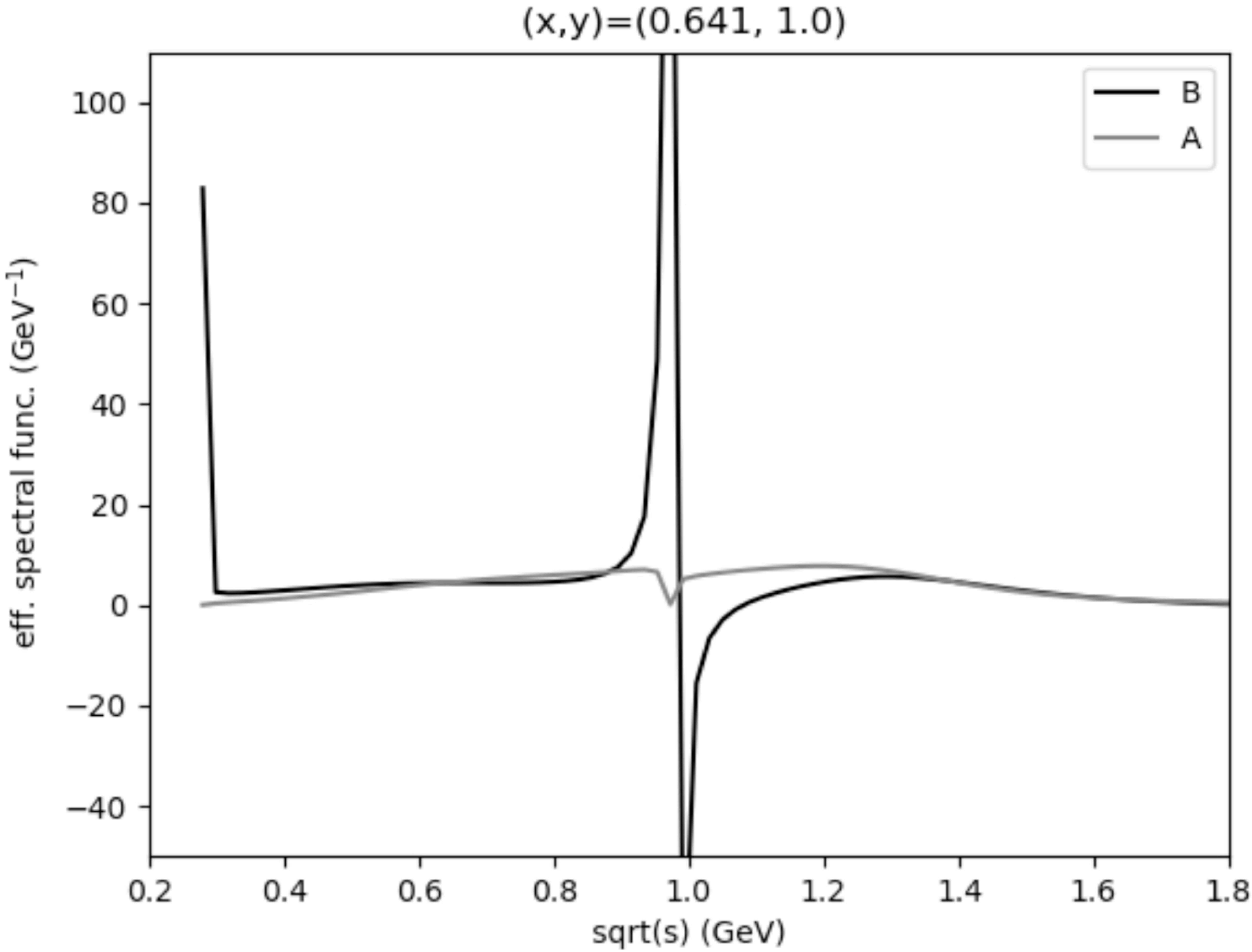

(x,y)=(0.001, 0.79)
eff. spectral func. (GeV^{-1})
sqrt(s) (GeV)
B
A
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8


(x,y)=(0.001, 1.0)
B
A
eff. spectral func. (GeV^{-1})
sqrt(s) (GeV)
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

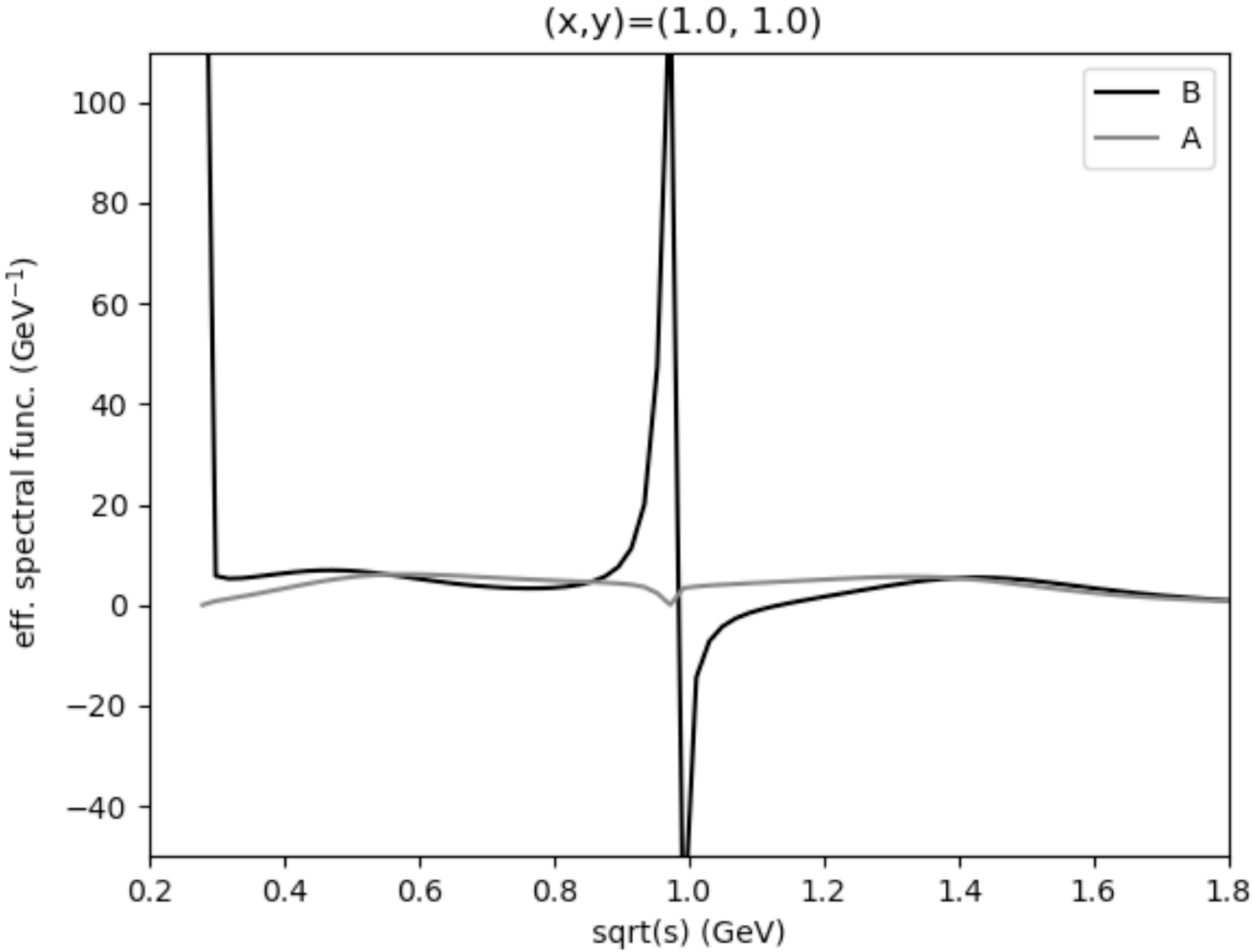


(x,y)=(0.129, 1.0)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)

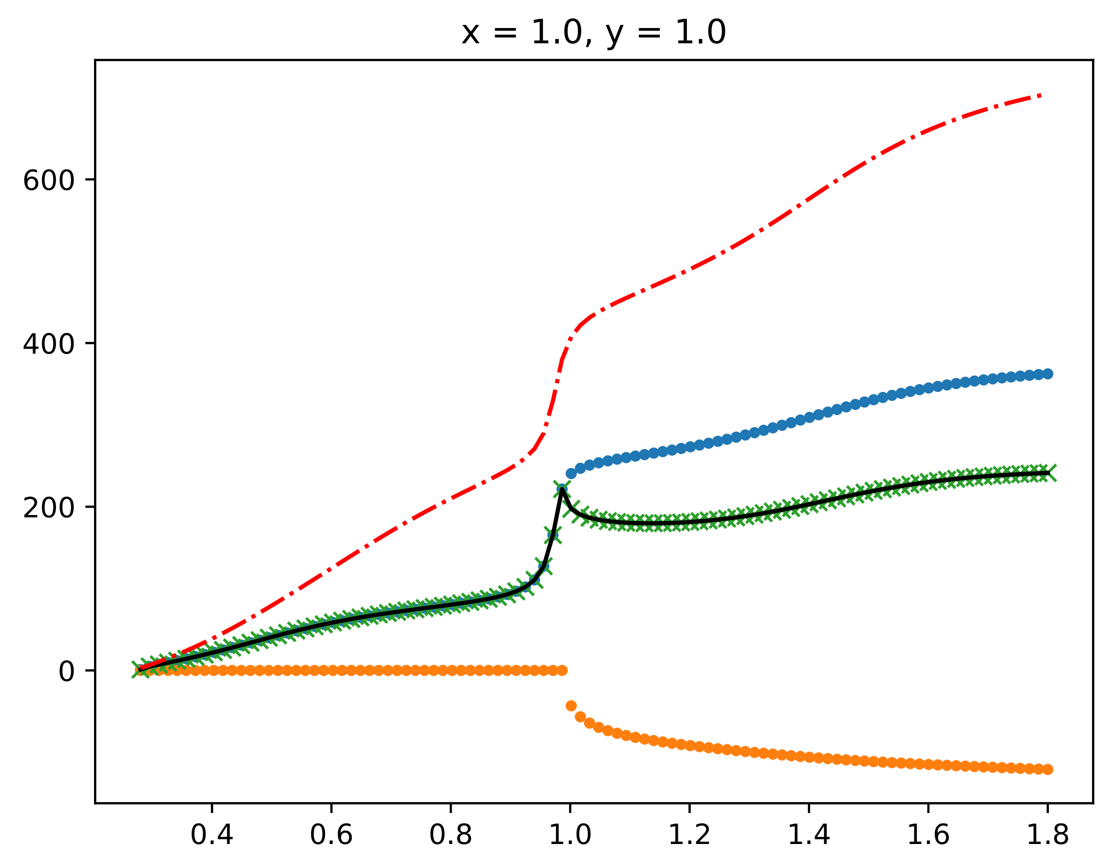

(x,y)=(0.36, 1.0)
B
A
eff. spectral func. (GeV^{-1})
sqrt(s) (GeV)
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8



(x,y)=(0.641, 1.0)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV⁻¹)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)



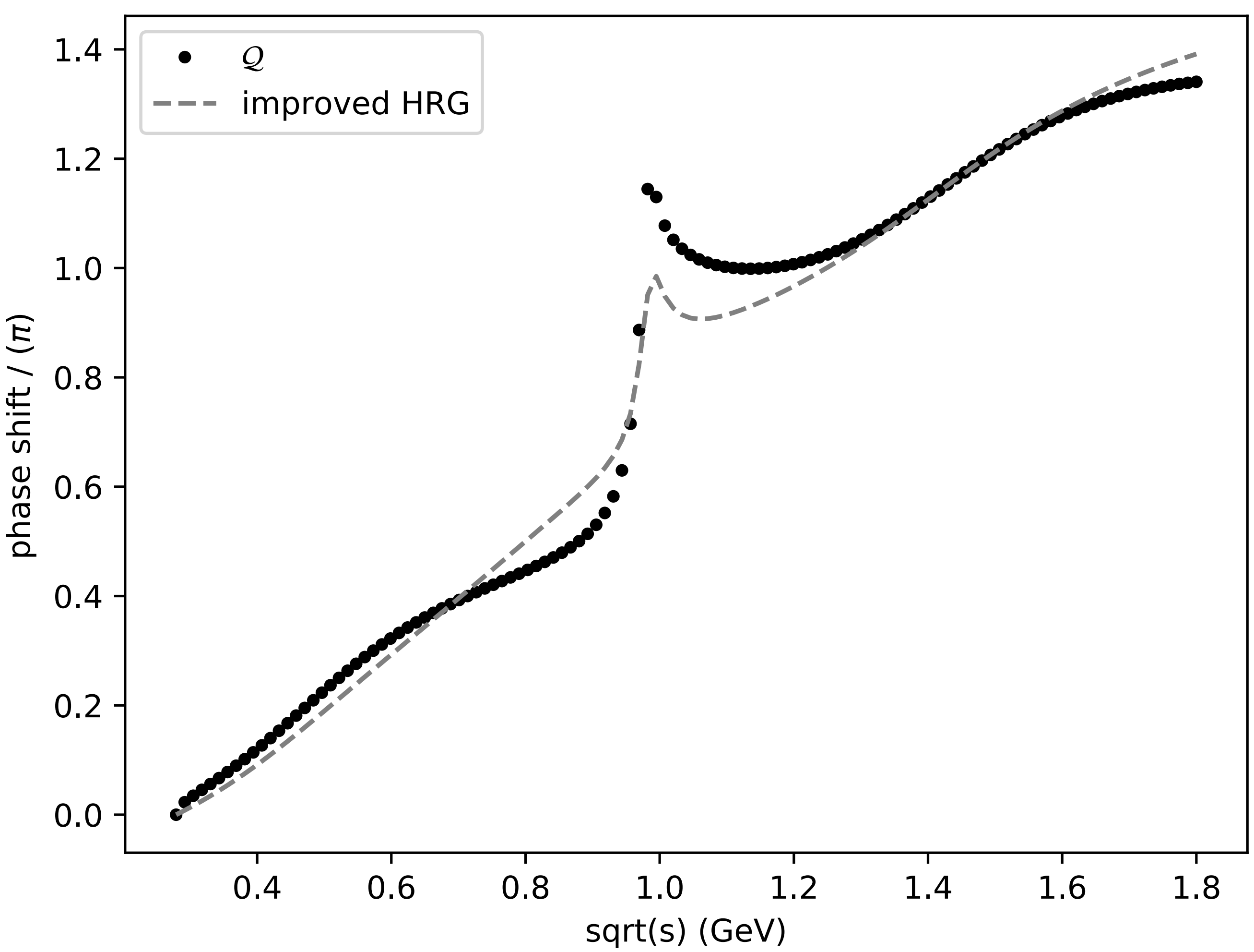
(x,y)=(1.0, 1.0)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)



f. [54, 55]

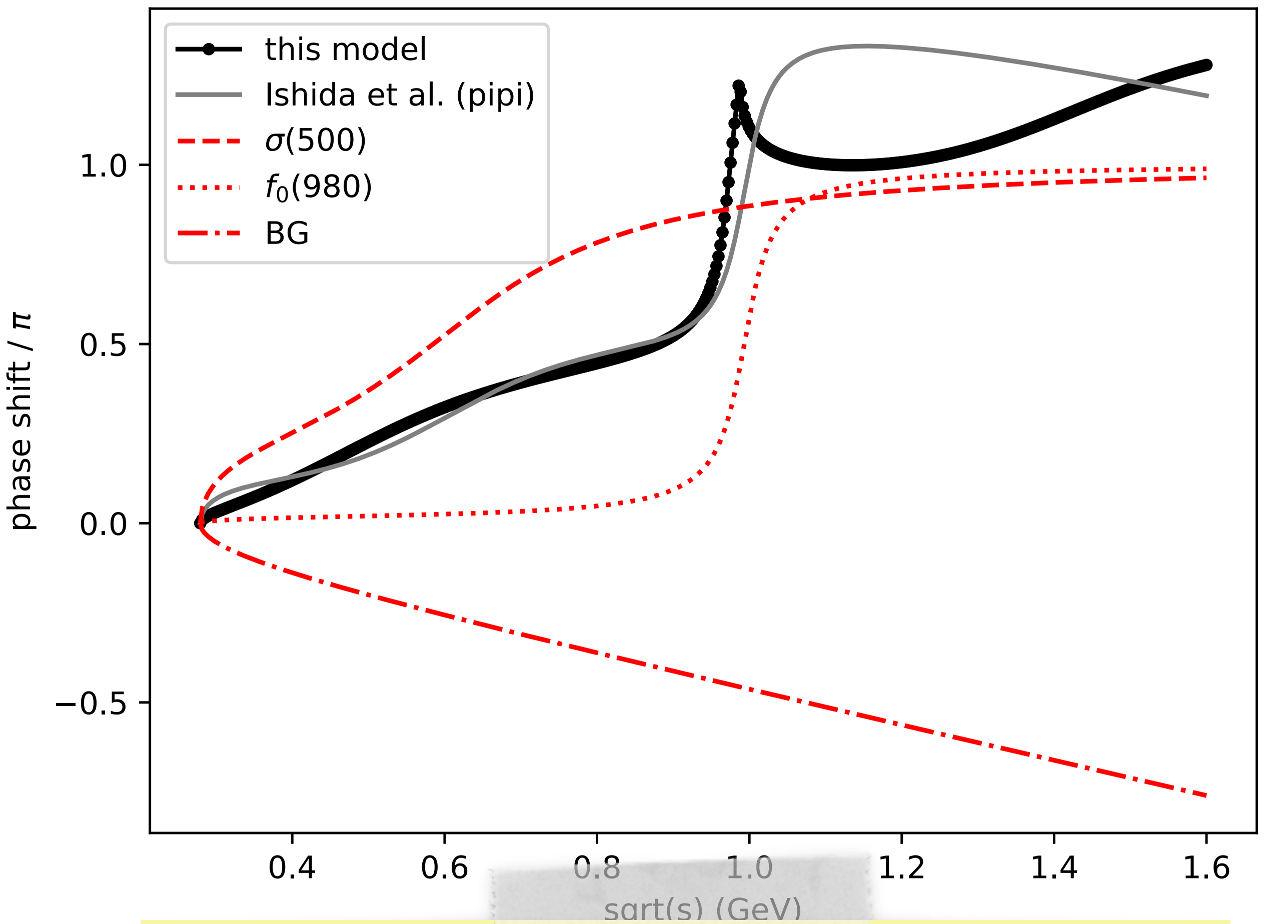
	Re $\sqrt{s}$	Im $\sqrt{s}$	sheet
p1	0.4637	-0.2357	II
p2	0.975	-0.0164	II
p3	1.401	-0.249	II
p4	0.6654	-0.2263	III
p5	1.4176	-0.2640	III
r1	0.787	+0.259	I
r2	1.410	+0.691	I
r3	0.981	-0.032	II
r4	1.393	-0.669	II
r5	0.918	+0.248	IV

II. Location of resonance poles (p_i) and roots (r_i)
l in the model.



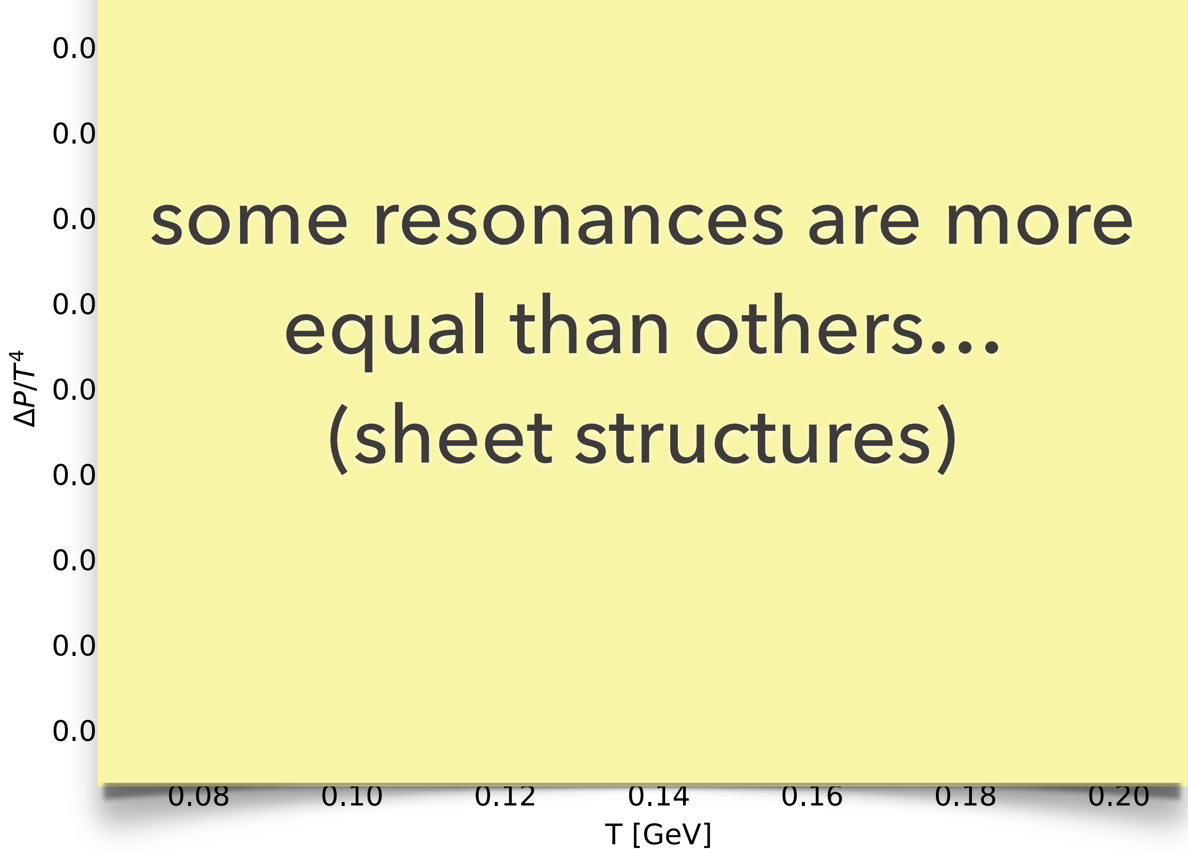
repulsive corrections in
HRG-like scheme:
via roots

	$\mathrm{Re}\sqrt{s}$	$\mathrm{Im}\sqrt{s}$	sheet

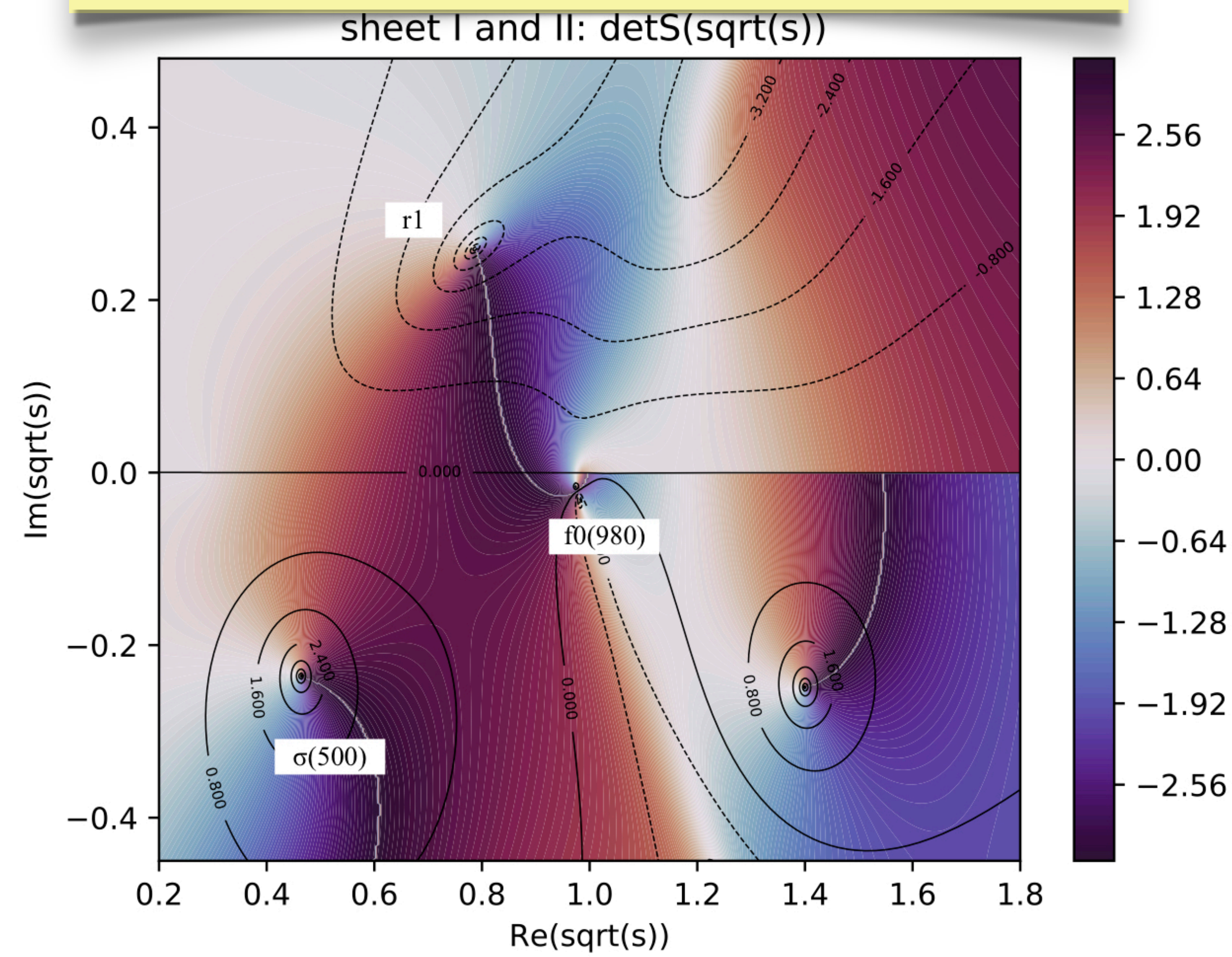


non-Briet-Wigner
=> poles and roots distribution

subtractive corrections

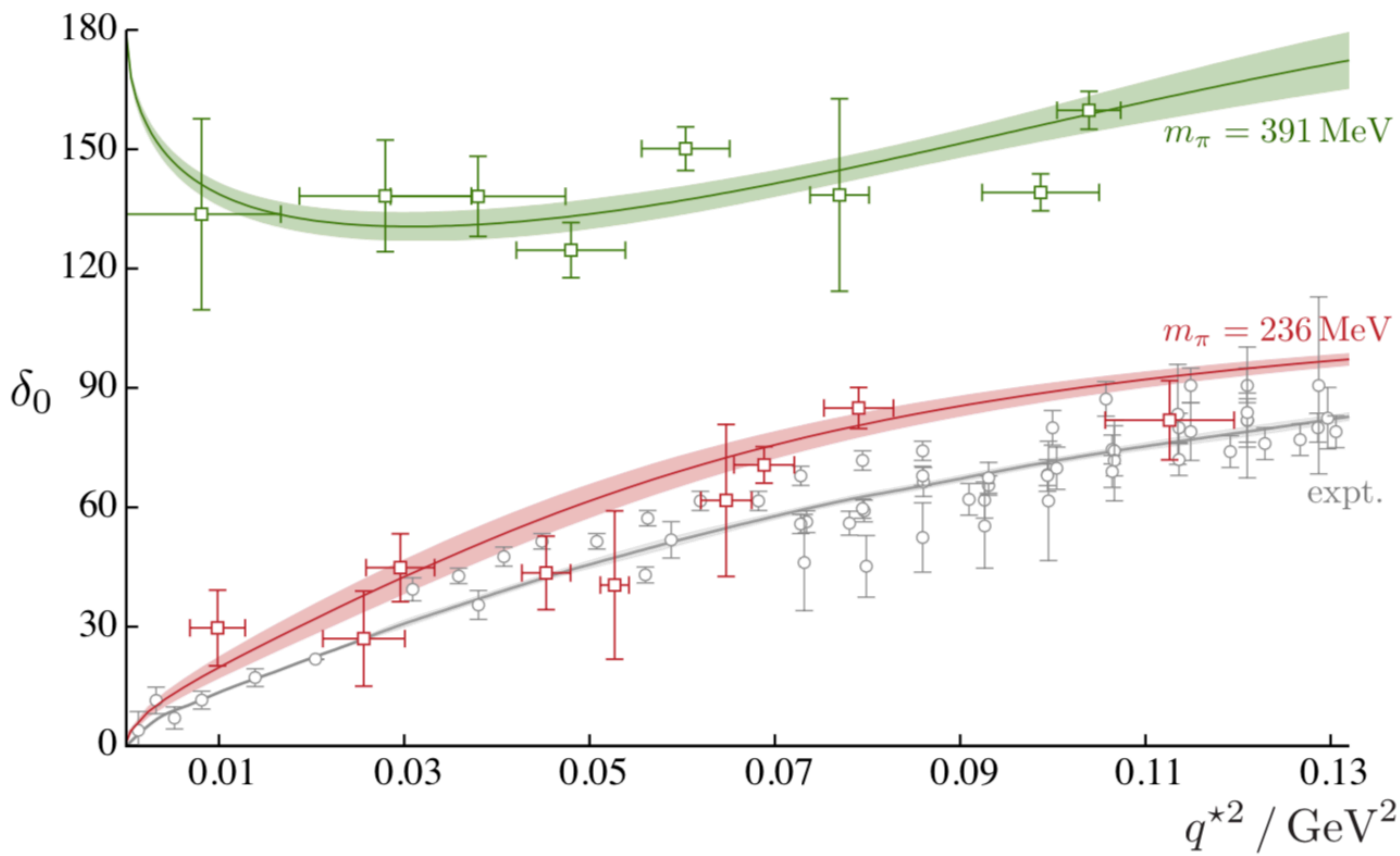


some resonances are more equal than others...
(sheet structures)



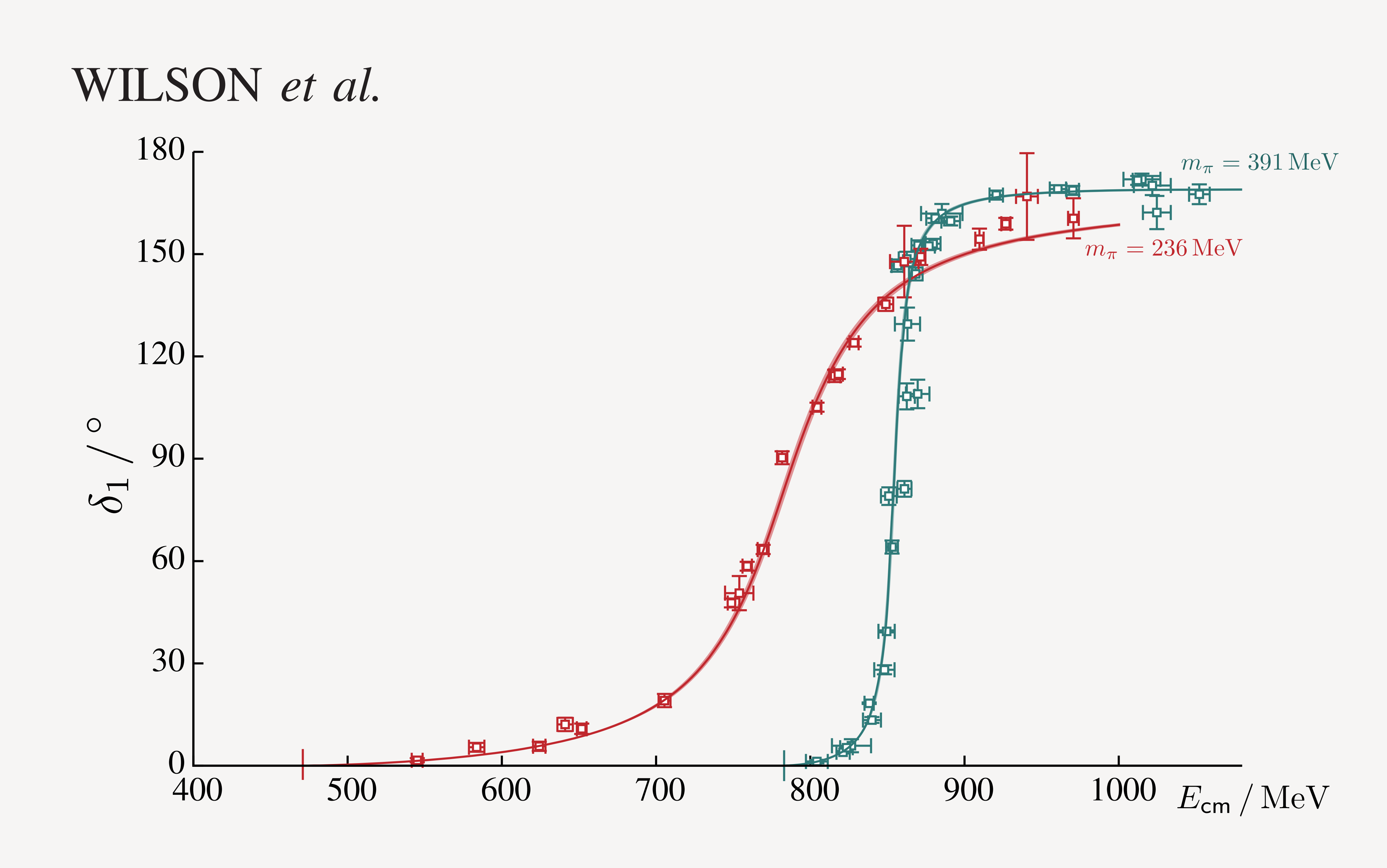
LATTICE COMPUTATIONS ON PHASE SHIFT

deuteron physics?



R. A. Briceno, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

LATTICE COMPUTATIONS ON PHASE SHIFT



NO SERIOUS MESON SPECTROSCOPY
WITHOUT SCATTERING*

George Rupp

CFIF, Instituto Superior Técnico, Universidade de Lisboa, 1049-001, Portugal

Eef van Beveren

CFC, Departamento de Física, Universidade de Coimbra, 3004-516, Portugal

Susana Coito

Institute of Modern Physics, CAS, Lanzhou 730000, China

(Received January 26, 2015)

CONTINUUM

$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

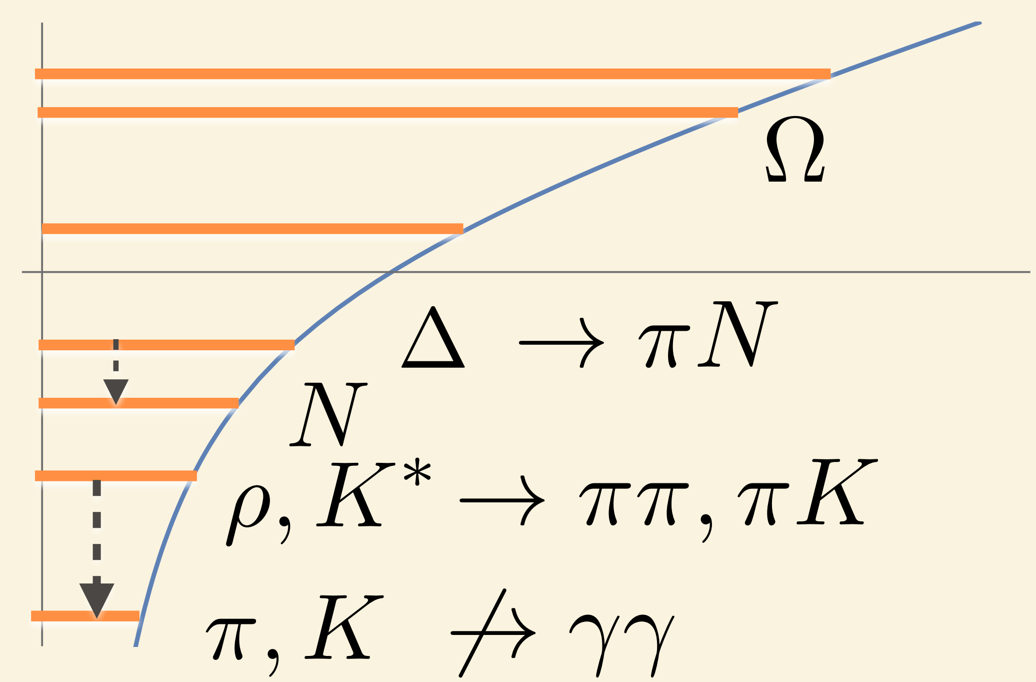
meson loops effects:

shift in hadron masses

Quark Model States:

mixing with continuum

QCD spectrum



IN-MEDIUM EFFECTS

VACUUM PHYSICS?

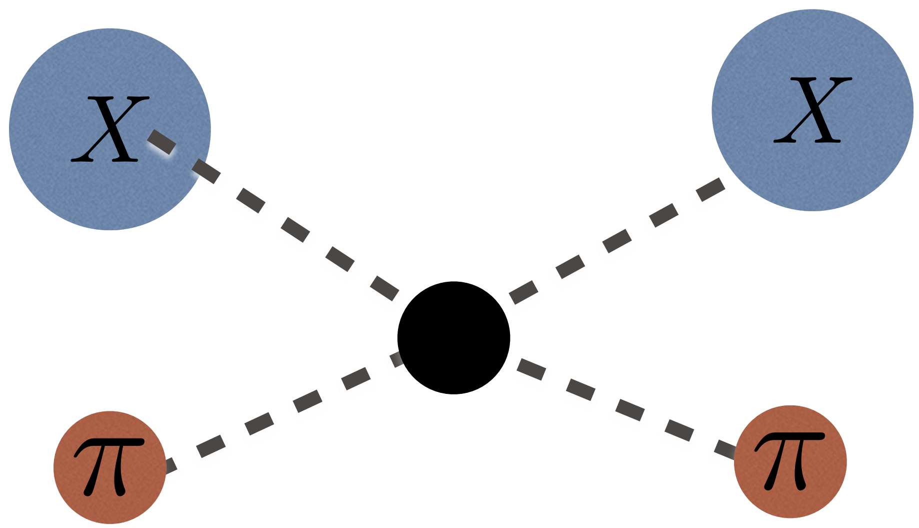
Quantum statistical mechanics of gases in terms of dynamical filling fractions and scattering amplitudes

André LeClair

Newman Laboratory, Cornell University, Ithaca, NY, USA

Received 22 November 2006, in final form 3 May 2007
Published 19 July 2007
Online at stacks.iop.org/JPhysA/40/9655

It helps to realize that at least in principle it is possible to decouple the zero temperature dynamics and the quantum statistical sums. The argument is simple: the computation of the partition function $Z = \mathrm{Tr}(\mathrm{e}^{-\beta \mathrm{H}})$ is in principle possible from the complete knowledge of the zero temperature eigenstates of the Hamiltonian H. In practice this is rather difficult and one resorts to perturbative methods such as the Matsubara method, which unfortunately entangles the zero temperature dynamics from the quantum statistical mechanics. However,



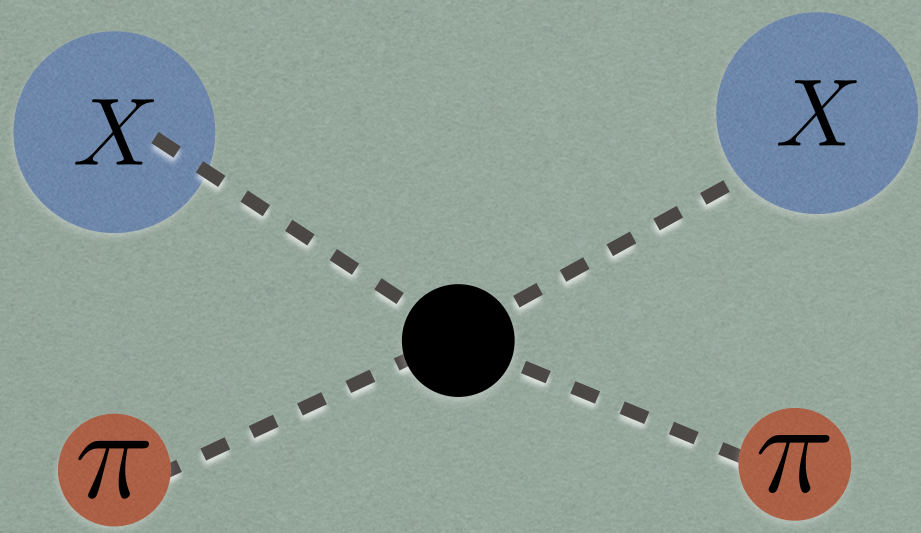
$$T_{\rm nr} \approx -\frac{4\pi f}{2m_{\rm red}}.$$

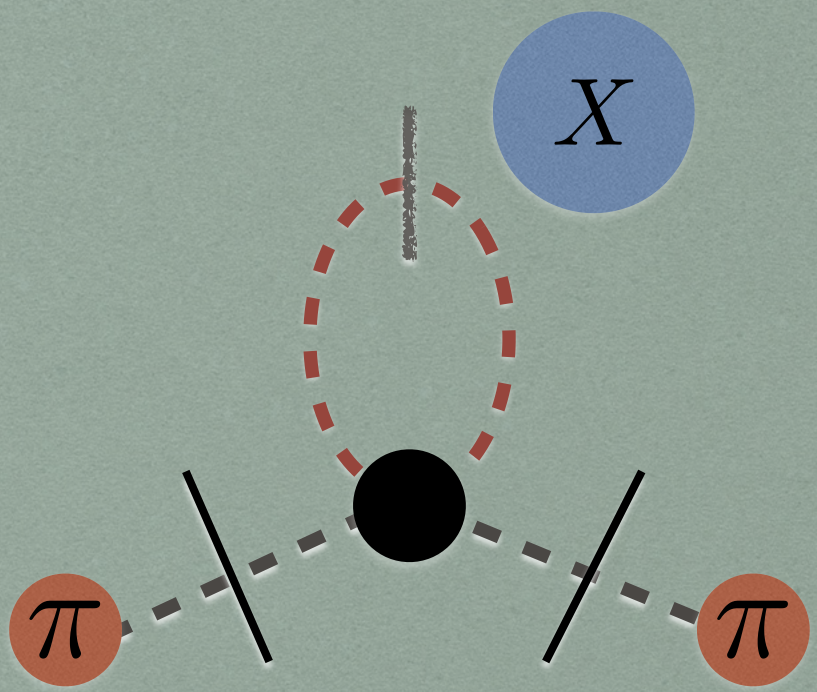
$$\begin{aligned}\Delta P &\approx \int \frac{d^3P}{(2\pi)^3}\frac{dE'}{(2\pi)}\, e^{-\beta(m_{\rm tot}+\frac{P^2}{2m_{\rm tot}}+E')} 2\mathcal{Q}(E') \\ &= \int \frac{d^3P}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\, e^{-\beta(m_{\rm tot}+\frac{P^2}{2m_{\rm tot}}+\frac{q^2}{2m_{\rm red}})}\,(-T_{\rm nr}) \\ &\approx N_{\rm th}^A N_{\rm th}^B \times (-T_{\rm nr}).\end{aligned}$$

$$\begin{aligned}\Delta P &\approx T\int \frac{d^3p_A}{(2\pi)^3}\, e^{-\beta(m_A+\frac{p_A^2}{2m_A})}\,(-\beta\Delta m_A) \\ &= -\Delta m_A N_{\rm th}^A \\ &= N_{\rm th}^A N_{\rm th}^B \times \frac{4\pi f}{2m_{\rm red}}.\end{aligned}$$

Change of pressure to due "Dressed mass"

IN-MEDIUM EFFECTS FROM S-MATRIX





$$\Delta P = N_{\rm th}^A \, N_{\rm th}^B \times \frac{4\pi f}{2m_{\rm red}}.$$

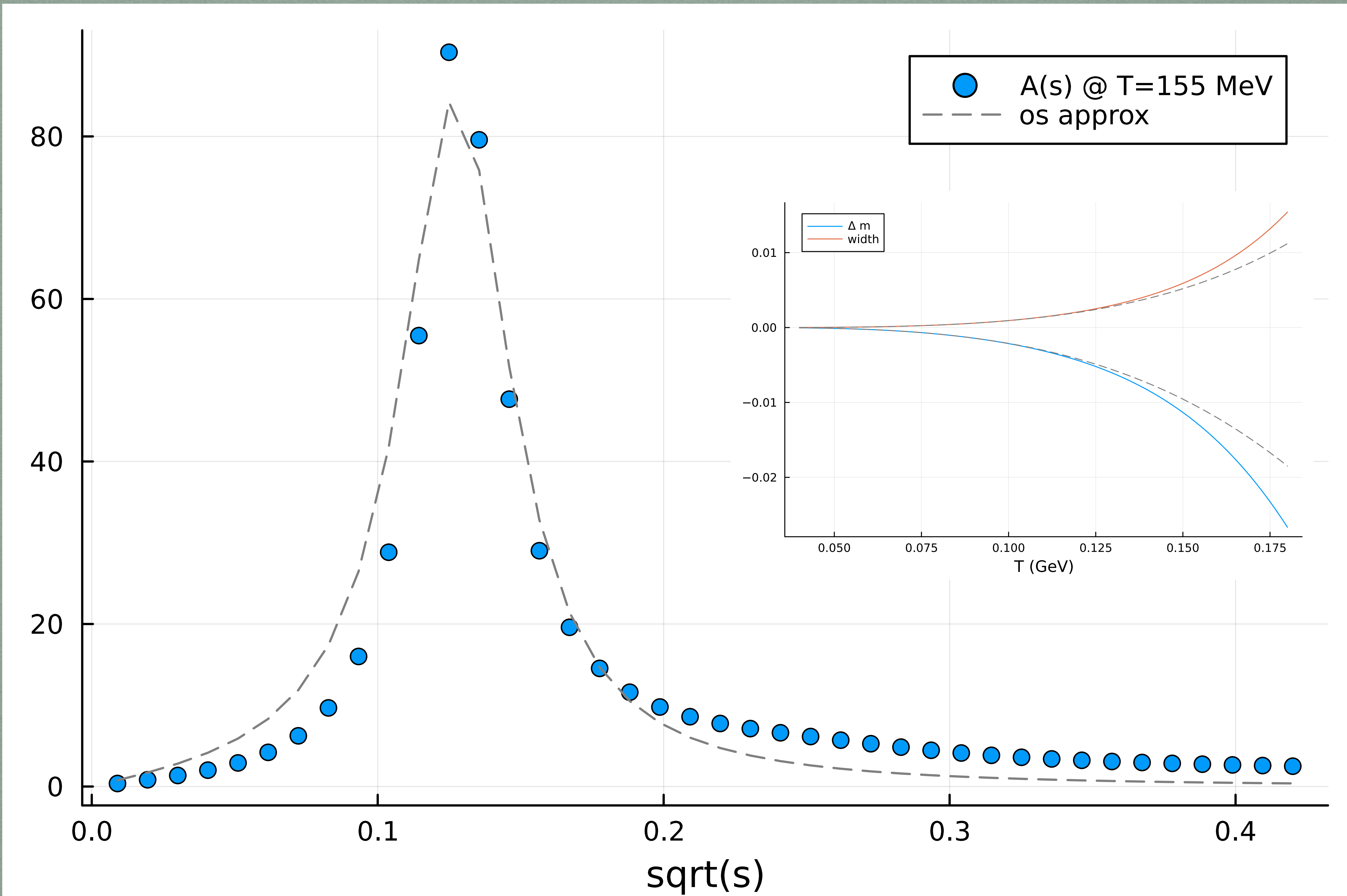
$$\Sigma_A(E_A) = \int \frac{d^3 k_B}{(2\pi)^3} \frac{1}{2E_B} \, n_{\rm th}(E_B) \, T(AB \to AB).$$

$$\Delta m_A = \frac{1}{2E_A} \, \mathrm{Re}\, \Sigma_A(p)$$

$$\approx N_{\rm th}^B \times \frac{-4\pi f}{2m_{\rm red}}.$$

A. Schenk NPB 363(1991)

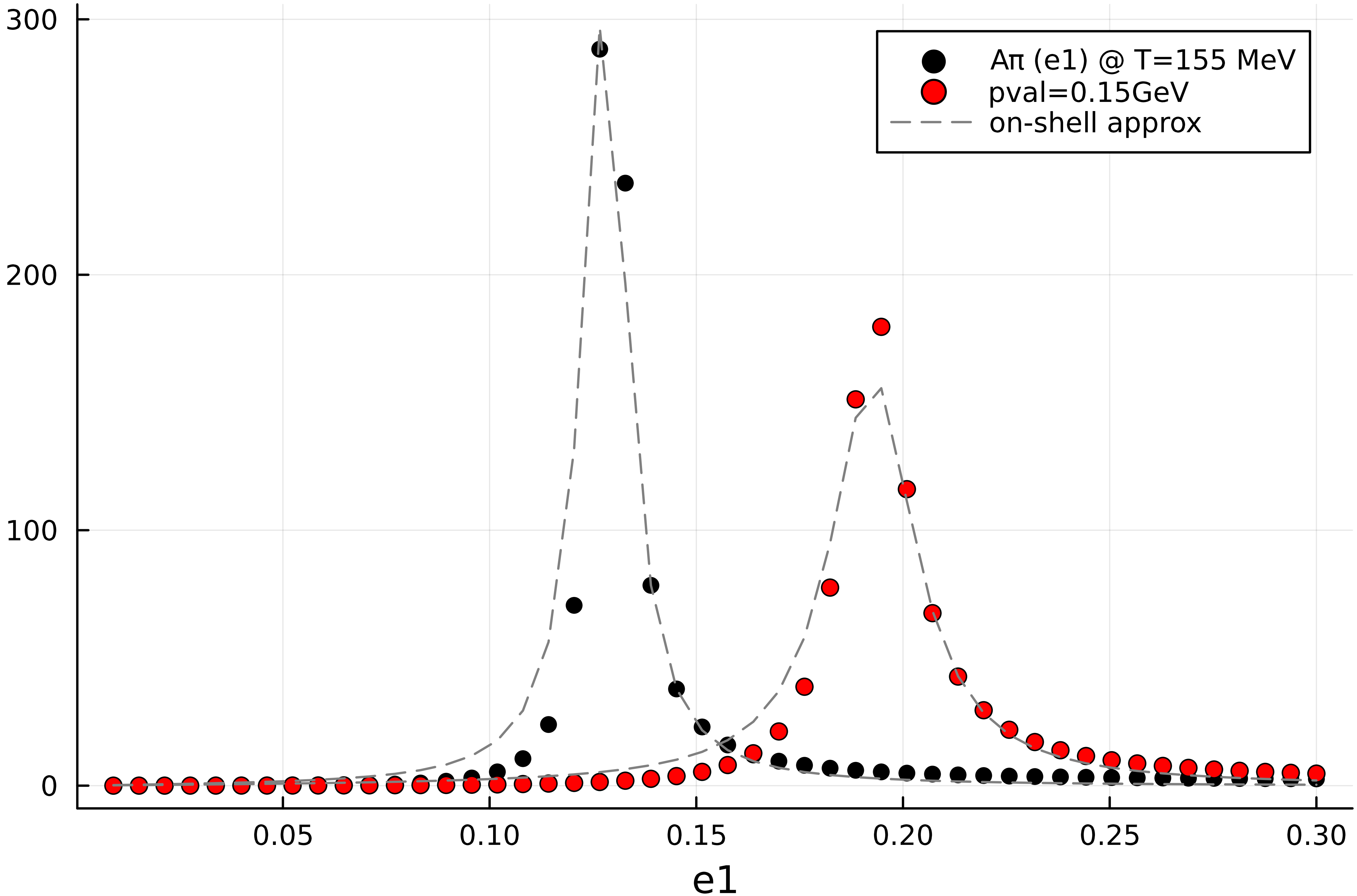
S. Jeon and P. J. Ellis PRD 58 045013 (1998)

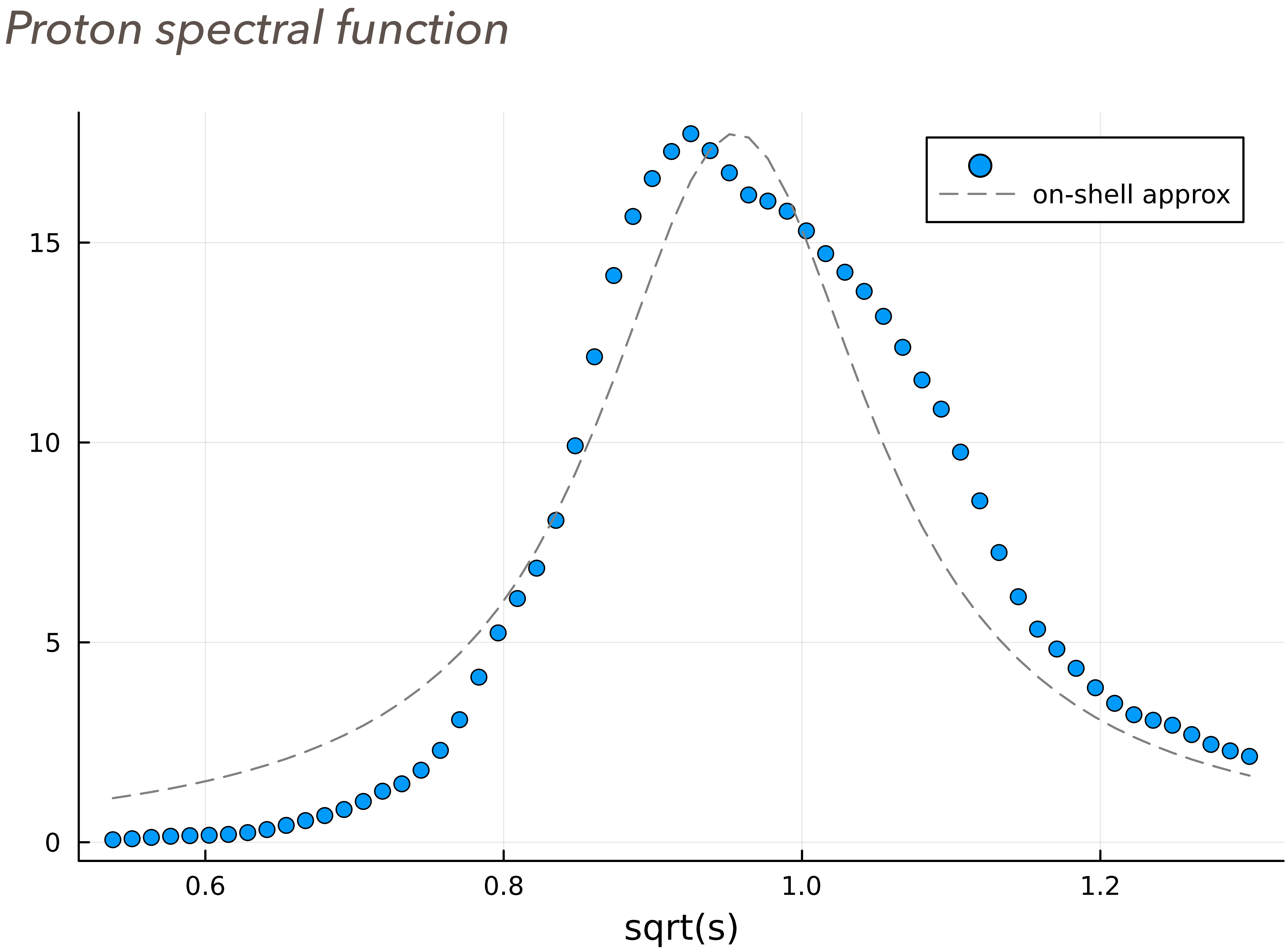


A. Schenk NPB 363(1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

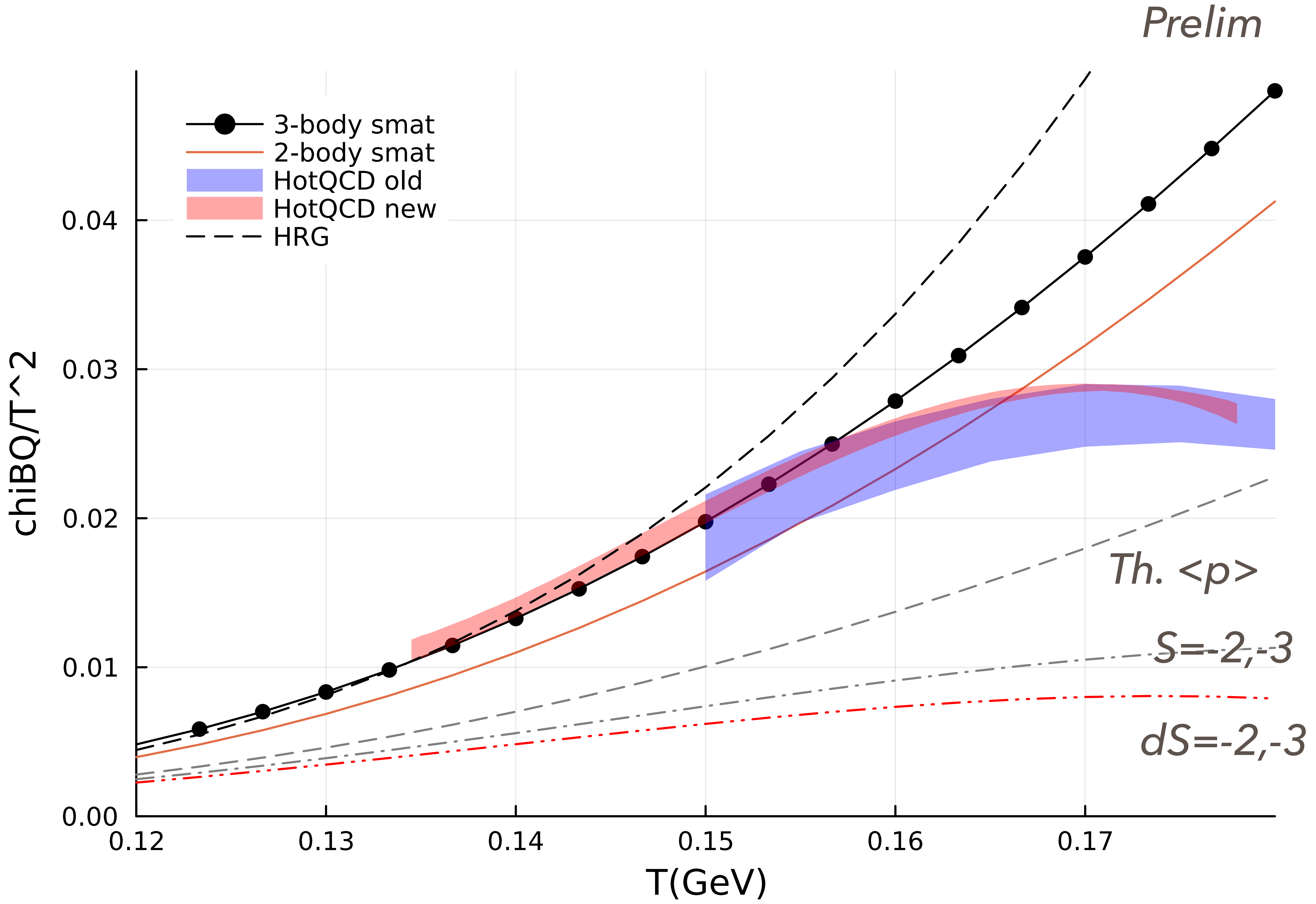
Pion spectral function

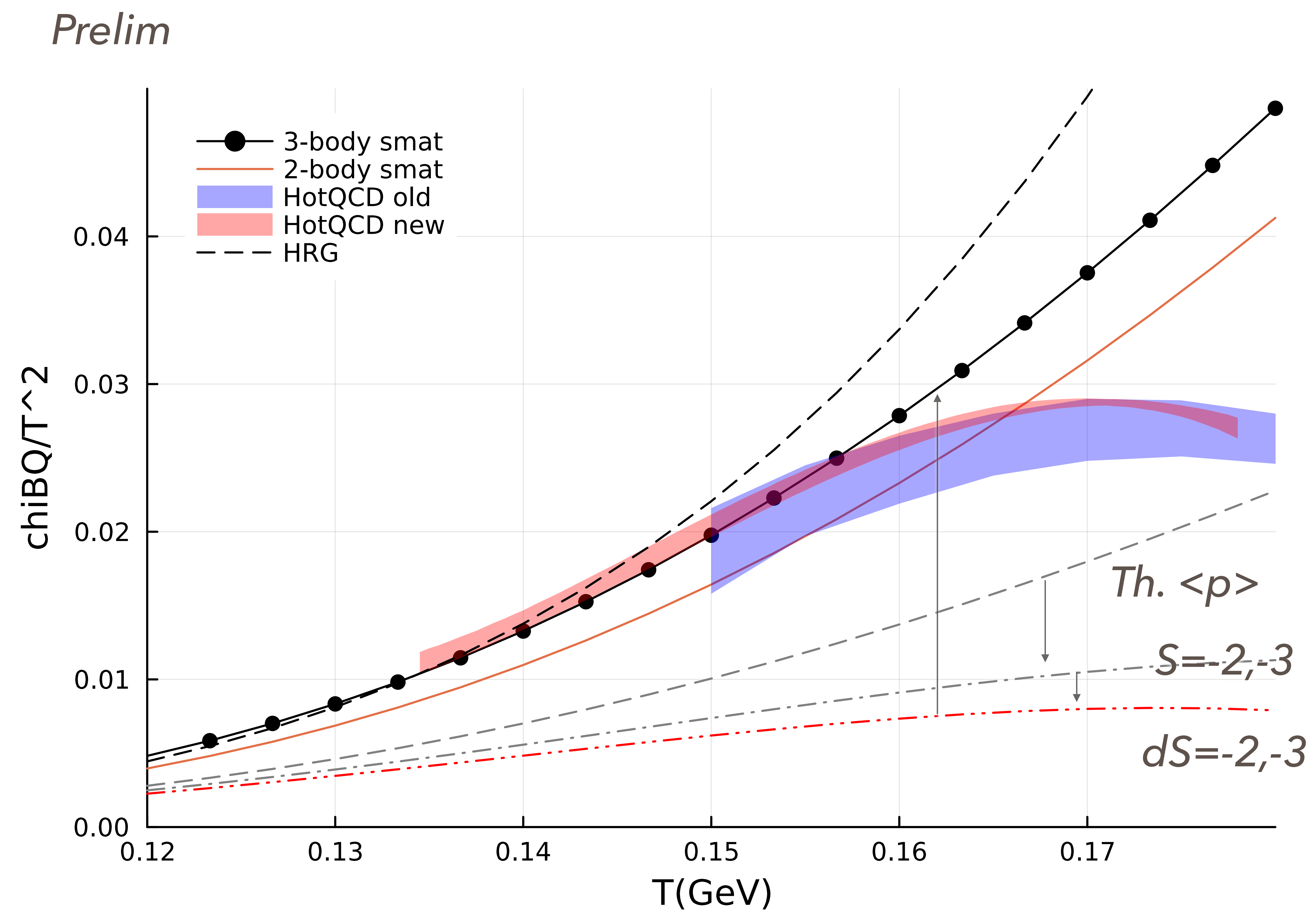




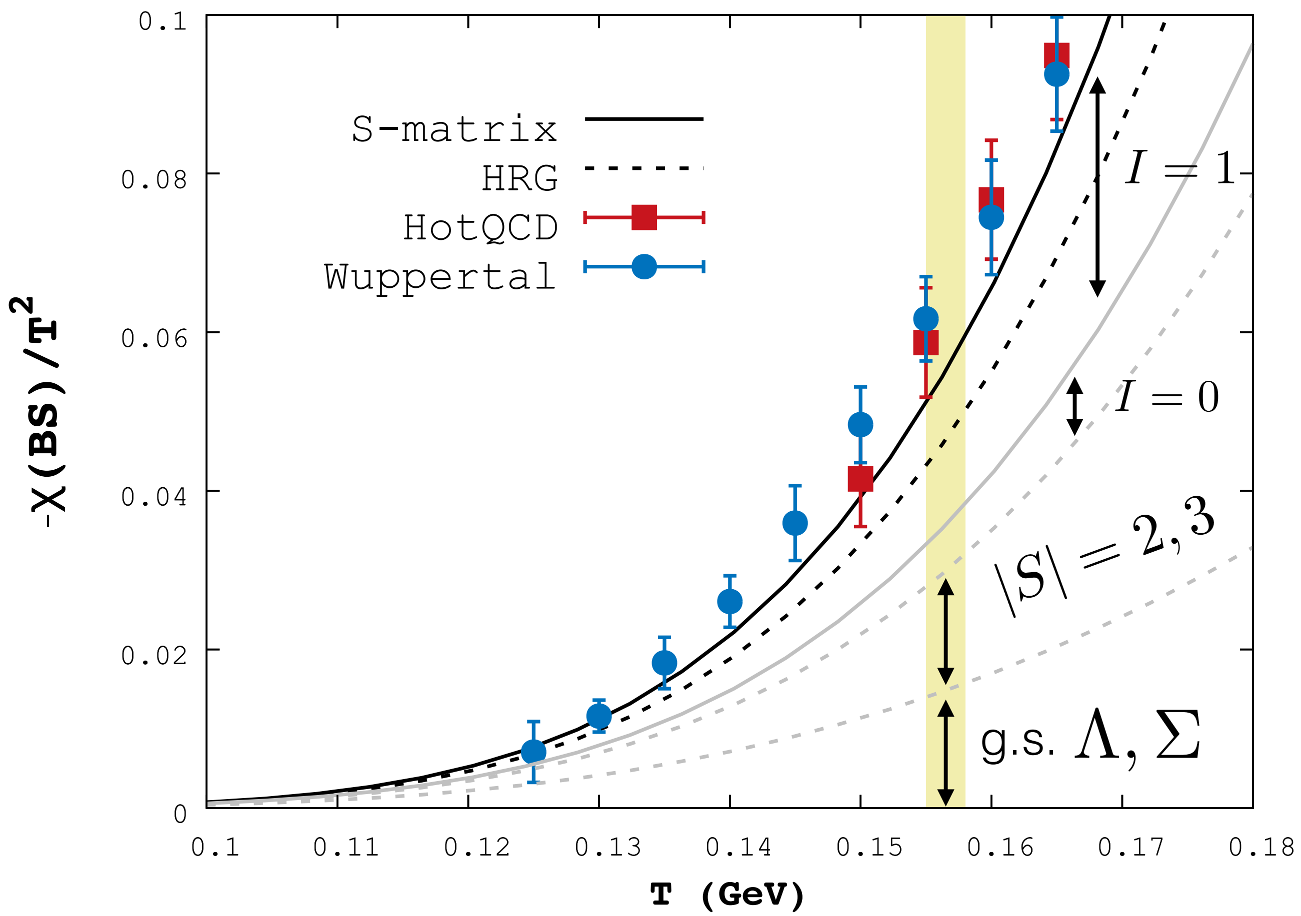
Proton spectral function
on-shell approx
15
10
5
0
0.6
0.8
1.0
1.2
sqrt(s)

A SMALL UPDATE

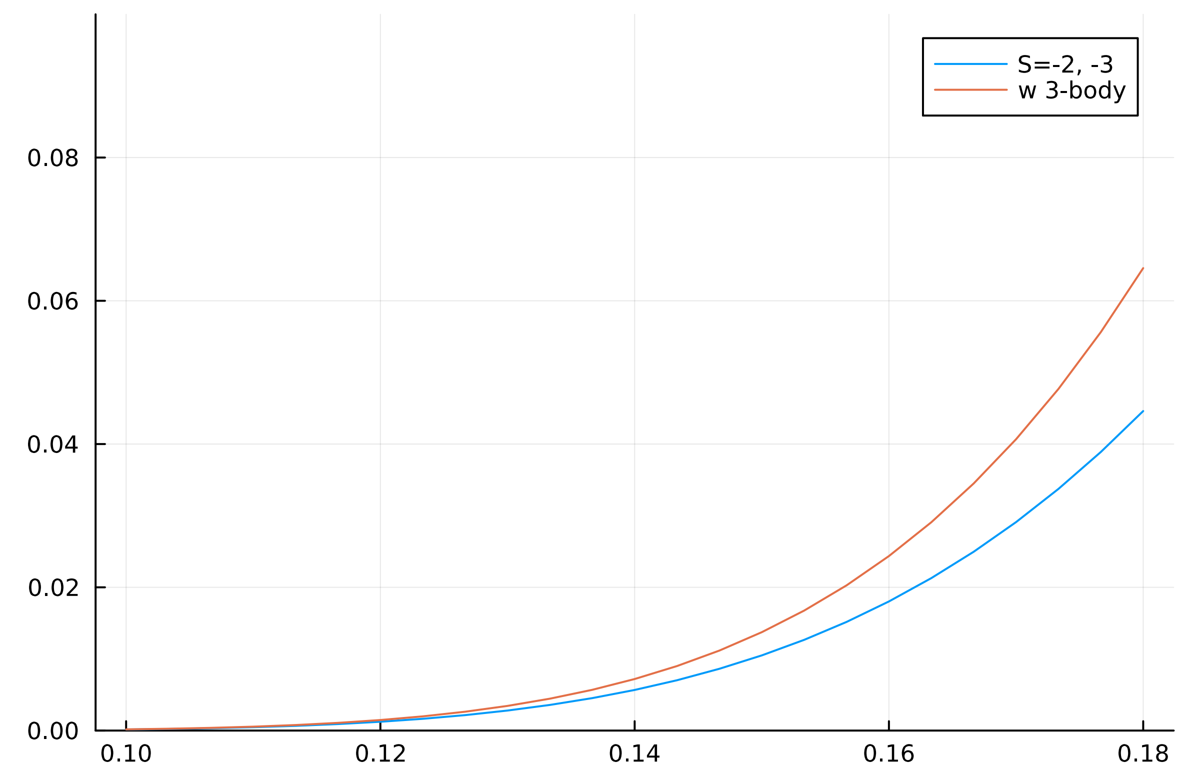

Prelim
3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
chiBQ/T^2
0.04
0.03
0.02
0.01
0.00
0.12
0.13
0.14
0.15
0.16
0.17
T(GeV)
Th. <p>
S=-2,-3
dS=-2,-3



Prelim
3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
Th. <p>
S=-2,-3
dS=-2,-3
chiBQ/T^2
T(GeV)
0.00
0.01
0.02
0.03
0.04
0.12
0.13
0.14
0.15
0.16
0.17



0.1
0.08
0.06
0.04
0.02
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
T (GeV)
-χ(BS)/T^2
S-matrix
HRG
HotQCD
Wuppertal
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ



0.08
0.06
0.04
0.02
0.00
0.10
0.12
0.14
0.16
0.18
S=-2, -3
w 3-body

THANK YOU!