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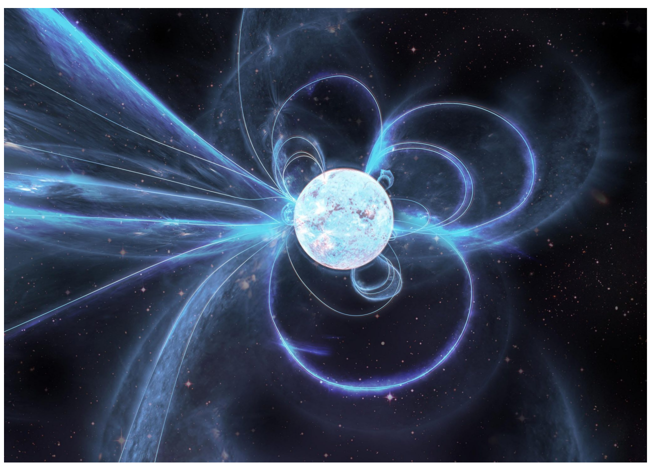
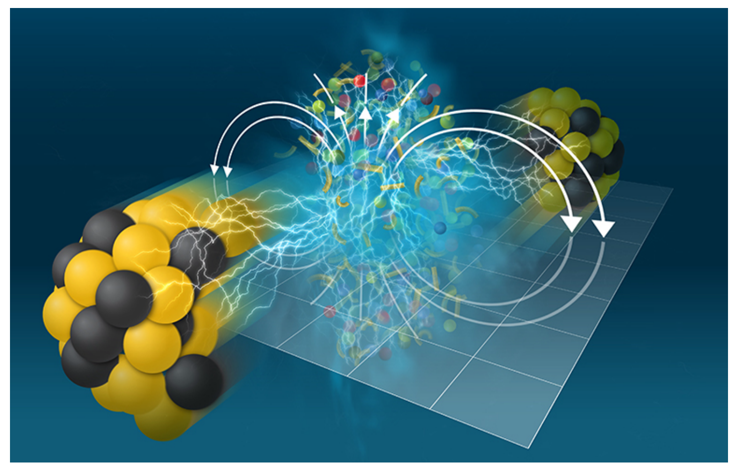
# Superconducting baryon crystal at strong magnetic field

G. W. Evans and A. Schmitt, JHEP 09, 192 (2022), JHEP 02, 041 (2024)



Carl Knox/OzGrav



T. Bowman and J. Abramowitz/BNL

This talk: QCD at nonzero $B$, $\mu$ ($T = 0$) via chiral perturbation theory

# Chiral perturbation theory with chiral anomaly

$N_f = 2$ chiral perturbation theory + electromagnetism + chiral anomaly
D.T. Son, M.A. Stephanov, PRD 77, 014021 (2008)
T. Brauner, N. Yamamoto, JHEP 04, 132 (2017)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\nabla_\mu\Sigma^\dagger\nabla^\mu\Sigma] + \frac{m_\pi^2 f_\pi^2}{4}\mathrm{Tr}[\Sigma + \Sigma^\dagger] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(A_\mu^B - \frac{e}{2}A_\mu\right) j_B^\mu$$

with $\nabla^\mu\Sigma = \partial^\mu\Sigma - i[\mathcal{A}^\mu, \Sigma]$ and anomalous baryon current
J. Wess, B. Zumino, PLB 37, 95 (1971); E. Witten, NPB 223, 422 (1983)
J. Goldstone, F. Wilczek, PRL 47, 986 (1981)

parametrize $\pi^0, \pi^\pm \to \alpha \in \mathbb{R}, \varphi \in \mathbb{C}$

anomalous current receives contributions from magnetic field and vorticity

$$j_B^\mu = -\frac{\epsilon^{\mu\nu\rho\lambda}}{4\pi^2}\partial_\nu\alpha\left(\frac{e}{2}F_{\rho\lambda} + \frac{\partial_\rho j_\lambda}{e f_\pi^2}\right)$$

# Chiral soliton lattice (page 1/2)

T. Brauner, N. Yamamoto, JHEP 04, 132 (2017)

free energy without charged pions

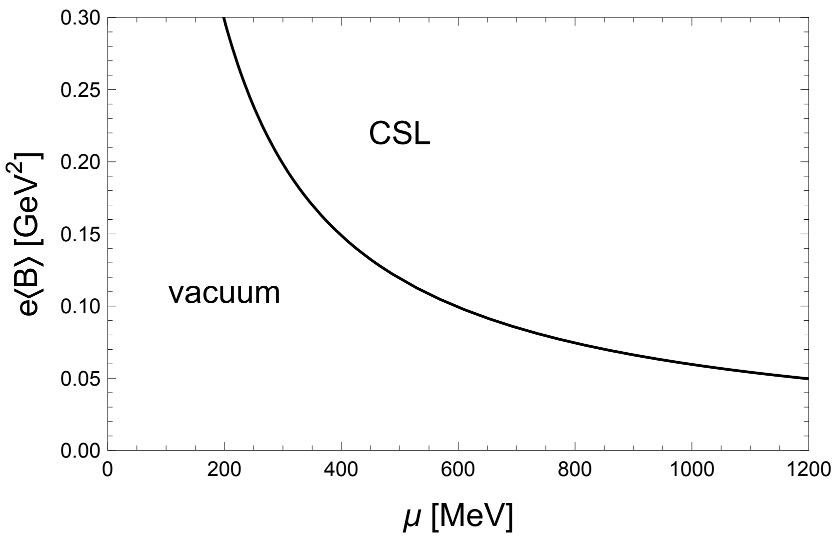
$$\Omega_0(\vec{x}) = \frac{f_\pi^2}{2}[(\nabla\alpha)^2 - 2m_\pi^2(\cos\alpha - 1)] + \frac{B^2}{2} - \frac{e\mu}{4\pi^2}\nabla\alpha\cdot\vec{B}$$

solve equation of motion (constant $\vec{B} = B\vec{e}_z$)

$$\Delta\alpha = m_\pi^2\sin\alpha \quad \Rightarrow \quad \alpha(\bar{z},p) = 2\arccos[-\mathrm{sn}(\bar{z},p^2)]\,, \qquad \bar{z} = \frac{zm_\pi}{p}\,.$$

minimize free energy with respect to $p$,
find critical magnetic field

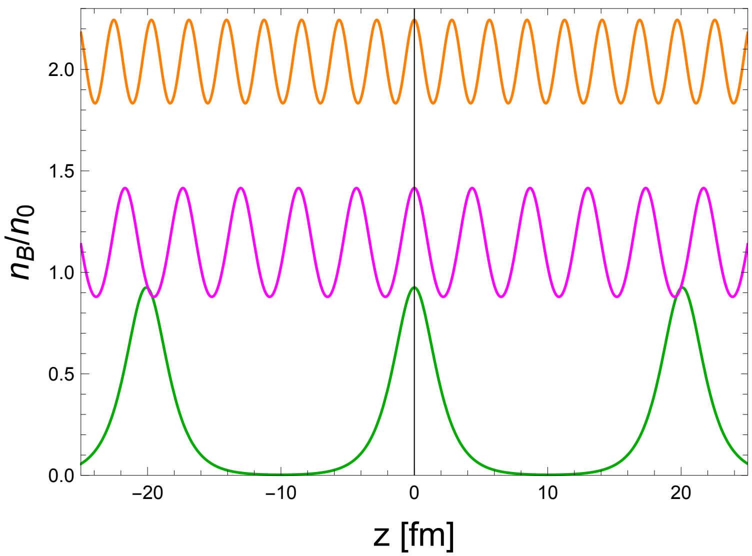
$$eB_{\mathrm{CSL}} = \frac{16\pi m_\pi f_\pi^2}{\mu}$$

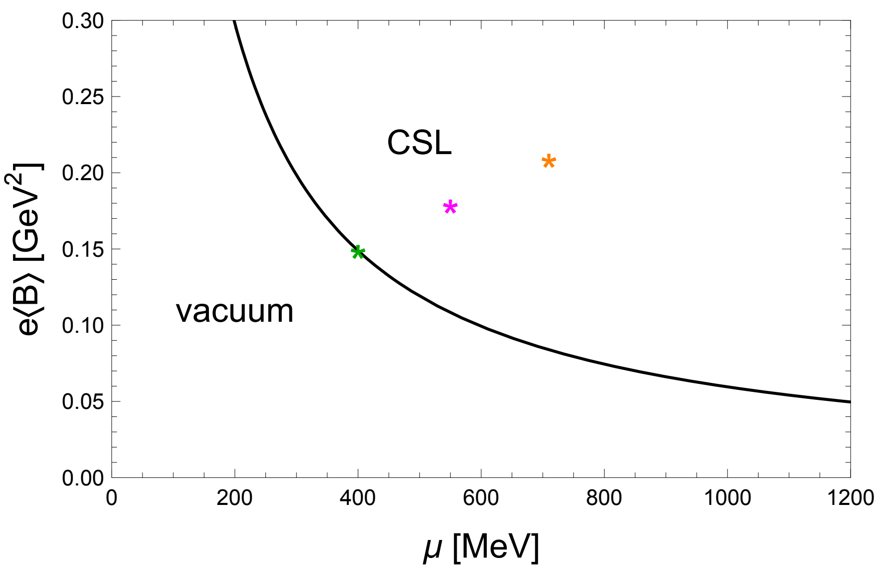


# Chiral soliton lattice (page 2/2)

T. Brauner, N. Yamamoto, JHEP 04, 132 (2017)

chiral soliton lattice = "stack of domain walls"
$\rightarrow$ "anomalous" baryon sheets, $n_B \propto \nabla\alpha$




chiral limit: $\nabla\alpha$ = const and CSL at any nonzero $B$ and $\mu$

from holography: A. Rebhan, A. Schmitt and S. A. Stricker, JHEP 05, 084 (2009)

# Instability towards pion condensation

T. Brauner, N. Yamamoto, JHEP 04, 132 (2017)

reinstate charged pions $\pi^{\pm}$ and compute their spectrum in the CSL background

$$Lf(\bar{z}) = \varepsilon f(\bar{z}) \,, \qquad \varepsilon = \frac{p^2}{m_\pi^2}[\omega^2 - (2n+1)eB] + 4 + p^2$$

with Lamé operator $L = -\partial_{\bar{z}}^2 + 6p^2\mathrm{sn}^2(\bar{z}, p^2)$

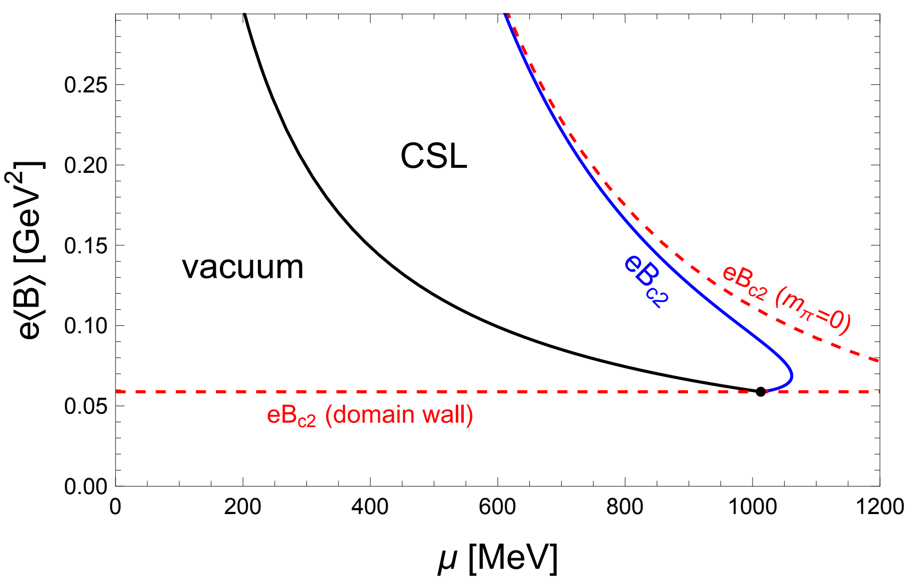
lowest eigenstate of $L$ and $n = 0$

$$\omega^2 = eB + m_\pi^2\left(1 - 2\frac{1+\sqrt{p^4 - p^2 + 1}}{p^2}\right) , \qquad \frac{E(p^2)}{p} = \frac{eB\mu}{16\pi m_\pi f_\pi^2}$$

instability ($\omega^2 < 0$) indicating
$\pi^{\pm}$ condensation at critical field $B_{c2}$

domain wall: $eB_{c2} = 3m_\pi^2$

chiral limit: $eB_{c2} = \dfrac{16\pi^4 f_\pi^4}{\mu^2}$



# Instability towards pion condensation

T. Brauner, N. Yamamoto, JHEP 04, 132 (2017)

reinstate charged pions $\pi^{\pm}$ and compute their spectrum in the CSL background

with I

lowest

> instability is analogous to type-II superconductivity
>
> G. W. Evans and A. Schmitt, JHEP 09, 192 (2022)
>
> → may use techniques from Ginzburg-Landau theory and Abrikosov lattice
>
> A.A. Abrikosov, Soviet Physics JETP 5, 1174 (1957)
>
> W.H. Kleiner, L.M. Roth, S.H. Autler, Phys. Rev. 133, A1226 (1964)

instability ($\omega^2 < 0$) indicating
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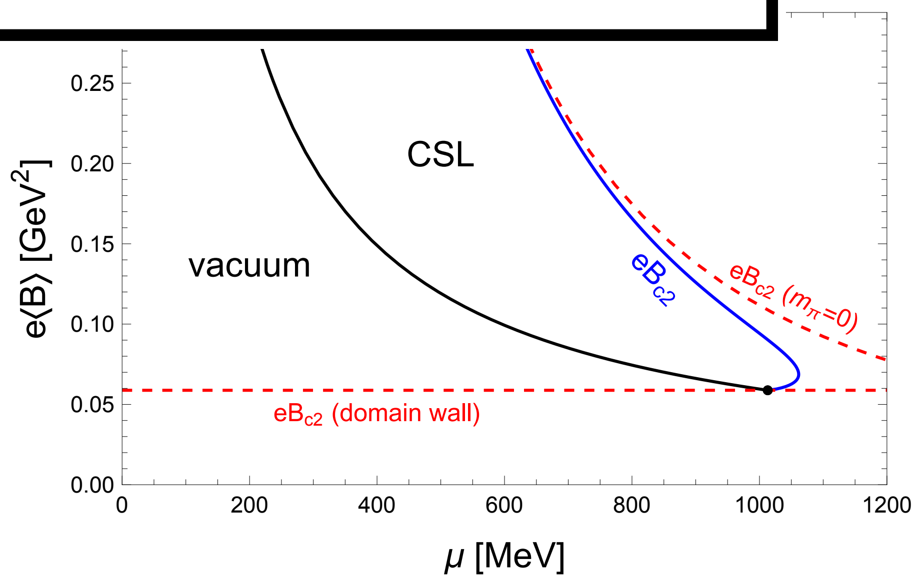
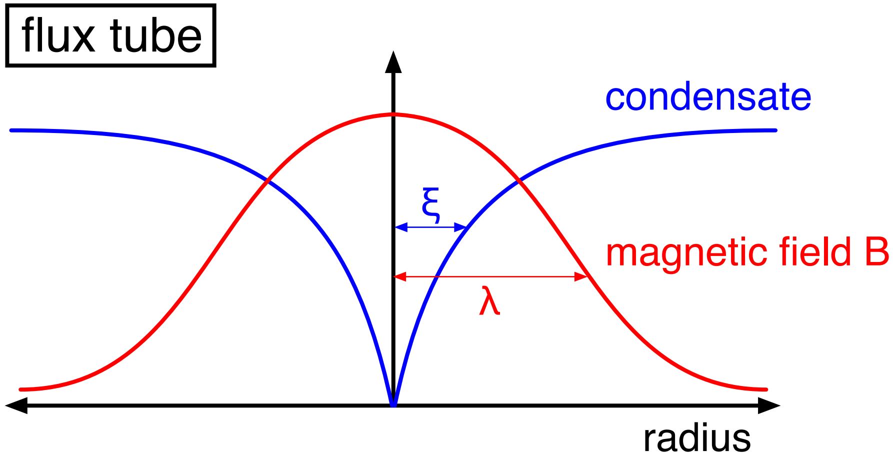


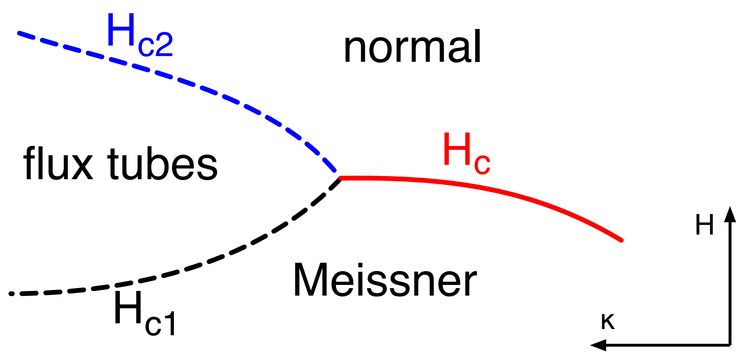
# Recall type-II superconductivity

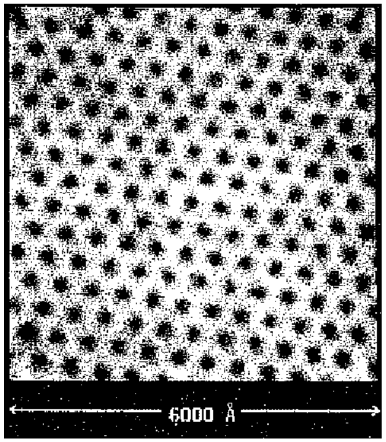



Ginzburg-Landau parameter

$$\kappa = \frac{\lambda}{\xi}$$

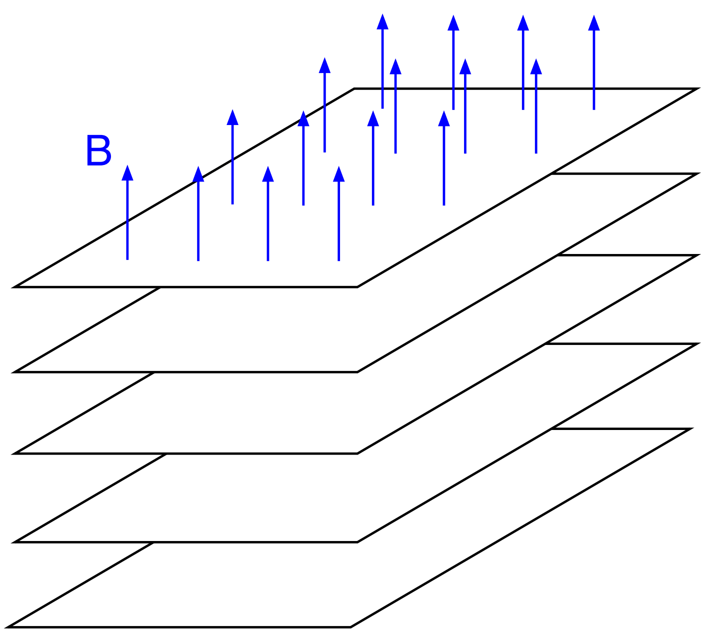
type-II superconductivity for $\kappa > 1/\sqrt{2}$: flux tube lattice for $H_{c1} < H < H_{c2}$

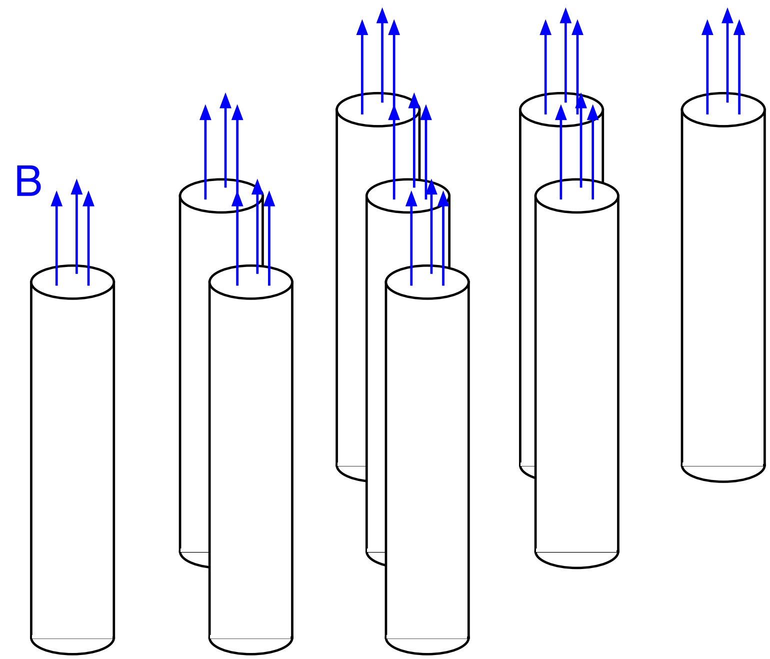



A.A. Abrikosov, RMP 76, (2004), Nobel lecture

# Pionic Abrikosov lattice?




chiral soliton lattice

1D modulation **along** B

standard type-II superconductor

2D modulation **perpendicular** to B

charged pion condensation

**3D crystal?**

# Compute crystal and free energy

method based on A.A. Abrikosov, Soviet Physics JETP 5, 1174 (1957)
W.H. Kleiner, L.M. Roth, S.H. Autler, Phys. Rev. 133, A1226 (1964)
chiral limit (2D crystal) G. W. Evans and A. Schmitt, JHEP 09, 192 (2022)
physical $m_\pi$ (3D crystal) G. W. Evans and A. Schmitt, JHEP 02, 041 (2024)

expand for small $\epsilon \sim \sqrt{|\langle B\rangle - B_{c2}|}$

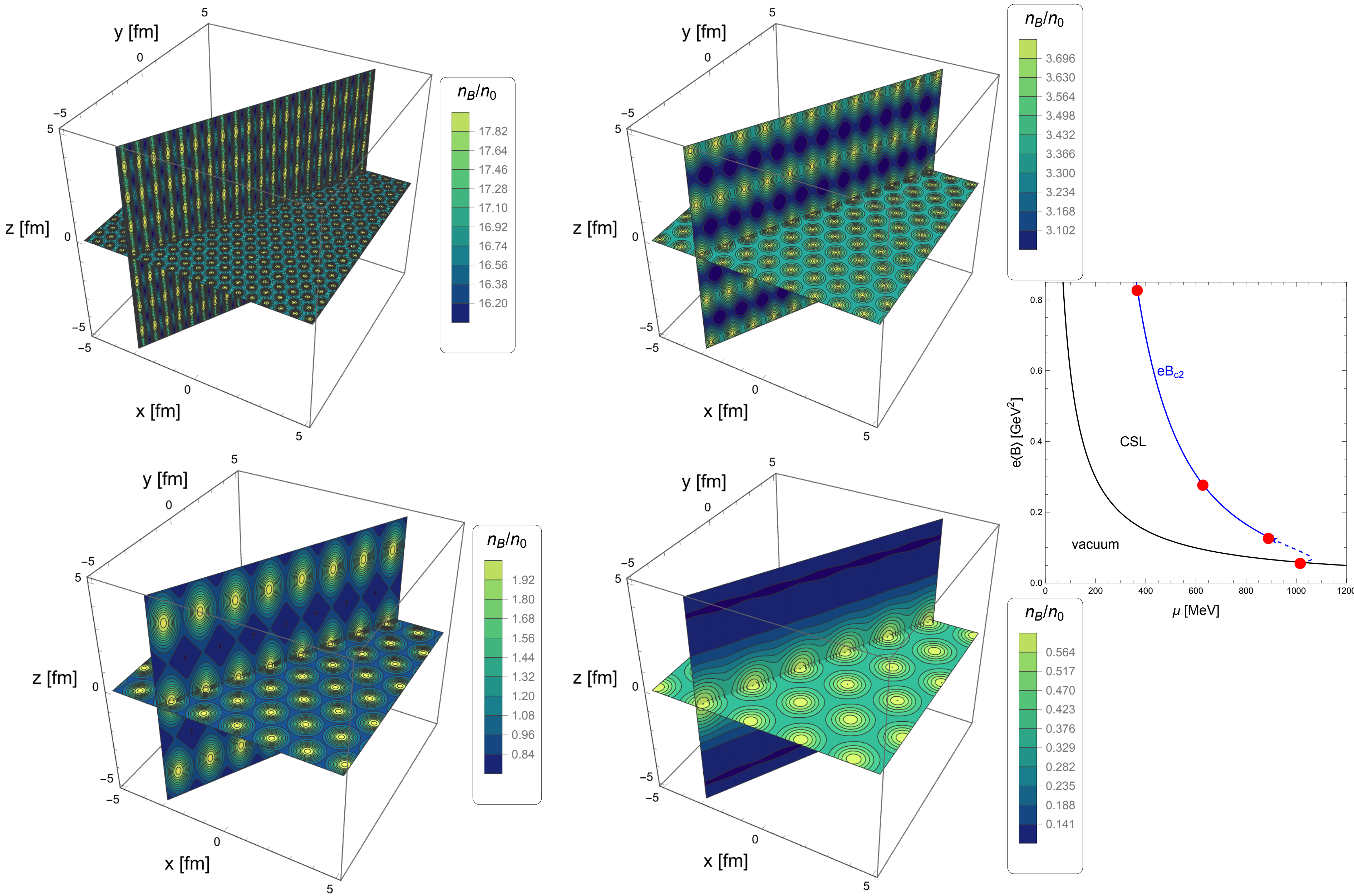
$$\alpha = \alpha_0 + \delta\alpha\,, \qquad \varphi = \varphi_0 + \delta\varphi\,, \qquad \vec{A} = \vec{A}_0 + \delta\vec{A}$$

solve leading-order and subleading-order equations of motion via Fourier space [factorization $(x,y)\times(z)$ *not* possible beyond leading order]

compute free energy density from solutions:

$$F = F_{\rm CSL} - \frac{\mathcal{G}^2}{2}\frac{(\langle B\rangle - B_{c2})^2}{\beta(2\kappa^2-1)+1+2\mathcal{H}_1-2\mathcal{H}_2}$$

# Superconducting baryon crystal



# When is the crystal preferred?

$$F = F_{\mathrm{CSL}} - \frac{\mathcal{G}^2}{2}\frac{(\langle B\rangle - B_{c2})^2}{\beta(2\kappa^2-1)+1+2\mathcal{H}_1-2\mathcal{H}_2}$$

$F_{\mathrm{CSL}}$ free energy density of CSL

$\kappa$ Ginzburg-Landau parameter

$$\beta \equiv \frac{\langle|\varphi_0|^4\rangle}{\langle|\varphi_0|^2\rangle^2} = \sum_{n,m} e^{-\frac{2\pi}{a}\left(m^2+nm+\frac{a^2+1}{4}n^2\right)}$$

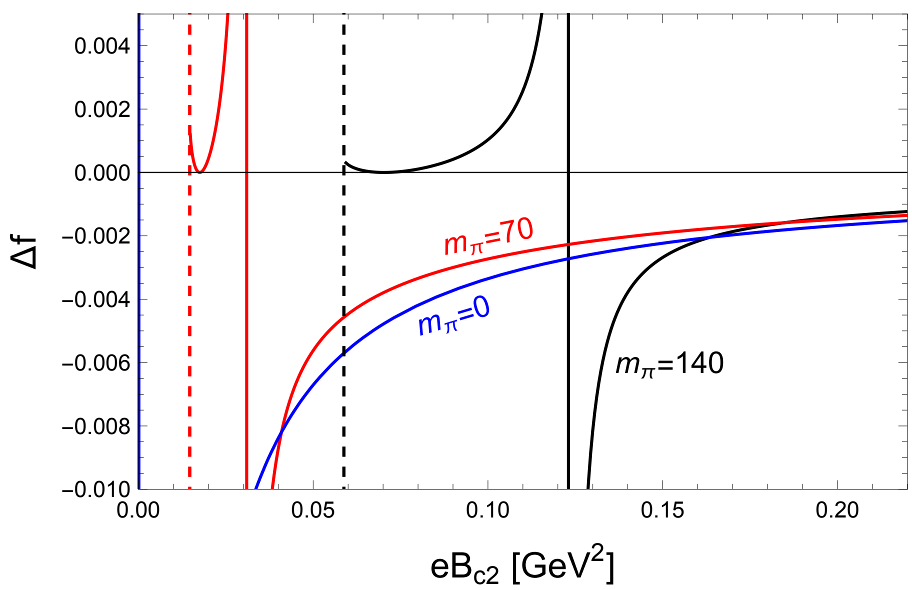
$a = 1$ quadratic lattice
$a = 1/\sqrt{3}$ hexagonal lattice (preferred!)

$\mathcal{G}, \mathcal{H}_1, \mathcal{H}_2$ functions of $a$, $p$
(Fourier sums)

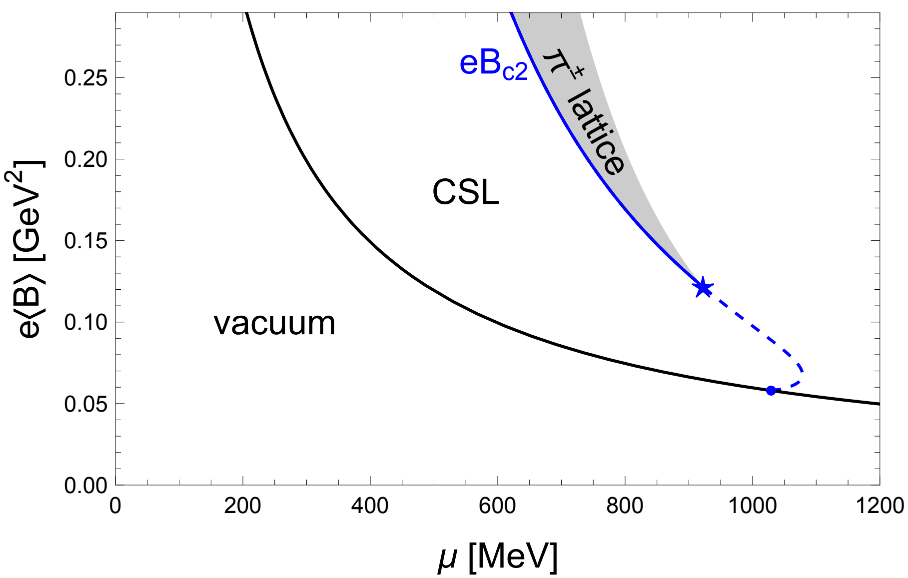
standard superconductor:
$\mathcal{G} = 1, \mathcal{H}_1 = \mathcal{H}_2 = 0$

sign change of $\Delta f$ along
CSL instability line

→ "type-I/type-II transition point"
(where expansion breaks down)



# Phase diagram: result

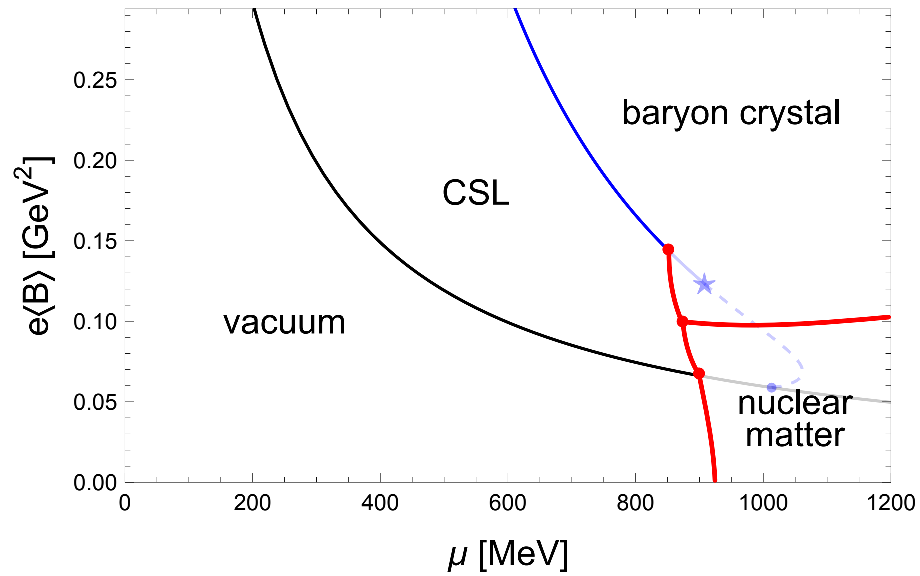



solid: 3D baryon crystal is the solution to the CSL instability

dashed: 3D baryon crystal is *not* the solution to the CSL instability

(our lattice is only a valid solution "before" the dashed line)

# Phase diagram: conjecture




CSL instability at dashed segment → discontinuous transition

zero magnetic field: first-order onset of nuclear matter

⇒ liquid-solid transition within nuclear matter

$(0.1\,\mathrm{GeV}^2 \sim 5\,m_\pi^2 \sim 5\times 10^{18}\,\mathrm{G})$

# Summary

- QCD at nonzero $B$ and $\mu$ exhibits a chiral soliton lattice, which becomes unstable at sufficiently large $B$ and/or $\mu$

- we have constructed the resulting phase: a 3D pion crystal with spatially inhomogeneous magnetic field and baryon number

- continuous transition is only realized for sufficiently large magnetic fields

# Outlook

- go beyond (leading order) chiral perturbation theory
  (support results with model calculation? holography?)

- connection to nuclear matter
  (liquid-solid transition from phenomenological model?)

- connection to Skyrmion approach?
  M. Eto, K. Nishimura and M. Nitta, JHEP 12, 032 (2023)

- include isospin chemical potential
  M.S. Grønli, T. Brauner, EPJ C 82, 354 (2022)
  Z. Qiu and M. Nitta, JHEP 06, 139 (2024)

- include nonzero temperatures
  T. Brauner, H. Kolešová, N. Yamamoto, PLB 823, 136737 (2021)

- relevance for neutron stars?