QCD phase diagram in the T − eB plane for varying pion mass

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In collaboration with Mahammad Sabir Ali and Rishi Sharma

Based on arXiv: 2407:14449

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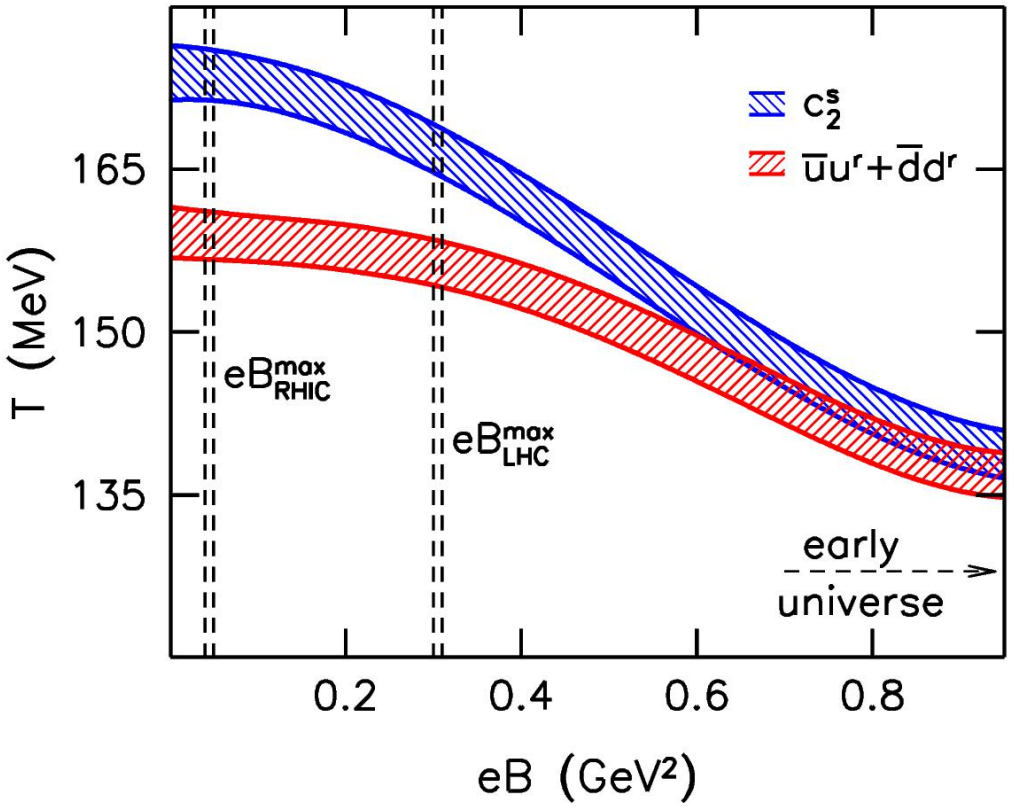
Motivation

Phase diagram

- QCD PD in the T-eB plane

[G. S. Bali et al., JHEP 02 (2012) 044]



- What happens if we increase the magnetic field further?

➡ There is a CP at high enough eB! [Endrődi 2015 and D'Elia et al., 2021]

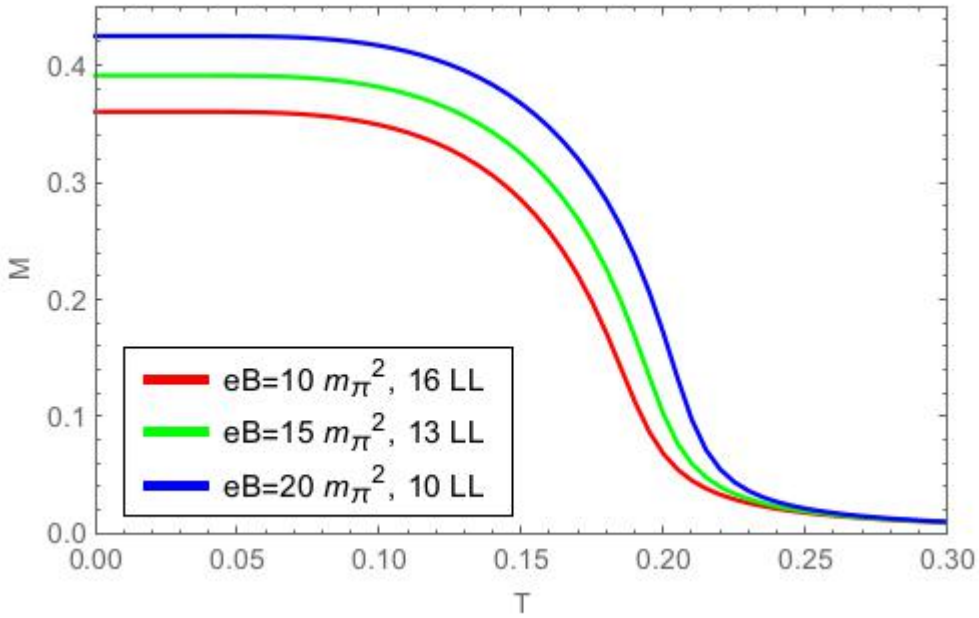
Recent calculation using holographic model [X. Cao and H. Liu, arXiv: 2408.00467]

Magnetic catalysis

- Did we always know about such a PD?

- Different effective model results and as well as lattice QCD calculations agreed on an increasing T_{CO} as a function of eB [V. P. Gusynin et al., NPB462, 249 (1996), M. D'Elia et al, PRD 82, 051501 (2010)]

For more such references: [G. S. Bali et al., PRD 86, 071502]



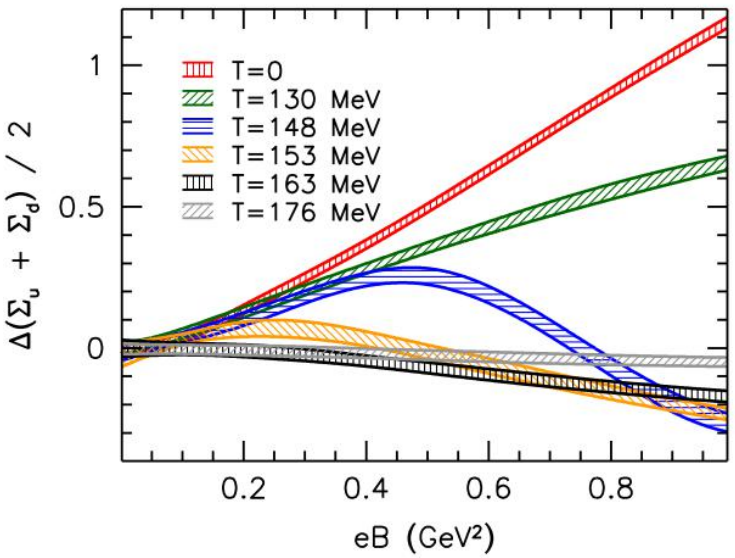
[CAI et al., PRD 99, 094028]

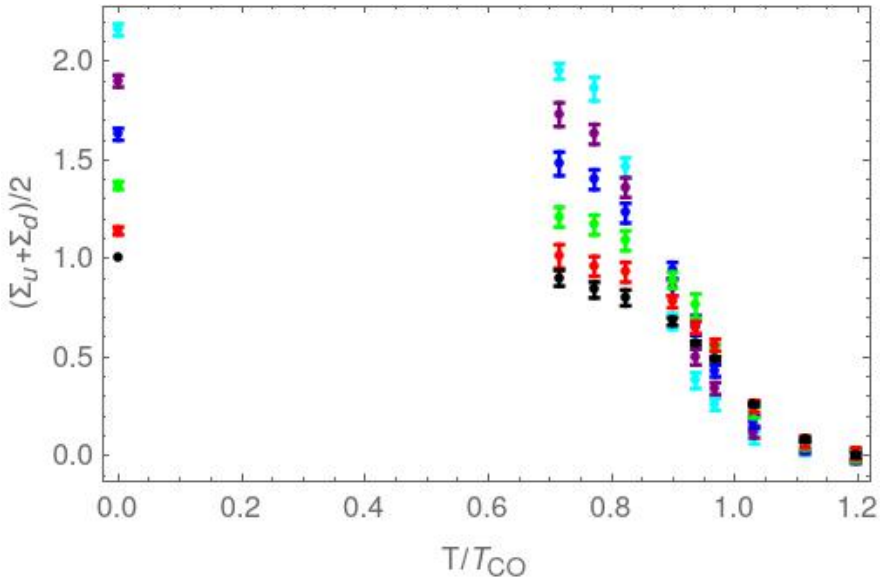
- Two things to observe:

I) Increasing T_{CO} and

II) Increasing condensate (MC) with eB

Inverse Magnetic catalysis

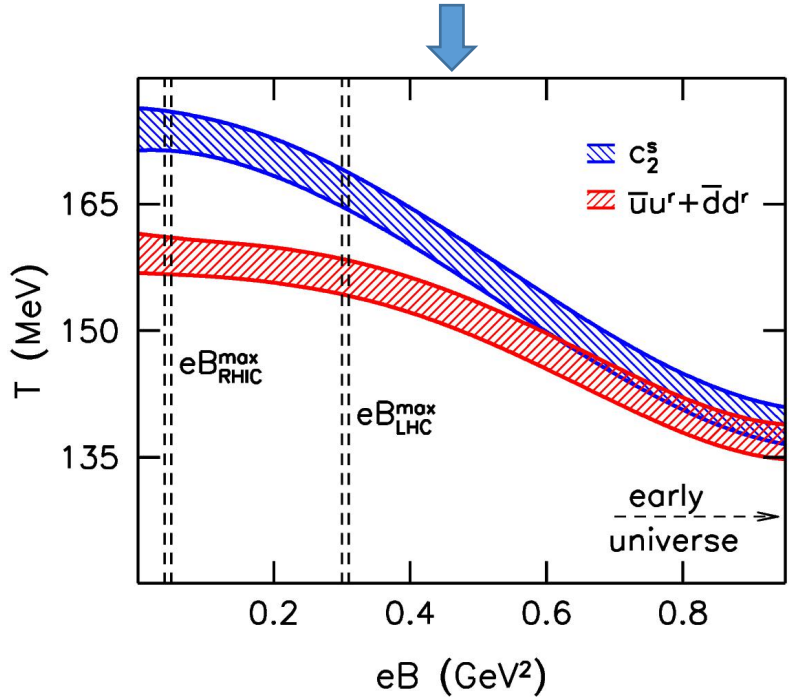




- IMC effect around the transition temperature with updated LQCD calculation [G. S. Bali et al., PRD 86, 071502]

- IMC effect and T_{CO} accompany each other!

- QCD PD in the T-eB plane [G. S. Bali et al., JHEP 02 (2012) 044]



Using effective models

- Models such (P)NJL, (P)QM etc using eB, T dependent coupling constant, going beyond mean fields, adding AMM, etc.

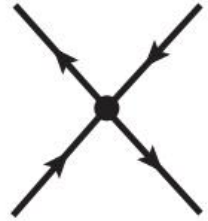
- A major point of these results is a field dependent coupling constant.

- Some examples using NJL model

An effective model perspective

- The standard NJL Lagrangian

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma_\mu\partial^\mu - m_0)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$



Local Four point interaction

- Cannot capture the IMC effect on its own. Needs an eB or eB and T dependent coupling constant [R. L. S. Farias et al., PRC 90, 025203; M. Ferreira et al., PRD 89 116011]

- Such examples:

a) $$G_S(eB,T) = c(eB)\left(1 - \frac{1}{1+e^{\beta(eB)(T_a(eB)-T)}}\right) + \gamma(eB)$$ [R. Farias et al., EPJA(2017) 53: 101

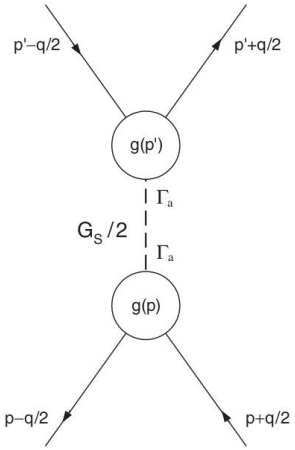
b) $$G_S(\xi) = G_S^0\frac{1+a\xi^2+b\xi^3}{1+c\xi^2+d\xi^4} \quad \text{with} \quad \xi = eB/\Lambda_{QCD}^2$$ [M. Ferreira et al., PRD 89 116011]

Non-local model

- On the other hand a non-local version can capture the IMC effect without tweaking the model further

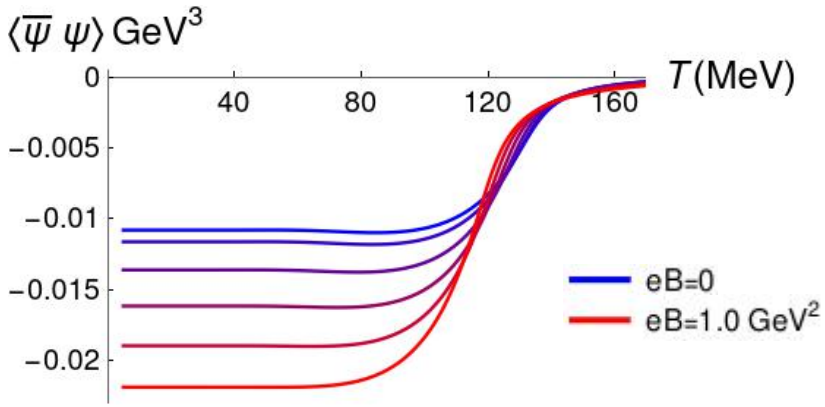
- The non-local interaction:

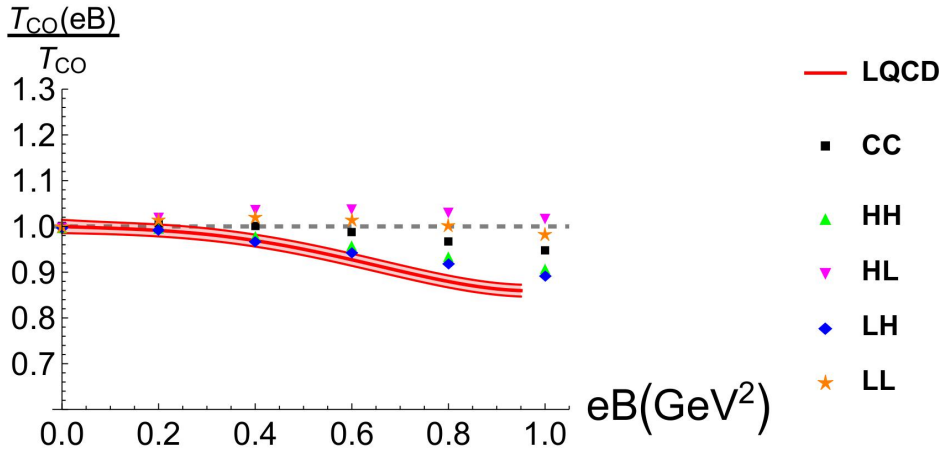
$$j_a(x) = \int d^4z g(z) \bar{\psi}\left(x + \frac{z}{2}\right) \Gamma_a \psi\left(x - \frac{z}{2}\right)$$



Difference

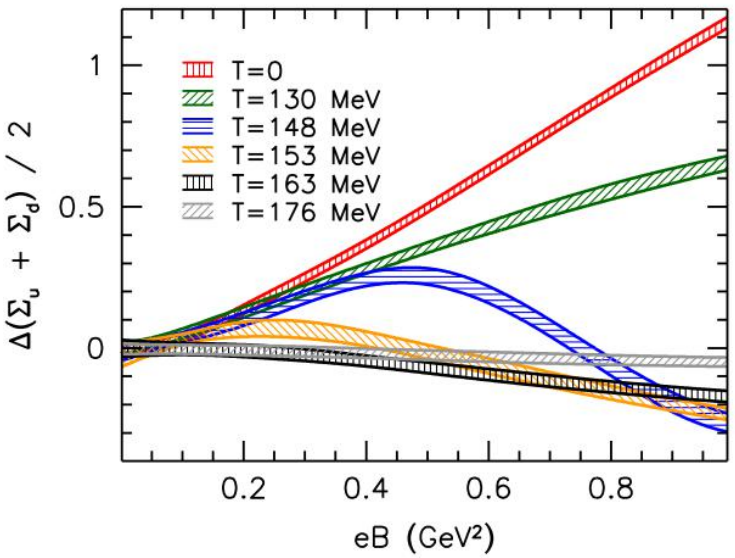
$g(z)$ is the non-local form factors [Birse et al., for example: NPA 582 655, Schmidt et al., PRC 50 435]

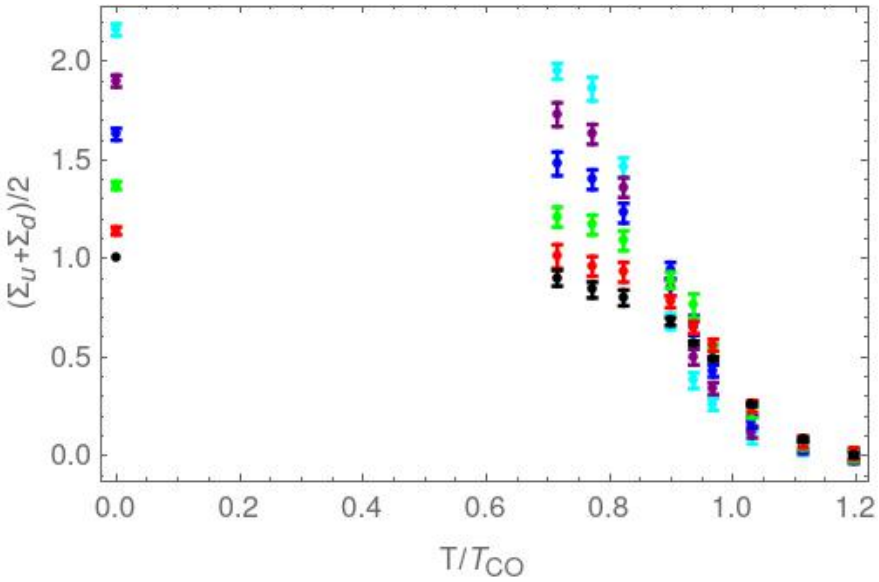




[Pagura et al., PRD 95, 034013; CAI et al., PRD 104, 114026]

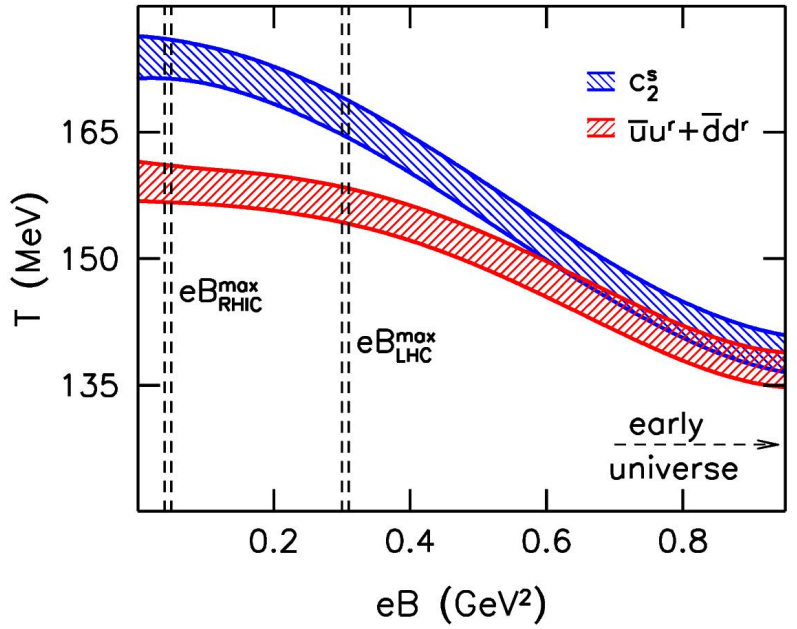
Inverse Magnetic catalysis



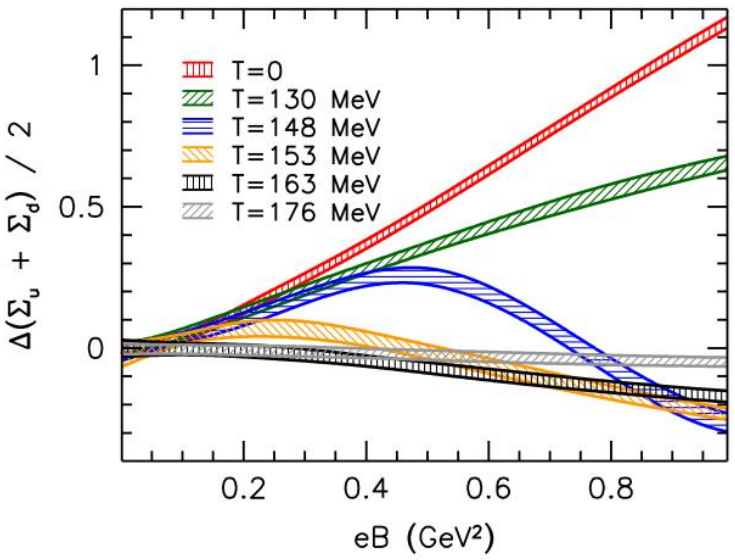


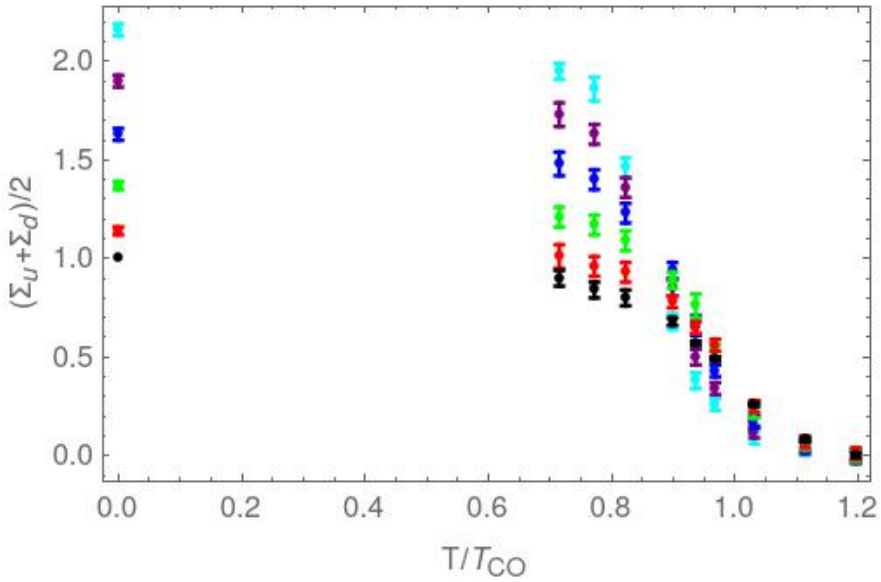
- IMC effect around the transition temperature with updated LQCD calculation [G. S. Bali et al., PRD 86, 071502]

- QCD PD in the T-eB plane
  [G. S. Bali et al., JHEP 02 (2012) 044]



Inverse Magnetic catalysis

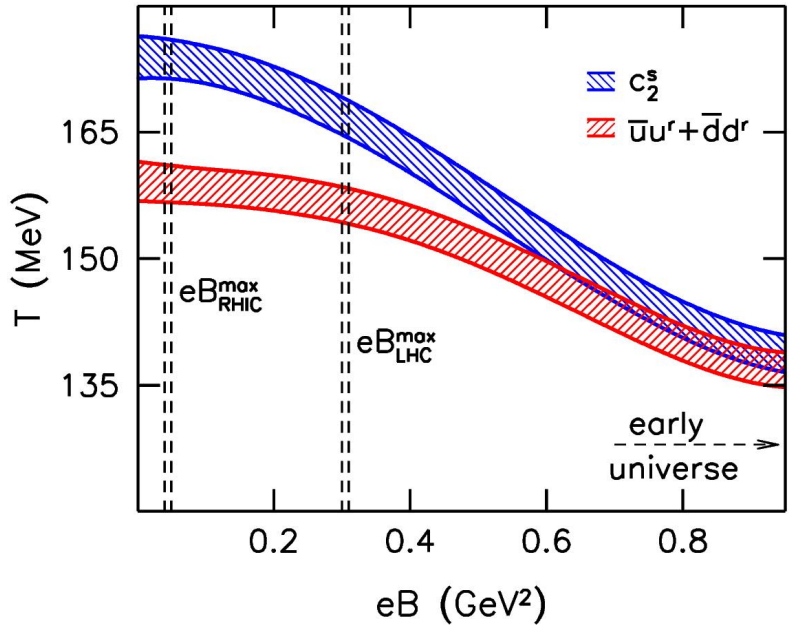




- IMC effect around the transition temperature with updated LQCD calculation [G. S. Bali et al., PRD 86, 071502]

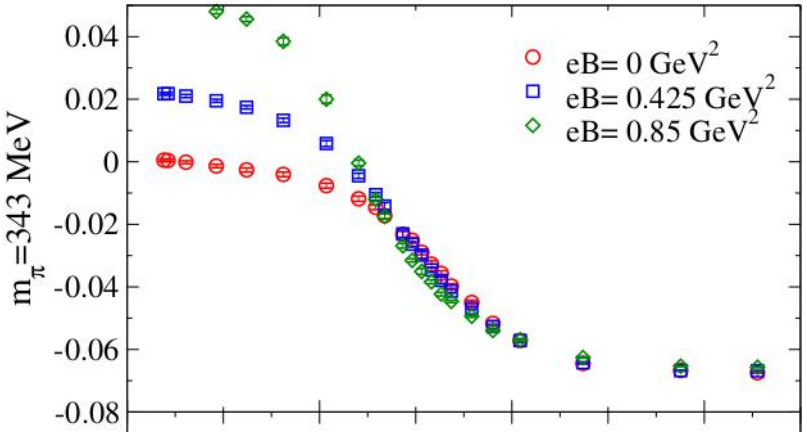
- Why such properties could not be observed in the previous lattice studies?

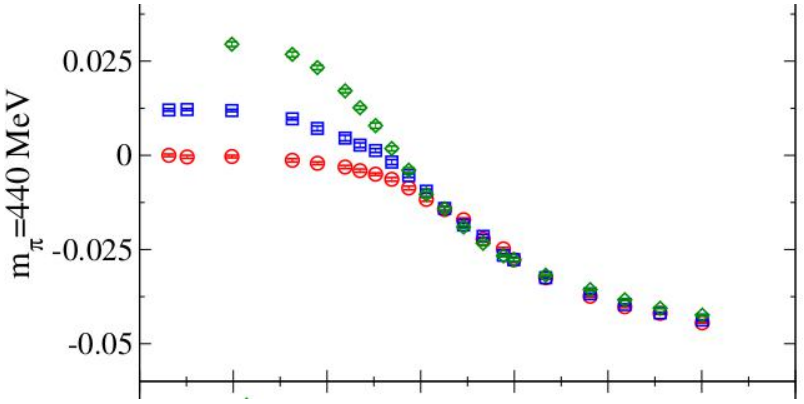
- QCD PD in the T-eB plane
  [G. S. Bali et al., JHEP 02 (2012) 044]

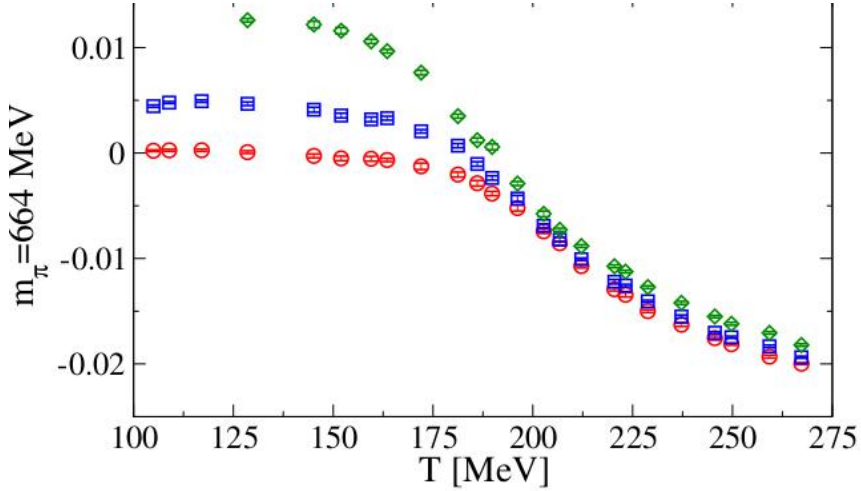


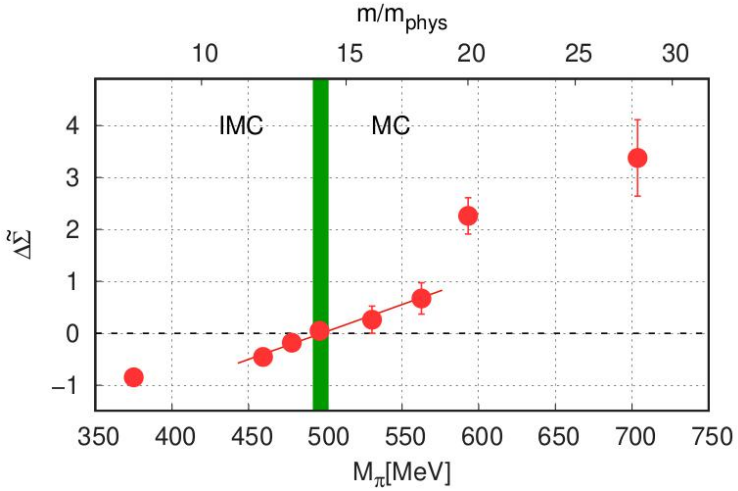
The role of pion mass

- The condensates with increasing pion mass [M. D'Elia et al., PRD 98, 054509]





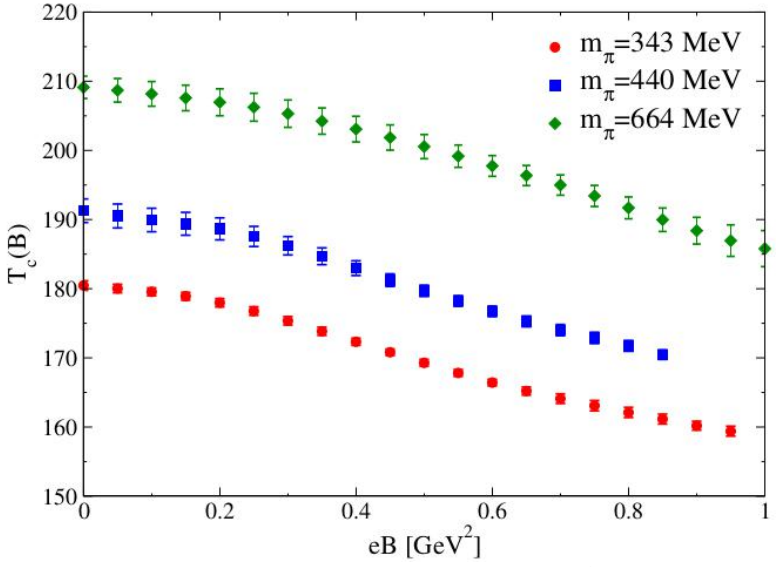




[G. Endroidi et al., JHEP07(2019)007]

- The PD with increasing pion mass

Conclusion: No IMC but decreasing T_{CO}



Question

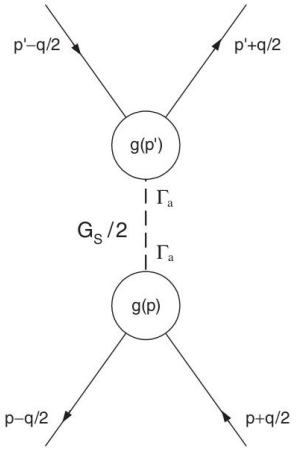
- Can effective models capture such beyond physical point properties in theT-eB plane?

- To answer this question we will focus on the NJL model.

Non-local model

- The non-local interaction:

$$j_a(x) = \int d^4 z g(z) \bar{\psi}\left(x + \frac{z}{2}\right) \Gamma_a \psi\left(x - \frac{z}{2}\right)$$



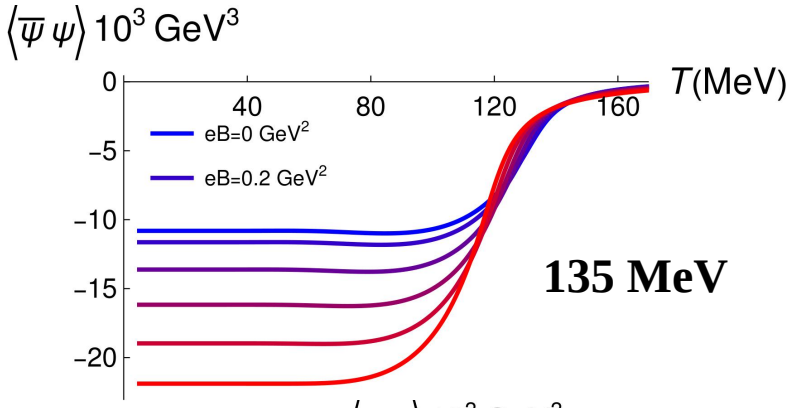
$g(z)$ is the non-local form factors [Birse et al., for example: NPA 582 655, Schmidt et al., PRC 50 435]

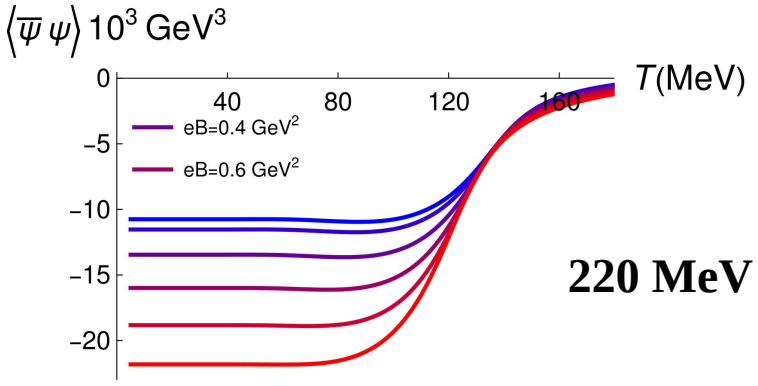
To go beyond physical pion mass

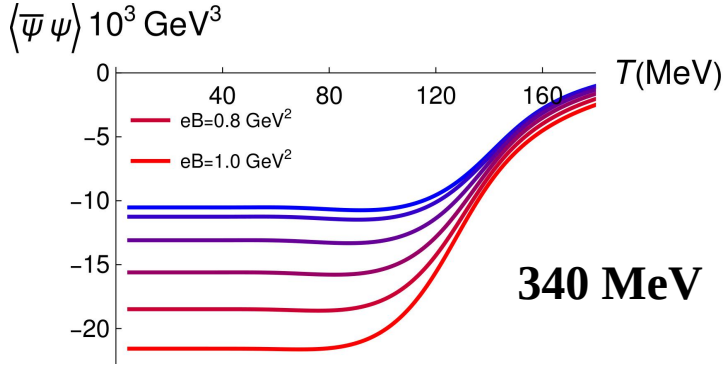
m_π (MeV)	m (MeV)
135	6.94
220	18.28
340	42.89
440	70.78

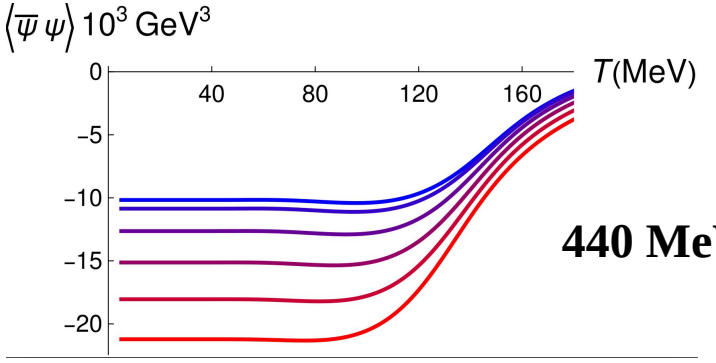
[CAI et al., arXiv: 2407:14449]

TABLE II. Parameter m for different pion masses, with Λ and G_0 kept fixed to their values at physical point ($\Lambda = 605.05$ MeV and $G_0 = 29.38/\Lambda^2$).







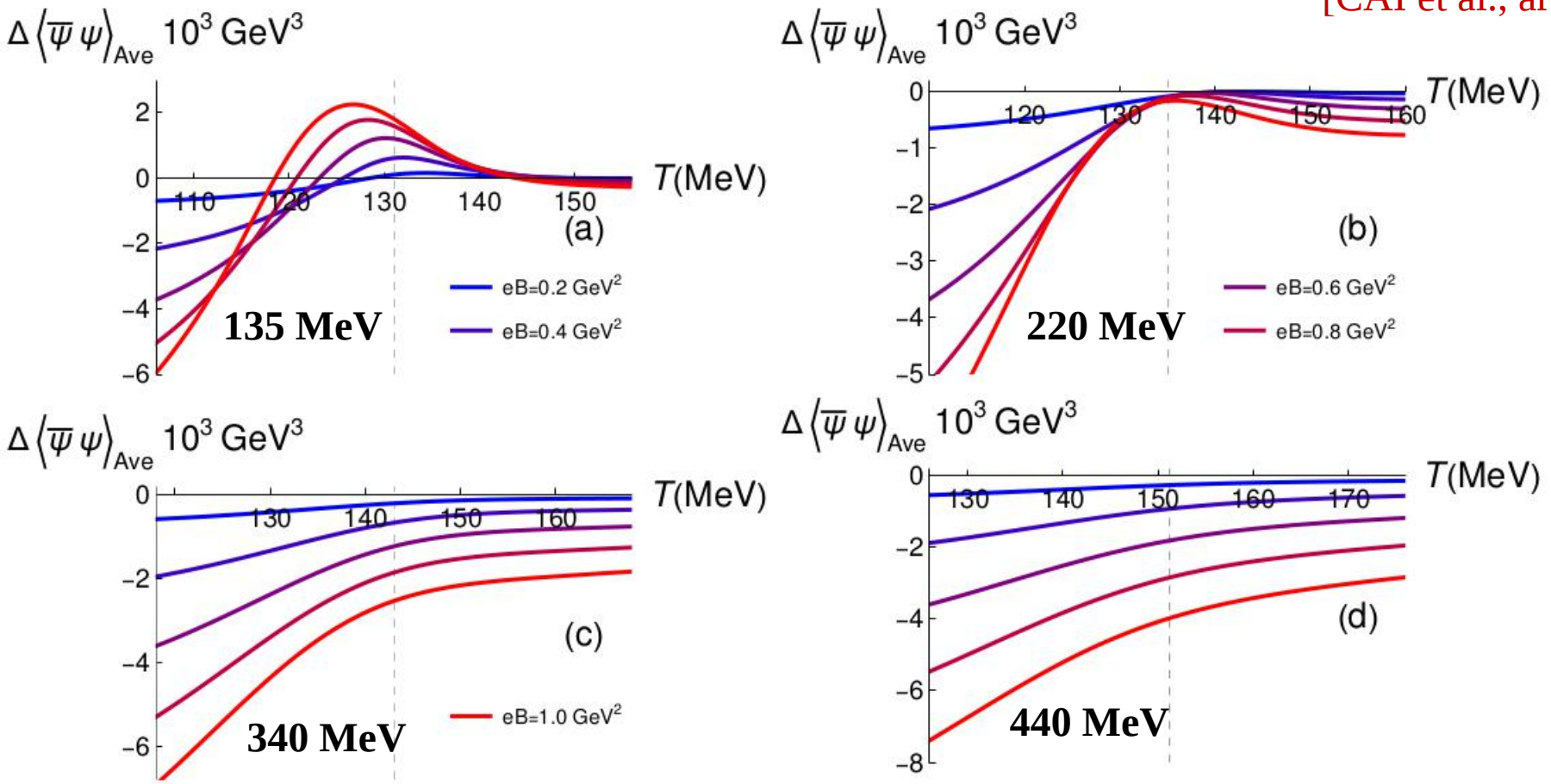


Condensate-average differences

- The condensate-average difference is defined as the average of the u and d quark condensates from which the value of the condensate average for $eB = 0$ is subtracted.

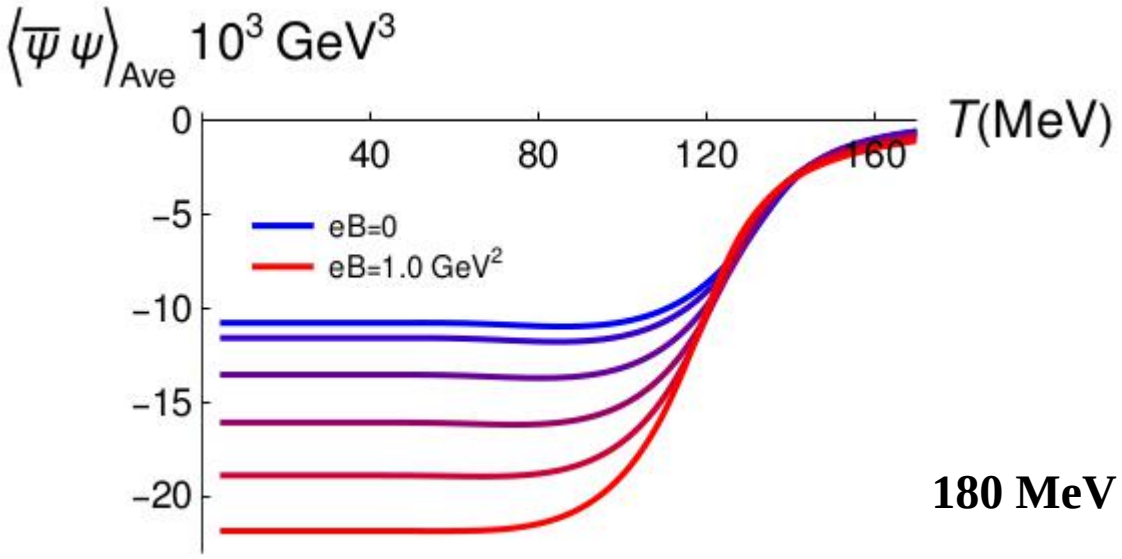
$$\Delta\langle\bar{\psi}\psi\rangle_{\mathrm{Ave}}(eB,T,m) = \langle\bar{\psi}\psi\rangle_{\mathrm{Ave}}(eB,T,m) - \langle\bar{\psi}\psi\rangle_{\mathrm{Ave}}(0,T,m)$$

[CAI et al., arXiv: 2407:14449]

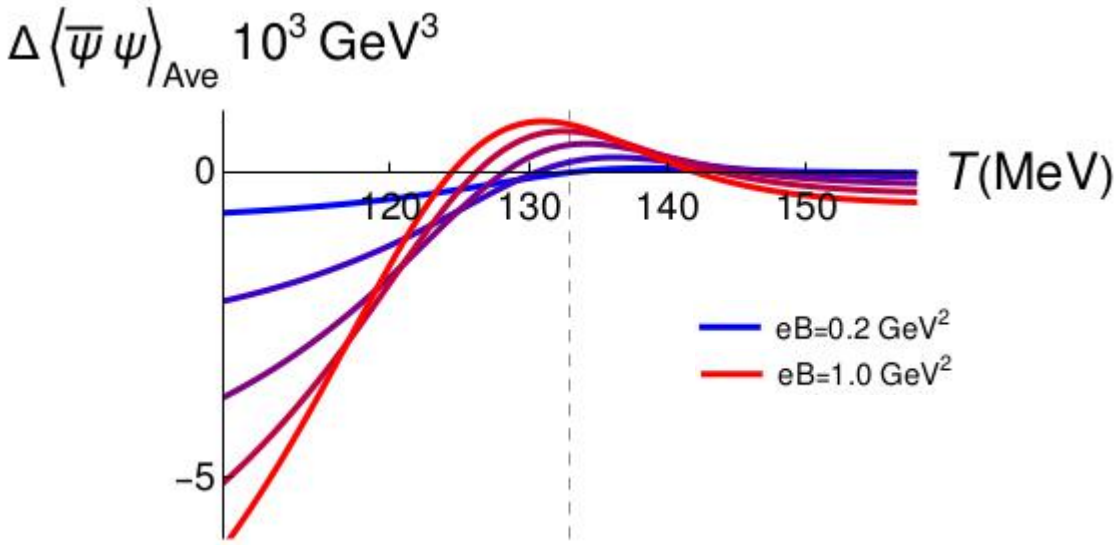


Pion mass value of 180 MeV

- Presence of the IMC effect for a higher pion mass value:

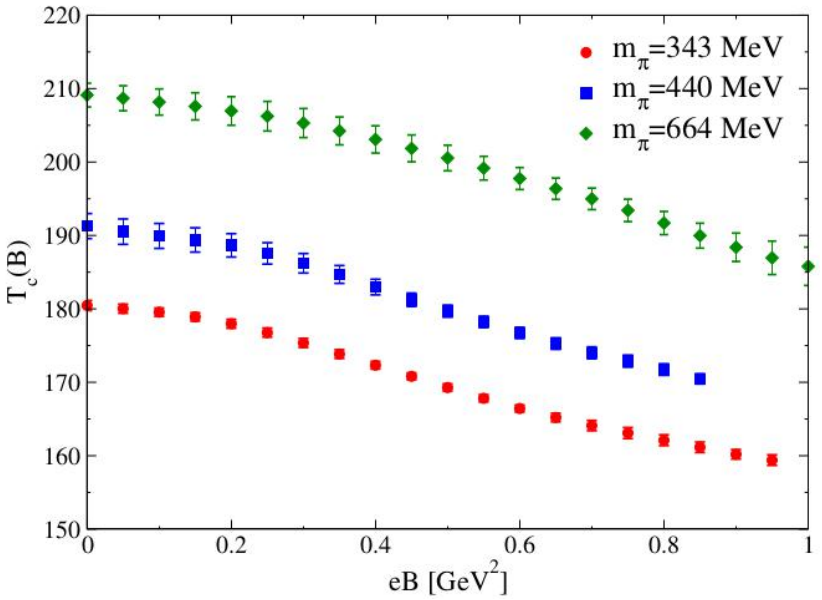


180 MeV



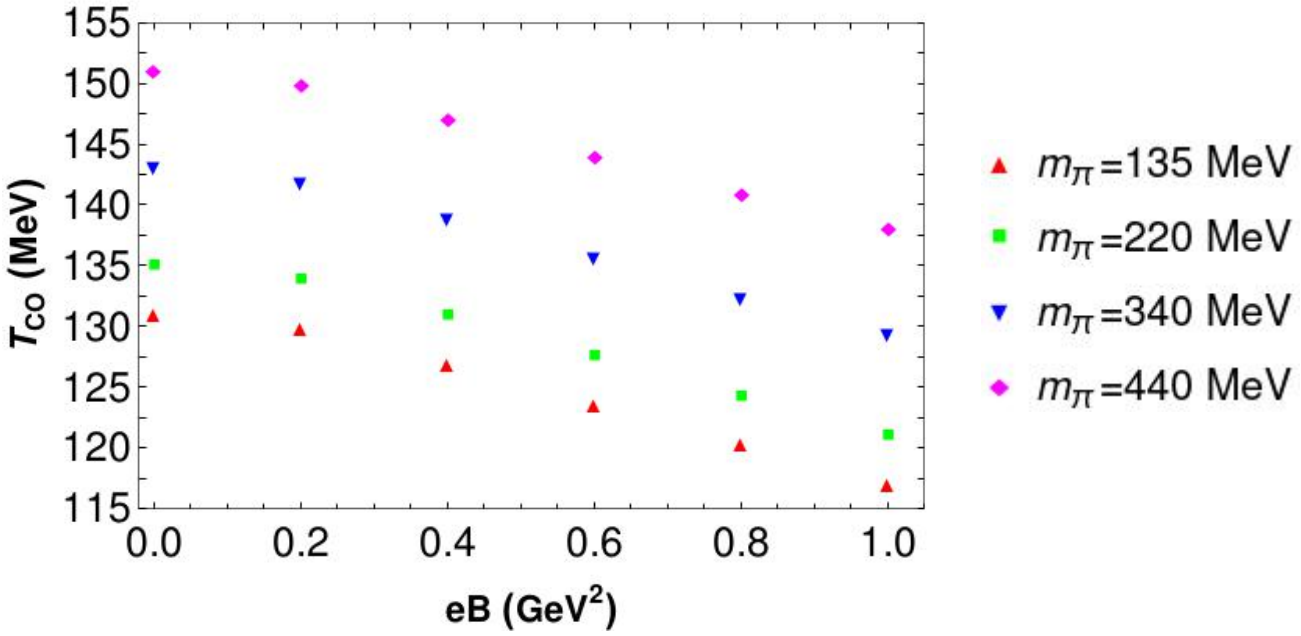
[CAI et al., arXiv: 2407:14449]

PD for beyond physical pion mass



Lattice QCD

[M. D'Elia et al., PRD 98, 054509]



Nonlocal NJL model

[CAI et al., arXiv: 2407:14449]

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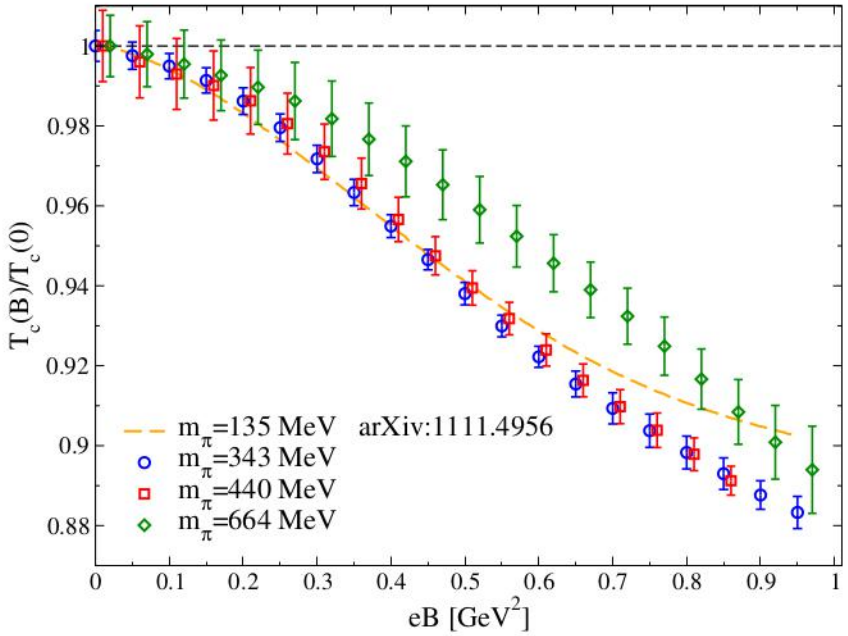

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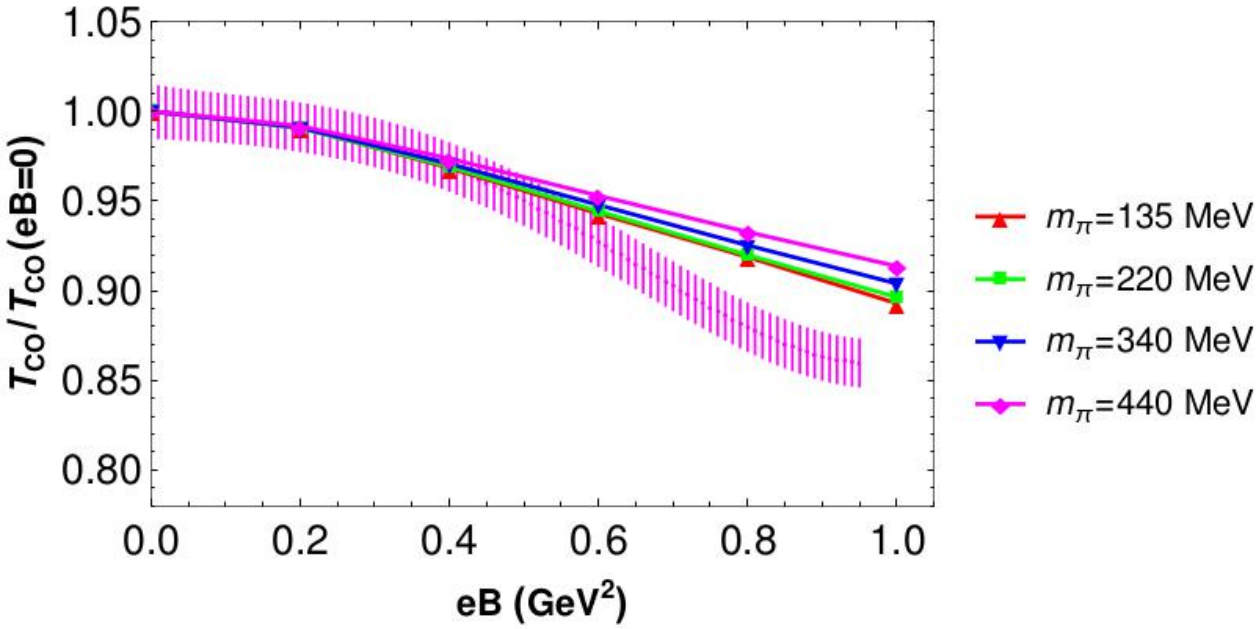


Nonlocal NJL model

[CAI et al., arXiv: 2407:14449]

Upshots

- We observe that with increasing pion mass, the IMC effect disappears.
- The decreasing trend of T_{CO} persists up to the tested pion mass values.
- This is qualitatively consistent with the LQCD results.
- The value of the pion mass at which the IMC effect goes away is lower than that found in the LQCD study.

2-flavour local model

- In the local 2-flavour model:

$$G_S(eB,T) = c(eB)\left(1 - \frac{1}{1+e^{\beta(eB)(T_a(eB)-T)}}\right) + \gamma(eB)$$

[R. Farias et al., EPJA(2017) 53: 101

m_π (MeV)	m (MeV)
135	5.5
220	13.2
340	32.2
440	54.2

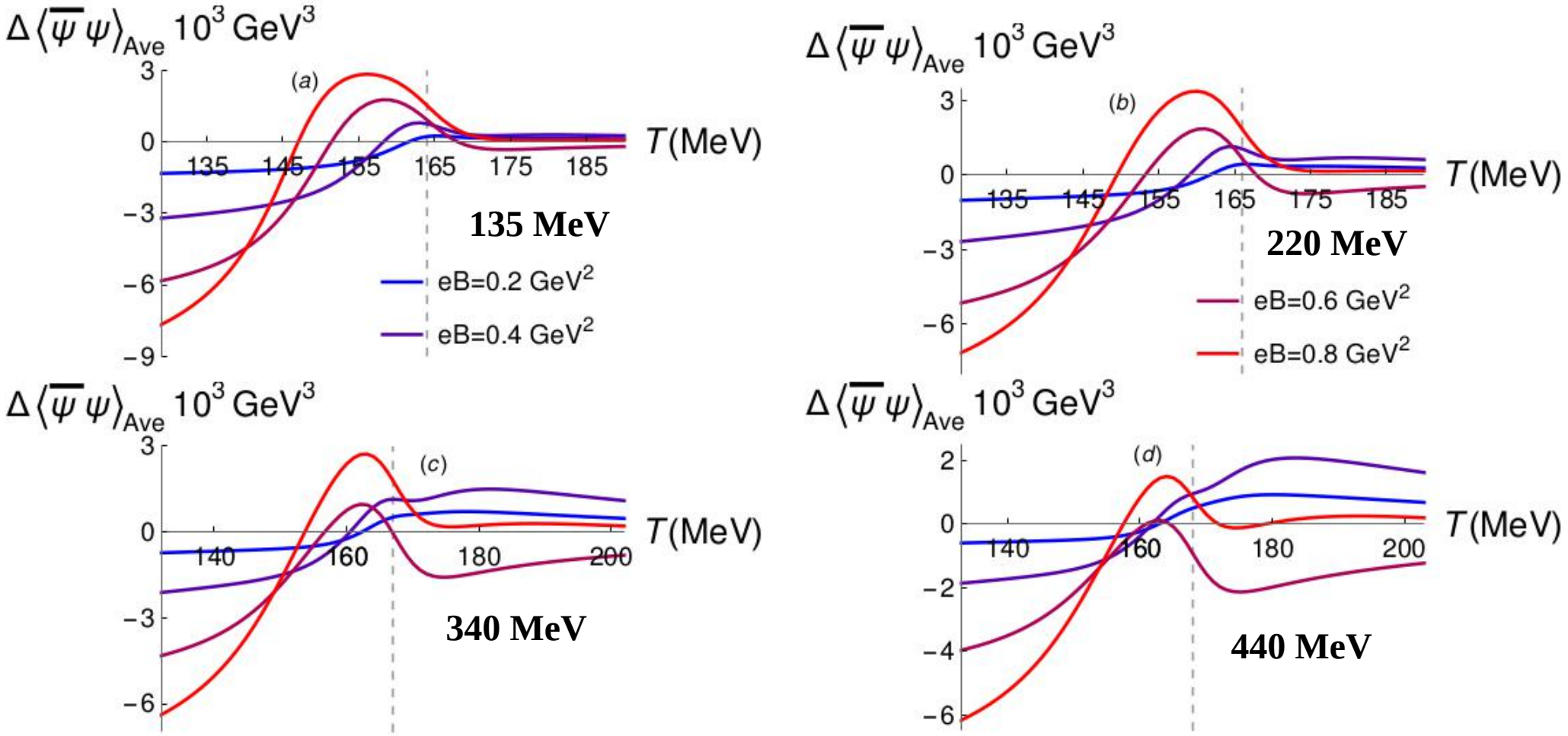
TABLE III. Parameter m for different pion masses, with Λ and G_0 kept fixed to their values at physical point ($\Lambda = 650$ MeV and $G_0 = 4.5$ GeV^{-2}).

[CAI et al., arXiv: 2407:14449]

2-flavour local model

- The IMC effect for different pion masses:

$$\Delta\langle\bar{\psi}\psi\rangle_{\rm Ave}(eB,T,m) = \langle\bar{\psi}\psi\rangle_{\rm Ave}(eB,T,m) - \langle\bar{\psi}\psi\rangle_{\rm Ave}(0,T,m)$$

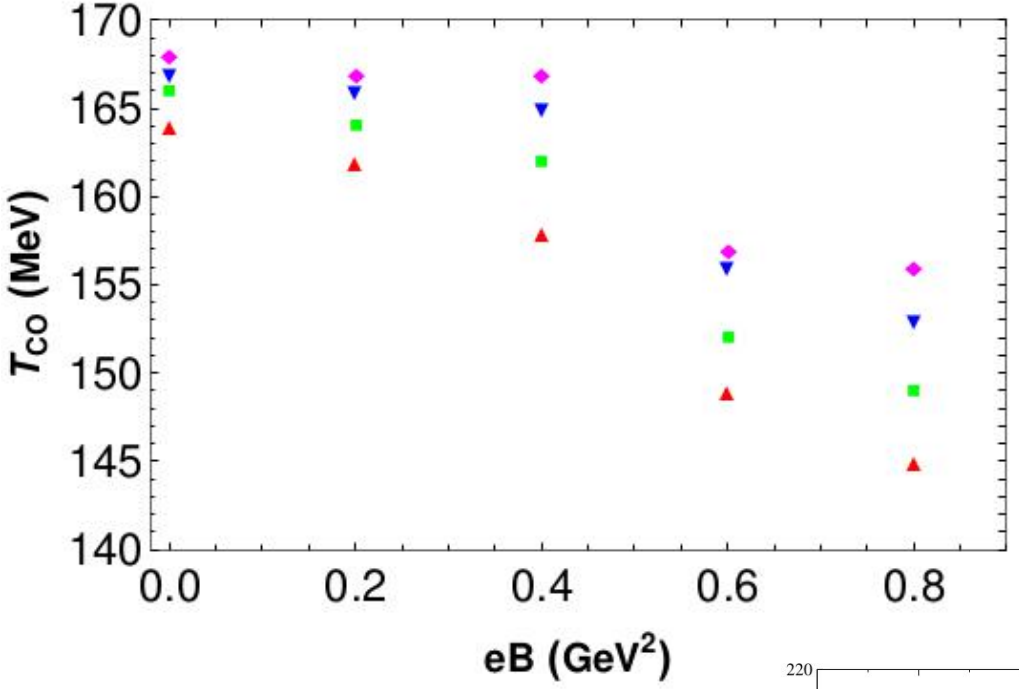


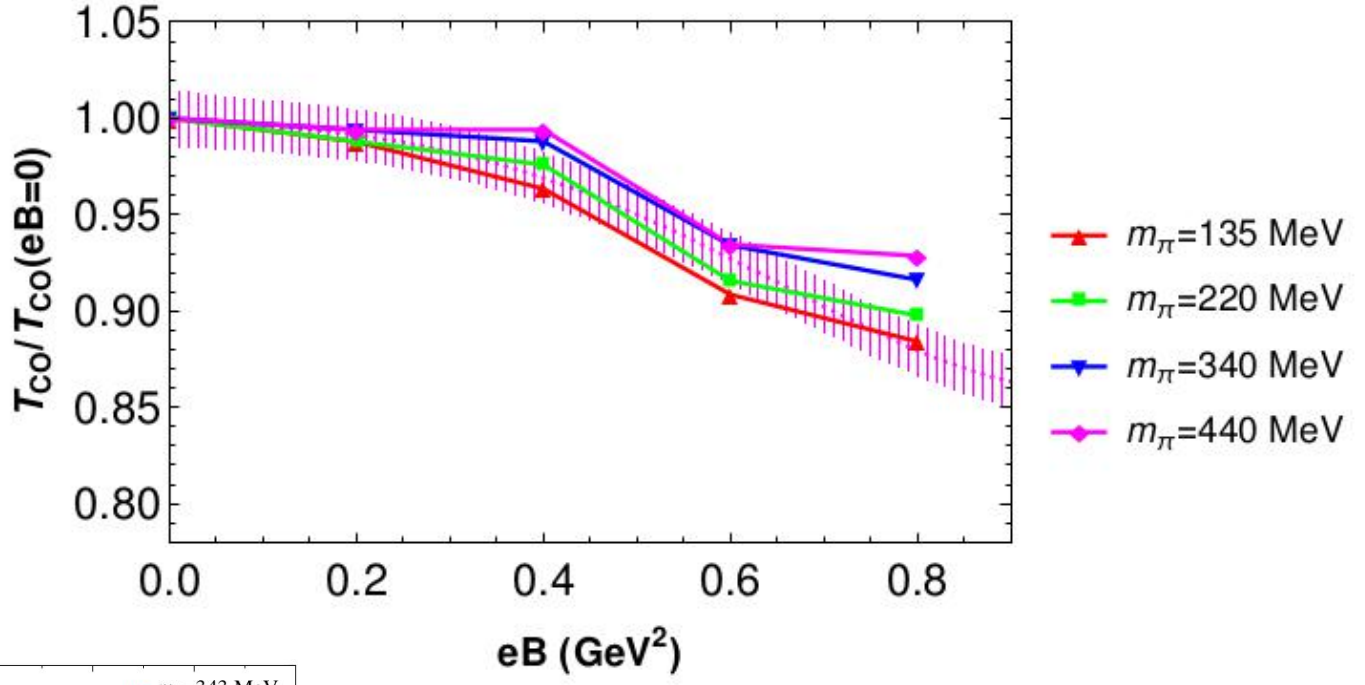
[CAI et al., arXiv: 2407:14449]

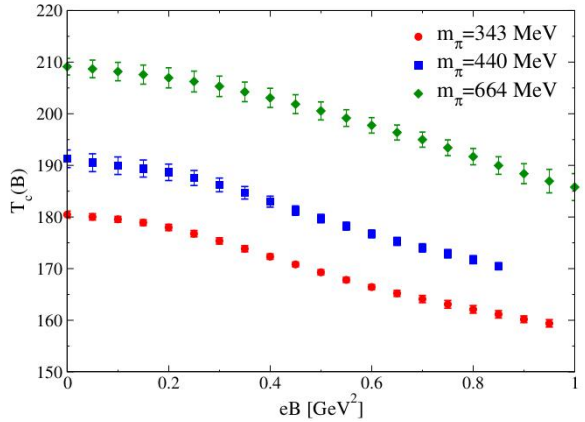
PD for beyond physical pion mass in 2-flavour local model

- The decreasing trend of T_{CO} for different pion masses:

[CAI et al., arXiv: 2407:14449]







[M. D'Elia et al., PRD 98, 054509]

2+1-flavour local model

- In the local 2+1-flavour model:

$$\mathcal{L}_{\rm sym} = G_1 \sum_{a=0}^{8} \left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}\, i\gamma_5\lambda_a\psi\right)^2 \right]$$

$$\mathcal{L}_{\rm det} = -G_2 \left[\det \bar{\psi}_i(1-\gamma_5)\psi_j + \det \bar{\psi}_i(1+\gamma_5)\psi_j \right]$$

$$G_1(\xi) = G_1^0 \frac{1 + a\xi^2 + b\xi^3}{1 + c\xi^2 + d\xi^4} \quad \text{with} \quad \xi = eB/\Lambda_{QCD}^2$$

[M. Ferreira et al., PRD 89 116011]

m_π (MeV)	m (MeV)
135	5.5
220	14.5
340	34.1
440	56.1

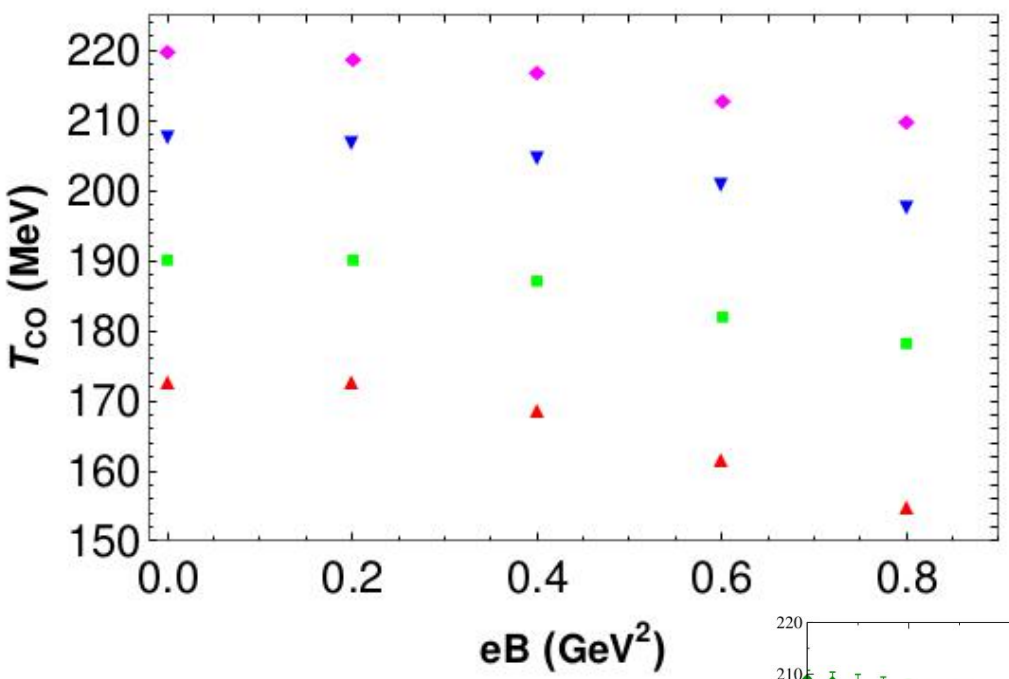
TABLE IV. Parameter m for different pion masses, with Λ, G_1 and G_2 kept fixed to their values at physical point ($\Lambda = 602.3$ MeV and $G_1 = 3.67/\Lambda^2$ and $G_2 = 12.36/\Lambda^5$). The strange-to-light quark mass ratio is kept at its physical value, 25.58.

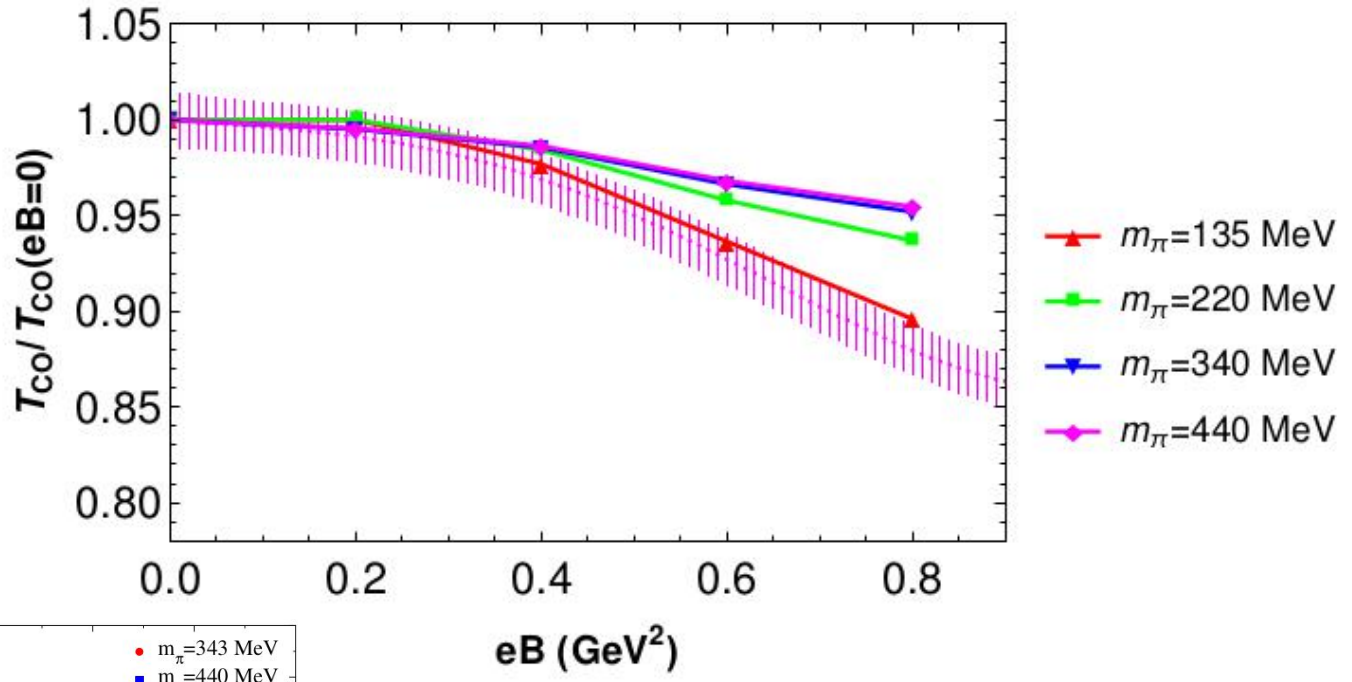
[CAI et al., arXiv: 2407:14449]

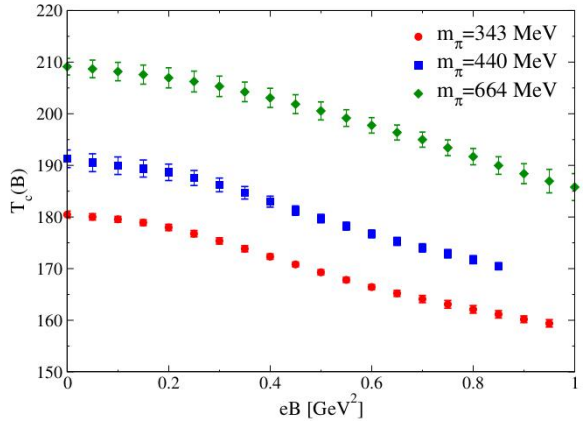
PD for beyond physical pion mass

- The decreasing trend of T_{CO} for different pion masses:

[CAI et al., arXiv: 2407:14449]





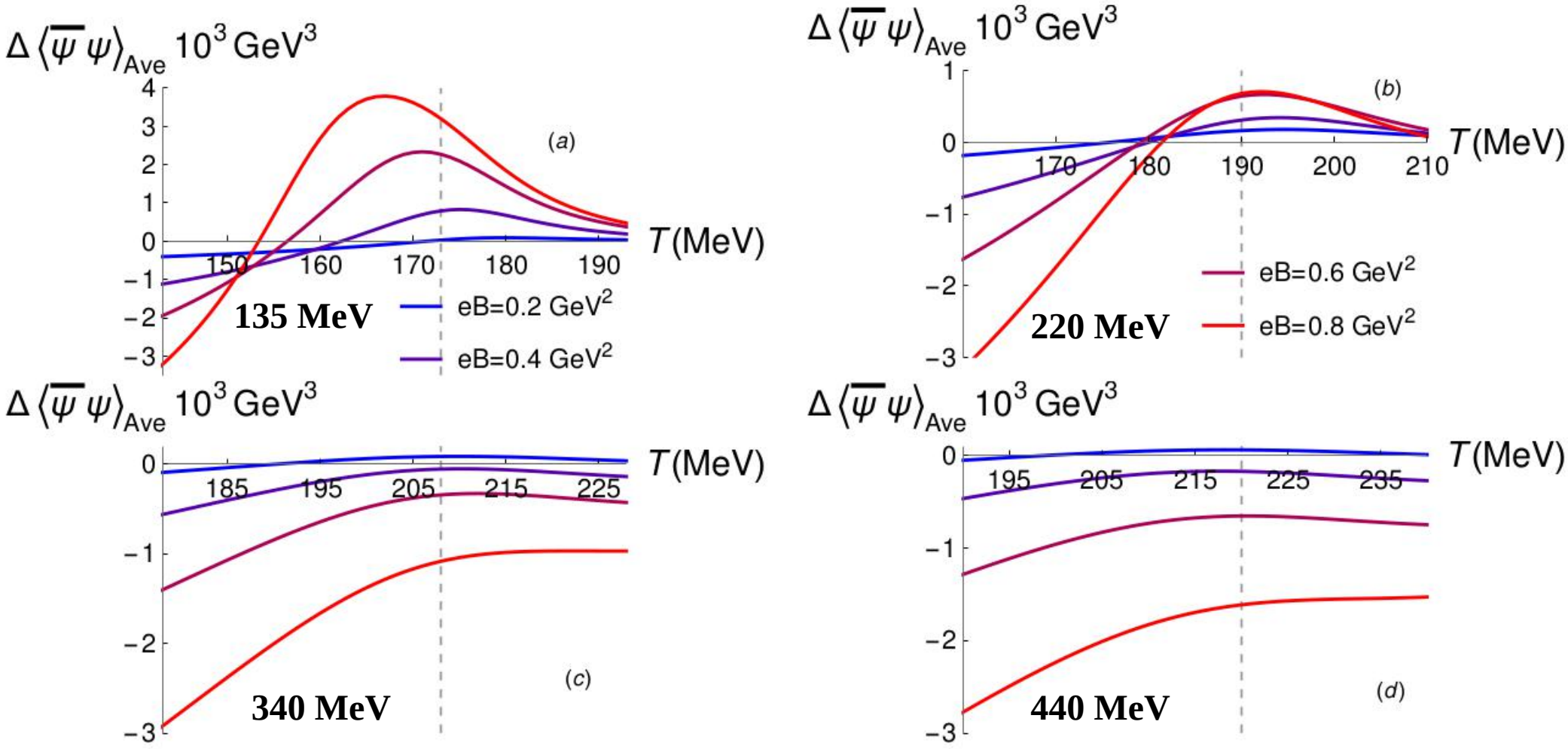


[M. D'Elia et al., PRD 98, 054509]

2+1-flavour local model

- The IMC effect for different pion masses:

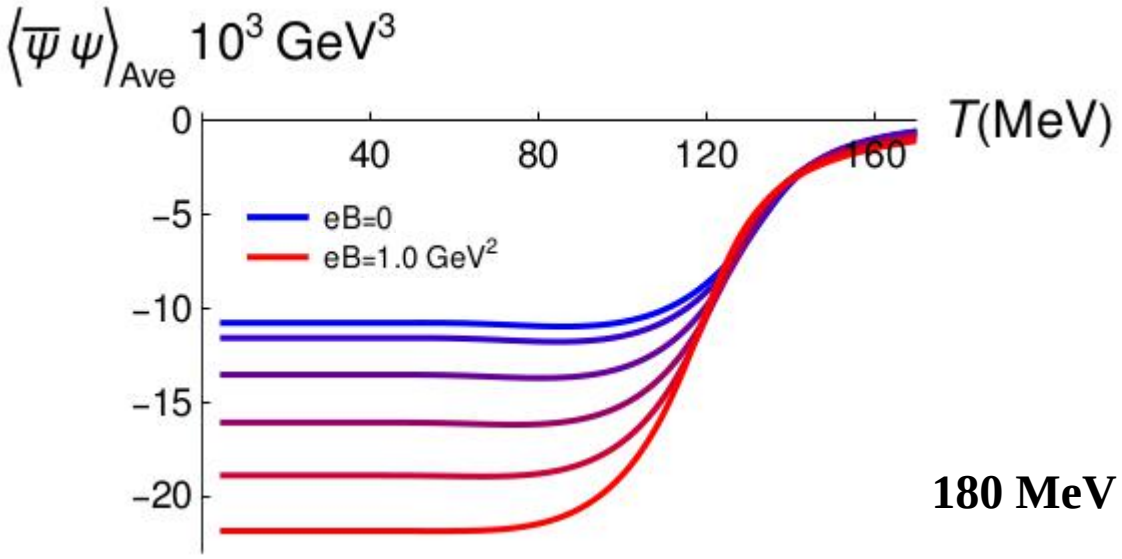
$$\Delta\langle\bar{\psi}\psi\rangle_{\rm Ave}(eB,T,m) = \langle\bar{\psi}\psi\rangle_{\rm Ave}(eB,T,m) - \langle\bar{\psi}\psi\rangle_{\rm Ave}(0,T,m)$$



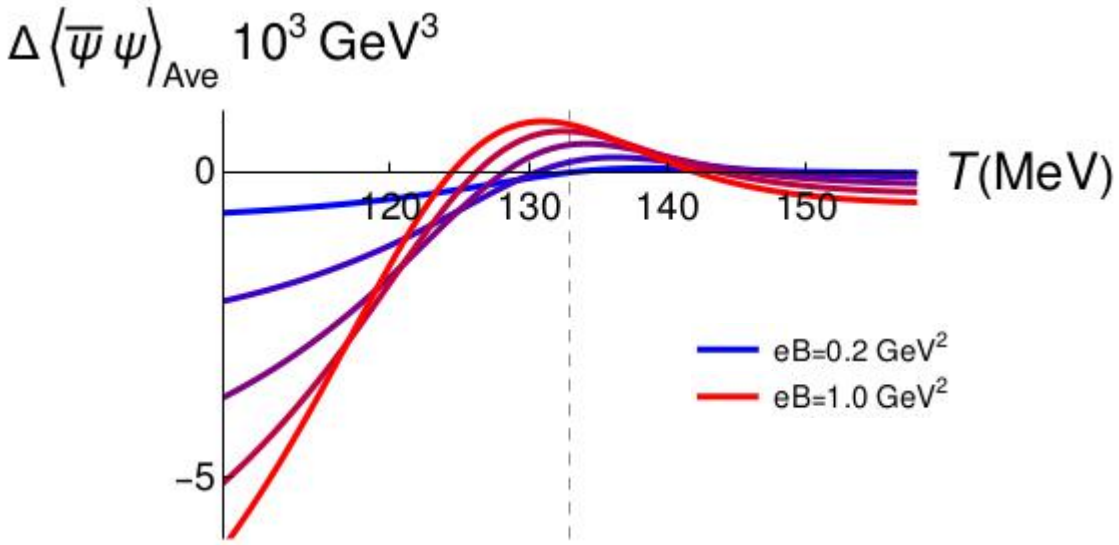
[CAI et al., arXiv: 2407:14449]

Flashback

- The consistent IMC effect for a higher pion mass value in a nonlocal framework:



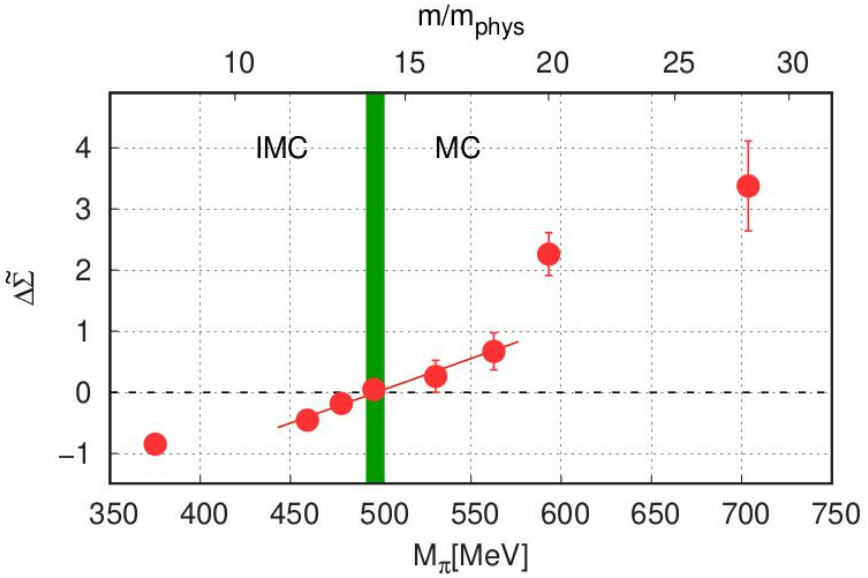
180 MeV



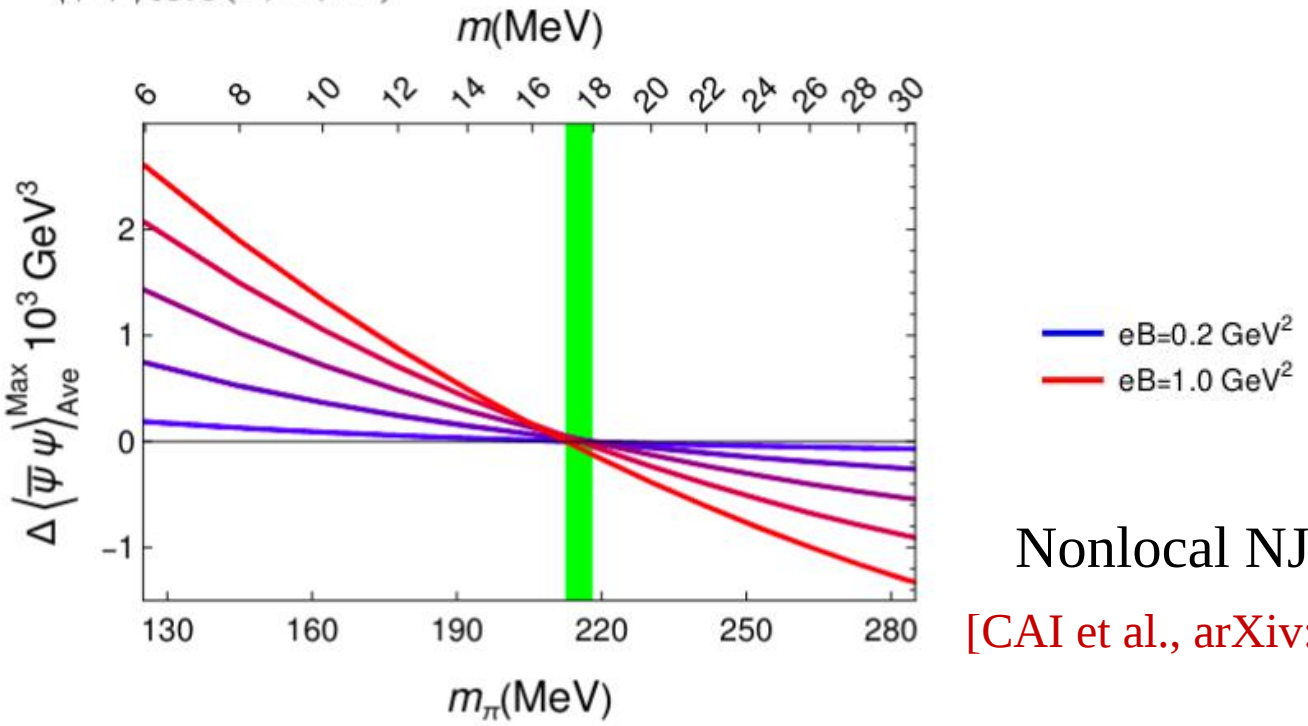
[CAI et al., arXiv: 2407:14449]

The pion mass beyond which the IMC disappears

$$\Delta\langle\bar{\psi}\psi\rangle_{\rm Ave}(eB,T,m) = \langle\bar{\psi}\psi\rangle_{\rm Ave}(eB,T,m) - \langle\bar{\psi}\psi\rangle_{\rm Ave}(0,T,m)$$



[G. Endroidi et al., JHEP07(2019)007]



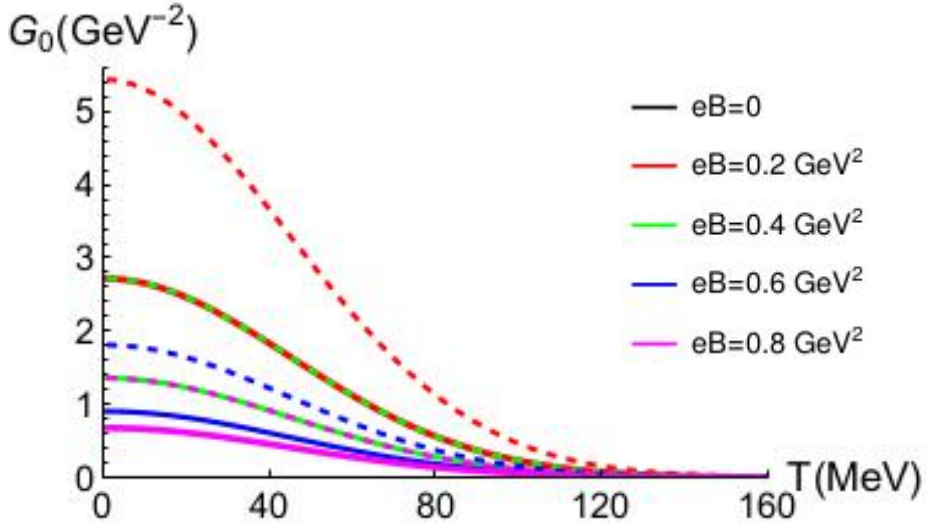
Nonlocal NJL model

[CAI et al., arXiv: 2407:14449]

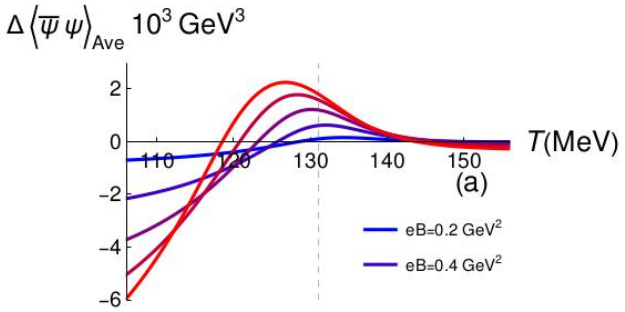
- However, the value is much lower than the LQCD calculated value, $\approx$ 500 MeV, calculated at a single value of eB= $0.6\ \mathrm{GeV}^2$.

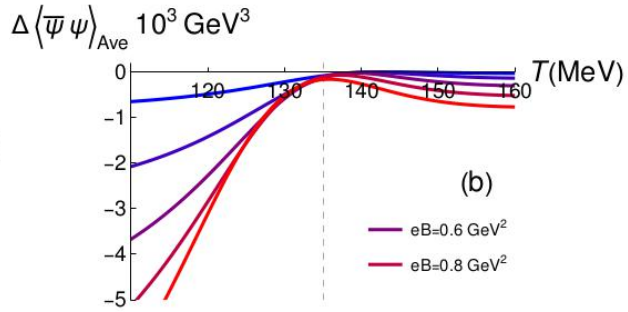
[G. Endroidi et al., JHEP07(2019)007]

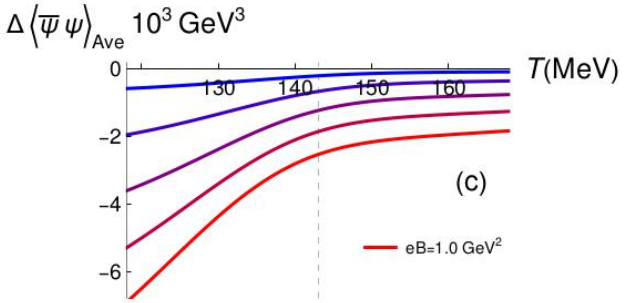
Behaviour of the coupling constant

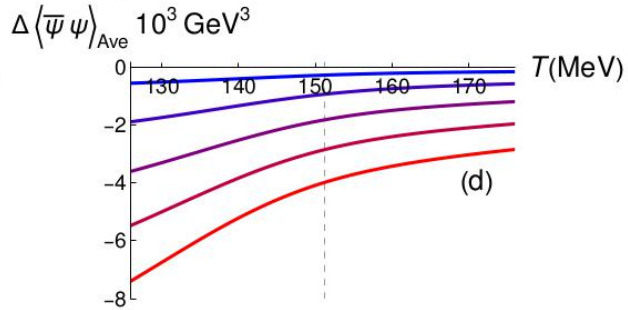


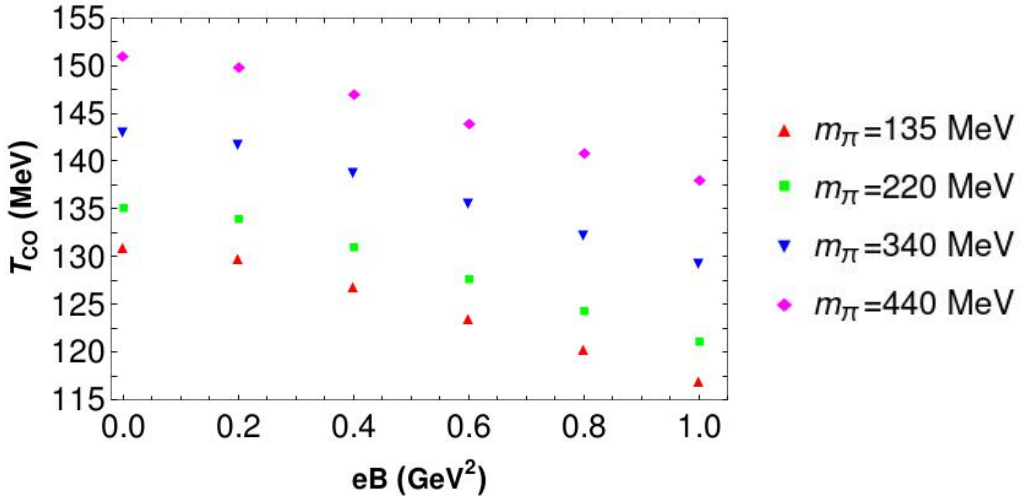
Nonlocal 2-flavour





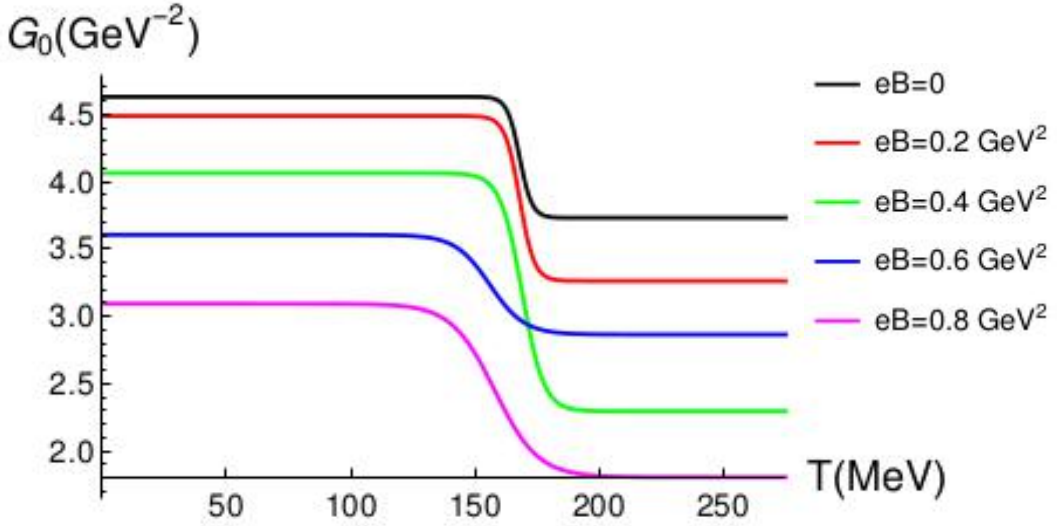




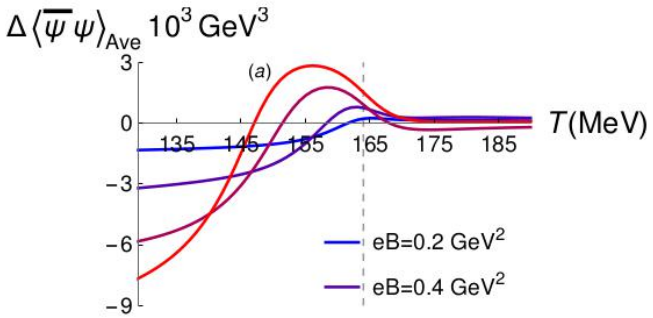


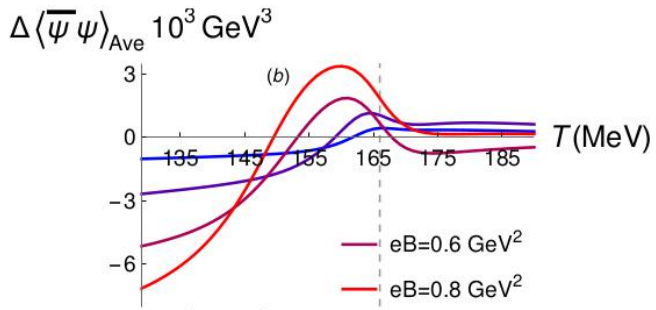
[CAI et al., arXiv: 2407:14449]

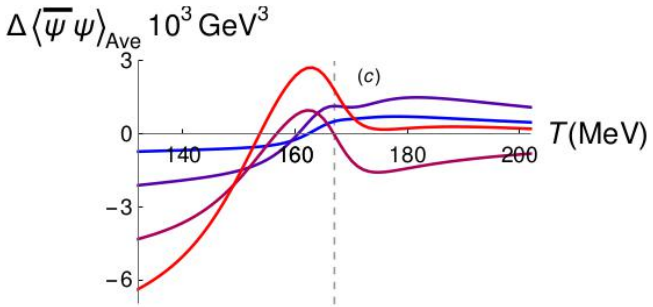
Behaviour of the coupling constant

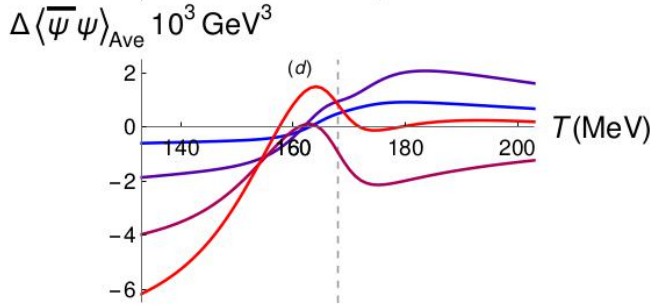


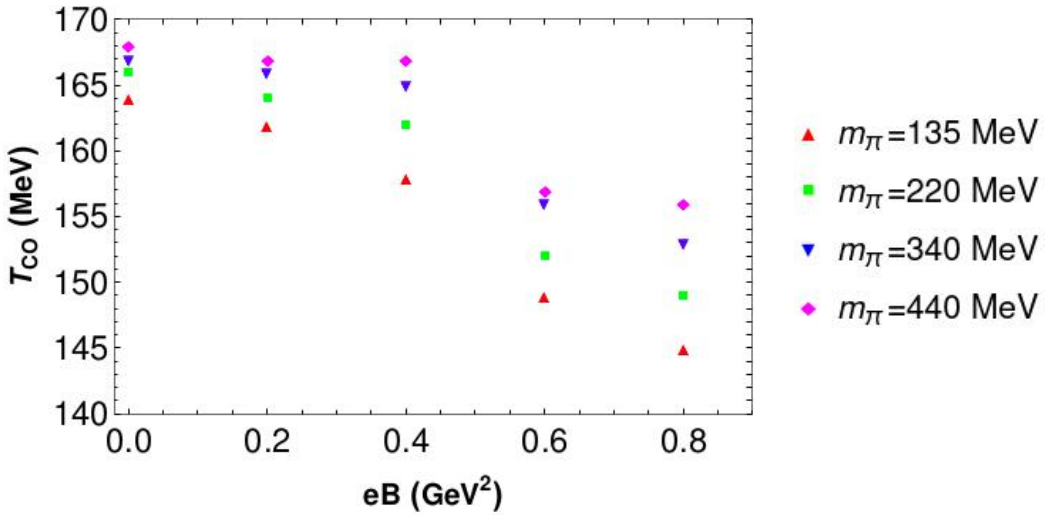
Local 2-flavour





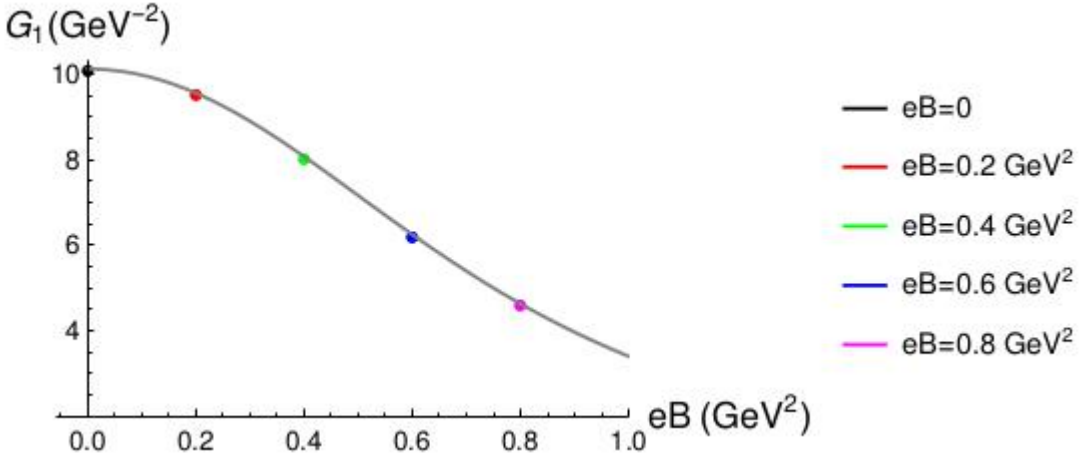




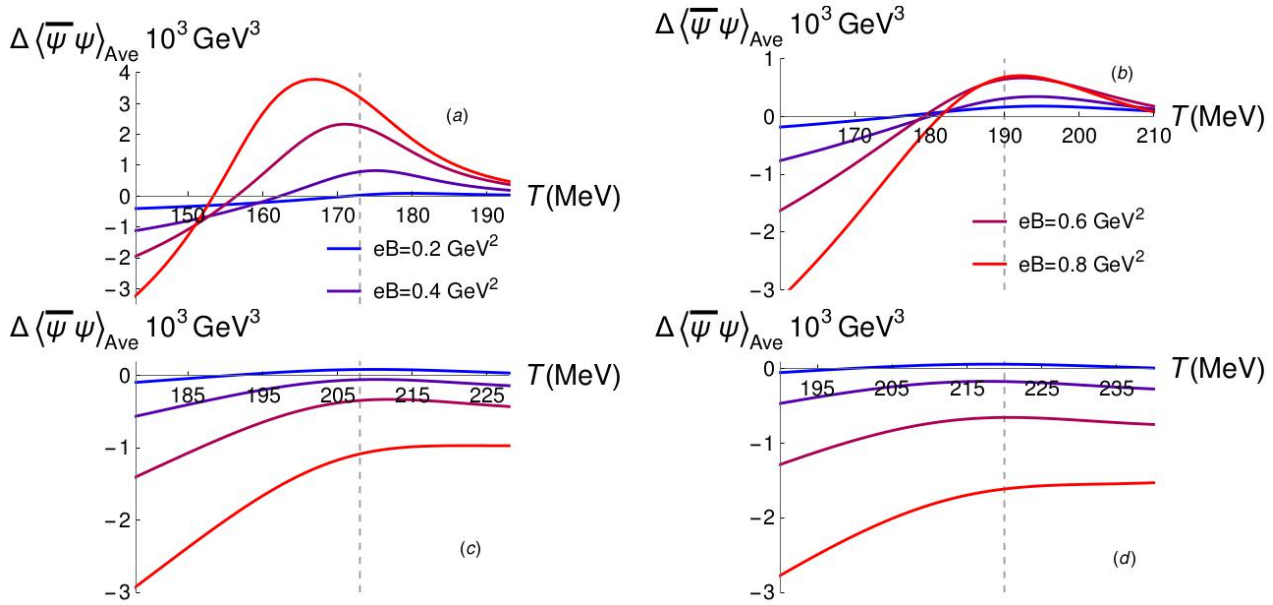


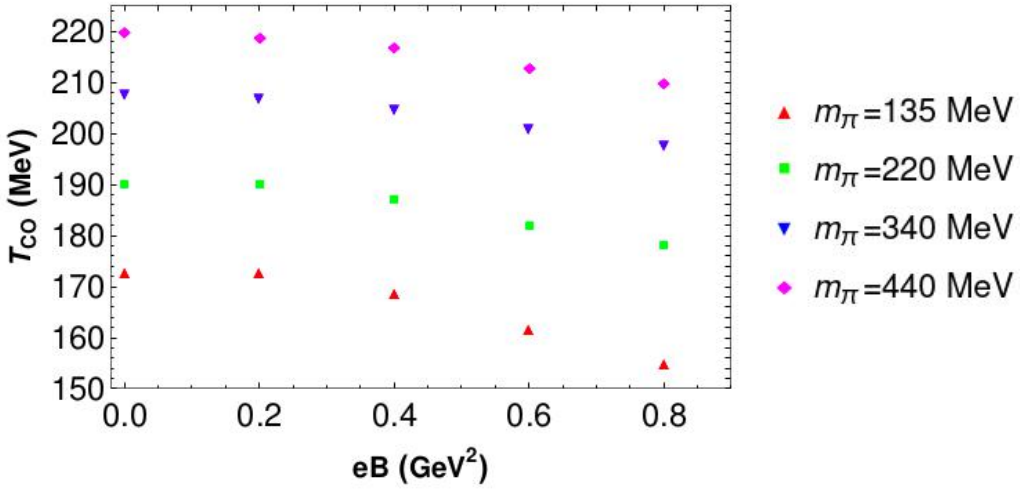
[CAI et al., arXiv: 2407:14449]

Behaviour of the coupling constants



Local (2+1)-flavour





[CAI et al., arXiv: 2407:14449]

Conclusion

- Lattice QCD shows that beyond certain unphysical pion mass the IMC effect disappears but the decreasing trend of the T_{CO} continues.
- We find out that the models are capable of capturing qualitatively the LQCD results for heavier (unphysical) pions.
- In this regard, the key feature in the models is the incorporation of the effect of a reduction in the coupling constant with increasing energy.
- For the local NJL model, this agreement depends on how the parameters of the model are fit at the physical point.
- The nonlocal version, on the other hand, captures the physics more naturally. There, we can also estimate the value of pion mass beyond which the IMC effect disappears.

Thank you

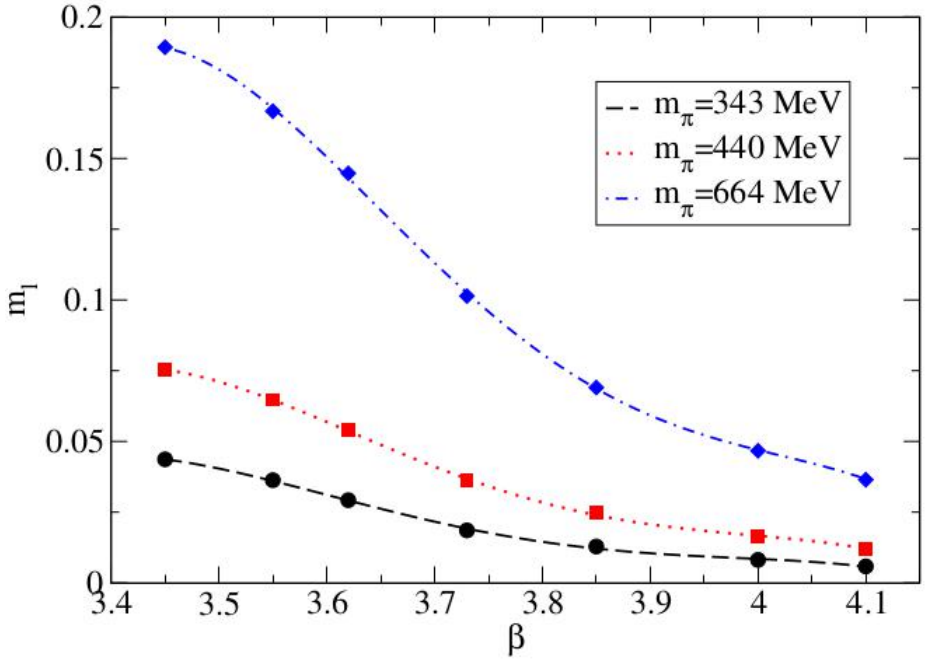


FIG. 1: Lines of constant pion mass in the $m_\ell - \beta$ plane.

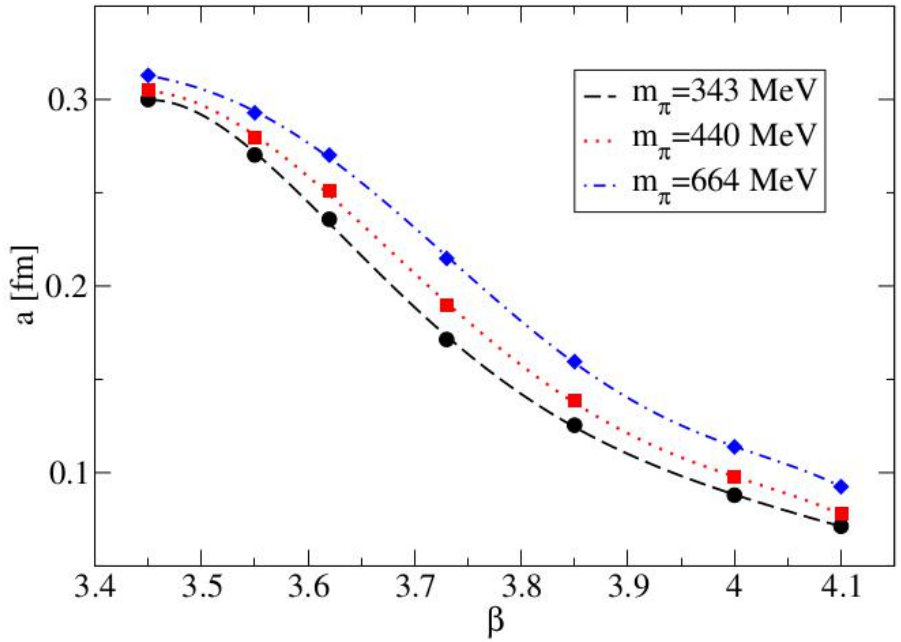


FIG. 2: Dependence of the lattice spacing a on β along the lines of constant pion mass.

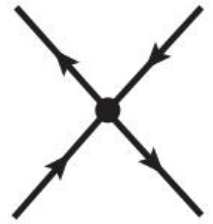
Condensate and f_π beyond physical pion mass

m_π (MeV)	m (MeV)	$\langle\bar{\psi}_i\psi_i\rangle^{1/3}$ (MeV)	f_π (MeV)
135	6.94	221.1	92.9
220	18.28	220.7	95.7
340	42.89	218.6	101.0
440	70.78	214.9	105.6

TABLE II. Parameter m for different pion masses, with Λ and G_0 kept fixed to their values at physical point ($\Lambda = 605.05$ MeV and $G_0 = 29.38/\Lambda^2$). The condensate and the pion decay constant are given for different pion masses.

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\gamma_\mu \partial^\mu - m_0)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau}\psi)^2]$$

The standard NJL Lagrangian



Local Four point interaction

Local version, the most popular one: advantages and disadvantages. Why do we need the non-local version?

Let's understand the basic working method of non-local version.

Let's first define a current and the corresponding action.

$$J_a^\mu = \bar{\psi}(x)\gamma^\mu \frac{\lambda_a}{2}\psi(x) \longrightarrow \mathcal{S}_{\mathrm{int}} = -\int d^4x \int d^4y J_a^\mu(x)\mathcal{G}_{\mu\nu}^{ab}(x-y)J_b^\nu(y)$$

Equivalent to gluonic field correlator

What should be the form of this correlator? And how to introduce it?

Non-local interactions are introduced in two alternative ways:

a) Instanton liquid picture: (Bowler and Birse, for example: NPA 582 (1995) 655-664 and others)

$$j_a(x) = \int d^4y d^4z r(y-x) r(x-z) \bar{\psi}(y) \Gamma_a \psi(z)$$

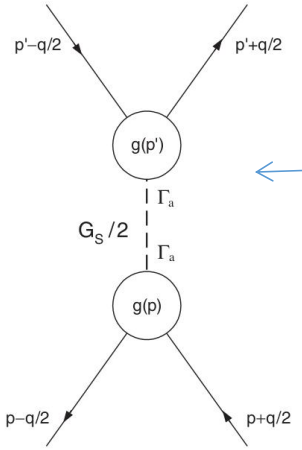
b) One gluon exchange: (Sebastian Schmidt et al, PRC 50 435 (1994))

$$j_a(x) = \int d^4z g(z) \bar{\psi}\left(x + \frac{z}{2}\right) \Gamma_a \psi\left(x - \frac{z}{2}\right)$$

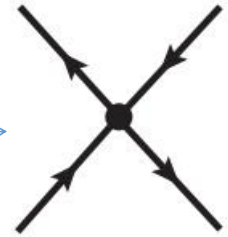
$r(x\text{-}y)$ and $g(z)$ are the non-local covariant form factors. Please see: N. N. Scoccola et al, PRD 74, 054026 (2006)

The major properties of these two methods are more or less the same. We will mainly focus on OGE method.

With the OGE method the non-local interaction looks like this:



← Difference →



Let's look at the action:

$$\frac{\mathcal{S}_{\mathrm{E}}^{\mathrm{MF}}}{V^{(4)}} = -4N_{\mathrm{c}} \int \frac{d^4p}{(2\pi)^4} \ln[p^2 + M^2(p)] + \frac{\bar{\sigma}^2}{2G}$$

The constituent mass is found from the gap equation: $M(p) = m_q + \mathcal{C}(p)\bar{\sigma}$ with the mean field $\bar{\sigma} = \langle\sigma\rangle$

By the principle of least action we can get the mean field:

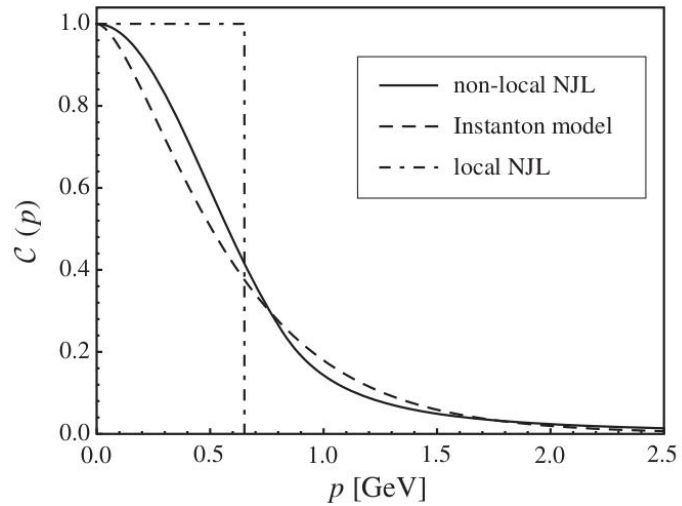
$$\bar{\sigma} = 8N_{\mathrm{c}}G \int \frac{d^4p}{(2\pi)^4} \mathcal{C}(p) \frac{M(p)}{p^2 + M^2(p)}$$

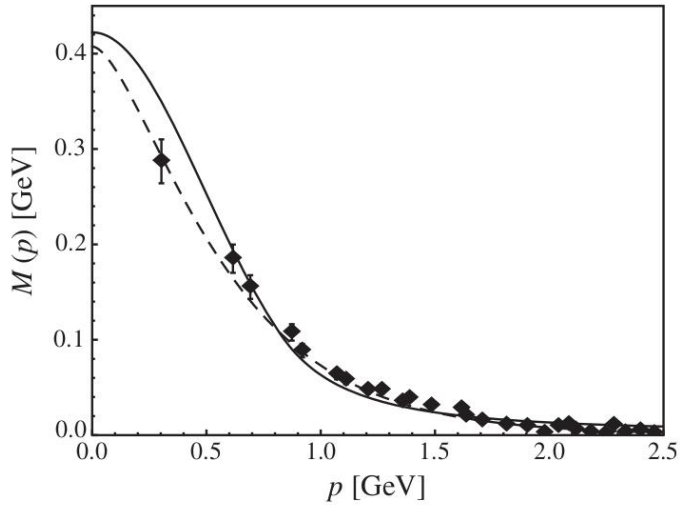
How to get the condensate:

$$\langle\bar{\psi}\psi\rangle = -4N_{\mathrm{f}}N_{\mathrm{c}} \int \frac{d^4p}{(2\pi)^4} \left[\frac{M(p)}{p^2 + M^2(p)} - \frac{m_q}{p^2 + m_q^2}\right]$$

Differences with the local version? $M = -G\langle\bar{\psi}\psi\rangle = \bar{\sigma}$

A good article to understand the basic
T. Hell et al, PRD 79 014022 (2009)





Advantages and disadvantages

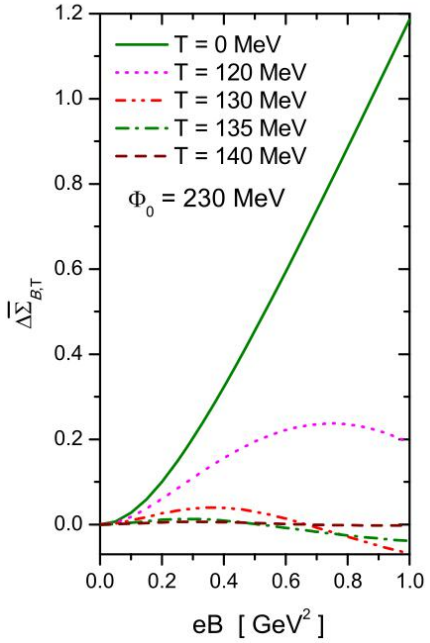
IMC appears: N. N. Scoccola et al PRD 95, 034013 (2017): $$j_a(x) = \int d^4z g(z) \bar{\psi}\left(x + \frac{z}{2}\right) \Gamma_a \psi\left(x - \frac{z}{2}\right)$$

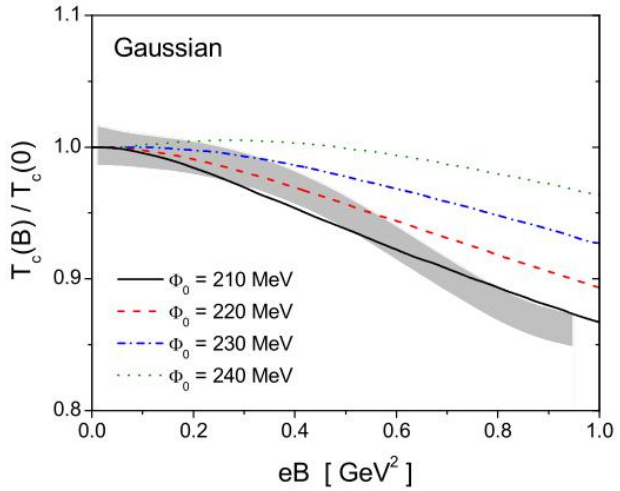
"In the case of the nonlocal model under consideration, the situation is more complicated since the inclusion of gauge interactions implies a change not only in the kinetic terms of the Lagrangian but also in the nonlocal currents."

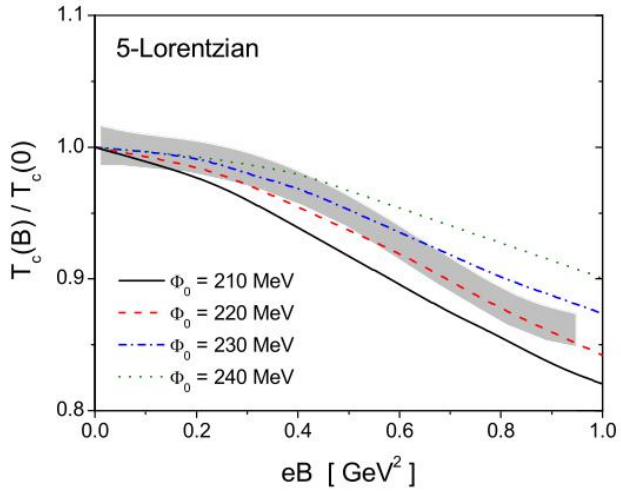
$$\psi(x - z/2) \to W(x, x - z/2)\psi(x - z/2)$$

Similarly for, $\overline{\psi}(x + z/2)$

$$W(s,t) = \mathrm{P}\ \exp\left[-i\hat{Q}\int_s^t dr_\mu \mathcal{A}_\mu(r)\right]$$







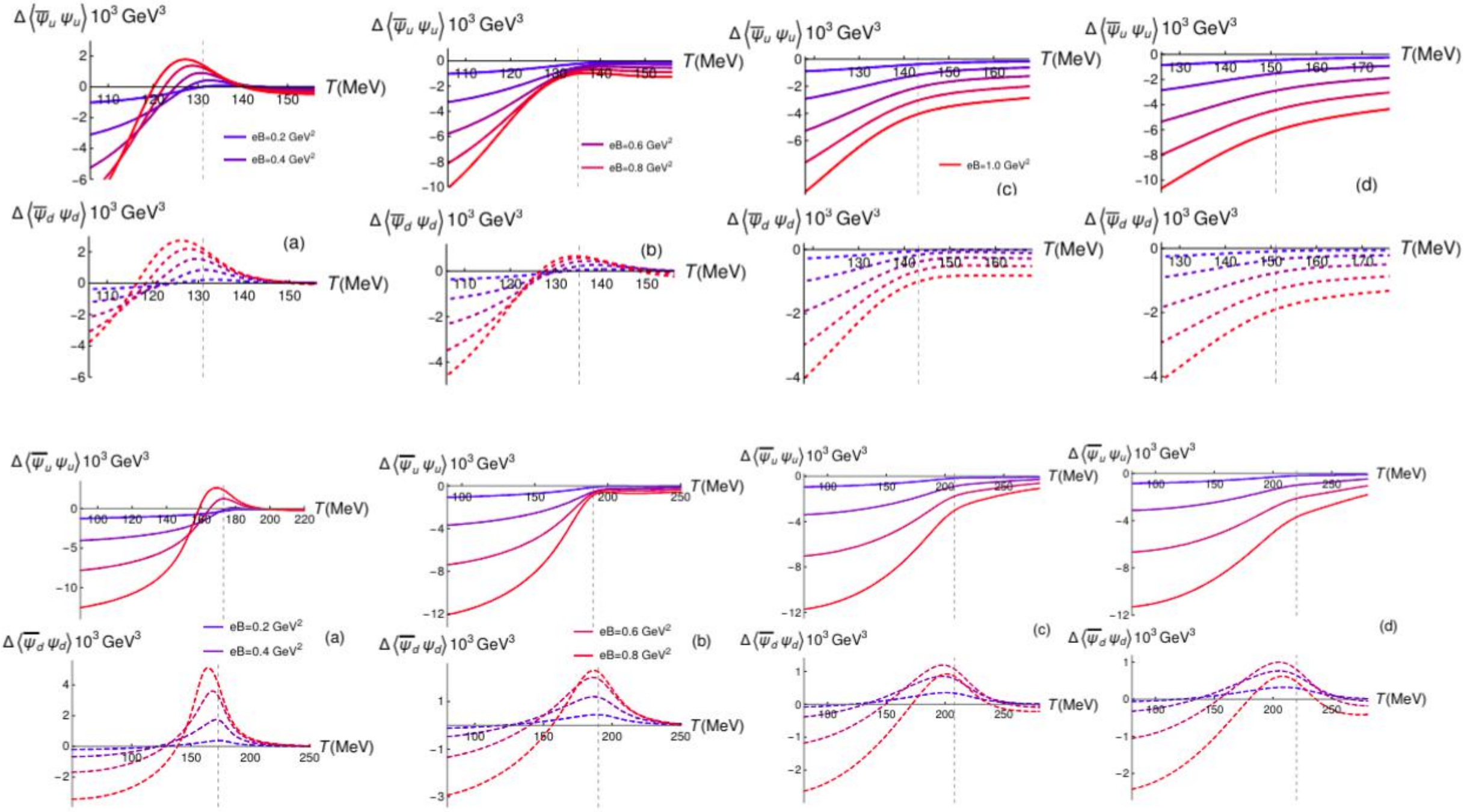
$$\langle \bar{\psi}_f(x)\psi_f(x)\rangle = \frac{\partial \Omega}{\partial m} = -\,4N_c \int \frac{d^4q}{(2\pi)^4}\left[\frac{M(q)}{q^2+M^2(q)} - \frac{m}{q^2+m^2}\right]$$

$$\mathcal{G}^{\pm}(p^2) = \frac{1}{G_0} - \,8\,N_c \int \frac{d^4q}{(2\pi)^4}\; h^2(q)\frac{[(q^+\cdot q^-) \mp M(q^+)M(q^-)]}{[(q^+)^2+M^2(q^+)]\,[(q^-)^2+M^2(q^-)]}$$

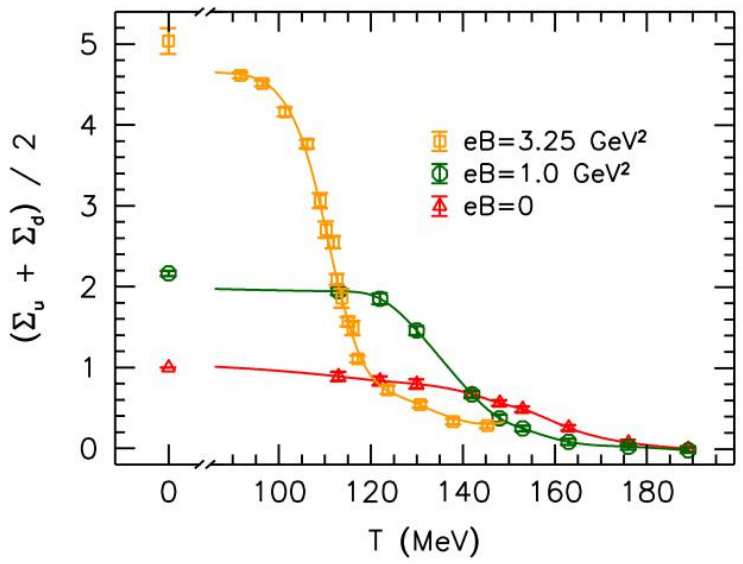
$$\mathcal{G}^{-}(-m_\pi^2) = 0$$

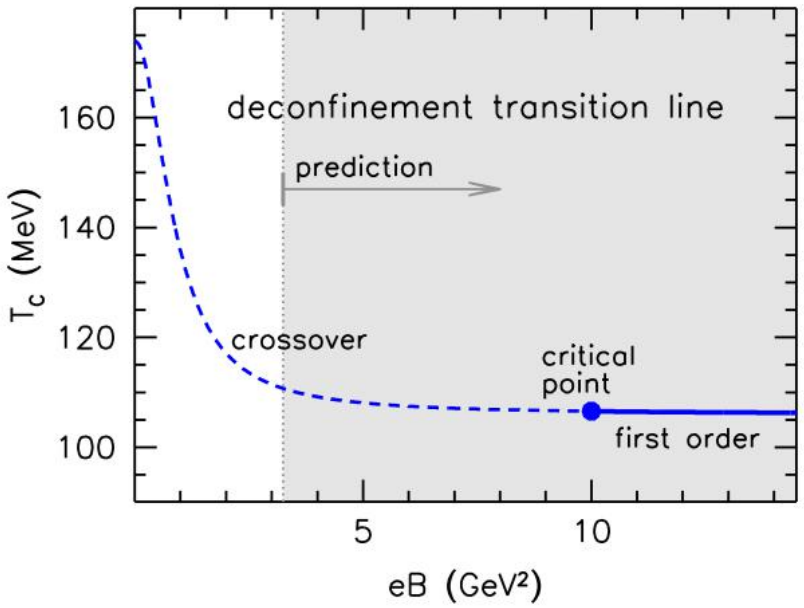
$$m_\pi^2\; F_\pi = m\; Z_\pi^{1/2}\; J(-m_\pi^2)$$

$$J(p^2) = \,8\,N_c \int \frac{d^4q}{(2\pi)^4}\; h(q)\frac{[(q^+\cdot q^-) + M(q^+)M(q^-)]}{[(q^+)^2+M^2(q^+)]\,[(q^-)^2+M^2(q^-)]}$$


$\Delta\langle\bar{\psi}_u\psi_u\rangle 10^3\,\mathrm{GeV}^3$
$\Delta\langle\bar{\psi}_d\psi_d\rangle 10^3\,\mathrm{GeV}^3$
T(MeV)
eB=0.2 GeV^2
eB=0.4 GeV^2
eB=0.6 GeV^2
eB=0.8 GeV^2
eB=1.0 GeV^2
(a)
(b)
(c)
(d)

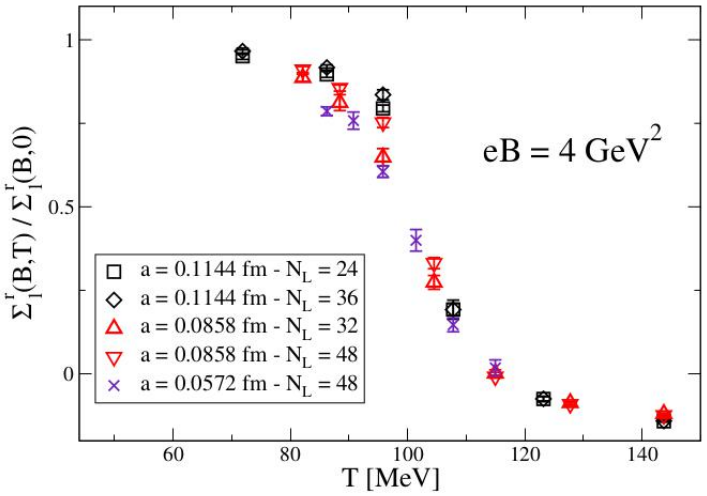
CP in the T-eB plane

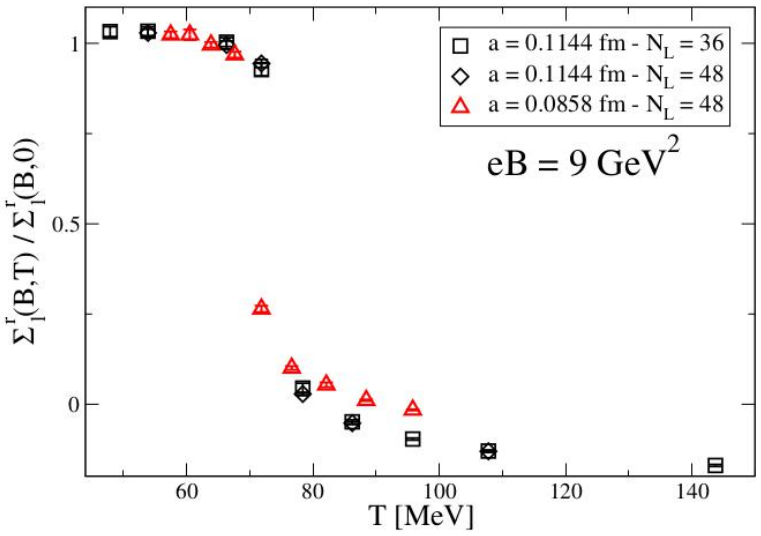


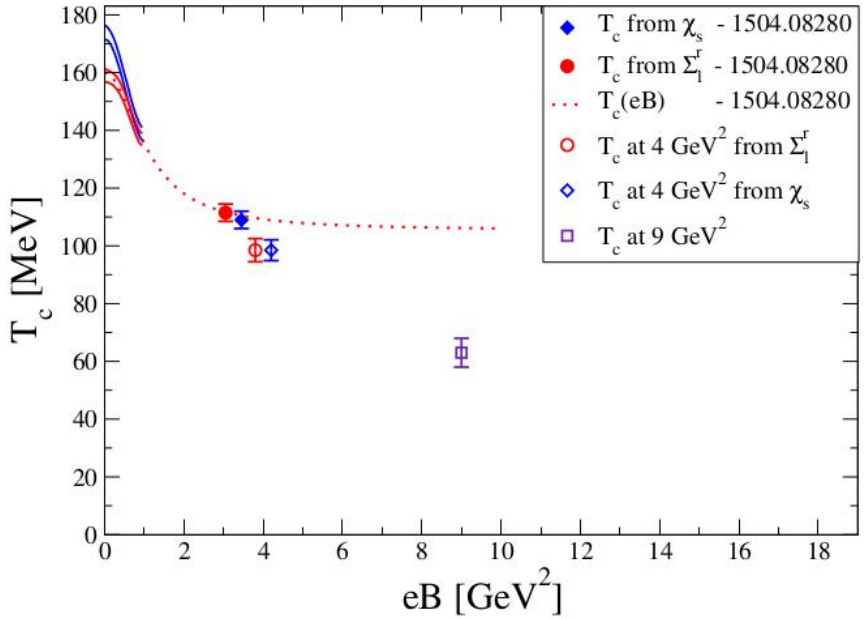


[Endrődi 2015]

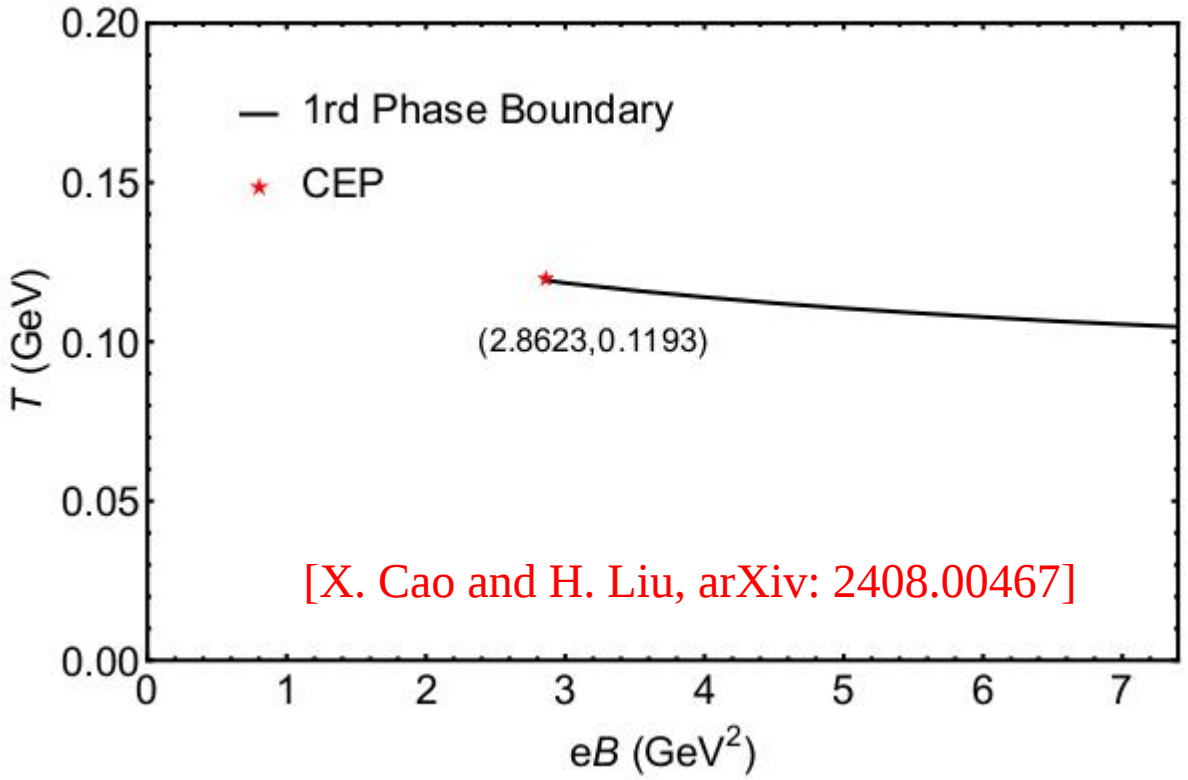
[Massimo et al 2015]







CP in the T-eB plane



[X. Cao and H. Liu, arXiv: 2408.00467]