The Roberge-Weiss transition in $N_f = 2+1$ QCD with background magnetic fields

K. Zambello[1,2] M. D'Elia[1,2] L. Maio[3] G. Zanichelli[1]

[1] University of Pisa [2] INFN - Sezione di Pisa

[3] CPT, Marseille

Trento, 12/09/2024

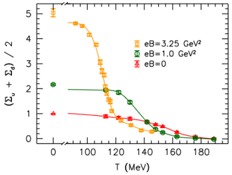
Introduction

Introduction (I)

- The phase diagram of QCD in the presence of strong magnetic fields has been actively studied during recent years, being relevant for understanding a wide range of physical phenomena, from the physics of the early universe to heavy-ion collision experiments

- Some interesting features:

• chiral symmetry breaking enhanced at zero T, but chiral condensate decreases around T_c (magnetic catalysis / inverse magnetic catalysis)



• strengthening of the chiral transition, but chiral restoration temperature T_c decreases as a function of eB

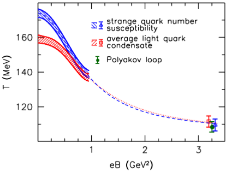


Fig. from JHEP 07, 173 (2015)

Introduction

Introduction (II)

- Some interesting features:

• the transition is a crossover at low eB, turns first order somewhere between 4 and 9 GeV^2

JHEP 07, 173 (2015) Phys.Rev.D 105, 034511 (2022)

but see also Phys.Rev.D 102, 054505 (2020)

• the (adimensional) curvature coefficient of the chiral crossover temperature $k_2 \sim 0.013(2)$ is weakly dependent on eB;

the (physical) curvature coefficient $A_2(eB) = k_2(eB)/T_c(eB)$ qualitatively changes behaviour at $eB \sim 0.6\ GeV^2$

$$\frac{T_c(eB,\mu_B)}{T_c(eB)} = 1 - k_2 \left(\frac{\mu_B}{T_c(eB)}\right)^2$$

Phys.Rev.D 100, 114503 (2019)

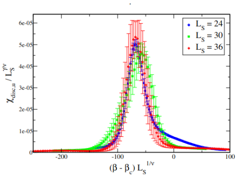
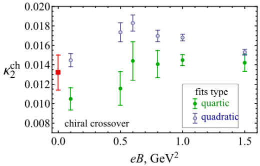
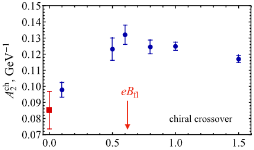


Fig. from Phys.Rev.D 105, 034511 (2022)





Figs. from Phys.Rev.D 100, 114503 (2019)

- In this work we investigate the Roberge-Weiss transition in the phase diagram at imaginary chemical potentials

• RW line at $\mu_B/T = i\pi$, whose end-point $(i\pi, T_{RW})$ is believed to be a second order critical point for physical quark masses

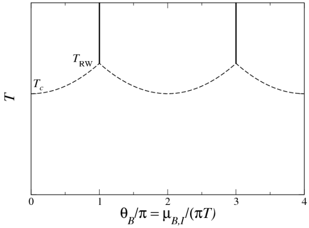
Phys.Rev.D 93, 074504 (2016)

Phys.Rev.D 105, 034513 (2022)

• indications that $T_{RW} \sim T_{Chiral}$ in the chiral limit

Phys.Rev.D 99, 014502 (2019)

Phys.Rev.D 106, 014510 (2022)



- Questions:

• What is the dependence of T_{RW} on eB?

• What is the fate of the transition at strong magnetic fields?

• Is there any relation between T_{RW} and the chiral restoration temperature?

Numerical set-up:

- $N_f = 2+1$, stout-staggered fermions with physical masses, tree-level Symanzik improved action
- $N_t = 6, 8$ lattices with different volumes
- We stay at constant chemical potential $\mu_f / T = i\pi$
- We use the (imaginary part of the) Polyakov loop $L = \langle |Im\ L| \rangle$ as the order parameter of the transition.

  For different eB we estimate T_{RW} as the inflection point of $L(T)$ and the peak of its susceptibility $\chi_L(T)$,

  $$L = \langle |Im\ L| \rangle$$
  $$\chi_L = N_t N_s^3 (\langle (Im\ L)^2 \rangle - \langle |Im\ L| \rangle^2)$$

  At fixed N_t and b_z, the temperature T is tuned by changing a. The magnetic field is $eB = \frac{6\pi b_z}{(aN_s)^2} = \frac{6\pi b_z N_t^2}{N_s^2} T^2$.
- Finite-size scaling analysis to determine the order of the transition

  $$\chi_L = N_s^{\frac{\gamma}{\nu}} \phi(t N_s^{\frac{1}{\nu}}),\ t = \frac{T - T_{RW}}{T_{RW}}$$

Transition at finite eB

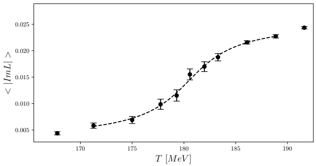
Transition at $0.2, 0.4, 0.6\ GeV^2$

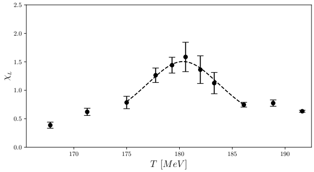
$N_t = 6$ runs at $eB = 0.2, 0.4, 0.6\ GeV^2$ (and $0\ GeV^2$)

- $N_s = 18, 24$ give similar results for T_{RW}, finite-size effects are tiny

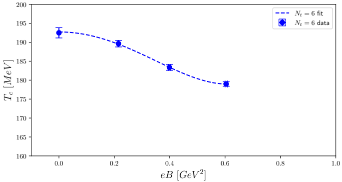
  i.e. $T_{RW}(N_s = 18, eB = 0.6\ GeV^2) = 180.38(69)\ MeV$

  $T_{RW}(N_s = 24, eB = 0.6\ GeV^2) = 178.99(59)\ MeV$





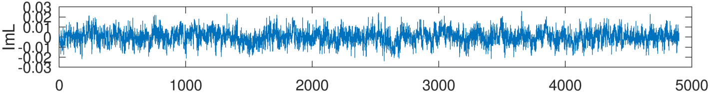
- We observe that T_{RW} decreases as a function of eB and the data fit well to a rational function $T_{RW}(eB) = T^0_{RW}\ \frac{1+a(eB)^2}{1+b(eB)^2}$

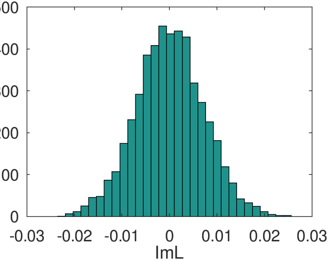


Transition at finite eB

Crosscheck for the RW line (low T)

Monte Carlo history and histogram of ImL at $\frac{\mu_f}{T} = i\pi$, $T < T_{RW}$

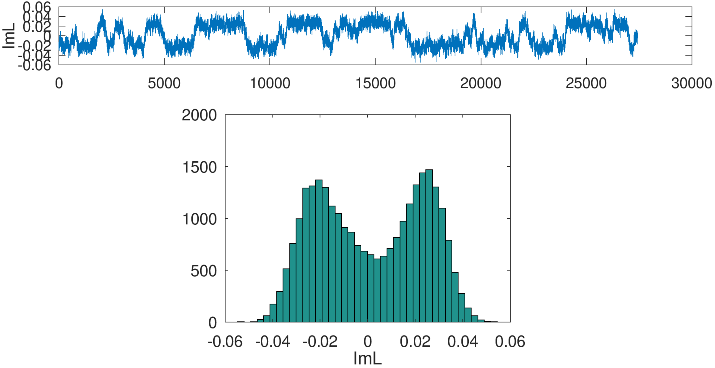




Transition at finite eB

Crosscheck for the RW line (high T)

Monte Carlo history and histogram of ImL at $\frac{\mu_f}{T} = i\pi$, $T \gtrsim T_{RW}$



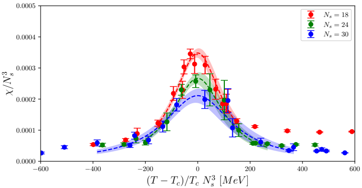
$\rightarrow$ first order phase transition between left and right sectors

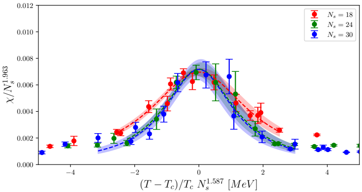
Transition at finite eB

FSS analysis at 1 GeV^2

$N_t = 6$ runs at $eB = 1\ GeV^2$

- Finite-size scaling analysis for $N_s = 18, 24, 30$, collapse plots:

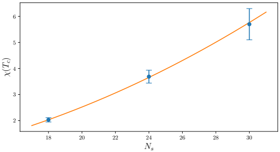




$\rightarrow$ compatible with a second order transition of the $Z(2)$ universality class

- Fitting the peaks of χ_L:

$$\chi_{max}(N_s) = \alpha\ N_s^{\frac{\gamma}{\nu}} \rightarrow \frac{\gamma}{\nu} = 2.04(19)$$

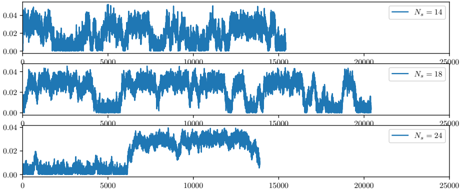


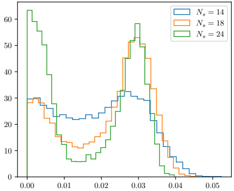
Transition at finite eB

Transition at 2.5 GeV^2 (I)

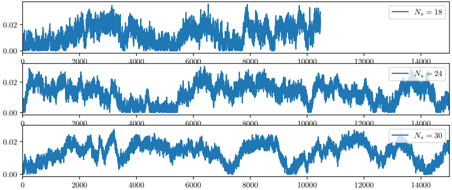
$N_t = 6$ runs at $eB = 2.5\ GeV^2$

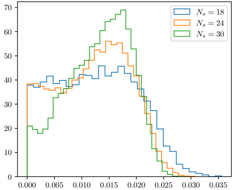
Histograms show a double peaked distribution, suggesting the presence of metastable states typical of a first order transition.





This can be compared with $eB = 1.0\ GeV^2$,



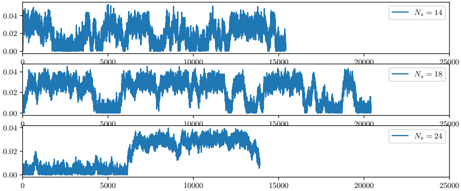


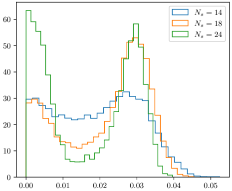
Transition at finite eB

Transition at 2.5 GeV^2 (II)

$N_t = 6$ runs at $eB = 2.5\ GeV^2$

Histograms show a double peaked distribution, suggesting the presence of metastable states typical of a first order transition.





But we expect large discretization effects.

$$eB = \frac{6\pi b_z N_t^2}{N_s^2} T^2, \text{ we want } \frac{b_z}{N_s^2} \ll 1$$

At 1 GeV^2, $b_z = 30$ and $N_s = 24 \rightarrow \frac{b_z}{N_s^2} \approx 0.05$

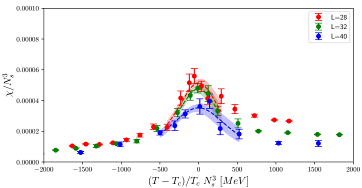
At 2.5 GeV^2, $b_z = 89$ and $N_s = 24 \rightarrow \frac{b_z}{N_s^2} \approx 0.15$

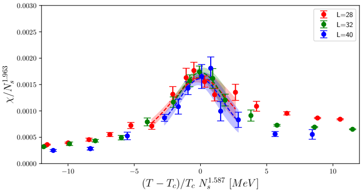
Towards the continuum limit

FSS analysis at 1 GeV^2

$N_t = 8$ runs at $eB = 1\ GeV^2$

- Finite-size scaling analysis for $N_s = 28, 32, 40$, collapse plots:

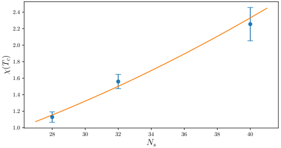




$\rightarrow$ compatible with a second order transition of the $Z(2)$ universality class

- Fitting the peaks of χ_L:

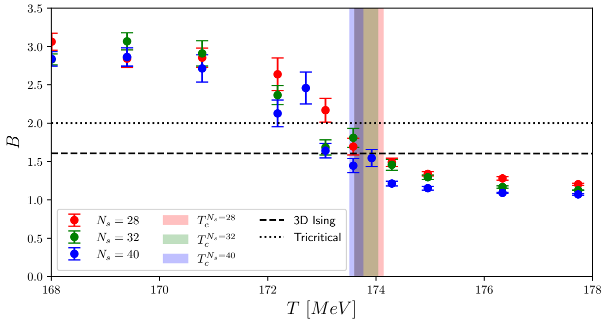
$$\chi_{max}(N_s) = \alpha\ N_s^{\frac{\gamma}{\nu}} \rightarrow \frac{\gamma}{\nu} = 1.97(28)$$



Towards the continuum limit
FSS analysis at 1 GeV^2

Binder cumulant:

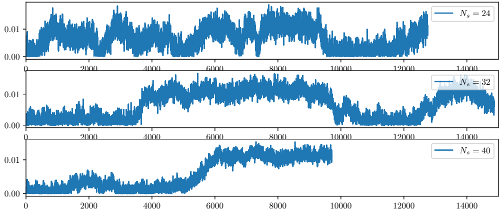
Results are compatibile with a critical point belonging to the $Z(2)$ universality class, but a tricritical point cannot be ruled out.

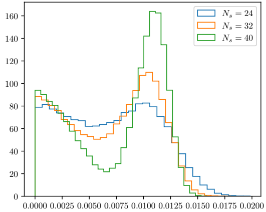


Towards the continuum limit
FSS analysis at 2.5 GeV^2

$N_t = 8$ runs at $eB = 2.5\ GeV^2$

Histograms still show a double peaked distribution, suggesting the presence of metastable states typical of a first order transition.





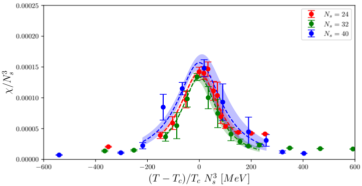
In this case $\frac{b_z}{N_s^2} \approx 0.08$.

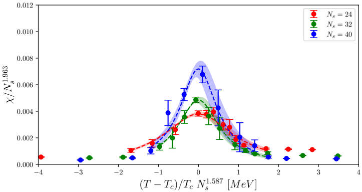
Towards the continuum limit

FSS analysis at 2.5 GeV^2

$N_t = 8$ runs at $eB = 2.5\ GeV^2$

- Finite-size scaling analysis for $N_s = 24, 32, 40$, collapse plots:

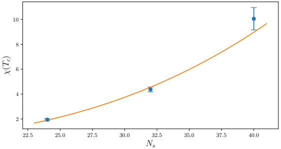




$\rightarrow$ compatible with a first order transition

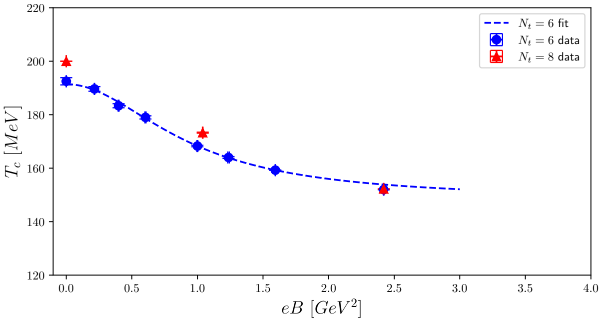
- Fitting the peaks of χ_L:

$$\chi_{max}(N_s) = \alpha\ N_s^{\frac{\gamma}{\nu}} \rightarrow \frac{\gamma}{\nu} = 3.03(18)$$



All in all, data fit well to a rational function
$T_{RW}(eB) = T^0_{RW} \, \frac{1+a(eB)^2}{1+b(eB)^2}$ up to 1.6 GeV^2.

Full data set (including 2.5 GeV^2) well parametrized by
$T_{RW}(eB) = T^0_{RW} \, \frac{1+a(eB)^2+c(eB)^4}{1+b(eB)^2+d(eB)^4}$.



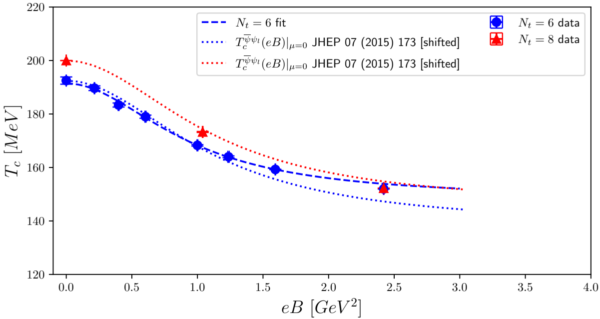
Curvature of the critical line (II)

We can Taylor expand the rational function ansatz around $eB = 0$:

$$T_{RW}(eB) = T^0_{RW} + k(eB)^2$$

Curvature close to the chiral critical line at $\mu = 0$ found by ref. JHEP 07, 173 (2015) from $\langle \bar{\psi}_l \psi_l \rangle$:

$$k \sim -44.8 \leftrightarrow k_{(eB)^2} = -50.0(3.5),\ k_{(eB)^4} = -56(10)$$



Conclusions

To summarize:

- We have studied the RW end-point in the presence of background magnetic fields
- The RW temperature decreases as a function of the strenght of the magnetic field
- We have found indications that the RW transition becomes first order between 1 and 2.5 GeV^2
- The curvature of the critical line is close to the curvature of the chiral critical line (from the light quark condensate) at $\mu = 0$

Thank you for listening!