New developments in studies of the QCD phase diagram

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QCD EoS in strong magnetic fields and non-zero baryon density

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Partially based on *Phys. Rev. Lett.* ***132***, *201903 (2024)* and ongoing work

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QCD EoS in strong magnetic fields and non-zero baryon density

Equilibrium description of strong interacting matter

$$p, \epsilon, \sigma, \ldots \equiv f(T, \mu, eB, \ldots)$$

thermodynamic obs. control parameters

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Early universe	Magnetars	Heavy ion-collision
Energy, evolution → Friedmann eq.	$m(r)$ of NS relations → TOV eq.	QGP → Hadronization → Freeze-out

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Early universe

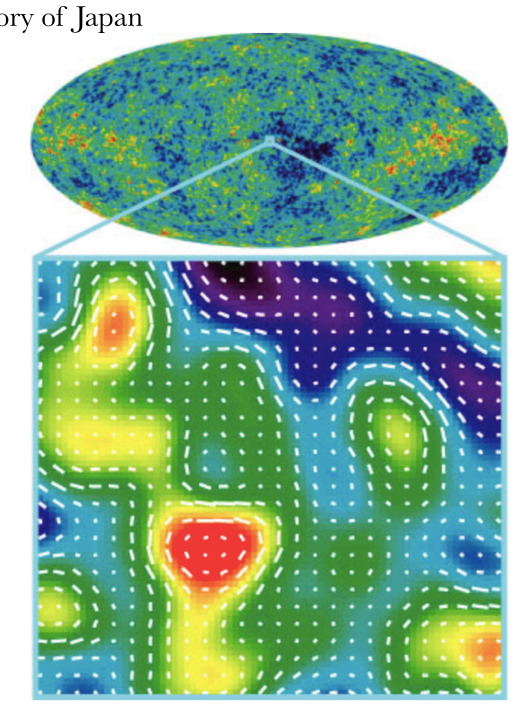
Energy, evolution → Friedmann eq.

Cosmological Magnetic Field: a fossil of density perturbations in the early universe

January 6, 2006 | Science

National Astronomical Observatory of Japan

Ichiki *et al.*, *Science*, ***311***, *827-829, 2006*

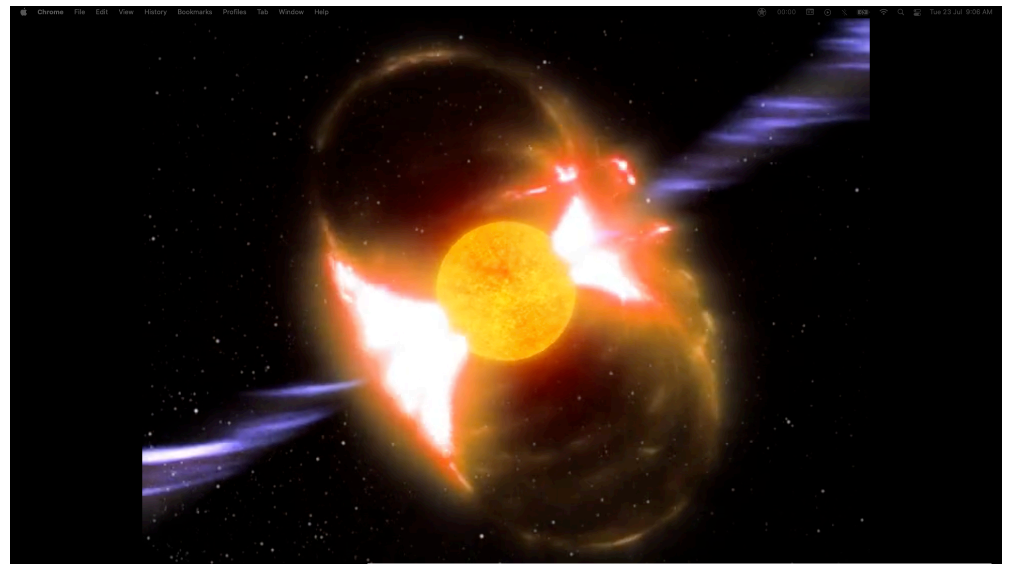


Vachaspati, *Phys. Lett. B* ***265*** *(1991)*

Enqvist, *Phys. Lett. B* ***319*** *(1993)*

Magnetars

$m(r)$ of NS relations → TOV eq.



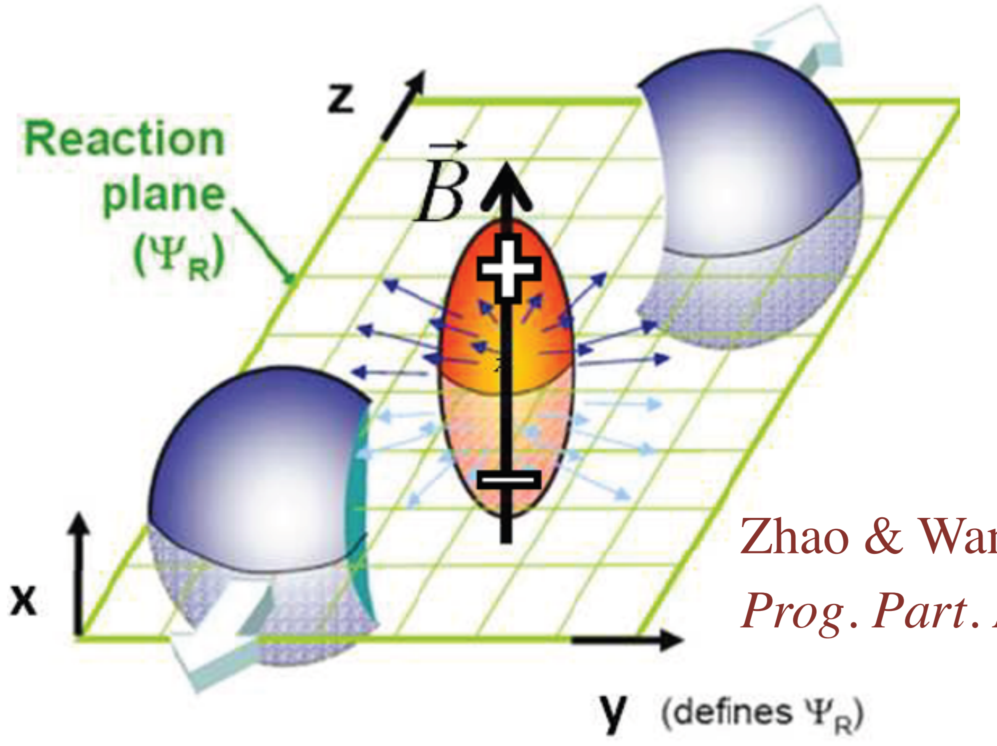
Schematic of XTE J1810-197

Duncan & Thompson, *Astrophys. J. Lett.* ***392*** *(1992) L9*

Anderson et al., *Phys. Rev. Lett.* ***100*** *(2008) 191101*

Heavy ion-collision

QGP → Hadronization → Freeze-out



Zhao & Wang, *Prog. Part. Nucl. Phys.* ***107*** *(2019)*

Kharzeev et al., *Nucl.Phys.A* ***803*** *(2008)*

Bali et al., *JHEP* ***07*** *(2020) 183*

Astrakhantsev et al., *PRD* ***102*** *(2020) 054516*

EoS and interplay with magnetic fields is ubiquitous!

QCD EoS in strong magnetic fields and non-zero baryon density

★ Interest in rich QCD phase structure at finite T and non-zero μ!

★ Pressure Taylor expanded as fluctuations of conserved charges $\mathscr{C} \in \{\mathrm{B}, \mathrm{Q}, \mathrm{S}\}$,

$$\hat{p}(T, eB, \hat{\mu}) \equiv \frac{p}{T^4} = \frac{1}{VT^3} \ln \mathscr{Z}_{\mathrm{GC}}(T, eB, V, \hat{\mu}_{\mathscr{C}})$$
$$= \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \boxed{\chi_{ijk}^{\mathrm{BQS}}} \hat{\mu}_{\mathrm{B}}{}^{i} \hat{\mu}_{\mathrm{Q}}{}^{j} \hat{\mu}_{\mathrm{S}}{}^{k}$$

(Lattice computable, theory meets experiment)

$$\chi_{ijk}^{\mathrm{BQS}} \equiv \chi_{ijk}^{\mathrm{BQS}}(T, eB) = \frac{\partial^{i+j+k}}{\partial \hat{\mu}_{\mathrm{B}}^{i} \partial \hat{\mu}_{\mathrm{Q}}^{j} \partial \hat{\mu}_{\mathrm{S}}^{k}} \hat{p}(T, eB, \hat{\mu}) \bigg|_{\hat{\mu}=0}$$

Sign-Problem

↓

Taylor Expand

$\mu_f \longleftrightarrow \mu_{\mathscr{C}}$ $\qquad \chi_{ijk}^{uds} \longleftrightarrow \chi_{ijk}^{\mathrm{BQS}}$

(2+1) QCD

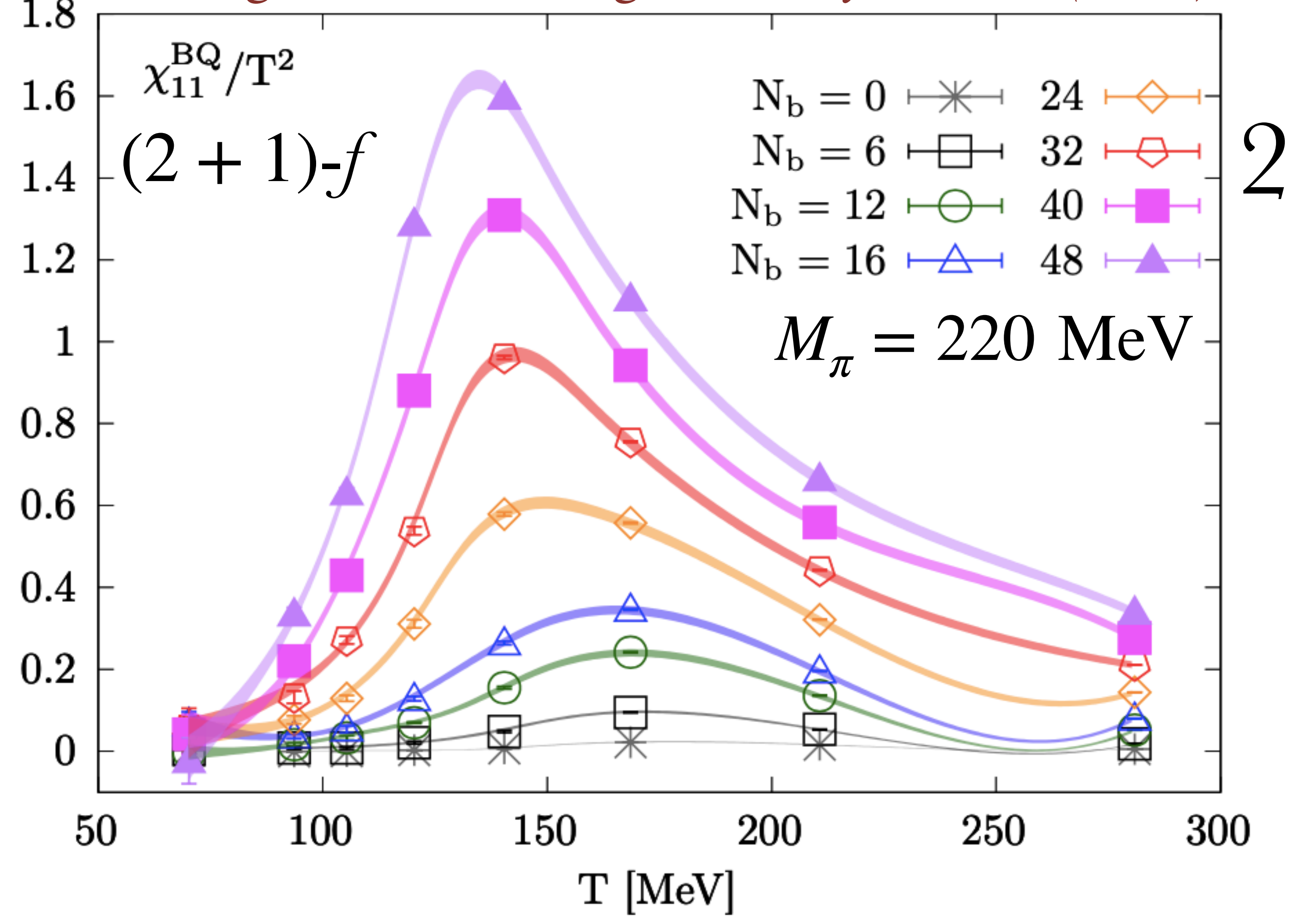
$$\mu_u = \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}}$$
$$\mu_d = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}}$$
$$\mu_s = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}}$$

Allton et al., *Phys. Rev. D* ***66*** *(2002) 074507*
HotQCD, *Phys. Rev. D* **95** *(2017) 054504*

Recent Lattice Works: Conserved Charges in Magnetic fields

χ_{ijk}^{BQS}

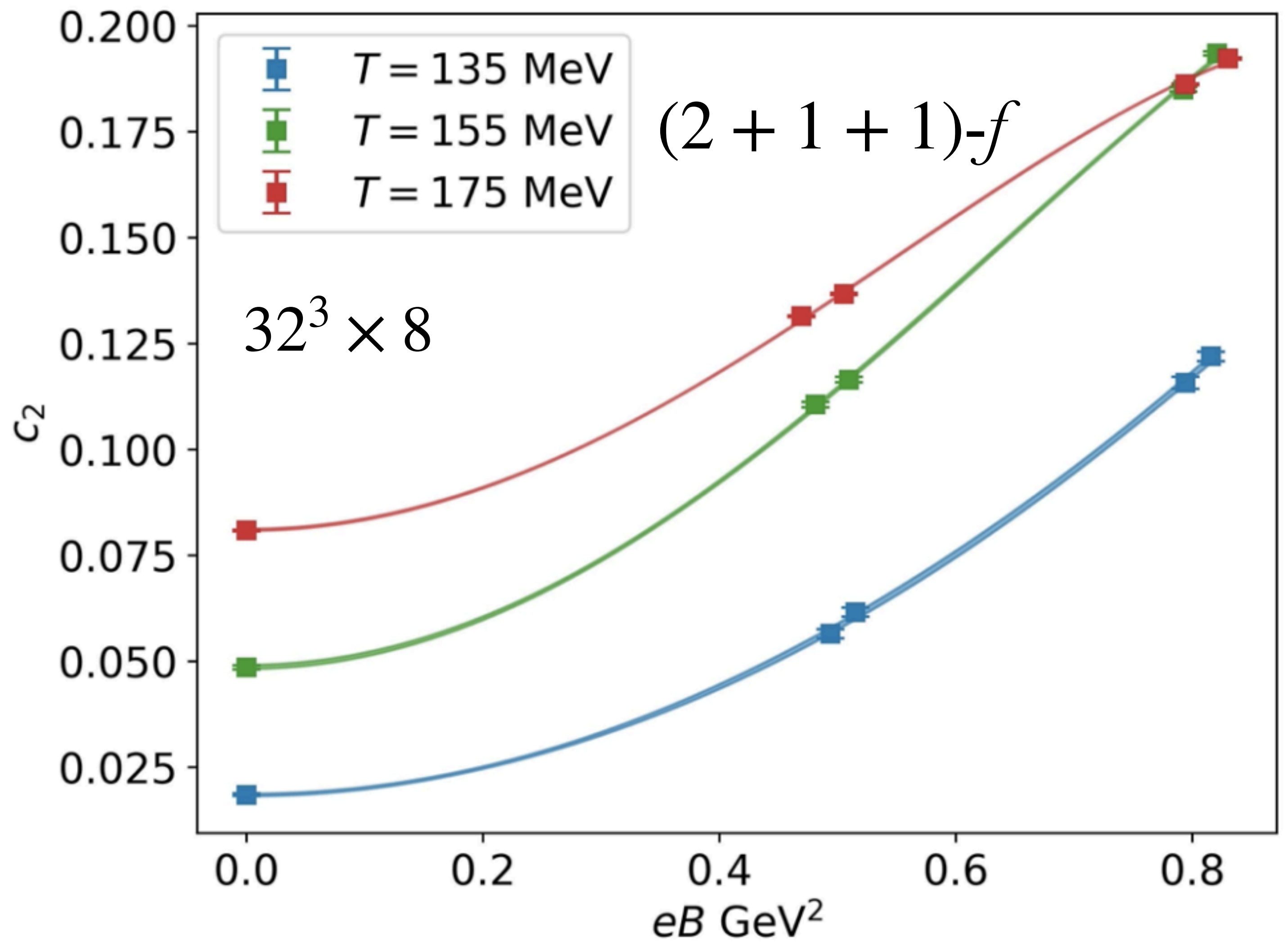
Ding, Li, Shi & Wang, *Eur. Phys. J.A* ***57*** *(2021) 6, 202*



2021

EoS: Pressure

Borsanyi et al., *PoS* ***LATTICE2023*** *(2024) 164*

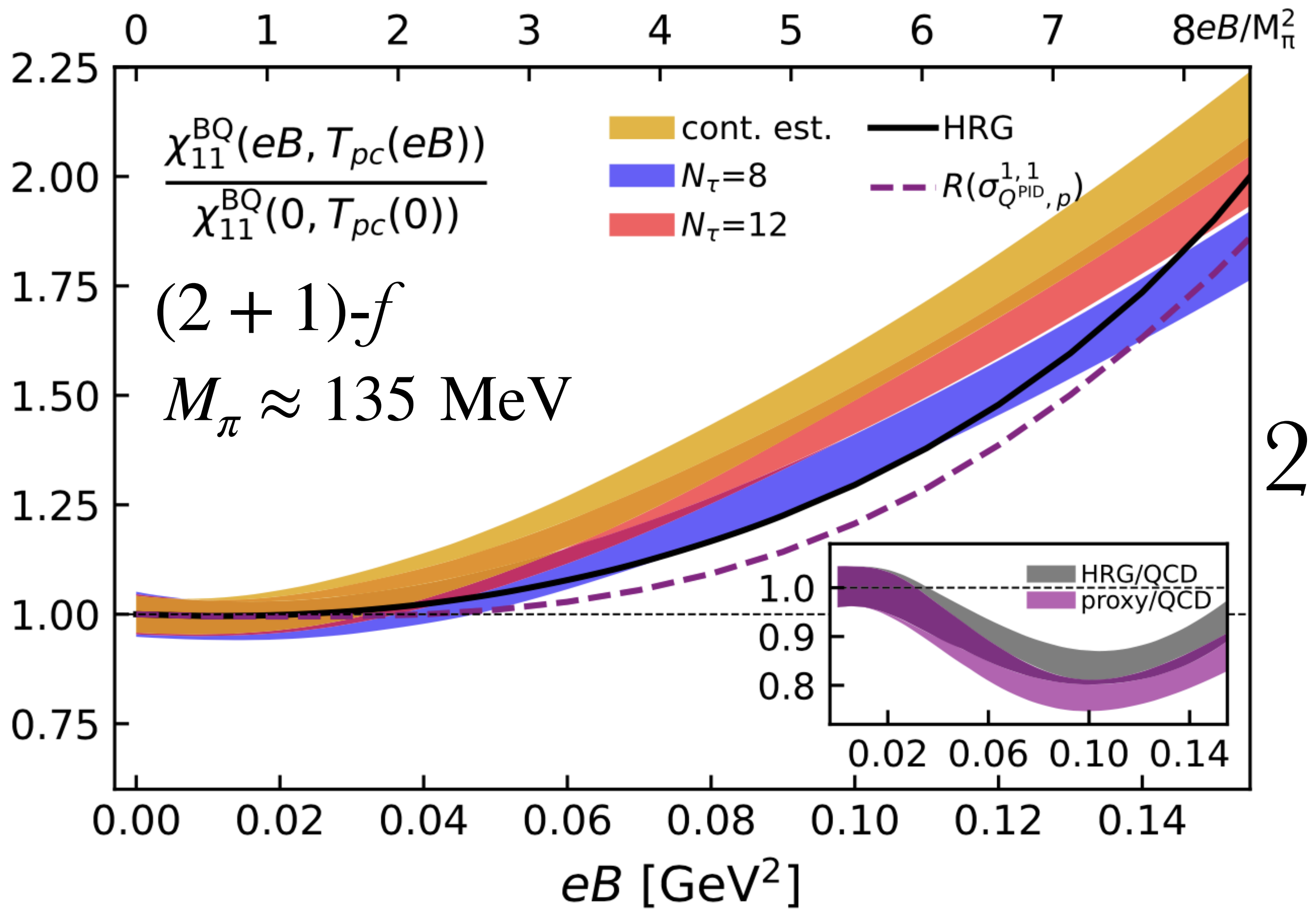


2023

★ Recent review article:

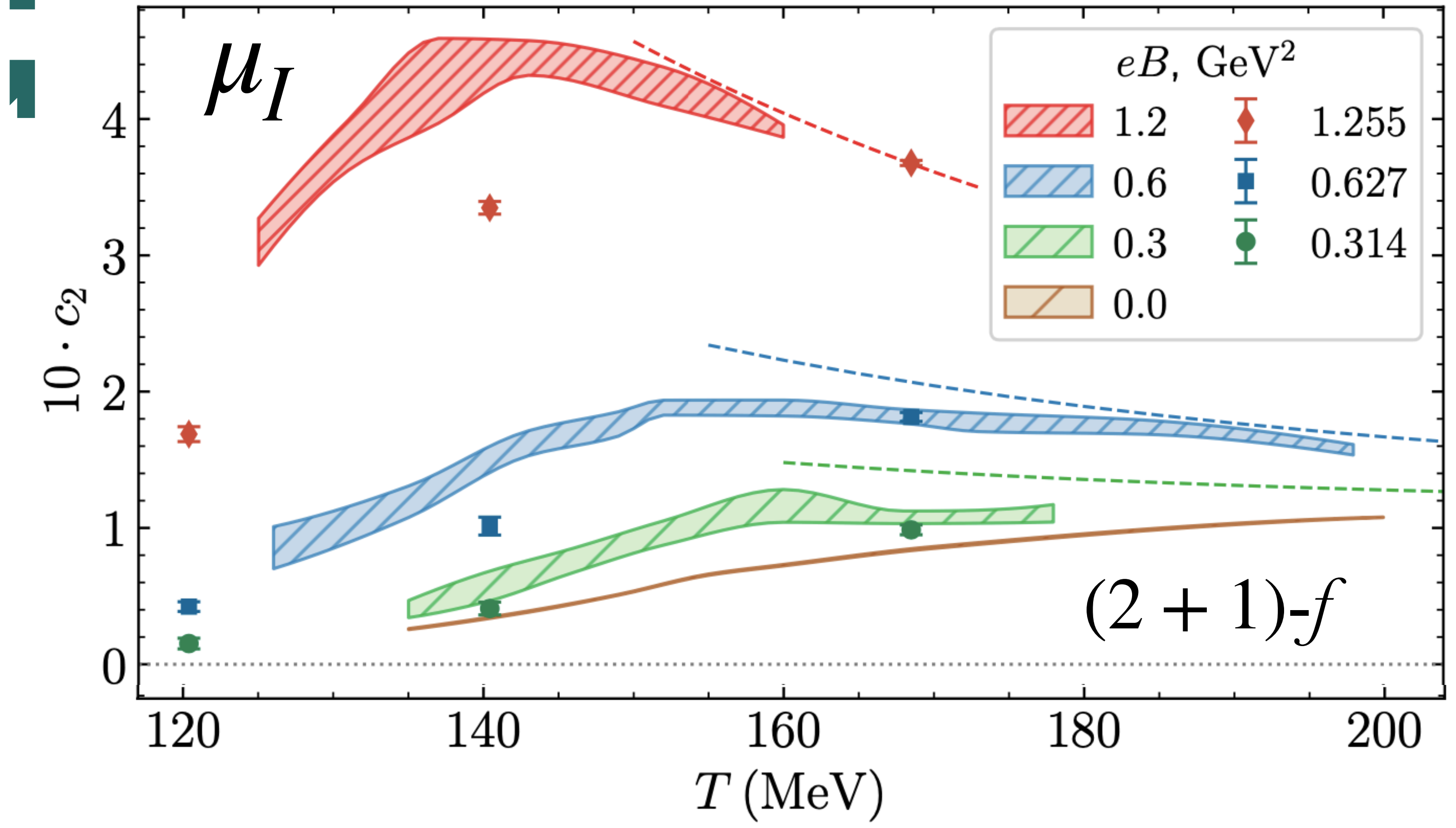
"QCD with background electromagnetic fields on the lattice: a review"

Endrodi, arXiv:2406.19780



2023

Ding, Gu, Kumar, Li & Liu, *Phys. Rev. Lett.* ***132****, 201903 (2024)*



2024

Astrakhantsev et al., *Phys.Rev.D* ***109*** *(2024) 9, 094511*

(2+1)-flavor QCD Lattice Ingredients

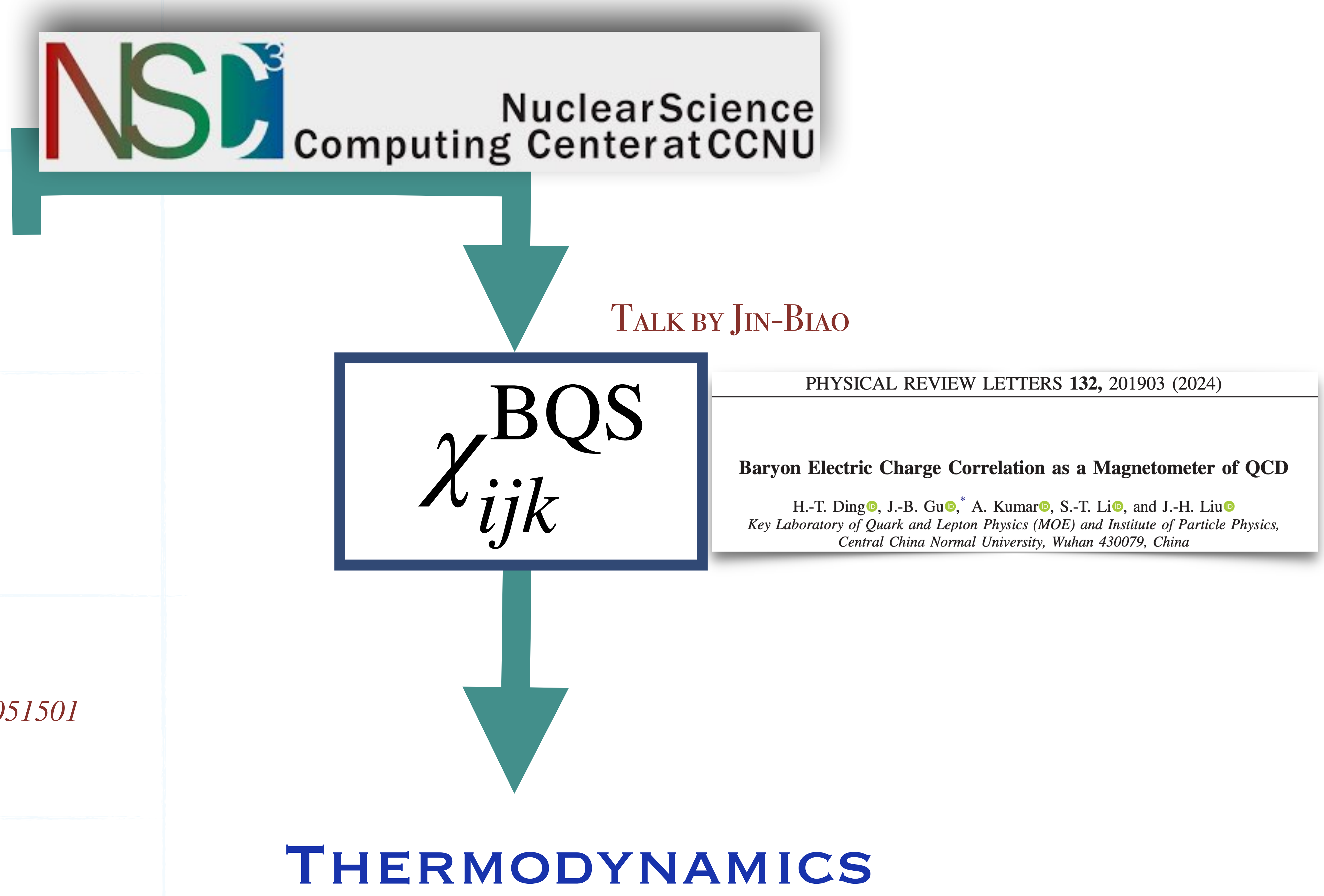
- HISQ & tree-level improved Symanzik gauge action
- Lattice: $N_\sigma / N_\tau = 4$ and $N_\tau = 8,\ 12 \rightarrow$ cont. est. (one additional $N_\tau = 16$)
- Physical pion mass: $m_s^{\text{phy}} / m_{u/d} = 27$, $M_\pi \approx 135$ MeV
- Non-zero μ and T: Taylor expansion, around T_{pc} $T \equiv [145 - 166)$ MeV
- Magnetic field: $\vec{B} = \vec{\nabla} \times \vec{A}$: no sign-problem! B_z : Landau gauge. Stokes theorem implies quantization: Elia et al., *Phys. Rev. D.* **82** *(2010) 051501*

$$eB = 6\pi N_b \, a^{-2} N_\sigma^{-2}$$

Fixed U(1) factor to links. PBC : constrains flux:

$$N_b = [1 - 32]$$

$$eB \equiv [M_\pi^2 - \ 45 M_\pi^2) \sim [0.02 - 0.8) \text{ GeV}^2$$



Thermodynamics: Observables of Interest $\mathcal{O}(T, eB, \hat{\mu})$

Pressure

$$\hat{p} = \sum_{ijk} \frac{1}{i!j!k!} \chi^{\mathrm{BQS}}_{ijk} \hat{\mu}^i_{\mathrm{B}} \hat{\mu}^j_{\mathrm{Q}} \hat{\mu}^k_{\mathrm{S}}$$

$$\hat{p}_{\mathrm{LO}} \equiv \frac{1}{2} \hat{\mu}^{\mathrm{T}} \chi^{\mathrm{BQS}}_{\mathrm{LO}} \hat{\mu}$$

$$\hat{\mu} \equiv \begin{pmatrix} \hat{\mu}_{\mathrm{B}} \\ \hat{\mu}_{\mathrm{Q}} \\ \hat{\mu}_{\mathrm{S}} \end{pmatrix} \qquad \chi^{\mathrm{BQS}}_{\mathrm{LO}} \equiv \begin{pmatrix} \chi^{\mathrm{B}}_2 & \chi^{\mathrm{BQ}}_{11} & \chi^{\mathrm{BS}}_{11} \\ \chi^{\mathrm{BQ}}_{11} & \chi^{\mathrm{Q}}_2 & \chi^{\mathrm{QS}}_{11} \\ \chi^{\mathrm{BS}}_{11} & \chi^{\mathrm{QS}}_{11} & \chi^{\mathrm{S}}_2 \end{pmatrix}$$

$\mathrm{LO} : i + j + k = 2$

$$\Xi^{\mathrm{BQS}}_{\mathrm{LO}} \equiv T \frac{\partial \chi^{\mathrm{BQS}}_{\mathrm{LO}}}{\partial T}$$

Number Density

$$\hat{n}^{\mathscr{C}} \equiv \frac{n_{\mathscr{C}}}{T^3} = \frac{\partial \hat{p}}{\partial \hat{\mu}_{\mathscr{C}}} = \sum_{ijk} \frac{\chi^{\mathrm{BQS}}_{ijk}}{i!j!k!} \frac{\partial}{\partial \mu_{\mathscr{C}}} \left(\hat{\mu}^i_{\mathrm{B}} \hat{\mu}^j_{\mathrm{Q}} \hat{\mu}^k_{\mathrm{S}} \right)$$

$$\hat{n}^{\mathscr{C}}_{\mathrm{LO}} \equiv \chi^{\mathrm{BQS}}_{\mathrm{LO}} \hat{\mu}$$

Energy and Entropy Density

Trace Anomaly $$\hat{\Delta} \equiv \frac{\epsilon - 3P}{T^4} = T \frac{\partial \hat{p}}{\partial T} = \sum_{ijk} \frac{\Xi^{\mathrm{BQS}}_{ijk}}{i!j!k!} \hat{\mu}^i_{\mathrm{B}} \hat{\mu}^j_{\mathrm{Q}} \hat{\mu}^k_{\mathrm{S}}$$

$$\hat{\epsilon} \equiv \hat{\Delta} + 3\hat{p} = \sum_{ijk} \frac{\Xi^{\mathrm{BQS}}_{ijk} + 3\chi^{\mathrm{BQS}}_{ijk}}{i!j!k!} \hat{\mu}^i_{\mathrm{B}} \hat{\mu}^j_{\mathrm{Q}} \hat{\mu}^k_{\mathrm{S}}$$

$$\hat{\sigma} \equiv \hat{\epsilon} + \hat{p} - \sum_{\mathscr{C}} \hat{\mu}_{\mathscr{C}} \hat{n}^{\mathscr{C}}$$

$$= \sum_{ijk} \frac{\Xi^{\mathrm{BQS}}_{ijk} + \left[4 - (i+j+k)\right] \chi^{\mathrm{BQS}}_{ijk}}{i!j!k!} \hat{\mu}^i_{\mathrm{B}} \hat{\mu}^j_{\mathrm{Q}} \hat{\mu}^k_{\mathrm{S}}$$

$$\hat{\epsilon}_{\mathrm{LO}} \equiv \frac{1}{2} \hat{\mu}^{\mathrm{T}} \left(\Xi^{\mathrm{BQS}}_{\mathrm{LO}} + 3\chi^{\mathrm{BQS}}_{\mathrm{LO}} \right) \hat{\mu}$$

$$\hat{\sigma}_{\mathrm{LO}} \equiv \frac{1}{2} \hat{\mu}^{\mathrm{T}} \left(\Xi^{\mathrm{BQS}}_{\mathrm{LO}} + 2\chi^{\mathrm{BQS}}_{\mathrm{LO}} \right) \hat{\mu}$$

Initial Nuclei Conditions

$$\hat{p}(T, eB, \hat{\mu}) = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \boxed{\chi_{ijk}^{\mathrm{BQS}}} \hat{\mu}_{\mathrm{B}}{}^{i}\, \hat{\mu}_{\mathrm{Q}}{}^{j}\, \hat{\mu}_{\mathrm{S}}{}^{k} \rightarrow \hat{\mu}_{\mathrm{B}}{}^{i+j+k}$$

Strangeness neutrality : $n^{\mathrm{S}} = 0$ + Isospin symmetry : $n^{\mathrm{Q}}/n^{\mathrm{B}} = r$

$\hat{\mu}_{\mathrm{Q/S}} \equiv \hat{\mu}_{\mathrm{Q/S}}(T, eB, \hat{\mu}_{\mathrm{B}})$

$$\xrightarrow[q_{2k-1},\, s_{2k-1}]{}$$

$$\mu_{\mathrm{Q}}/\mu_{\mathrm{B}} = q_1 + q_3\hat{\mu}_{\mathrm{B}}^2 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^4) + \dots$$

$$\mu_{\mathrm{S}}/\mu_{\mathrm{B}} = s_1 + s_3\hat{\mu}_{\mathrm{B}}^2 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^4) + \dots$$

$$q_1 = \frac{r\left(\chi_2^{\mathrm{B}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{BS}}\right) - \left(\chi_{11}^{\mathrm{BQ}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{QS}}\right)}{\left(\chi_2^{\mathrm{Q}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{QS}}\chi_{11}^{\mathrm{QS}}\right) - r\left(\chi_{11}^{\mathrm{BQ}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{QS}}\right)} \qquad s_1 = -\frac{\left(\chi_{11}^{\mathrm{BS}} + q_1\chi_{11}^{\mathrm{QS}}\right)}{\chi_2^{\mathrm{S}}}$$

★ $P_2 \equiv f(\chi_{ijk}^{\mathrm{BQS}}, q_1, s_1)$

HotQCD, *Phys. Rev. Lett.* ***109*** *(2012) 192302*

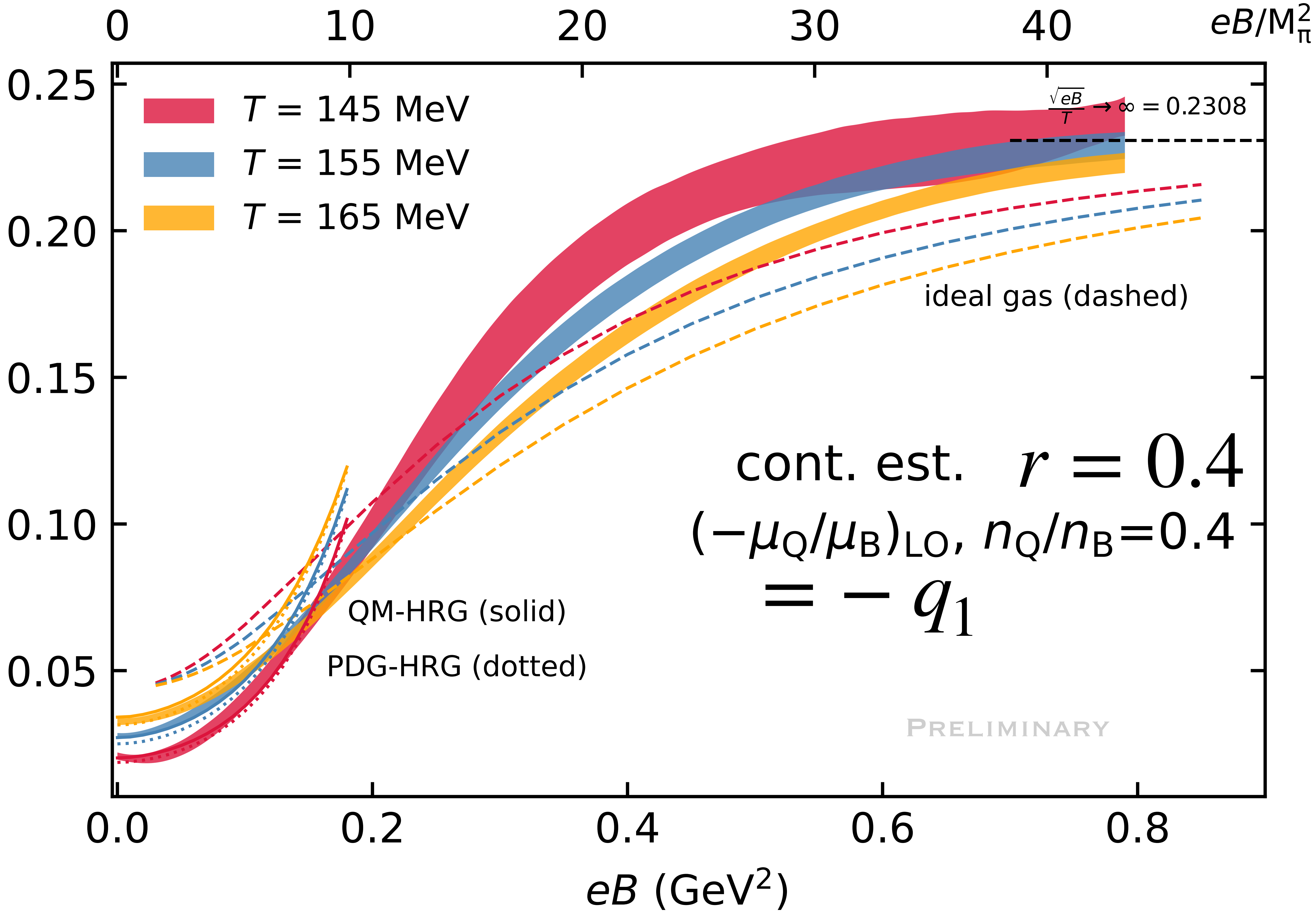
Fukushima & Hidaka, *Phys. Rev. Lett.* ***117****, 102301*

r

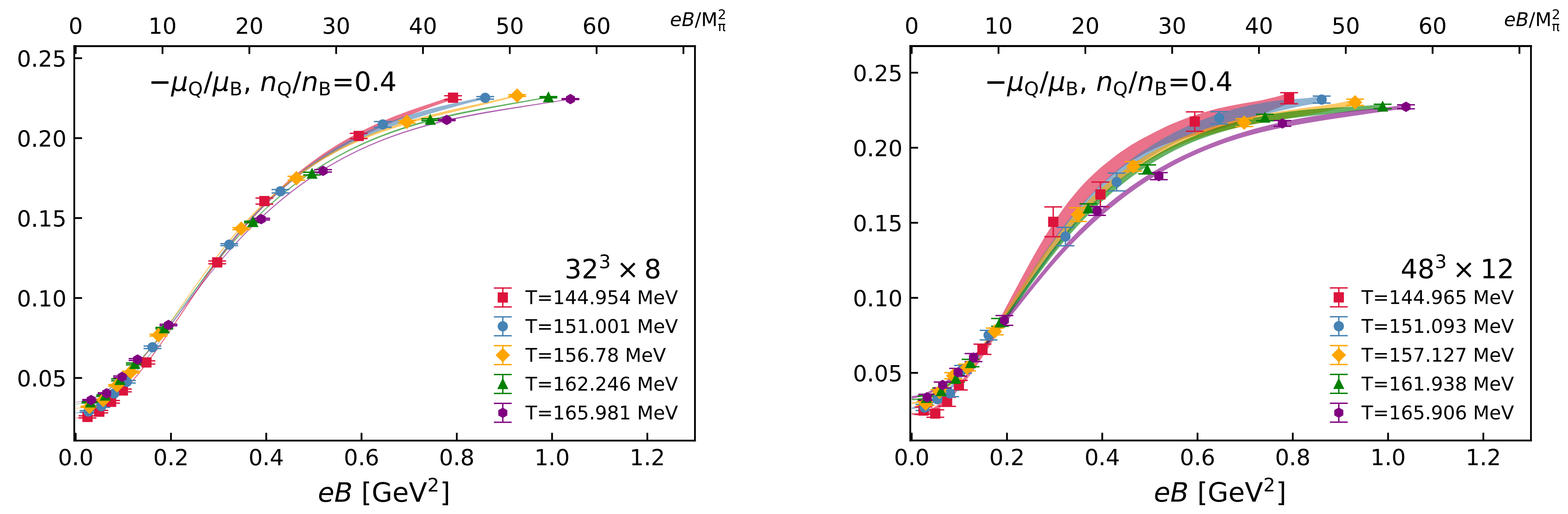
- 1/2 Isospin symmetry, $\mu_{\mathrm{Q}} = 0$ at $eB = 0$
- ✓ 0.4 Pb-Pb HIC ~ Isospin asymmetry
- 0.0 Electric charge neutral, $n_{\mathrm{Q}} = 0$

μ_Q/μ_B in presence of eB

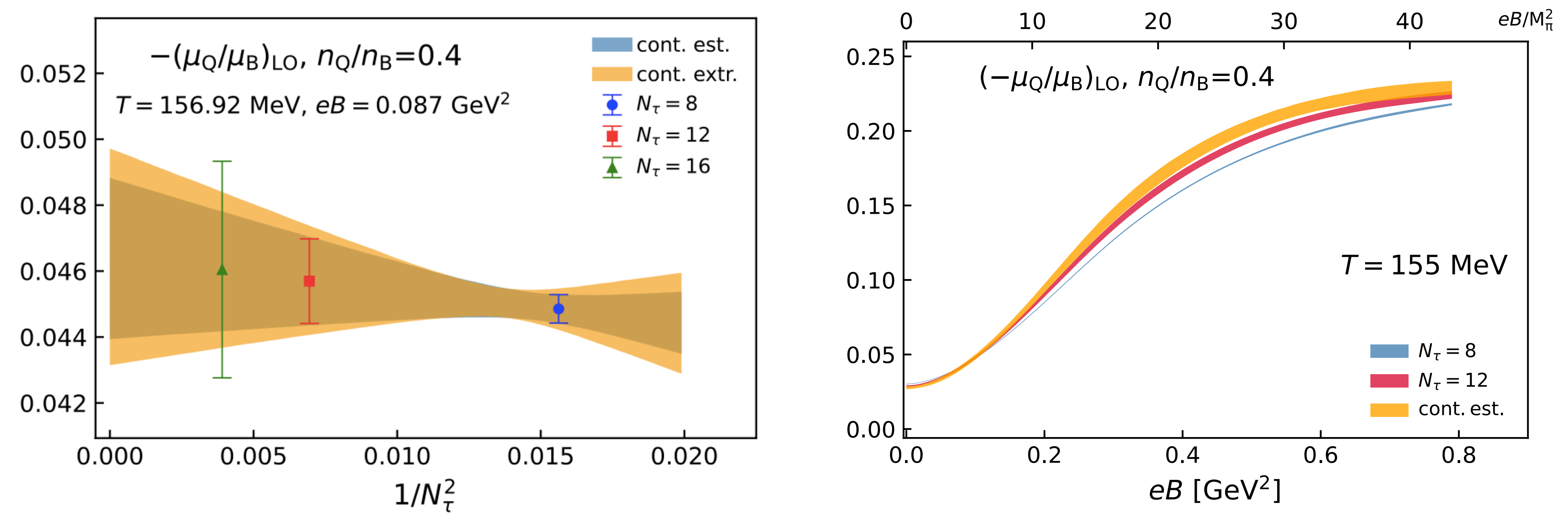
$$q_1 = \frac{r\left(\chi_2^{B}\chi_2^{S}-\chi_{11}^{BS}\chi_{11}^{BS}\right)-\left(\chi_{11}^{BQ}\chi_2^{S}-\chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^{Q}\chi_2^{S}-\chi_{11}^{QS}\chi_{11}^{QS}\right)-r\left(\chi_{11}^{BQ}\chi_2^{S}-\chi_{11}^{BS}\chi_{11}^{QS}\right)}$$



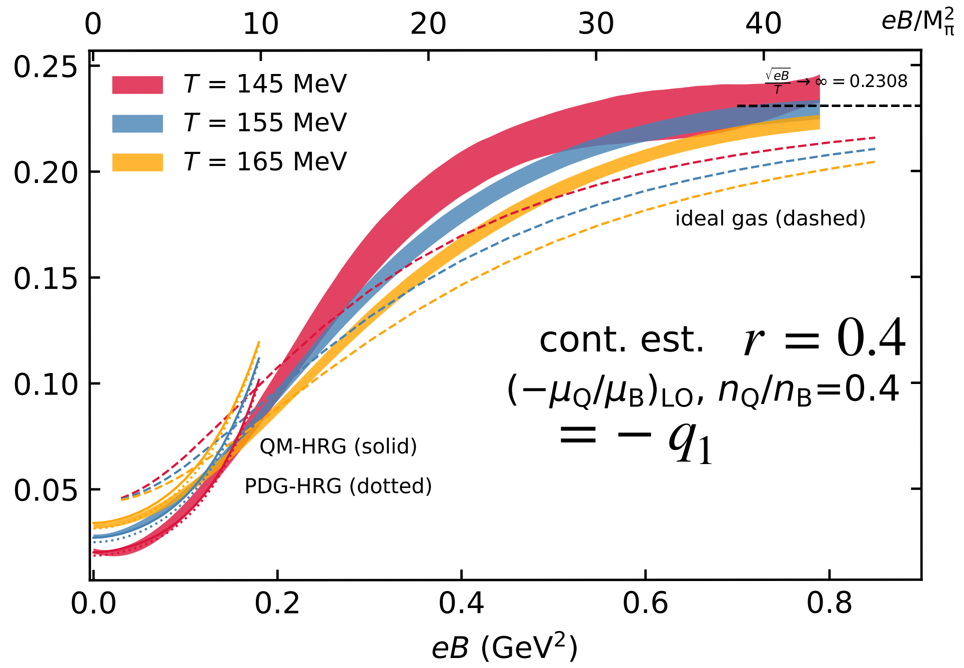
A. Lattice data + spline interpolation



B. Continuum estimates



μ_Q/μ_B IN PRESENCE OF eB



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$T \in [145 - 166)$ MeV

$eB \in [0.0 - 0.8)$ GeV^2

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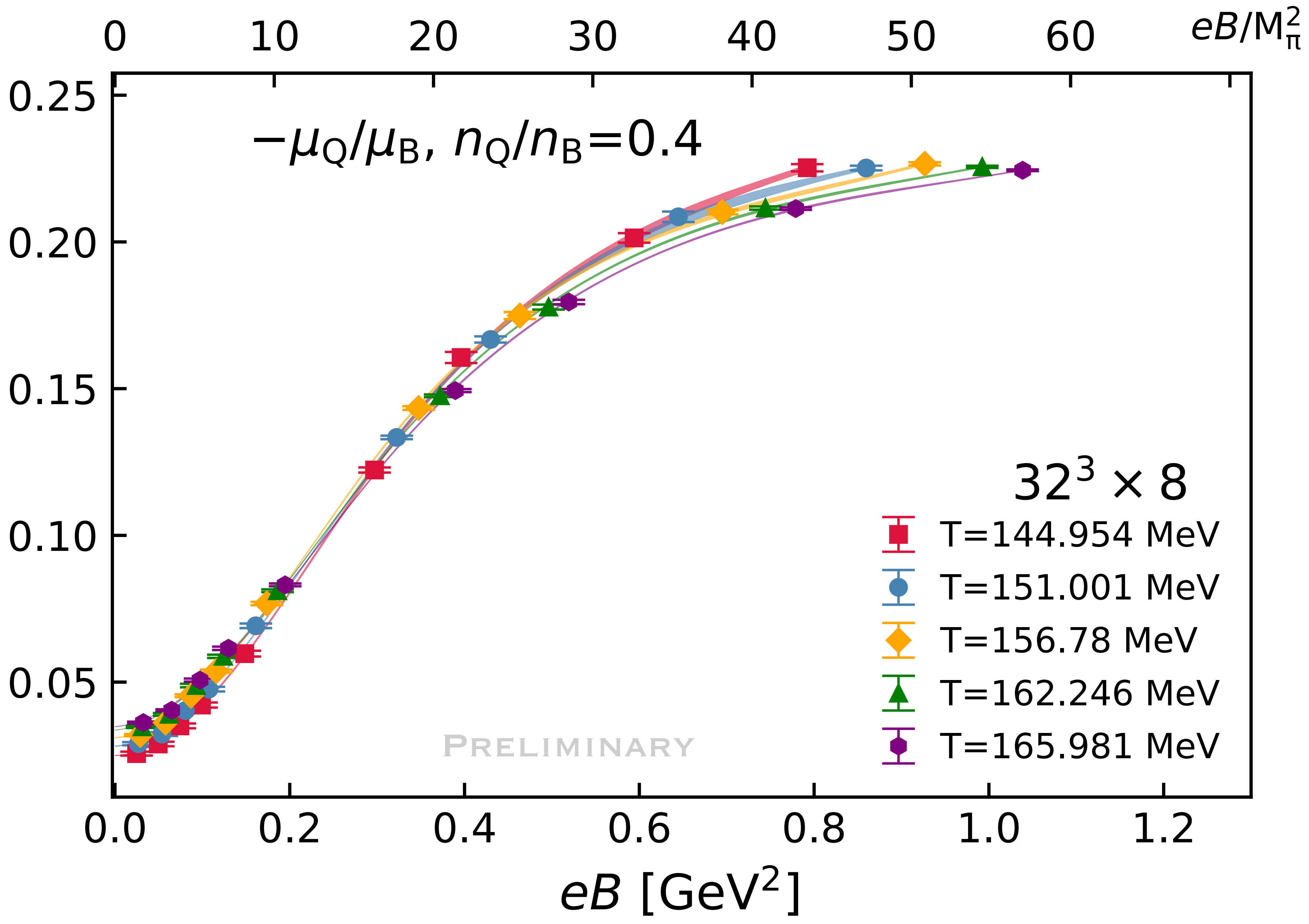
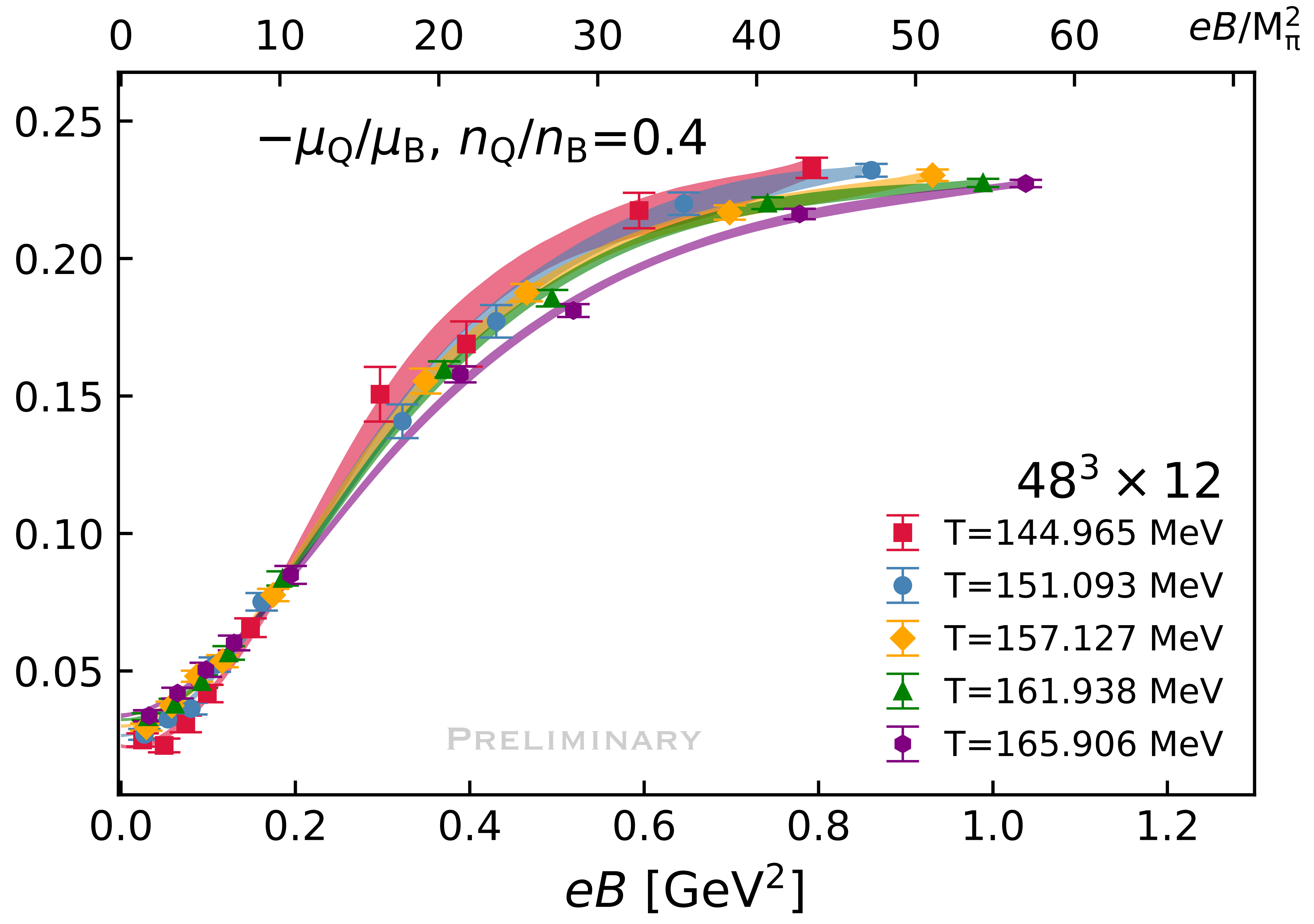
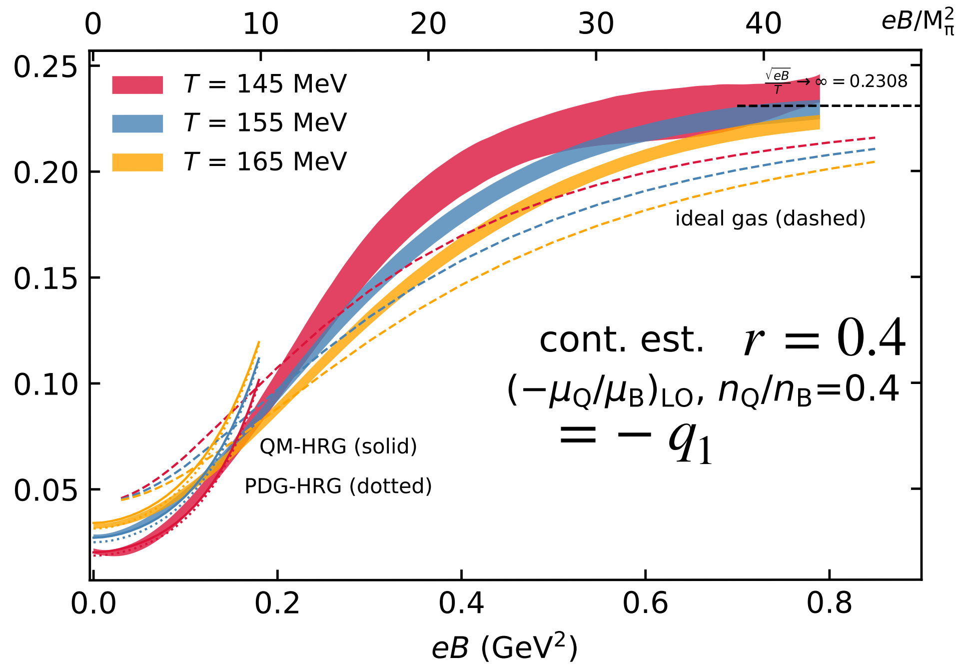




μ_Q/μ_B in presence of eB



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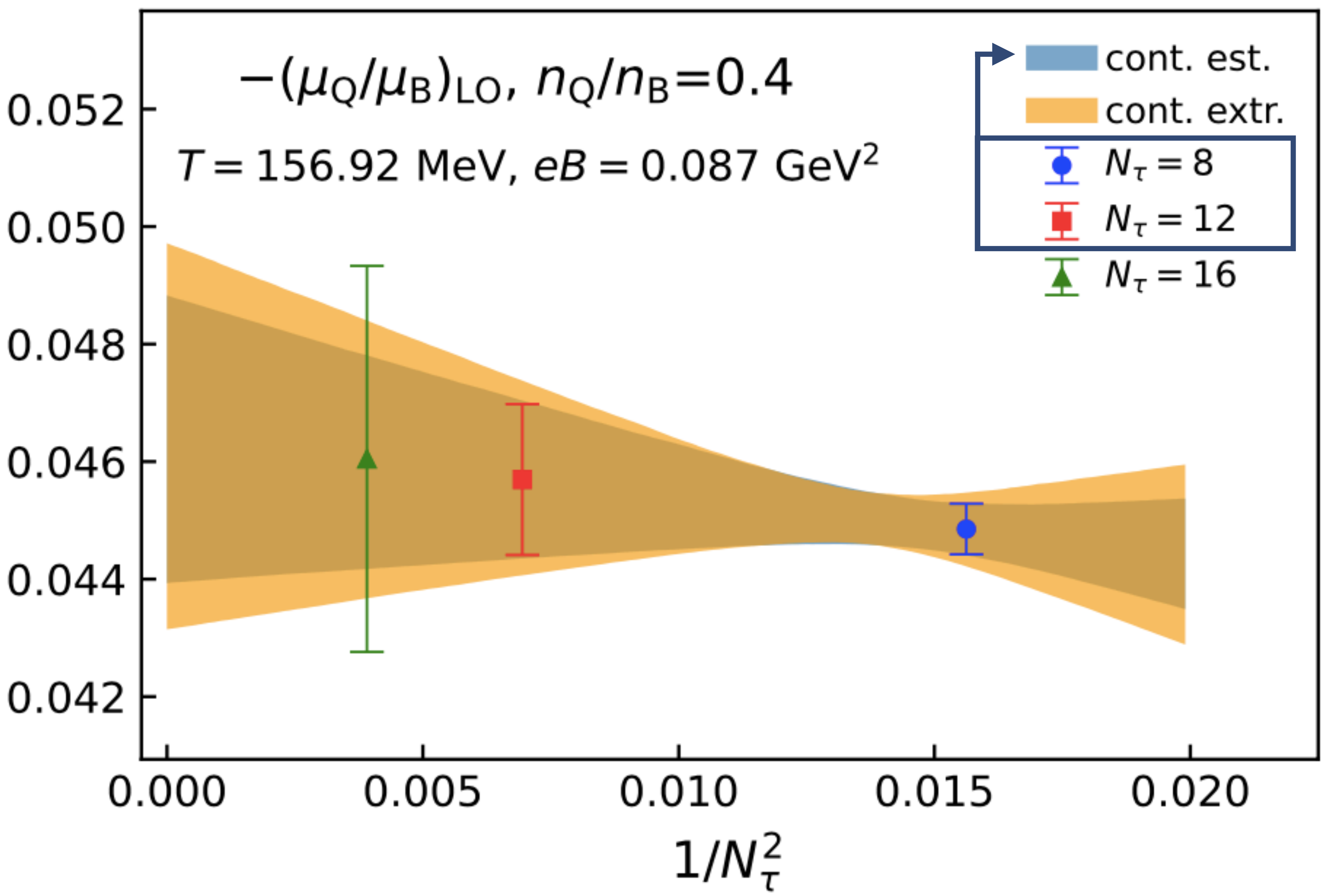

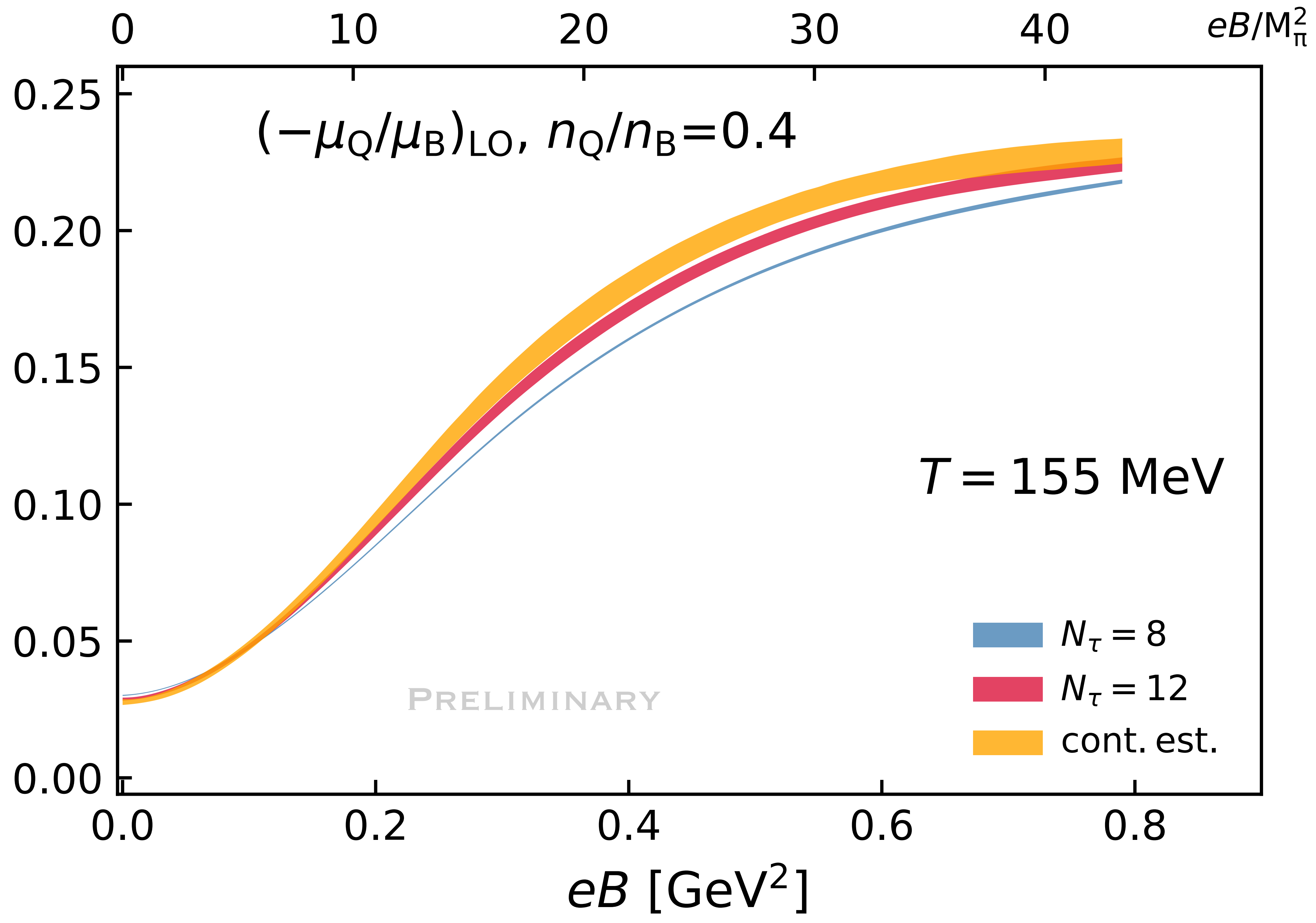

μ_Q/μ_B in presence of eB

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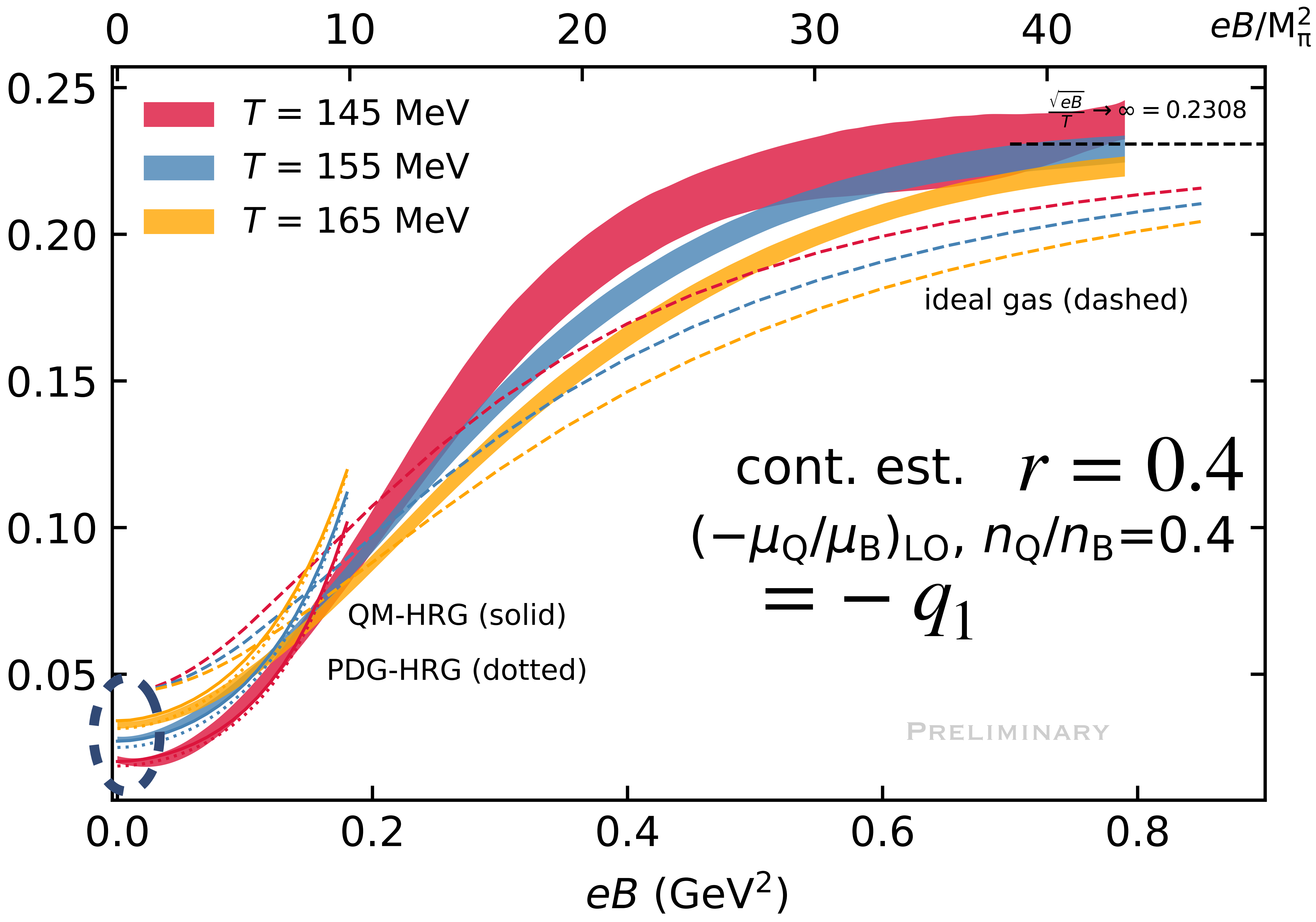
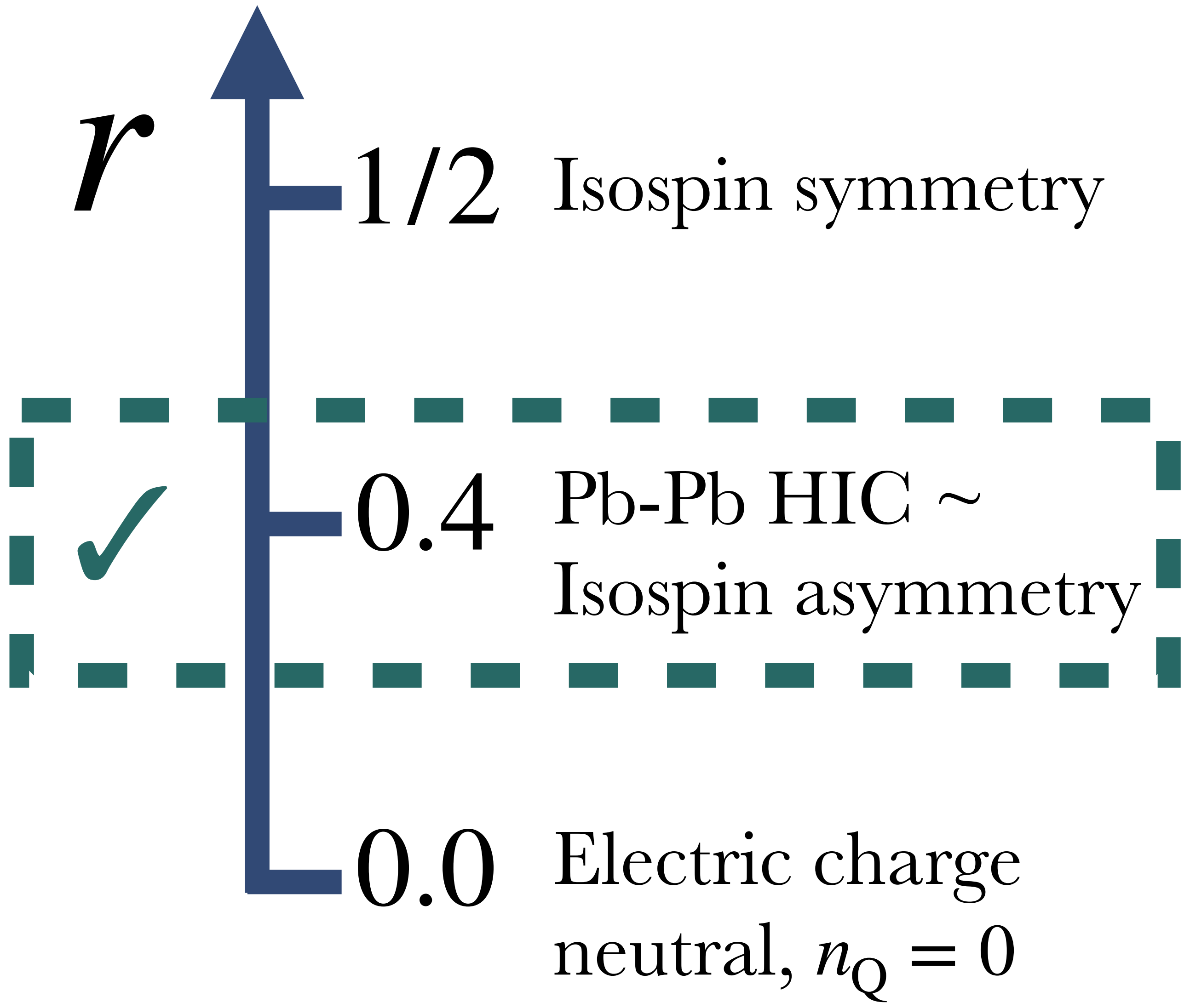


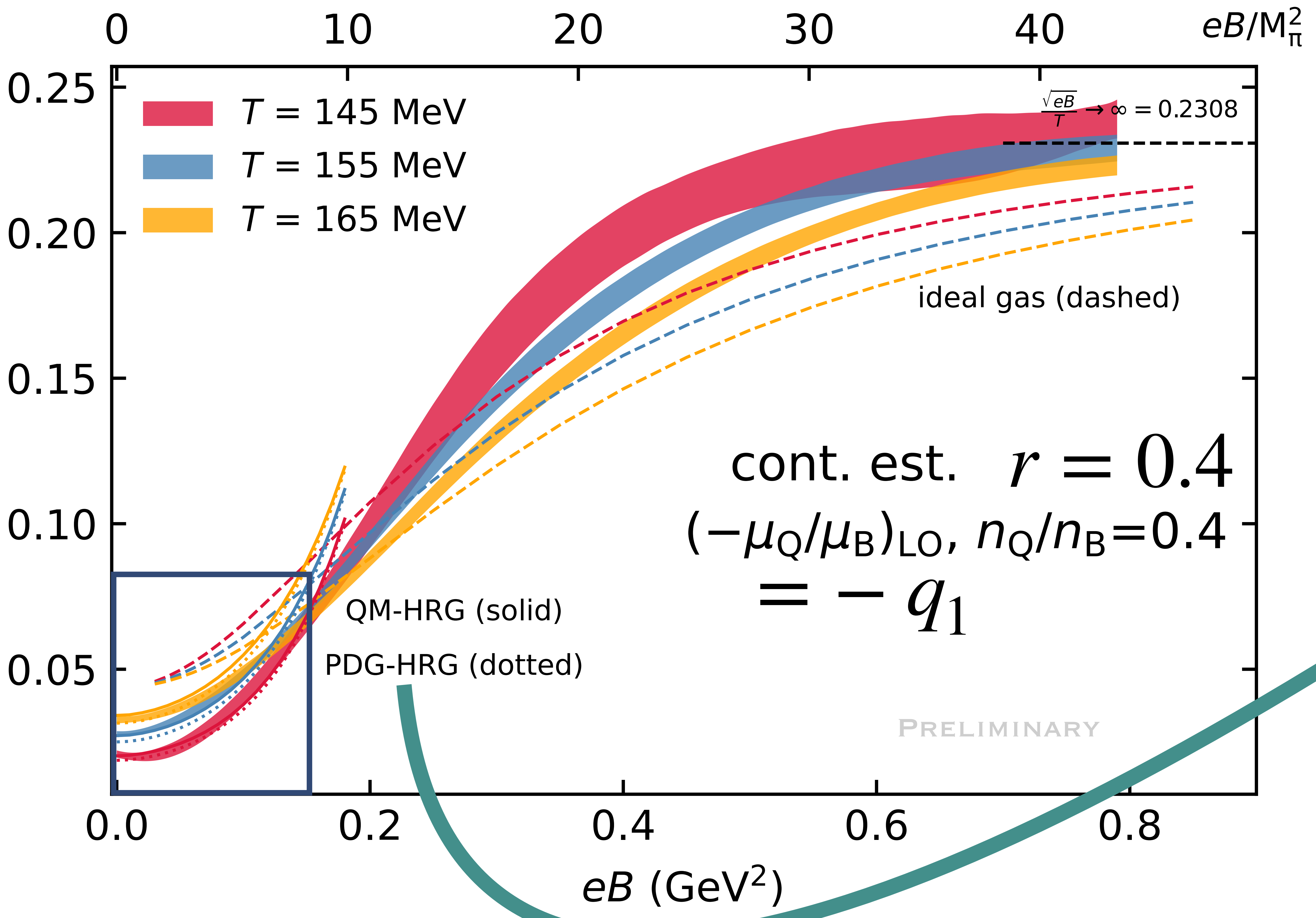
A. q_1 is negative!
Grows/more -ve with eB!



		$eB = 0$
1/2 Isospin symmetry	$\#p = \#n$	$\mu_Q = 0$
0.4 Pb-Pb HIC ~ Isospin asymmetry	$\#p < \#n$	$\mu_Q < 0$
0.0 Electric charge neutral, $n_Q = 0$		

μ_Q/μ_B IN THE PRESENCE OF eB

$$\boxed{q_1 = \frac{r\left(\chi_2^{B}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^{Q}\chi_2^{S} - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{QS}\right)}}$$



A. q_1 is negative!
Grows/more -ve with eB!

B. Good agreement with PDG-HRG and QM-HRG for smaller eB and low T

$$\frac{p^c_{\rm HRG}}{T^4} = \frac{|q_i|B}{2\pi^2T^3}\sum_{s_z=-s_i}^{s_i}\sum_{l=0}^{\infty}\epsilon_0$$

$$\times\sum_{k=1}^{\infty}(\pm 1)^{k+1}\frac{e^{k\mu_i/T}}{k}K_1\left(\frac{k\epsilon_0}{T}\right)$$

where $\epsilon_0 = \sqrt{m_i^2 + 2|q_i|B(l + 1/2 - s_z)}$

Fukushima & Hidaka, *Phys. Rev. Lett.* ***117***, *102301*

Ding, Li, Shi & Wang, *Eur. Phys. J.A* ***57*** *(2021) 6, 202*

μ_Q/μ_B IN THE PRESENCE OF eB

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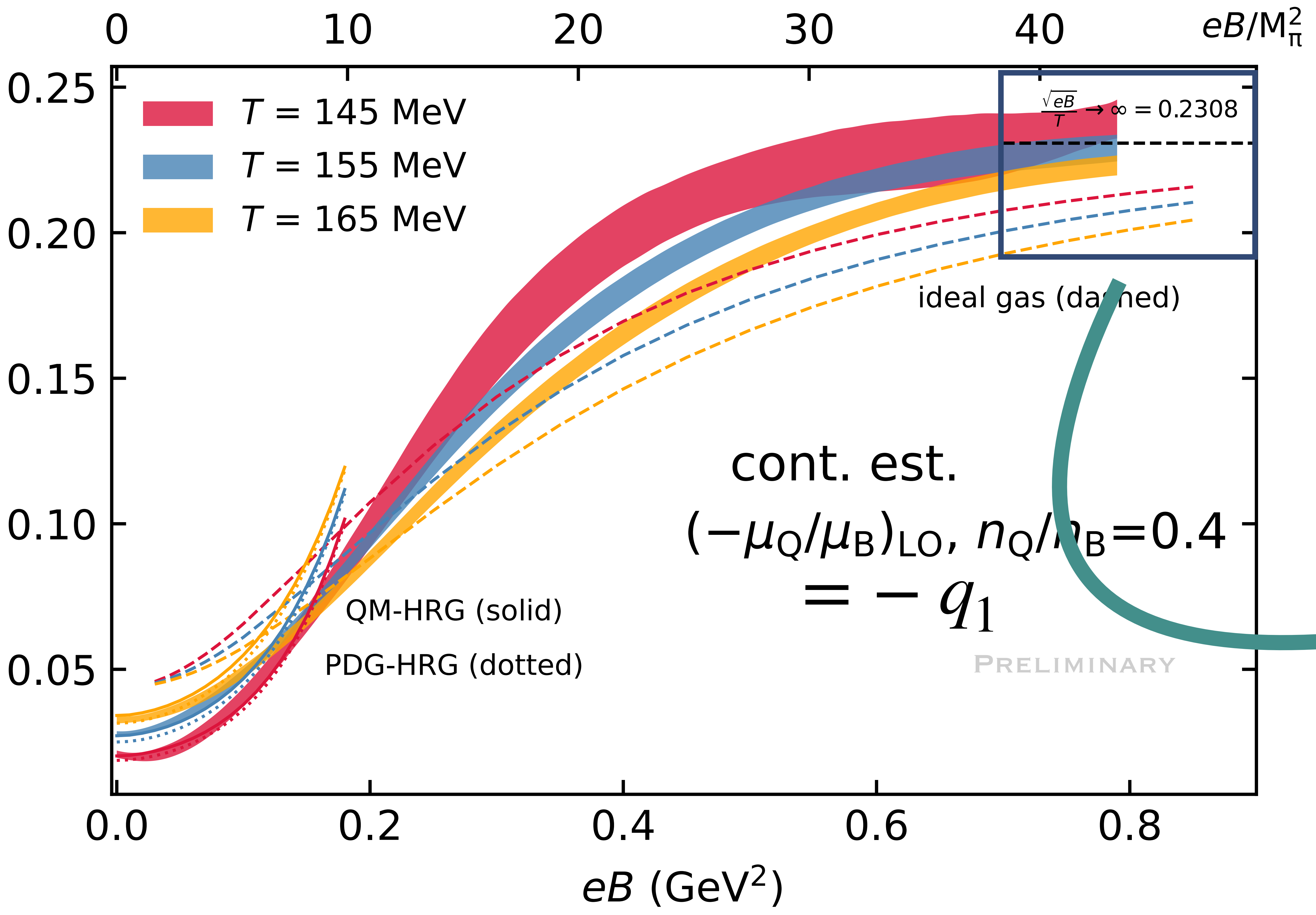
$$



A. q_1 is negative!
Grows/more -ve with eB!

B. Good agreement with PDG-HRG and QM-HRG for smaller eB and low T

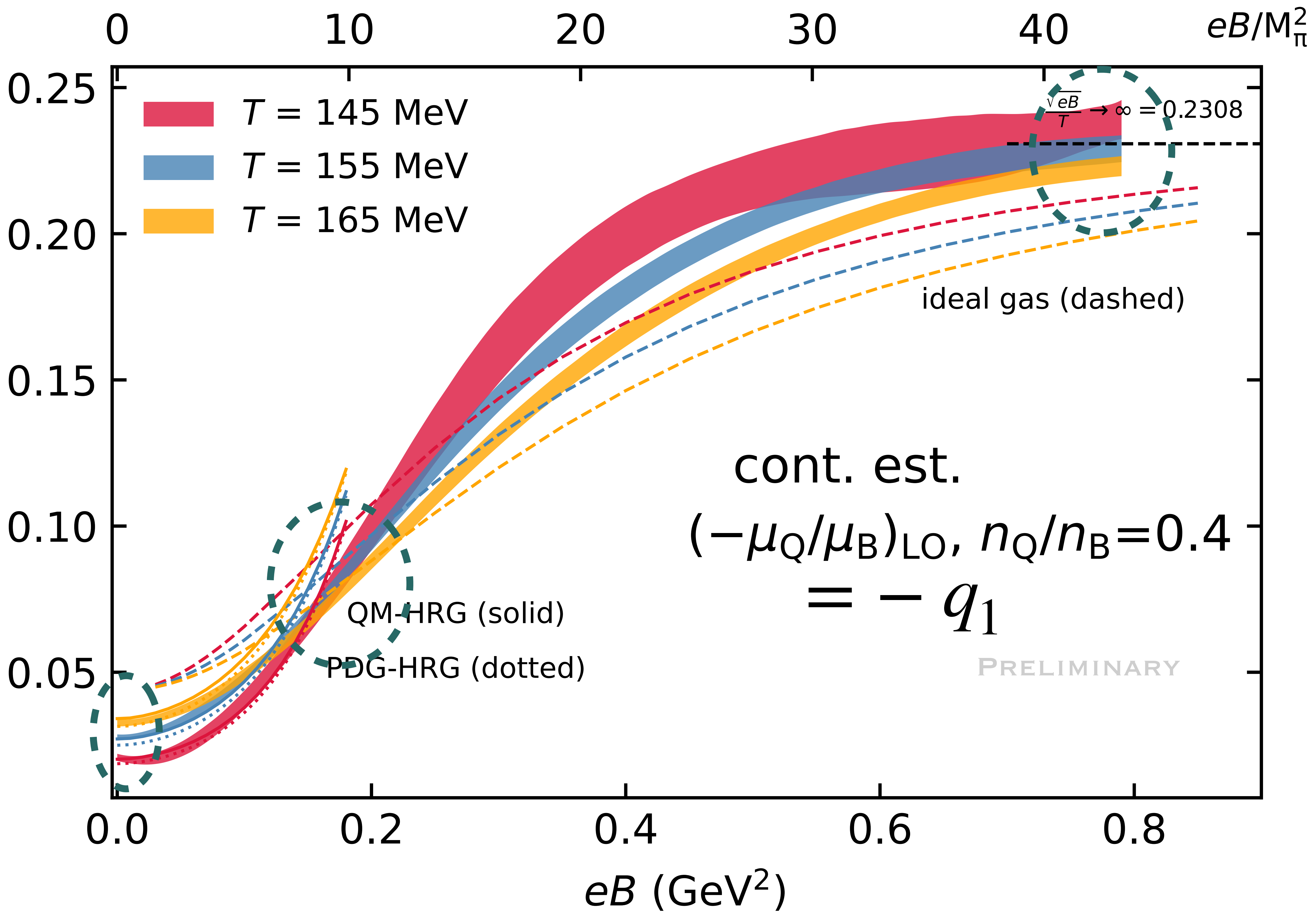
C. At very strong eB saturation to free limit

$$
\frac{p_{\rm ideal}}{T^4} = \frac{8\pi^2}{45} + \sum_f \frac{3\,|q_f|\,B}{\pi^2 T^2}\left[\frac{\pi^2}{12} + \frac{\hat{\mu}_f^2}{4} + p_f^{ll}(B)\right]
$$

Ding, Li, Shi & Wang, *Eur. Phys. J.A* **57** *(2021) 6, 202*

μ_Q/μ_B in the presence of eB

$$q_1 = \frac{r\left(\chi_2^{B}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^{Q}\chi_2^{S} - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^{S} - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$



A. q_1 is negative!
   Grows/more -ve with eB!

B. Good agreement with PDG-HRG and QM-HRG for smaller eB and low T

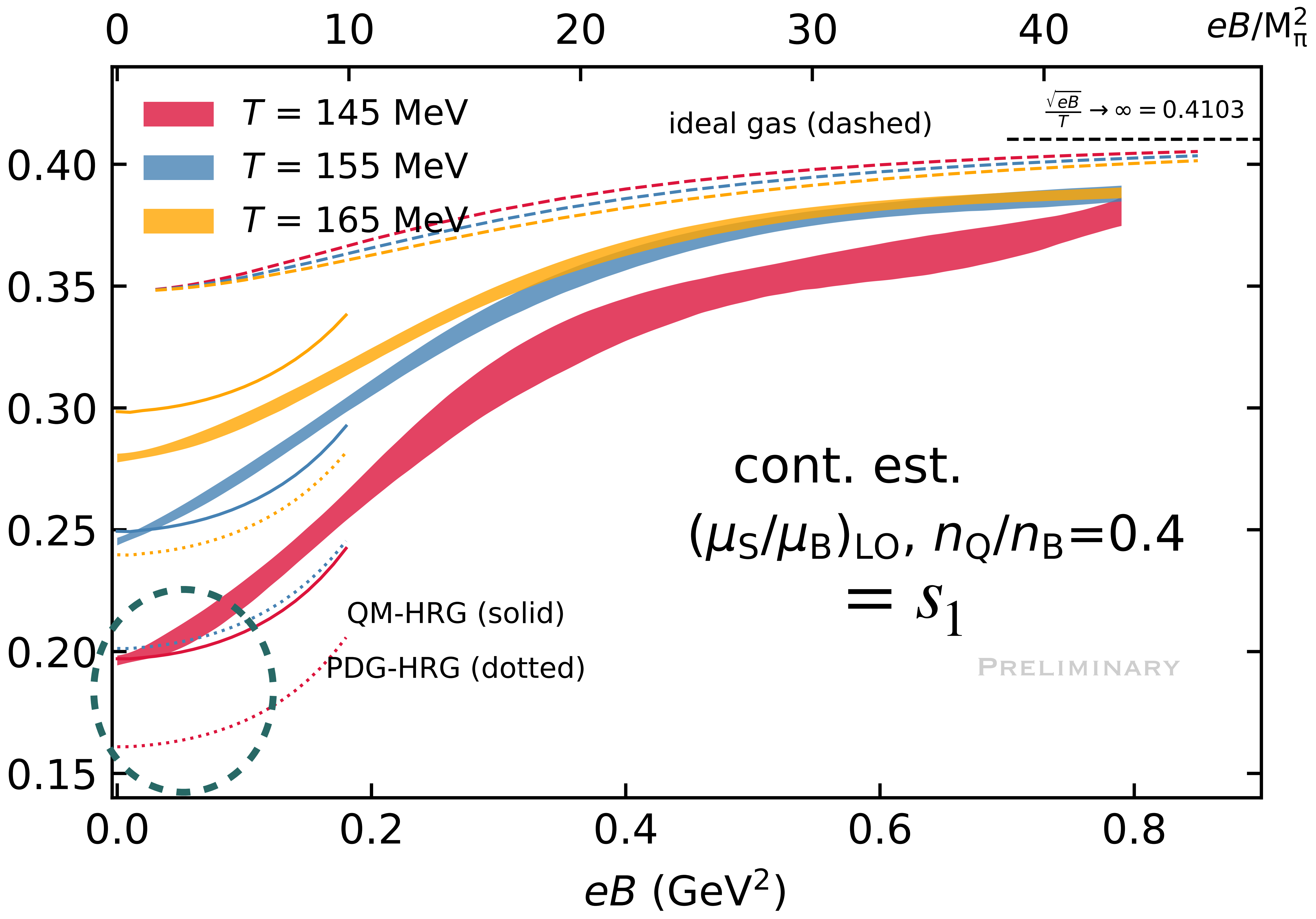
C. At very strong eB saturation to free limit

D. Crossing in T & sign of slope changes at strong enough eB

   near HRG: low $T \to$ small q_1
   near ideal: low $T \to$ large q_1

μ_S/μ_B IN PRESENCE OF eB

$$\boxed{s_1 = -\frac{\left(\chi_{11}^{BS} + q_1\chi_{11}^{QS}\right)}{\chi_2^{S}}}$$



★ Lattice results better agreement with QM-HRG than PDG-HRG

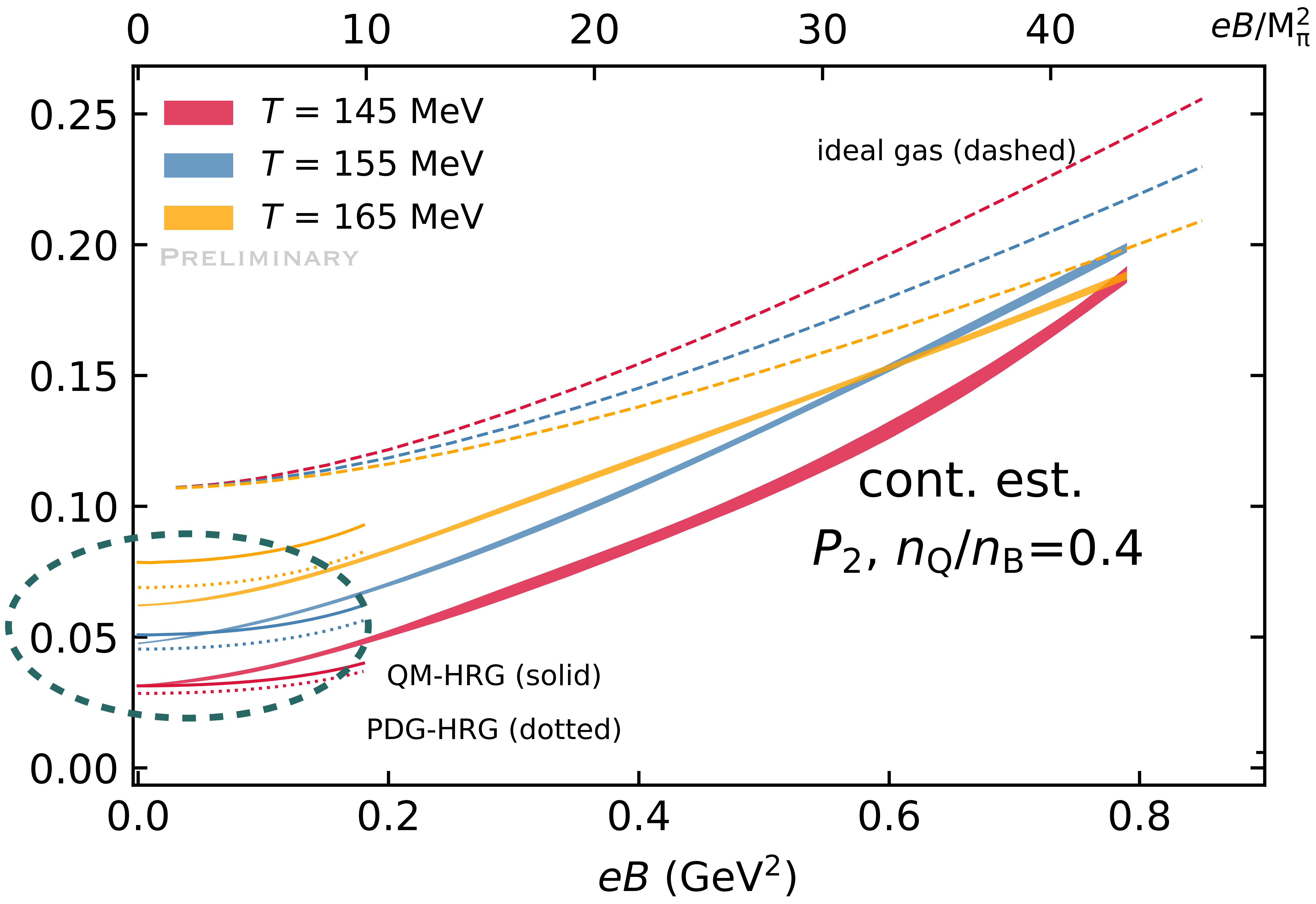
Magnetic EoS: Pressure

$$\hat{\mu} \equiv \begin{pmatrix} \hat{\mu}_B \\ q_1\ \hat{\mu}_B \\ s_1\ \hat{\mu}_B \end{pmatrix} \qquad \hat{p}_{LO} \equiv \frac{1}{2}\hat{\mu}^T \chi_{LO}^{BQS} \hat{\mu}$$

$$\Delta\hat{p} \equiv \hat{p}(T, eB, \mu_B) - \hat{p}(T, eB, 0) = \sum_{k=1}^{\infty} P_{2k}(T, eB)\ \hat{\mu}_B^{2k}$$

$$P_2(T, eB) = \frac{1}{2}\left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2\right) + \chi_{11}^{BQ} q_1 + \chi_{11}^{BS} s_1 + \chi_{11}^{QS} q_1 s_1$$

A. HRG agreement? Subject to smaller eB and low T



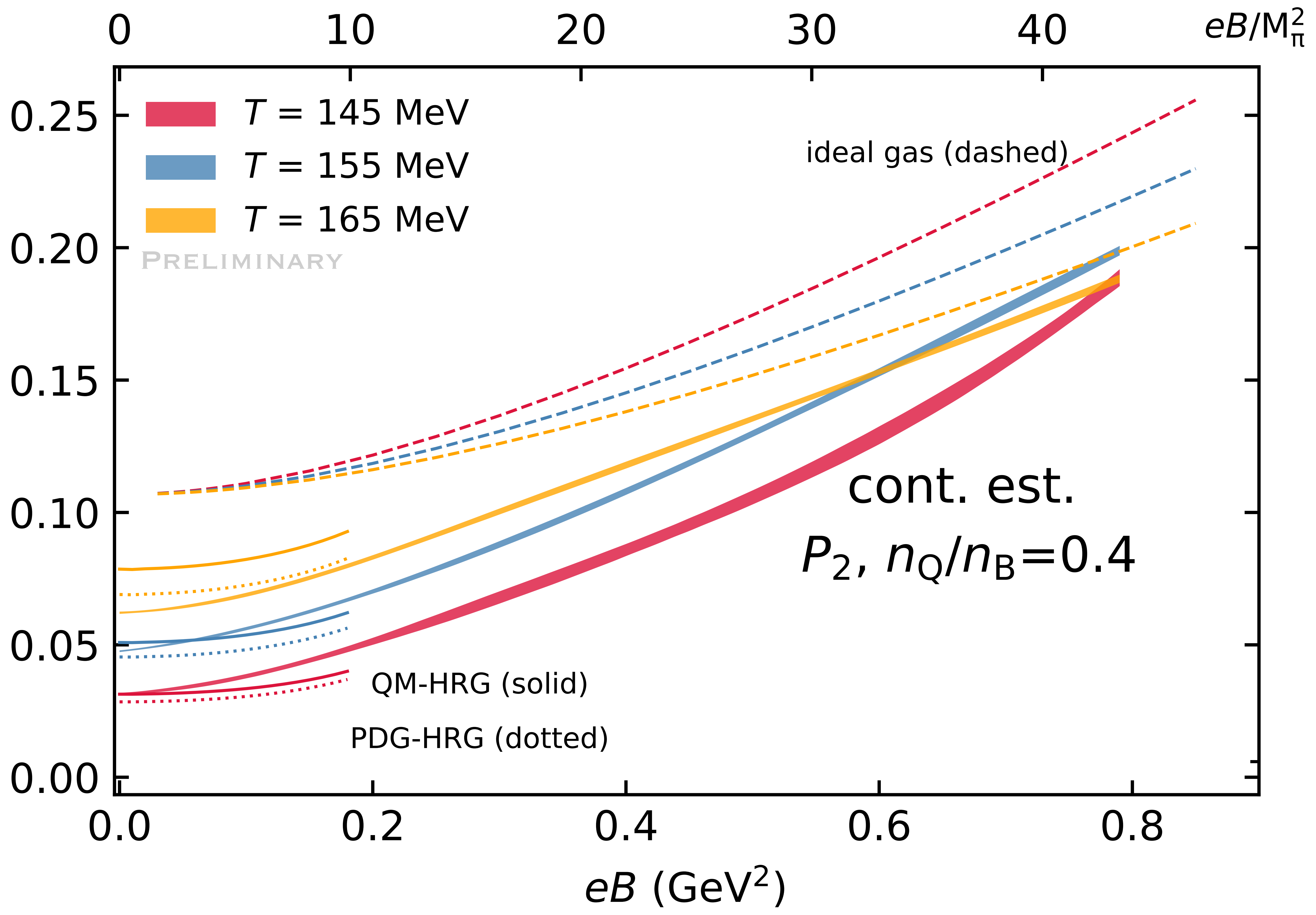
Magnetic EoS: Pressure

$$\Delta\hat{p} \equiv \hat{p}(T, eB, \mu_B) - \hat{p}(T, eB, 0) = \sum_{k=1}^{\infty} P_{2k}(T, eB)\ \hat{\mu}_B^{2k}$$

$$P_2(T, eB) = \frac{1}{2}\left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2\right) + \chi_{11}^{BQ} q_1 + \chi_{11}^{BS} s_1 + \chi_{11}^{QS} q_1 s_1$$

A. HRG agreement? Subject to smaller eB and low T

B. P_2 grows with eB, ideal gas saturation for fixed eB expected at very high T



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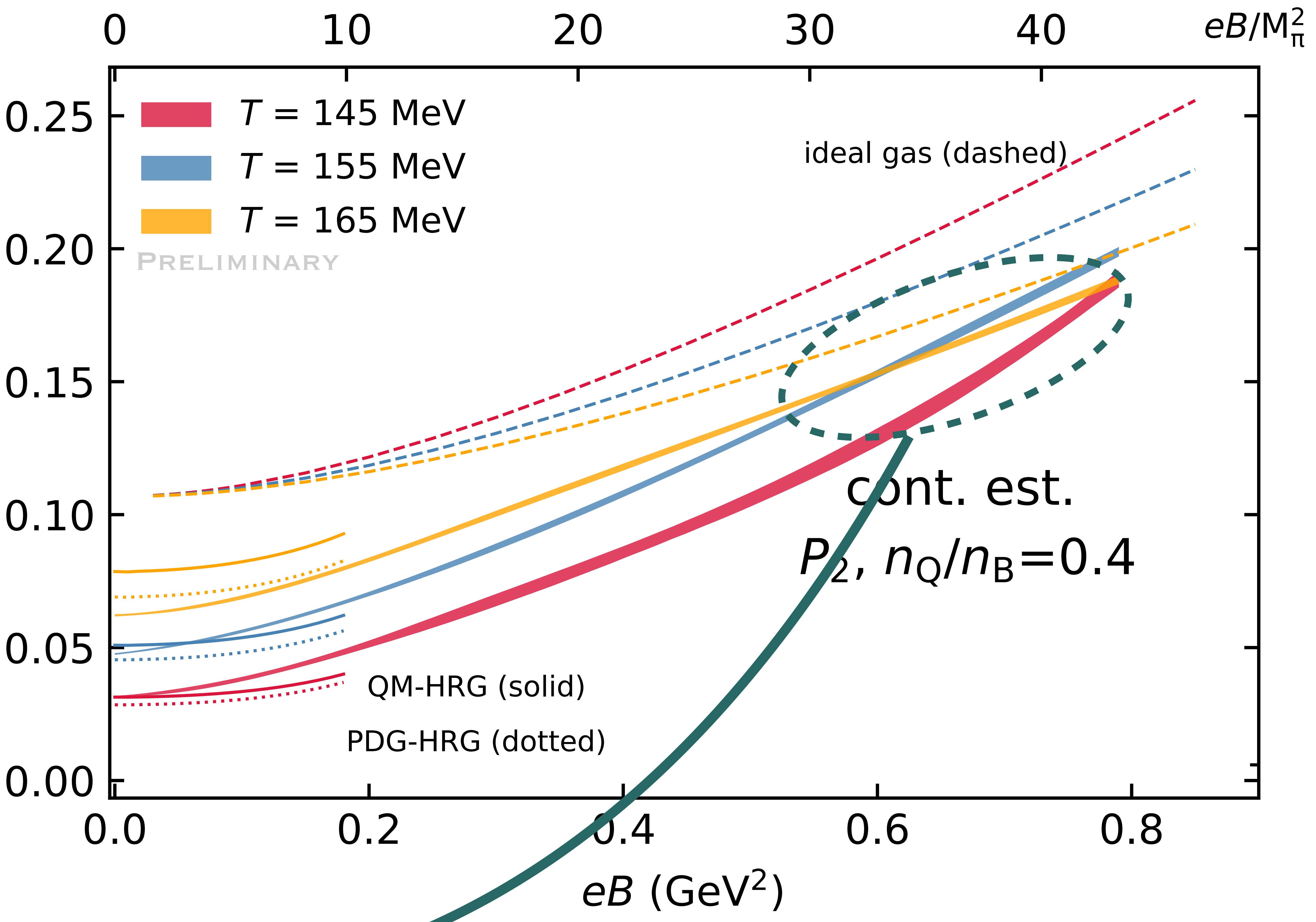
A. HRG agreement? Subject to smaller eB and low T

B. P_2 grows with eB, ideal gas saturation for fixed eB expected at very high T

C. After $eB \sim 0.6$ GeV^2, signs of T crossing

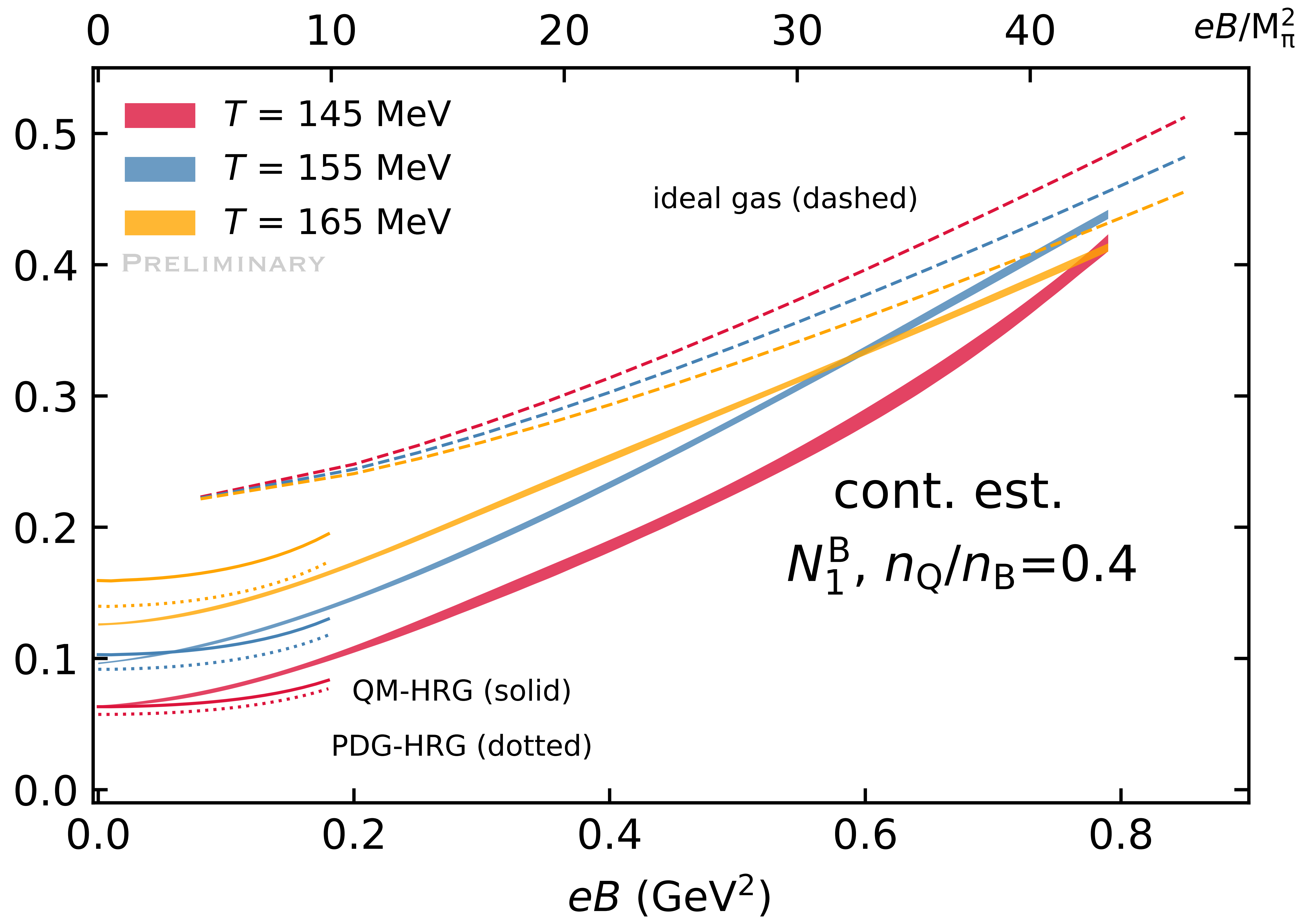


Magnetic EoS: Baryon Density

$$\hat{n}^{\mathscr{C}} \equiv \partial_{\hat{\mu}_{\mathscr{C}}} \hat{p} = \sum_{k=1}^{\infty} N_{2k-1}^{\mathscr{C}}(T, eB)\ \hat{\mu}_{\mathrm{B}}^{2k-1}$$

$$\boxed{N_1^B(T, eB) = \chi_2^{\mathrm{B}} + q_1 \chi_{11}^{\mathrm{BQ}} + s_1 \chi_{11}^{\mathrm{BS}}}$$

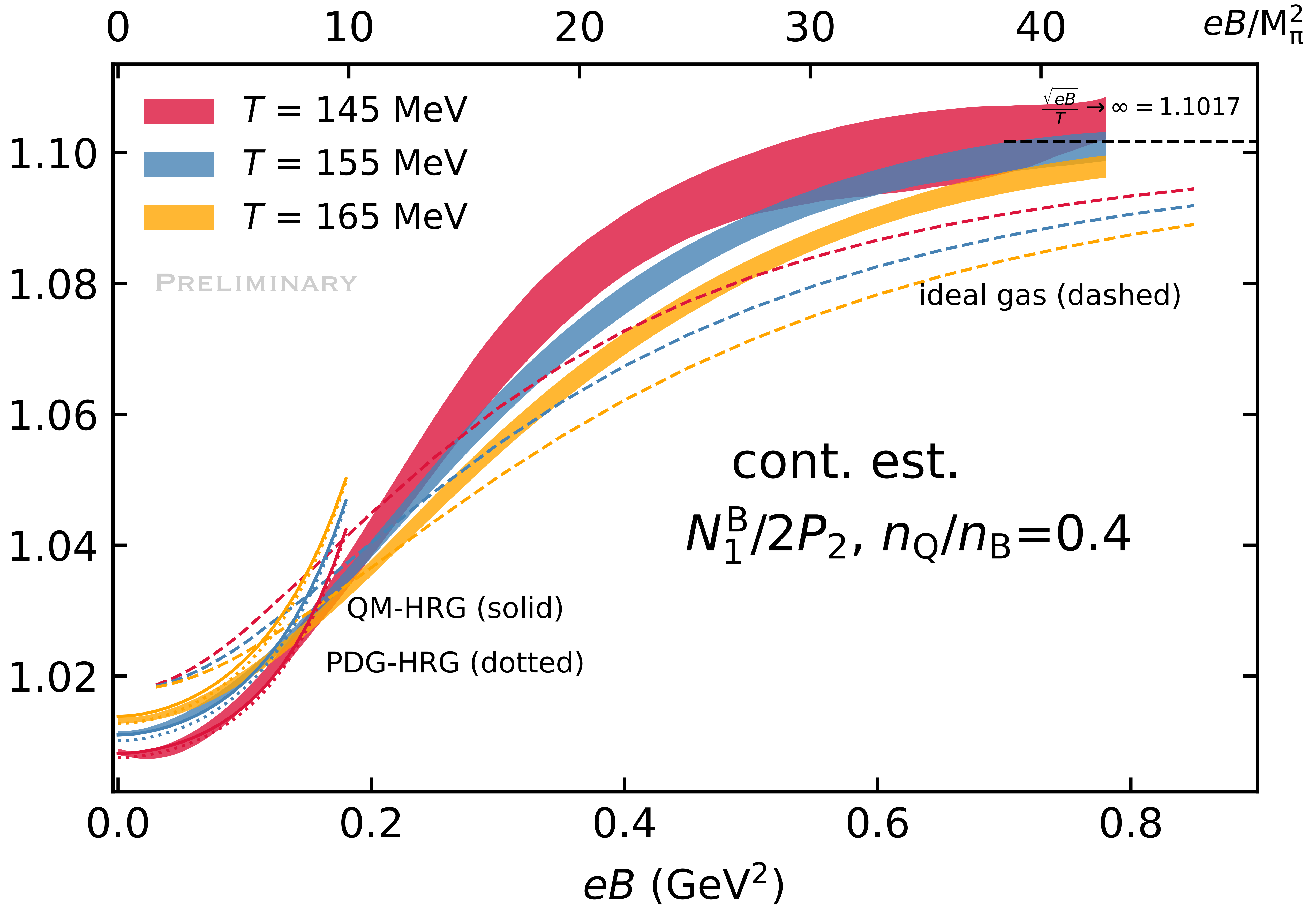
- ★ Similar eB and T dependence as pressure
- ★ Magnitude appears to be shifted from $2P_2$



Magnetic EoS: Baryon Density to Pressure

$$2mP_{2m} = N^{\mathrm{B}}_{2m-1} + r\sum_{j=1}^{m}(2j-1)\ q_{2j-1}N^{\mathrm{B}}_{2m-2j+1}$$

$$\boxed{\frac{N_1^B}{2P_2} = \frac{1}{1+rq_1}}$$

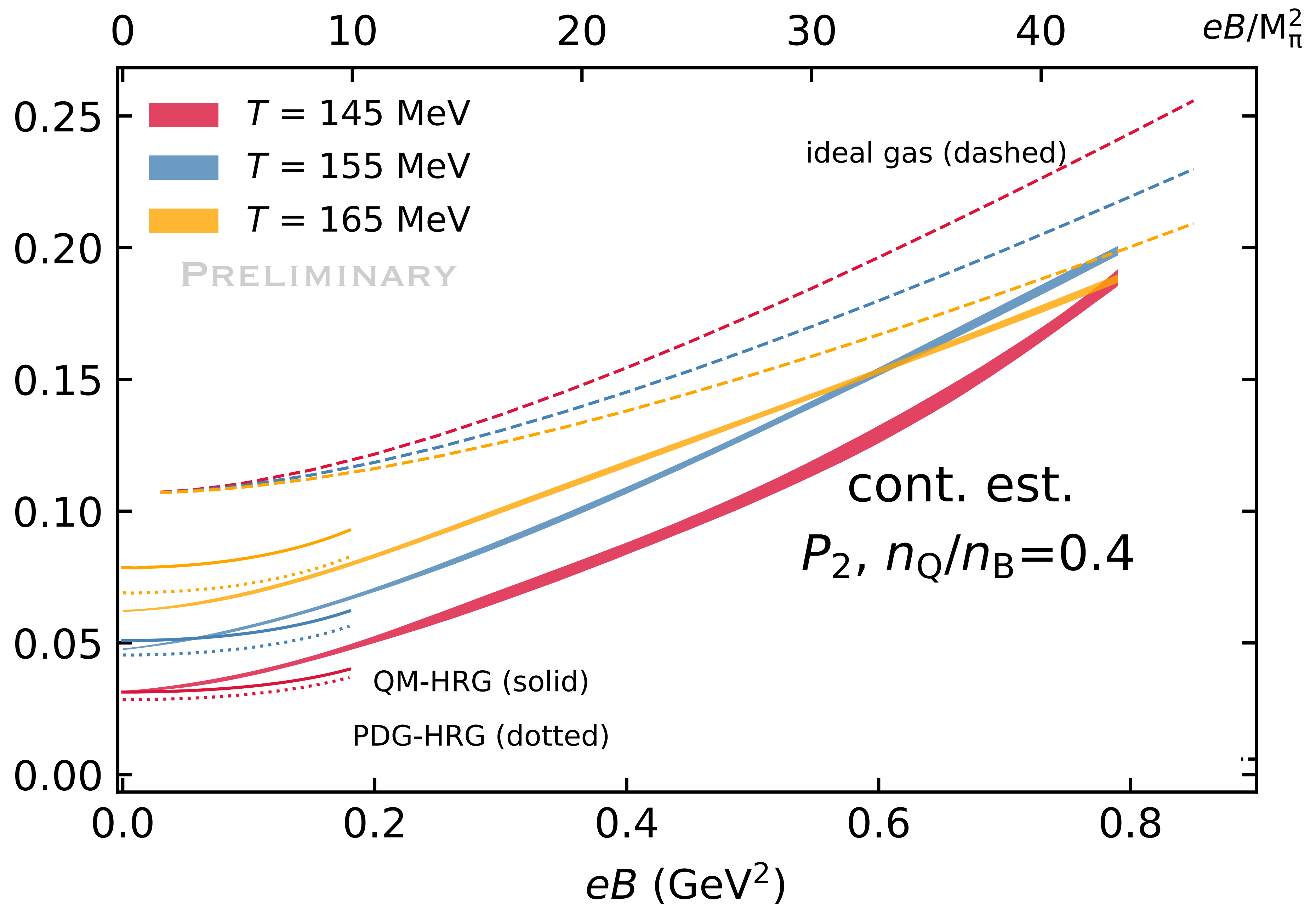


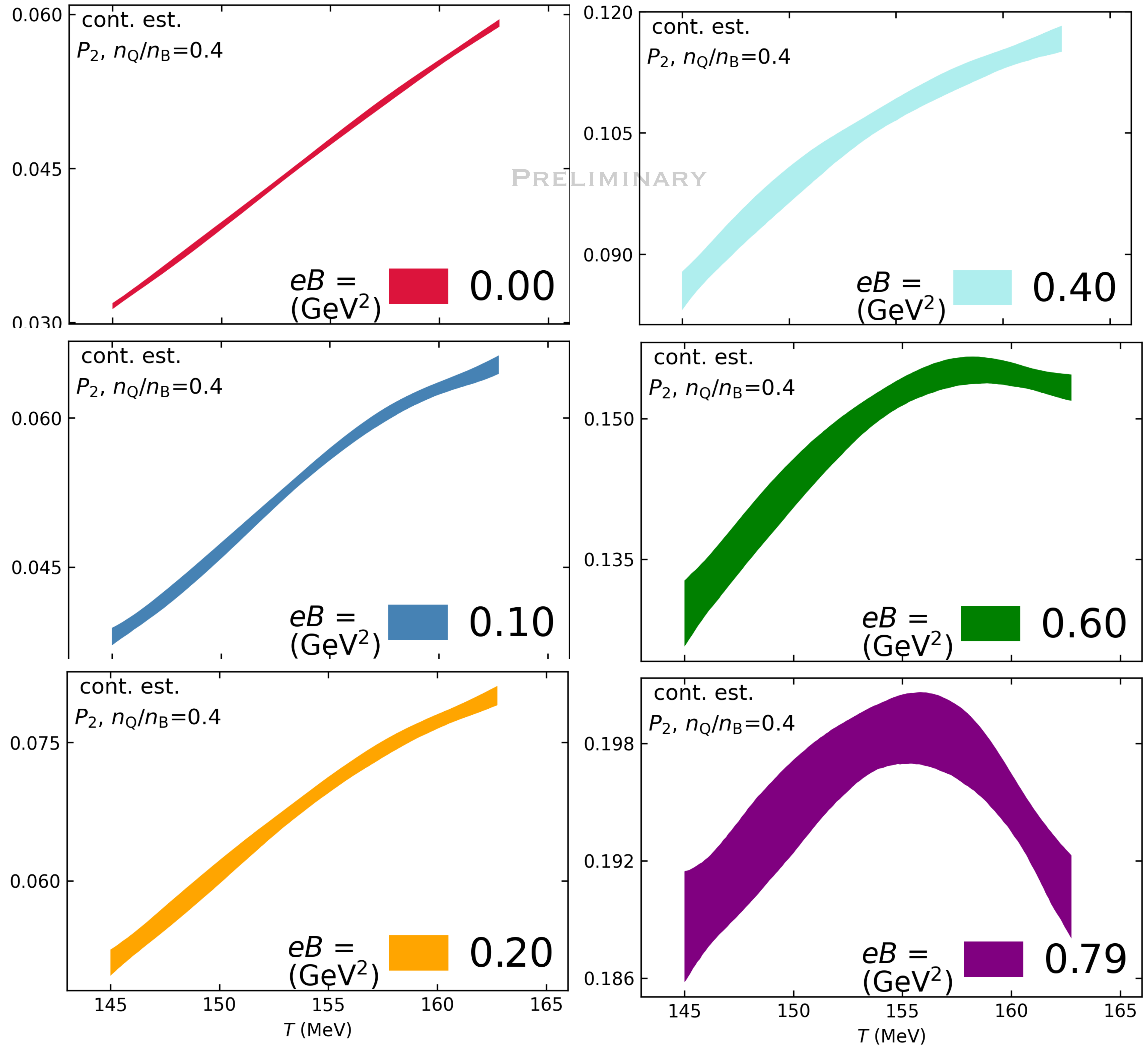
★ Deviation from unity, reflects isospin symmetry breaking by rq_1 factor

★ $N_1^{\mathrm{B}}/2P_2$ saturates at very strong eB

Magnetic EoS: Pressure vs T

- ★ Mild peak structure forms in P_2 and appears to have shifted towards low T as eB grows.
- ★ Hints of T_{pc} lowering!





Magnetic EoS: Energy and Entropy Density

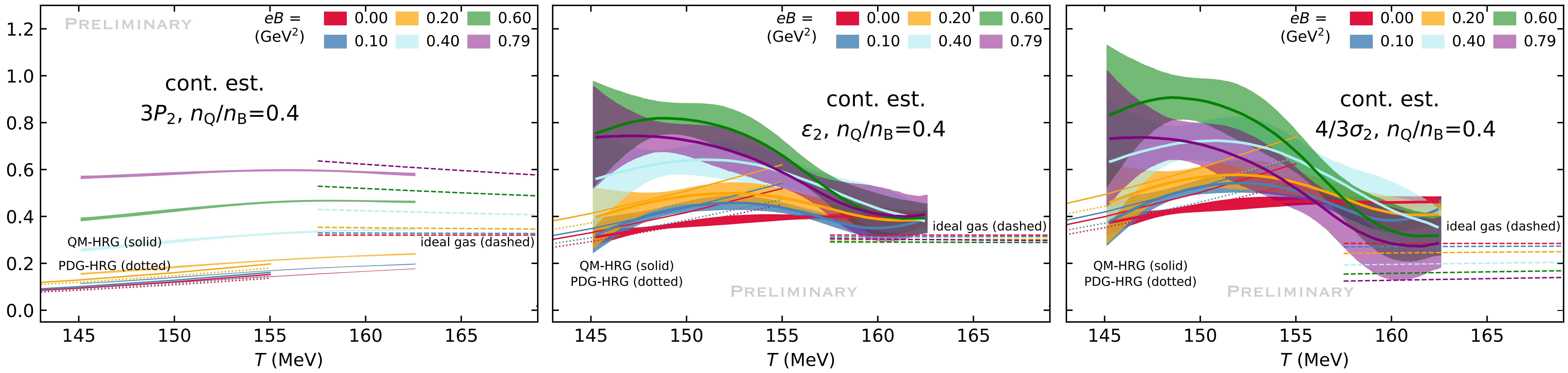
$$\Delta\hat{\epsilon} \equiv \hat{\epsilon}(T,\mu_{\mathrm{B}}) - \hat{\epsilon}(T,0) = \sum_{k=1}^{\infty} \epsilon_{2k}(T,eB)\ \hat{\mu}_{\mathrm{B}}^{2k} \quad \& \quad \Delta\hat{\sigma} = \sum_{k=1}^{\infty} \sigma_{2k}(T,eB)\ \hat{\mu}_{\mathrm{B}}^{2k}$$

$$\epsilon_2(T,eB) = 3P_2 + TP_2' - rTq_1'N_1^{\mathrm{B}}$$

$$\sigma_2(T,eB) = \epsilon_2 + P_2 + TP_2' - (1 + rq_1)N_1^{\mathrm{B}}$$

★ Clearly, very strong eB modifies the T dependence of P_2, ϵ_2 and σ_2

★ Peak structure developed in P_2, corresponds to decrease in magnitude of ϵ_2 and σ_2

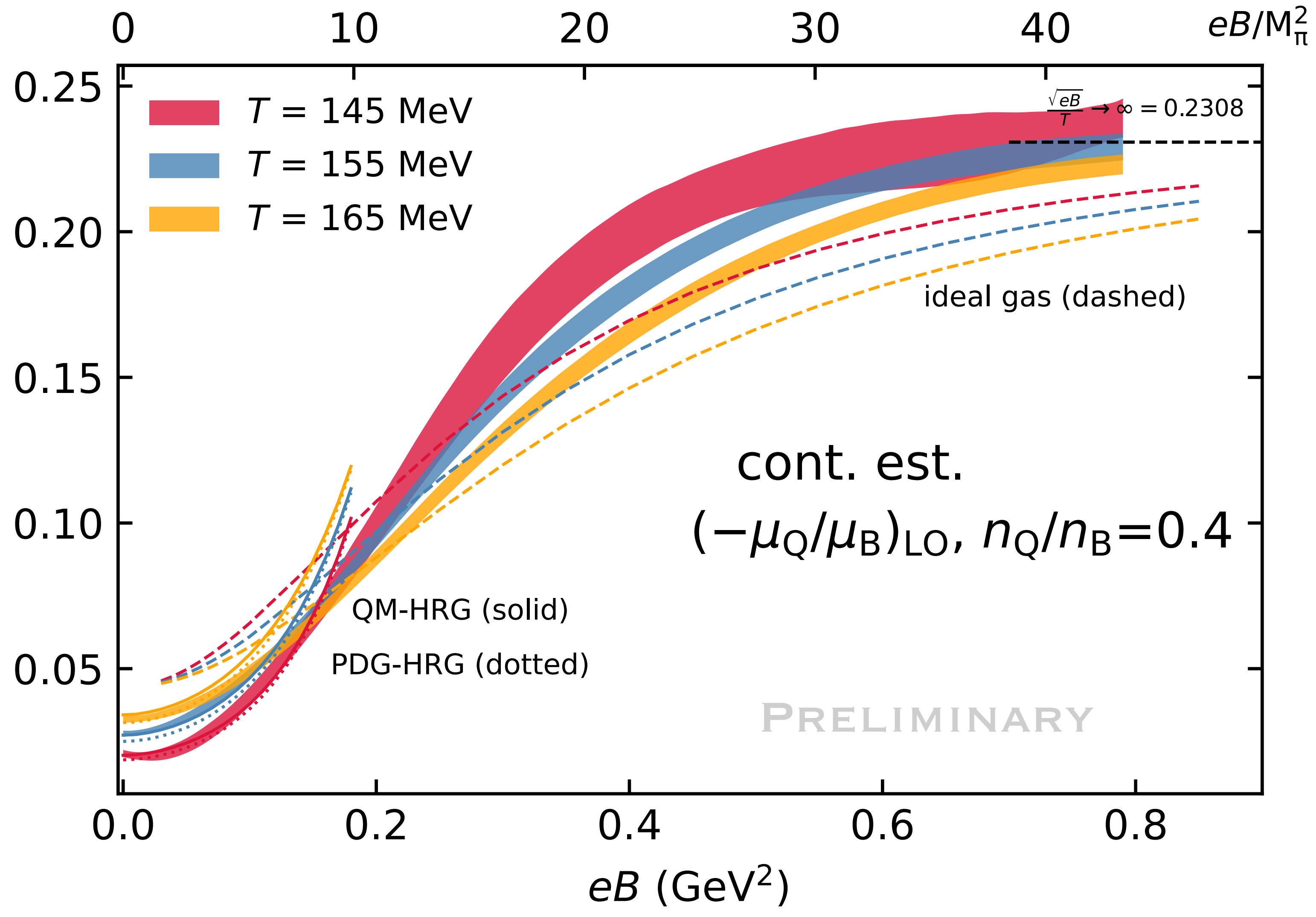


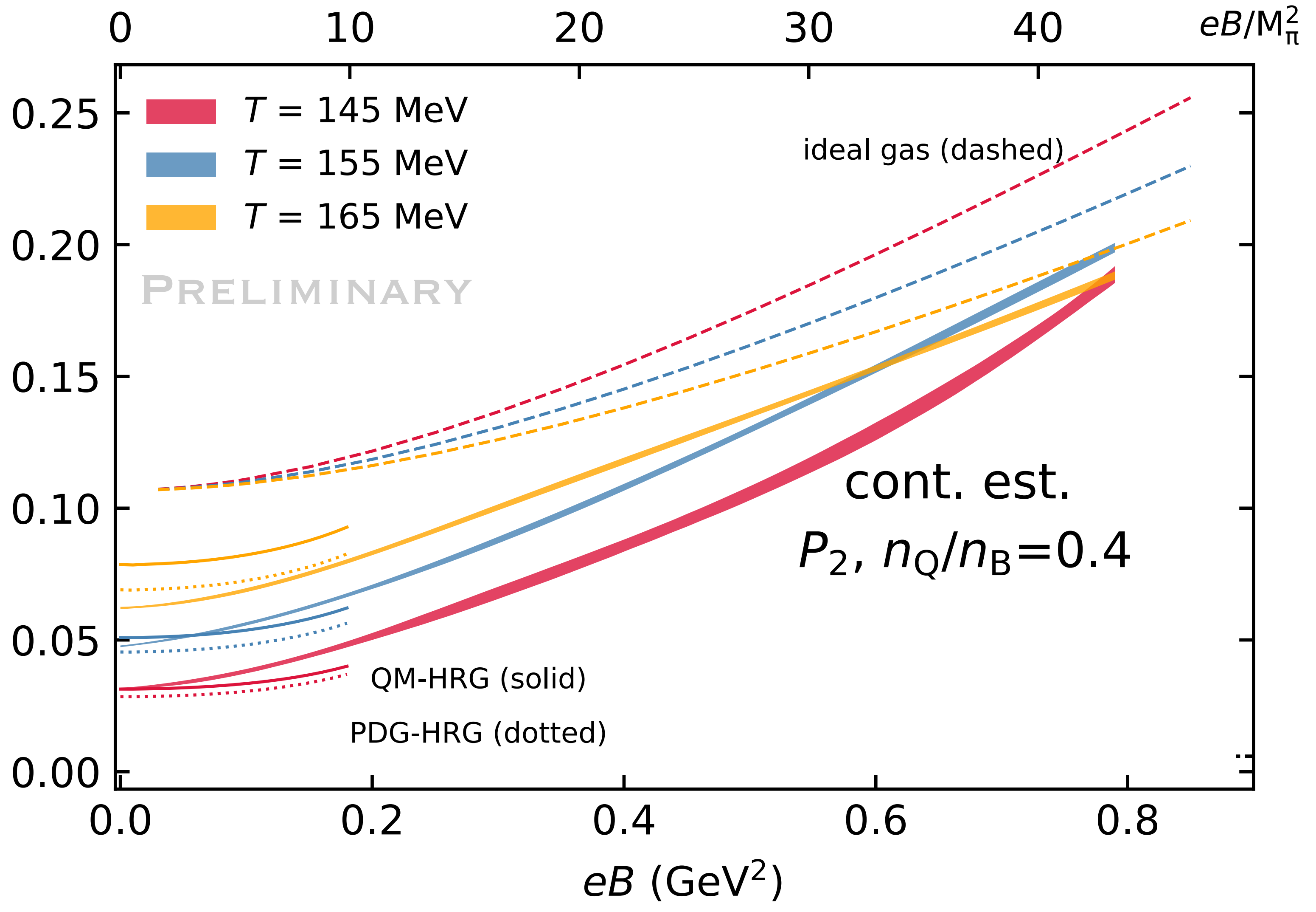
Take Home Message

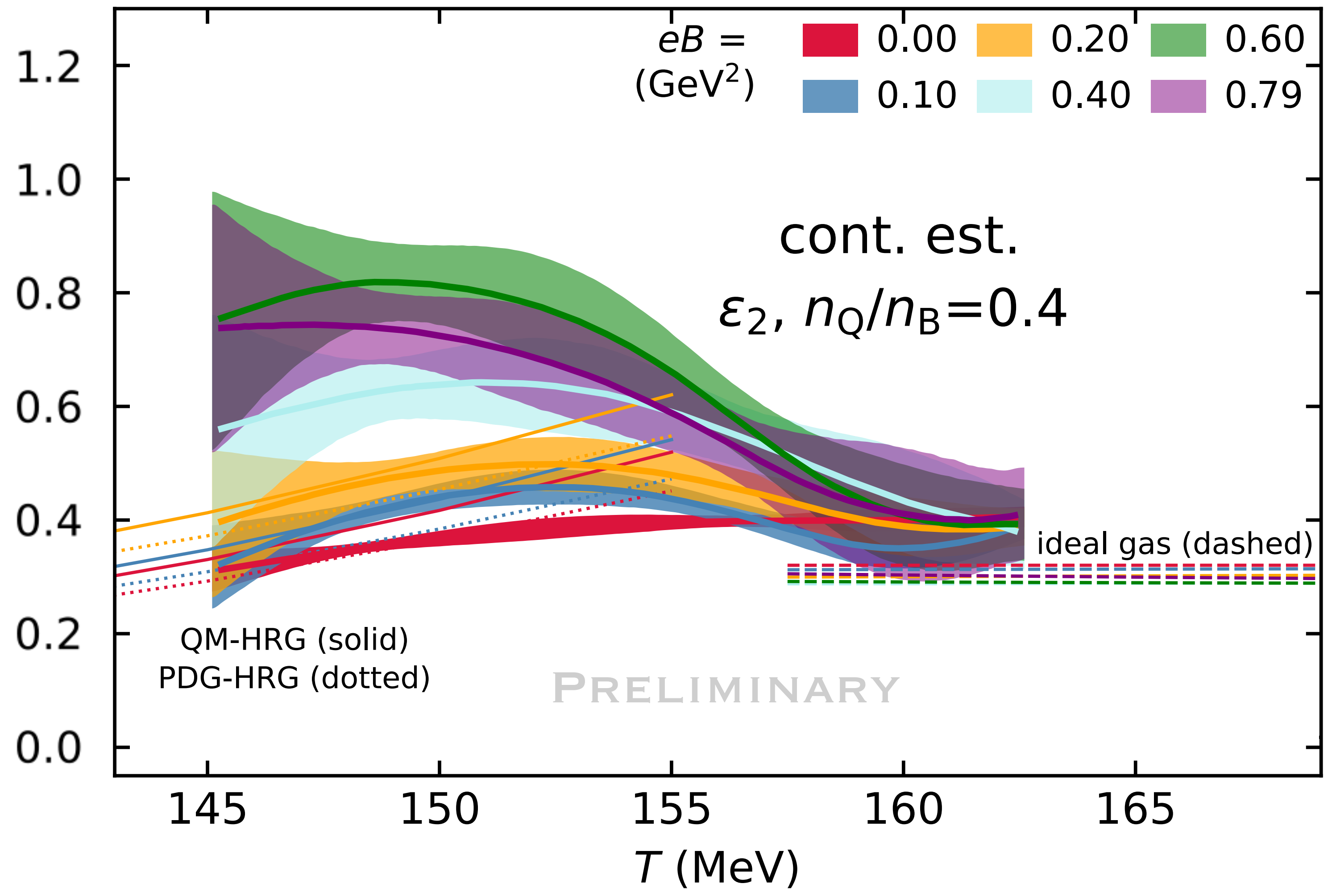
★ Explored $(2+1)$-f QCD magnetic EoS at non-zero density, upto leading order, from first principle lattice calculation using Taylor expansion

★ HRG breaks down in strong eB regime. For smaller eB, good agreement with QM-HRG subject to lower T

★ Different growth rates of bulk observables with eB. Crossing in T, and mild peak shift of P_2 towards low T as eB grows; T_{pc} lowering





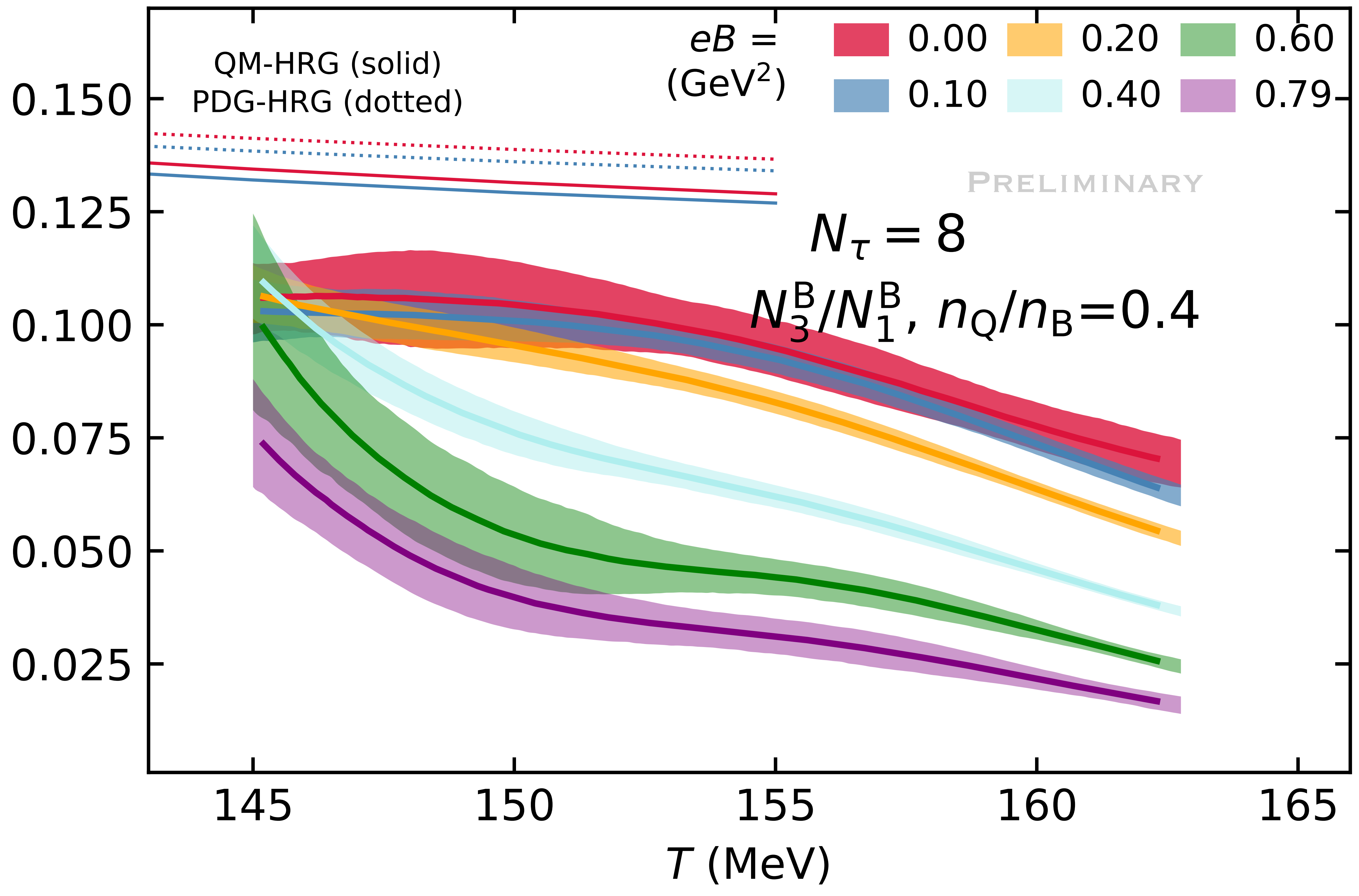


Thank You for your time & attention!

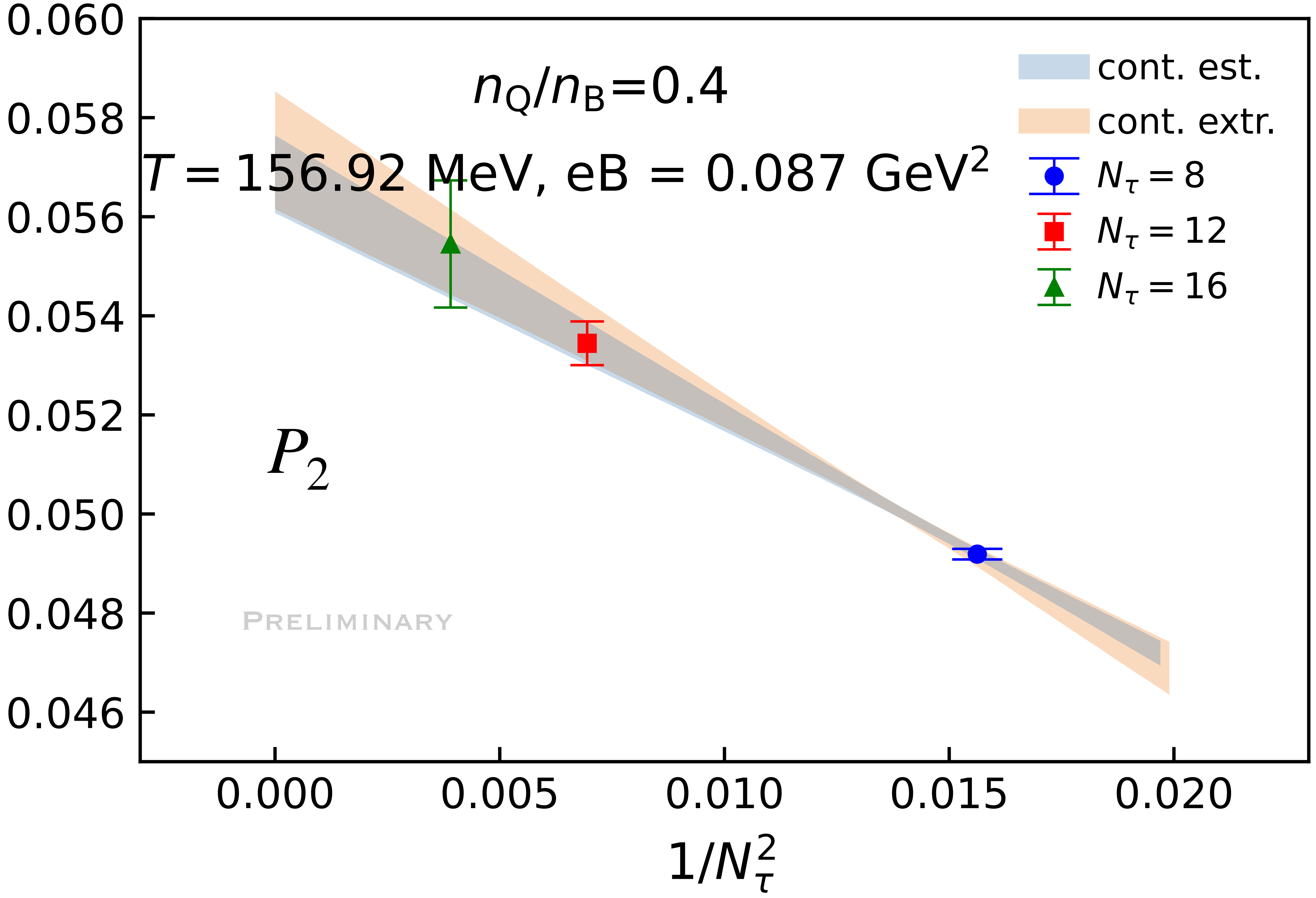
Some Backups!

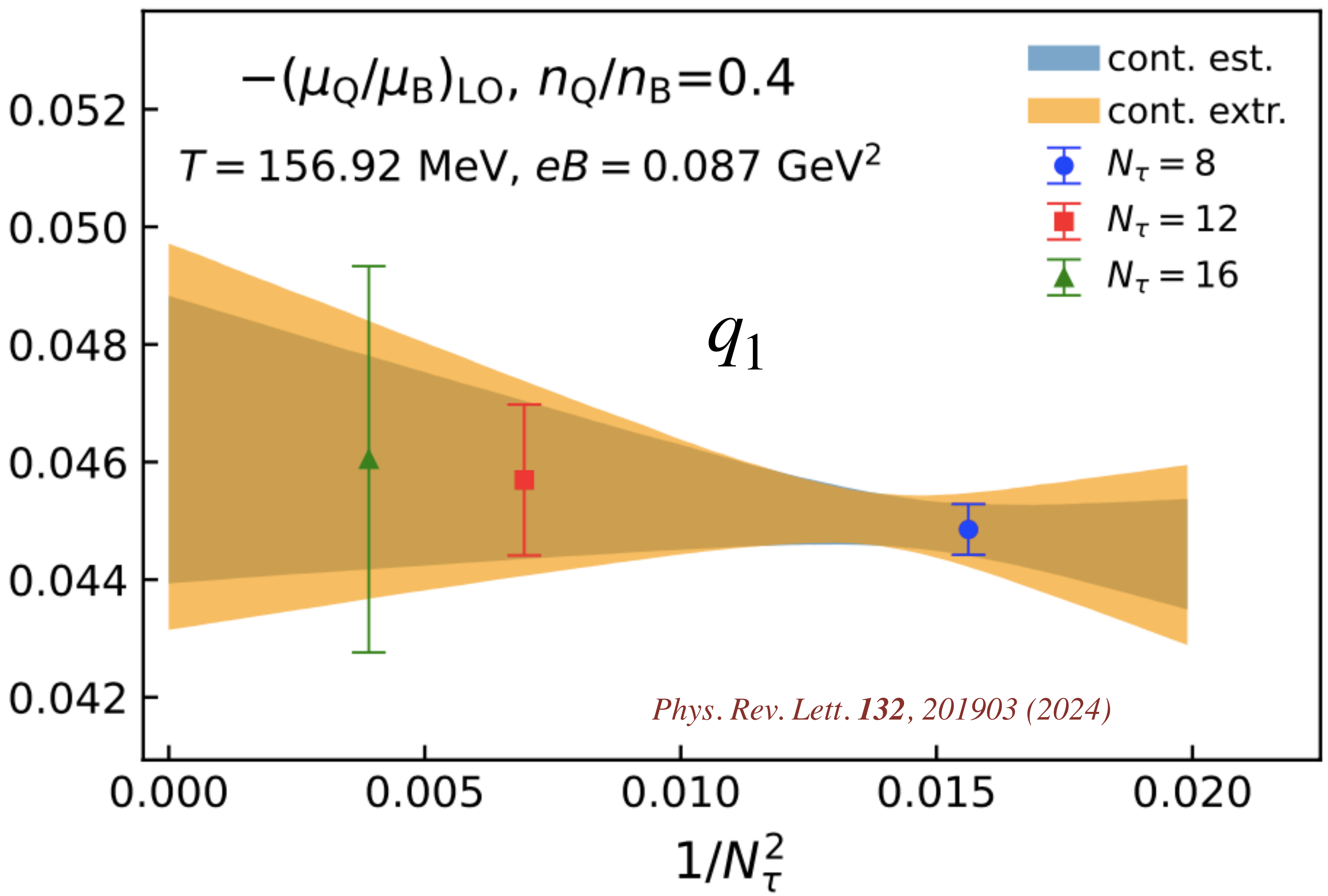
Next-to-leading order

★ Ongoing work: insights on next-to-leading order contributions. n^{B} dominant to Δp (factor ~ 2), but interestingly as eB grows contributions reduce drastically.

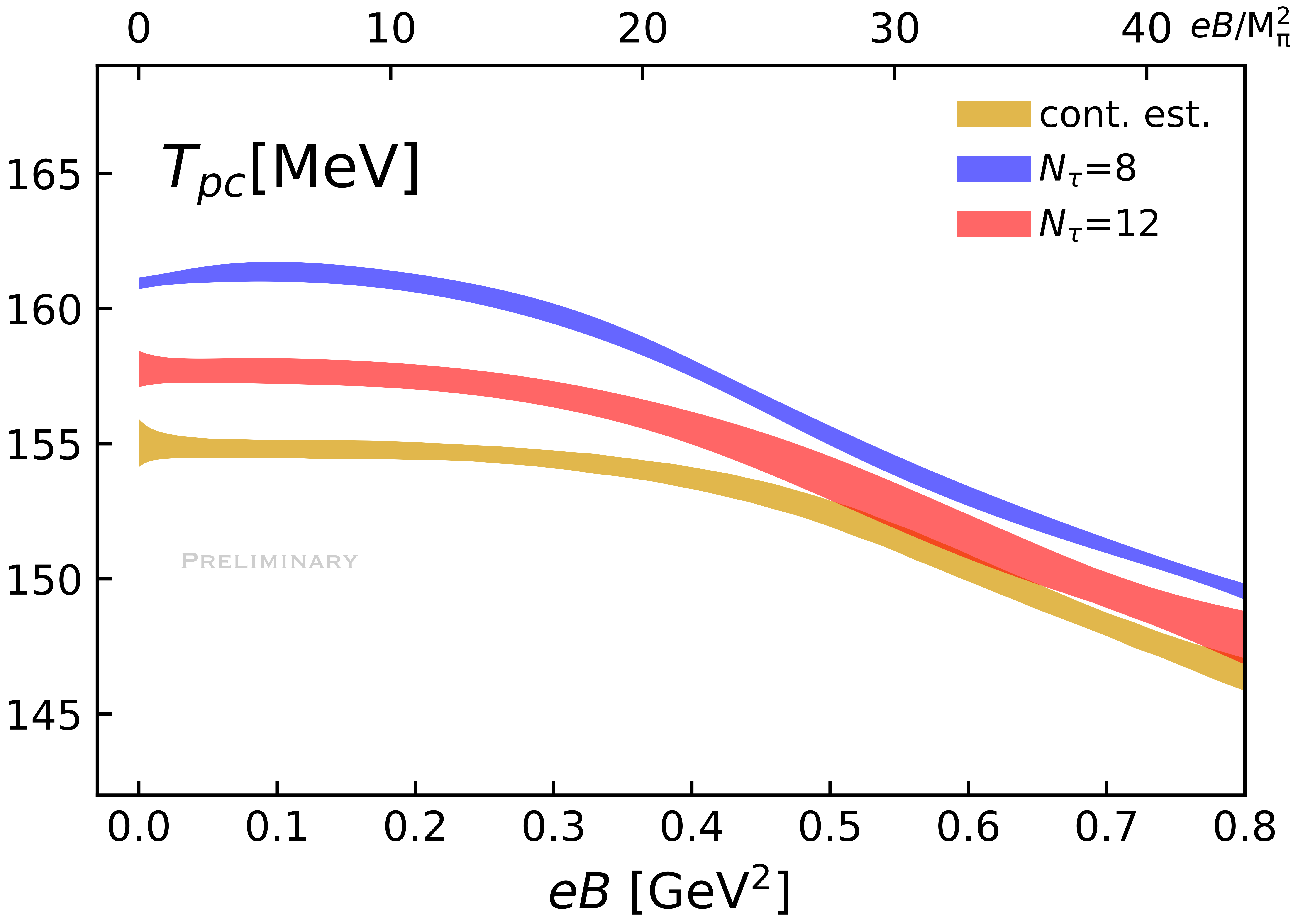


Continuum Estimates vs Extrapolations





Transition Line and Chiral Susceptibility



★ Finding the peak location of χ_M at each value of eB to determine $T_{pc}(eB)$

$$M = \frac{1}{f_K^4}\left[m_s\left(\langle\bar\psi\psi\rangle_u + \langle\bar\psi\psi\rangle_d\right) - \left(m_u + m_d\right)\langle\bar\psi\psi\rangle_s\right]$$

$$\chi_M(eB) = \frac{m_s}{f_K^4}\left[m_s\chi_l(eB) - 2\langle\bar\psi\psi\rangle_s(eB=0) - 4m_l\chi_{su}(eB=0)\right]$$