NuclearScience
Computing CenteratCCNU

Fluctuations of conserved charges in strong magnetic fields with (2+1)-flavor QCD

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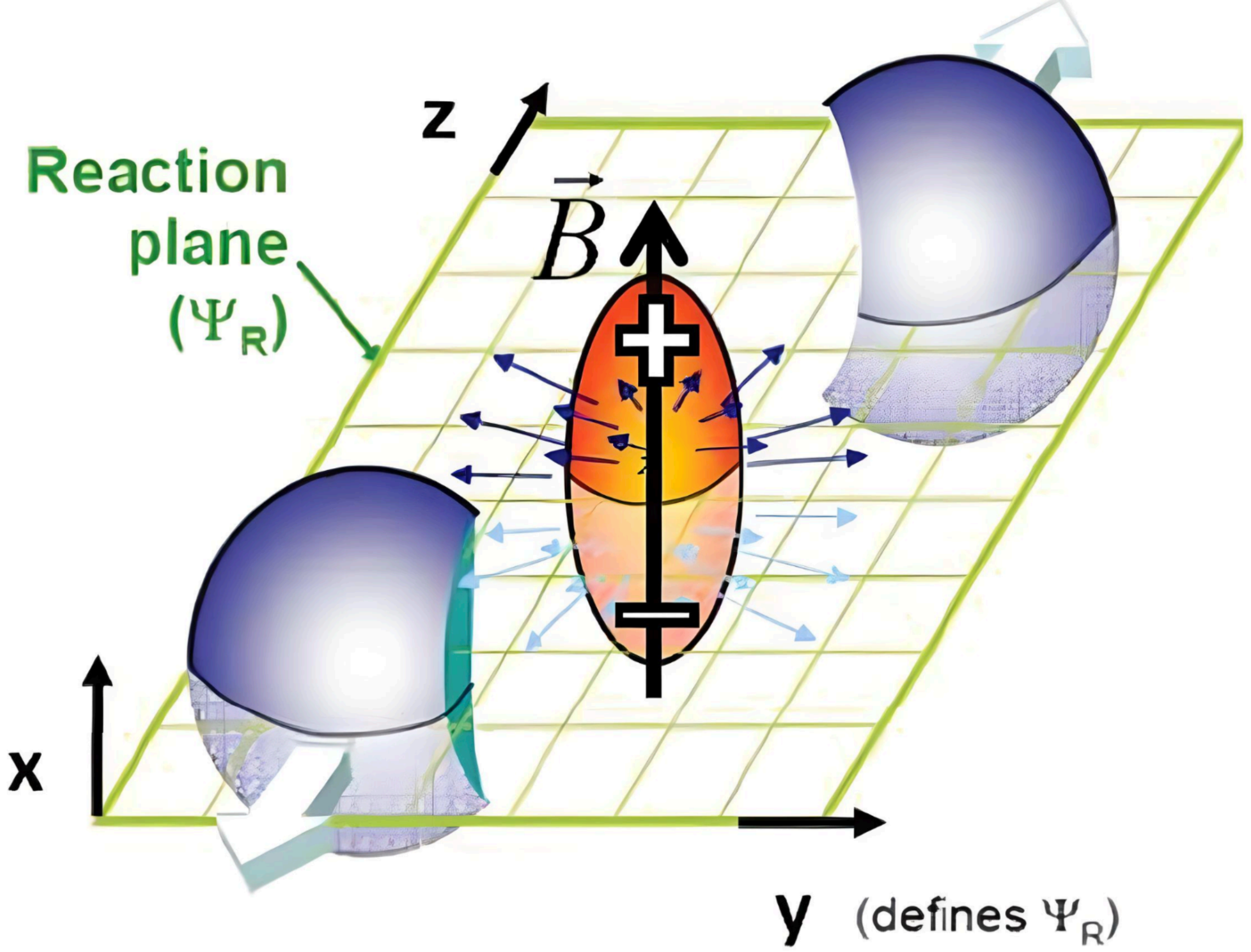
Based on Phys. Rev. Lett. **132**, 201903 (2024) and work in progress

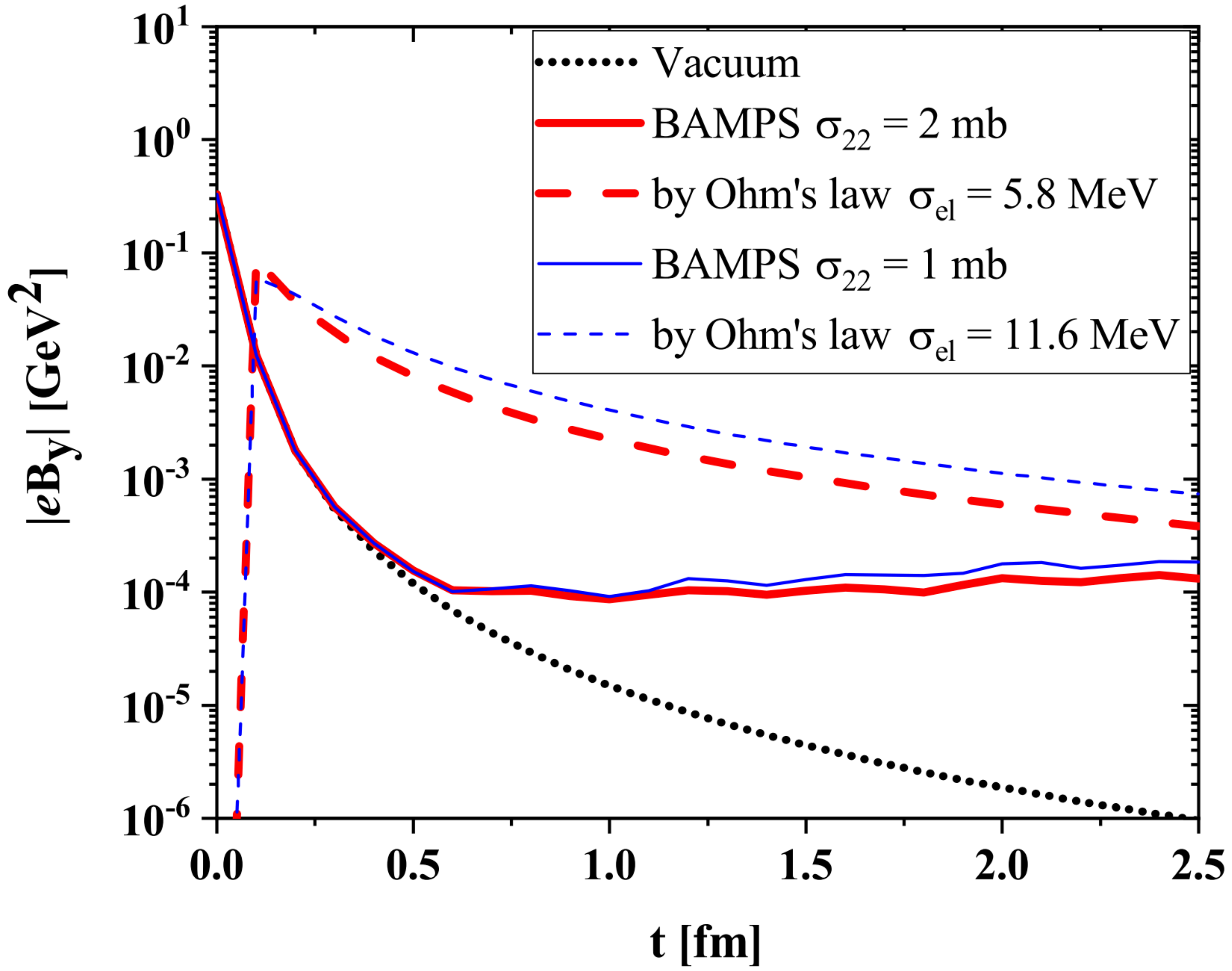
In collaboration with H.-T. Ding, A. Kumar, S.-T. Li and J.-H. Liu

New developments in studies of the QCD phase diagram, Sep. 9th - 13th, 2024 @ ECT*, Italy

Strong magnetic fields in heavy-ion collisions



J. Zhao and F. Wang, Prog.Part.Nucl.Phys. 107 (2019) 200-236



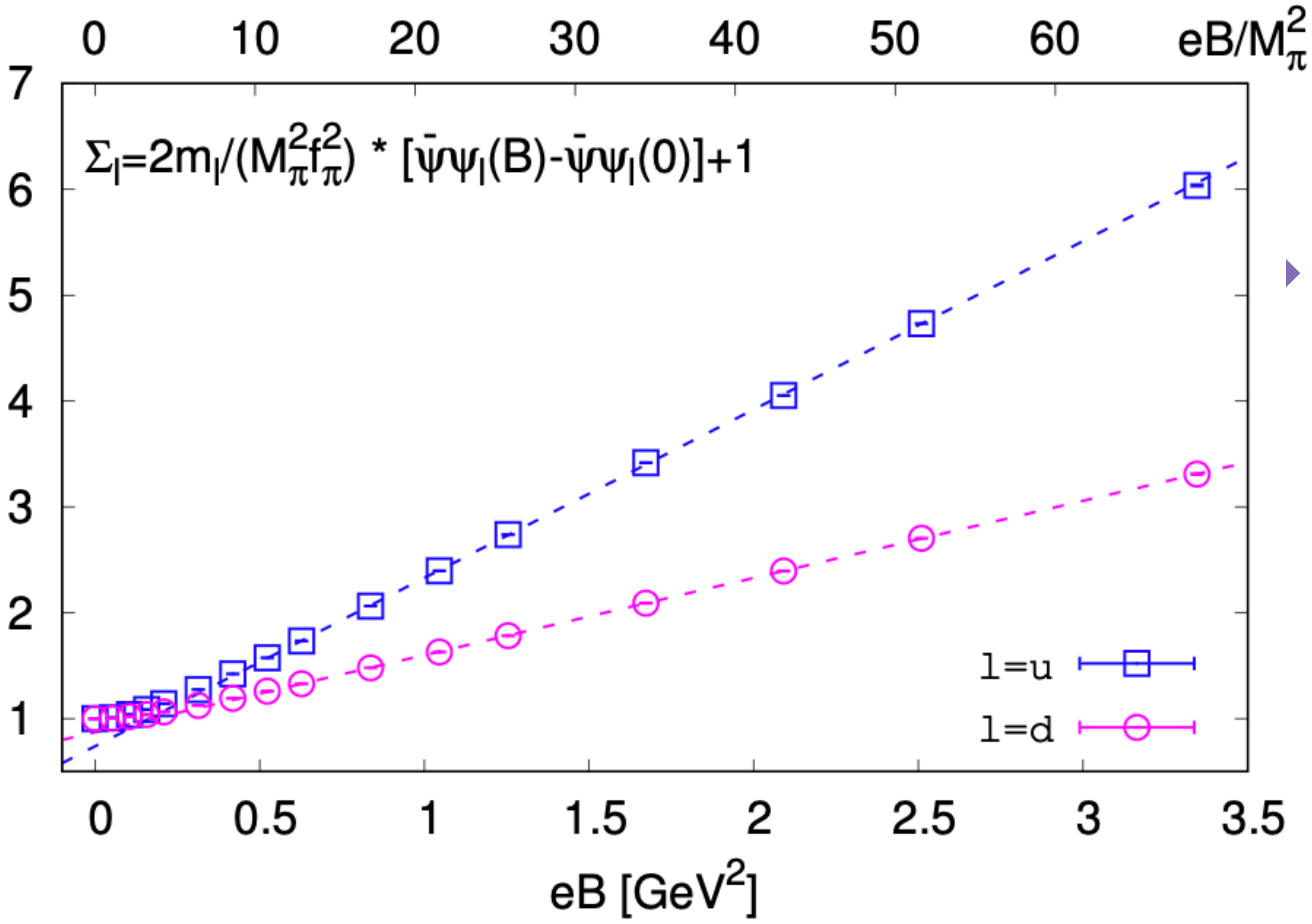
Z. Wang et al., Phys.Rev.C 105 (2022) L041901

$eB_{\tau=0} \sim 5\, M_\pi^2$ in RHIC, $eB_{\tau=0} \sim 70\, M_\pi^2$ in LHC

W.-T.Deng, X.-G. Huang, Phys.Rev.C 85, 044907 (2012)

Does this strongly decaying magnetic field manifest itself in the final stage of HIC?

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



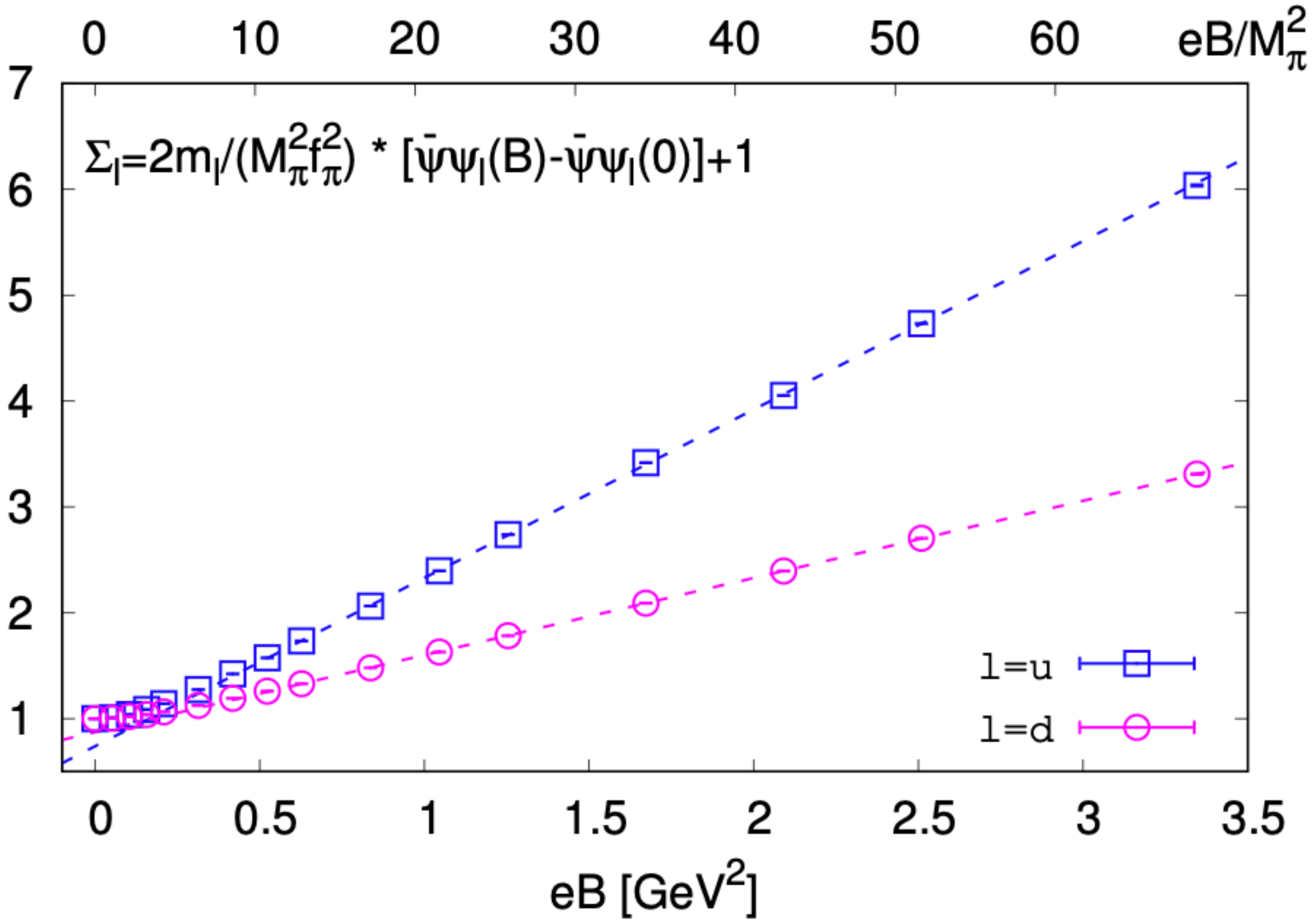
H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104, 014505 (2021)

See also in e.g. Bali et al., Phys.Rev.D 86, 071502(2012)

- Sign of isospin symmetry breaking:

  Non-degeneracy of *u* and *d* condensates at strong magnetic fields

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104, 014505 (2021)

See also in e.g. Bali et al., Phys.Rev.D 86, 071502(2012)

A clear effect but Not accessible in HIC experiments!

Fluctuations of net baryon number (B), electric charge (Q) and strangeness (S)

Taylor expansion of the QCD pressure:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}\left(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s\right) = \sum_{i,j,k=0}^{\infty} \boxed{\frac{\chi_{ijk}^{\mathrm{BQS}}}{i!j!k!}} \left(\frac{\mu_{\mathrm{B}}}{T}\right)^i \left(\frac{\mu_{\mathrm{Q}}}{T}\right)^j \left(\frac{\mu_{\mathrm{S}}}{T}\right)^k$$

C. Allton et al., Phys.Rev. D 66, 074507 (2005)

Taylor expansion coefficients at $\mu = 0$ are computable in LQCD

$$\chi_{ijk}^{uds} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k}\right|_{\mu_{u,d,s}=0}$$

$$\chi_{ijk}^{\mathrm{BQS}} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_{\mathrm{B}}/T\right)^i \partial\left(\mu_{\mathrm{Q}}/T\right)^j \partial\left(\mu_{\mathrm{S}}/T\right)^k}\right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}}$$

$$\mu_d = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}}$$

$$\mu_s = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}}$$

LQCD: H.-T.Ding, F. Karsch, S.Mukherjee, Int. J. Mod. Phys. E 24, no.10, 1530007 (2015)
Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112

At $eB \neq 0$ a lot more need to be explored

HRG: *G. Kadam et al., JPG 47, 125106 (2020); M. Ferreira et al., Phys. Rev. D 98, 034003 (2018); K. Fukushima and Y. Hidaka, Phys. Rev. Lett. 117, 102301 (2016); A. Bhattacharyya et al., EPL 115, 62003 (2016); M. Marczenko et al., arXiv:2405.15745*

PNJL: *W.-J. Fu, Phys. Rev. D 88, 014009 (2013)*

Lattice Setup

- Highly improved staggered fermions and a tree-level improved Symanzik gauge action
- $N_f = 2 + 1$
- Lattice sizes : $32^3 \times 8,\ 48^3 \times 12;\ \ 64^3 \times 16$
- $m_s^{\mathrm{phy}}/m_l = 27,\ M_\pi(eB = 0) \approx 135\ \mathrm{MeV}$
- T window : $(144\ \mathrm{MeV},\ 166\ \mathrm{MeV})$, i.e. $(0.9T_{pc},\ 1.1T_{pc})$
- eB window: $eB \lesssim 45M_\pi^2 \sim 0.8\ \mathrm{GeV}^2$

$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}, \quad N_b = 1,2,3,4,6,12,16,24,32$$

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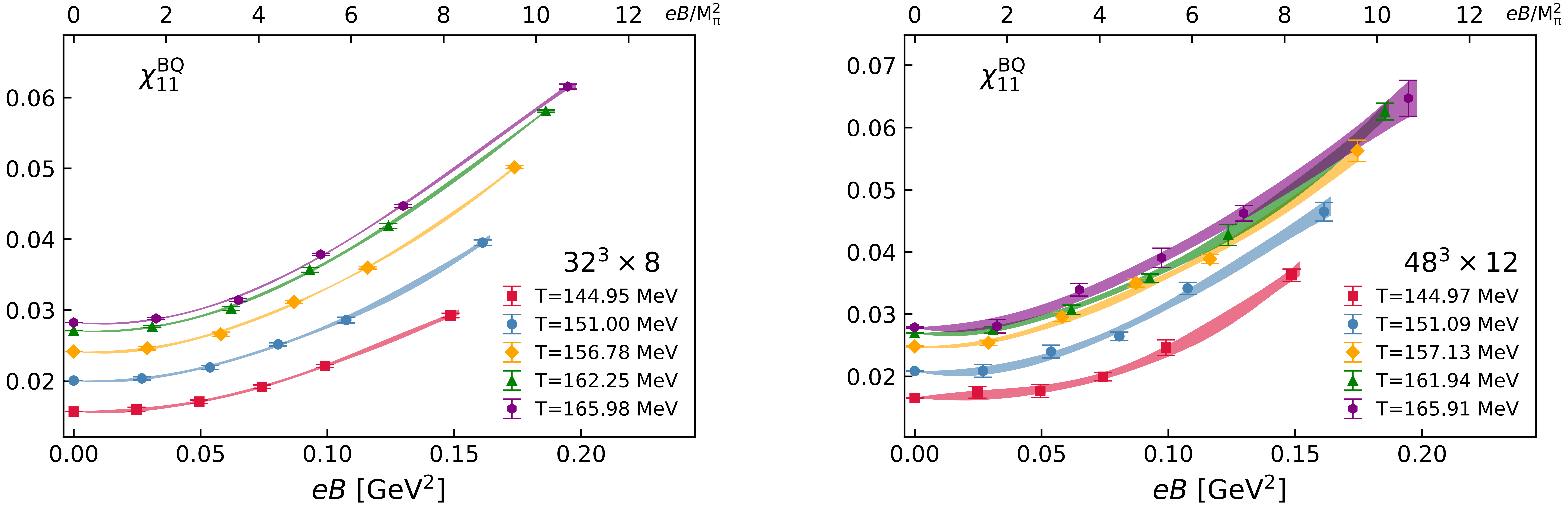
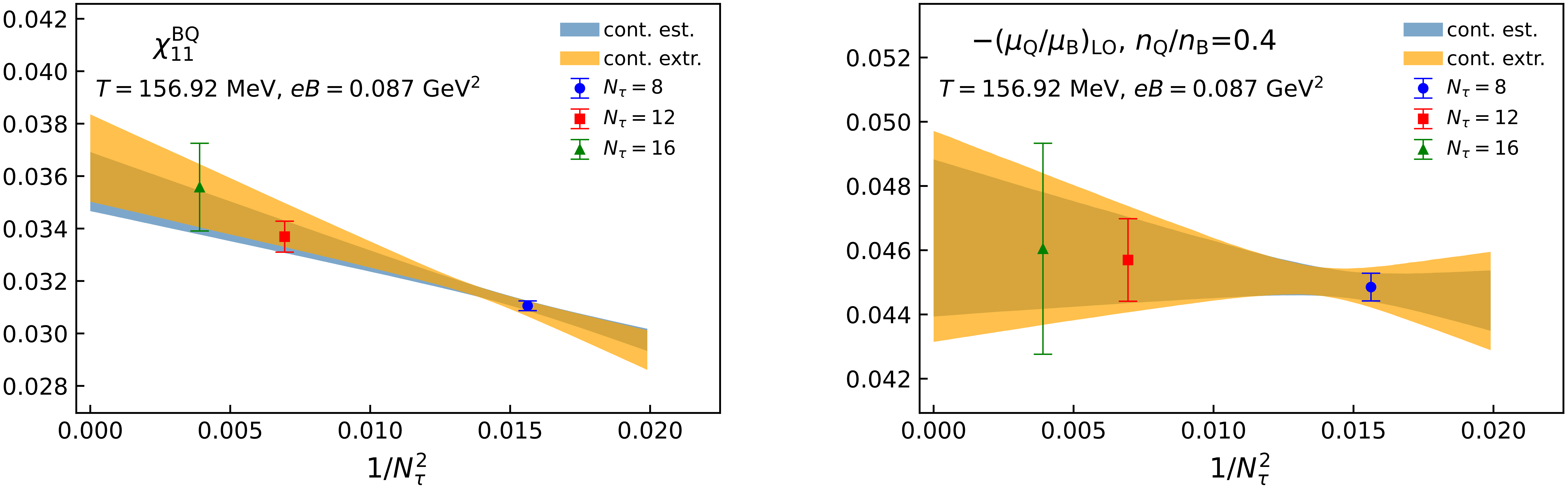
Lattice data on $N_\tau = 8$ and 12 lattices



H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

The zero magnetic field data comes from D. Bollweg et al., Phys. Rev. D 104, 074512 (2021)

Continuum estimate and extrapolation

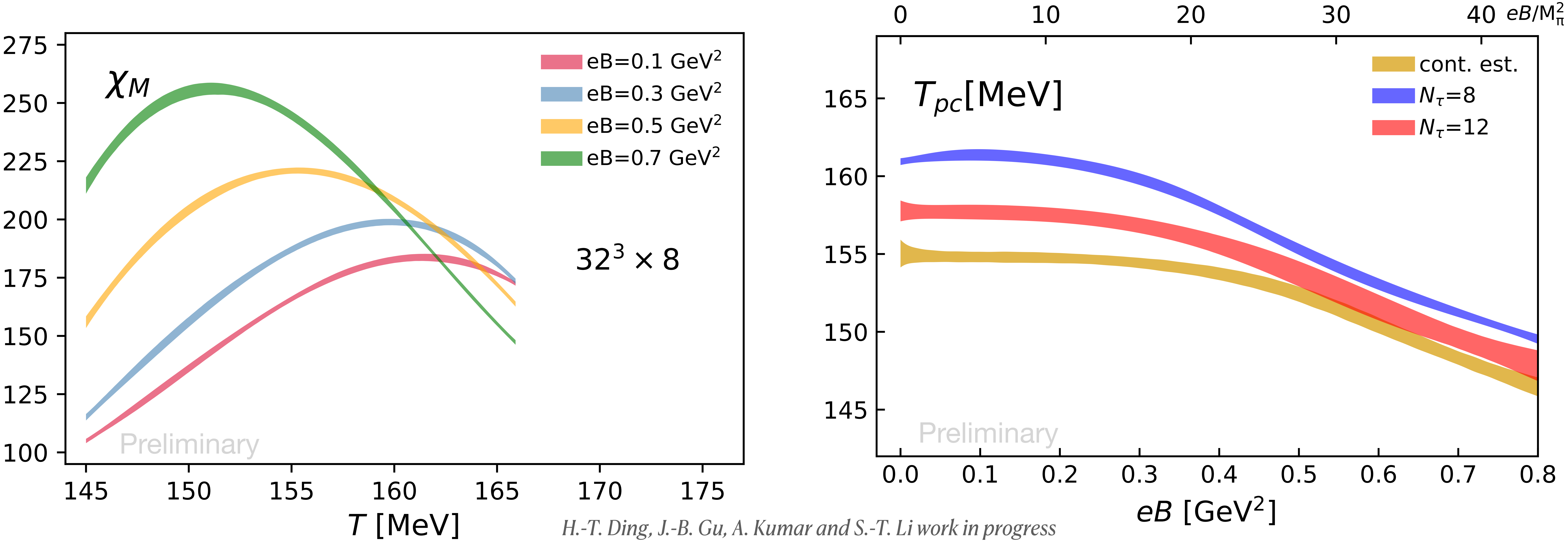


H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

$$\mathcal{O}\left(T, eB, N_\tau\right) = \mathcal{O}(T, eB) + \frac{c}{N_\tau^2} \quad \begin{cases} \text{Continuum estimate} \\ \text{Continuum extrapolation} \end{cases} \text{obtained from} \quad \begin{matrix} N_\tau = 8 \text{ and } 12 \\ N_\tau = 8,\ 12, \text{ and } 16 \end{matrix}$$

Continuum estimate and continuum extrapolation are consistent within uncertainty

Transition line on $T - eB$ plane



H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li work in progress

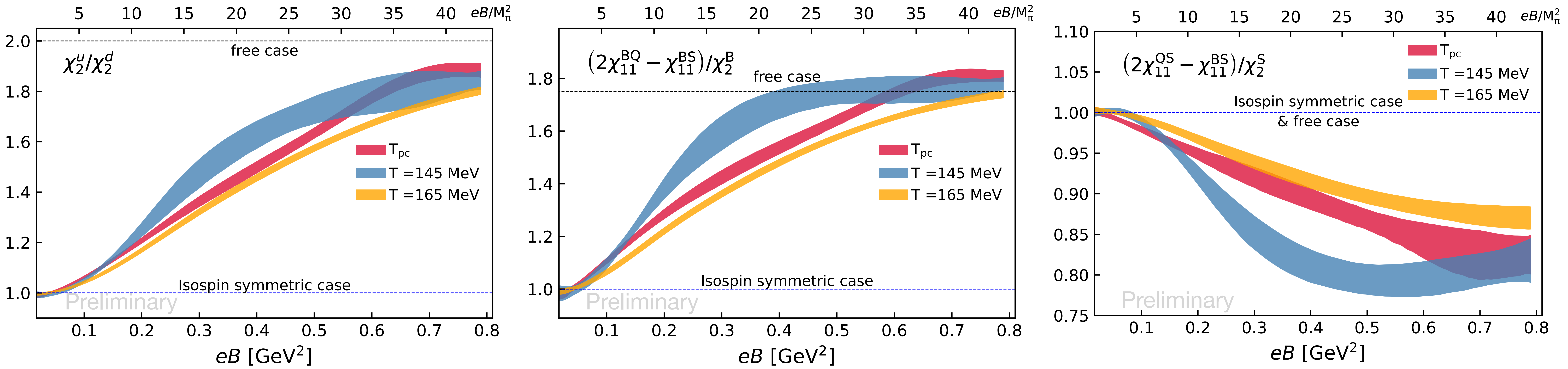
$$\chi_M(eB) = \frac{m_s}{f_K^4}\left[m_s\chi_l(eB) - 2\langle\bar{\psi}\psi\rangle_s(eB=0) - 4m_l\chi_{sl}(eB=0)\right]$$

Finding the peak location of chiral susceptibility (χ_M) at each eB value to determine $T_{pc}(eB)$

T_{pc}:
- almost independent of eB, $eB \lesssim 0.3\ \mathrm{GeV}^2$
- decreases with eB, $eB \gtrsim 0.3\ \mathrm{GeV}^2$

H.-T. Ding et al., Phys. Rev. Lett. 123, 062002 (2019)

Isospin symmetry breaking at non-zero magnetic field

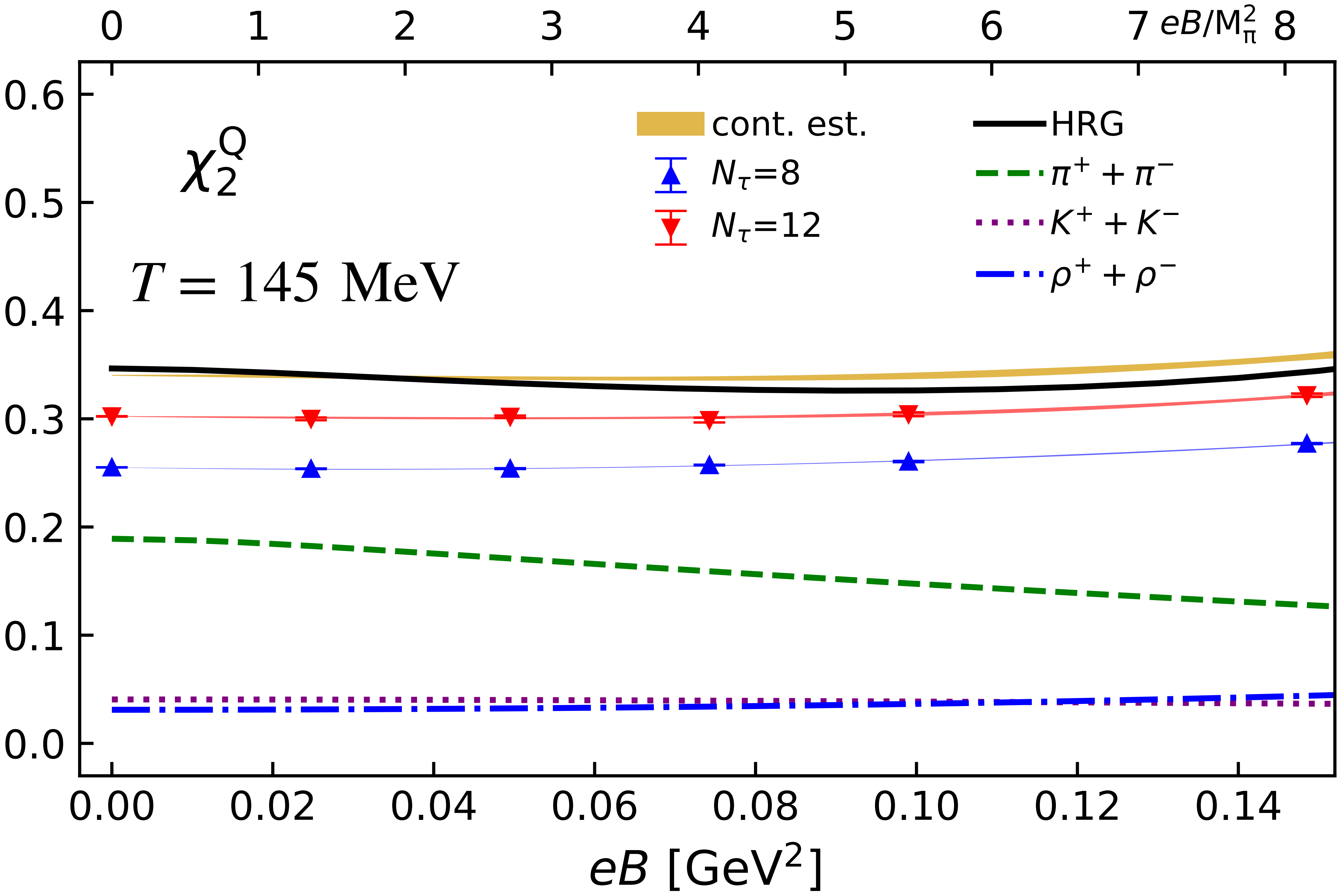


H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li work in progress

At zero magnetic field, isospin symmetry system:

$$\begin{cases} \chi_2^u = \chi_2^d \\ \chi_{11}^{us} = \chi_{11}^{ds} \end{cases} \Longrightarrow \begin{cases} 2\chi_{11}^{\mathrm{BQ}} - \chi_{11}^{\mathrm{BS}} = \chi_2^{\mathrm{B}} \\ 2\chi_{11}^{\mathrm{QS}} - \chi_{11}^{\mathrm{BS}} = \chi_2^{\mathrm{S}} \end{cases}$$

Electric charge fluctuations at $T = 145$ MeV



H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

- χ_2^Q almost independent of eB

- Hadron Resonance Gas model (HRG): Pressure arising from charged hadrons ($eB \neq 0$):

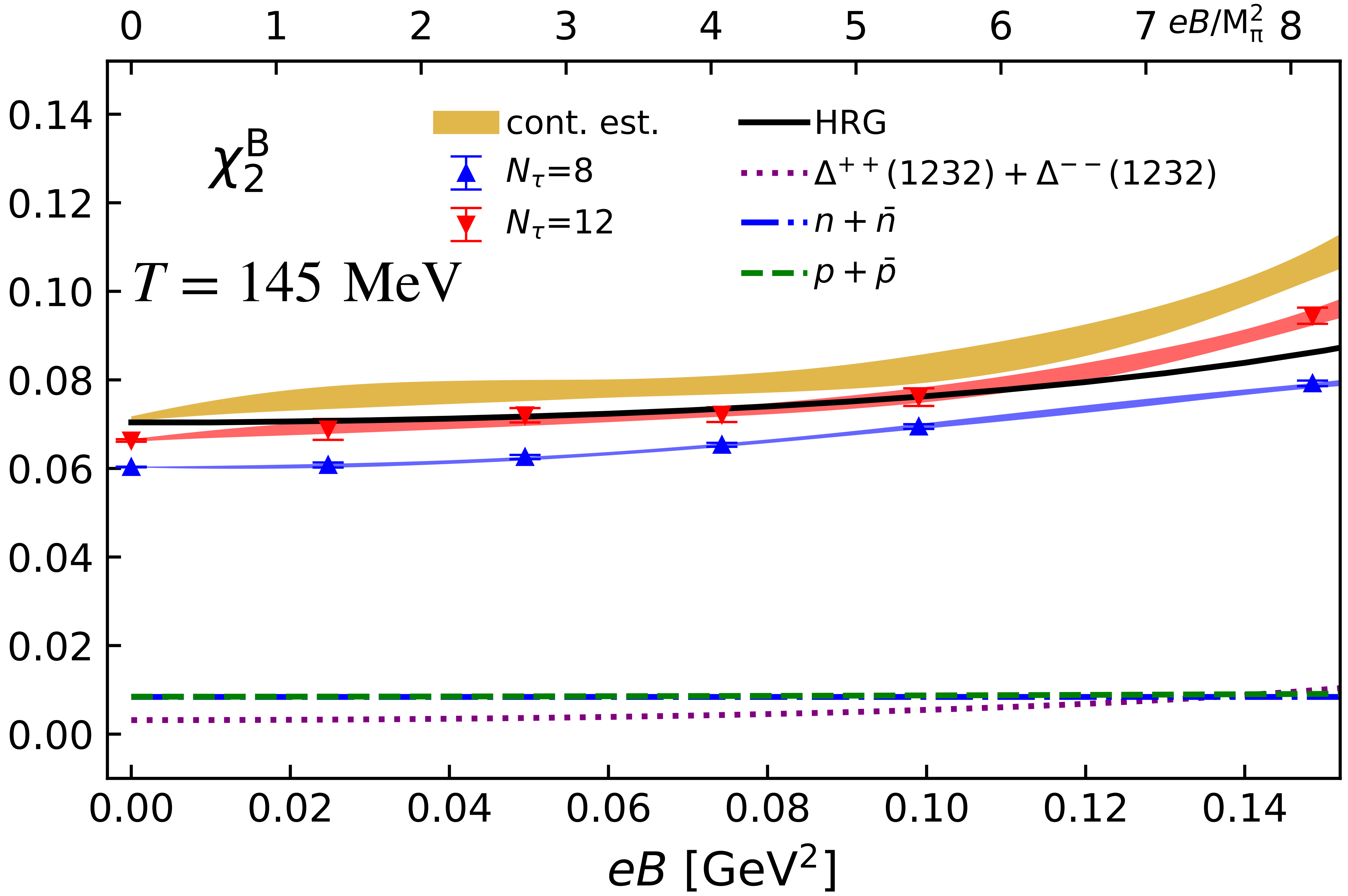
$$\frac{p_c^{M/B}}{T^4} = \frac{|q_i|\,B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1\left(\frac{n\varepsilon_0}{T}\right)$$

where $\varepsilon_0 = \sqrt{m_i^2 + 2\,|q_i|\,B\,(l + 1/2 - s_z)}$,

K_1 is the first-order modified Bessel function of the second kind

H.-T. Ding et al., Eur. Phys. J. A 57 (2021) 6, 202

Baryon number fluctuations at $T = 145\ \mathrm{MeV}$



H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

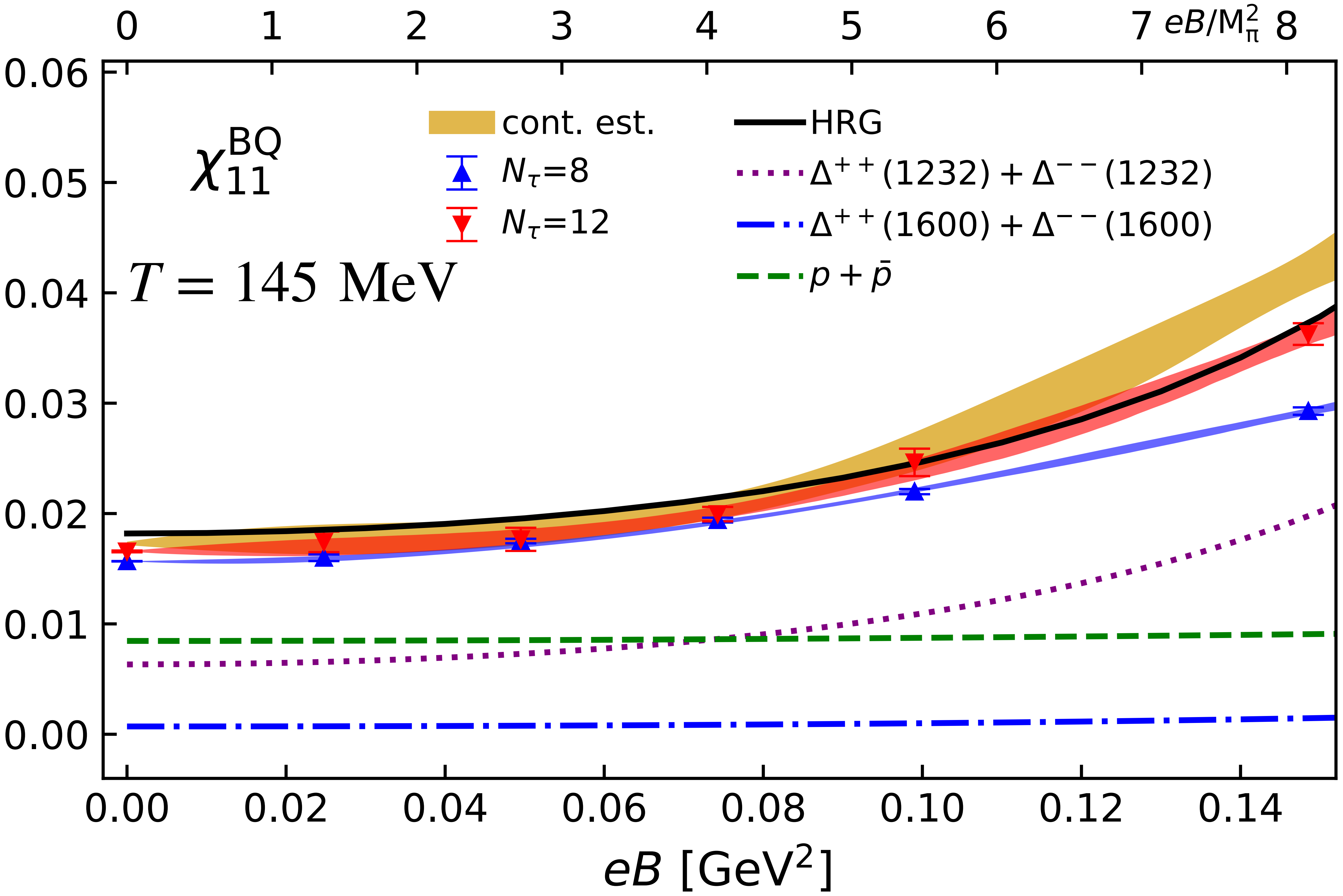
- χ_2^B increases $\sim 45\%$ at $eB \sim 8M_\pi^2$
- Hadron Resonance Gas model (HRG): Pressure arising from charged hadrons ($eB \neq 0$):

$$\frac{p_c^{M/B}}{T^4} = \frac{|q_i|\,B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1\left(\frac{n\varepsilon_0}{T}\right)$$

- χ_2^B receives contributions also from neutral baryons

H.-T. Ding et al., Eur. Phys. J. A 57 (2021) 6, 202

Baryon electric charge correlation at $T = 145$ MeV



H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

- ✦ χ_{11}^{BQ} increases $\sim$ **140%** at $eB \sim 8M_\pi^2$, **Magnetometer of QCD**
- ✦ The results of HRG model are consistent with LQCD up to $eB \sim 5M_\pi^2$
- ✦ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ give most of the contributions of magnetic field dependence of χ_{11}^{BQ}
- ✦ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ are not measurable in HIC experiments

Proxy construction based on the HRG

$\Delta^{++}(1232) \rightarrow p + \pi^+$: branching ratio almost **100%** !

HRG: Fluctuations expressed in terms of stable hadronic states:

$$\chi_{ijk}^{\mathrm{BQS}}\left(T, \hat{\mu}_{\mathrm{B}}, \hat{\mu}_{\mathrm{Q}}, \hat{\mu}_{\mathrm{S}}\right) = \sum_R \mathrm{B}_R^i \mathrm{Q}_R^j \mathrm{S}_R^k \frac{\partial^l p_R / T^4}{\partial \hat{\mu}_R^l}$$

net- B : $\boxed{\tilde{p}} + \tilde{n} + \tilde{\Lambda} + \tilde{\Sigma}^+ + \tilde{\Sigma}^- + \tilde{\Xi}^0 + \tilde{\Xi}^- + \tilde{\Omega}^-$

net- Q : $\boxed{\tilde{\pi}^+ + \tilde{K}^+ + \tilde{p}} + \tilde{\Sigma}^+ - \tilde{\Sigma}^- - \tilde{\Xi}^- - \tilde{\Omega}^-$

net- S : $\boxed{\tilde{K}^+} + \tilde{K}^0 - \tilde{\Lambda} - \tilde{\Sigma}^+ - \tilde{\Sigma}^- - 2\tilde{\Xi}^0 - 2\tilde{\Xi}^- - 3\tilde{\Omega}^-$

$\mathrm{B}_R, \mathrm{Q}_R, \mathrm{S}_R$ are the baryon number, electric charge and strangeness of the species R

R. Bellwied et al., Phys. Rev. D 101, 034506 (2020)

In HIC, fluctuations are related to the variance or covariance of net-multiplicity for Identified π, K, p

STAR, Phys.Rev.C 100, 014902 (2019); STAR, Phys.Rev.C 105, 029901 (2019)

$\sigma_{Q^{\mathrm{PID}},p}^{1,1}$ as proxy for χ_{11}^{BQ}:

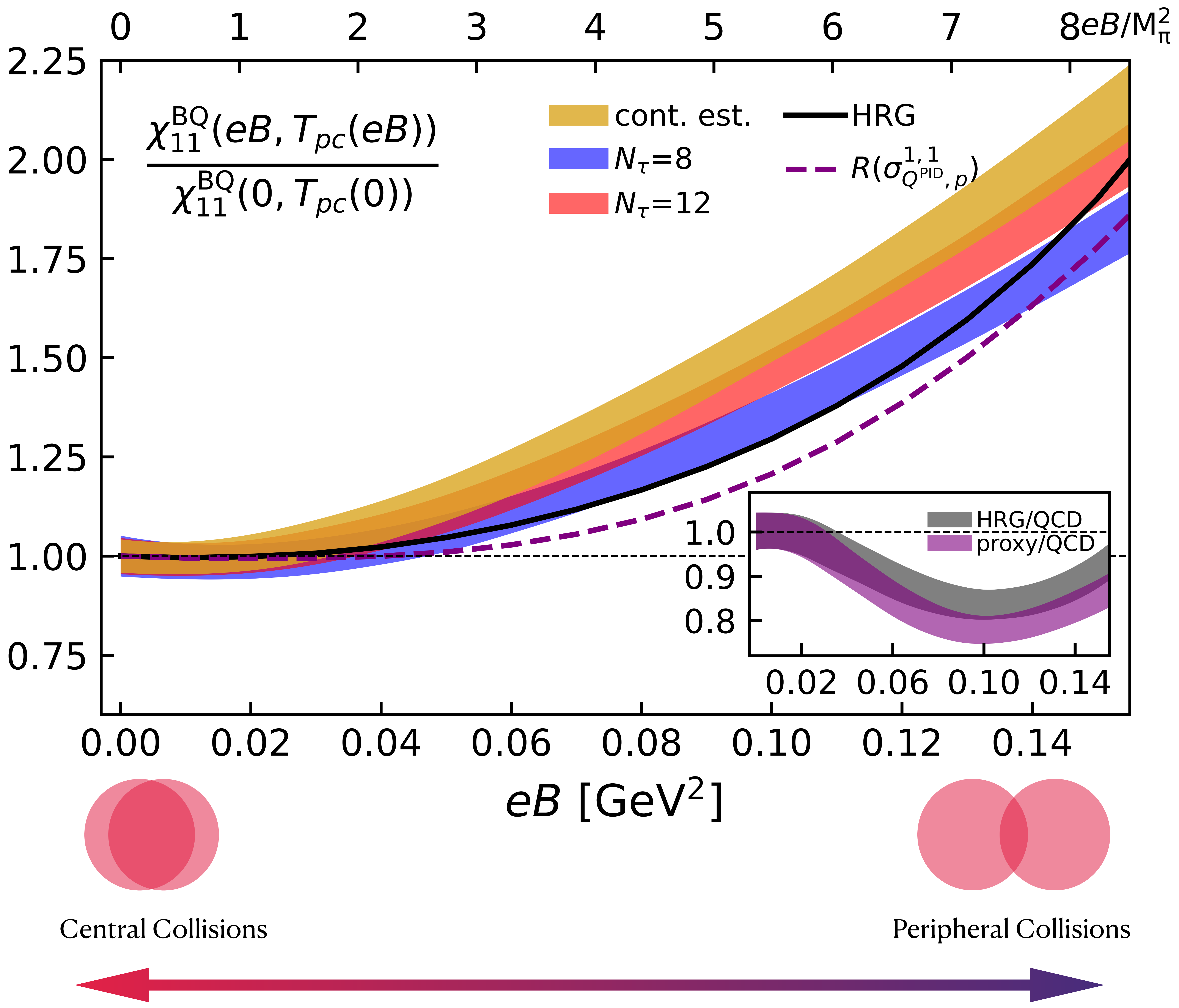
$$\sigma_{Q^{\mathrm{PID}},p}^{1,1} = \sum_R \left(P_{R\rightarrow\tilde{p}}\right)\left(P_{R\rightarrow Q^{\mathrm{PID}}}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}_R^2} + \frac{\partial^2 p_{\tilde{p}}/T^4}{\partial\hat{\mu}_{\tilde{p}}^2}$$

where $P_{R\rightarrow i}$ represents number of particle *i* produced by resonance *R* after the **entire decay chain**,

Q^{PID} : $\tilde{\pi}^+$, $\tilde{K}^+$, $\tilde{p}$

In proxy, contributions from all resonance decays are considered!

Proxy for χ_{11}^{BQ} along the transition line



H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

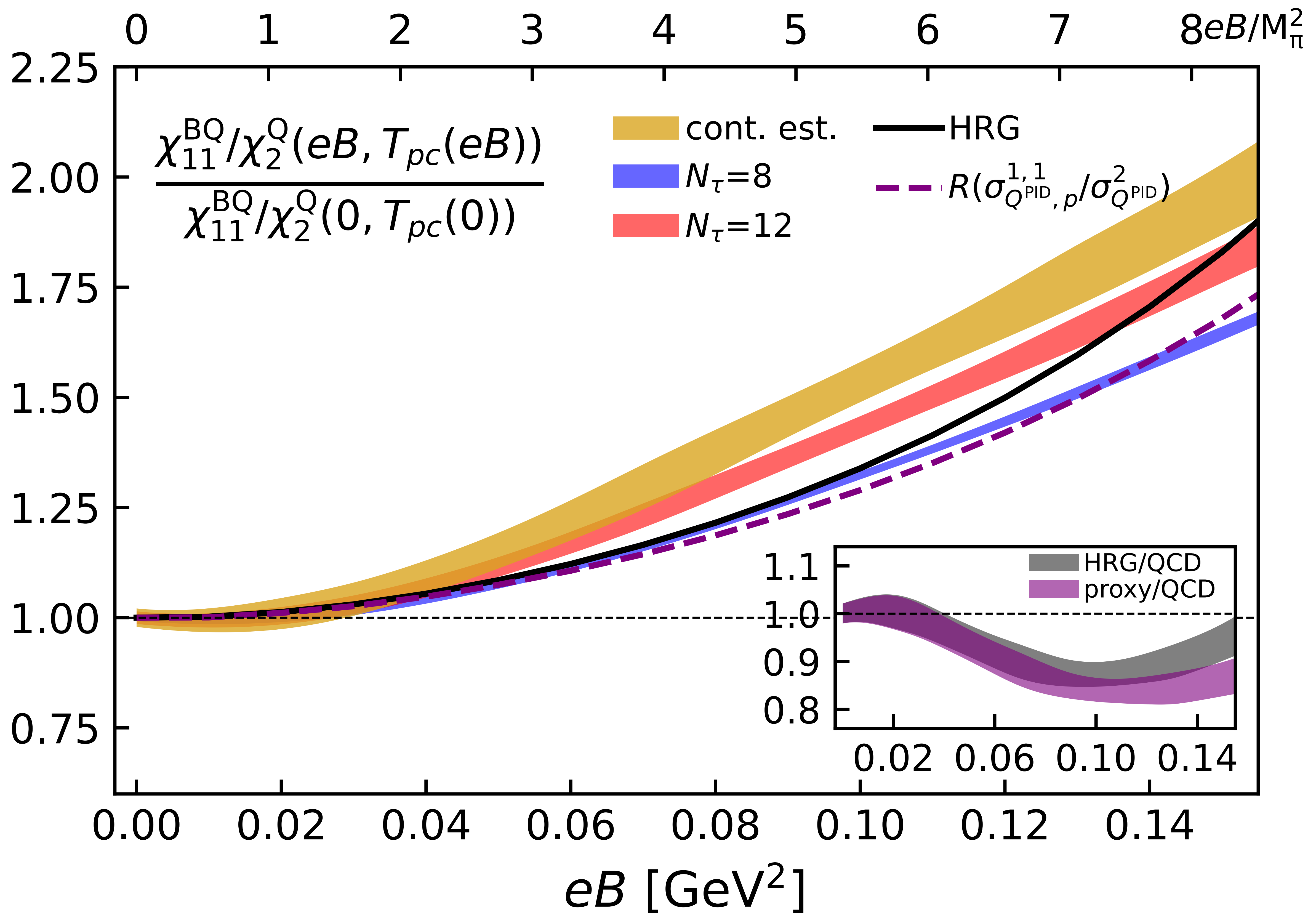
✦At $eB \simeq 8M_\pi^2$, ratio of $\chi_{11}^{\mathrm{BQ}} \sim$ **2.1**

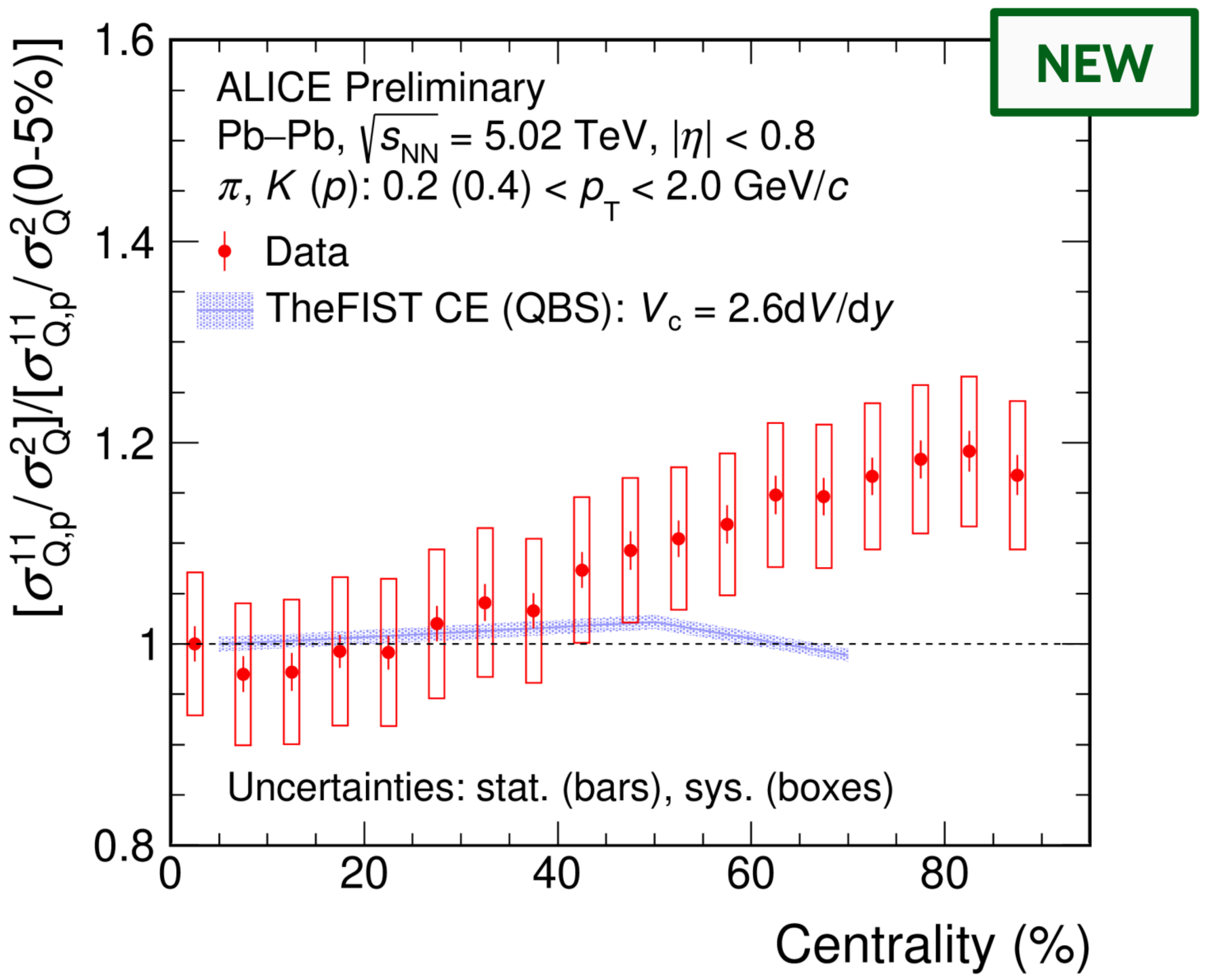
$$R(\sigma^{1,1}_{Q^{\mathrm{PID}},p}) = \sigma^{1,1}_{Q^{\mathrm{PID}},p}(eB)/\sigma^{1,1}_{Q^{\mathrm{PID}},p}(eB=0)$$

✦The proxy $R(\sigma^{1,1}_{Q^{\mathrm{PID}},p})$ can represent **80～85%** of the LQCD results

✦$R(\sigma^{1,1}_{Q^{\mathrm{PID}},p})$ is a reasonable proxy for χ_{11}^{BQ}

LQCD meets experiment





ALI-PREL-573205

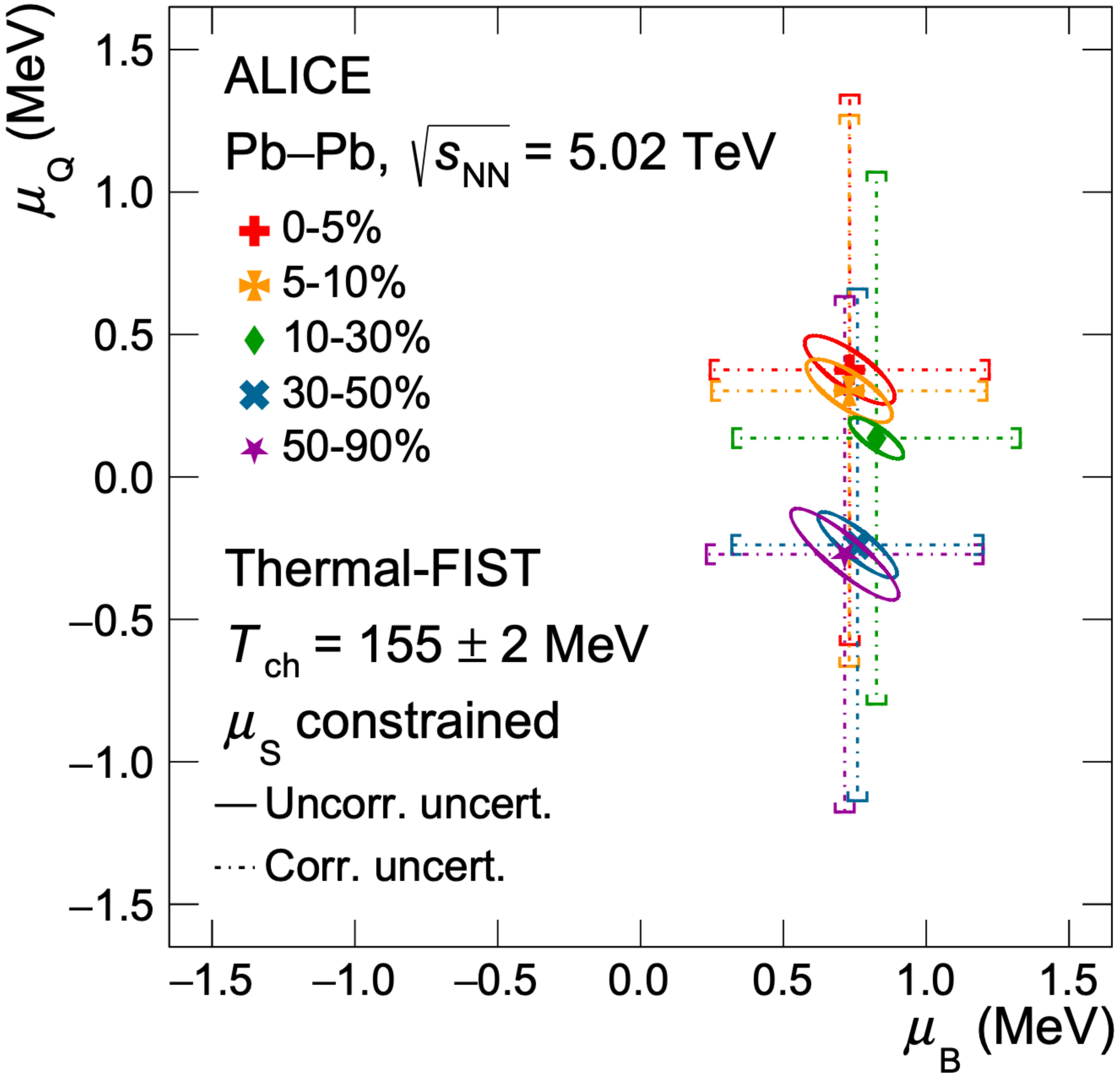
S. Saha for the ALICE collaboration @ SQM 2024

Central Collisions

Peripheral Collisions

H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

Electric charged chemical potential over baryon chemical potential



ALICE, Phys. Rev. Lett. 133, 092301 (2024)

- μ_Q/μ_B can be obtained from the thermal statistics fits to particle yields

- μ_Q/μ_B also can be obtained from fluctuations of B, Q, S

$$\mu_Q/\mu_B = \boxed{q_1} + q_3\ \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

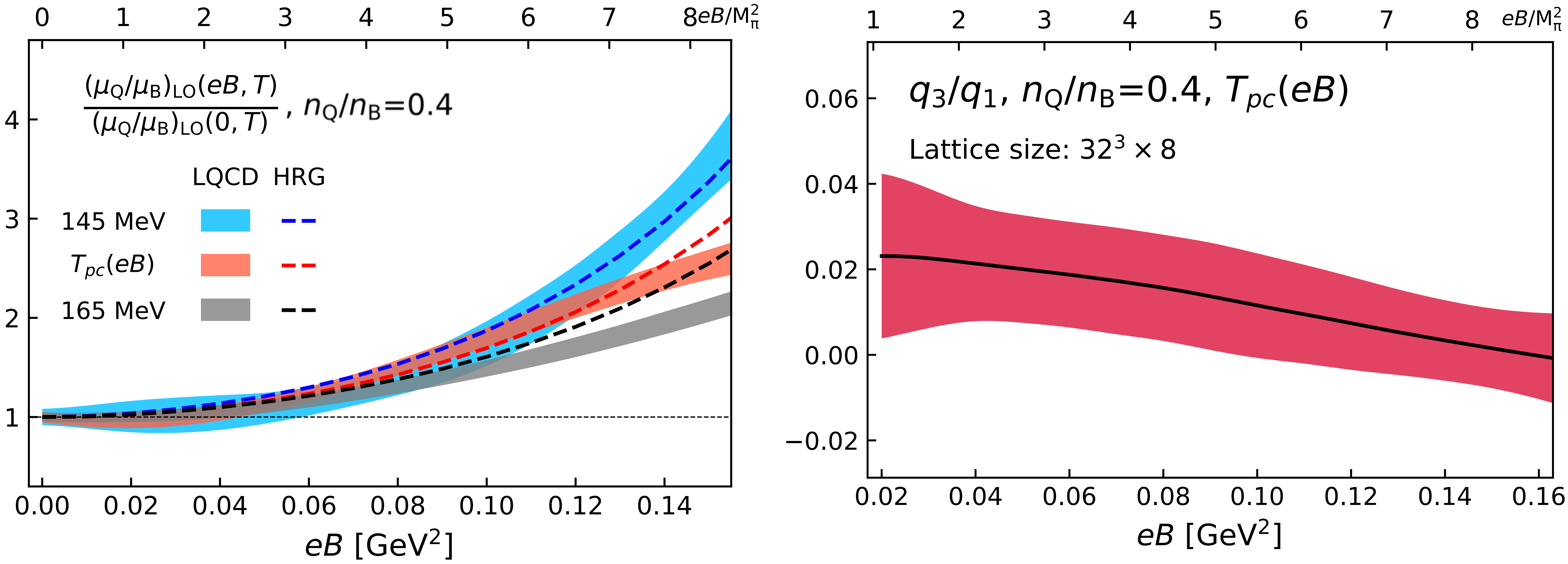
$$\boxed{q_1} = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$

with constraints: $r = n_Q/n_B,\ n_S = 0$

HotQCD, Phys. Rev. Lett. 109 (2012) 192302

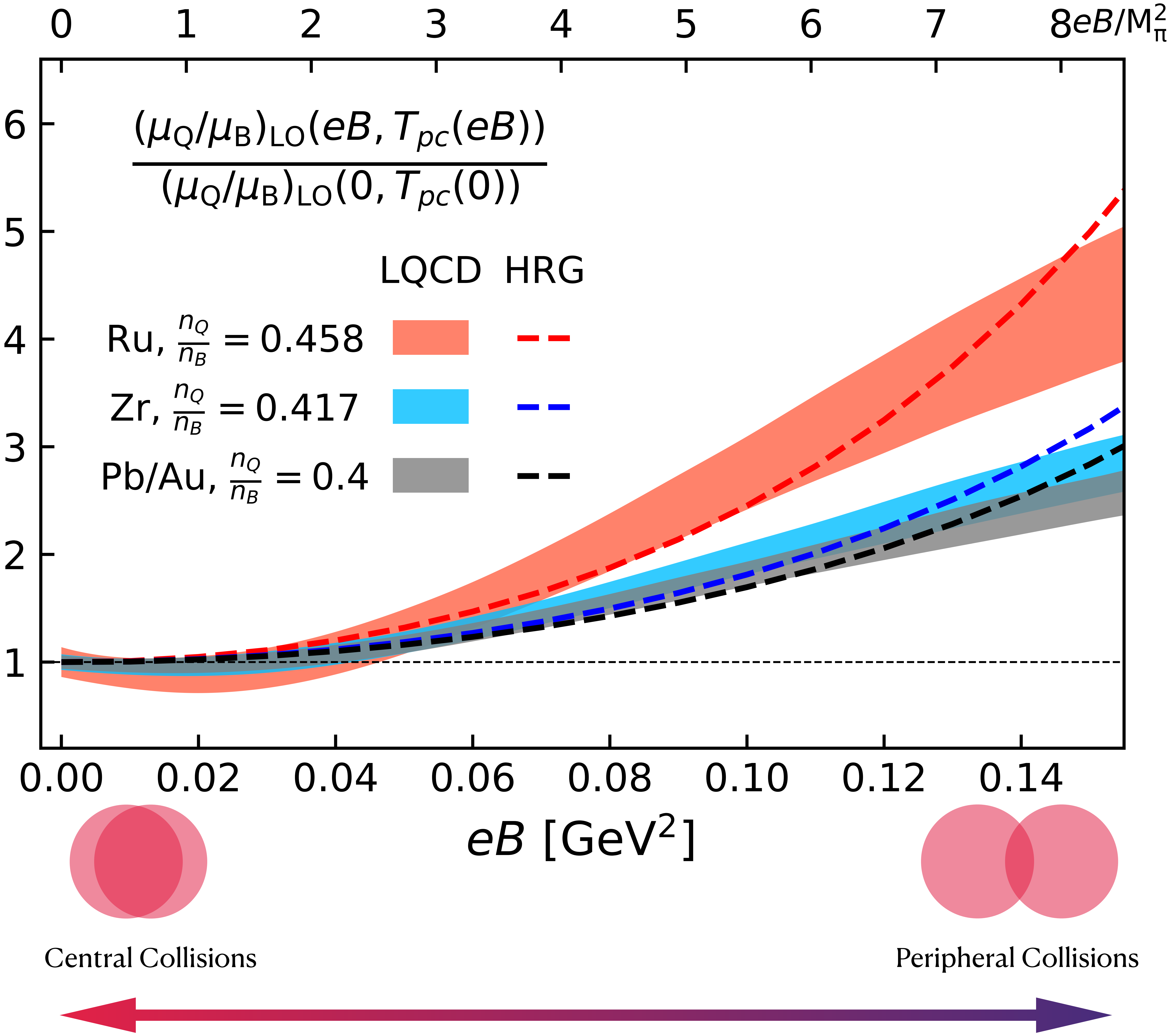
Dependence of μ_Q/μ_B on the magnetic field

$$\mu_Q/\mu_B = q_1 + q_3\ \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$



H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

$(\mu_Q/\mu_B)_{LO}$ in different collision system



H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

$^{96}_{44}\mathrm{Ru} + ^{96}_{44}\mathrm{Ru}$: r = 0.458

$^{96}_{40}\mathrm{Zr} + ^{96}_{40}\mathrm{Zr}$: r = 0.417

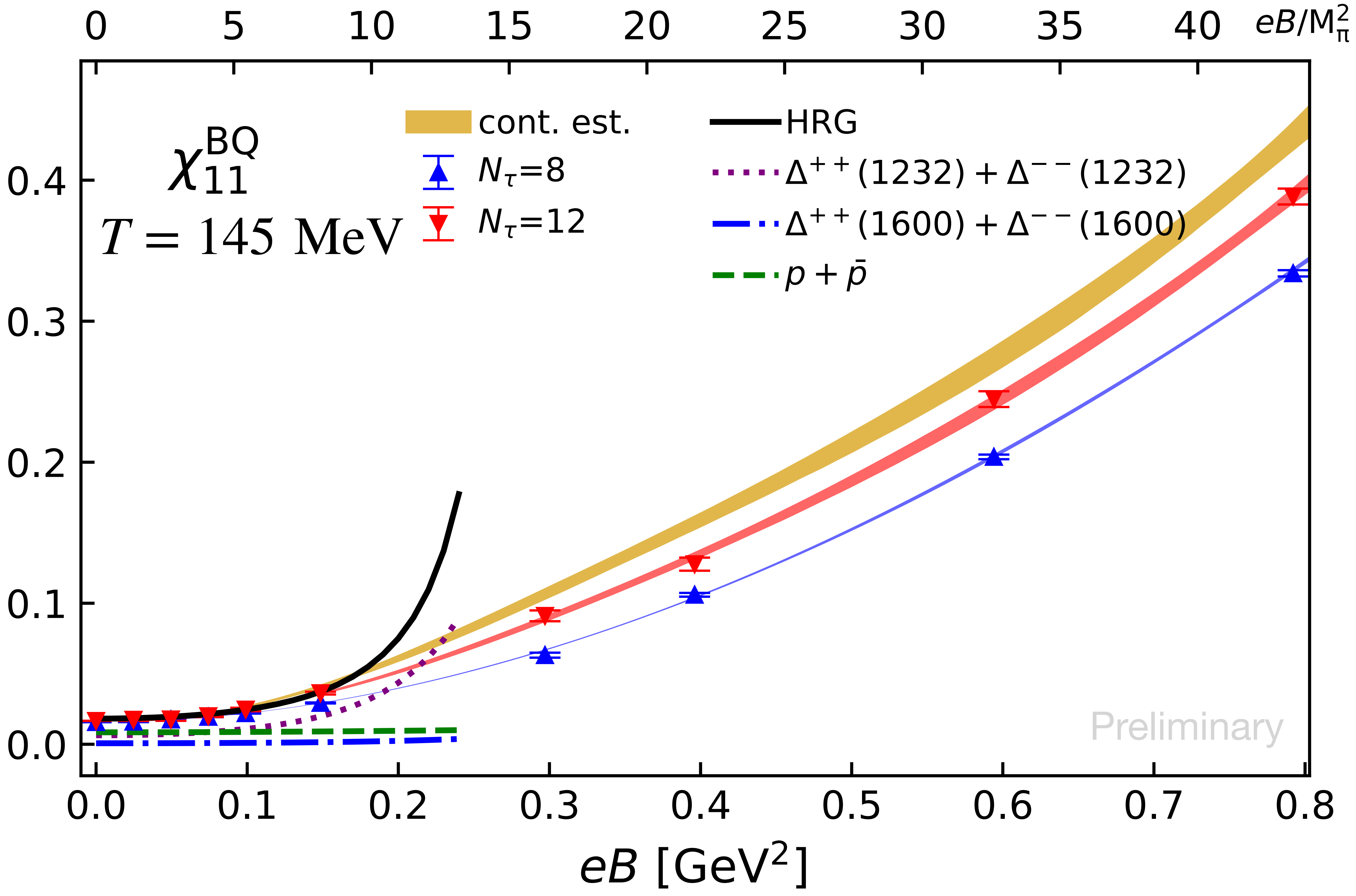
$^{208}_{82}\mathrm{Pb} + ^{208}_{82}\mathrm{Pb}$: r = 0.4

✦ At $eB \simeq 8M_\pi^2$,

Ratio of $(\mu_Q/\mu_B)_{LO}$ for Pb, Au, Zr ~2.4

Ratio of $(\mu_Q/\mu_B)_{LO}$ for **Ru ~ 4**

The breaking down of HRG in very strong magnetic fields



H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li work in progress

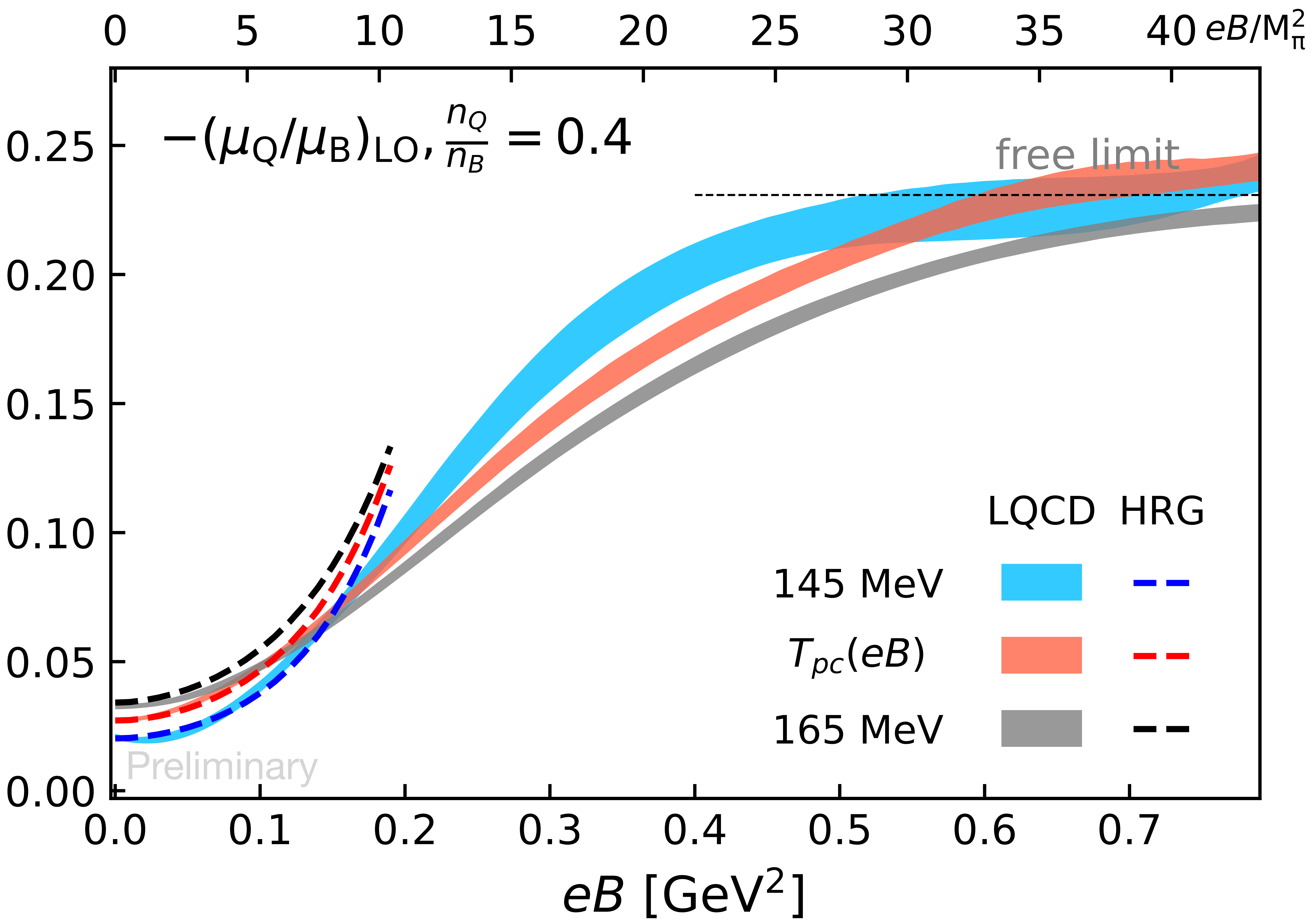
In very strong magnetic fields:

- χ_{11}^{BQ} keeps increasing
- HRG are not applicable

$$\varepsilon_0 = \sqrt{m_i^2 + 2\left|q_i\right| B\left(l + 1/2 - s_z\right)}$$

H.-T. Ding et al., Eur. Phys. J. A 57 (2021) 6, 202

$(\mu_Q/\mu_B)_{LO}$ in very strong magnetic field



H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li work in progress

$$\mu_Q/\mu_B = \boxed{q_1} + q_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$\boxed{q_1} = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$

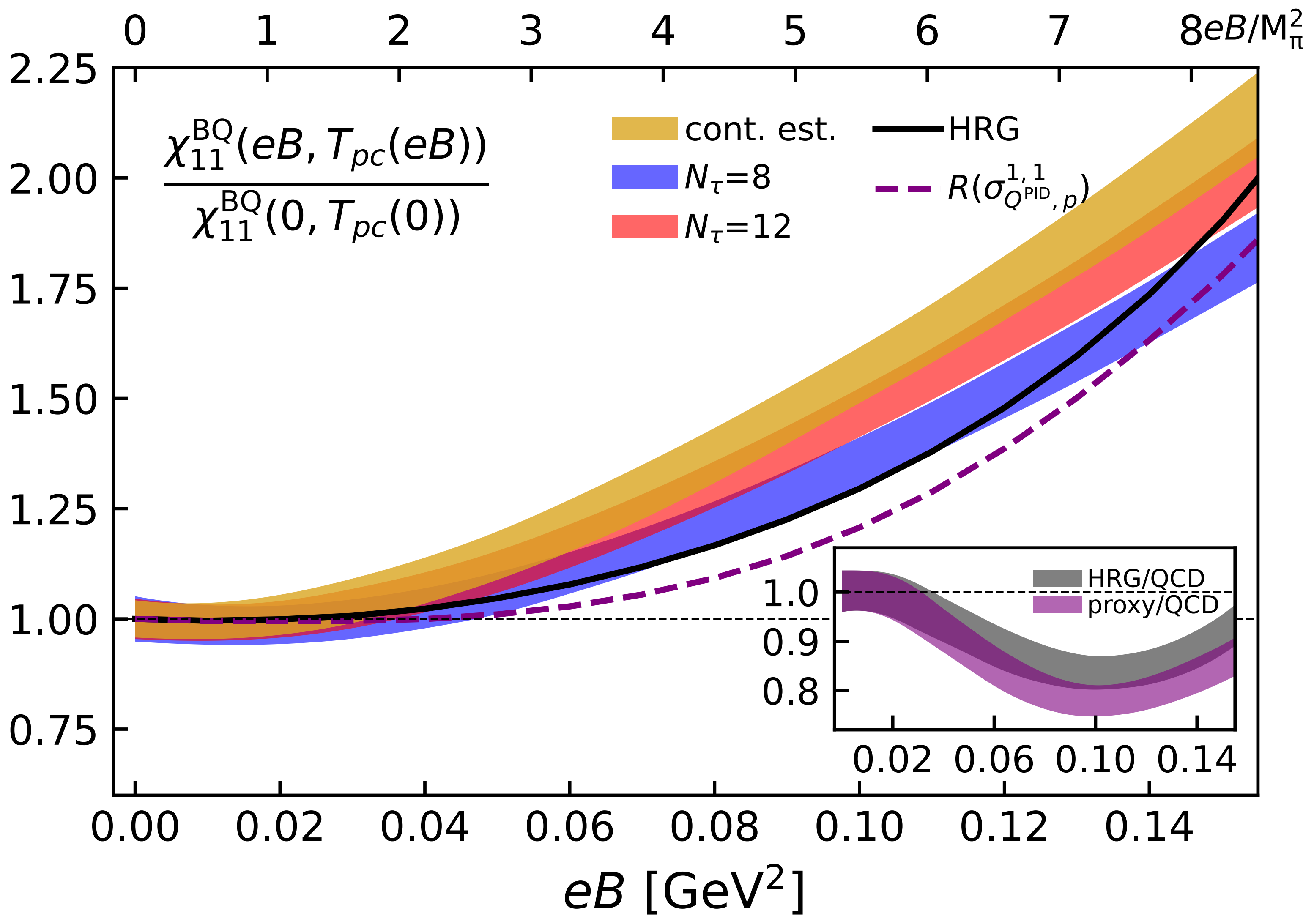
$r = n_Q/n_B = 0.4$ for Pb/Au collision

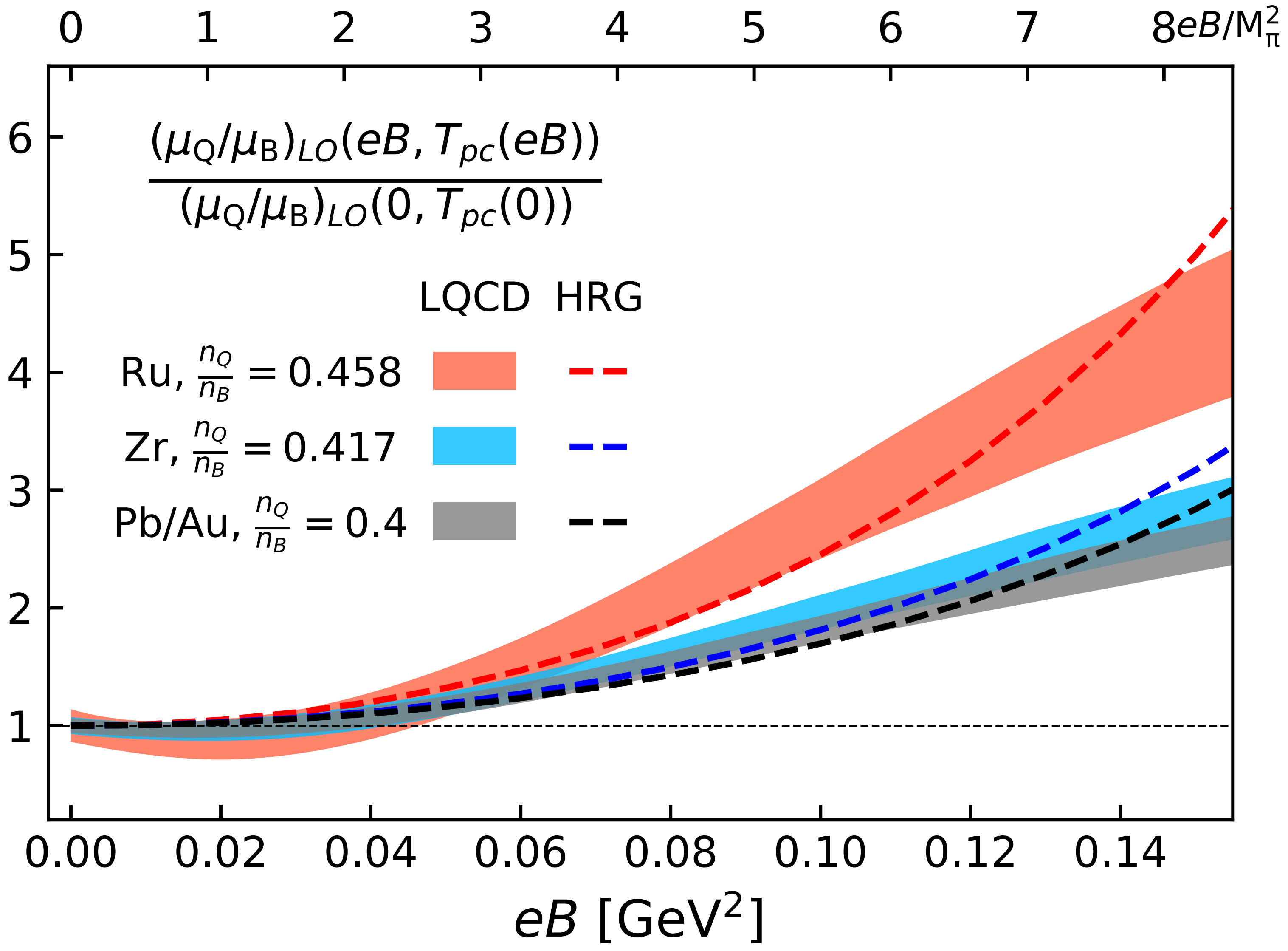
- ✦ q_1 approaching saturates in very strong magnetic field

Also see Arpith's talk @Thursday, 14:30

Summary

- QCD benchmarks are provided for the 2nd order fluctuations of conserved charges based on LQCD computation on $N_\tau = 8$ and 12 lattices
- χ_{11}^{BQ} is strongly affected by eB, and a reasonable proxy is provided for measurement in HIC
- The $\mu_\mathrm{Q}/\mu_\mathrm{B}$ depends significantly on the magnetic field and is sensitive to the initial $n_\mathrm{Q}/n_\mathrm{B}$





Thank you!

Backup

Lattice QCD in strong magnetic fields

B pointing along the z direction

$$u_x\left(n_x,n_y,n_z,n_\tau\right)=\begin{cases}\exp\left[-iqa^2BN_xn_y\right] & \left(n_x=N_x-1\right)\\ 1 & \text{(otherwise)}\end{cases}$$

$$u_y\left(n_x,n_y,n_z,n_\tau\right)=\exp\left[iqa^2Bn_x\right]$$

$$u_z\left(n_x,n_y,n_z,n_\tau\right)=u_t\left(n_x,n_y,n_z,n_\tau\right)=1$$

No sign problem !

Quantization of the magnetic field

$$q_u = 2/3\ e$$
$$q_d = -1/3\ e$$
$$q_s = -1/3\ e$$

$$\longrightarrow \quad eB=\frac{6\pi N_b}{N_xN_y}a^{-2}$$

a is changed to get the targeted T, $T=\dfrac{1}{aN_\tau}$

- Statistics($eB \neq 0$): N_τ=8: ~40000 ($N_{\rm rv}$: 603)
  N_τ=12: ~5000 ($N_{\rm rv}$: 102 ~ 705)
  N_τ=16: ~3000 ($N_{\rm rv}$: 603)

Landau gauge
G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

Proxy in experiment

✦ Conserved charges susceptibilities in experiment:

$$\chi^2_\alpha = \frac{1}{VT^3}\kappa^2_\alpha, \quad \chi^{1,1}_{\alpha,\beta} = \frac{1}{VT^3}\kappa^{1,1}_{\alpha,\beta}$$

the second-order cumulants(κ) are the variance or covariance(σ) of the net-multiplicity N:

$$\kappa^2_\alpha = \sigma^2_\alpha = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)^2\rangle$$

$$\kappa^{1,1}_{\alpha,\beta} = \sigma^{1,1}_{\alpha,\beta} = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)(\delta N_\beta - \langle\delta N_\beta\rangle)\rangle$$

with $\delta N_\alpha = N_{\alpha^+} - N_{\alpha^-}$ and $\alpha, \beta = p, Q^{PID}, k$

- p : a proxy for the net-baryon
- k : a proxy for the net-strangeness
- Q^{PID}: identified π, k and p

STAR, Phys.Rev.C 100 (2019) 1, 014902

$$\sigma^{1,1}_{Q^{\mathrm{PID}},p} = \sigma^2_p + \sigma^{1,1}_{p,\pi} + \sigma^{1,1}_{p,K}$$

$$\sigma^2_p = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{p}}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}^2_R}$$

$$\sigma^{1,1}_{p,\pi} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{\pi}^+}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}^2_R}$$

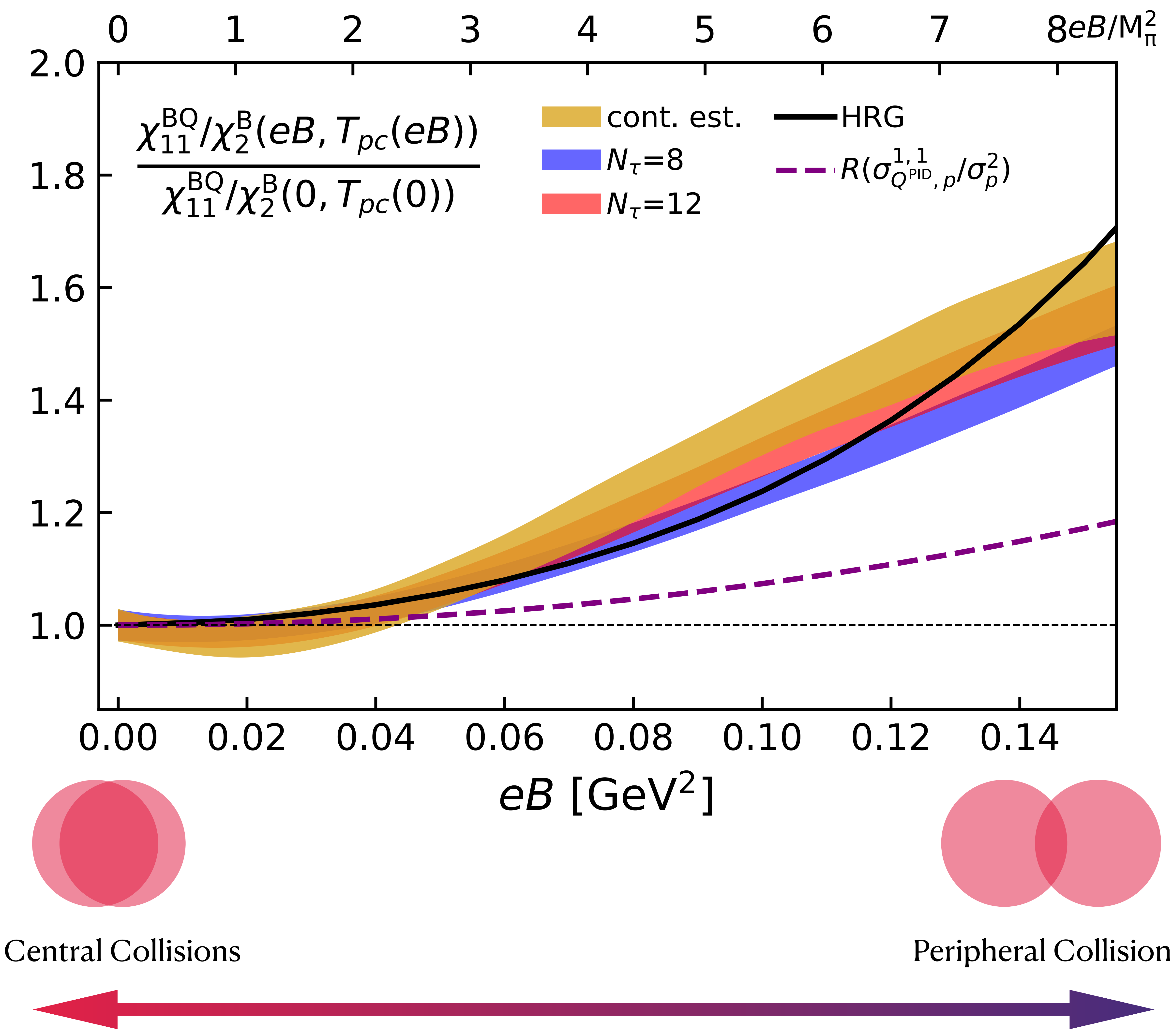
$$\sigma^{1,1}_{p,K} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{K}^+}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}^2_R}$$

where $P_{R\to i} = \sum_\alpha \mathrm{N}^\alpha_{R\to i} n^R_{i,\alpha}$

$n^R_{i,\alpha}$: numbers of i produced by R in decay channel α

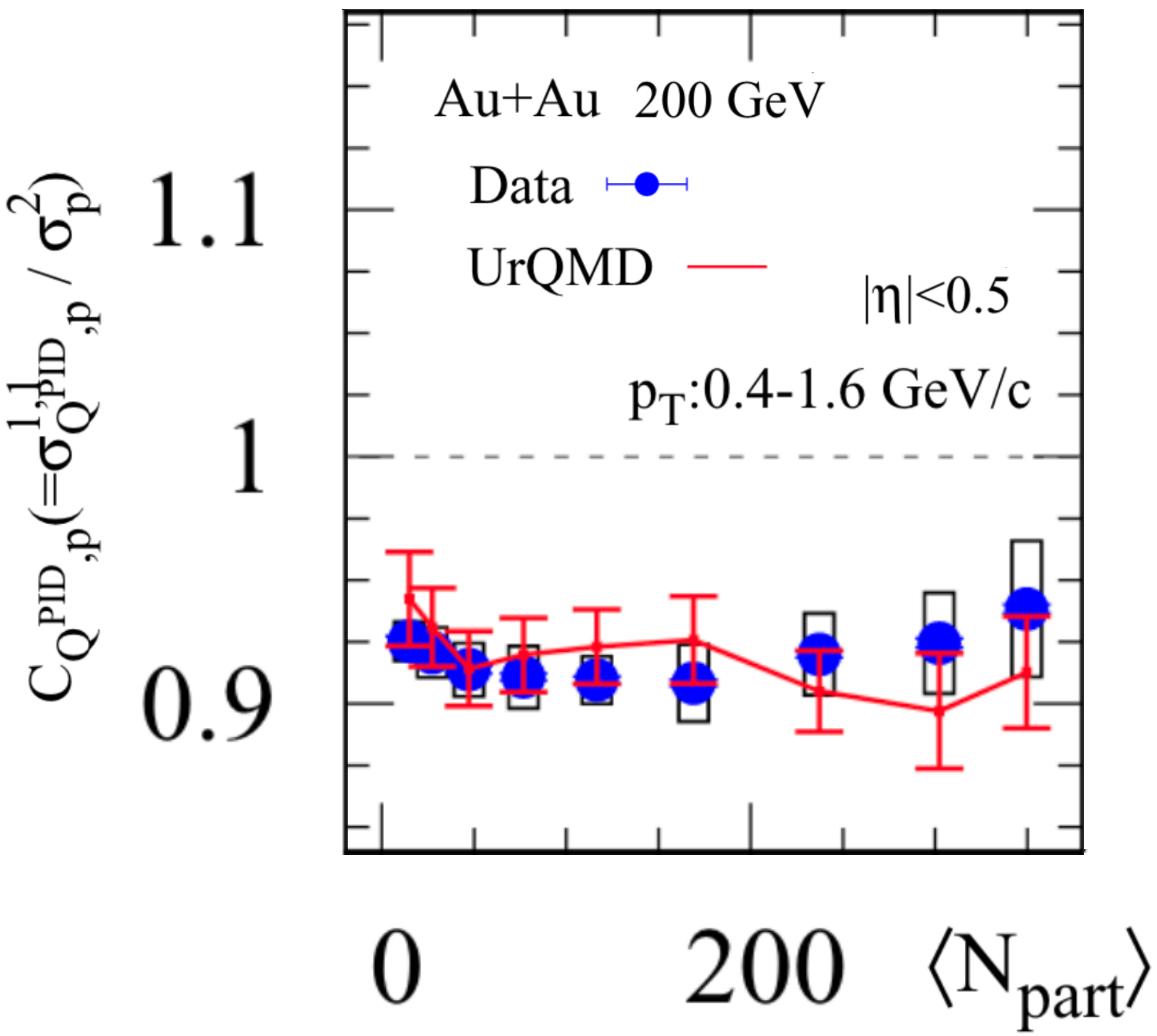
$\mathrm{N}^\alpha_{R\to i}$: Branching ratio of channel α

χ_{11}^{BQ}/χ_2^B along the transition line



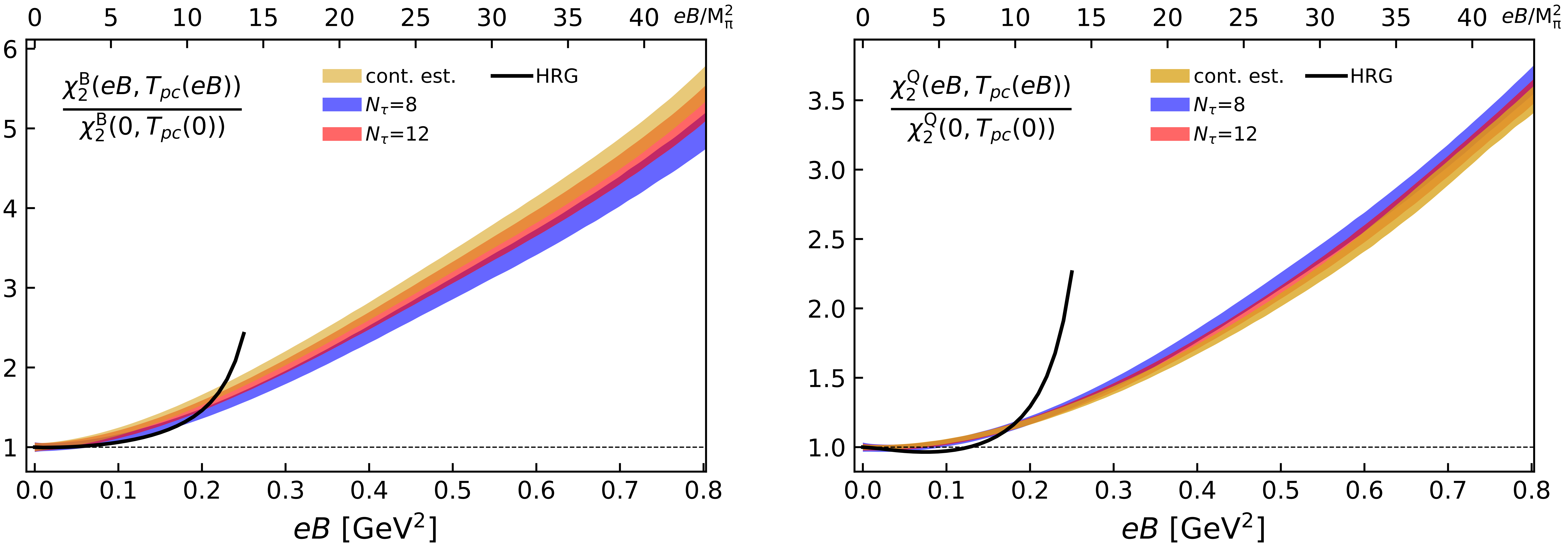
H.-T. Ding, J.-B. Gu et al., Phys. Rev. Lett. 132, 201903 (2024)

Proxy for χ_{11}^{BQ}/χ_2^B



STAR, Phys. Rev. C 105 (2022) 029901(E)

Diagonal fluctuations in very strong magnetic fields



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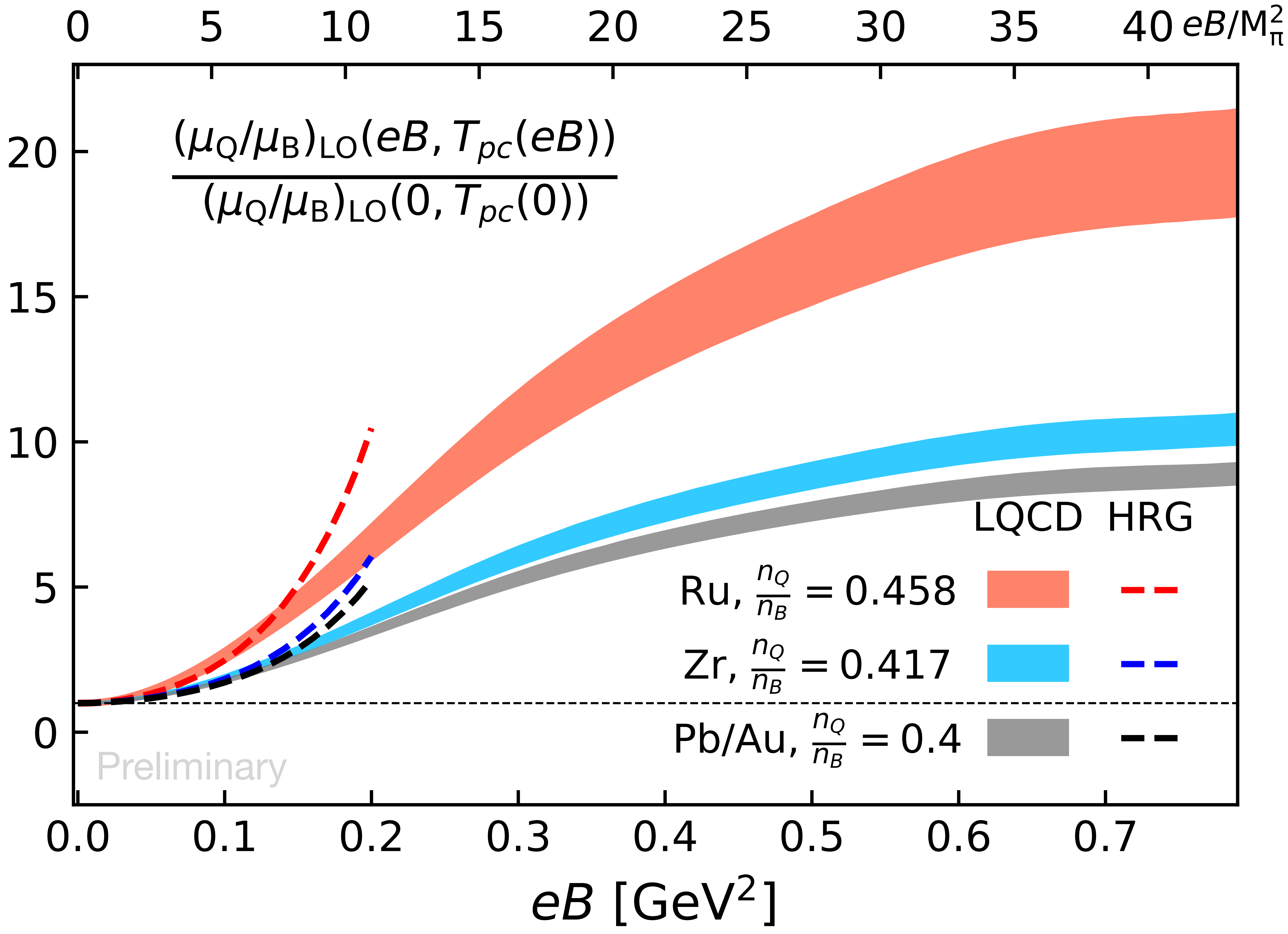
Dependence of $(\mu_Q/\mu_B)_{LO}$ on the magnetic field in the large magnetic field range



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$$\mu_Q/\mu_B = \boxed{q_1} + q_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$\boxed{q_1} = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$

$$r = n_Q/n_B$$

✦At $eB \simeq 40M_\pi^2$,

Ratio of $(\mu_Q/\mu_B)_{LO}$ for Pb, Au, Zr ~ 9

Ratio of $(\mu_Q/\mu_B)_{LO}$ for **Ru ~ 20**