Finite temperature QCD from lattice simulations with overlap fermions

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JÜLICH
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New developments in studies of the QCD phase diagram, Trento, 2024

Chiral fermions on the lattice

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Nielsen-Ninomiya «no-go» theorem:

- Lattice chiral fermions $\Longrightarrow$ fermion doubling:

  equal number of left- and right- handed particles

[Nielsen and Ninomiya, 1981]

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[Neuberger, 1998]

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Also [Y. Aoki, Wed, 10.50]

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- Keep $Q = \mathrm{const}$ ($Q = 0$)
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- Everything is preliminary!

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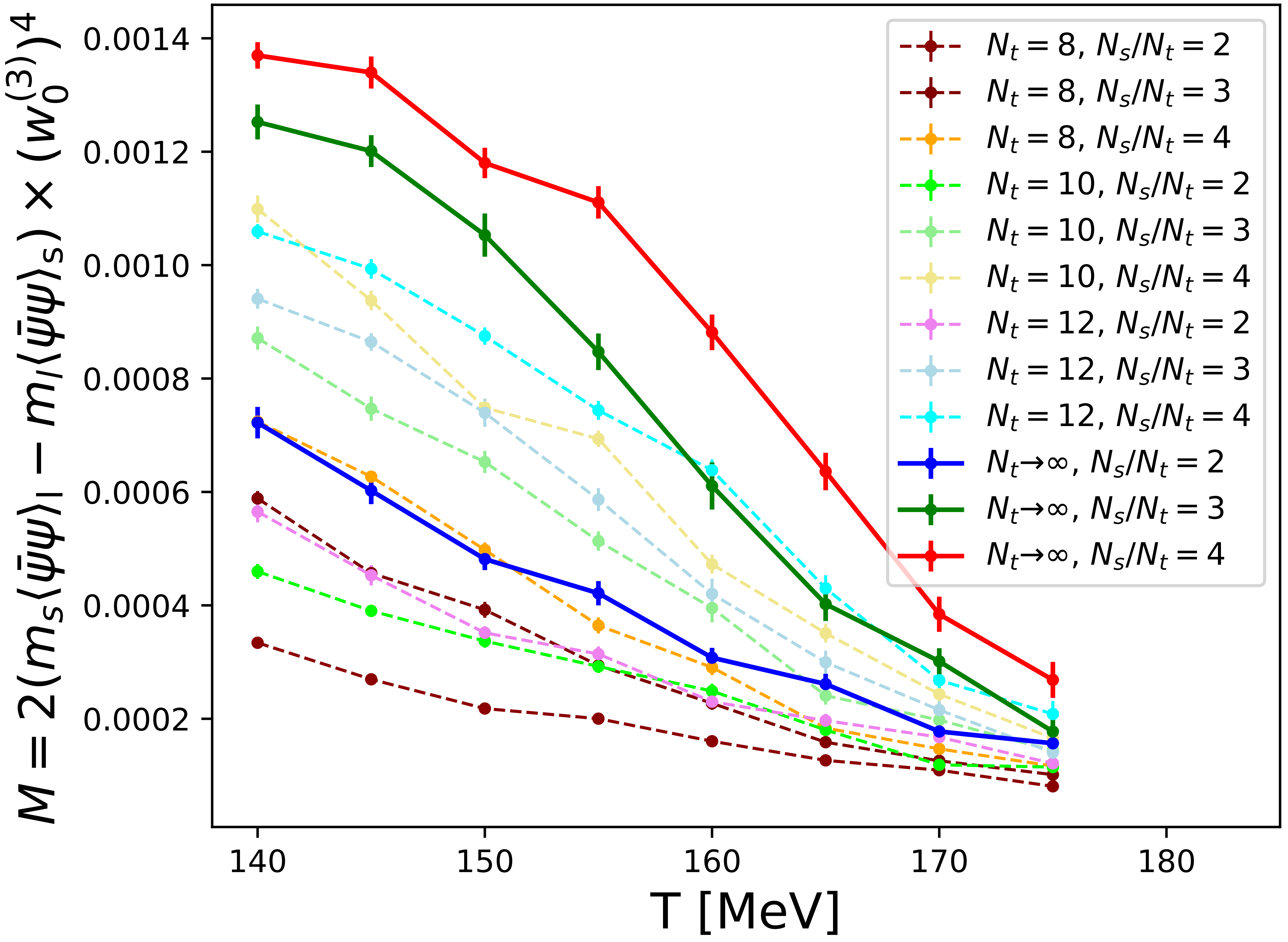
Chiral condensate

$Q = 0$ sector

- $M = 2\left(m_s\langle\bar{\psi}\psi\rangle_l - m_l\langle\bar{\psi}\psi\rangle_s\right)$
- Large cutoff effects and FV effects
- $T_{pc} \sim 160$ MeV
- $N_s/N_t = 2$ is completely off



Chiral susceptibility

$Q = 0$ sector

$$\chi_M = m\partial_m M$$

- $N_s/N_t = 2$ is completely off
- Same for staggered [Borsanyi et al., 2024]

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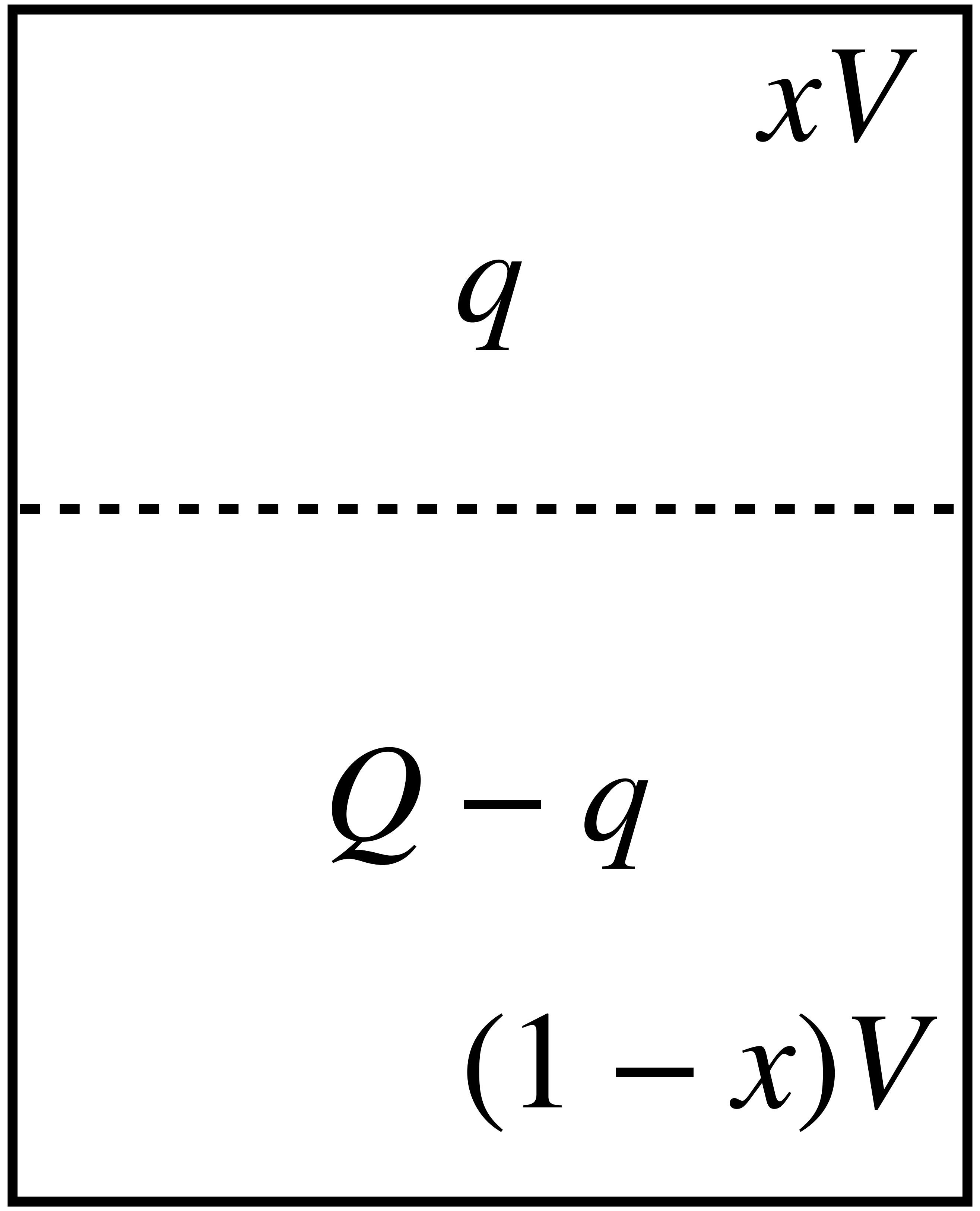
  - Weights Z_Q/Z_0 (or topological susceptibility χ) - is also possible

Topological susceptibility from simulations at fixed Q

Slab method

$$Q, V \equiv V_4$$



$$p(q, Q - q) \propto p_1(q) p_2(Q - q) \propto$$

$$e^{-\frac{q^2}{2\chi V x}} e^{-\frac{(Q-q)^2}{2\chi V(1-x)}} \propto e^{-\frac{1}{2\chi V}\frac{q'^2}{x(1-x)}}$$

$$q' = q - xQ$$

$$\langle q \rangle = xQ$$

$$\langle q'^2 \rangle = \langle q^2 \rangle - x^2 Q^2 \propto \chi V x(1-x)$$

Up to boundary effects: $V \to \infty$

[Bietenholz, de Forcrand and Gerber, 2015]

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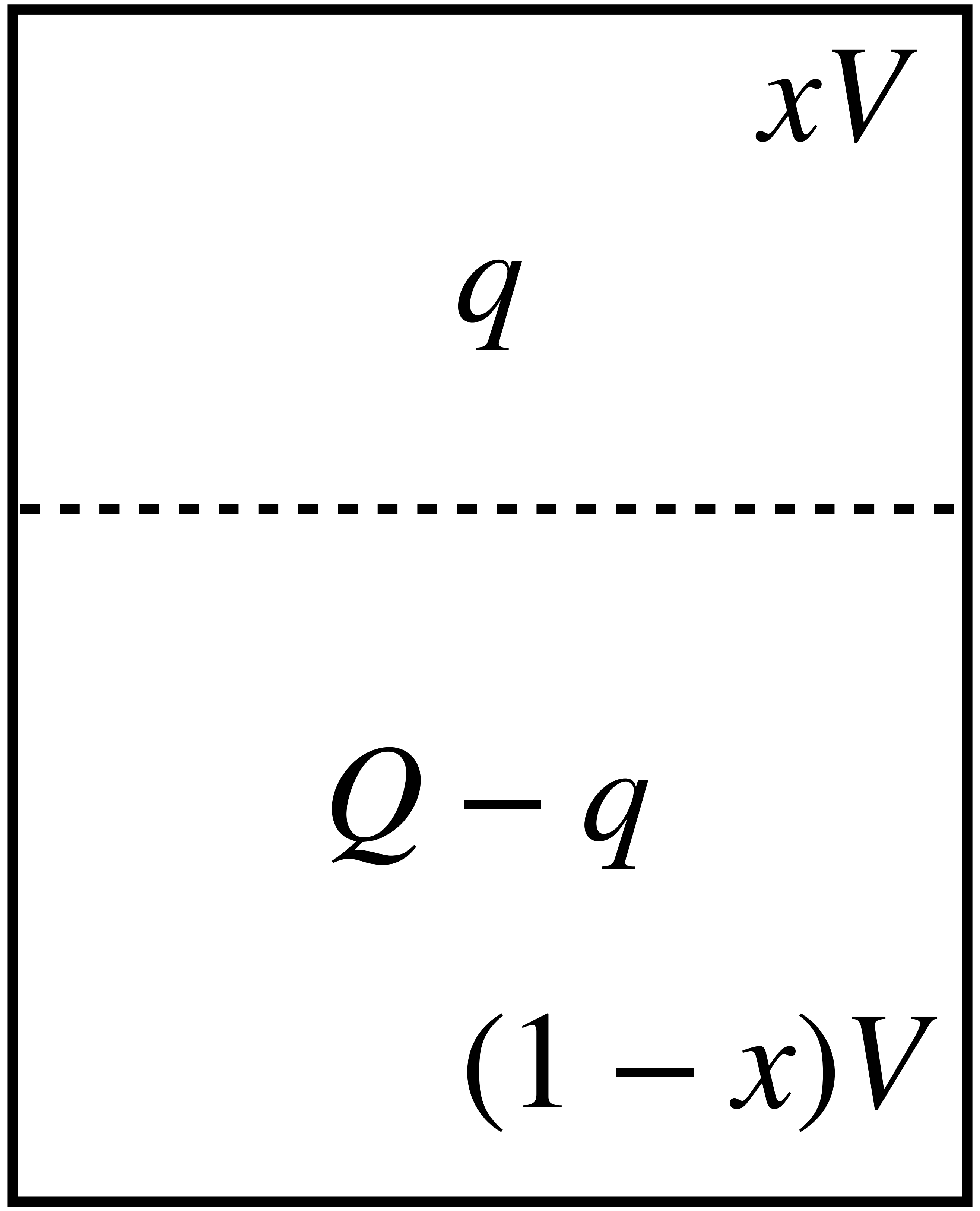
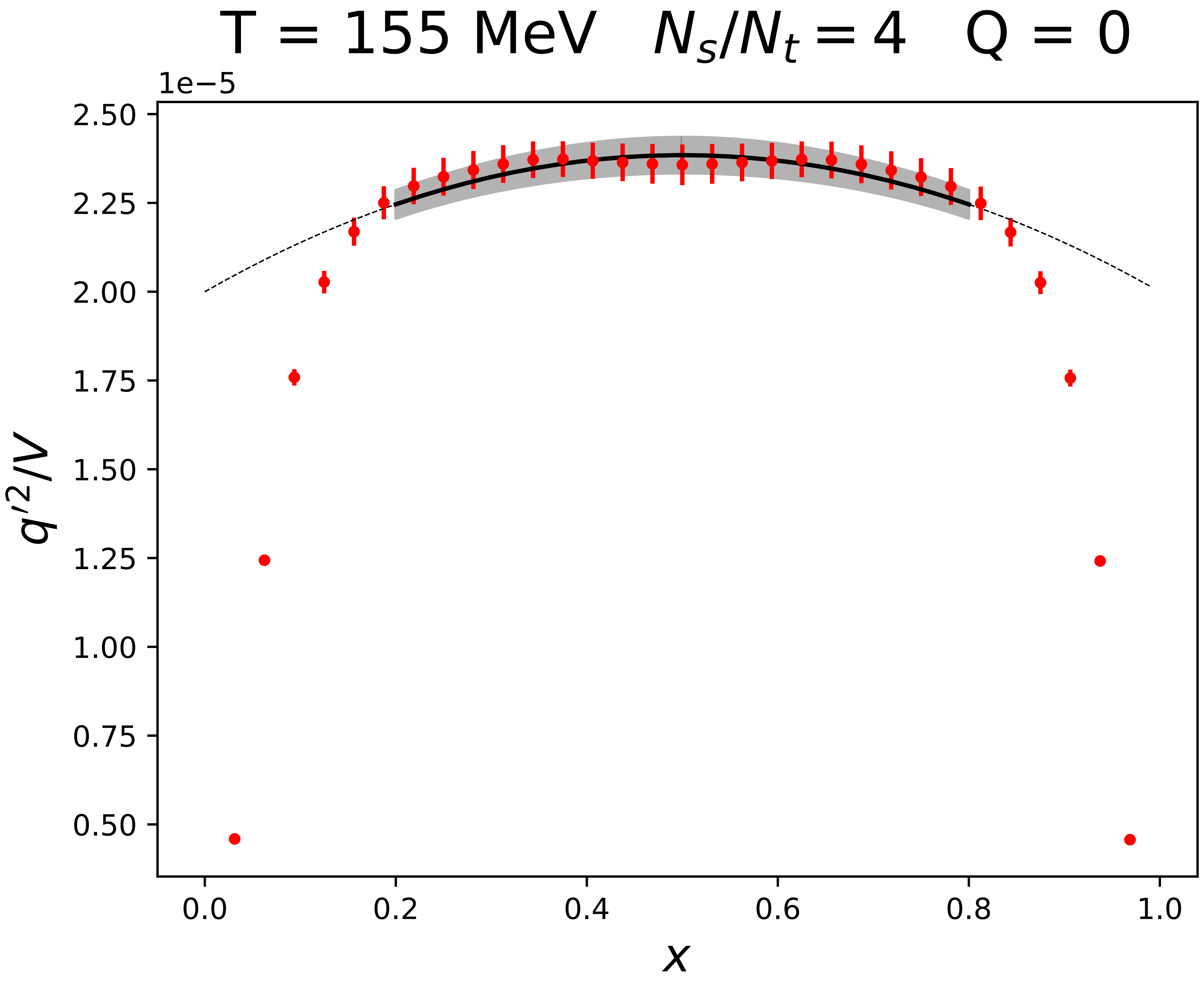
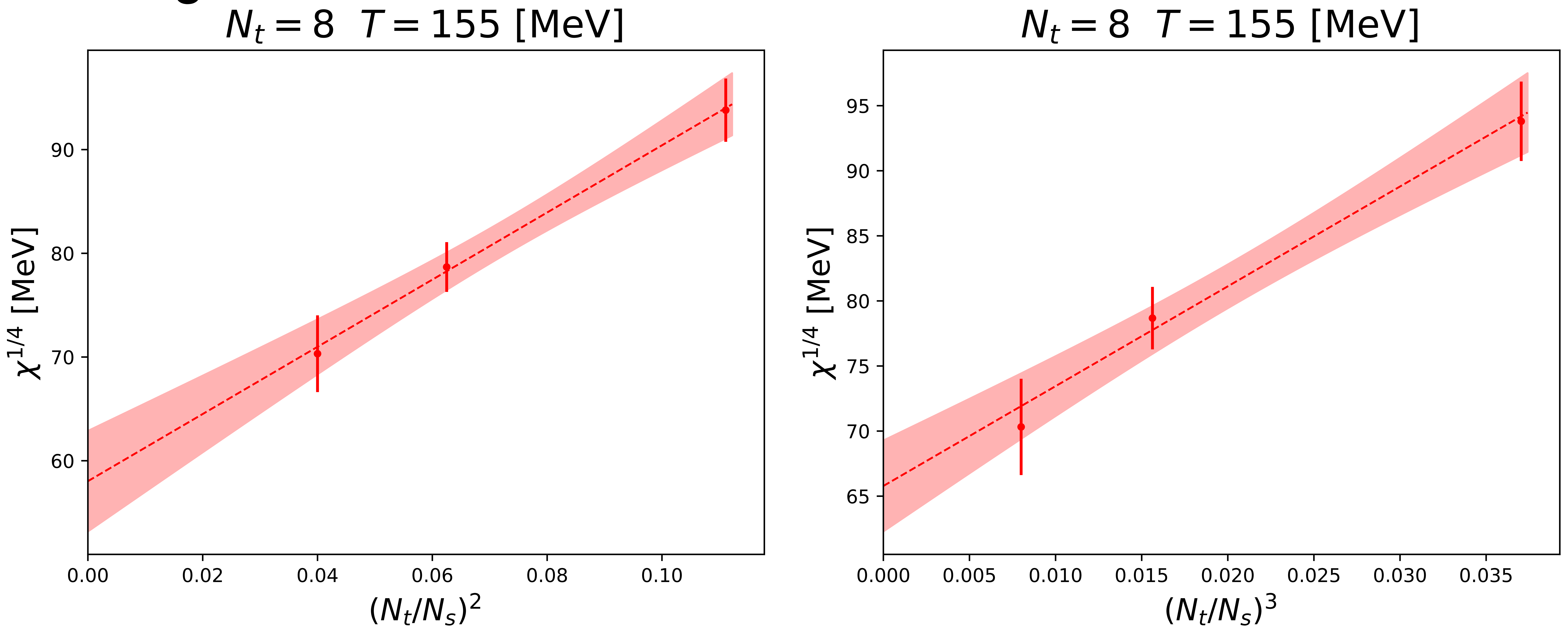




$$\langle q'^2 \rangle \propto \chi V x(1-x)$$

Topological susceptibility from fixed Q

Taking $V \to \infty$



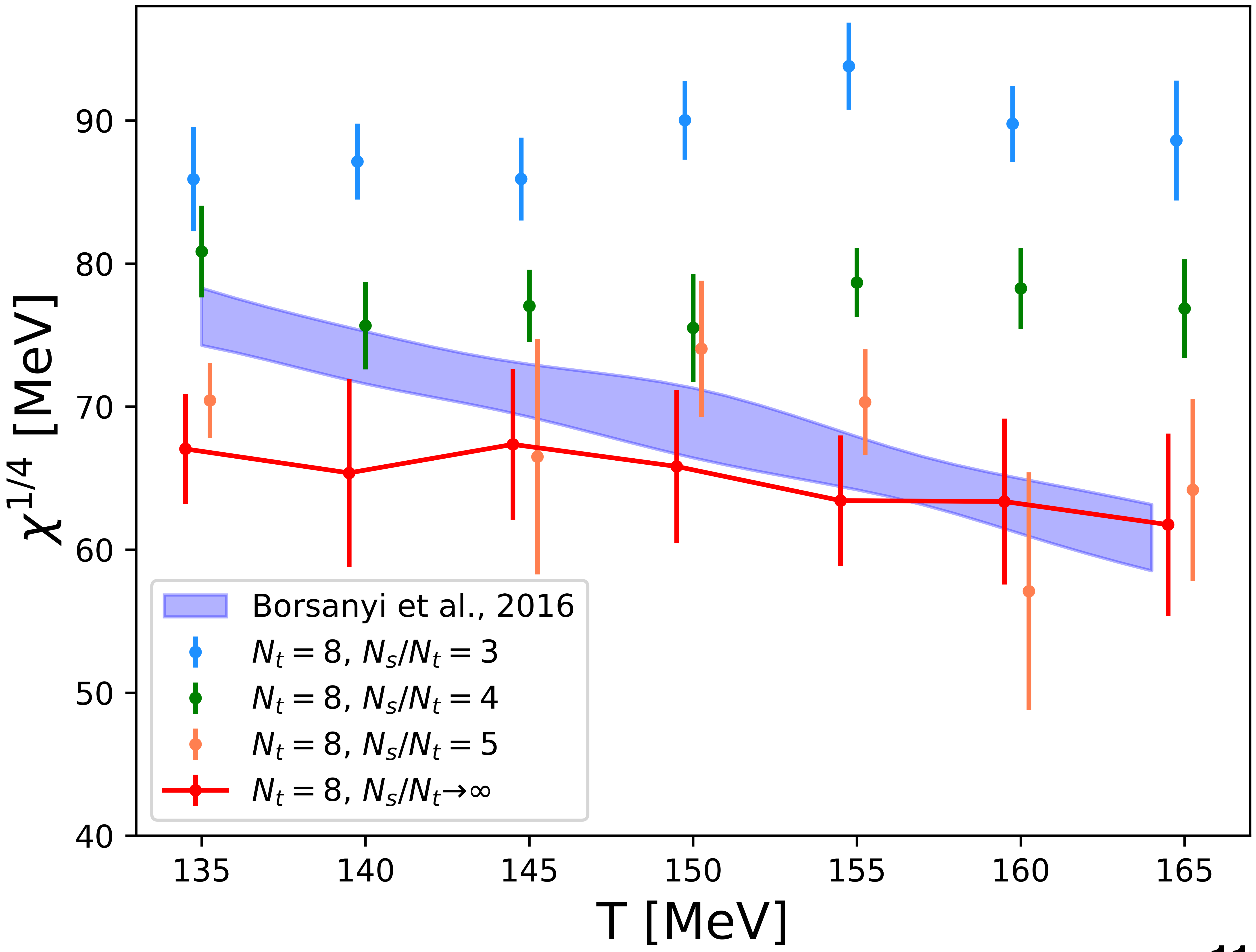
Take half-sum $\sim (N_s/N_t)^2$ and $\sim (N_s/N_t)^3$: systematic uncertainty

Topological susceptibility from simulations at fixed Q

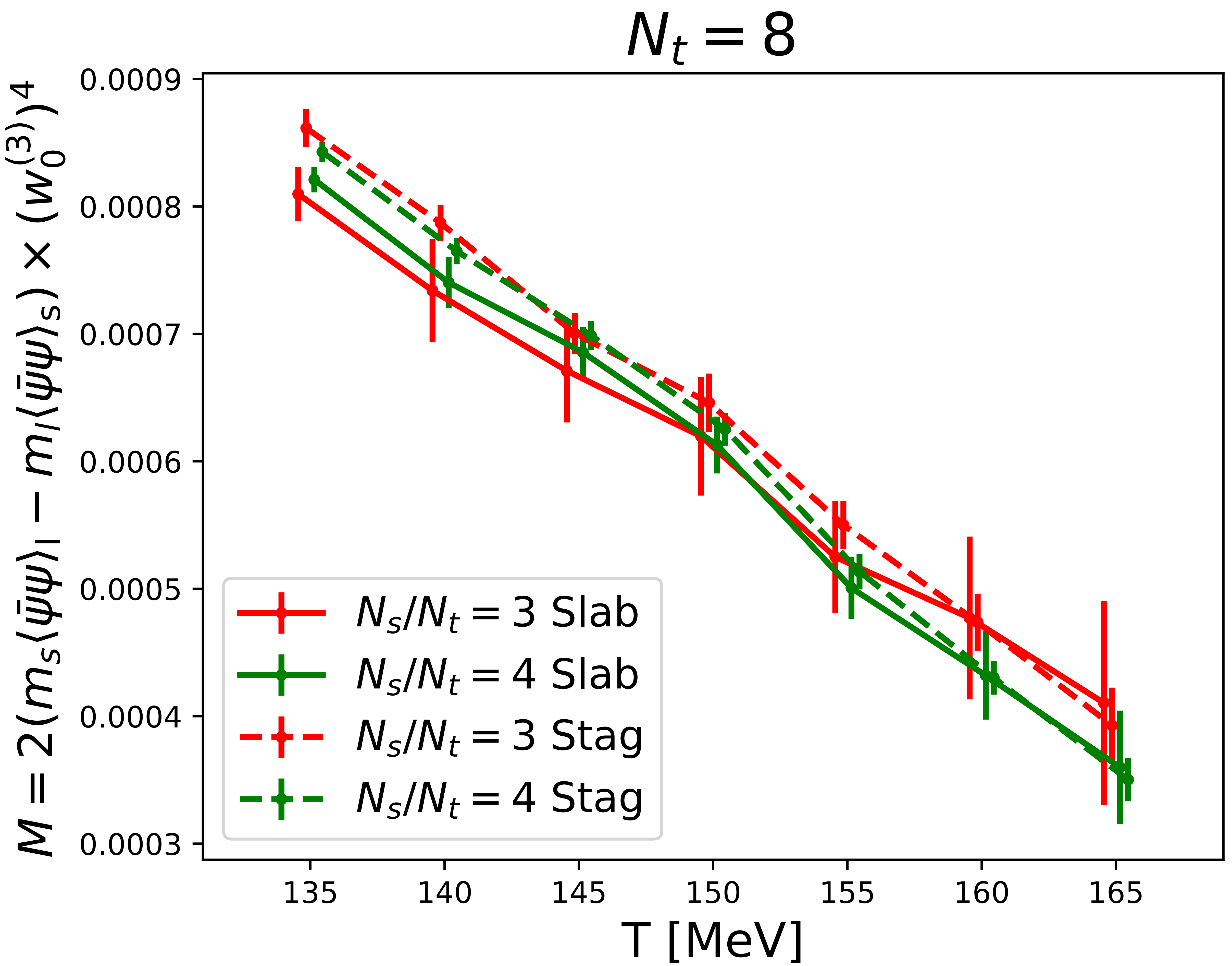
Slab method $N_t = 8$

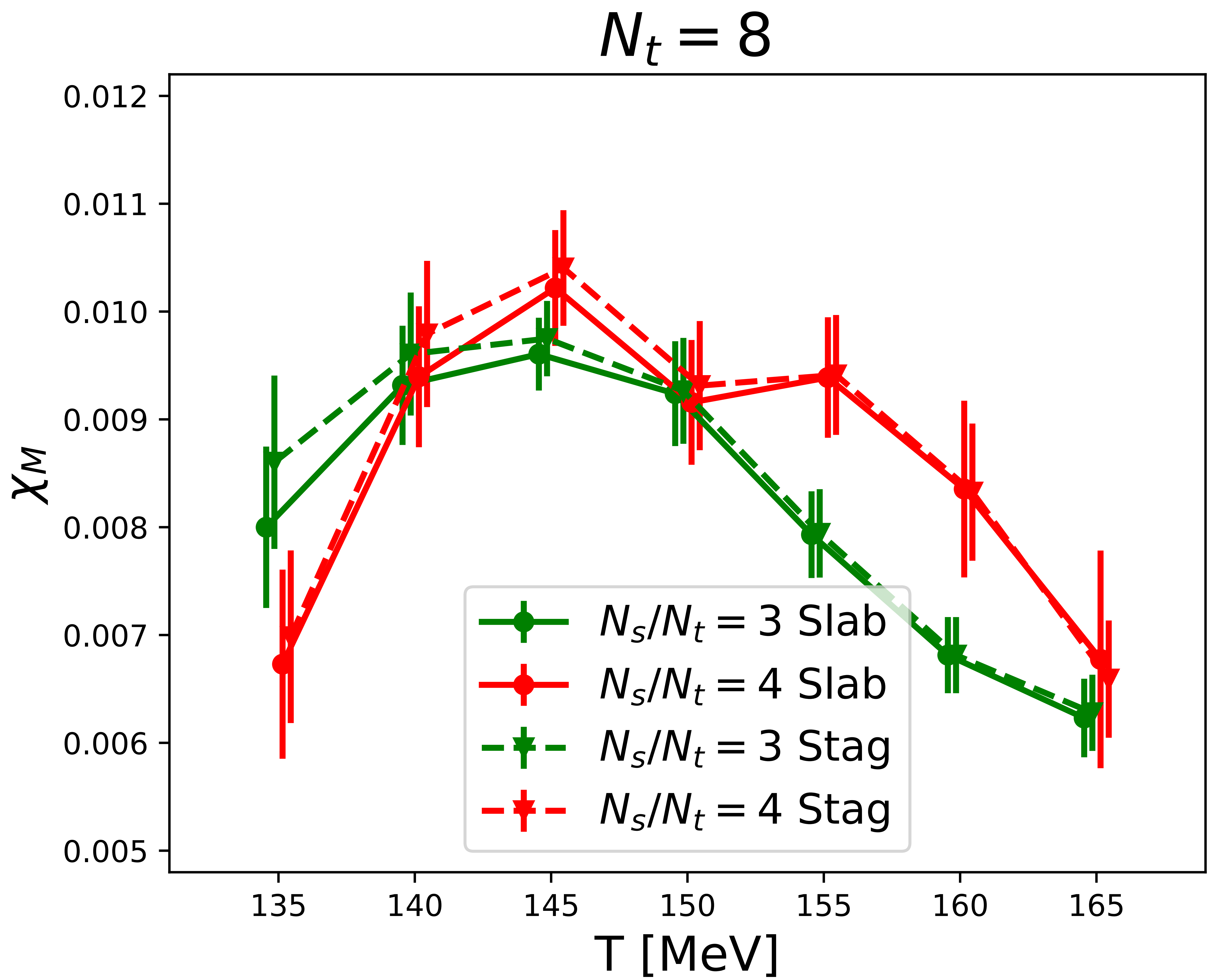
- Noisy
- Consistent with

  [Borsanyi et al., 2016]
- Local topological fluctuations



Summing over topological sectors





χ_Q from:

- Stag: [Borsanyi et al., 2016]
- Slab: overlap results at fixed Q

Dirac operator spectrum

$D_{\rm ov}^\dagger D_{\rm ov}\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$

- Chiral symmetry (Banks-Casher relation):

$$\bar\psi\psi \propto \int \frac{m}{\lambda^2+m^2}\rho(\lambda) \xrightarrow[m\to 0]{} \rho(\lambda=0)$$

- Axial symmetry:

$$\chi_A = \chi_\pi - \chi_\delta \propto \int d\lambda \frac{m^2}{(m^2+\lambda^2)^2}\rho(\lambda)$$

- Talks: [I. Horvath, Tue, 11.30] [T. Kovacs, Tue, 14.30] [W.-P. Huang, Tue, 15.10]

Dirac operator spectrum, $T = 145\ \mathrm{MeV}$

$$D_{\mathrm{ov}}^{\dagger} D_{\mathrm{ov}} |e_i\rangle = \lambda_i^2 |e_i\rangle$$

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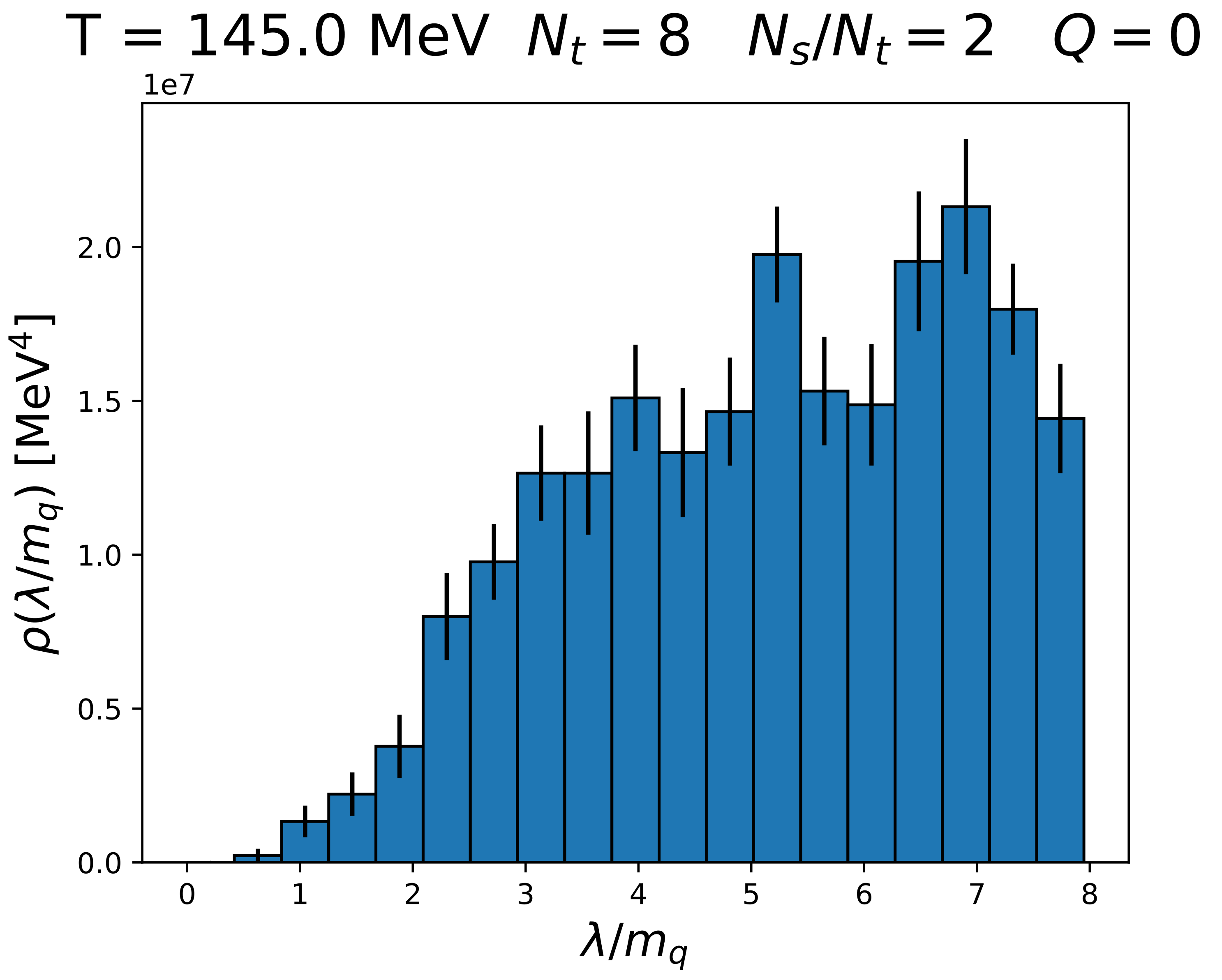

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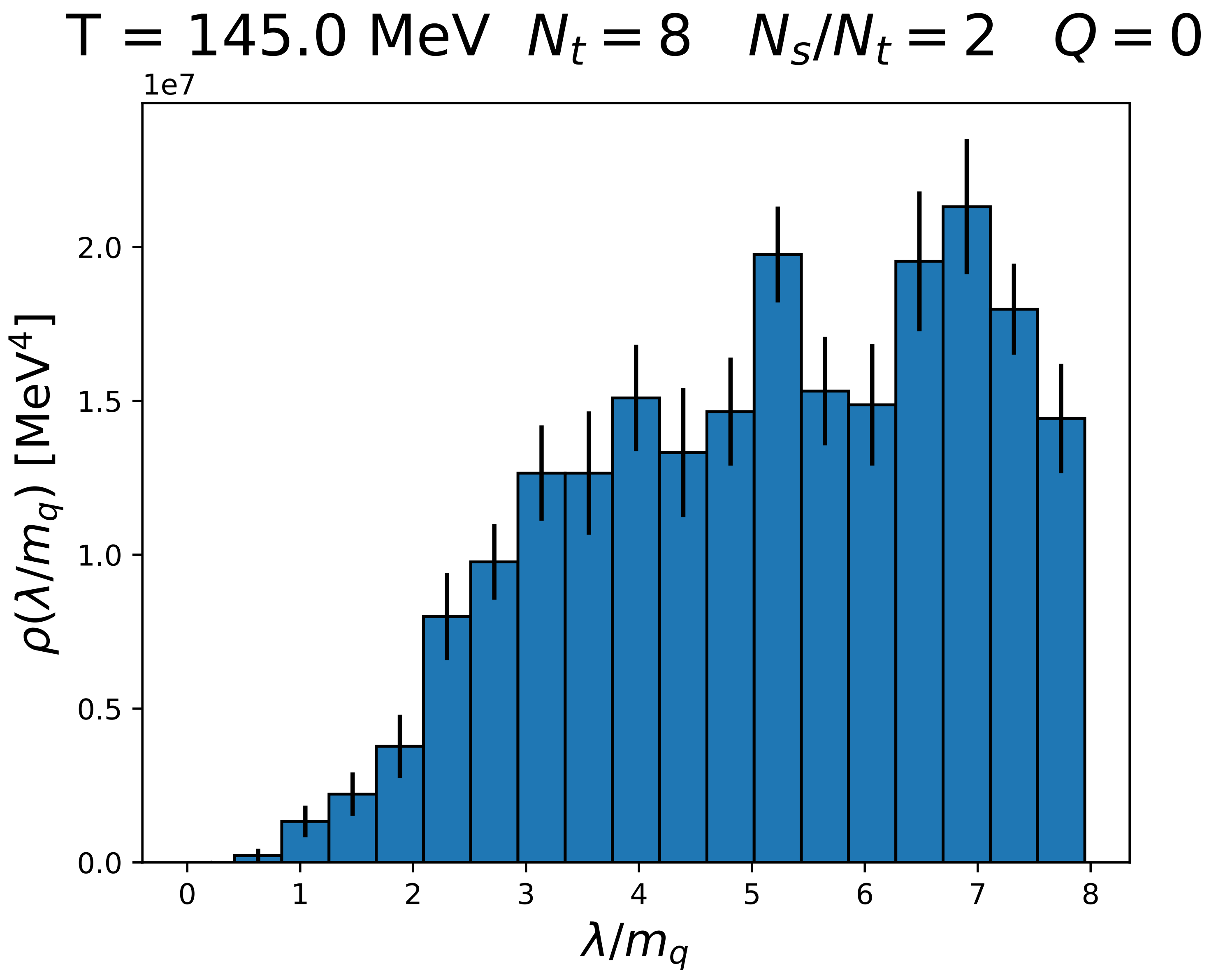

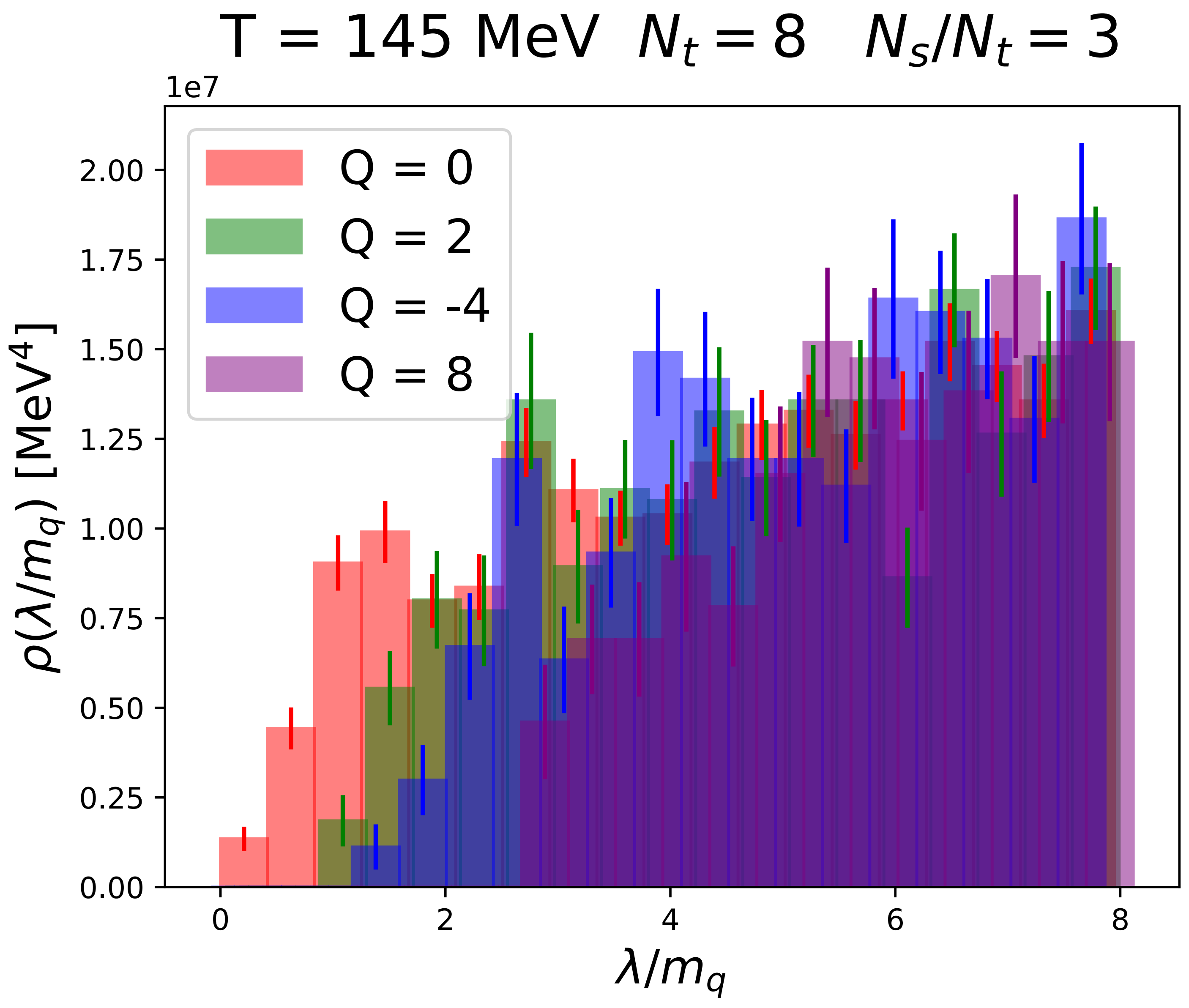

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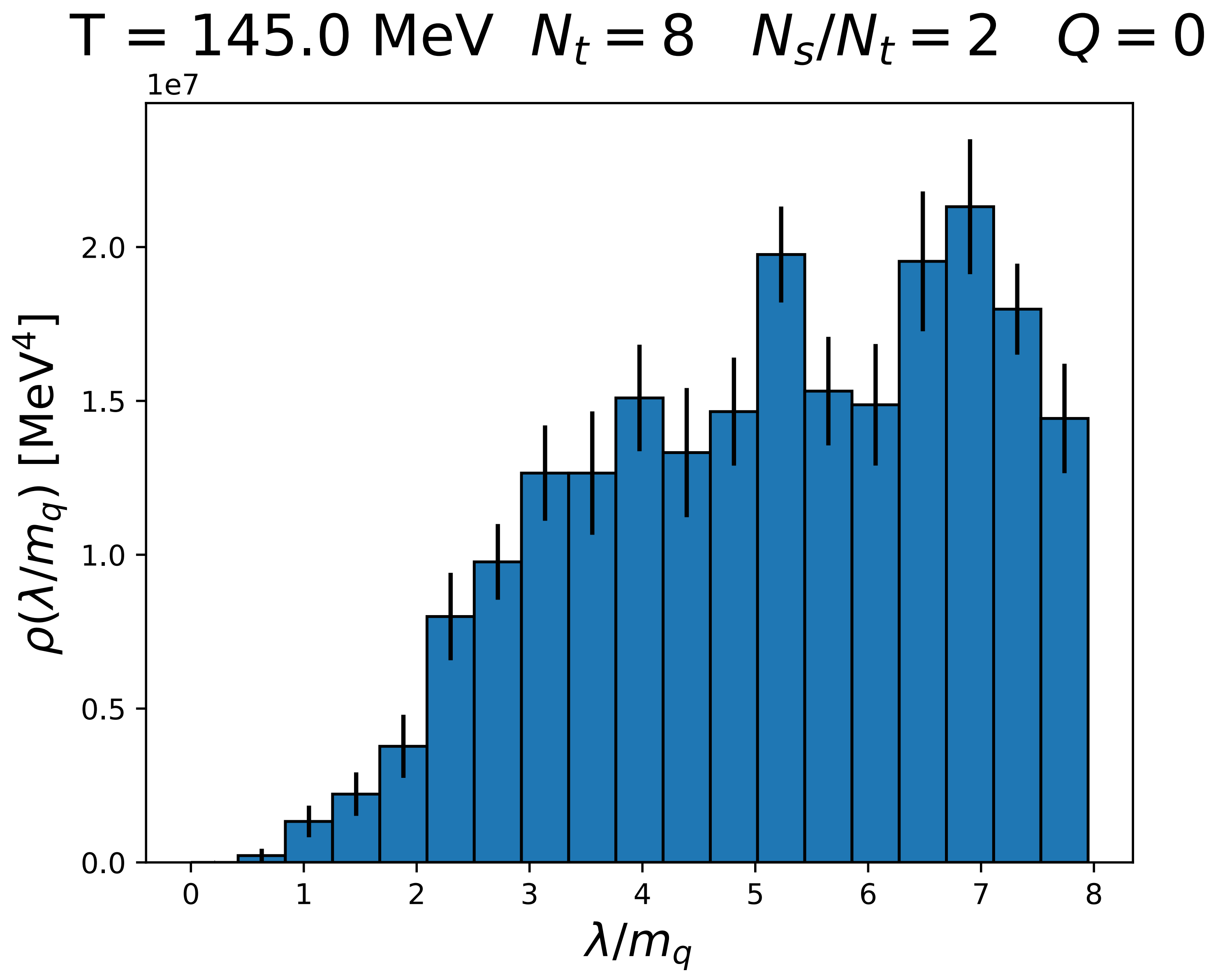

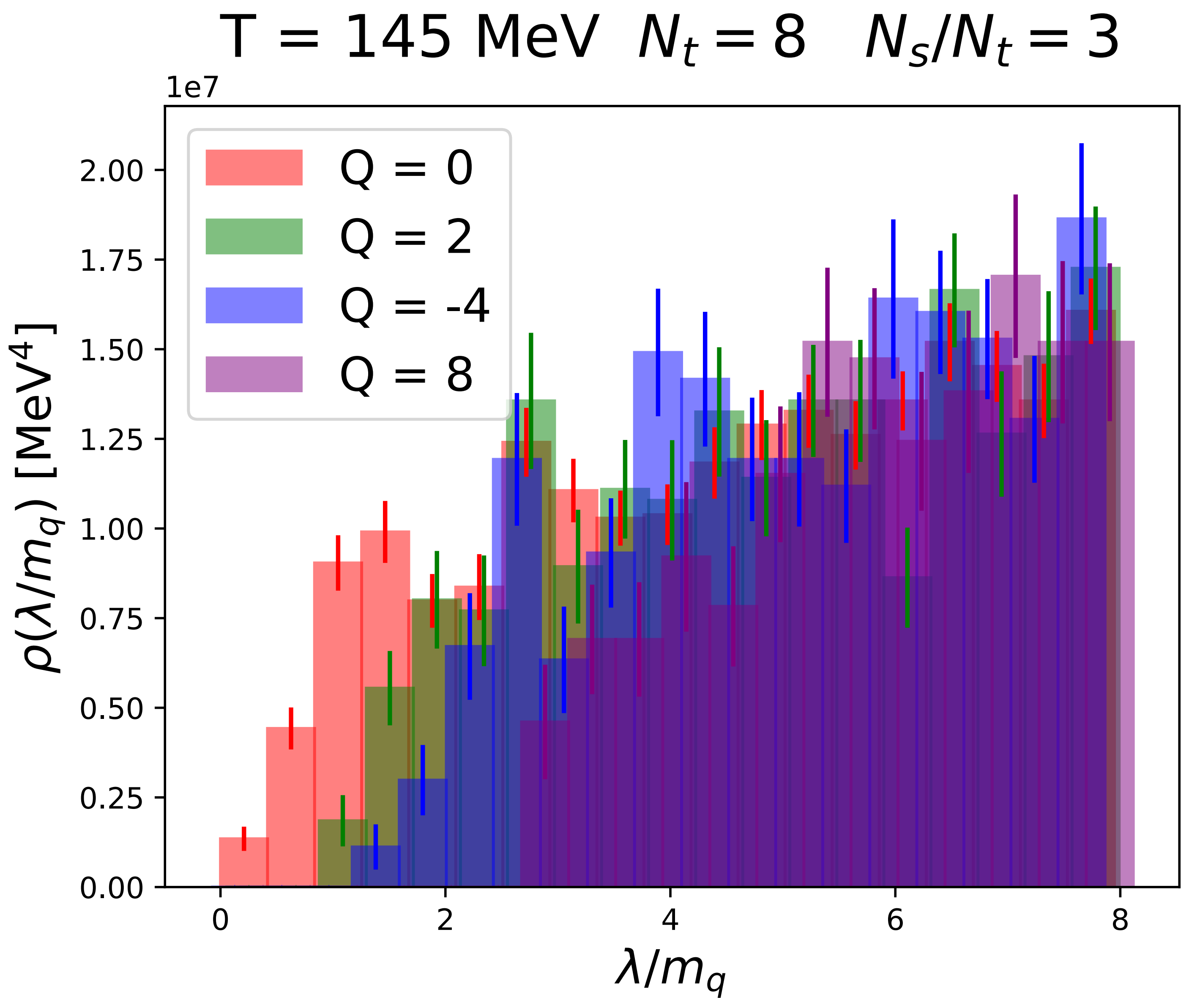

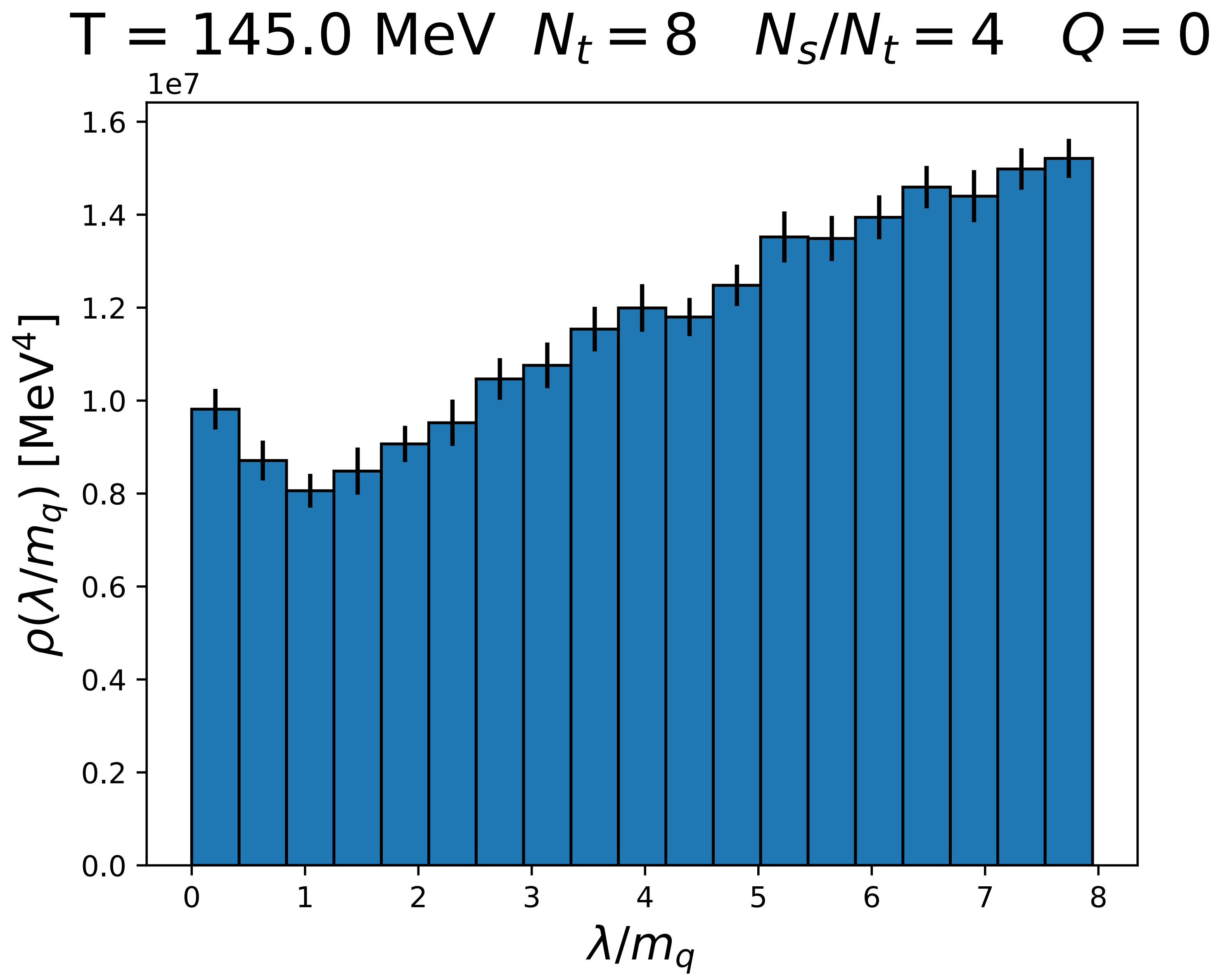

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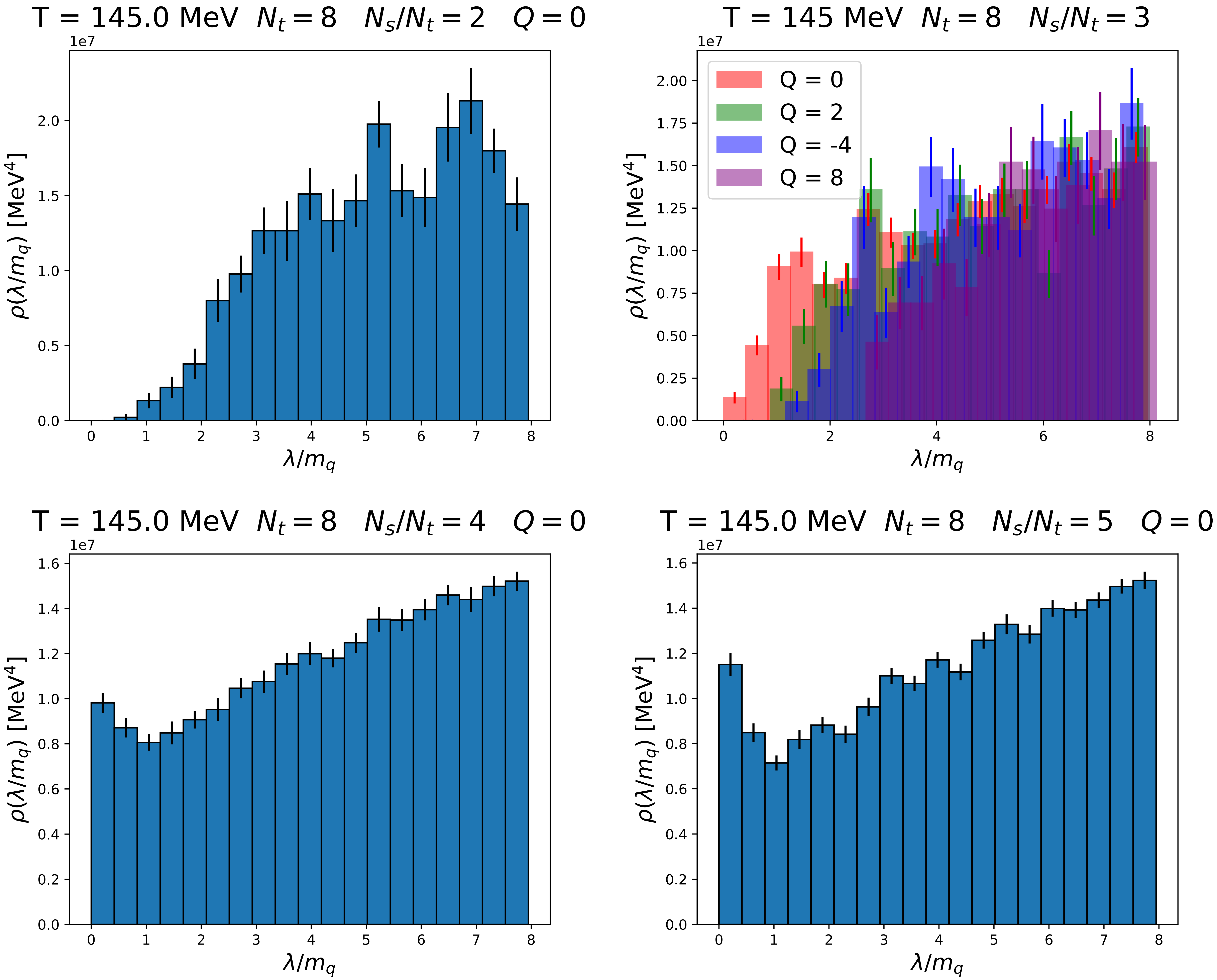

Dirac operator spectrum, $T = 170\ \mathrm{MeV}$

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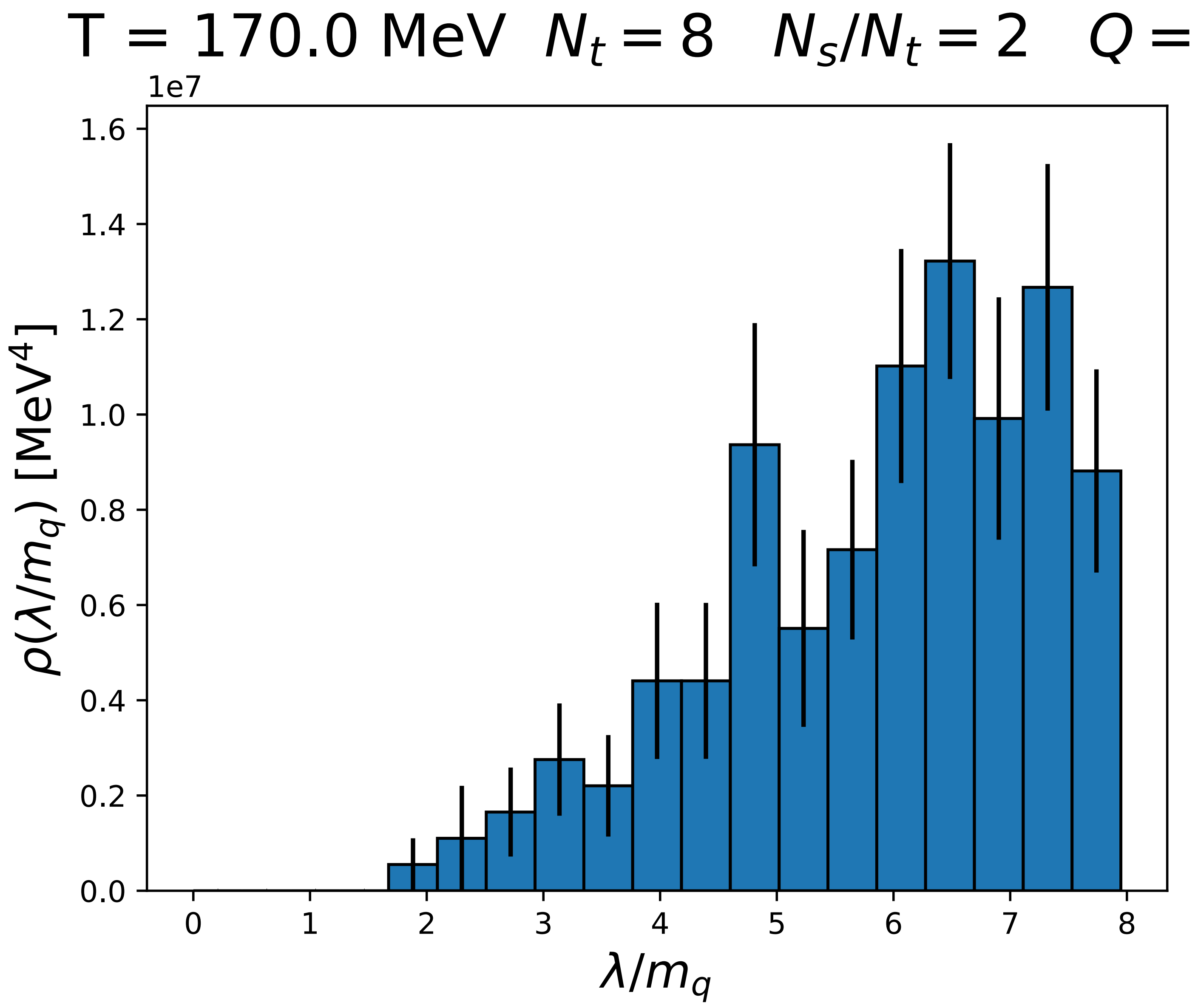

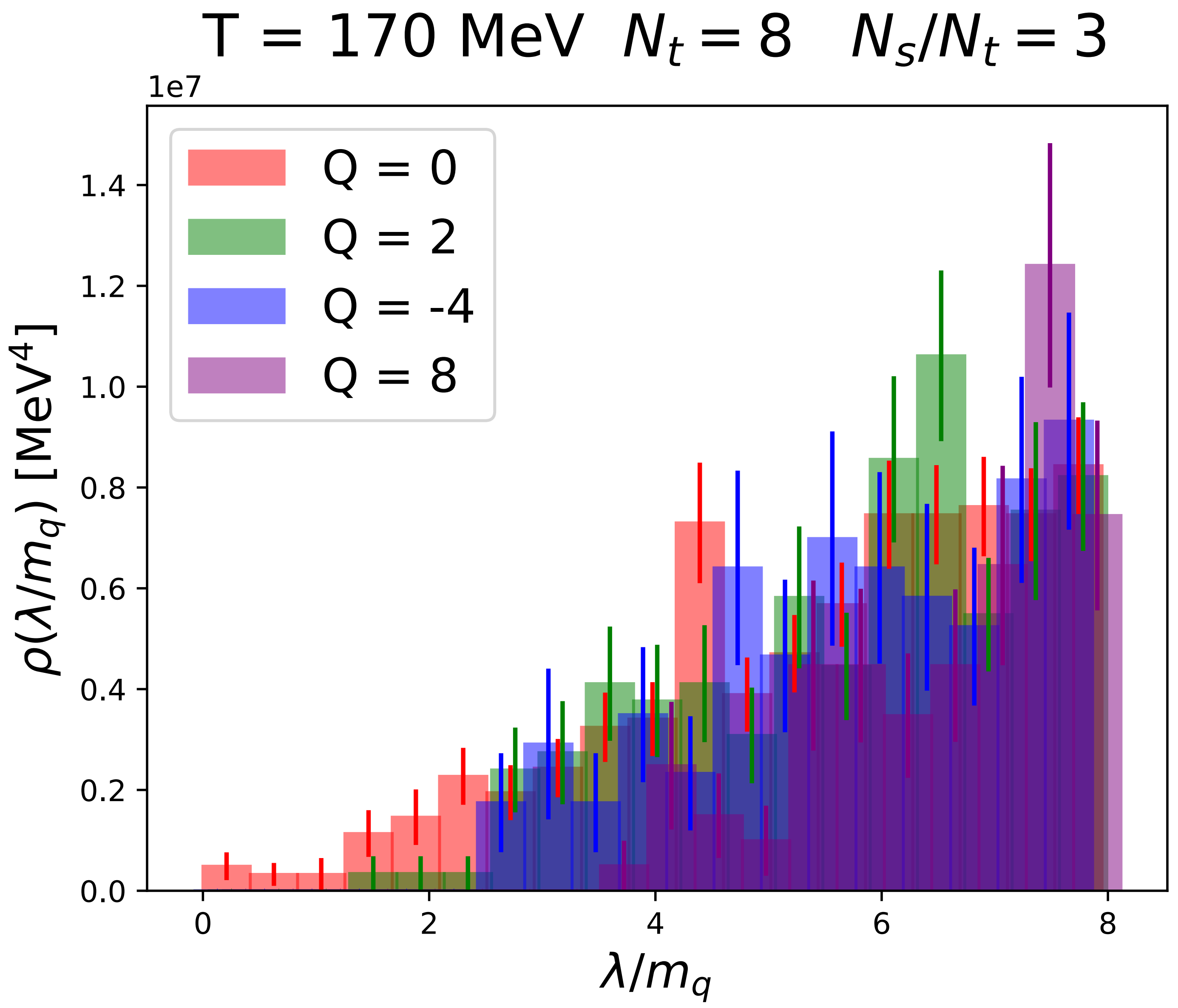

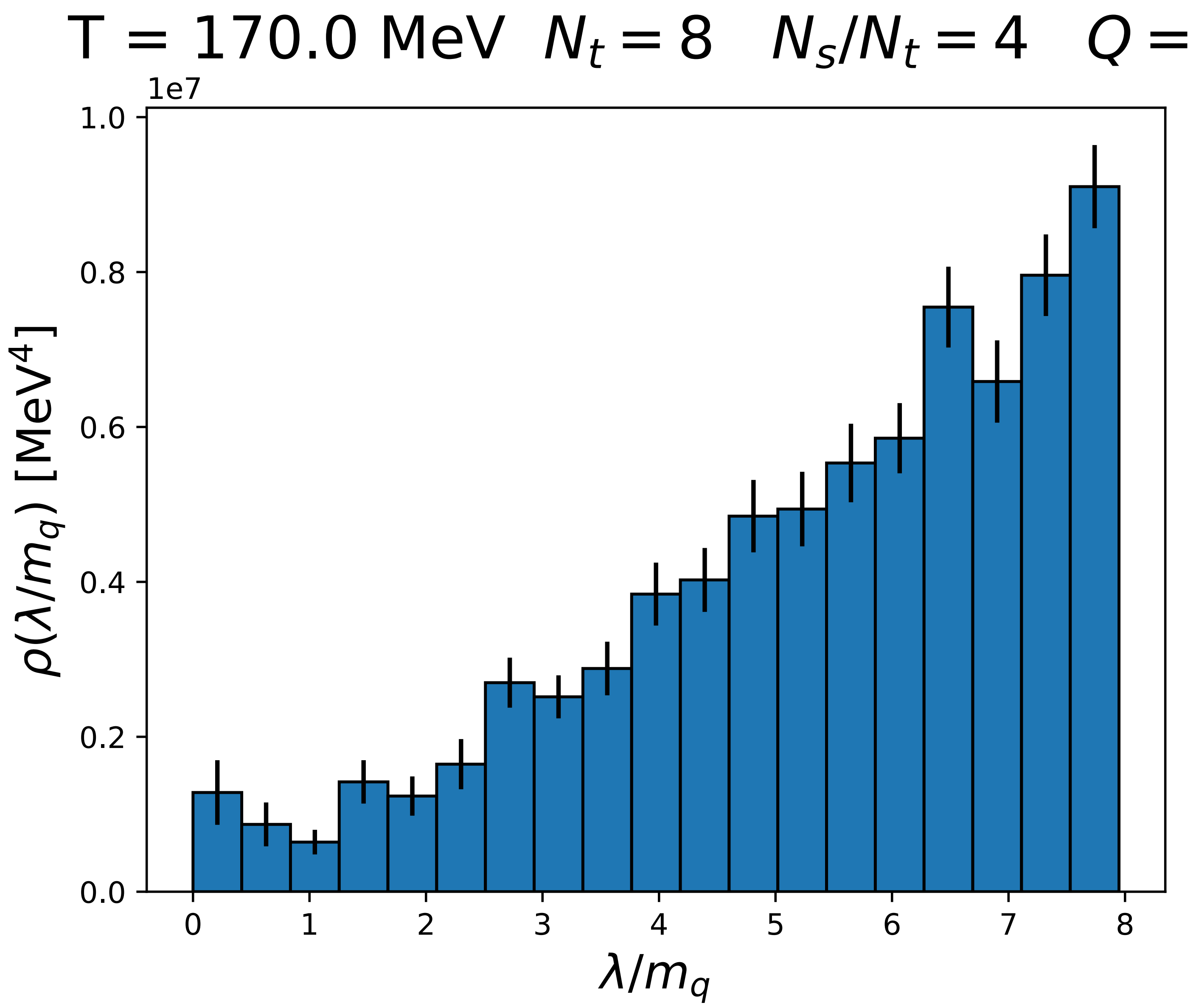

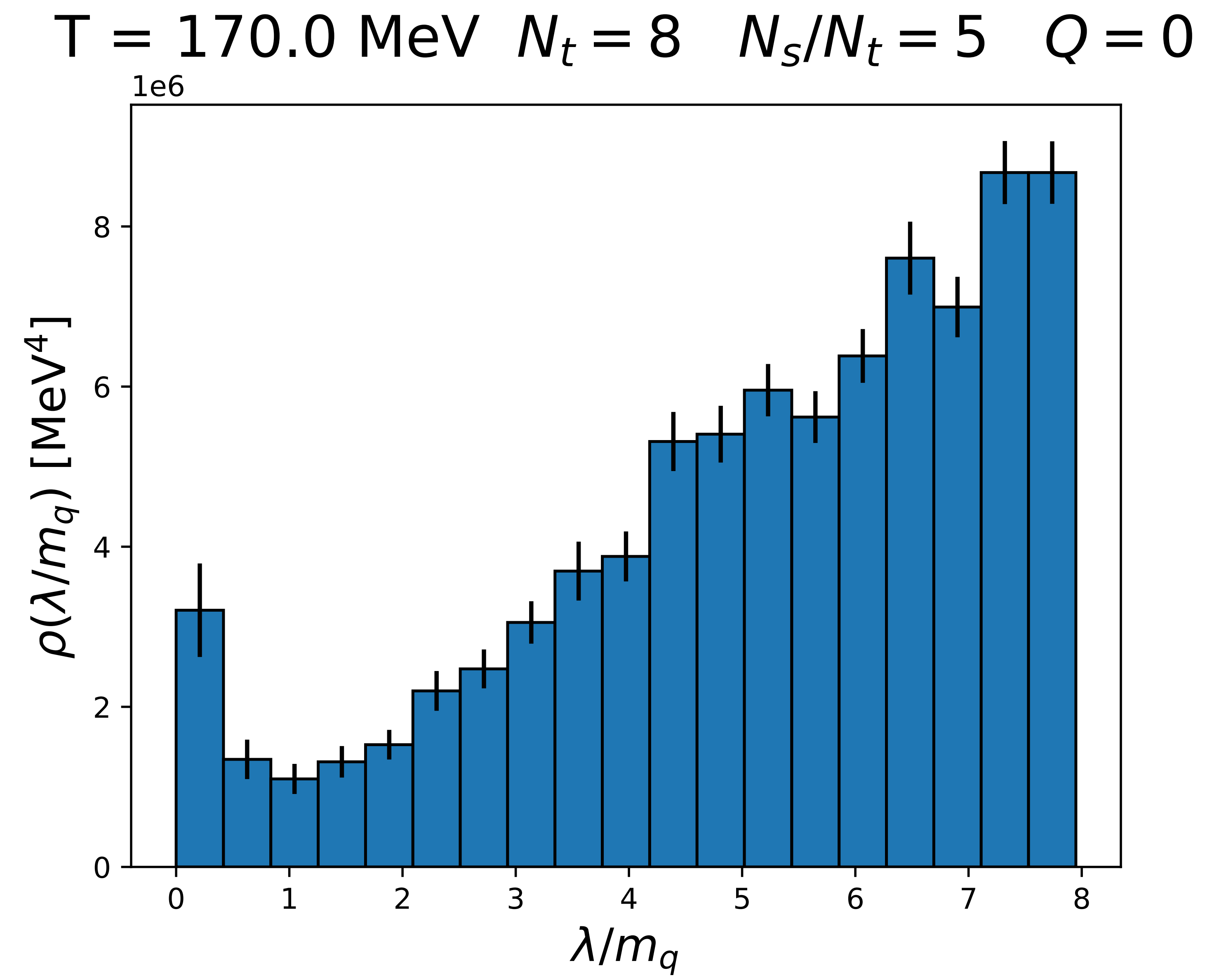

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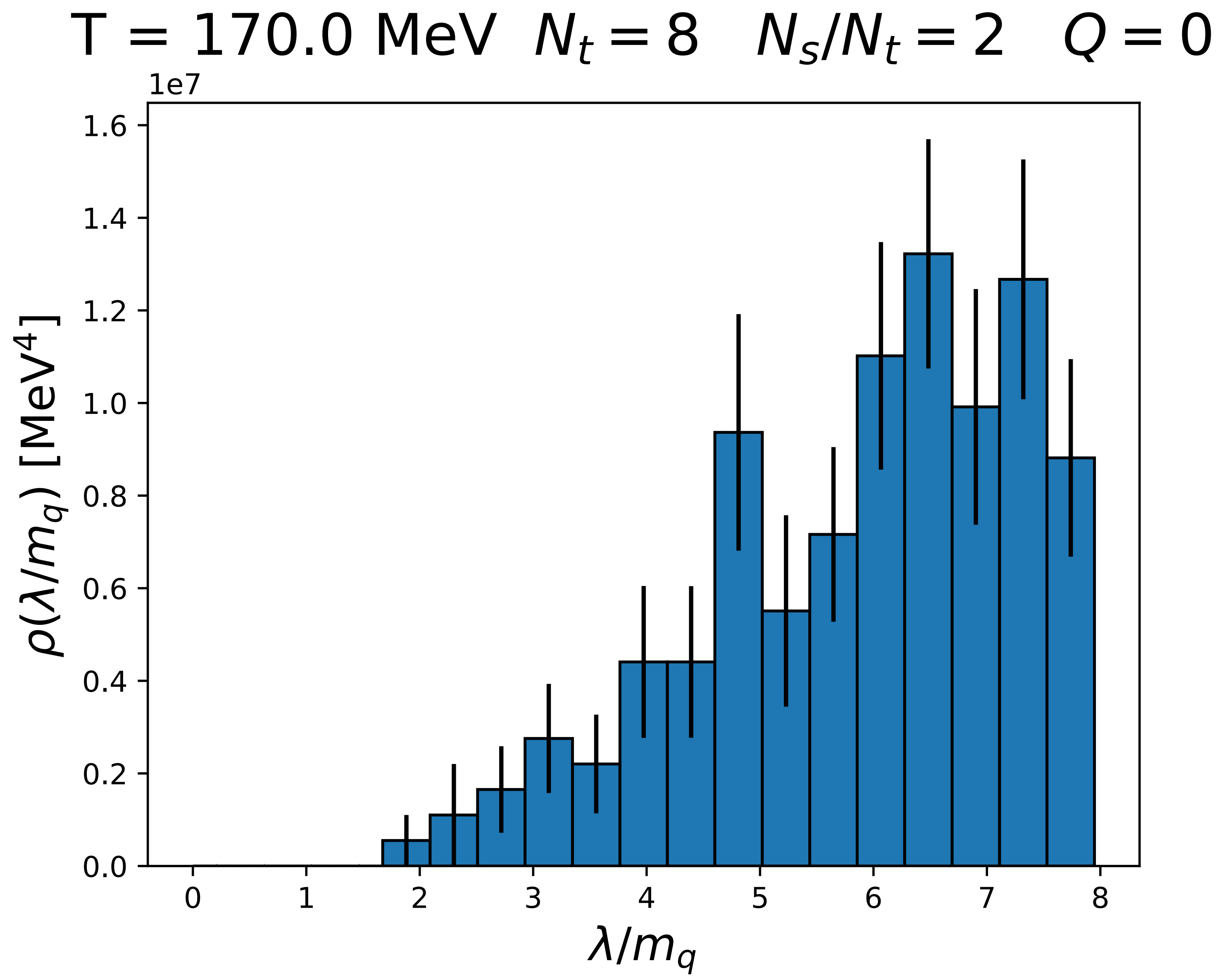

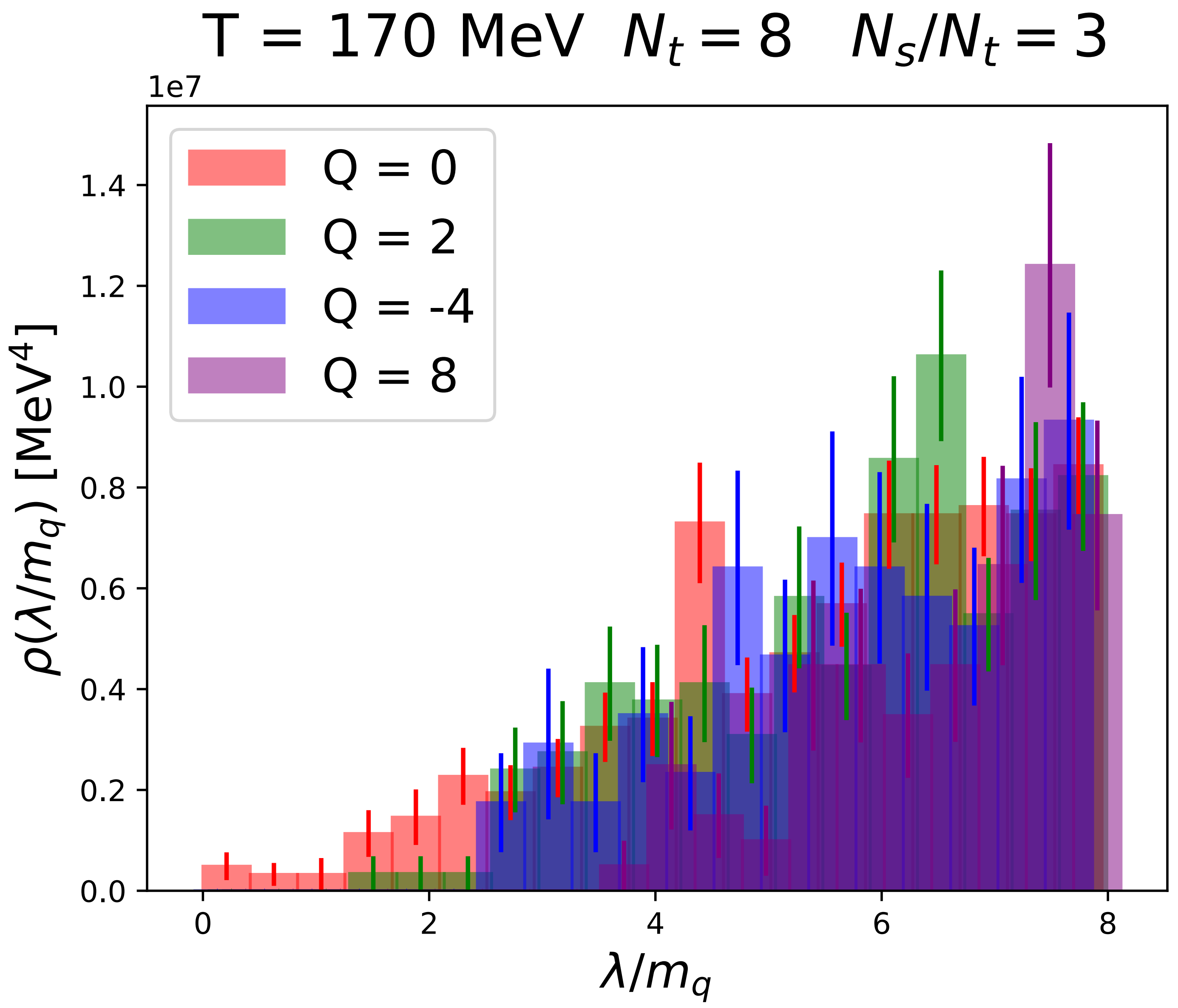

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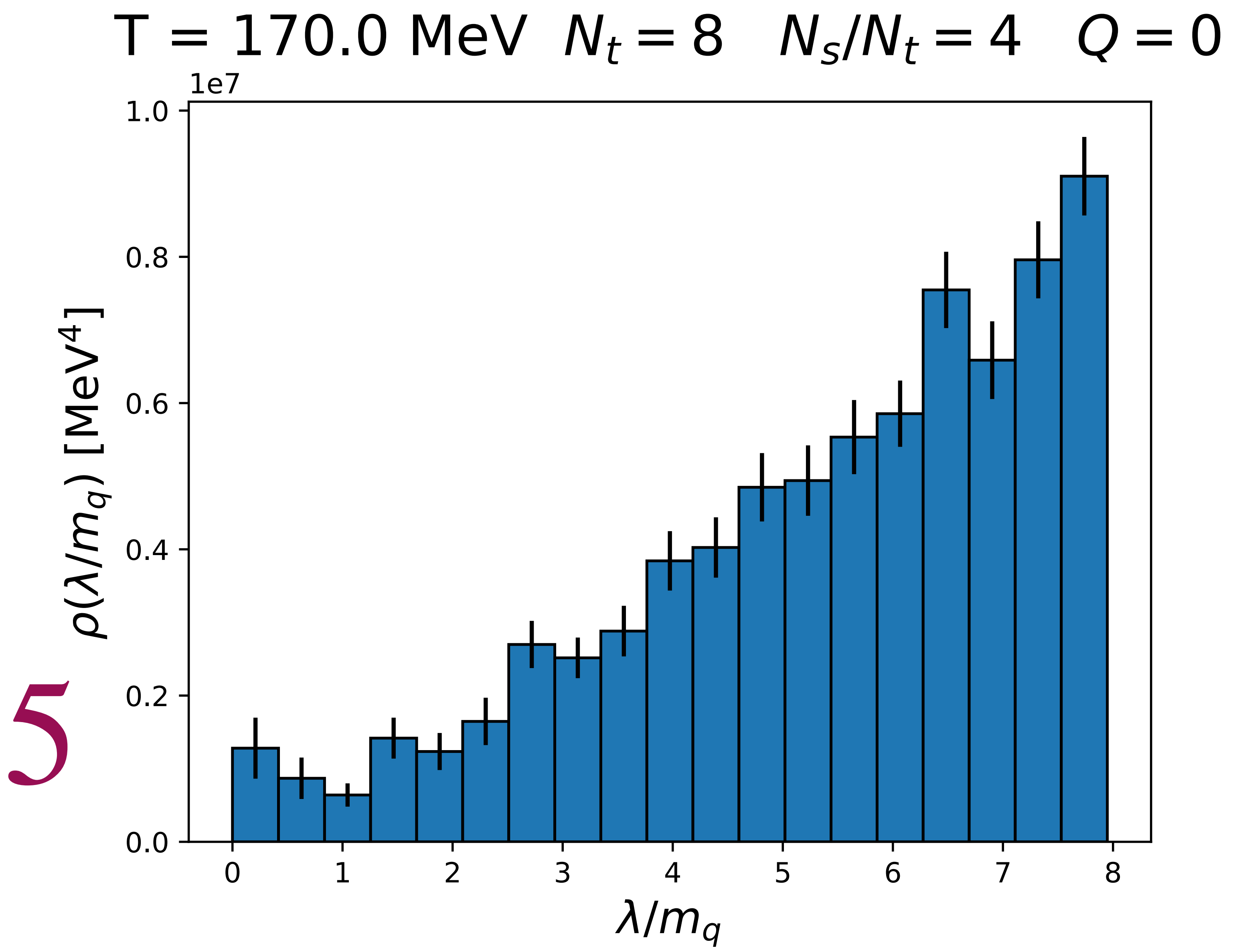

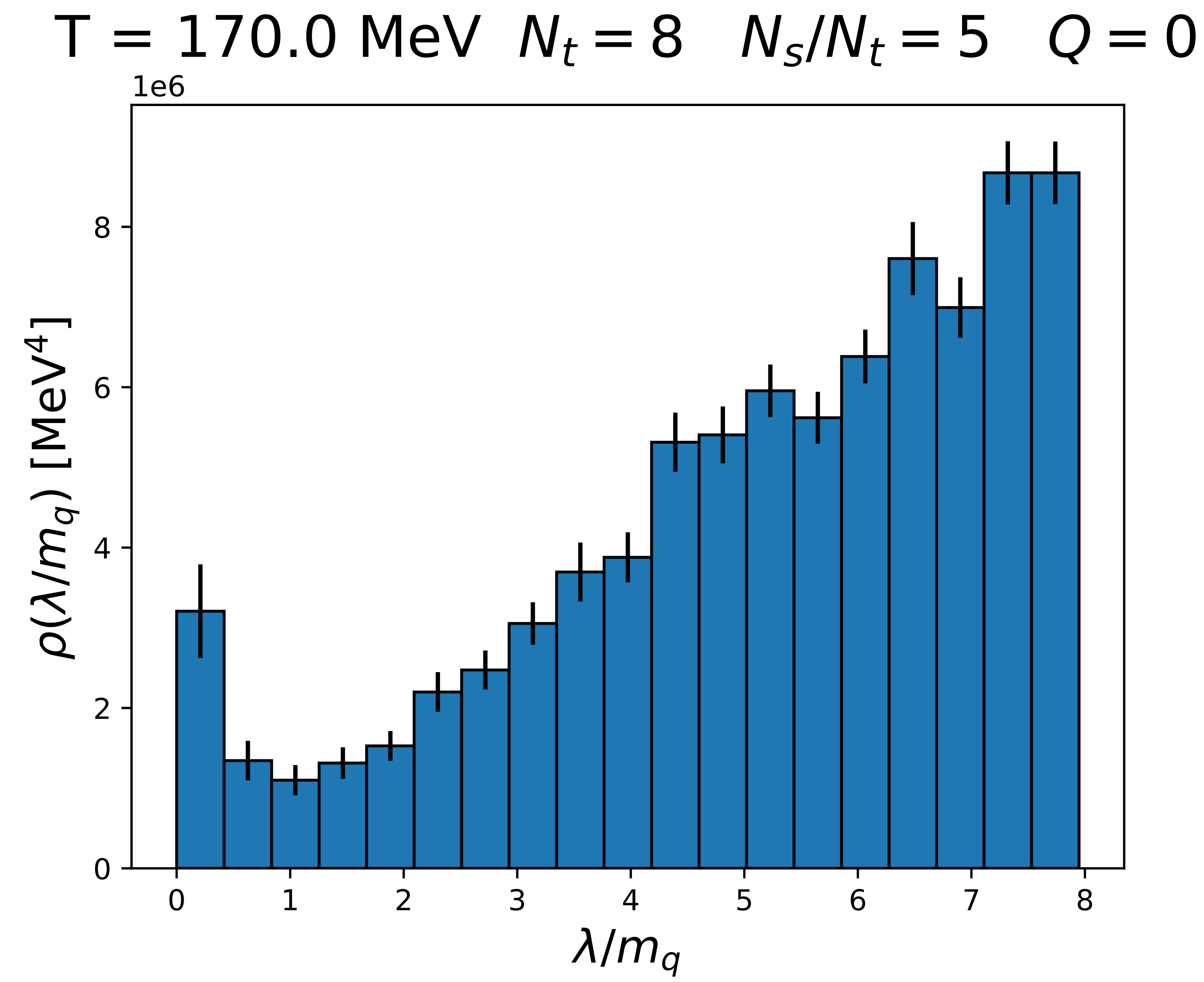

Peak $\rho(\lambda \to 0)$:
Large $N_s/N_t \gtrsim 4 - 5$

Summary

- Dynamical overlap fermions at $m_\pi = m_\pi^{\mathrm{phys}}$
  - Preliminary data around T_{pc}, mainly $N_t = 8$
  - Simulations at fixed Q
  - Summation over Q
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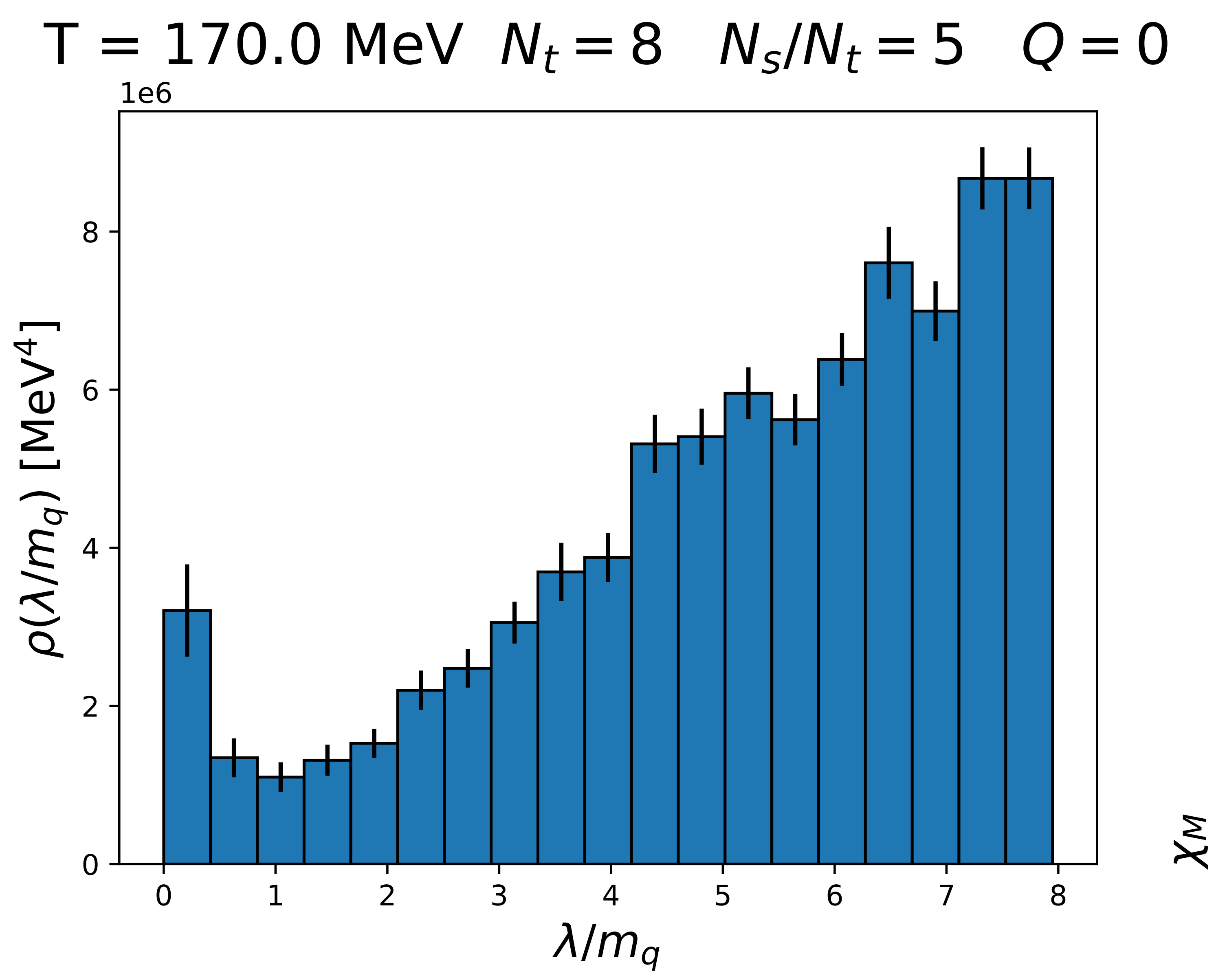
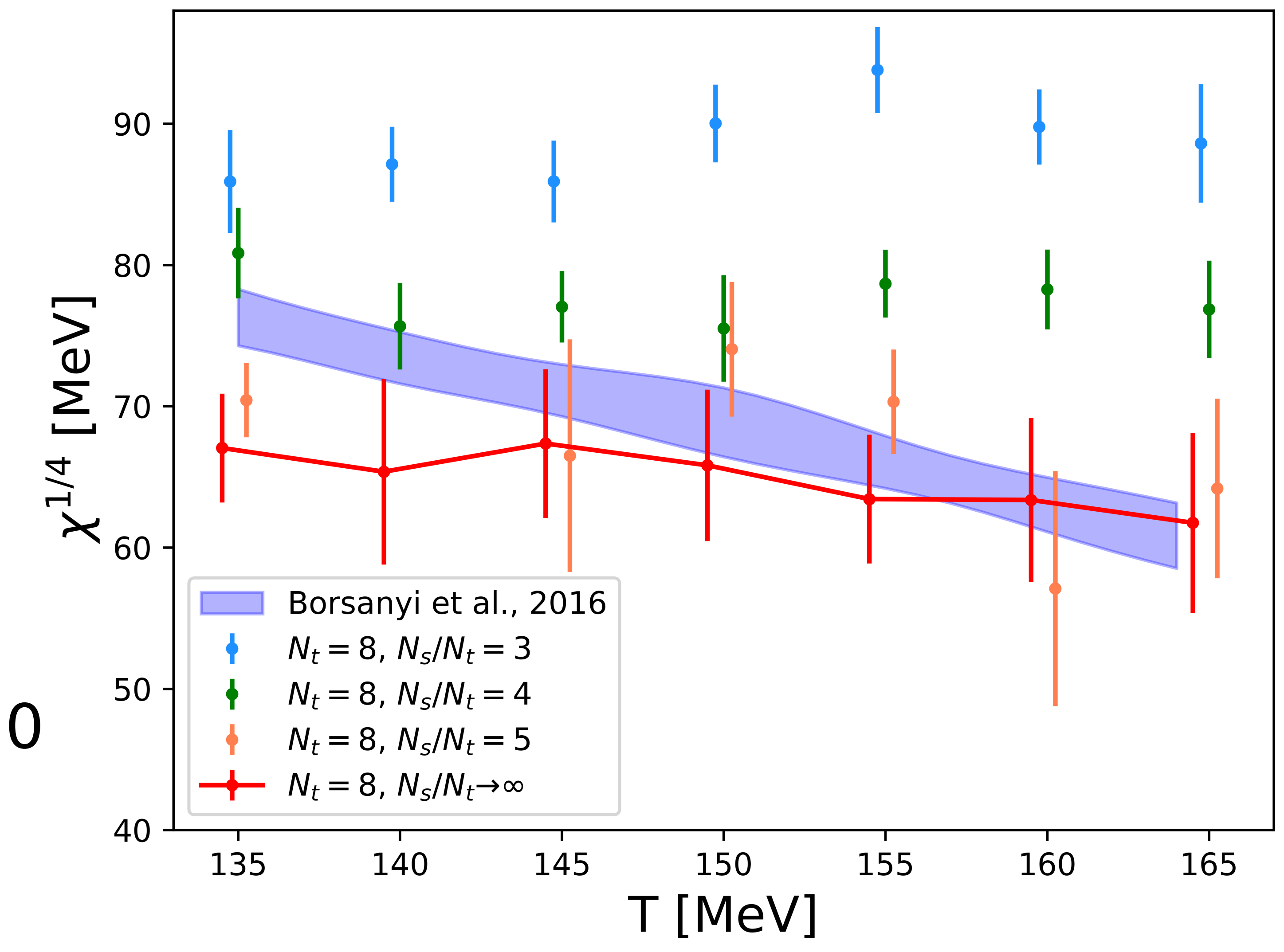
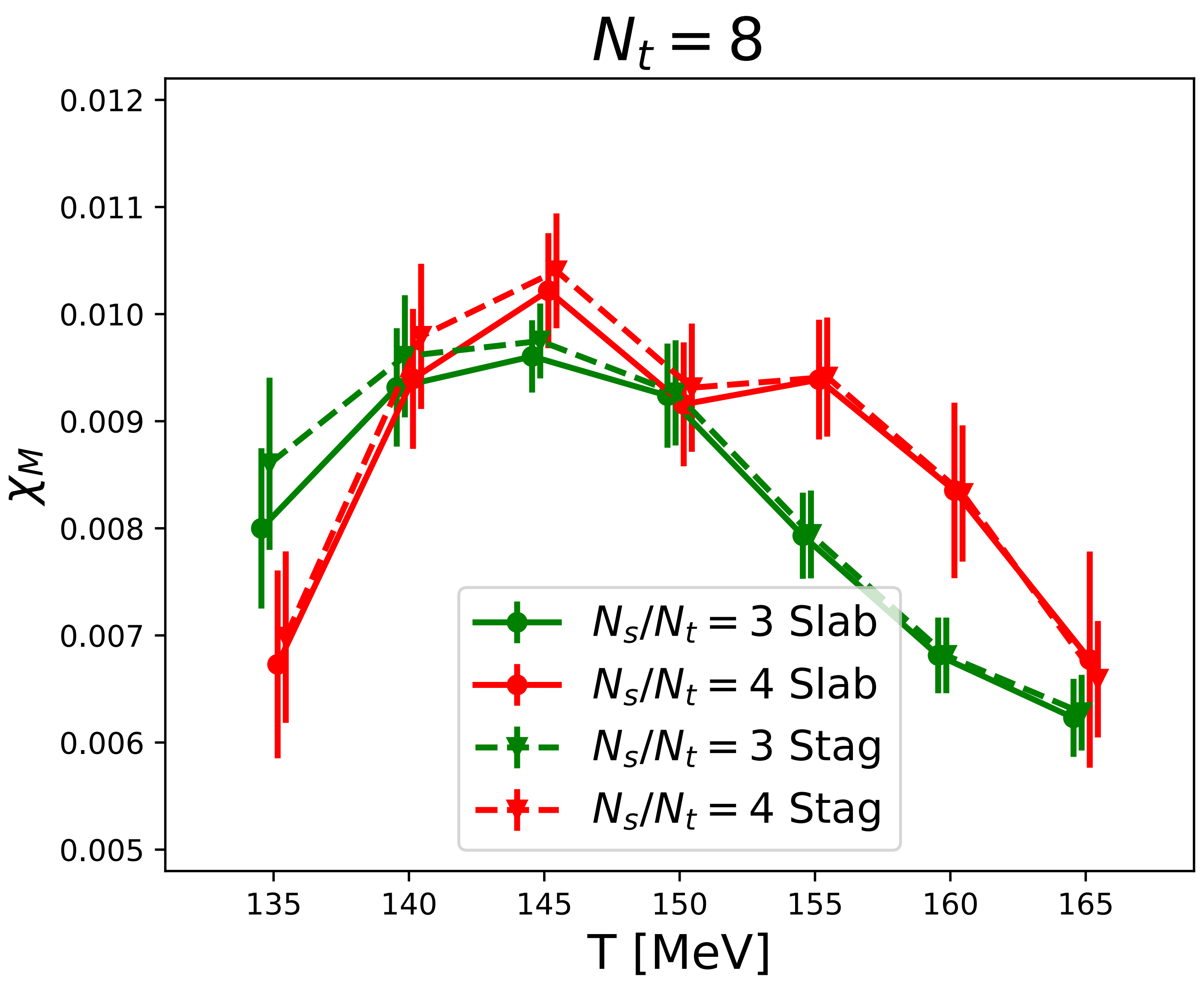
- Dirac spectrum: peak at $\rho(\lambda \to 0)$

  for $N_s/N_t \gtrsim 4-5$ at $T \gtrsim T_{\mathrm{pc}}$







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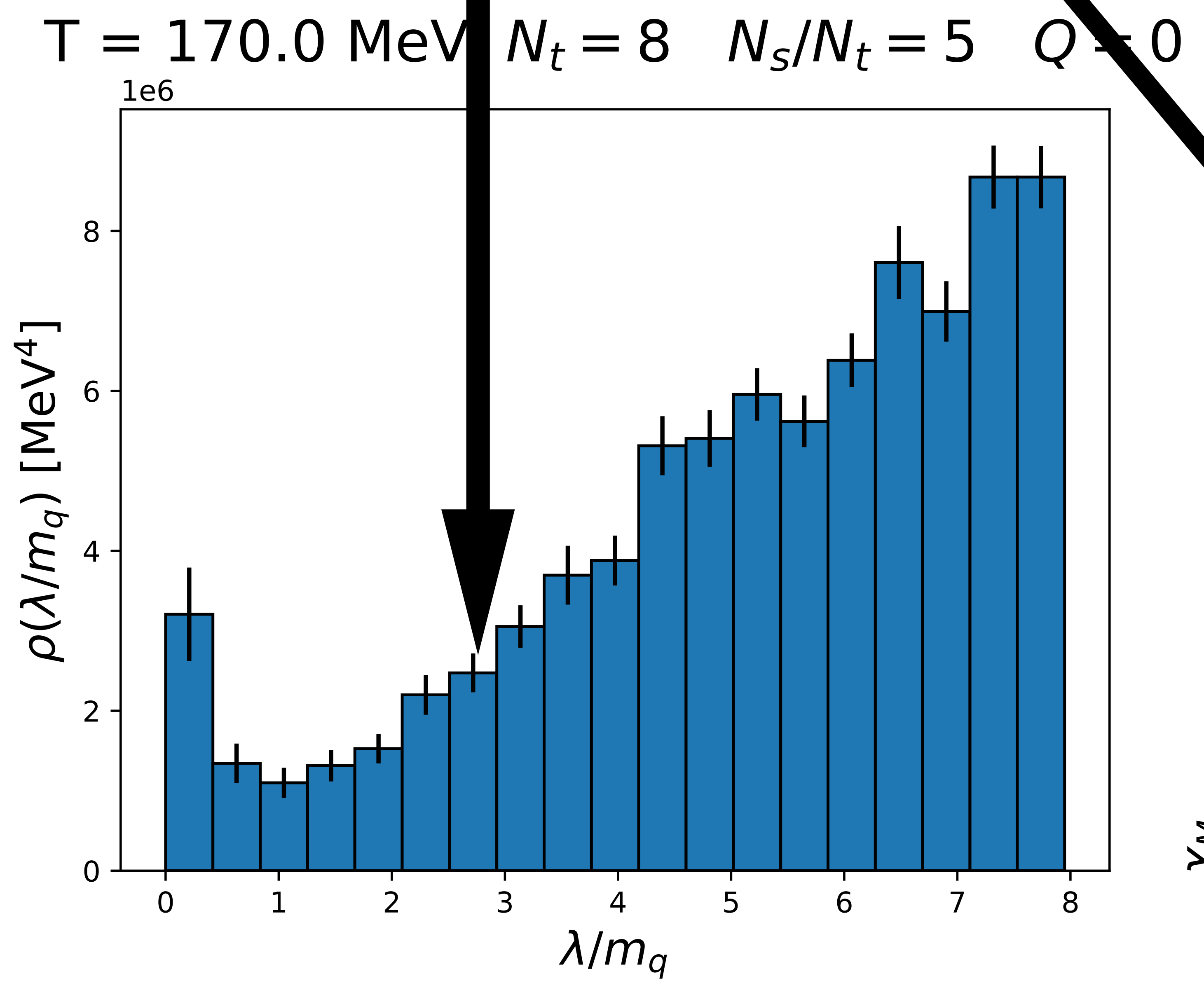

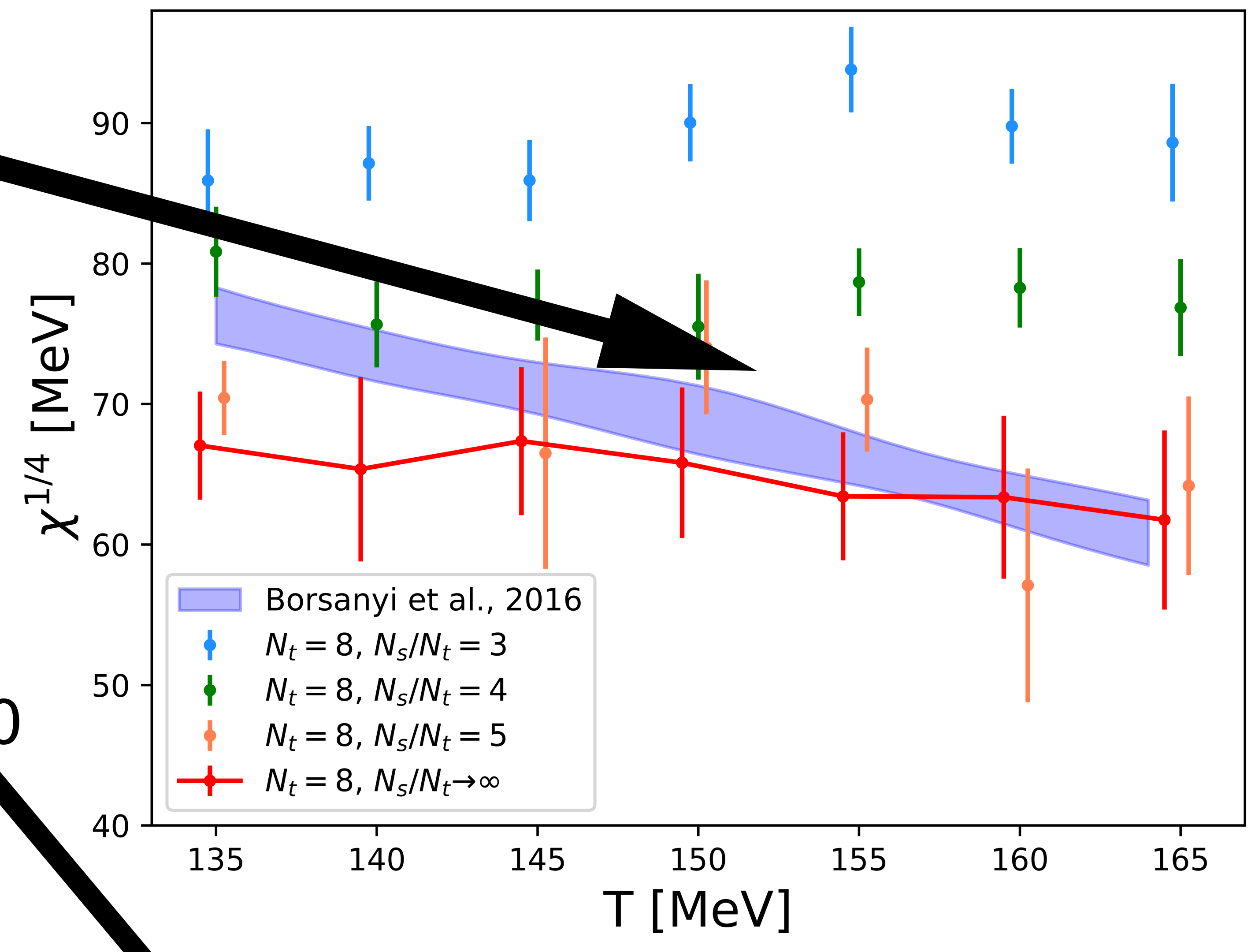

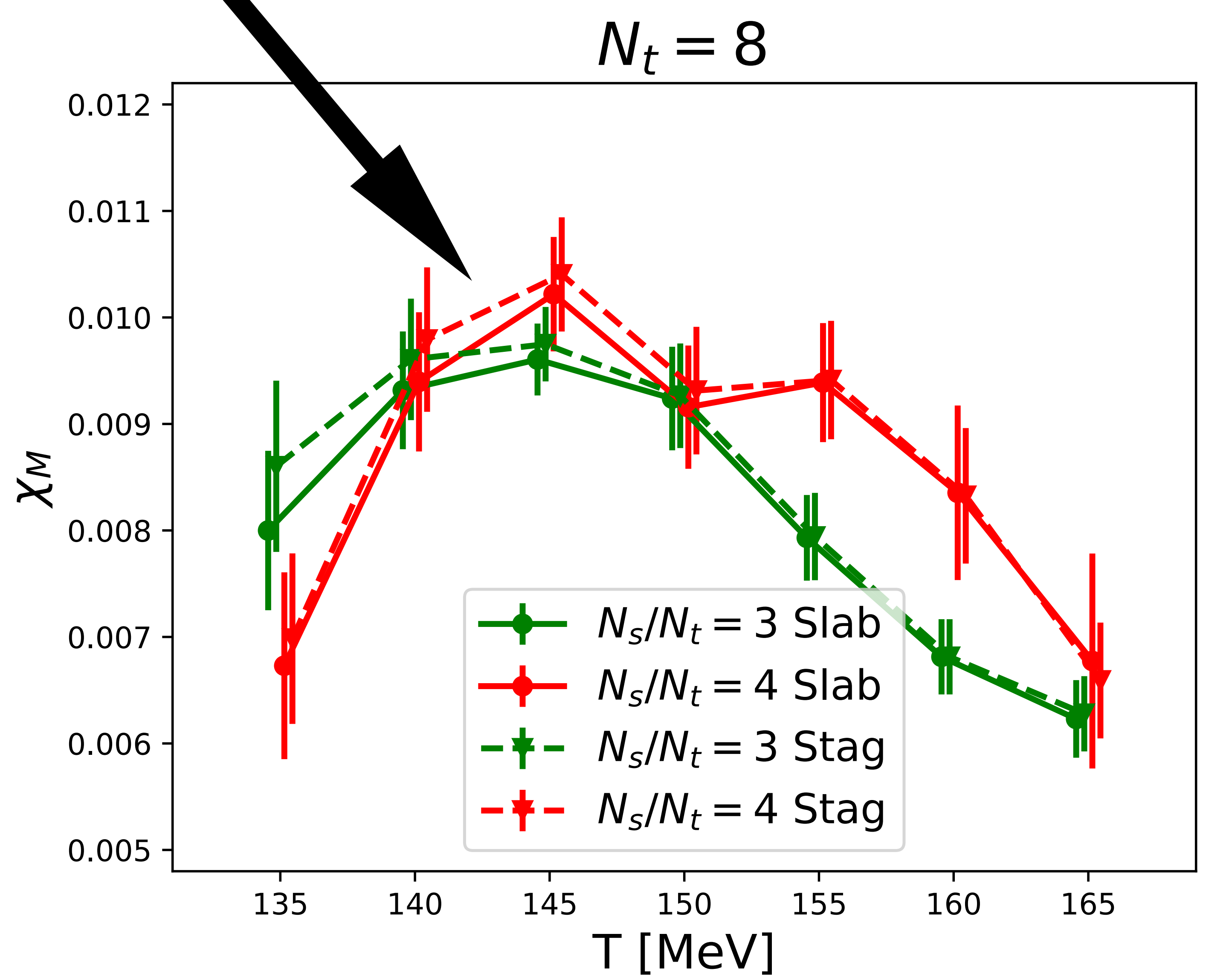

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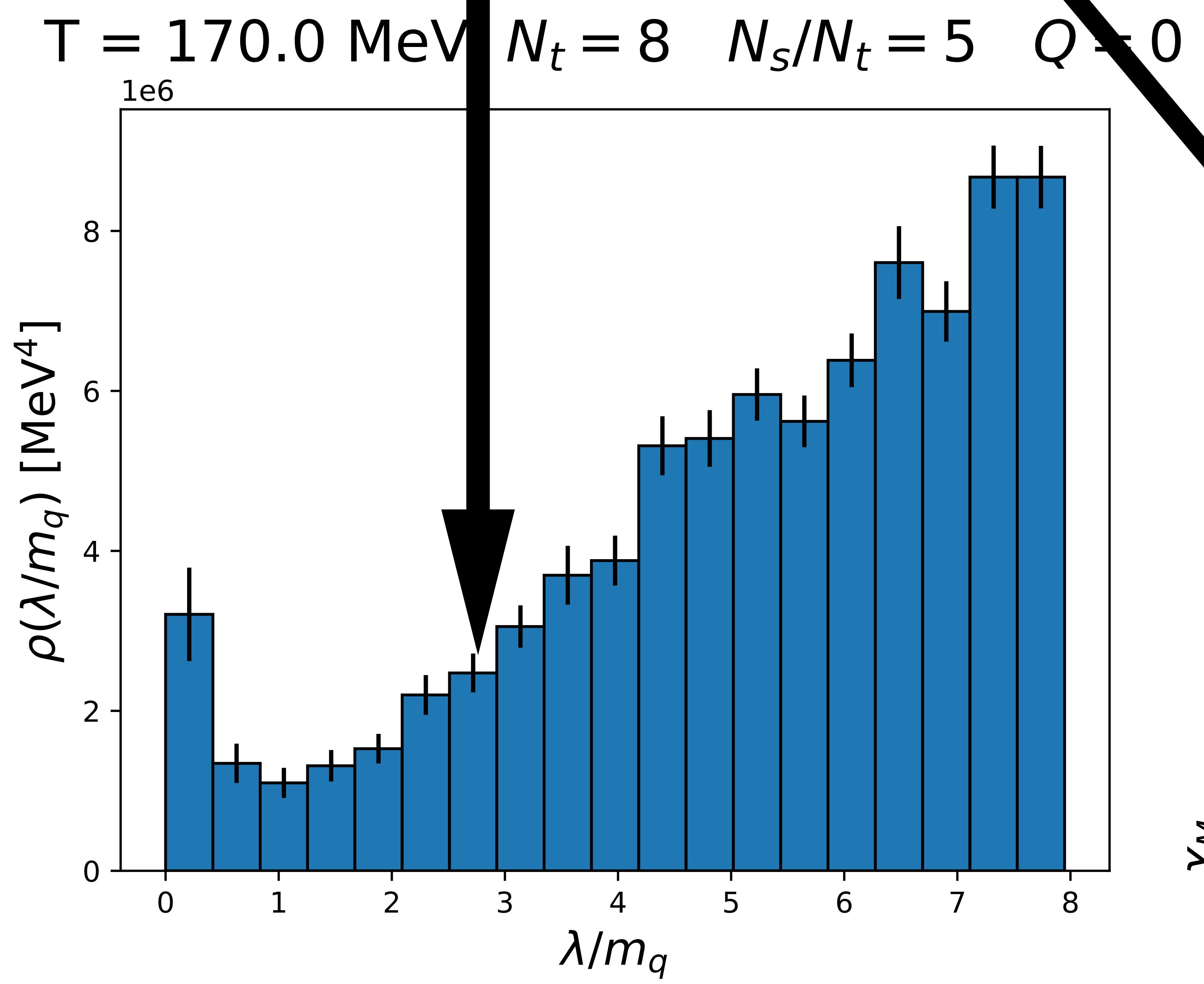
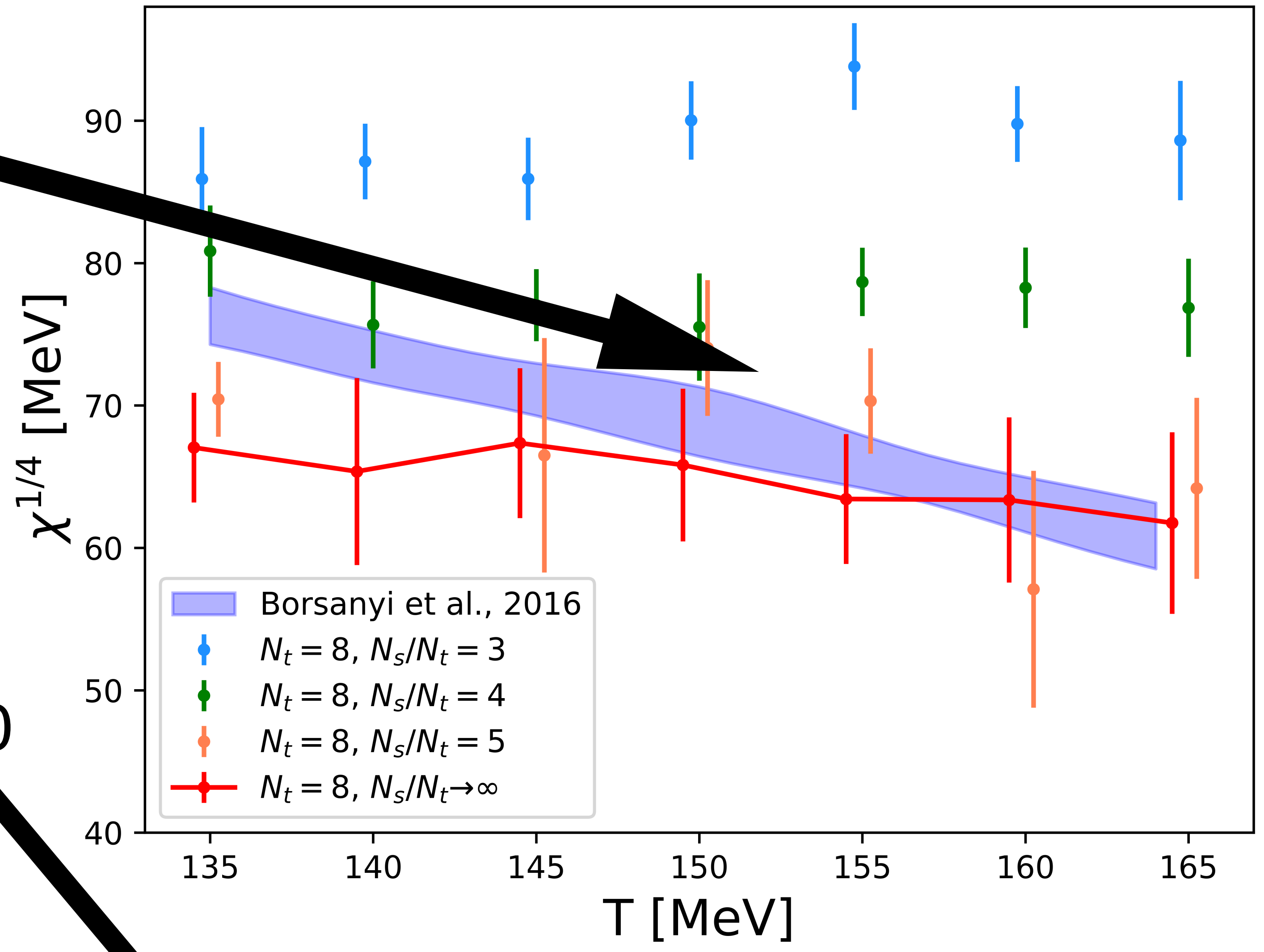
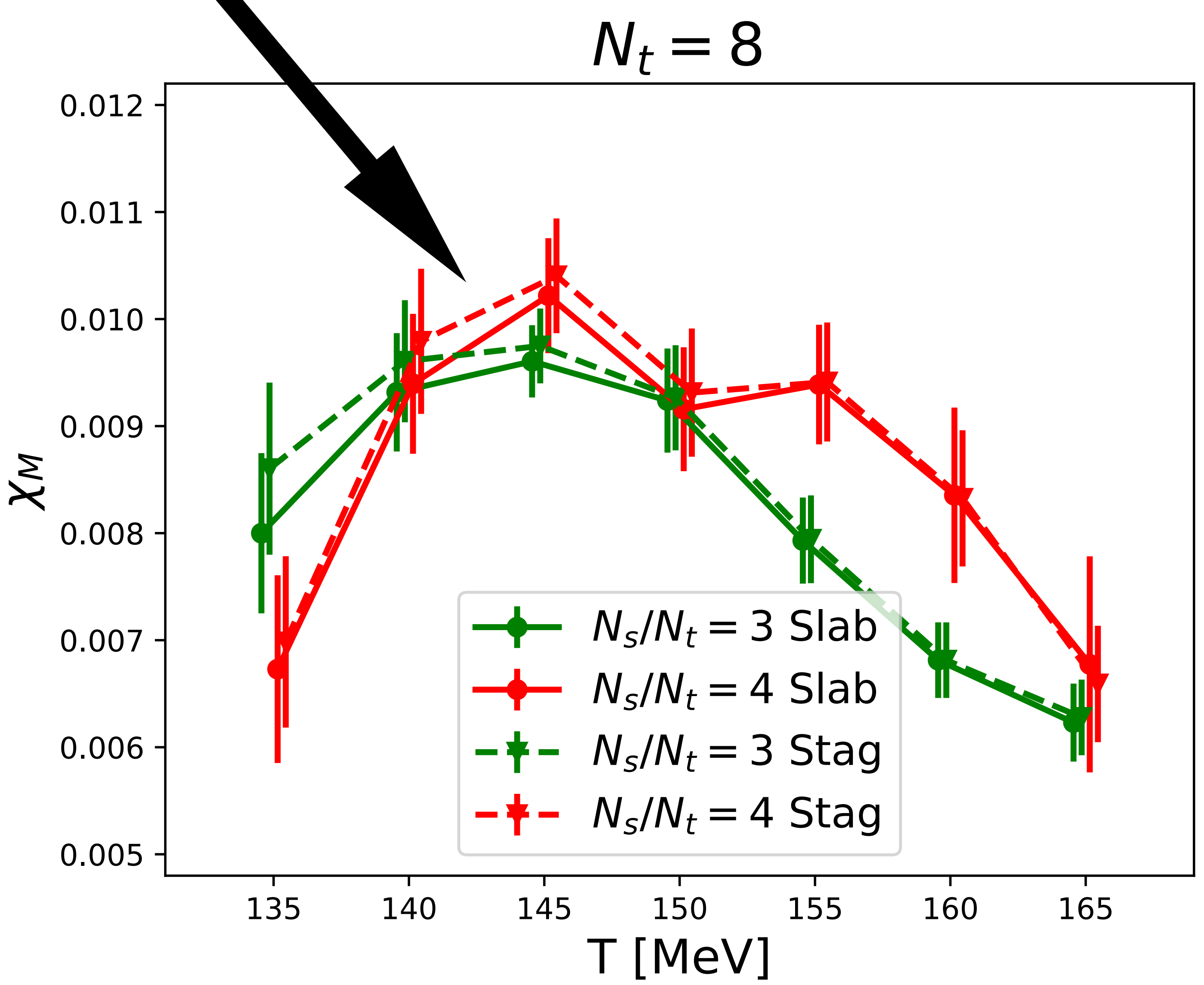






Thank you for your attention!

Backup

Action details

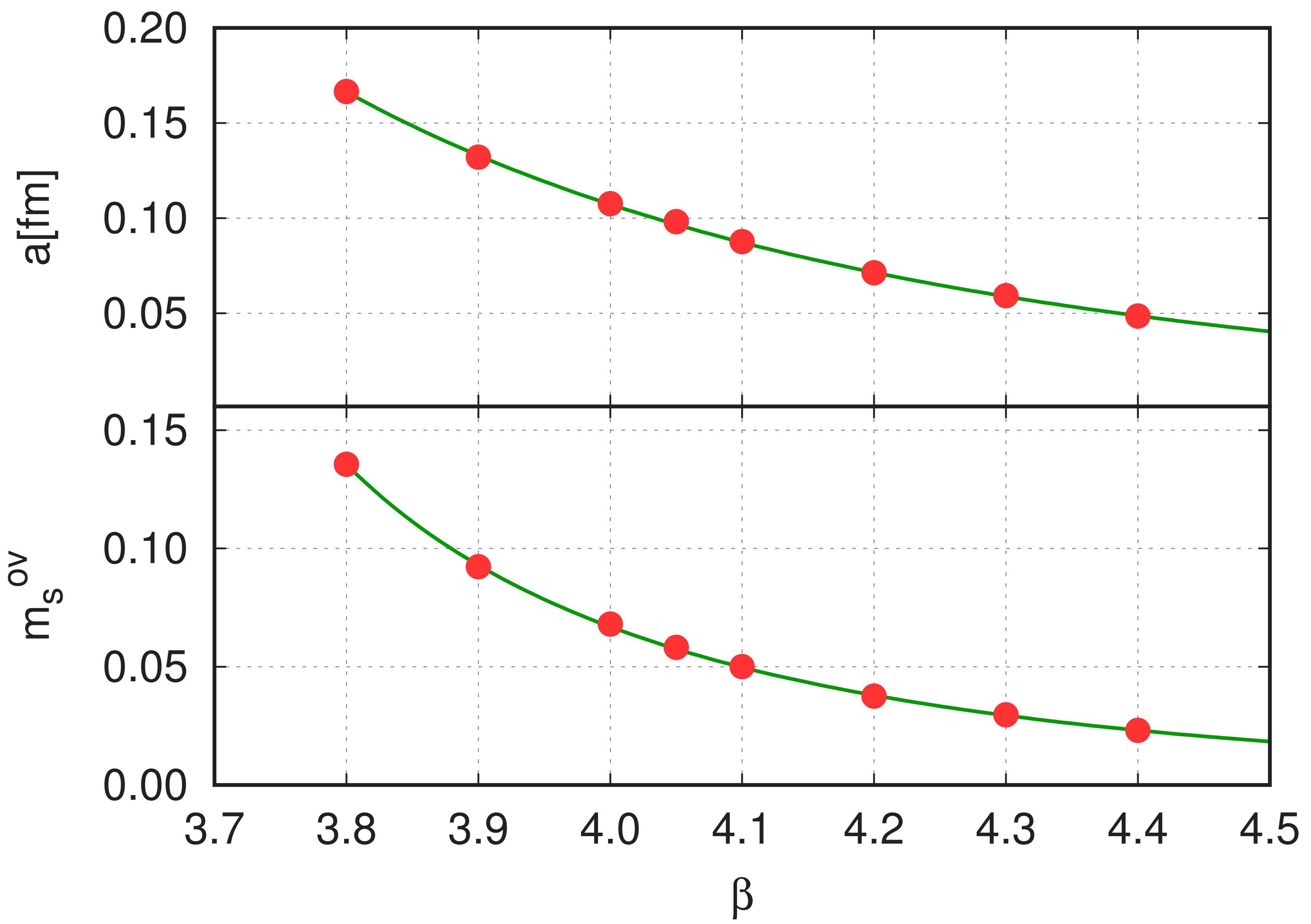
- Symanzik improved gauge action
- Fermion sector: 2 steps of HEX smeared gauge fields
- $N_f = 2 + 1$ flavours of overlap quarks, physical masses
- 2 flavours of Wilson fermions with mass $-m_W$
- Two boson fields with mass $m_B a = 0.54$
- $O(1000 - 10000)$ MD trajectories per point (Q, T, L)

- $a \to 0$: irrelevant
- Keep $Q = \text{const}$ $(Q = 0)$
- Make calculations faster

[Fukaya et al., 2006]

Lattice details, scale setting

Scale setting from simulations with large m_π



- Simulations are done along the LCP
- Scale setting: require $T = 0$ simulations
- $N_f = 3$ staggered simulations, $T = 0$, $w_0^{(3)} = 0.153(1)$ fm, $m_\pi^{(3)} = 712(5)$ MeV
- $N_f = 3$ overlap simulations, $T = 0$, at each β tune m_s^{ov} to have $m_\pi w_0 \equiv m_\pi^{(3)} w_0^{(3)}$
- $N_f = 2 + 1$ overlap simulations, $T \neq 0$: $m_s = m_s^{\text{ov}}, \quad m_{ud} = R m_s^{\text{ov}}, \quad a = w_0^{(3)}/w_0^{\text{ov}}$
- Physical point: $m_{ud} = m_{ud}^{(phys)}, \quad m_s = m_s^{(phys)}$

[Borsanyi et al., 2016]

Implementing odd number of flavours

Exploiting $Q = \text{const}$

- Monte Carlo: determinant of a hermitian operator $H^2 = D_{\text{ov}} D_{\text{ov}}^\dagger$: $N_f = 2$

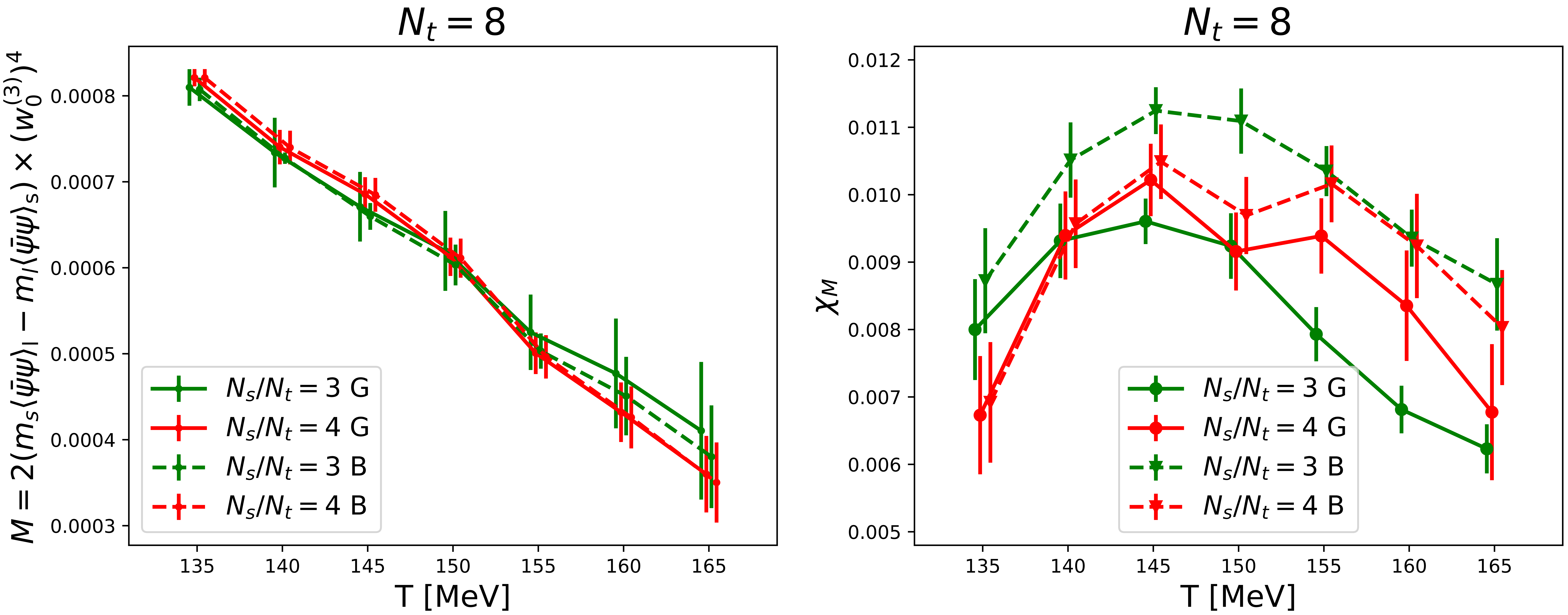
- To simulate $\underline{N_f = 1}$ (strange quark): need to take the square root

- Chirality projectors: $P_\pm = \dfrac{1 \pm \gamma_5}{2}$, $H_\pm^2 = P_\pm H^2 P_\pm$

- Fixed topology $Q = \text{const}$:
  $\det H^2 \sim \det H_+^2 \det H_-^2 \sim (\det H_+^2)^2 \sim (\det H_-^2)^2$

- Take $\det H_+^2$ or $\det H_-^2$

Summing over topological sectors



- <u>Gaussian</u>: $Z_Q/Z_0 = e^{-Q^2/(2\chi V)}$ - central limit theorem
- <u>Bessel</u>: $Z_Q/Z_0 = I_V(\chi V)$ - motivated by free instanton-antiinstanton gas