The chiral phase transition in many flavour QCD

Owe Philipsen

- Chiral phase transition in massless limit constrains the QCD phase diagram
- New picture emerging after 40 years of "common wisdom": 2nd order for all N_f
- Connection to the conformal window

With: F. Cuteri, A. D'Ambrosio, M. Fromm, R. Kaiser, J-P. Klinger, A. Sciarra

CRC-TR 211 Strong-interaction matter under extreme conditions
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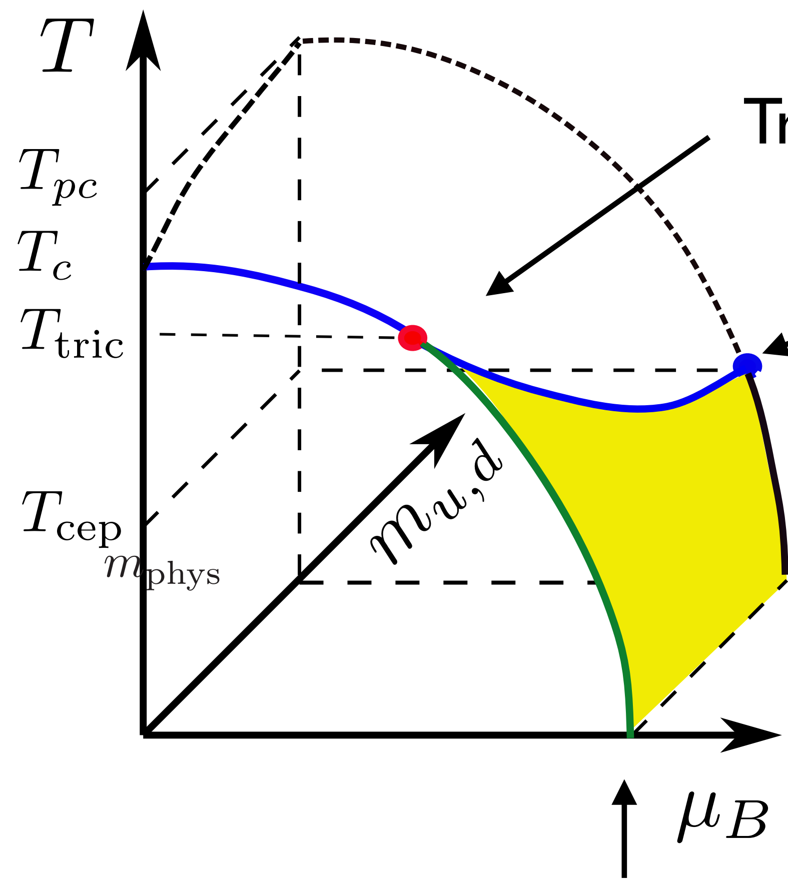
History: motivation for the critical endpoint

[Rajagopal 95, Halasz et al., PRD 98, Stephanov, Rajagopal, Shuryak PRL 98, Rajagopal, Wilczek 00, Hatta, Ikeda, PRD 03,…]

Breaking/restoration of exact chiral symmetry requires a (non-analytic) phase transition

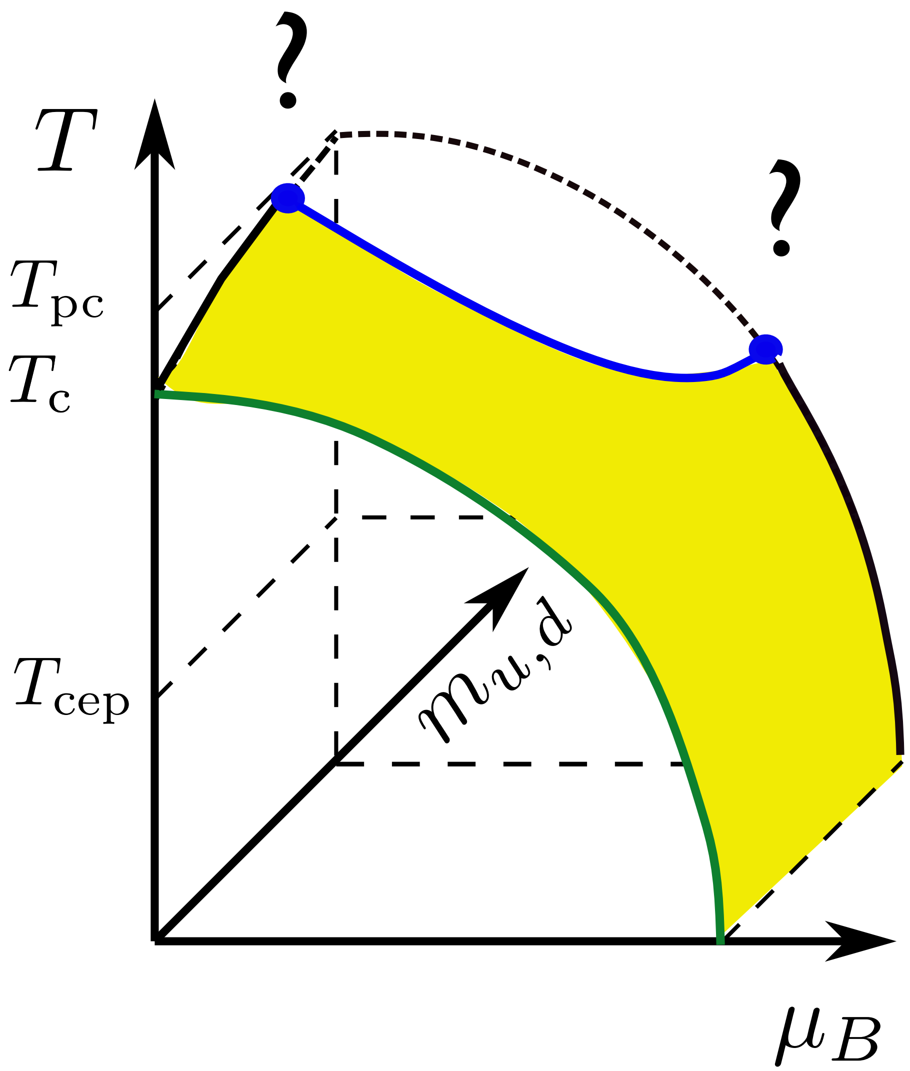
$N_f = 2:$

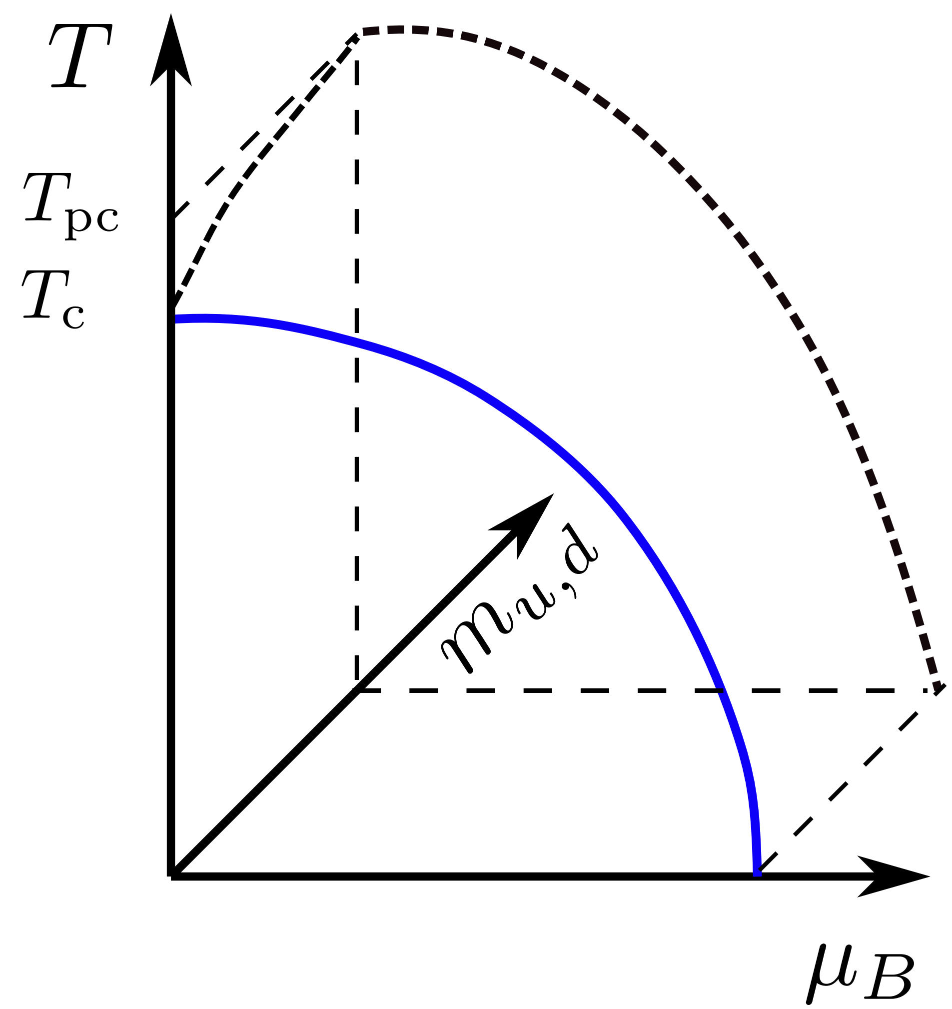
Model predictions, early lattice results →



Model predictions, no QCD information

Other (mostly ignored) possibilities



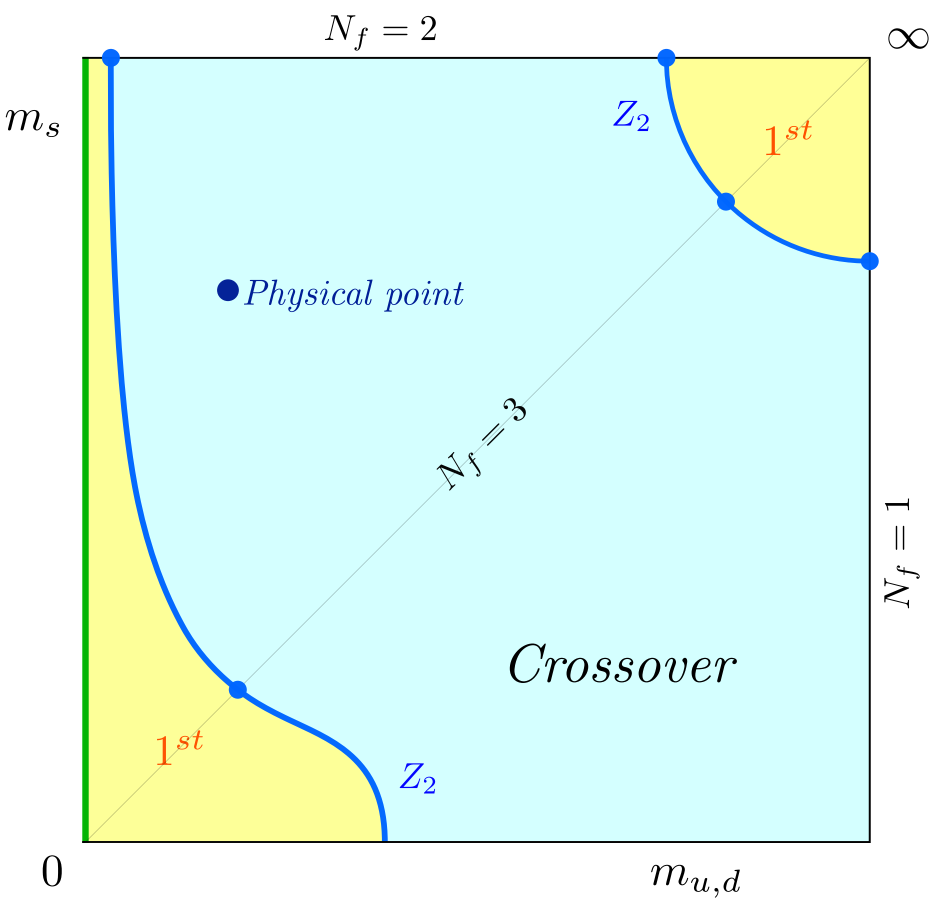


The order of the chiral phase transition at $\mu_B = 0$ narrows down possibilities

Nature of the QCD thermal transition at zero density

$N_f = 2+1$

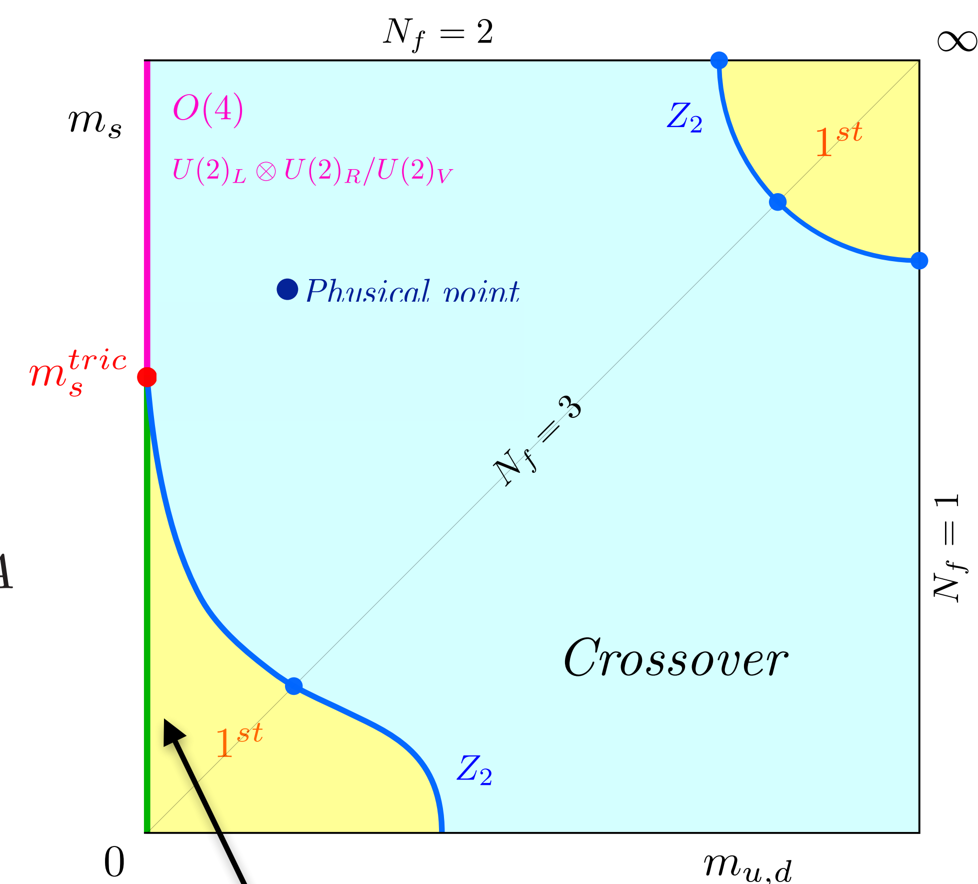
deconfinement p.t.:
breaking of global $Z(3)$ symmetry



[Pisarski, Wilczek, PRD 84]:
(Linear sigma model in 3d)

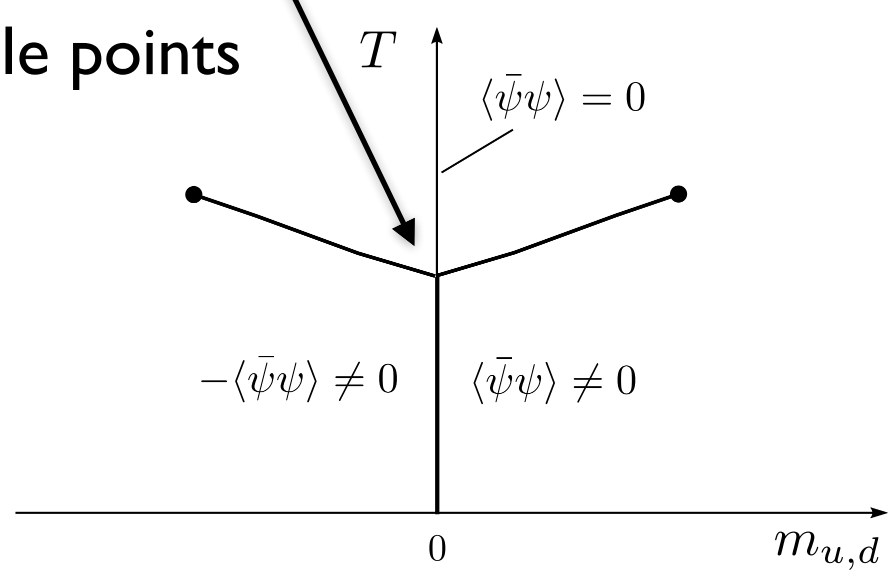
$N_f = 2$ depends on $U(1)_A$

restored broken



$N_f \geq 3$ 1st order

triple points



chiral p.t.
restoration of global symmetry in flavour space

$SU(2)_L \times SU(2)_R \times U(1)_A$

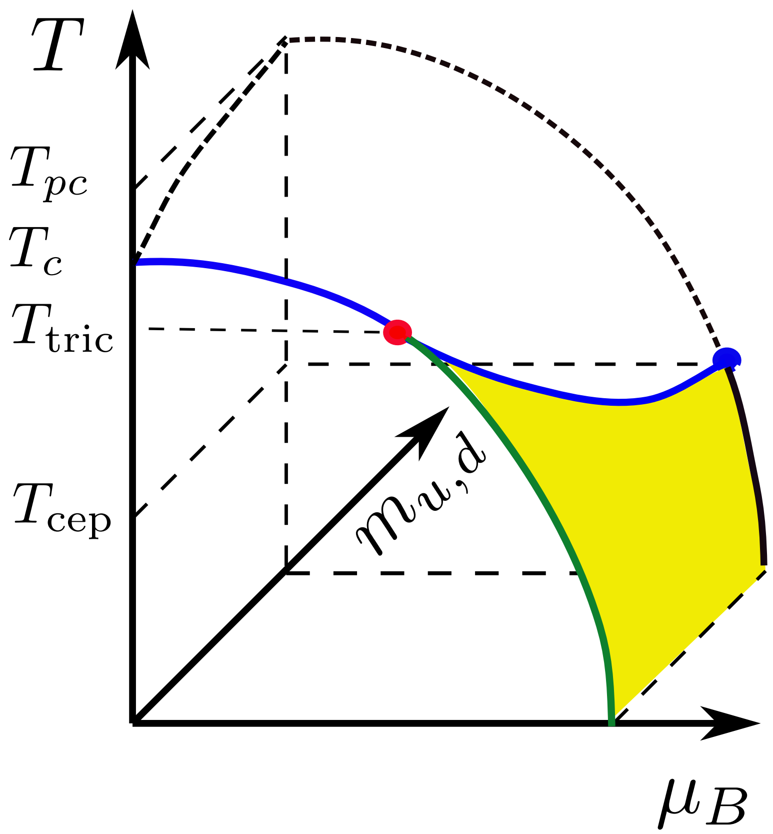
anomalous

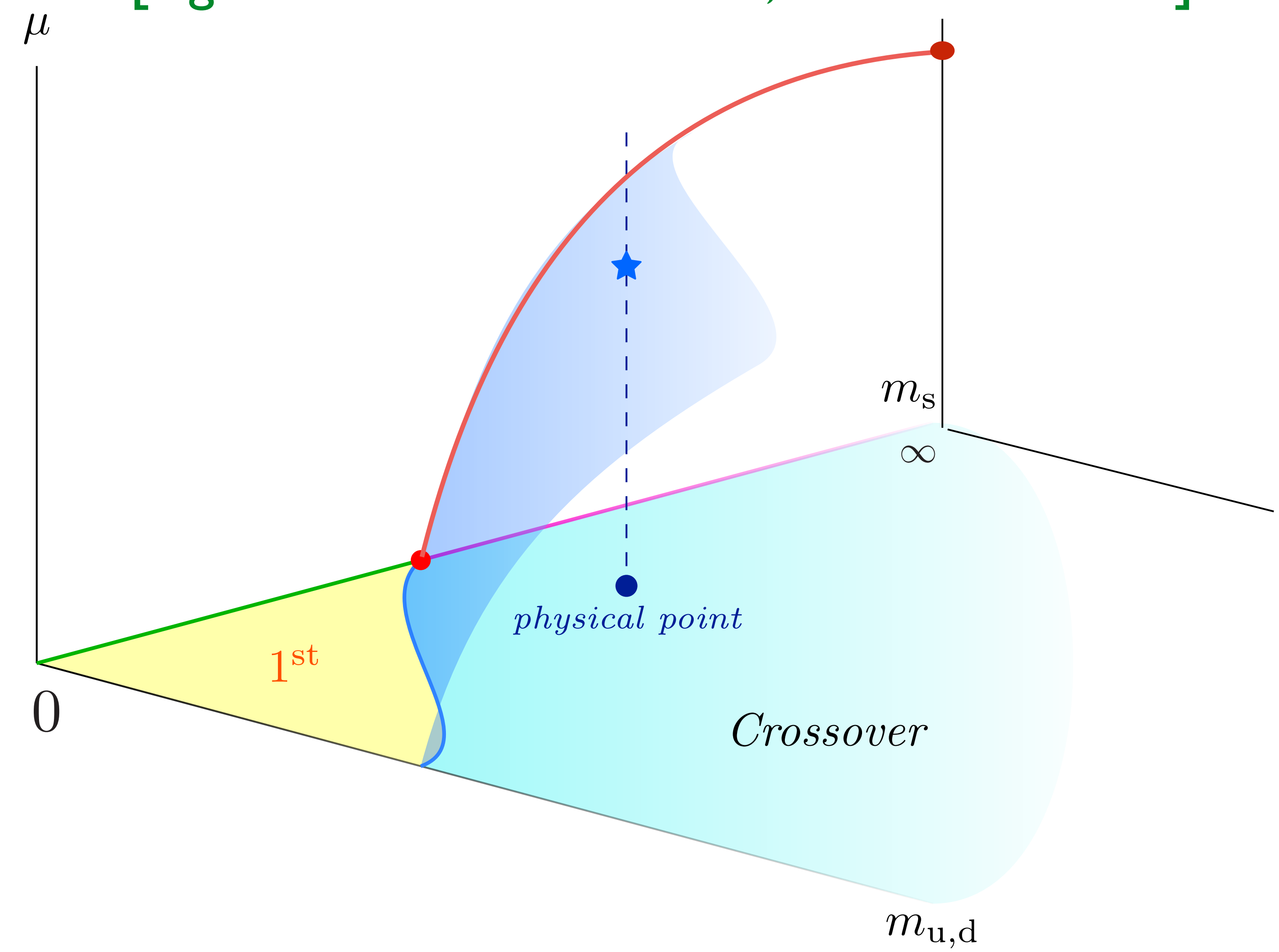
The Columbia plot with chemical potential

$N_f = 2$

$N_f = 2+1$

[Figure edited from Sciarra, PhD thesis 2016]



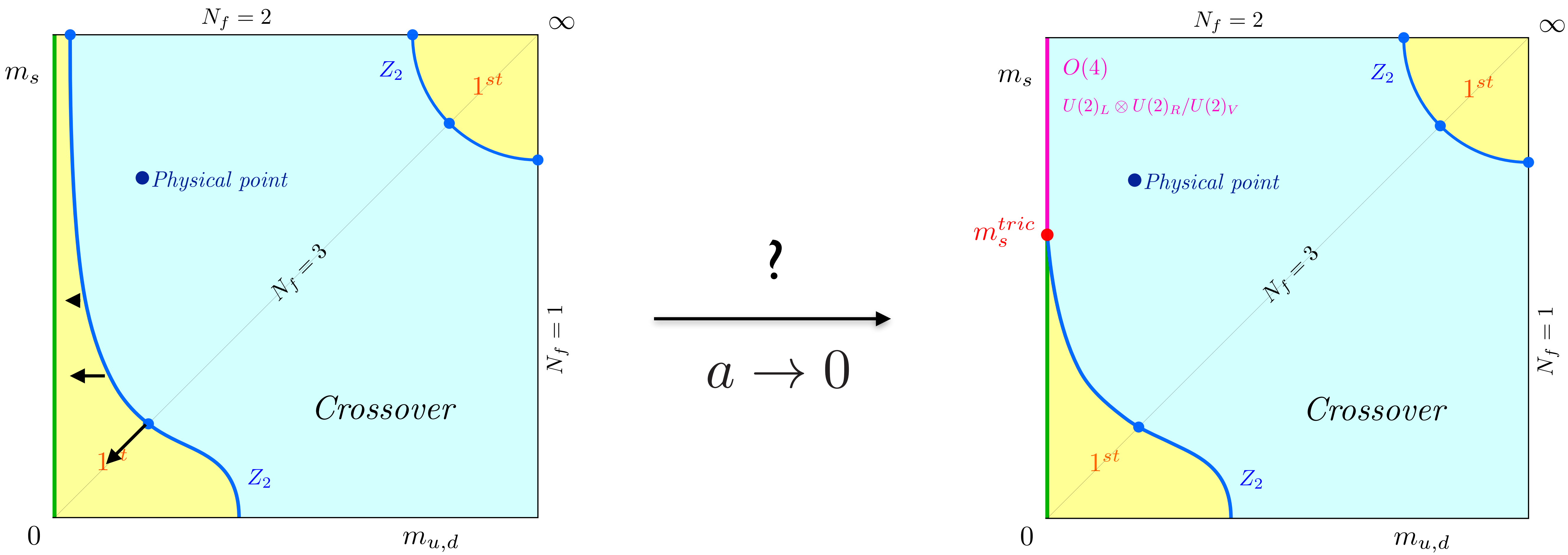


[Stephanov, Rajagopal, Shuryak PRL 98]: (based on models, early lattice)

"As m_s is reduced from infinity, the tricritical point … moves to lower μ until it reaches the T-axis and can be identified with the tricritical point in the (T, m_s)-plane"

The nature of the QCD chiral transition at zero density

…is elusive, massless limit not simulable!

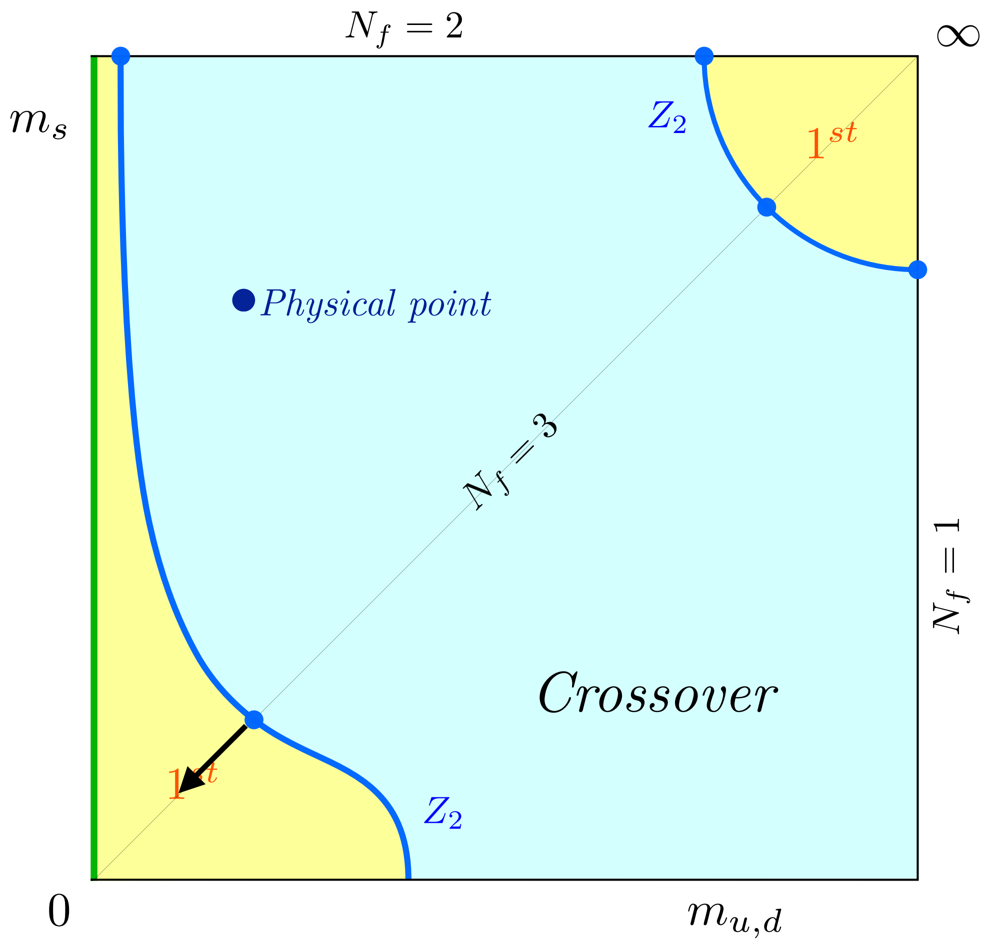


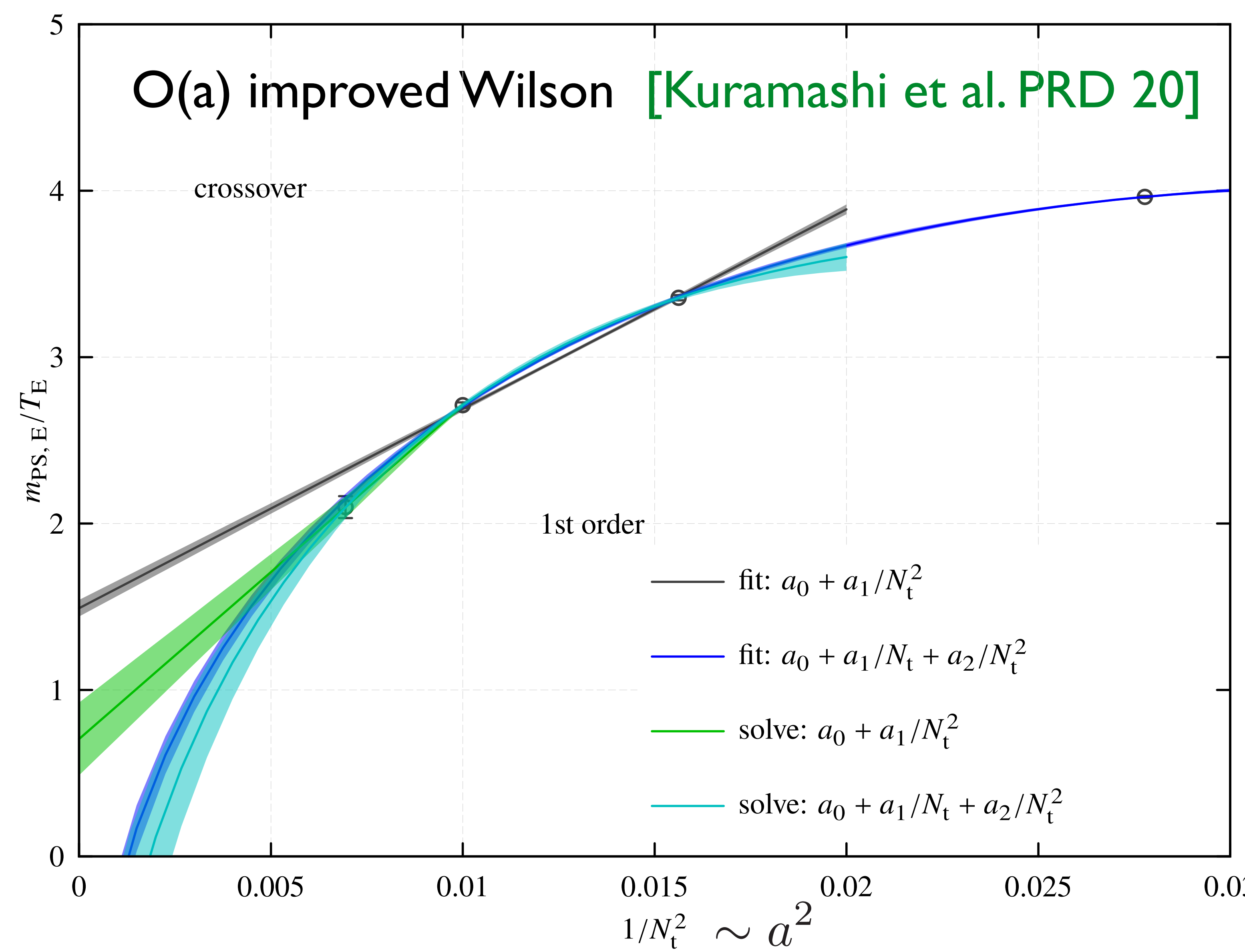
- Coarse lattices or unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \to 0$, no 1st order for improved staggered actions
- For fixed lattice spacing: apparent contradictions between different lattice actions

Details and references: [O.P., Symmetry 13, 2021]

The nature of the QCD chiral transition, Nf=3

…has enormously large cut-off effects!

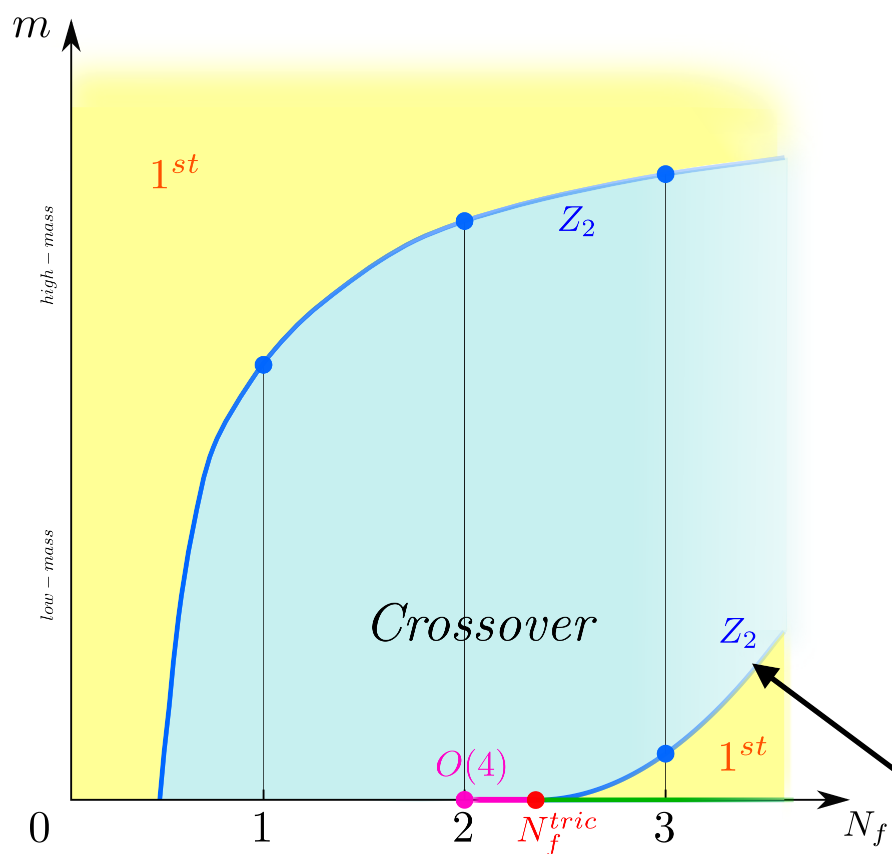


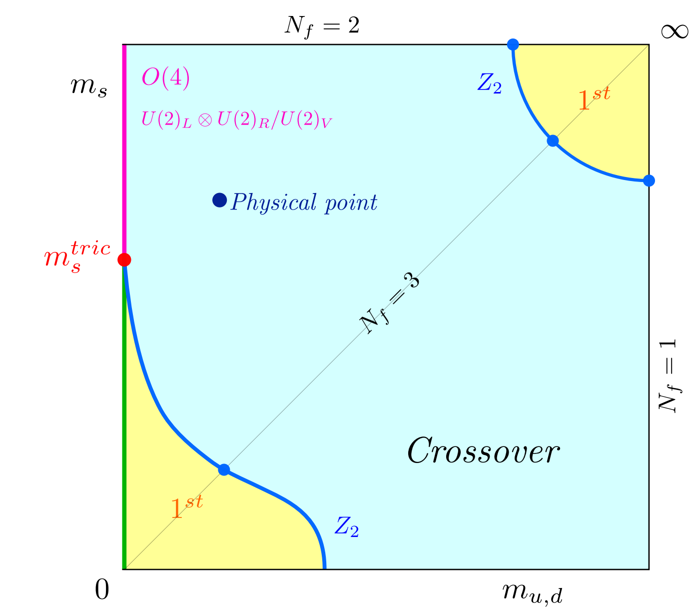


O(a)-improved Wilson:
1st order region shrinks for $a \to 0$

$m_\pi^c \leq 110$ MeV $\quad N_\tau = 4, 6, 8, 10, 12$

Different view point: mass degenerate quarks





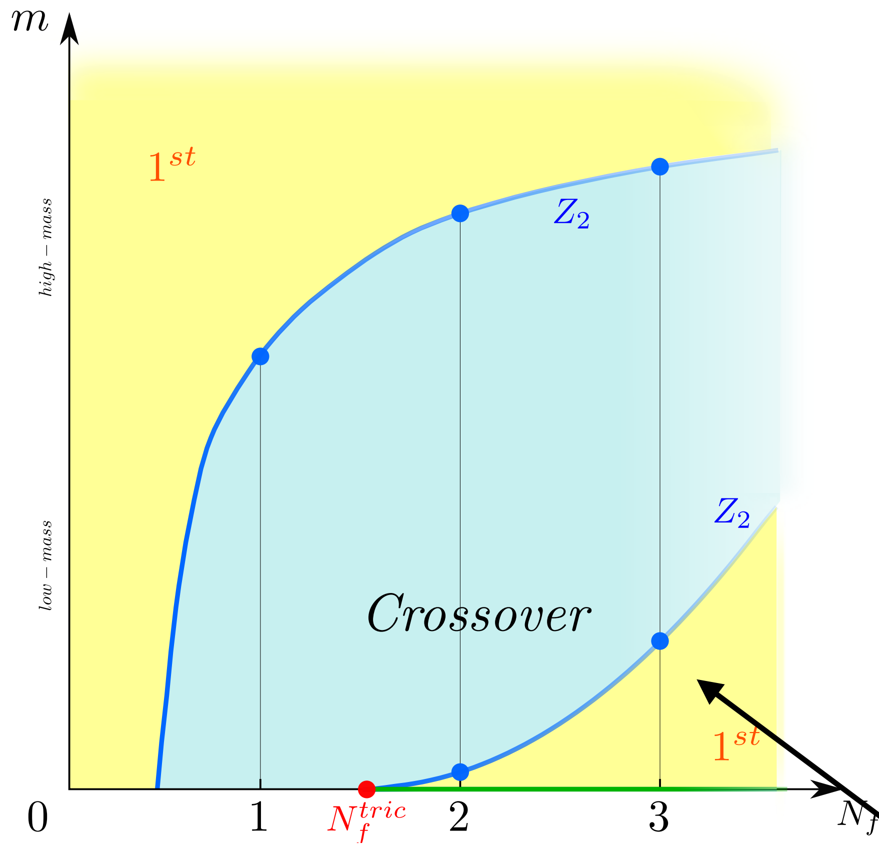
$$Z(N_\mathrm{f}, g, m) = \int \mathcal{D}A_\mu \; (\det M[A_\mu, m])^{N_\mathrm{f}} \; e^{-\mathcal{S}_\mathrm{YM}[A_\mu]}$$

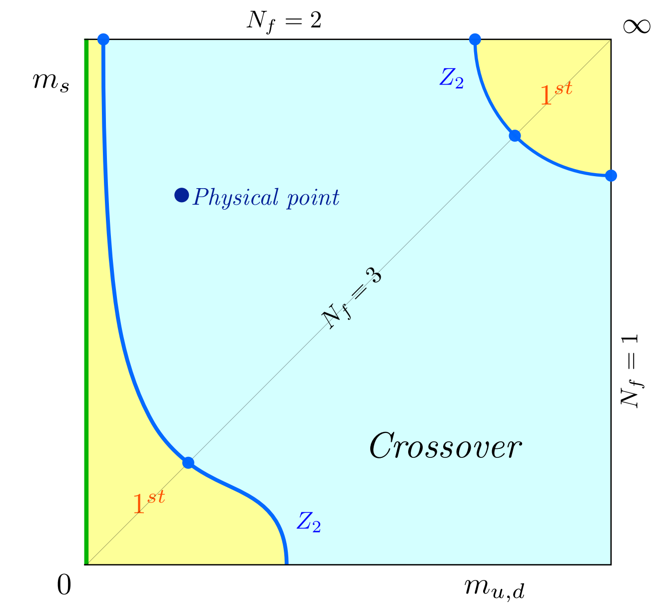
$$N_\mathrm{f}^c(am) = N_\mathrm{f}^\mathrm{tric} + \mathcal{B}_1 \cdot (am)^{2/5} + \mathcal{O}\left((am)^{4/5}\right)$$

- Consider analytic continuation to continuous N_f
- Tricritical point guaranteed to exist if there is 1st order at any N_f
- Known exponents for critical line entering tric. point!
- Continuation to $a \neq 0$: Z(2) surface ends in tricritical line

[Cuteri, O.P., Sciarra PRD 18]

Different view point: mass degenerate quarks





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[Cuteri, O.P., Sciarra PRD 18]

Methodology to determine order of transition

Finite size scaling of generalised cumulants

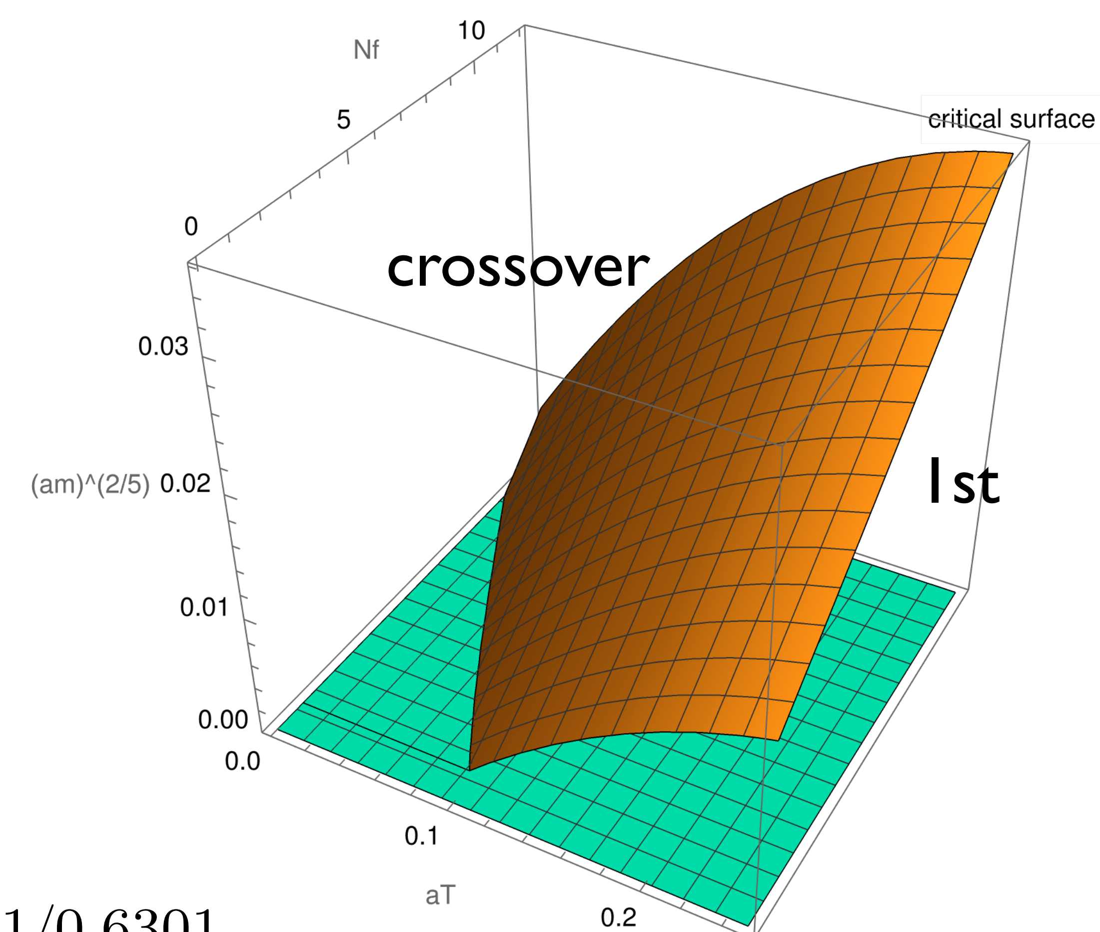
$$B_n = \frac{\langle (\bar\psi\psi - \langle\bar\psi\psi\rangle)^n \rangle}{\langle (\bar\psi\psi - \langle\bar\psi\psi\rangle)^2 \rangle^{n/2}}$$

Standard staggered fermions, bare parameters: β, am, N_f, N_τ

(Pseudo-critical) phase boundary: $B_3 = 0$ 3d manifold

Second-order 3d Ising:

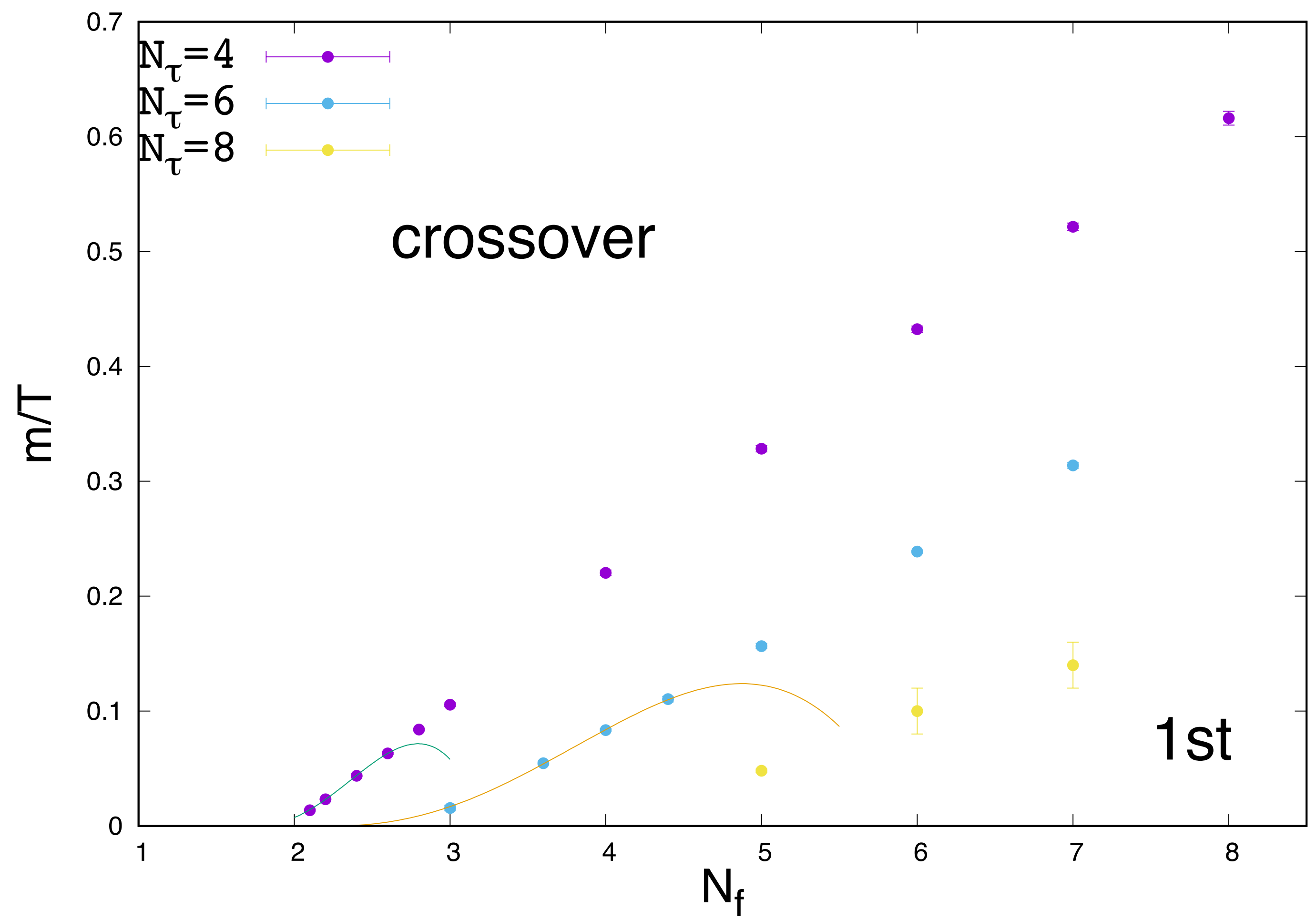
2d chiral critical surface separates 1st order from crossover

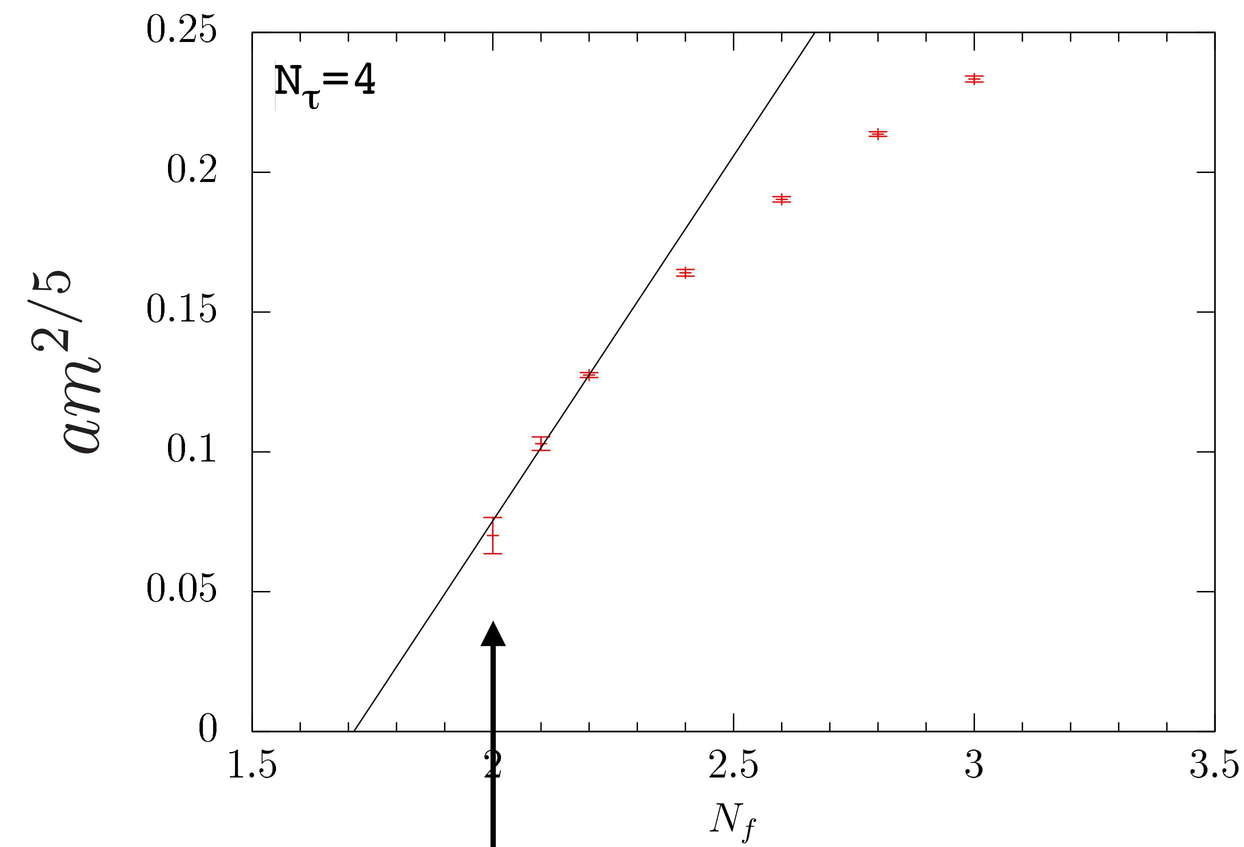


$$B_4(\beta_c, am, N_\sigma) \approx 1.604 + c\,(am - am_c)\,N_\sigma^{1/0.6301}$$

Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21] ~120 M Monte Carlo trajectories with light fermions, aspect ratios 3,4,5

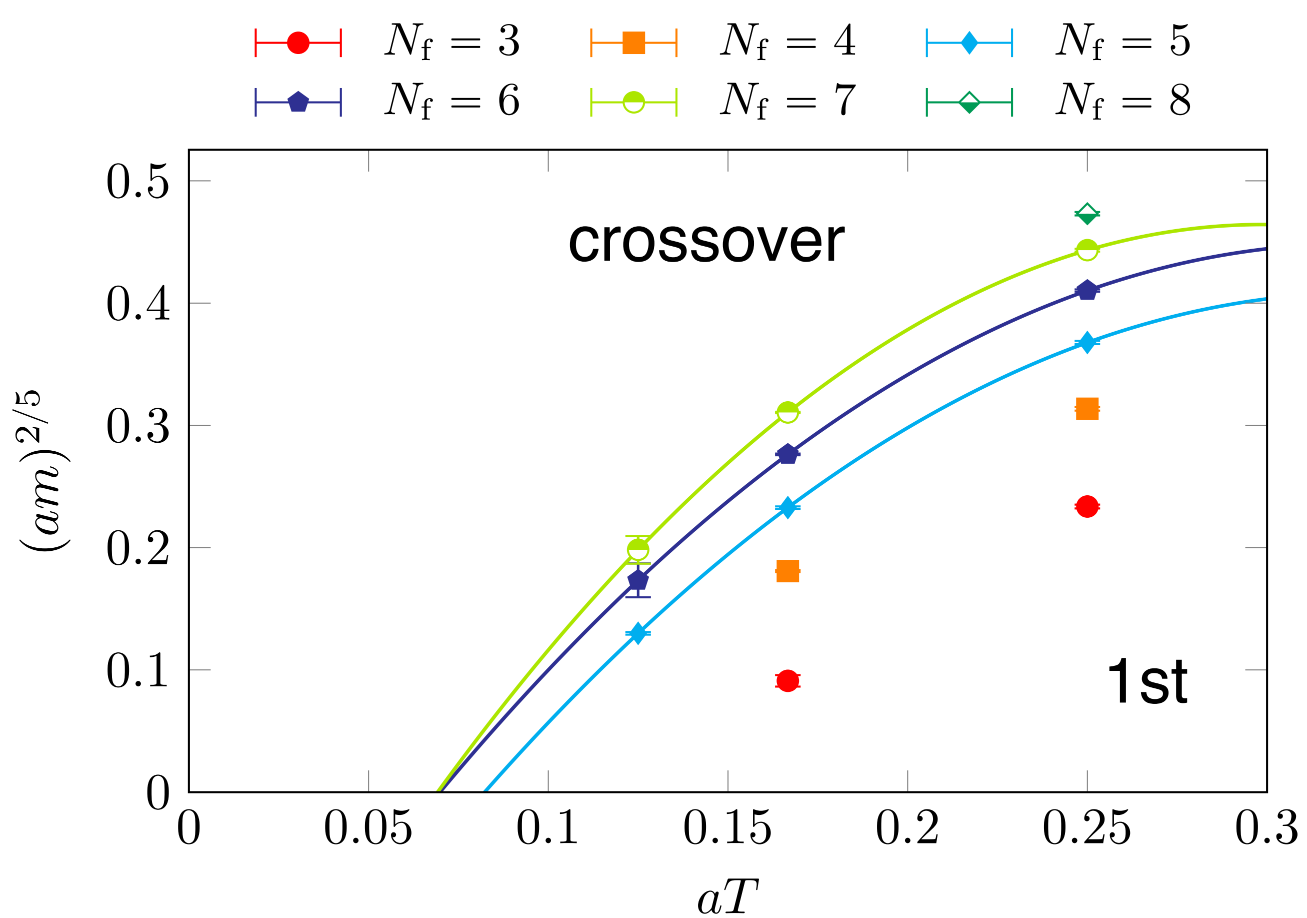




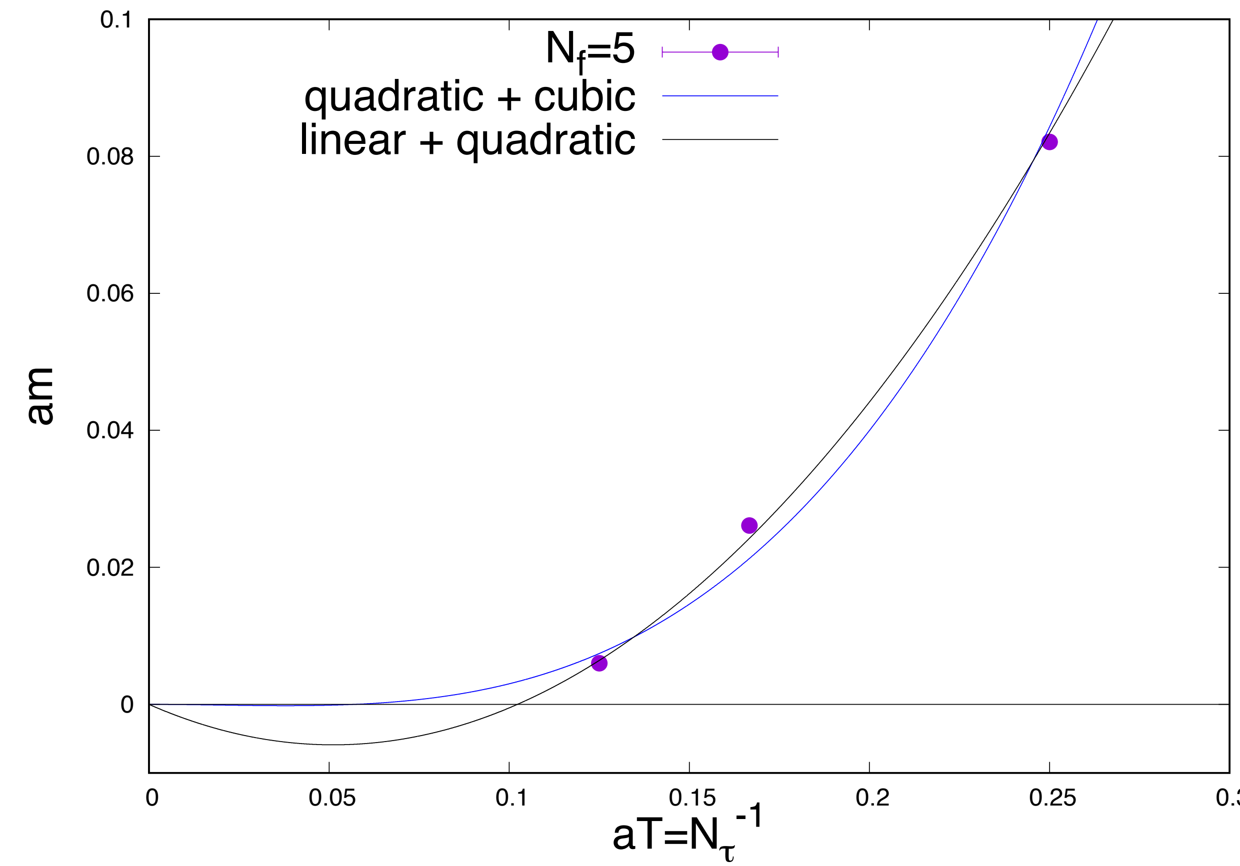
- Tricritical scaling observed in different variable pairings
- Consistent with tric. scaling from finite imaginary μ [Bonati et al. PRD 14]
- Old question: $m_c/T = 0$ or $\neq 0$? Answered for $N_f = 2$
- New question: will $N_f^{\rm tric}$ slide beyond $N_f = 3$?

Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]



1st order scenario does not fit!

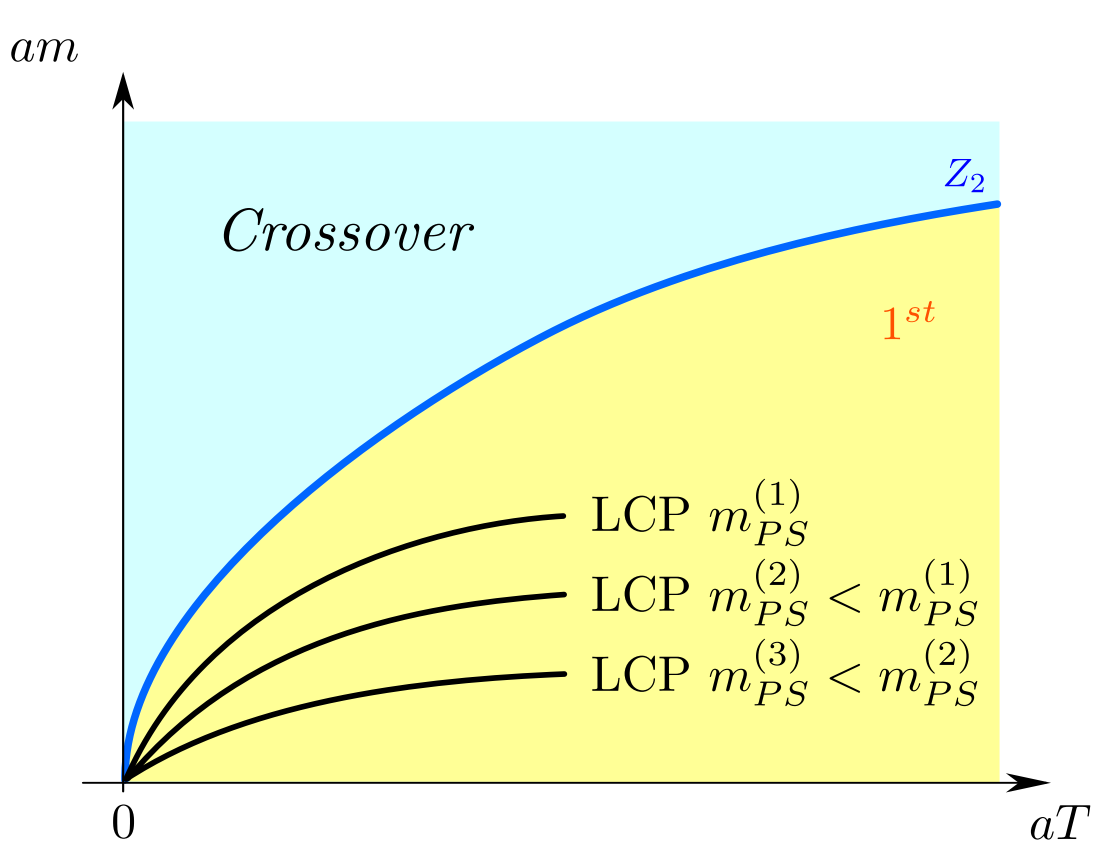


- Tricritical scaling observed also in plane of mass vs. lattice spacing
- Allows extrapolation to lattice chiral limit, tricritical points $N_\tau^{\text{tric}}(N_f)$
- 1st order scenario: $m_c(a) = m_c(0) + c_1(aT) + c_2(aT)^2 + \ldots$

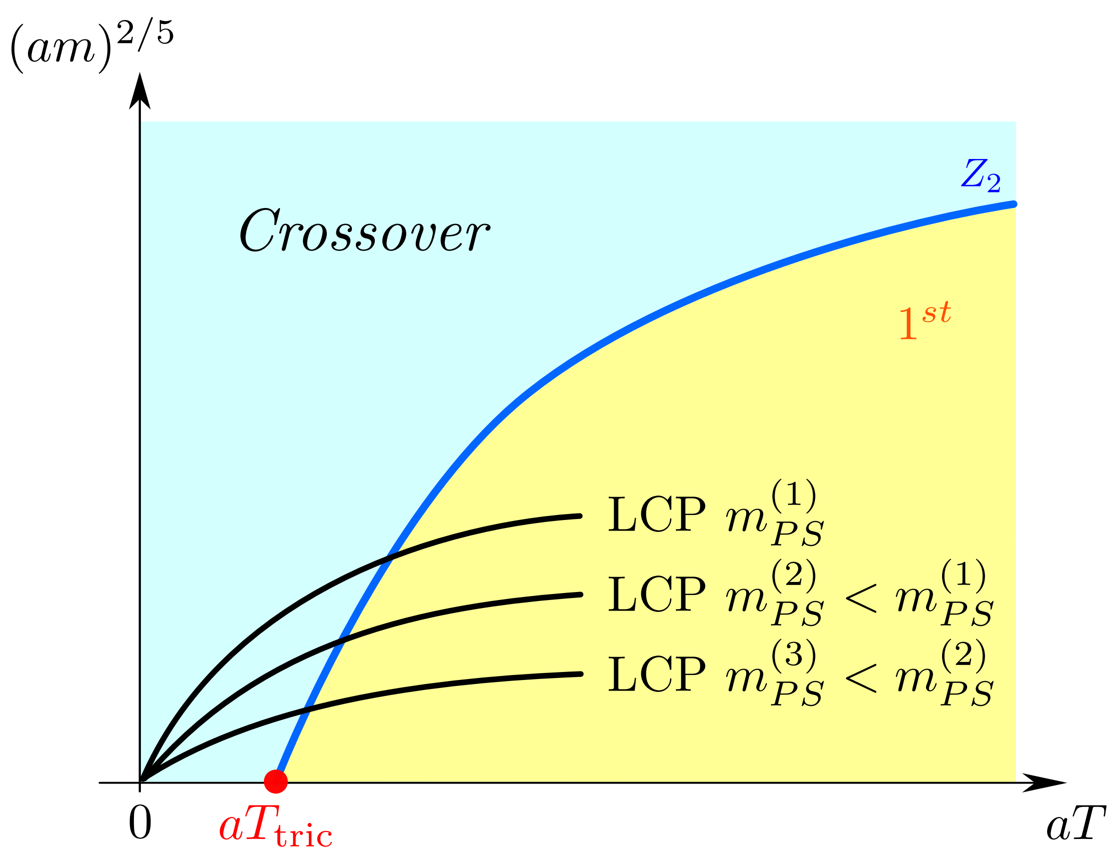
Incompatible with data! $\chi^2_{\text{dof}} > 10$

Implications for the continuum

- Finite $N_\tau^{\rm tric}(N_f)$ implies that 1st order transition is not connected to continuum
- Approaching continuum first, then chiral limit:
  Continuum chiral phase transition second-order!



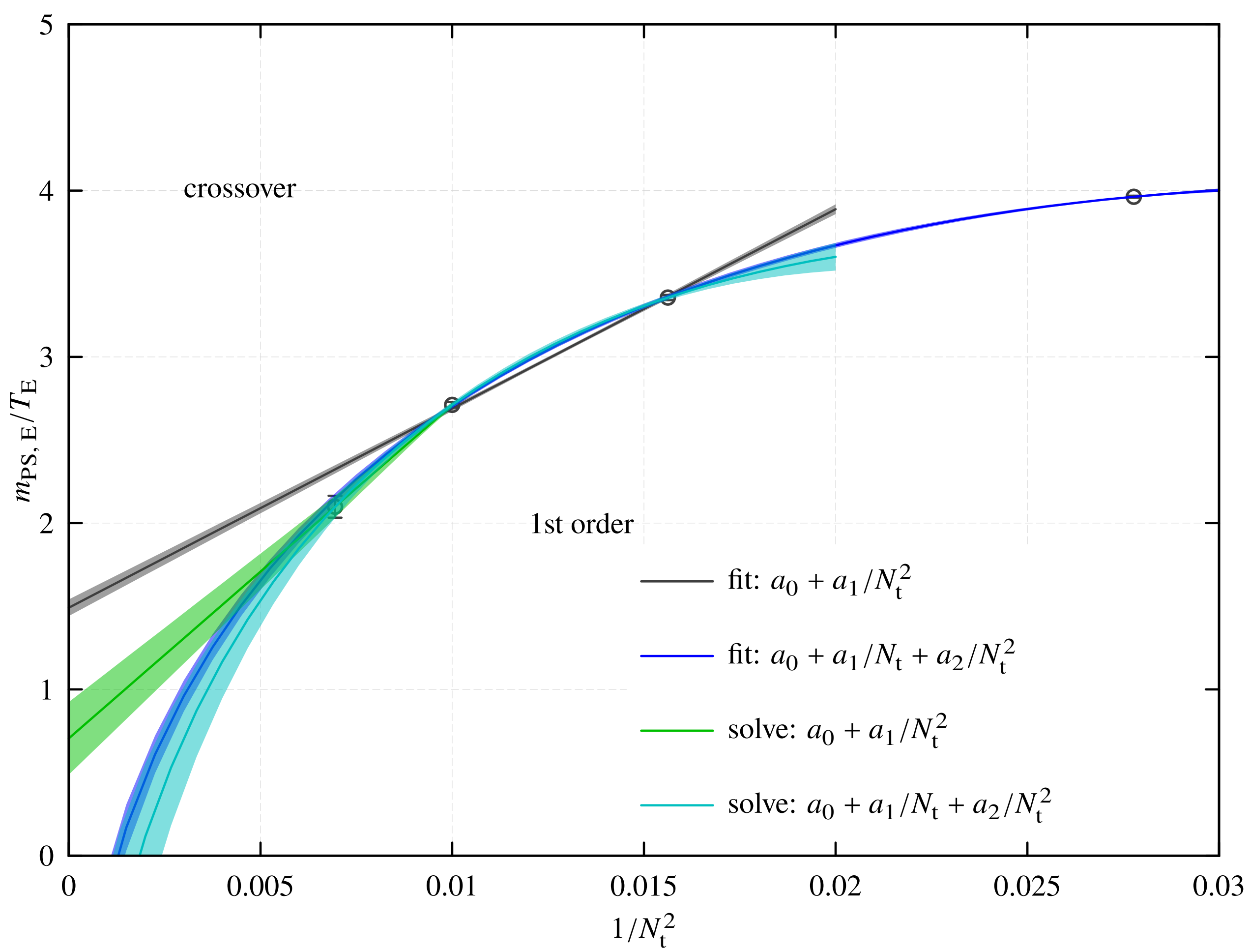
1st order scenario



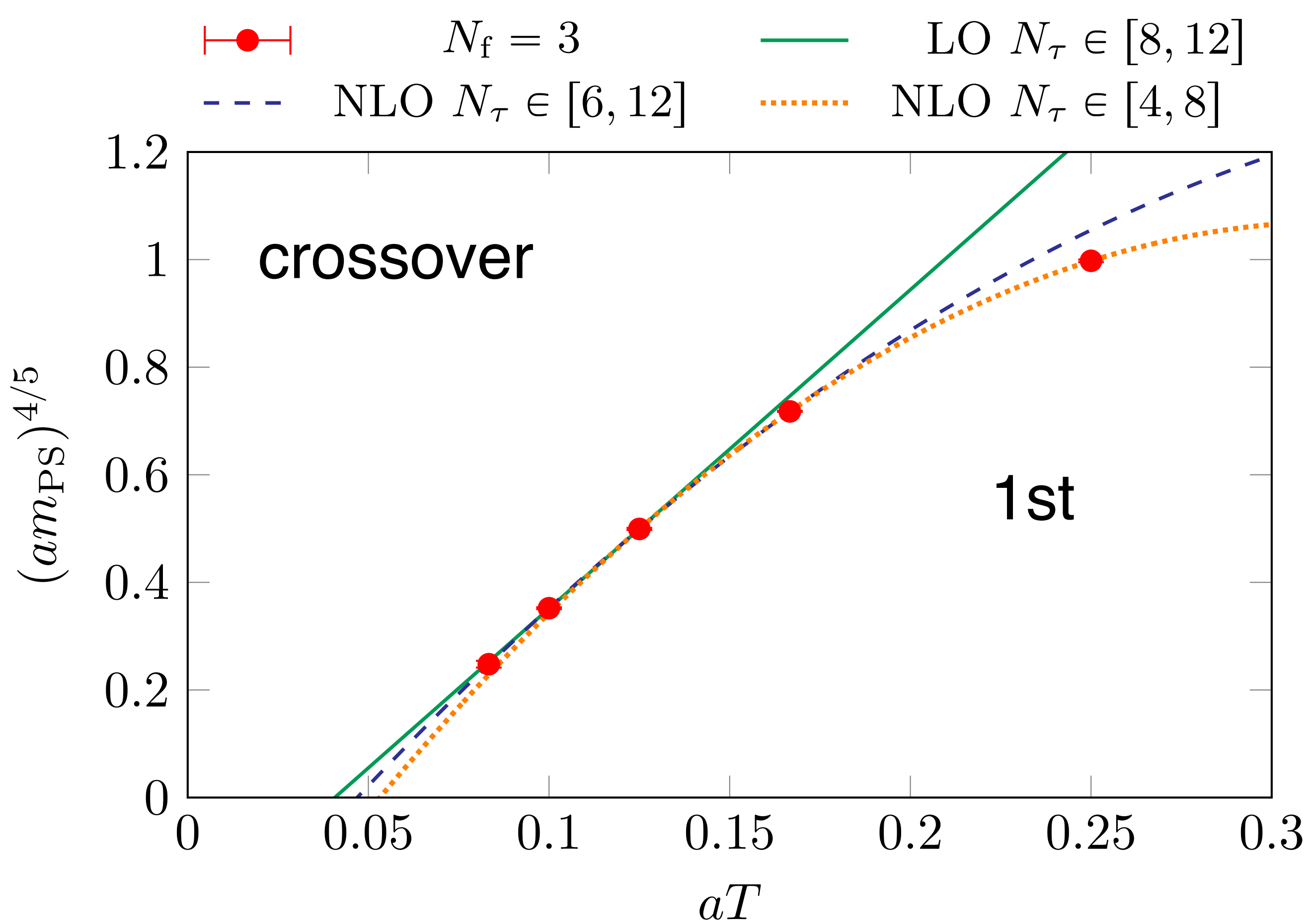
2nd order scenario

Nf=3 O(a)-improved Wilson fermions

[Kuramashi et al. PRD 20] $m_\pi^c \leq 110\ \mathrm{MeV} \quad N_\tau = 4, 6, 8, 10, 12$



Re-analysis using: $am_{PS}^2 \propto am_q$



[Cuteri, O.P., Sciarra, JHEP 21]

Tricritical scaling!

Nf=3 consistent with staggered, 2nd order in chiral continuum limit!

The Columbia plot in the continuum

[Cuteri, O.P., Sciarra JHEP 21]

[HotQCD PRL 19]

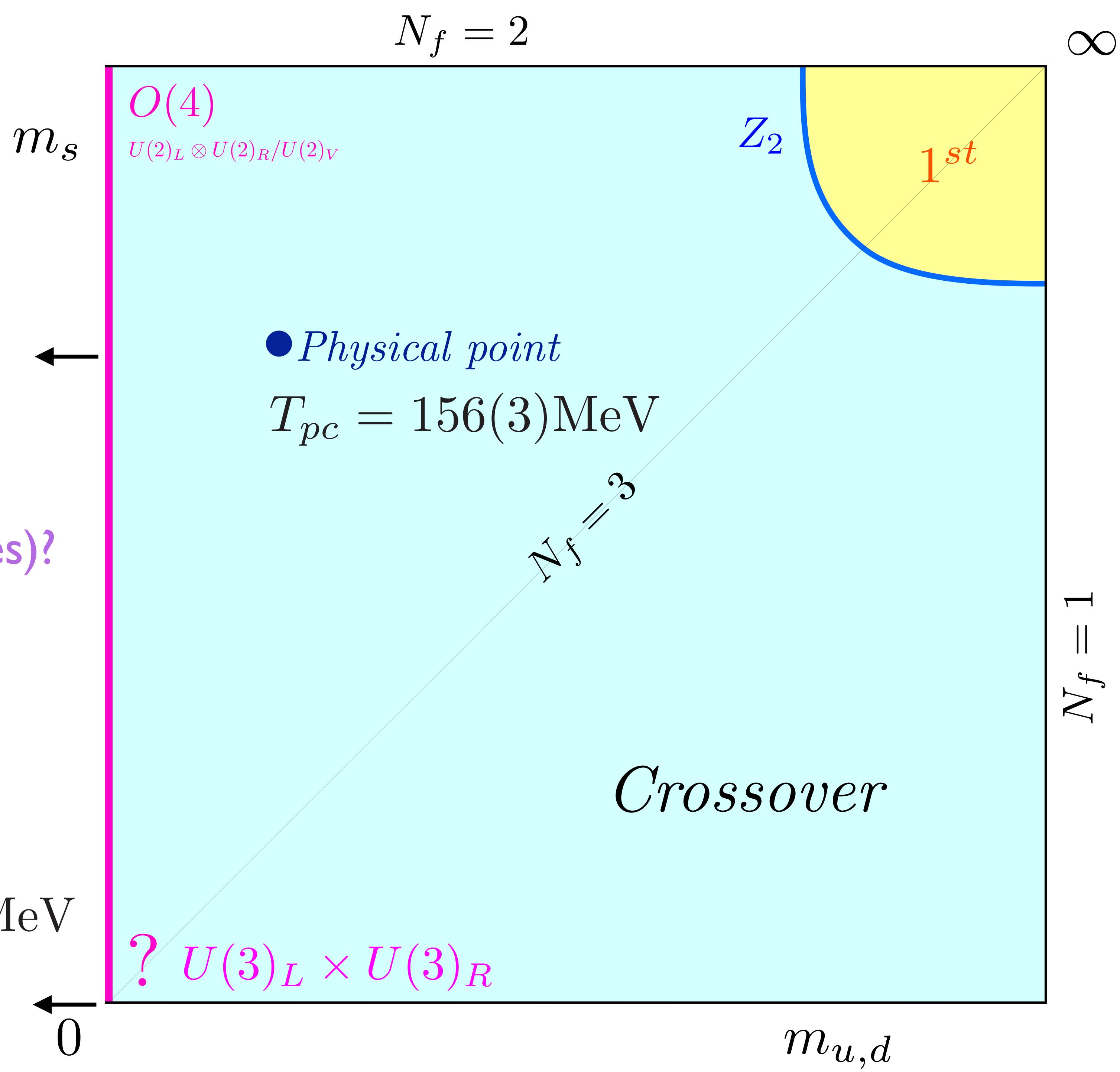
$T_c^0 = 135(8)\mathrm{MeV}$

Universality class(es)?

[HotQCD PRD 22]

HISQ, crossover down to $m_\pi \approx 45\mathrm{MeV}$

$T_c^0 = 98(6)\mathrm{MeV}$



Crossover for DW fermions, Nf=3, $m_q \sim m_{phys}$ [Zhang et al., PoS LAT22, 23]

QCD with imaginary chemical potential

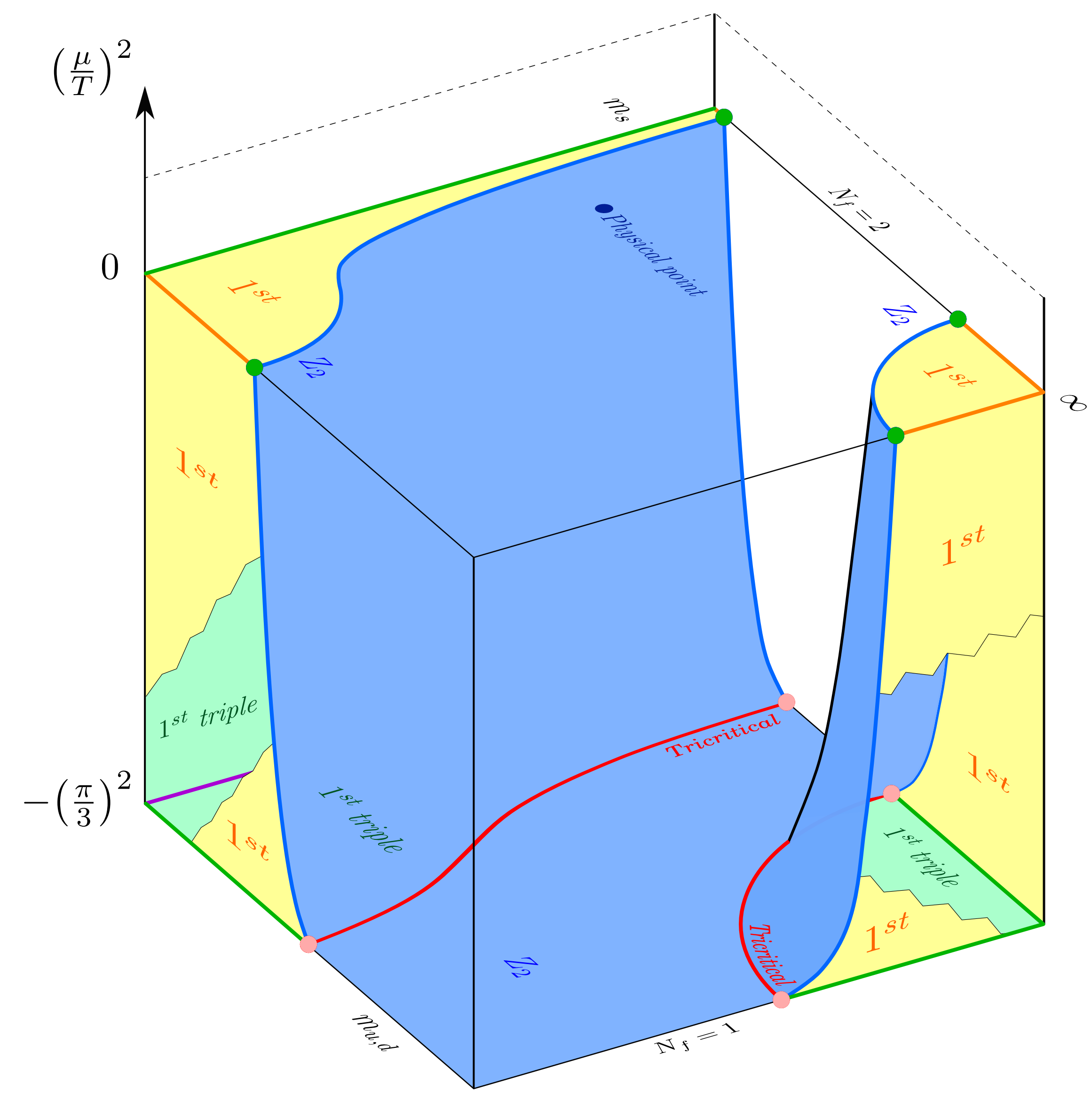
Motivation: no sign problem!

Roberge-Weiss (center) symmetry: $Z\left(T, i\frac{\mu_i}{T}\right) = Z\left(T, i\frac{\mu_i}{T} + i\frac{2n\pi}{N_c}\right)$

Results from coarse lattices: $N_\tau = 4$

Chiral critical surface analytic around $\mu_B = 0$, negative curvature
[de Forcrand, O.P. 07]

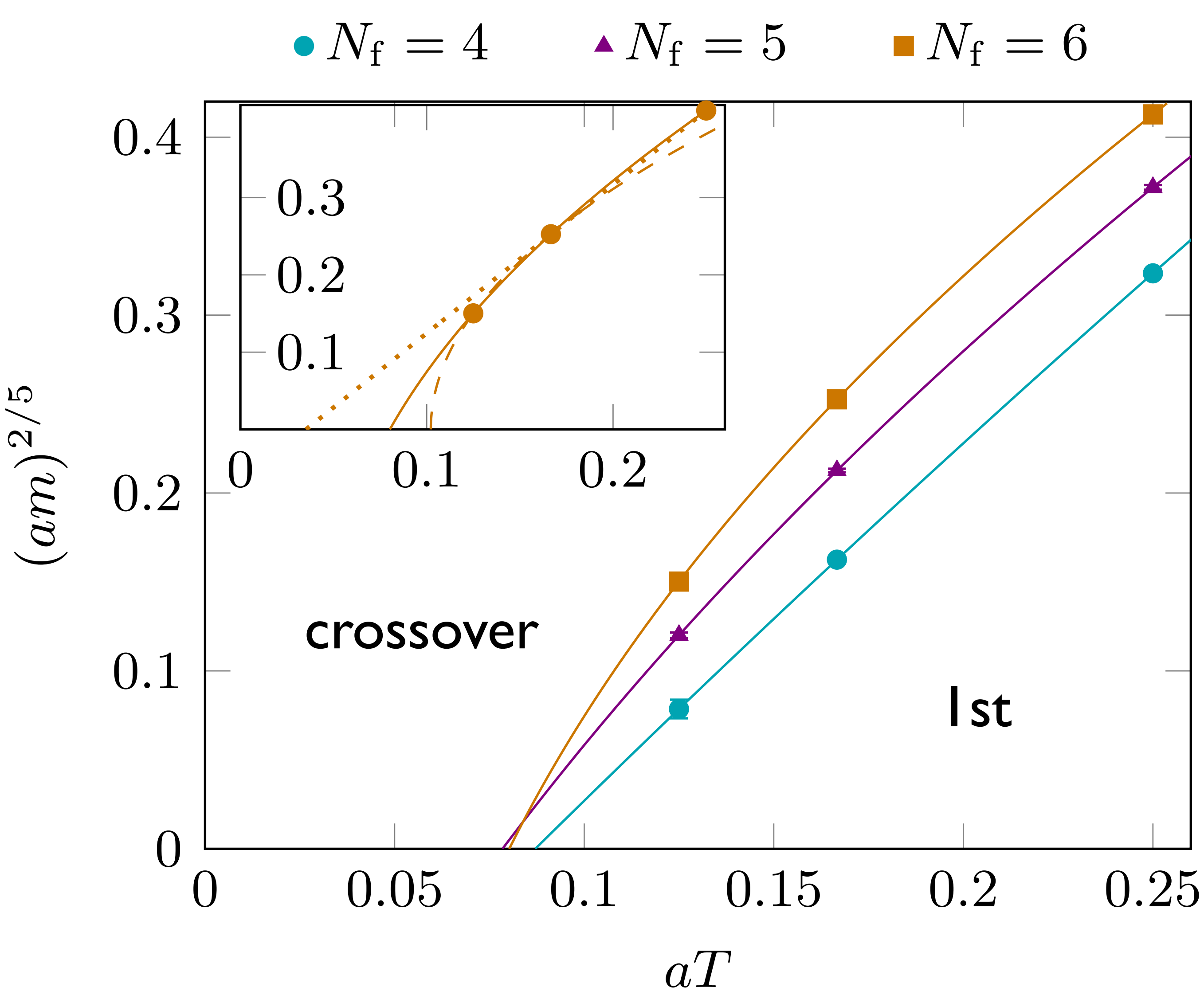
Details and reference list:
[O.P., Symmetry 13, 2021]



From (Philipsen and Sciarra 2020)

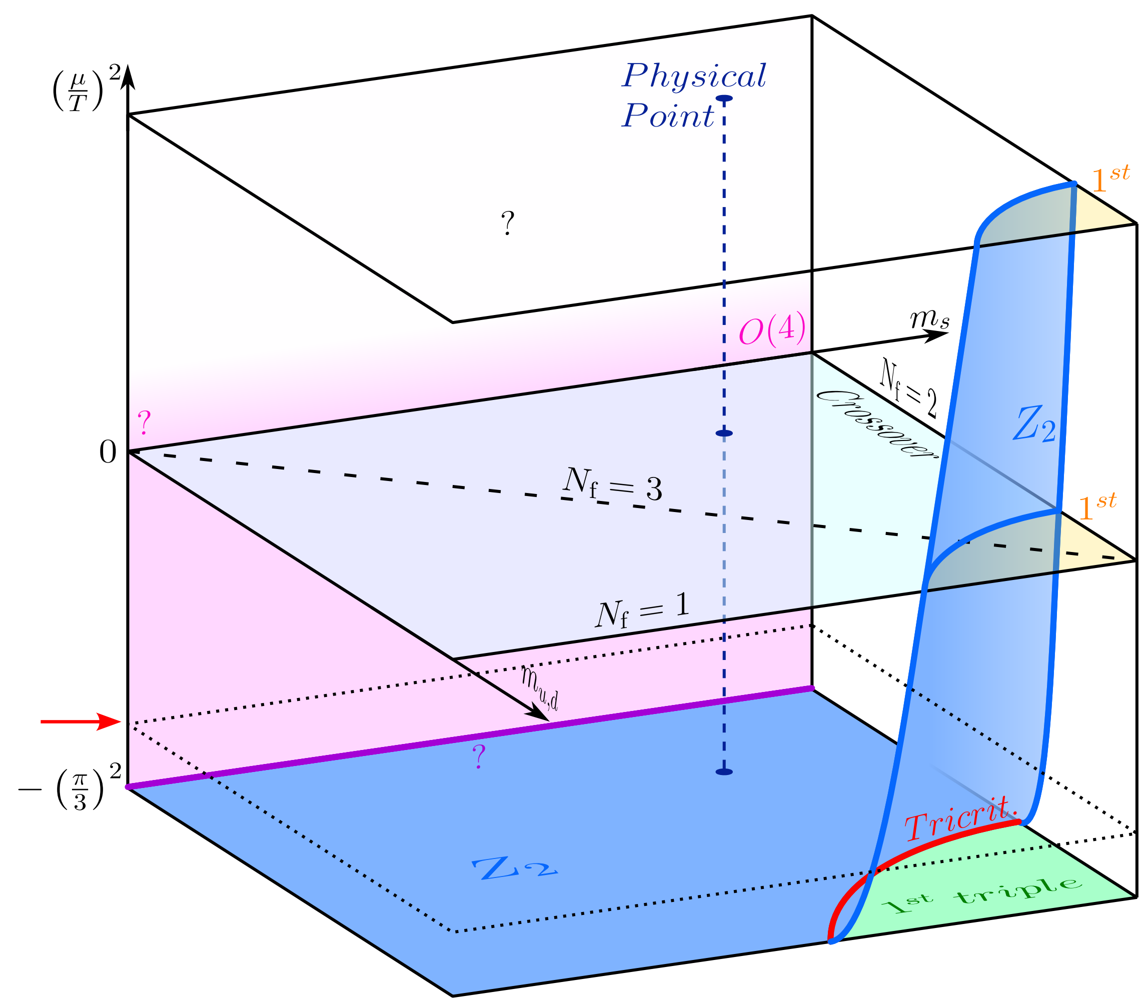
Imaginary chemical potential: cutoff effects

Repeat study of Columbia plot with $\mu = i\,0.81\pi T/3$



Same situation as $\mu = 0$

1st-order region not connected to continuum limit!



Imaginary chemical potential, improved actions

$\mu = i\pi T/3$ Roberge-Weiss boundary

- [Bonati et al., PRD 19]
  stout-smeared staggered $N_\tau = 4$

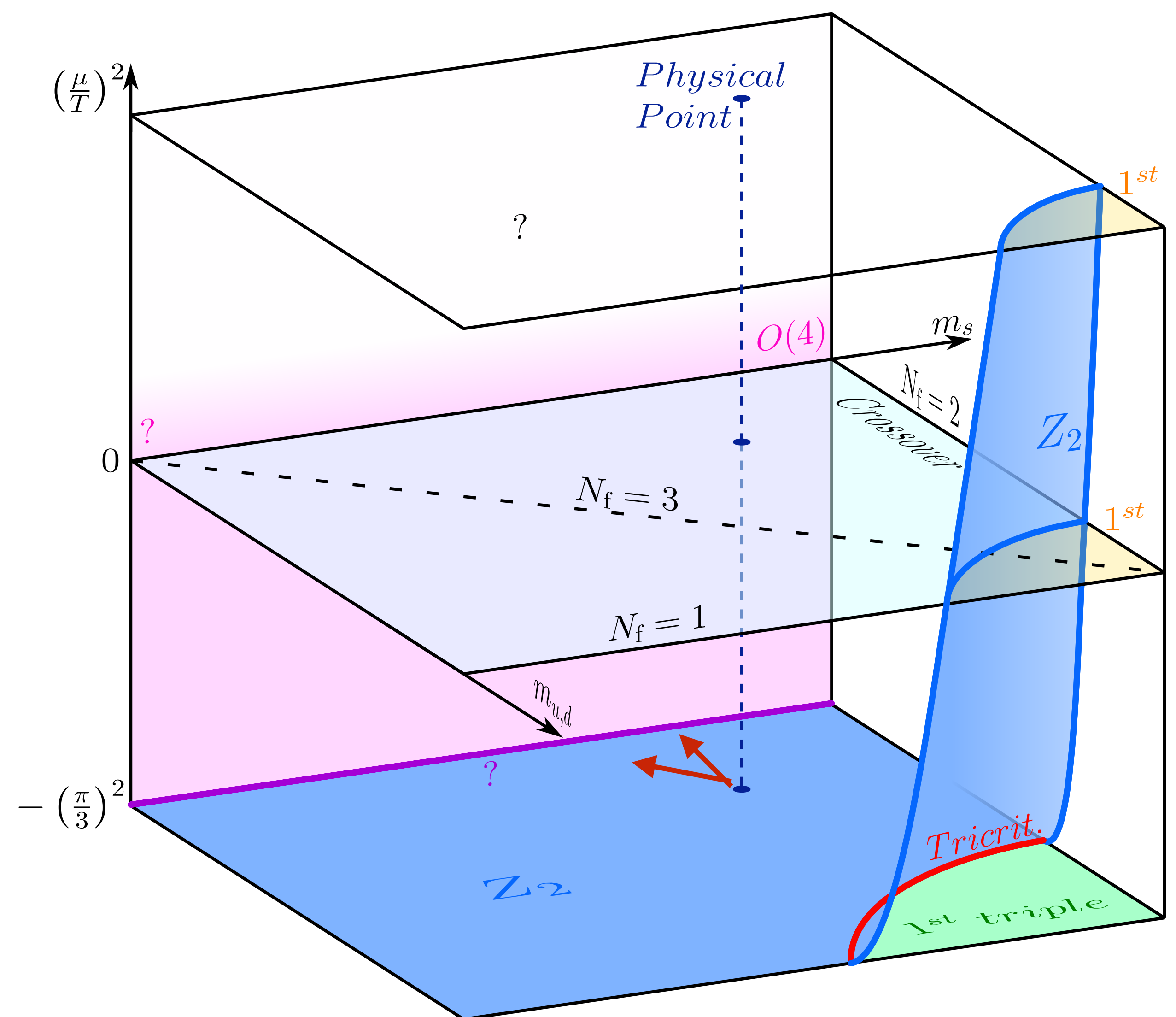
  quark mass scan down to $m_\pi \approx 50$ MeV
  fixed m_{ud}/m_s

- [Bielefeld+Frankfurt, PRD 22]
  HISQ $N_\tau = 4$

  quark mass scan down to $m_\pi \approx 55$ MeV
  fixed m_s

- No sign of 1st-order phase transition!

- Entire chiral critical surface moves to massless limit



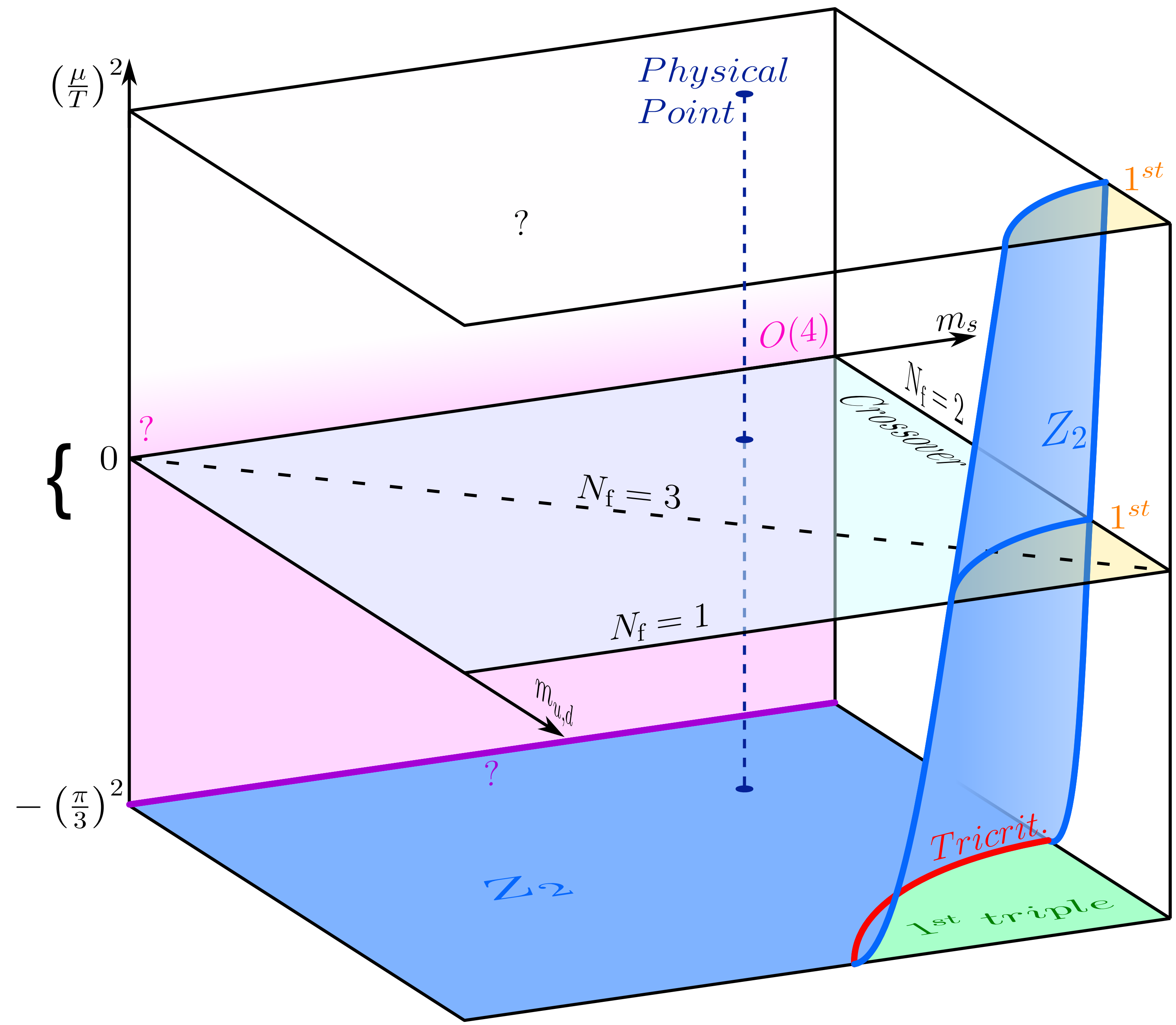
Columbia plot with chemical potential, continuum

[Bernhardt, Fischer, PRD 23]

Dyson-Schwinger eqs. $|\mu| \leq 30\text{MeV}$

Same picture

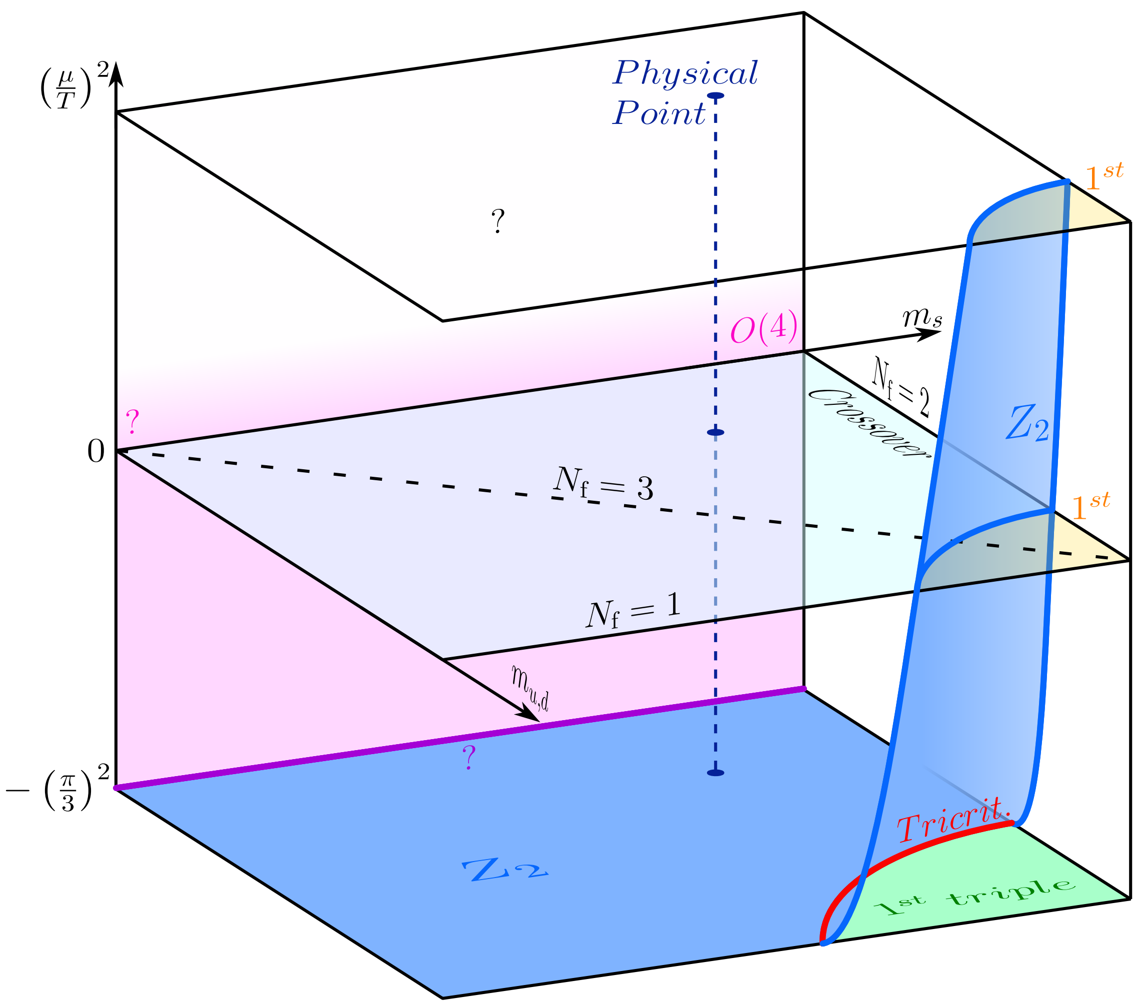
Columbia plot analytic around $\mu = 0$



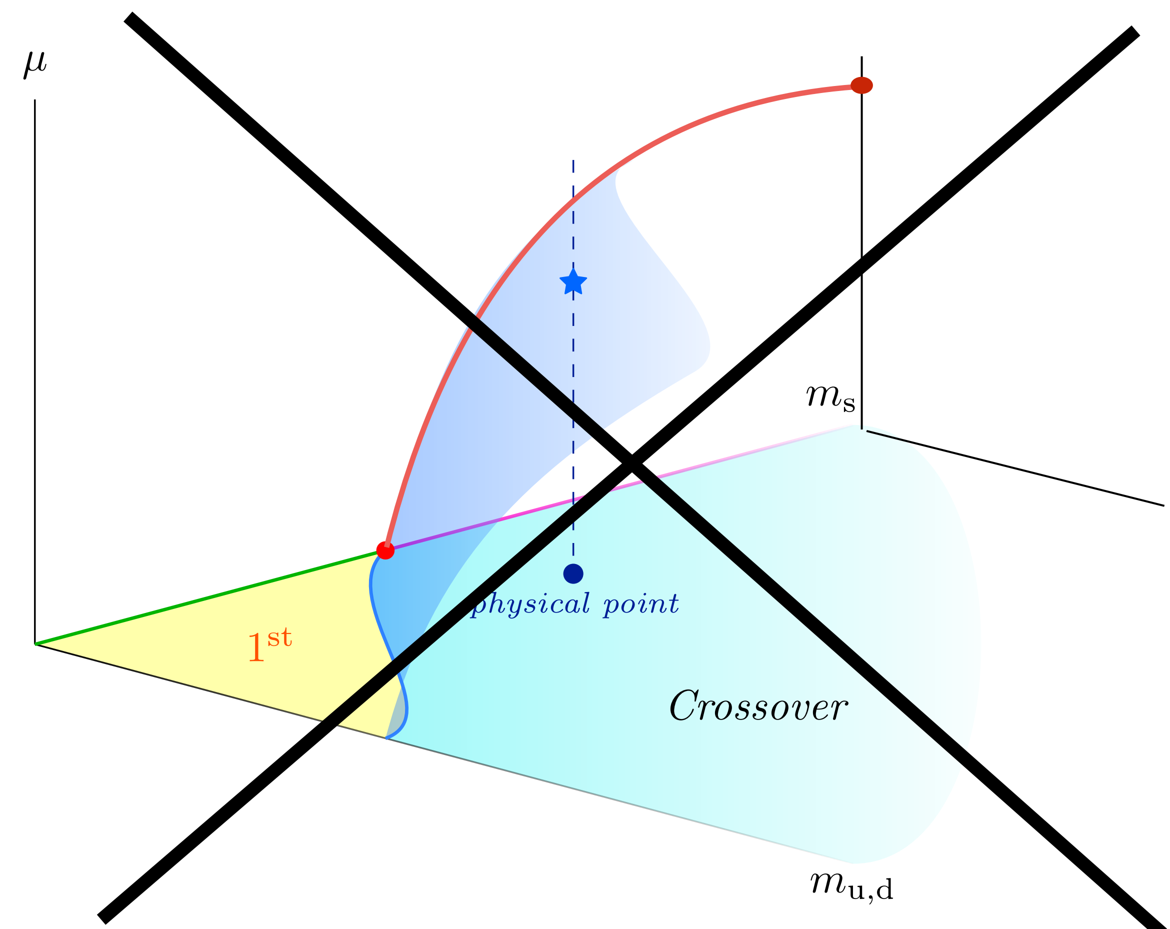
Columbia plot with chemical potential, continuum

Critical point not ruled out

But requires additional critical surface



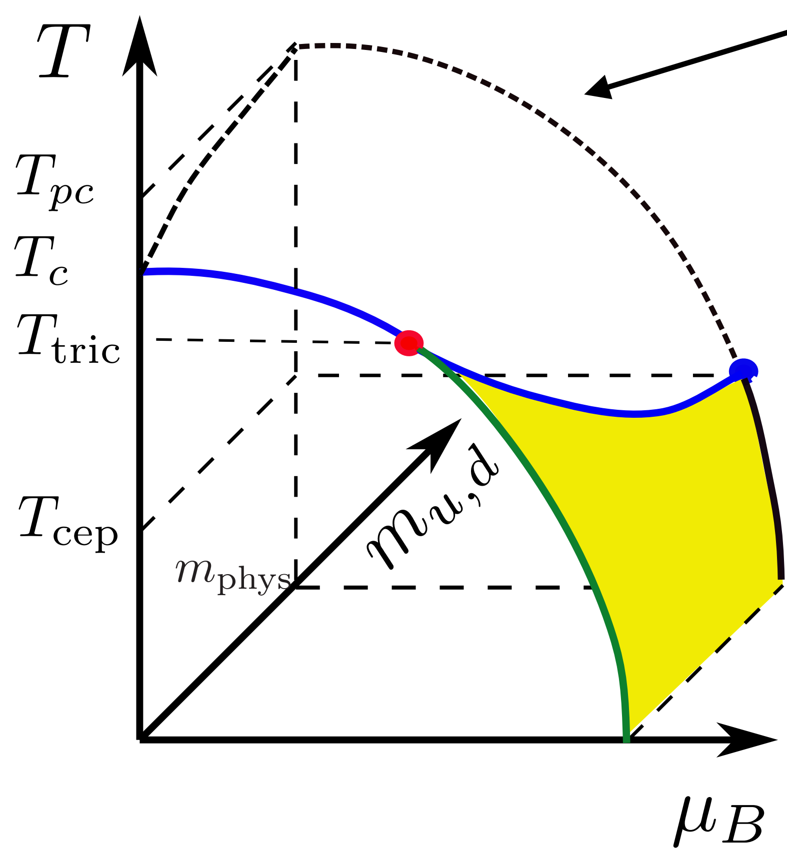
Class of low energy models now ruled out!



$\nexists$ Tuning of parameters for $N_f = 2 + 1$ theory with critical point at $\mu = 0$!

From the chiral limit back to the physical point

The "standard scenario":

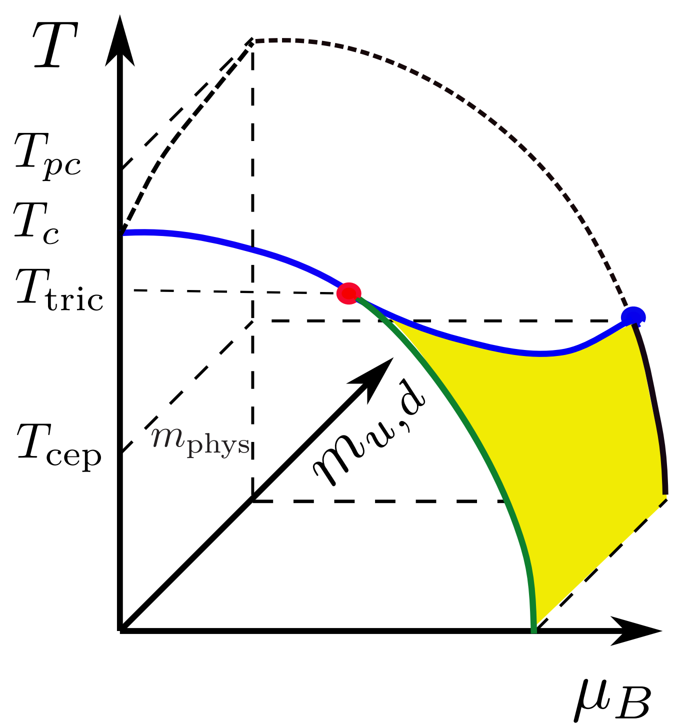


$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_{pc}(0)}\right)^2 + \dots$$

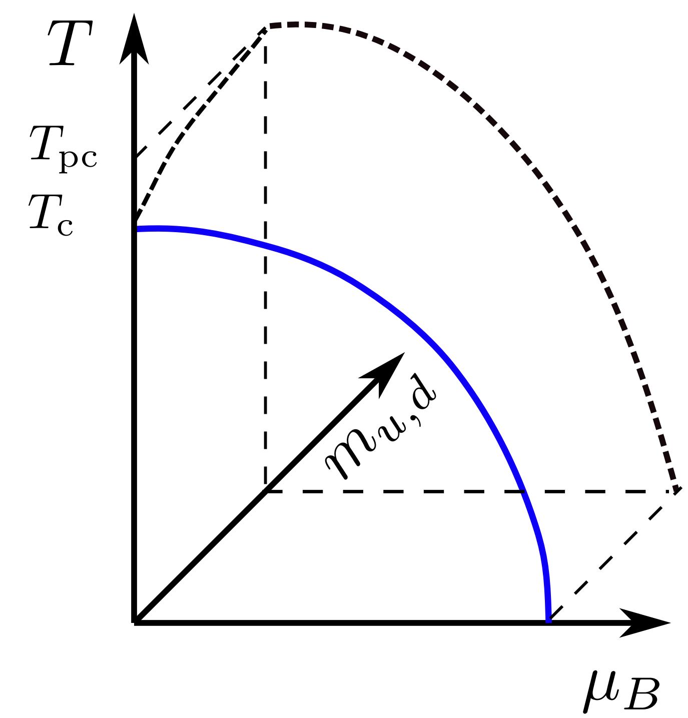
κ_2	Action	
0.0158(13)	imag. μ, stout-smeared staggered	[Bellwied et al, PLB 15]
0.0135(20)	imag. μ, stout-smeared staggered	[Bonati et al, NPA 19]
0.0145(25)	Taylor, stout-smeared staggered	[Bonati et al, PRD 18]
0.016(5)	Taylor, HISQ	[HotQCD, PLB 19]

$T_{pc} > T_c > T_{\rm tric} > T_{\rm cep}$ → $\mu_B^{\rm cep} > 3.1\ T_{pc}(0) \approx 485$ MeV

Summary: constraints on the critical point



?



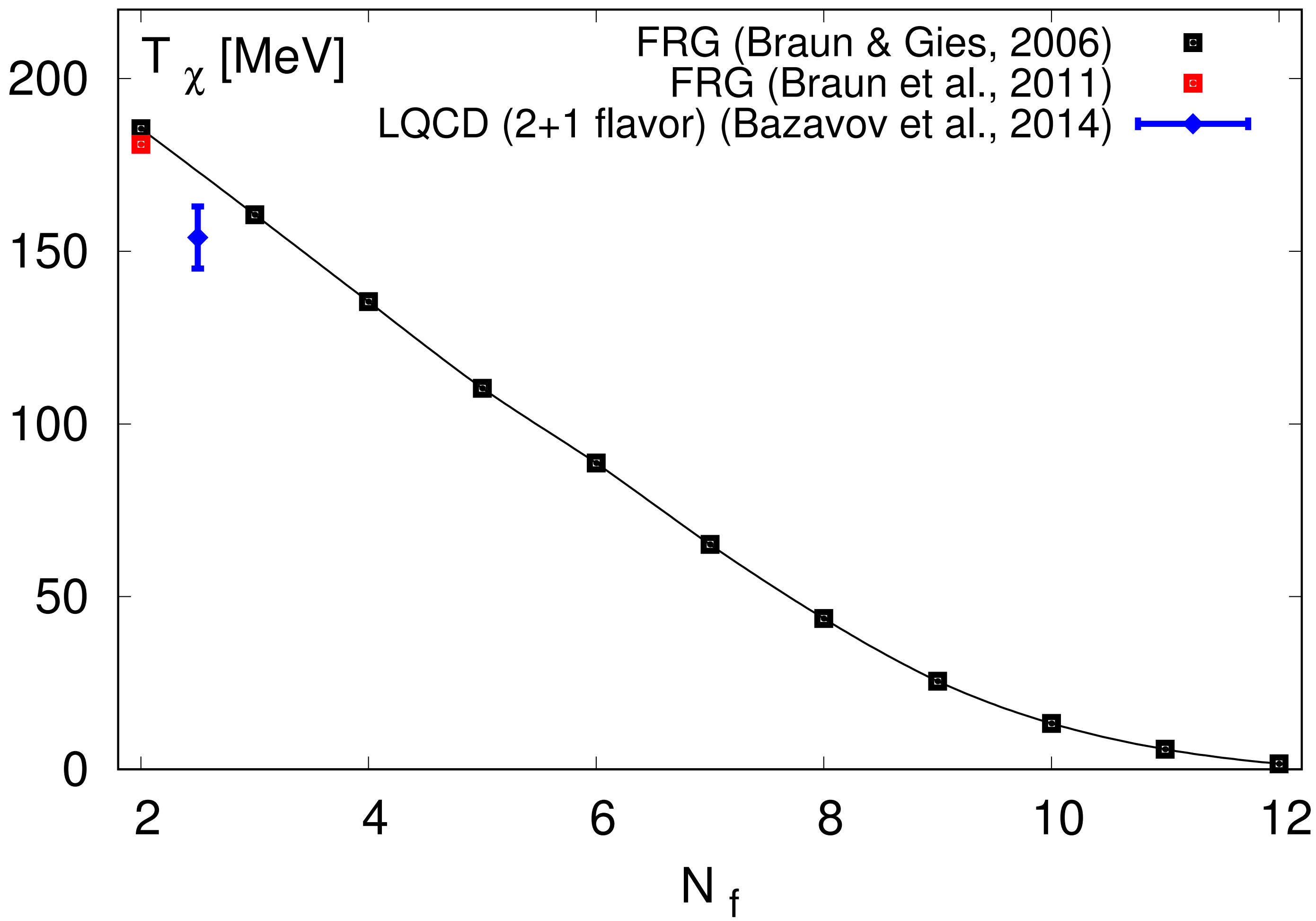
- Ordering of critical temperatures $\mu_B^{\mathrm{cep}} > 3.1\ T_{pc}(0) \approx 485\ \mathrm{MeV}$ [O.P. Symmetry 21]
- Cluster expansion model of lattice fluctuations $\mu_B^{\mathrm{cep}} > \pi T$ [Vovchenko et al. PRD 18]
- Singularities, Pade-approx. fluctuations $\mu_B^{\mathrm{cep}} > 2.5T, T < 125\ \mathrm{MeV}$ [Bollweg et al. PRD 21]
- Direct simulations with refined reweighting $\mu_B^{\mathrm{cep}} > 2.5T$ [Wuppertal-Budpest collaboration, PRD 21]

- Consistent with DSE, fRG [Fischer PPNP 19; Fu, Pawlowski, Rennecke PRD 20; Gao, Pawlowski PRD 21]

CEP seen at larger density, but "not yet controlled" $(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (98, 643)\,\mathrm{MeV}$

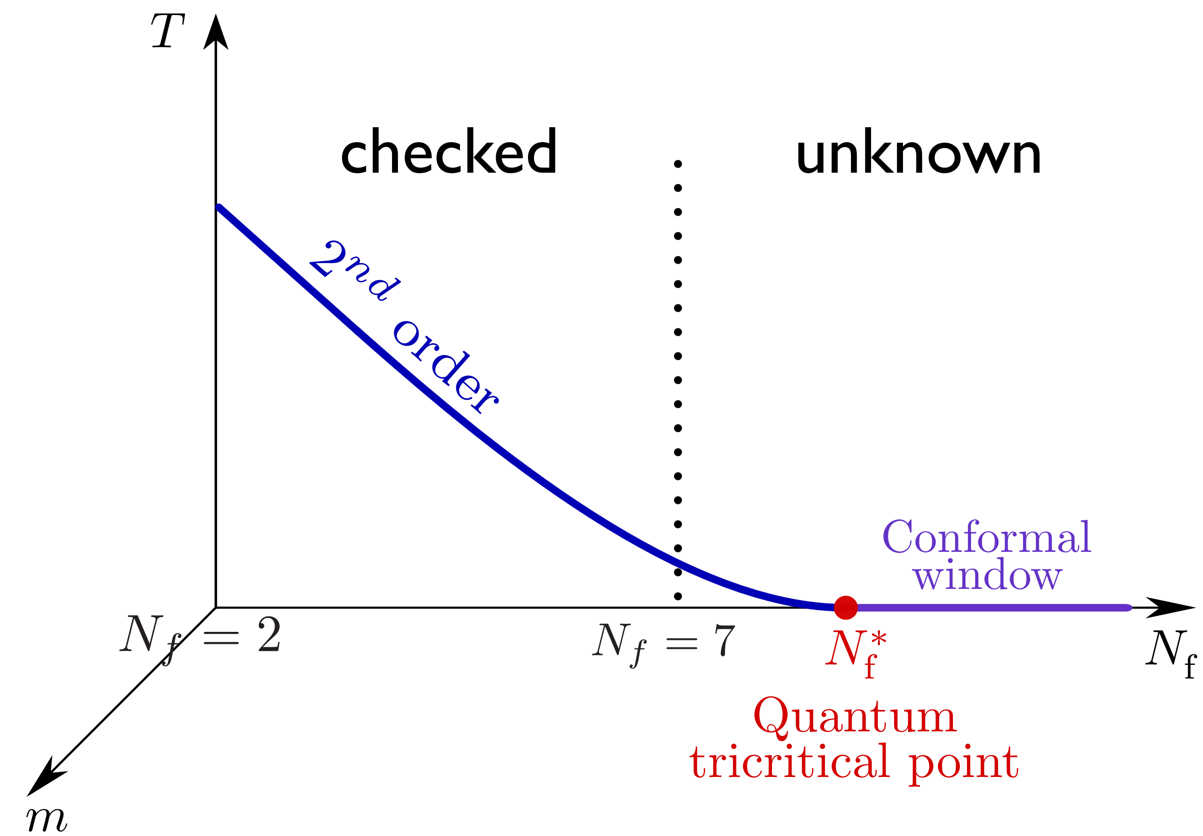
The chiral phase transition for different N_f

Temperature dependence:



For lattice, see [Miura, Lombardo, NPB 13]

Order of the transition:



[Cuteri, O.P., Sciarra, JHEP 21]

The chiral phase transition in the massless limit is likely second-order for all N_f

Consistent with [Fejos, Hatsuda PRD 24, Pisarski, Renneke PRD 24] with conditions on anomaly

Towards the conformal window, $N_f > 6$

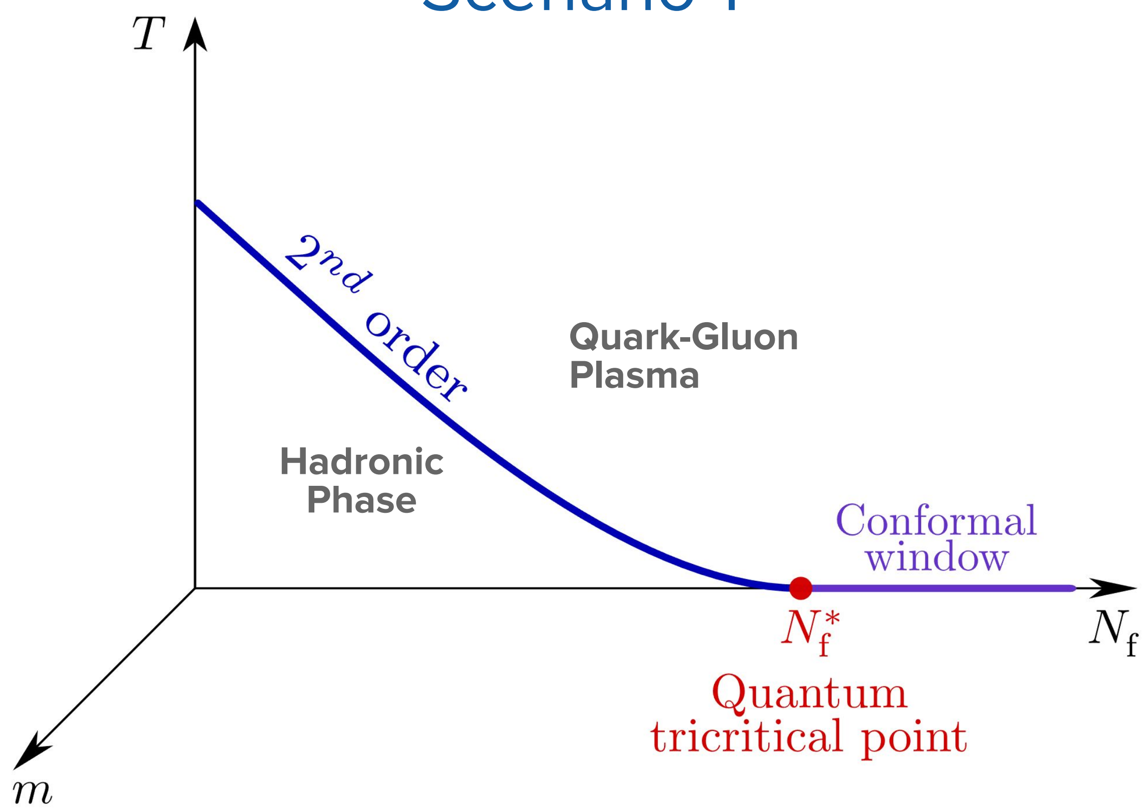
<u>Onset of conformal window Nf*:</u>

$10 \lesssim N_f^* \lesssim 12$ { **[Braun, Gies 11]**, **[Lombardo, Pallante, Deuzeman 13]**

$8 \lesssim N_f^* \lesssim 9$ **[Hasenfratz et al. 23]**

What is the value of N_f^* ?

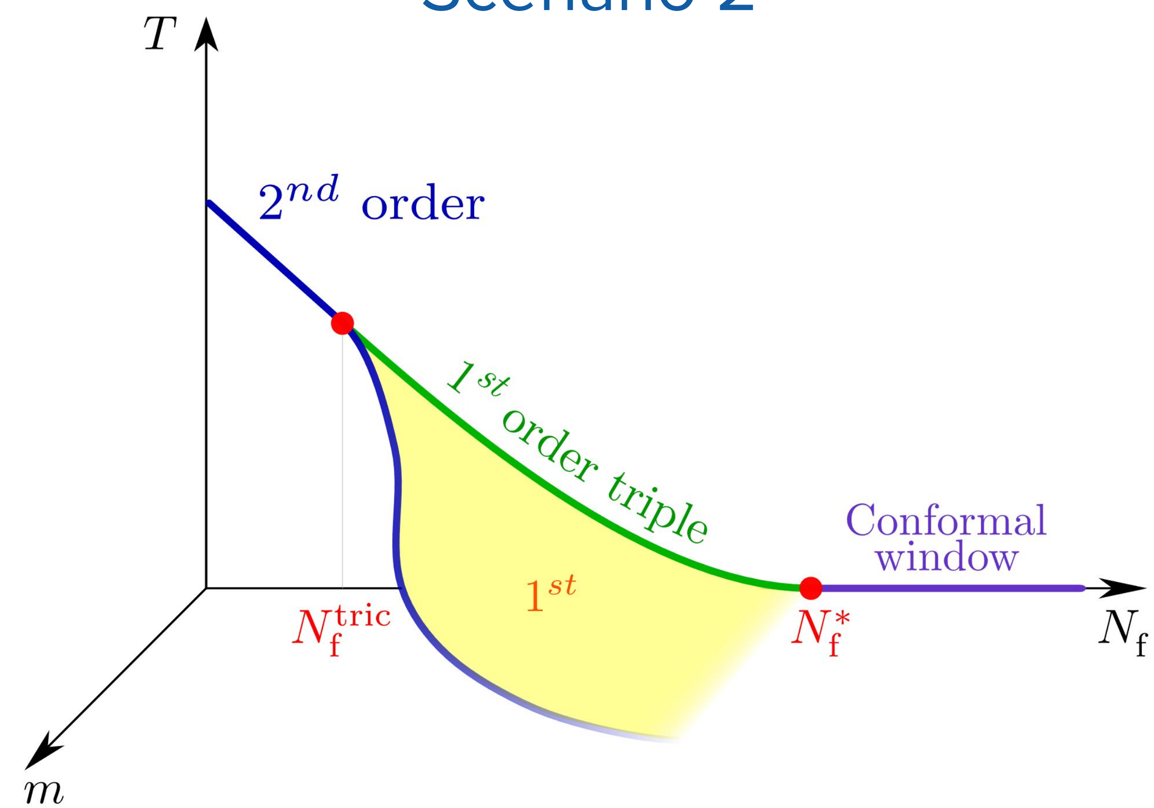
Scenario 1



- 2nd order for all N_f
- $N_f^{tric} = N_f^*$

N_f^{tric} at $T = 0$

Scenario 2

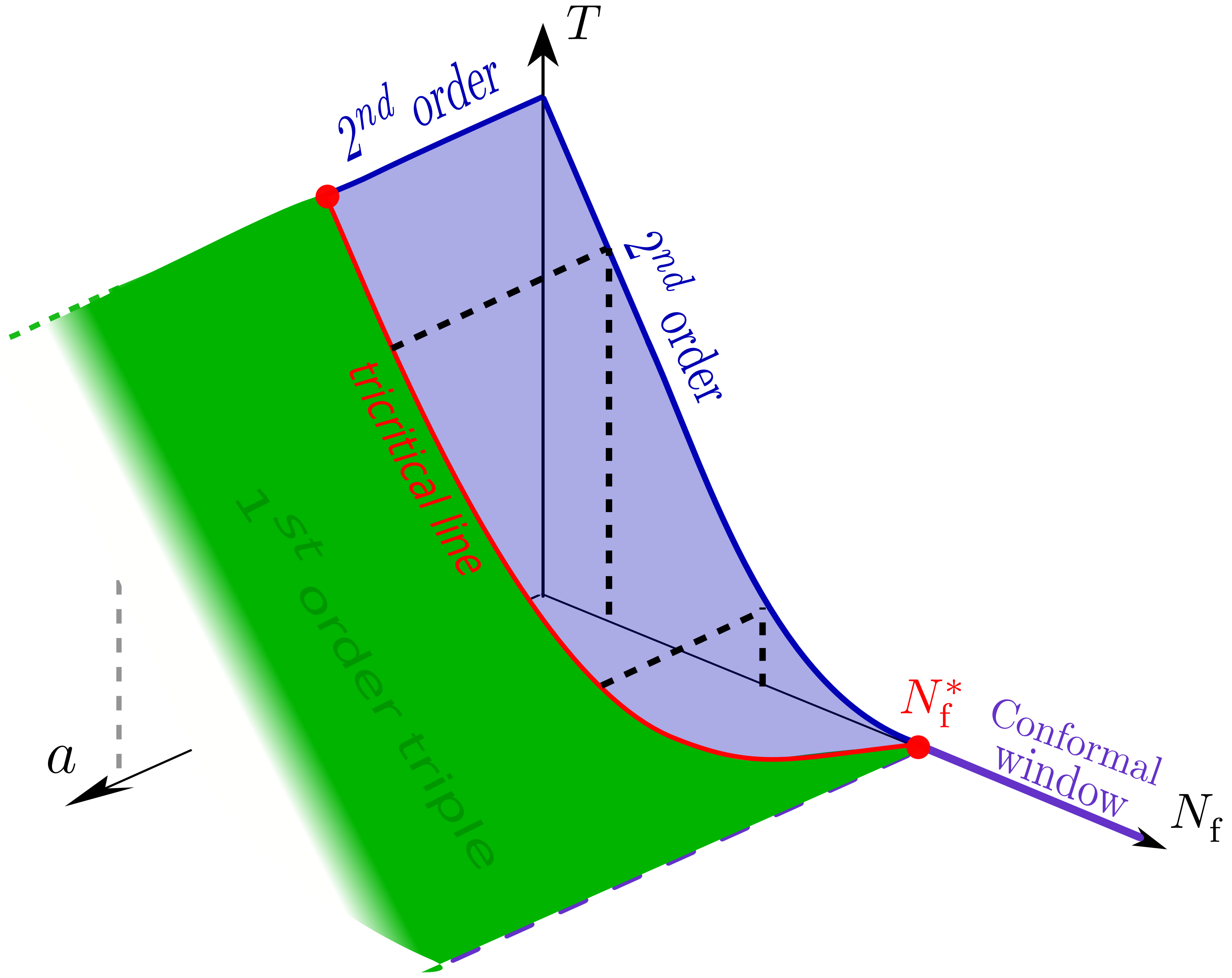


- 2nd order turns into 1st order at N_f^{tric}
- $6 < N_f^{tric} < N_f^*$

N_f^{tric} at $T > 0$

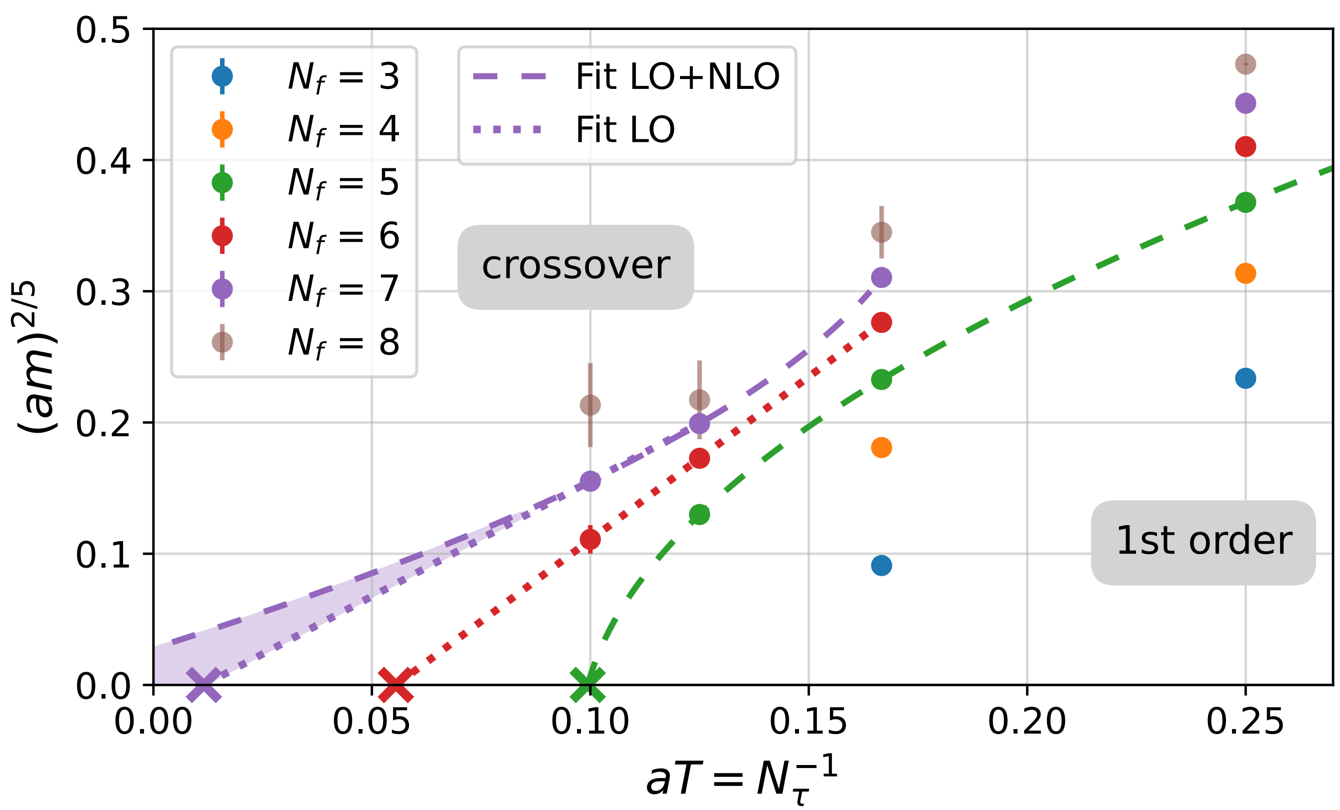
Towards the conformal window: our approach

We can reliably determine tricritical points in the lattice chiral limit (fixed a)

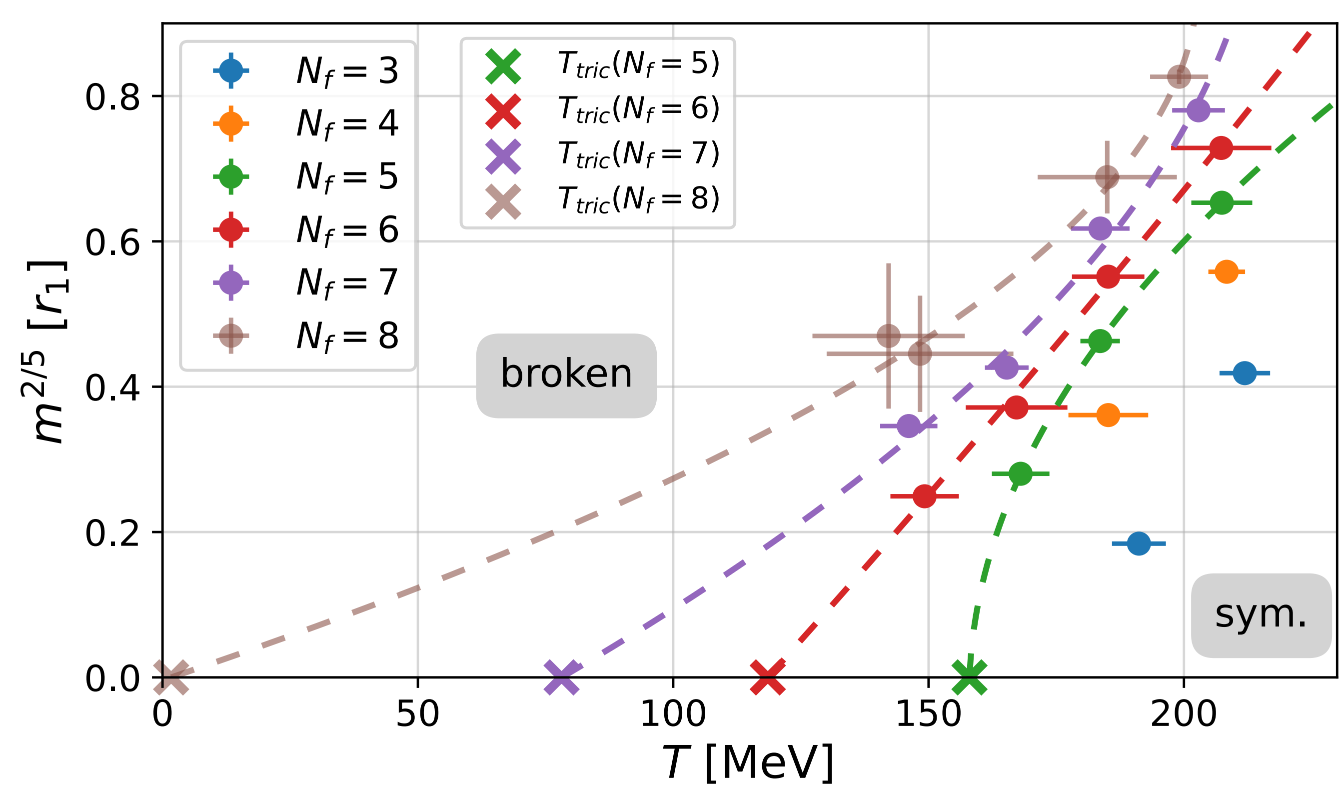


$N_f > 6$,   preliminary

- Additional lattice spacing, $N_\tau = 10$
- Scale setting for temperature: Sommer scales r_0, r_1
- Quantitative values of T not important, but when is T=0?
- N_f^* is boundary for tricritical scaling (conformal scaling beyond!)



$$T_c(m) = T_{tric} + A \cdot m^{2/5} + B \cdot m^{4/5}$$



Preliminary result: $7 < N_f^* < 9$

Conclusions

- Chiral transition at zero density is second order for Nf=2-6 massless quark flavours
- So far consistent between all lattice discretisations + DSE
- Imaginary chemical potential has no effect on the order of the chiral transition
- Lesson from cutoff effects:

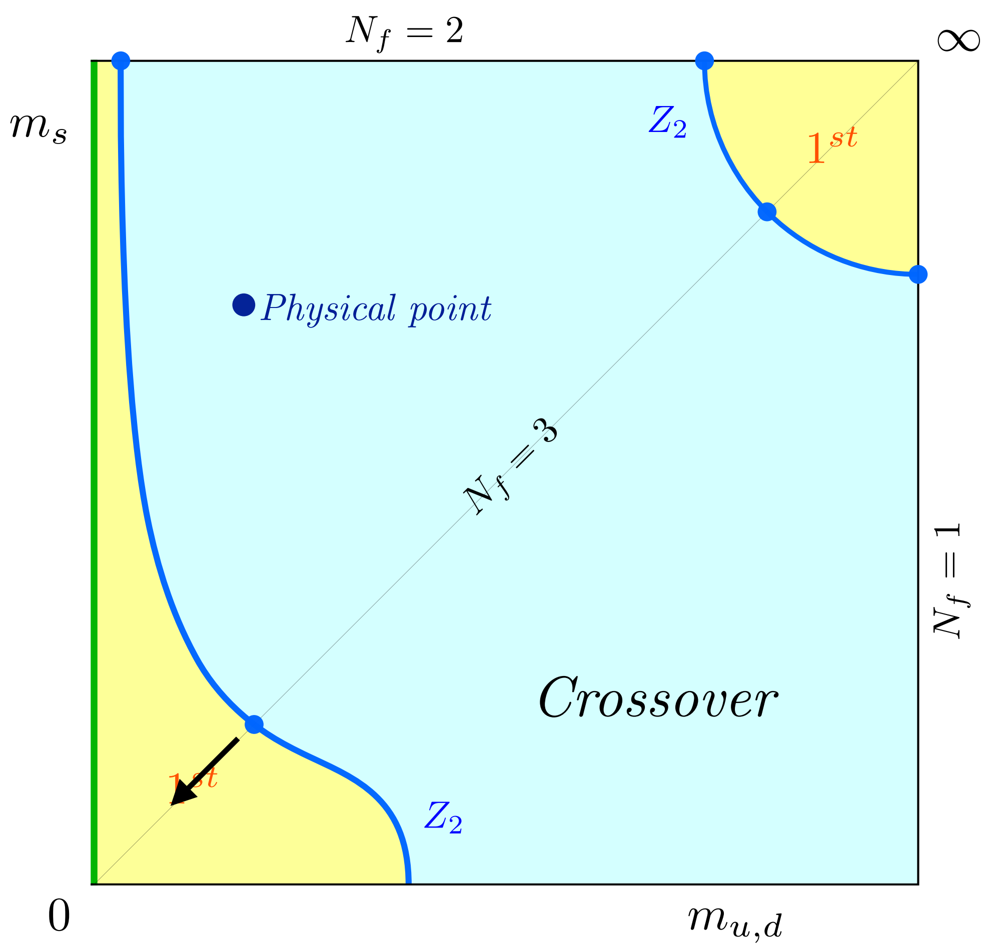
  Correct UV sector of a theory is crucial for its phase diagram!

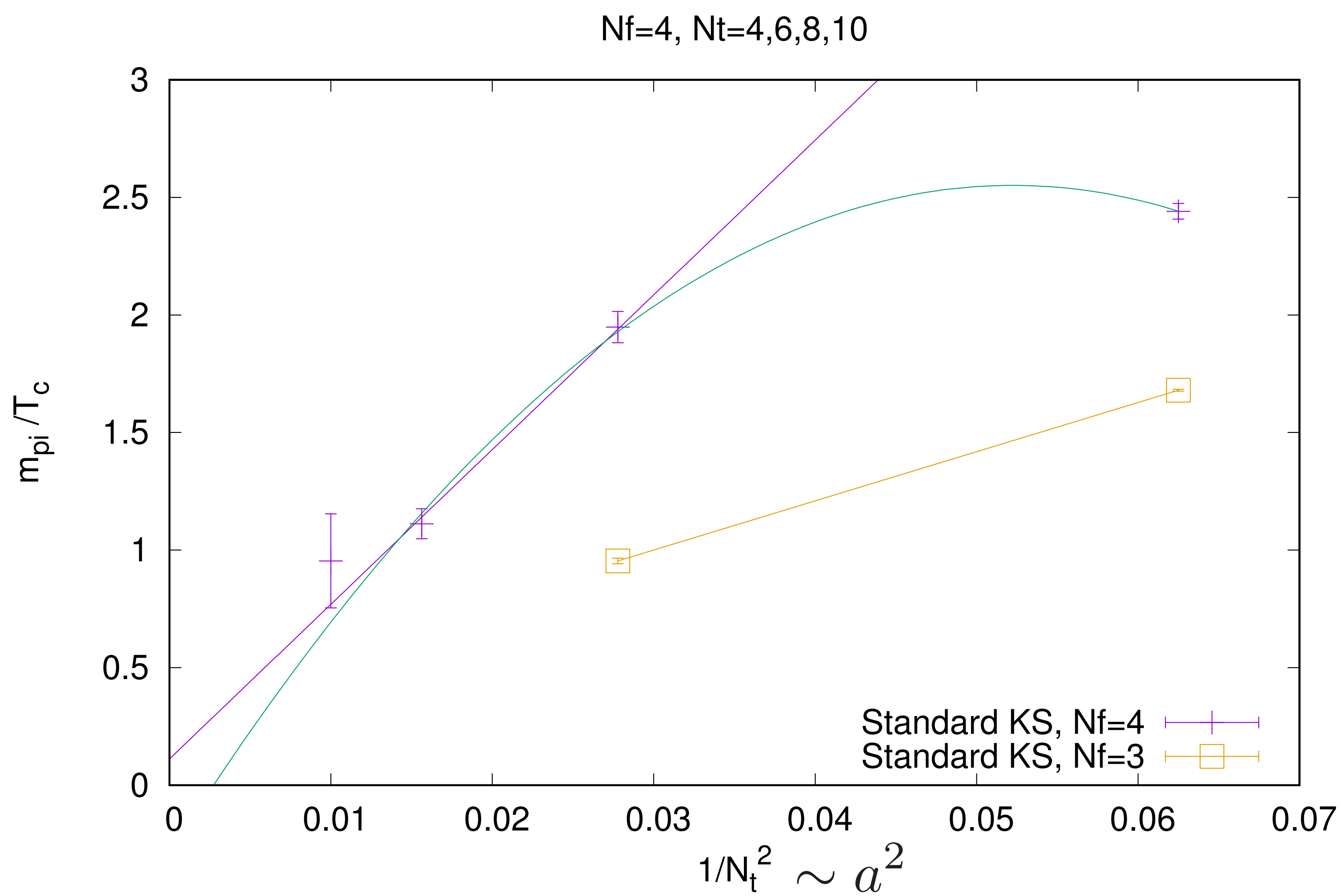
  "Low energy effective models" can be deceiving
- Onset of conformal window in reach

Backup slides

The nature of the QCD chiral transition, Nf=3,4

…has enormously large cut-off effects!





Unimproved staggered:
1st order region shrinks for $a \to 0$, both for $N_f = 3, 4$ [de Forcrand, D'Elia, PoS LAT 17]

No first-order region at all for HISQ fermions [HotQCD PRD 19, 22]

Machines and computing approach

Goethe-HLR (Goethe U.) and VIRGO cluster (GSI),   AMD-GPU cluster

Scans of parameter space parallel, one lattice per GPU, strictly zero communication overhead

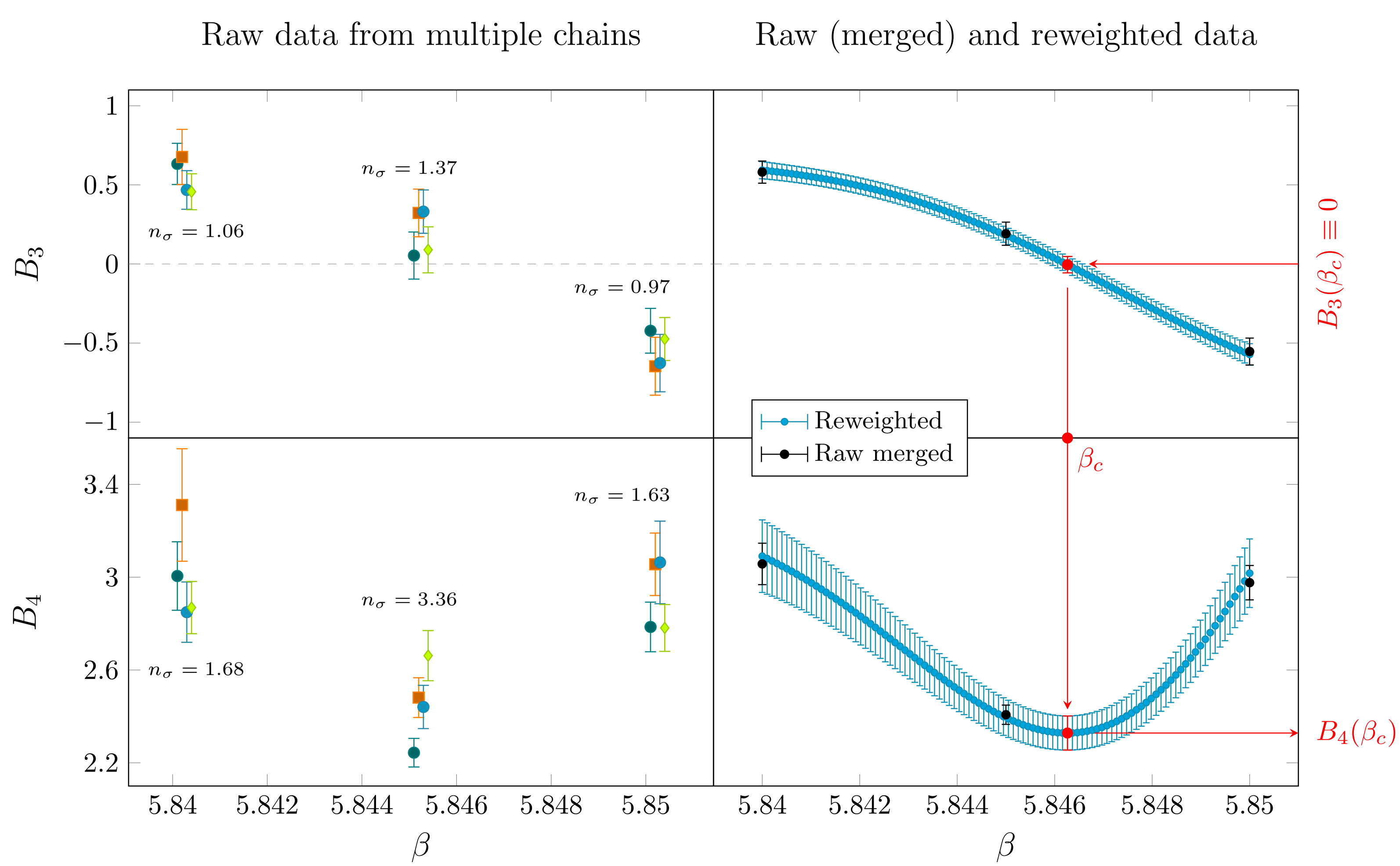
Search for phase boundary:

3-4 coupling values with multiple simulation chains

Good control over autocorrelation; merge independent chains

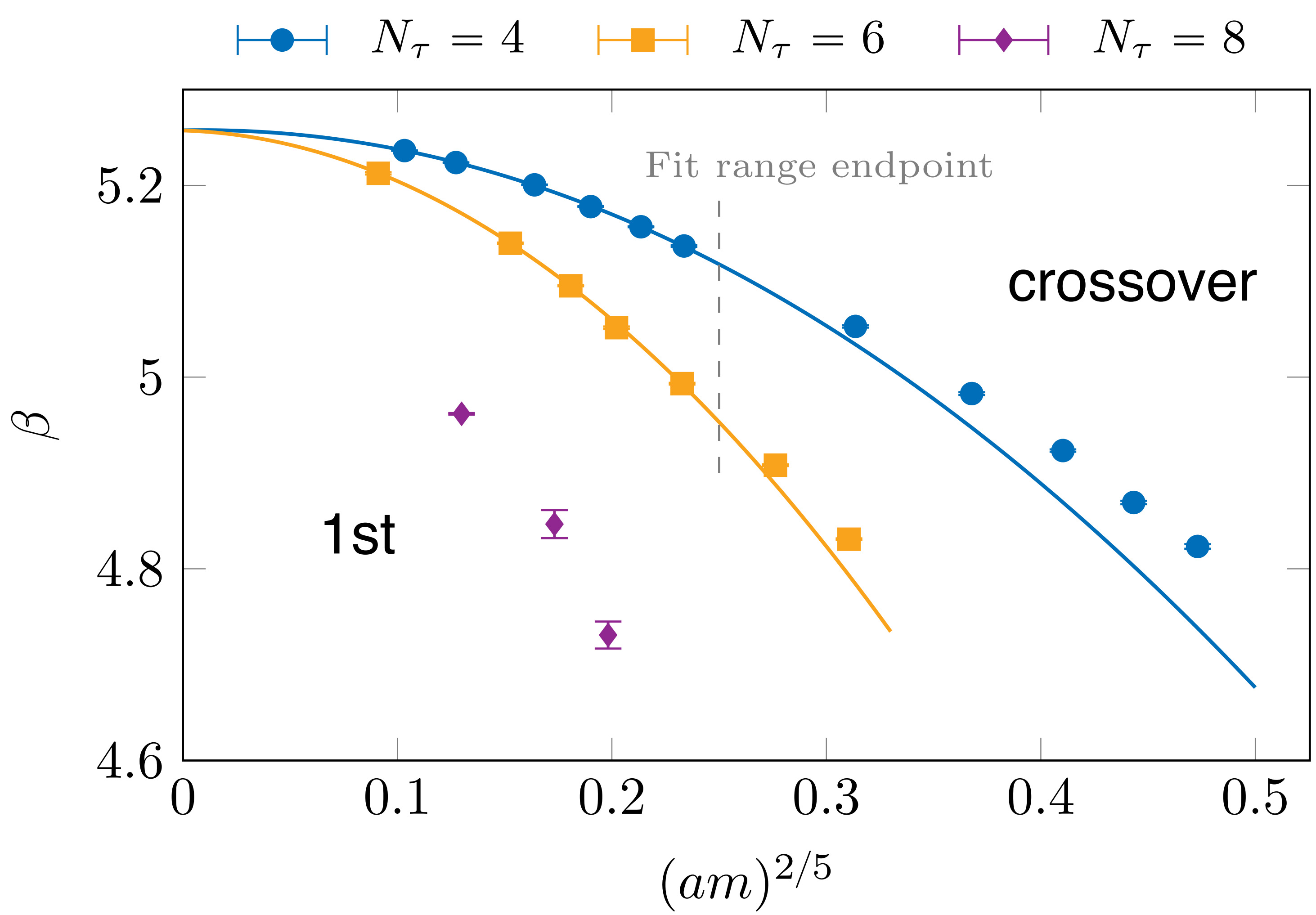
Repeat for different masses

Repeat for different volumes



Bare parameter space of unimproved staggered LQCD

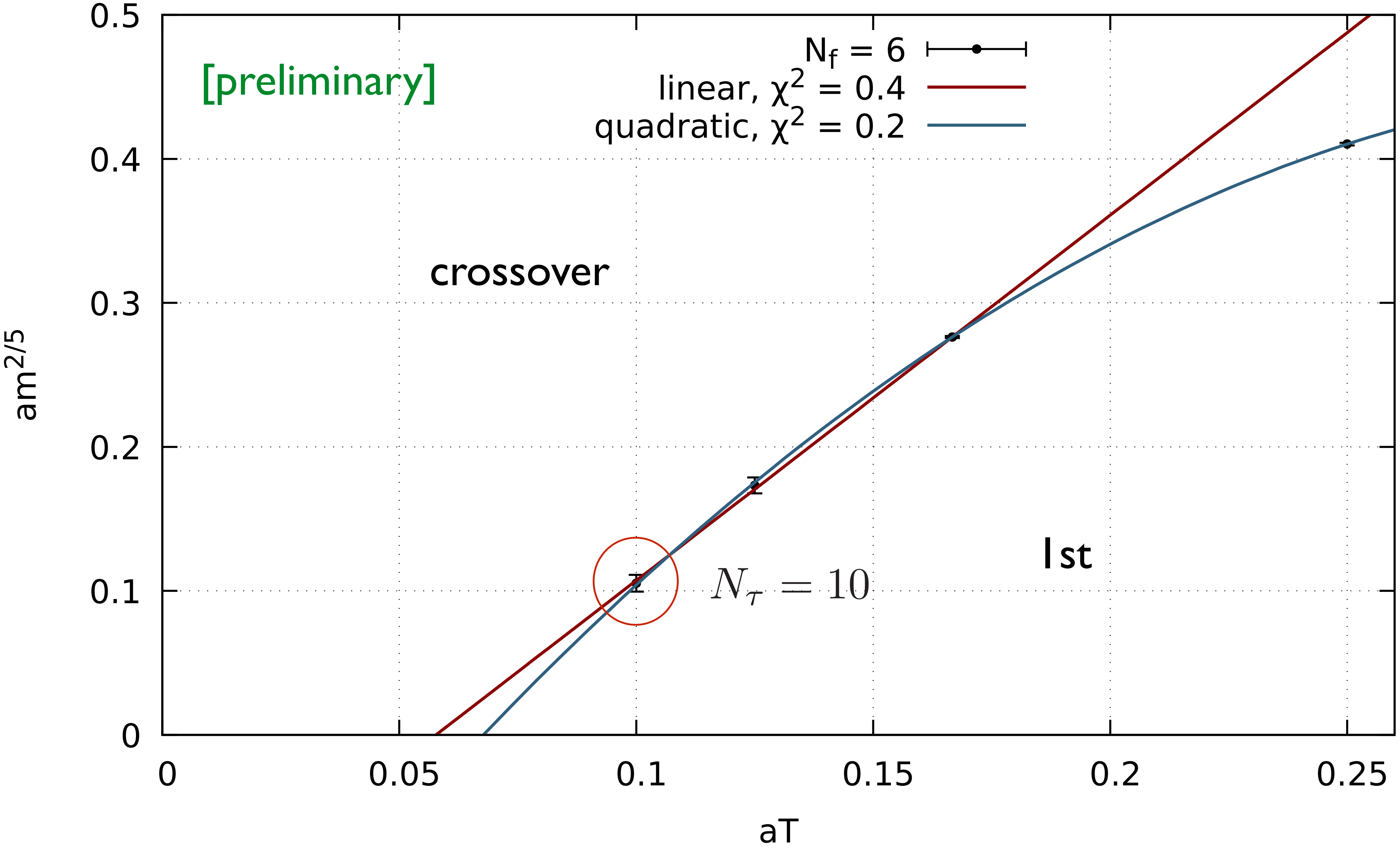
[Cuteri, O.P., Sciarra JHEP 21] ~120 M Monte Carlo trajectories with light fermions, aspect ratios 3,4,5



- Data points implicitly labeled by Nf
- Tricritical scaling observed in lattice bare parameter space
- Tricritical extrapolation always possible!

Meanwhile in Frankfurt…

progressing to finer lattices



New $N_\tau = 10$ result on predicted scaling curve!

What about Pisarski, Wilczek 1984?

- 3d ϕ^4 - Ginzburg-Landau-Wilson theory for chiral condensate plus t'Hooft term
- Epsilon expansion about $\epsilon = 1$
- All conclusions confirmed by [Butti, Pelisetto, Vicari, JHEP 03]
  (High order perturbative expansion in fixed d)
- Support also from simulation of 3d sigma model [Gausterer, Sanielovici, PLB 88]

Suggested resolution: ϕ^6 term, in 3d renormalisable; even higher powers…..?

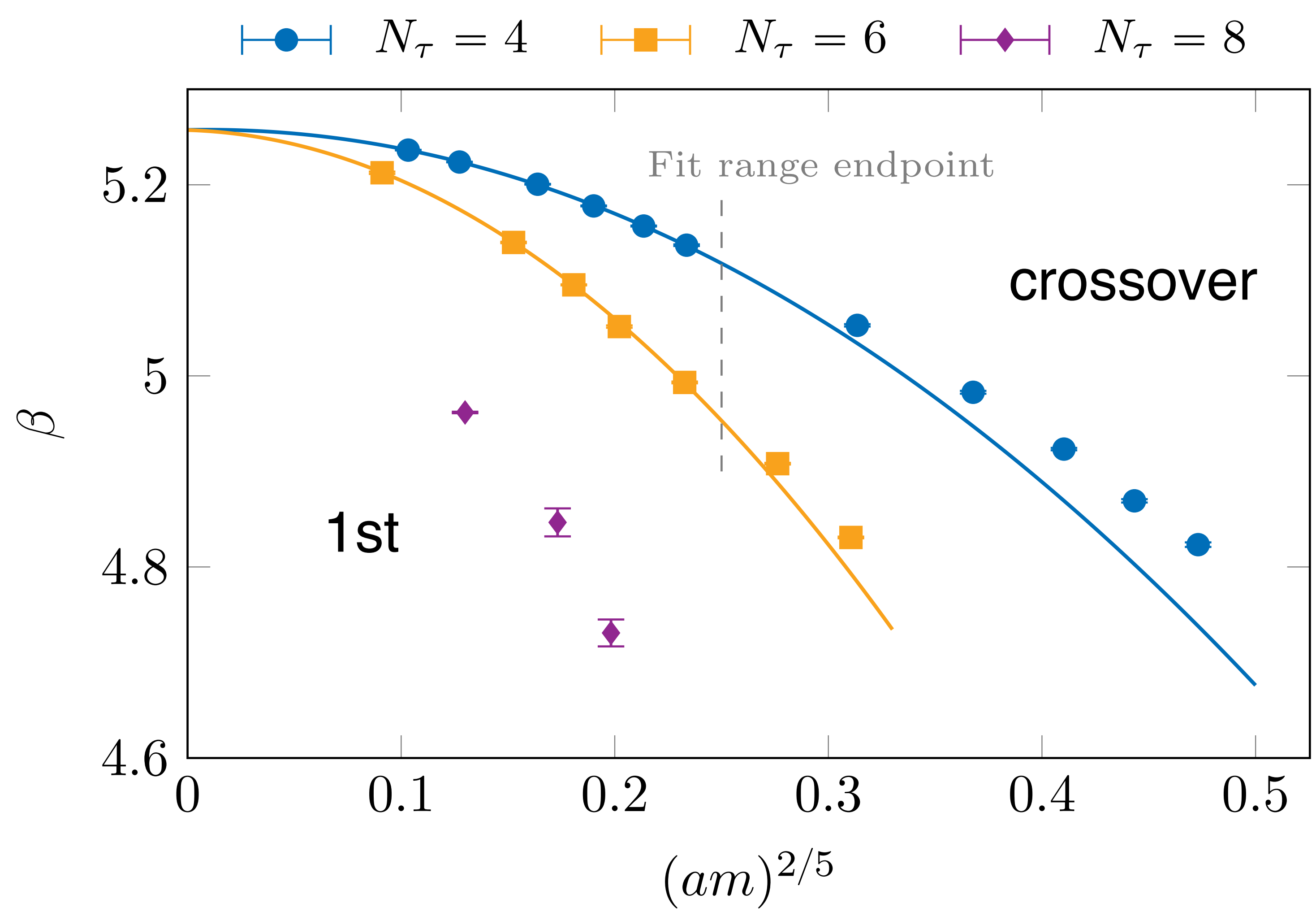
[Fejos, PRD 22] 3d ϕ^6 with t'Hooft term, functional RG study:
IR-stable fixed point, 2nd order transition for restored anomaly

[Kousvos, Stergiou, SciPost 23] Numerical conformal bootstrap:
U(3)xU(3) displays IR stable fixed point

No contradictions!

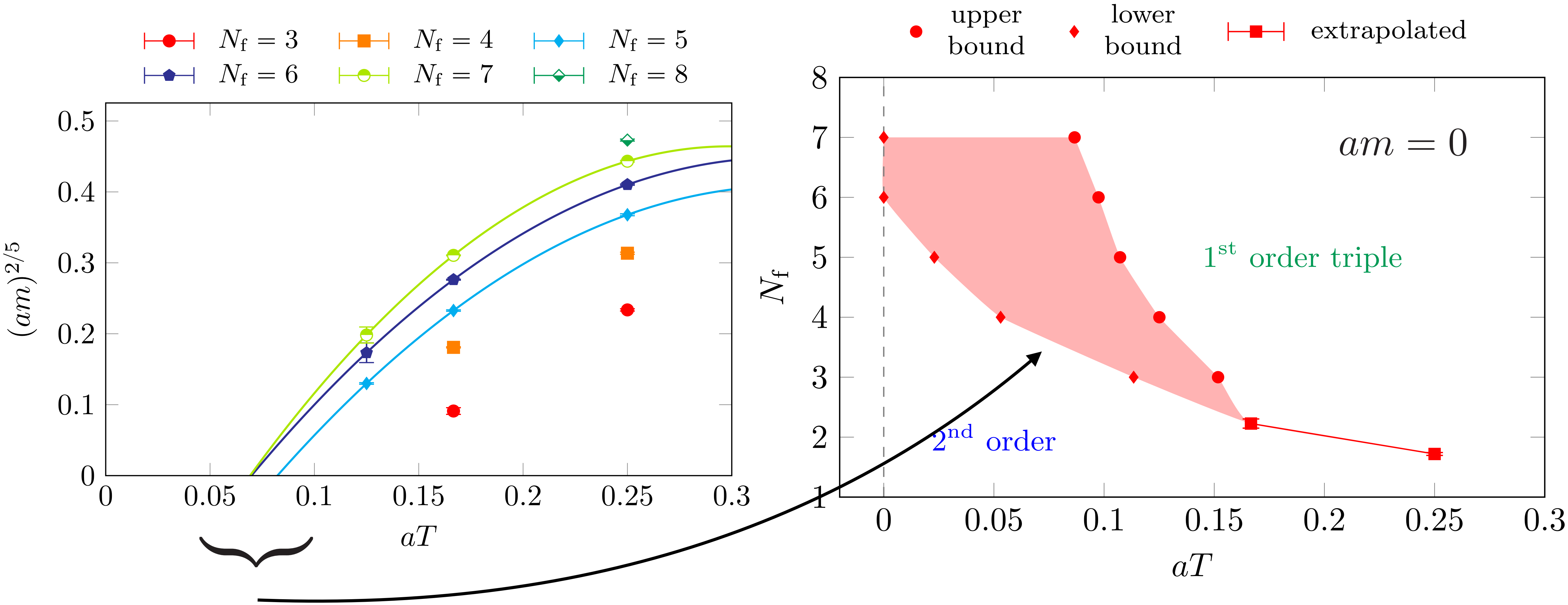
Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra JHEP 21] ~120 M Monte Carlo trajectories with light fermions, aspect ratios 3,4,5



- Data points implicitly labeled by Nf
- Tricritical scaling observed in lattice bare parameter space
- Tricritical extrapolation always possible!

Digression: tricritical points as function of Nf



- $N_\tau^{\text{tric}}(N_f)$ increasing function
- Tricritical line in the plane of the lattice chiral limit, separates 1st from 2nd
- Is there a tricritical point in the continuum?