Quark pairing condensate and the strongly coupled quark-gluon matter

Yi Lu

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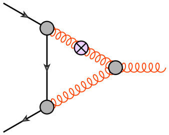
Based on: Fei Gao, **YL** and Yu-Xin Liu, arXiv:2403.16816

fQCD Collaboration:
Braun, Chen, Fu, Gao, Ihssen, Geissel, Huang, Lu, Pawlowski, Rennecke, Sattler, Schallmo, Stoll, Tan, Toepfel, Turnwald, Wen, Wessely, Wink, Yin, Zheng, Zorbach



ECT* Trento, Sept. 10, 2024

Introducing the pairing condensate

In cold and dense matter - attractive interaction and Cooper instability near the Fermi surface: Cooper, Phys. Rev. 104: 1189 (1956).

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BCS-theory: the true ground state has an energy gap in the excitation spectrum:

$$\omega_\pm^2(\vec{p}) = (E(\vec{p}) \pm \mu)^2 + |\Delta|^2,$$

found by minimising the free energy - gap equation:

$$\Delta = \int \frac{d^3\vec{p}}{(2\pi)^3} g^2 \Delta \left(\frac{1}{\omega_-(\vec{p})} + \frac{1}{\omega_+(\vec{p})} \right).$$

Bardeen, Cooper and Schrieffer, Phys. Rev. 108, 1175 (1957).

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The gap corresponds to the pairing condensate $\Delta \sim \langle \psi_{\uparrow,\vec{k}}\, \psi_{\downarrow,-\vec{k}} \rangle$;
Physics outcome: (low-temperature) superconductivity.

Analogy in QCD and the color-superconductivity (CSC):

- 2nd color superconductivity (2SC), i.e. *u*-*d* diquark condensate:

$$\langle \bar{q}_{\mathcal{C}} \gamma_5 \tau_2 \lambda_2 q \rangle, \qquad q_{\mathcal{C}} = \mathcal{C} q^* = \gamma_2 \gamma_4 q^*;$$

or in matrix representation - see e.g. Alford, Berges and Rajagopal, Nucl. Phys. B 1999:

$$\mathcal{M}_{\mathrm{2SC}} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & \cdots \\ -1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1 & \cdots \\ 0 & 0 & 0 & 1 & 0 & \cdots \\ \cdots & & & & & \end{pmatrix},$$

with the color-flavor basis:
$\{\boldsymbol{(r,u)}, \boldsymbol{(g,d)}, (b,s), \boldsymbol{(r,d)}, \boldsymbol{(g,u)}, (r,s), (b,u), (g,s), (b,d)\}$.

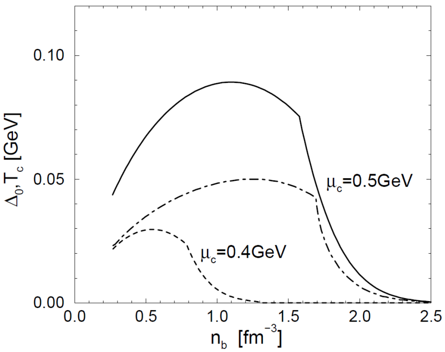
- Higher density may also enable the interplay of strange quarks: $\{u, d, s\}$ color-flavor locking (CFL).

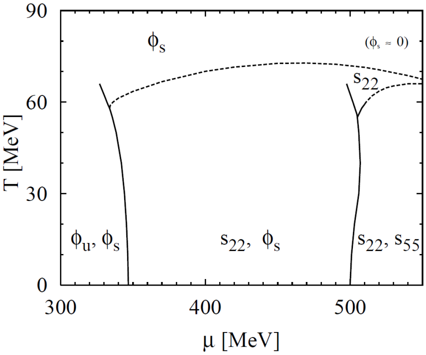
Verifying the color-superconductivity phase is still not a simple task.

- Primarily: mean field models with contact interaction:

  Rapp, Schäfer, Shuryak and Velkovsky, PRL 1997;

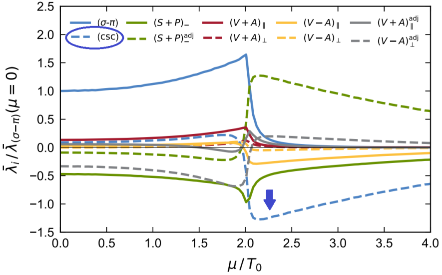
  Alford, Berges and Rajagopal, Nucl.Phys.B 1999; Buballa, Phys.Rept. 2005.

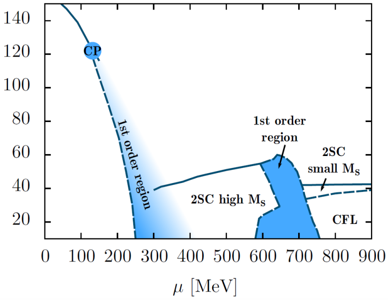




Verifying the color-superconductivity phase is still not a simple task.

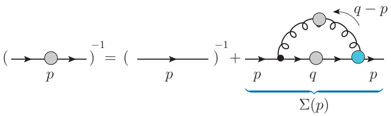
- Particular focus on the improvements of interaction vertices:
  - RG flow of the 4-quark interactions - Braun, Leonhardt and Pospiech, PRD 2020.
  - Dyson-Schwinger equations (DSEs): - Müller, Buballa and Wambach, 2016; Nickel, Wambach and Alkofer, PRD 2008 and 2006; Hou, Wang and Rischke, PRD 2004.
  - Other improvements on the interaction: Alford, Pangeni and Windisch, PRL 2018.

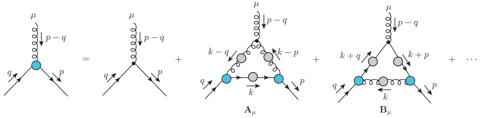




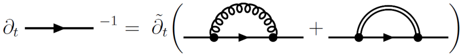
QCD interaction vertex from refined truncations

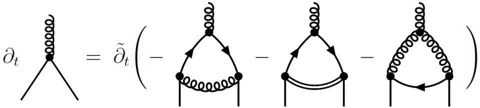
Highlights of functional QCD approaches:





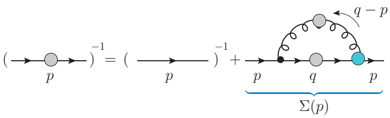
DSE: Gao, Papavassiliou and Pawlowski, PRD 103: 094013 (2021);
Williams, Fischer and Heupel, PRD 93: 034026 (2016); Williams, EPJA 51: 5 (2015).

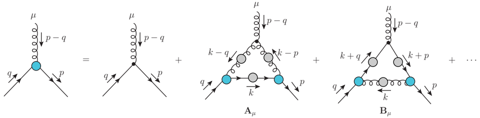




fRG: Ihssen, Pawlowski, Sattler and Wink, arXiv:2408.08413; see also in Jan's talk on the LEGO principle.
Fu, Pawlowski and Rennecke, PRD 101: 054032 (2020);
Cyrol, Mitter, Pawlowski and Strodthoff, PRD 97: 054006 (2018).

- Systematic error estimates: identify leading tensor structures.
- Self-consistency: couple the matter sector with the interaction sector.





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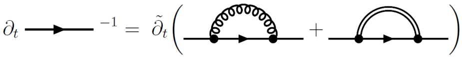

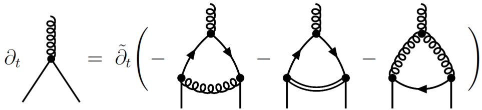

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Success in QCD vacuum - dynamical mass generation:

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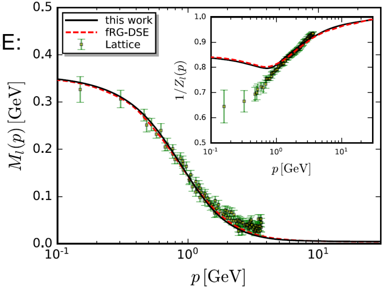

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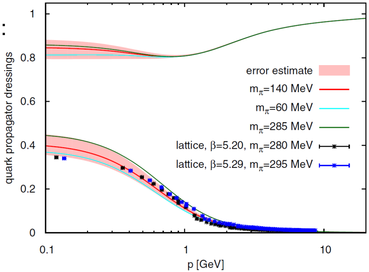
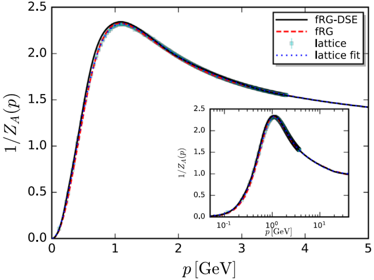
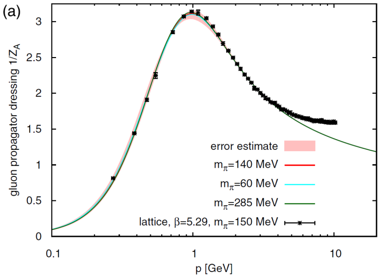
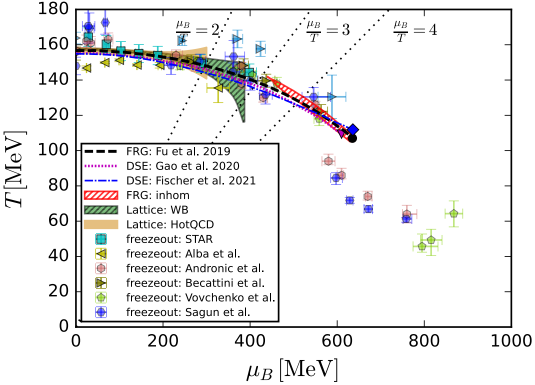






- And at finite T and μ_B: chiral phase structure of QCD.



$$\frac{T_c}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 + \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \cdots$$

Functional:

Gao and Pawlowski, PLB 820: 136584 (2021),

Gunkel and Fischer, PRD 104, 054022 (2021),

Gao and Pawlowski, PRD 102, 034027 (2020),

Fu, Pawlowski and Rennecke, PRD 101: 054032 (2020);

Lattice:

Borsanyi et al. (WB Collab.) PRL 125: 052001 (2020),

Bazavov et al. (hotQCD Collab.), PLB 795: 15–21 (2019).

- Thermodynamic quantities (e.g. fluctuations): see Wei-jie's talk.

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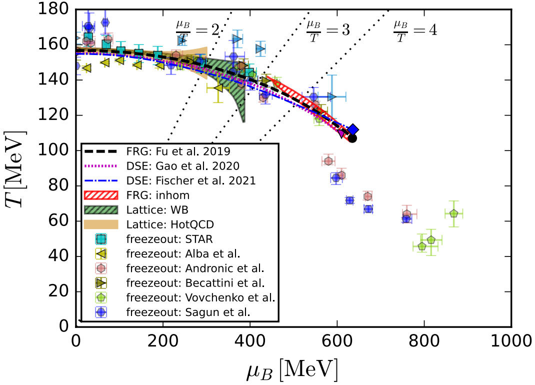

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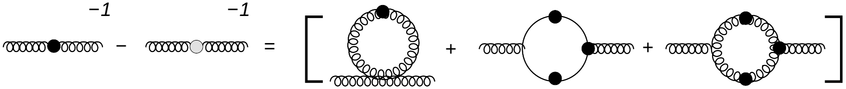
- Thermodynamic quantities (e.g. fluctuations): see Wei-jie's talk.
- Apply the refined scheme of interactions to the diquark condensate study.

We follow the DSEs approach; some implications:

- Gluon propagator is relatively separable - vacuum + (T, μ_B, N_f) correction.



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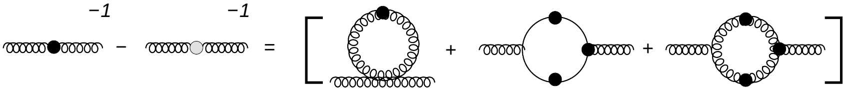
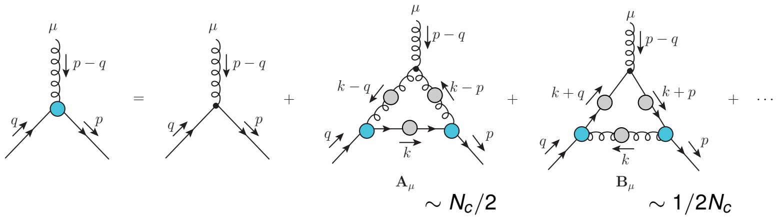


- Non-Abelian vertex diagram A_μ is found to be dominating:



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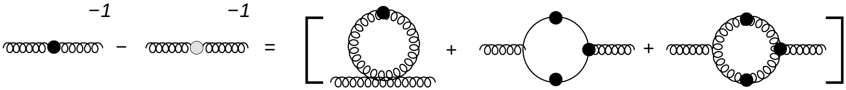

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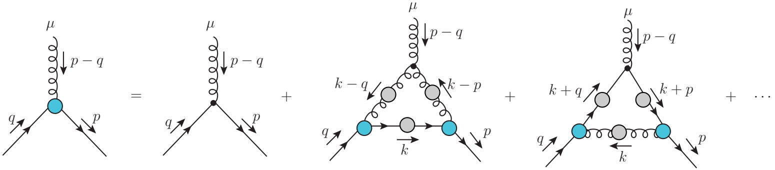
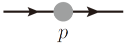


$\mathbf{A}_\mu$ $\mathbf{B}_\mu$

- Complicated tensor structures drop out in the chiral symmetric phase:

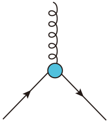
$$\Gamma_\mu(q, p; k) \simeq r_1\gamma_\mu \simeq Z_c(k)\gamma_\mu, \qquad Z_c(k) = G_c(k)/k^2.$$

DSEs in the Nambu-Gor'kov formalism



$$\boldsymbol{G}_q^{-1}(p) = \begin{pmatrix} G_q^{(+)} & \Delta_q^{(-)} \\ \Delta_q^{(+)} & G_q^{(-)} \end{pmatrix} \simeq \begin{pmatrix} \mathrm{i}\gamma\cdot\tilde{p} & \Delta^* \\ \Delta & \mathrm{i}\gamma\cdot\tilde{p}^* \end{pmatrix}.$$

$$\tilde{p} = (\omega_p + \mathrm{i}\mu, \vec{p}), \quad p = (\omega_p, \vec{p}).$$



$$\boldsymbol{\Gamma}_\mu^a = \begin{pmatrix} \Gamma_{\mu+}^a & \Xi_{\mu-}^a \\ \Xi_{\mu+}^a & \Gamma_{\mu-}^a \end{pmatrix} \simeq \begin{pmatrix} r_1\,\gamma_\mu \frac{\lambda_a}{2} & ? \\ ? & -r_1\,\gamma_\mu \frac{\lambda_a^T}{2} \end{pmatrix}.$$

- Chiral symmetric phase - simplifications for diagonal parts.
- Exploring the "off-diagonal" interaction Ξ_μ^a.
- 2-SC diquark condensate for simplicity: $\Delta_q^{(+)} = \Delta\,\gamma_5 \mathcal{M}_{2\mathrm{SC}}$.

- Slavnov Taylor identity in Nambu-Gor'kov (N-G) formalism:

$$ik_\mu \boldsymbol{\Gamma}^a_\mu \simeq Z_c(k)\left[\frac{\boldsymbol{\Lambda}^a}{2}\,\boldsymbol{G}_q^{-1}(q) - \boldsymbol{G}_q^{-1}(p)\,\frac{\boldsymbol{\Lambda}^a}{2}\right], \qquad \boldsymbol{\Lambda}^a = \begin{pmatrix} \lambda^a & 0 \\ 0 & -(\lambda^a)^T \end{pmatrix};$$

longitudinal constraint (see e.g. Müller, Buballa and Wambach, 2016):

$$\Xi^a_\mu \sim k_\mu(\lambda^a \mathcal{M} + \mathcal{M}(\lambda^a)^T)\,\Delta;$$

With $\propto \Delta$ contribution to self energy, it is still a BCS-type diquark generation:

$$\text{when} \quad G_A(k) \to G_0, \quad \delta\boldsymbol{\Sigma} \to \sum\!\!\!\!\!\!\int_q \frac{G_0}{\tilde{q}^2 + \Delta^2}\Delta.$$

Analysing the off-diagonal vertex

- Slavnov Taylor identity in Nambu-Gor'kov (N-G) formalism:

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- Transversal part - to identify the leading structures.

Full transversal basis of the vertex:

$$\Xi^T_\mu(q,p;k) = \sum P_{\mu\nu}(k)\mathcal{T}^{(i)}_\nu \gamma_5\, t_i.$$

$$\begin{aligned}
[\bar{q}\not{D}q]: \quad & \mathcal{T}^{(1)}_\mu = -\mathrm{i}\,\gamma_\mu, \\
[\bar{q}\not{D}^2 q]: \quad & \mathcal{T}^{(2)}_\mu = (q+p)_\mu, \\
& \mathcal{T}^{(3)}_\mu = (\not{q}+\not{p})\gamma_\mu, \\
& \mathcal{T}^{(4)}_\mu = \not{k}\gamma_\mu, \\
[\bar{q}\not{D}^3 q]: \quad & \mathcal{T}^{(5)}_\mu = \mathrm{i}\,\not{k}(q+p)_\mu, \\
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& \mathcal{T}^{(7)}_\mu = -\frac{\mathrm{i}}{2}\,[\not{q},\not{p}]\gamma_\mu, \\
[\bar{q}\not{D}^4 q]: \quad & \mathcal{T}^{(8)}_\mu = \frac{1}{2}\,[\not{q},\not{p}](q+p)_\mu.
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Full transversal basis of the vertex: $\not{D}$ ^ odd structures: not for 2-SC (scalar);

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$\not{D}$ ˆ odd structures: not for 2-SC (scalar);

$\not{D}$ ˆ even structures: $\mathcal{T}^{(4)}$ is particularly interesting: follows from the hierarchy of the full tensor structures:

$\{1,4,7\} \gg \{2,5,6\} \gg \{3,8\}$.

Gao, Papavassiliou and Pawlowski, PRD 2021,
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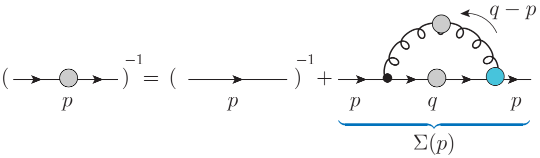
We then focus on:

$$[\Xi^{(+)}_\mu]^a = t_4 P_{\mu\nu}(k)\mathcal{T}^{(4)}_\nu \gamma_5\,\mathcal{K}^a_+,$$

color-flavor structure - from TWTI:
Qin and Roberts and Schmidt, PLB 2014 in N-G.

$$\mathcal{K}^a_+ = \frac{1}{2}[(\lambda^a)^T\mathcal{M} - \mathcal{M}\lambda^a].$$

Contribution of the relevant vertex structures to the diquark gap equation:



$$, \quad \mathbf{\Gamma}_\mu \sim \begin{pmatrix} Z_c\,\gamma_\mu & t_4^*\,T_\nu^{(4)} + \Xi^{\mathrm{STI}}_{\nu,-} \\ t_4\,T_\nu^{(4)} + \Xi^{\mathrm{STI}}_{\nu,+} & Z_c\,\gamma_\mu \end{pmatrix}.$$

$$\Delta \doteq \alpha_d \Delta + \alpha_{\text{NG-STI}} \Delta + \alpha_4 t_4 .$$

Quark DSE in Nambu-Gor'kov

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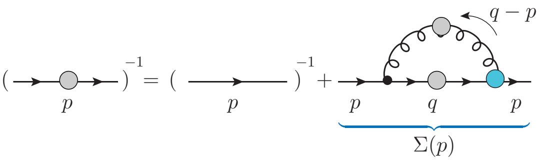

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$$\Delta \doteq \alpha_d \Delta + \alpha_{\text{NG-STI}} \Delta + \alpha_4 t_4 .$$

For $N_c = 3$:

$$\alpha_d \Delta = 2 g_s^2 \sum\!\!\!\!\!\!\!\int_q \frac{q^2 + \mu^2}{\tilde{q}^2 \tilde{q}^{*2}} Z_c(q-p) G_A(q-p)\,\Delta ,$$

and:

$$\alpha_4 t_4 = \frac{9}{2} g_s^2 \sum\!\!\!\!\!\!\!\int_q \frac{\tilde{q}\cdot(\tilde{q}-\tilde{p})}{\tilde{q}^2} G_A(q-p)\, t_4(q,p).$$

Vertex DSE in Nambu-Gor'kov

Gluon self-interaction is one distinct feature in QCD from QED;
Self-consistency for the off-diagonal vertex - specific focus on the diagram A:

$$[\Xi_\mu^{(+)}]^a(q,p) = \int_k f^{abc}[S^{3A}_{\mu\mu_1\mu_2}](p-q, q-k, k-p)\,[G_A]_{\mu_1\nu_1}(k-q)$$
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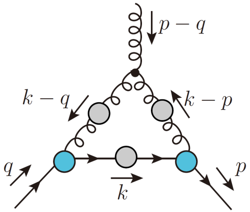

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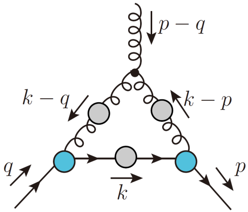
- Approximation: $t_4(q,p)$ at the symmetric point $q = p = \epsilon$, and take the "infrared" limit $\epsilon \to \epsilon_0 = (\pi T, \vec{0})$.

  Vertex strength t_4 is singled out; DSE turns into a polynomial equation for t_4:



$$t_4 = \beta_d \Delta + \beta_4 \Delta^*\, t_4^2,$$

$$\beta_d = \frac{4}{3} g_s^2 \sum\!\!\!\!\!\!\int_k \frac{\vec{k}^2}{(k-\epsilon_0)^2} Z_c^2(k-\epsilon_0) G_A^2(k-\epsilon_0), \qquad \beta_4 = 8 g_s^2 \sum\!\!\!\!\!\!\int_k \frac{\vec{k}^2(k^2+\mu^2)}{\tilde{k}^2 \tilde{k}^{*2}} G_A^2(k-\epsilon_0).$$

From weakly- to strongly- coupled diquarks

$t_4 \doteq \beta_d \Delta + \beta_4 \Delta^* \, t_4^2$. With small Δ : $t_4^{(w)} \simeq \beta_d \Delta$ or $t_4^{(s)} \simeq (\beta_4 \Delta^*)^{-1}$.

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The first scenario again gives the BCS-type diquark condensate:

$$\Delta^{(w)} \doteq (\alpha_4 \beta_d + \alpha_d + \alpha_{\text{NG-STI}}) \Delta^{(w)};$$

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$$\alpha_4 = \frac{3}{2} \langle g_s^2 A_\mu^2 \rangle - \frac{9}{2} g_s^2 \sum\!\!\!\!\!\!\int_q \frac{\tilde{q} \cdot \epsilon_0}{\tilde{q}^2} G_A(q - \epsilon_0), \qquad \epsilon_0 = (\pi T, \vec{0}).$$

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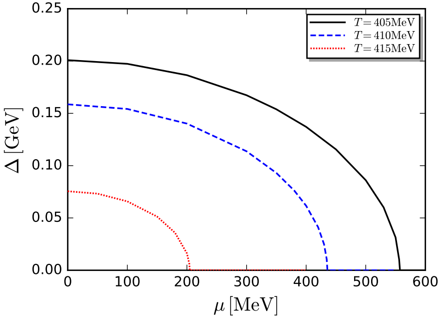
  - and when $\alpha_d + \alpha_{\text{NG-STI}} < 1$.

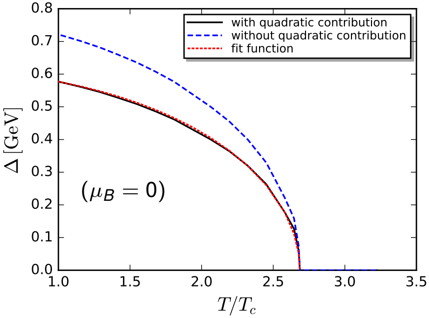
Numerical results with QCD inputs

- Lattice QCD result of dim-2 gluon condensate (vacuum): $\langle g_s^2 A_\mu^2 \rangle = 4.4\,\text{GeV}^2$, renormalised at $10\,\text{GeV}^2$. [Arriola, Bowman and Broniowski, PRD 70: 097505 (2004)]
- 2+1-flavor gluon propagator (vacuum, Landau gauge).

  We compute: finite T and μ_B corrections for $\langle g_s^2 A_\mu^2 \rangle$ and $[G_A]_{\mu\nu}$.

  Results for the second scenario:

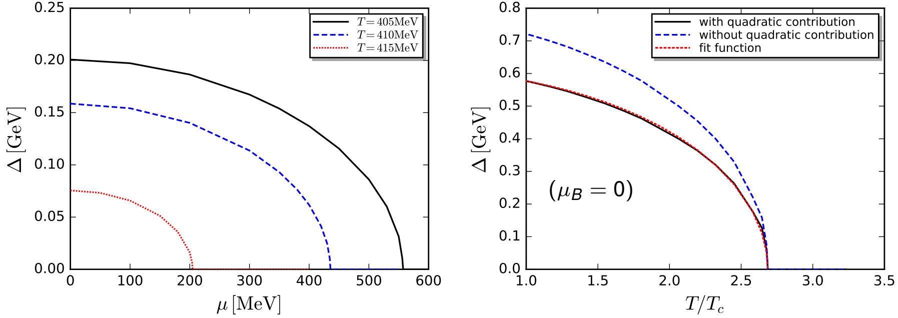




Numerical results with QCD inputs

- The second scenario $|\Delta^{(s)}|^2 \propto \alpha_4(T)$ becomes dominant at small μ_B;
- $\Delta^{(s)}$ persists even towards zero chemical potential: $\alpha_4(T_\chi = 155\,\mathrm{MeV}) > 0$: strongly coupled pairing condensate from the non-Abelian interactions.
- Sign change of α_4 at some higher T_Δ; beyond that there is no $\Delta^{(s)}$.

$T_\Delta = 416\,\mathrm{MeV}$. fit at zero μ_B: $|\Delta^{(s)}|^2 \propto \alpha_4(T) \propto 1 - (T/T_\Delta)^a$, $a = 2.16$.



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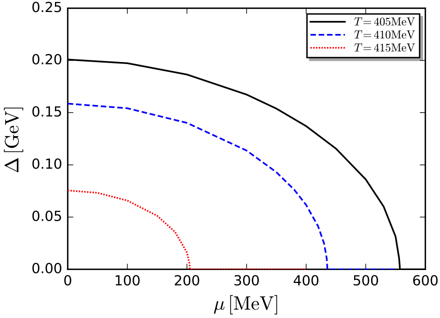
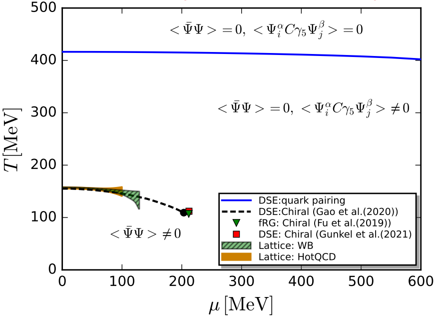
simplifation, or over-simplifcation?





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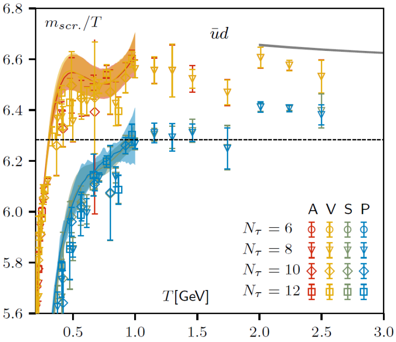
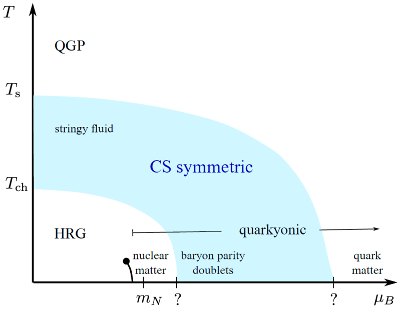
Glozman, Philipsen and Pisarski, Eur. Phys. J. A 58: 247 (2022).

Predicted temperature window $T_\chi < T < T_s$ by lattice QCD.
- Chiral symmetry restored; the dynamics is inconsistent with partonic DOFs.





Discussion and outlook

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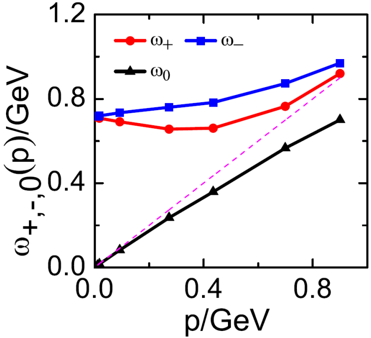

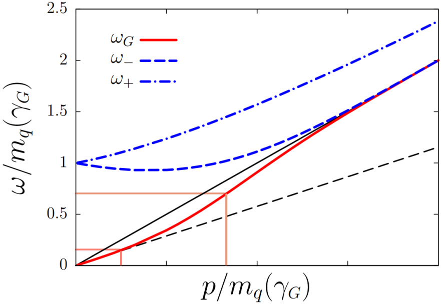

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    Thermodynamic quantities are afar from the asymptotic limit at a few times of T_χ.

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Thanks for your attention!!

Back-up

Symmetry relations in Nambu-Gor'kov

Transformation of the charge conjugate $\mathcal{C}$ (D. Müller, 2013):

$$\boldsymbol{G}_q = \begin{pmatrix} G_q^{(+)} & \Delta_q^{(-)} \\ \Delta_q^{(+)} & G_q^{(-)} \end{pmatrix}; \qquad \boldsymbol{\Gamma}_\mu^a = \begin{pmatrix} \Gamma_{\mu+}^a & \Xi_{\mu-}^a \\ \Xi_{\mu+}^a & \Gamma_{\mu-}^a \end{pmatrix}.$$

$$G_q^{(\pm)}(p) = -\mathcal{C} G_q^{(\mp)}(p)^T \mathcal{C}, \qquad \Delta_q^{(\pm)}(p) = -\mathcal{C}\Delta_q^{(\mp)}(-p)^T \mathcal{C}.$$

$$\Gamma_\mu^{a,(+)}(p,q) = -\mathcal{C}\Gamma_\mu^{a,(-)}(-q,-p)^T \mathcal{C}, \quad \Xi_\mu^{a,(+)}(p,q) = -\mathcal{C}\Gamma_\mu^{a,(-)}(-q,-p)^T \mathcal{C}.$$

In the presence of diquarks and with chiral symmetry at $\mu_B = 0$?

$$\boldsymbol{G}_q \sim \begin{pmatrix} \mathrm{i}\gamma \cdot p & \Delta\gamma_5\mathcal{M} \\ \Delta\gamma_5\mathcal{M} & \mathrm{i}\gamma \cdot p \end{pmatrix}.$$