Towards a parameter-free determination of QCD critical exponents and chiral phase transition temperature

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Strong-interaction matter under extreme conditions

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Collaborators and outline of talk

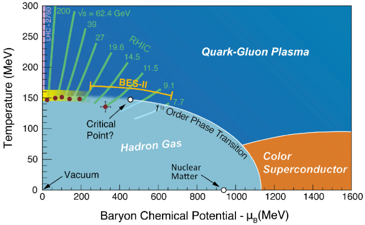
An ongoing work with my collaborators :

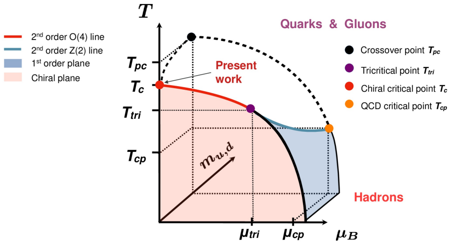
Frithjof Karsch , Sipaz Sharma , Christian Schmidt , Mugdha Sarkar

A brief outline of the talk :

- Motivation : Setting up the stage
- What's New in this work ?
- Ingredients and details of the work
- Some New Results
- Conclusions and outlook

Motivation : QCD phase diagram





- Grand aim : Find the **QCD critical point** $(\mu_{cp}, T_{cp}, m_{phys})$, where **crossover** $\longleftrightarrow$ **1^{st} ord. phase transition** (PT) lines.
- One way : **Constrain** T_{cp} using $\xleftarrow[\text{analysis}]{\text{3-d model}}$ $T_{cp} < T_{tri} < T_c < T_{pc}$.
- Hence, knowledge of T_c $\rightarrow$ **very important** to determine T_{cp}
- From Lattice QCD $\xrightarrow[\text{BMW}]{\text{HotQCD}}$ $T_{pc} \approx$ **157** MeV, $T_c \approx$ **133** MeV
- In this work $\rightarrow$ also estimate T_c on lattice. Then, what's NEW ??

What's new

Aim of this work : we try to find

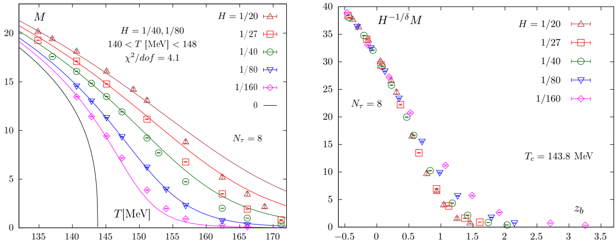
- Chiral phase transition temperature $\boldsymbol{T_c}$, and
- Critical exponent $\boldsymbol{\delta}$ of this phase transition, considering this is **2^{nd} order** with a **well-defined** universality class (UC)

What's new ? → All these in a NEW way which is **independent** of **fitting** and **input parameters**. These are

- **Critical exponents** (CE) of the UC of chiral phase tr.
- **Scaling parameters** → present in scaling variables appearing in scaling functions (impt. & appear in critical behaviour analysis)

Why is this important ? → Because in related previous works and also in a recent paper [Ding et. al. 2403.09390], to get T_c, one used

What was there before ?



[Ding et. al. 2403.09390]

- **O(2)** CE's β, δ in scaling functions $\rightarrow$ fitting parameters for
- fitting the order parameter M vs T data (A detailed definition of M will be given later). From this, one
- Obtains scaling ansatz $\leftarrow$ using $T_c \pm 3\%$ data for fit range.
- Observes good scaling behaviour $\leftarrow$ **only in this** T range.
- Thus $\Rightarrow$ **global fits** $\rightarrow$ **inconclusive**. So, in this work $\cdots$

This work: Aim

We try estimating T_c in a way free of fits. This is done by

1. **without using at all**, **any** O(2) or O(4) universality class **fitting parameter** or **any other properties** of this class, and
2. with **no knowledge** of **critical exponents** β, δ at the **input level**

- Ensuring $\Rightarrow$ **fit-independent**, "**parameter-free**" approach
- The **only** pre-assumption here is (kind of established) **chiral PT** is a **second order** PT having a **universality class**

A quick revision on chiral phase transition $\cdots$

Universality class, critical exponents and scaling

- Chiral Ph. tr. (cPT) $\rightarrow$ **2^{nd}** *order* [Pisarski, Wilczek], where
  $SU(2)_L \times SU(2)_R$ [**QGP**] $\xrightarrow[\textbf{breaking}]{\textbf{spontaneous}}$ $SU(2)_V$ [**hadrons**]
- This PT $\in$ ***O*(4)** univ. class $\xleftarrow[\text{by}]{\text{indicated}}$ $O(4)$ crit. expts.
  (2 independent CE's : β, δ) $\leftarrow$ hyperscaling relations.
- Order parameter $\rightarrow$ light quark chiral condensate $\langle\bar{\psi}\psi\rangle_\ell$,
  source field / symmetry-breaking field $\rightarrow$ light quark mass m_ℓ
- From Scaling relation $\rightarrow$ $\langle\bar{\psi}\psi\rangle_\ell \sim m_\ell^{1/\delta}$ at $T = T_c$
- We use **staggered fermions** $\xrightarrow[\text{preserve}]{\text{which}}$ O(2) rather than full O(4) chiral symmetry for <u>non-zero</u> lattice spacing.

Ingredients and Formulae $\cdots$

In scaling region

As $T \approx T_c \Rightarrow$ scaling region (SR), we can write bare quantities as :

$$\langle\bar{\psi}\psi\rangle_\ell = M_\ell = h^{\frac{1}{\delta}}\, f_G(z) + M_{\ell,regular}$$
$$\chi_\ell = \frac{\partial\langle\bar{\psi}\psi\rangle_\ell}{\partial m_\ell} = \frac{1}{\delta}\, h^{\frac{1}{\delta}-1} f_\chi(z) + \chi_{\ell,regular}\ , \quad \text{where} \tag{1}$$

$$z = t\, h^{-1/\beta\delta}, \qquad t = \frac{1}{t_0}\left[\frac{T}{T_c} - 1\right], \qquad h = \frac{1}{h_0}\left[\frac{m_\ell}{m_s}\right] \tag{2}$$

- Here, scaling variable $\rightarrow z$; non-universal parameters $\rightarrow t_0, h_0$
- $\langle\bar{\psi}\psi\rangle_\ell,\ \chi_\ell \rightarrow$ dimensionless (in lattice spacing units)
- $f_G(z),\ f_\chi(z) \rightarrow$ scaling functions. However, one finds $\langle\bar{\psi}\psi\rangle_\ell$ has additive (UV) and multiplicative (log) divergences.
- So, we use an **improved** order parameter in our work. But $\cdots$

Brief about our work setup

Before showing the improved parameter, some details about the present setup of our work. In this work, we use the following :

- The same values of main parameters and statistics, as was collected for [H.-T. Ding et al, 2403.09390]
- $\mathcal{O}(a^2)$ improved staggered fermions (HISQ)
- $\mathcal{O}(a^2)$ Symanzik improved gauge action
- quark masses m_ℓ, m_s and $H = m_\ell/m_s$. We consider $\mathbf{1/20} \leq H \leq \mathbf{1/160}$, corresponding to $m_\pi \in [55 : 160]$ (MeV)
- $N_s^3 \times 8$ lattices with $N_s = 32, 56$ varying with H, such that,

  $m_\pi \cdot N_s > 3.8$ for $20 \leq H^{-1} < 160$, and

  $m_\pi \cdot N_s = 2.7$ for $H^{-1} = 160$.

Improved order parameter

The **Improved** order param. M is given: $\big($Used by [Unger, PhD thesis Bielefeld, 2010], [Kotov et.al., 2105.09842], [Dini et.al. 2111.12599]$\big)$

$$M = M_\ell - H\,\chi_\ell \quad \text{where} \tag{3}$$

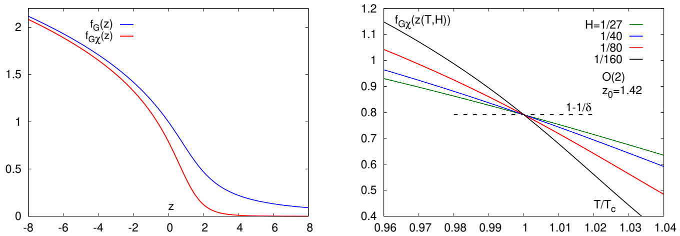
$$M_\ell = \frac{m_s}{f_K^4}\,\langle\bar{\psi}\psi\rangle_\ell\,, \qquad \chi_\ell = m_s\,\frac{\partial M_\ell}{\partial m_\ell} \quad \text{which makes}$$

$$M = h^{\frac{1}{\delta}}\,\boldsymbol{f_{G\chi}(z)} \quad \text{with} \qquad f_{G\chi}(z) = f_G(z) - f_\chi(z) \tag{4}$$

Eqn.(4) **only**, is $\rightarrow$ universal scaling part of M. In this improved M

- **No** $\mathcal{O}(a^{-2})$ add. div. $\rightarrow$ **well-defined** in **continuum limit**
- **Mult. renorm.** by m_s $\rightarrow$ **no** log div. $\rightarrow$ **well-def.** in **chiral limit**
- **No** $\mathcal{O}(H)$ regular terms in $M \Rightarrow$ **reduced** $\big(M_{reg} < M_{\ell,reg}\big)$
- Also **directly related** to scal. func. $\boldsymbol{f_{G\chi}(z)}$. Its properties :

Plot of the scaling function



- Inflection pt. of $f_{G\chi}(z)$ **closer** to $z = 0$ than $f_G(z)$. For O(2) UC, $z_{t,G\chi} = 0.629\,(10) < \; z_{t,G} = 0.7991\,(96)$ [Ding. et. al., 2403.09390].
- So, $f_{G\chi}(z) \rightarrow$ **weaker** H, m_ℓ dependence of T_c. We observe :
- a **unique** intersection point in $f_{G\chi}$-T plane $\forall\ H \neq 0$. This pt.
- precisely **at** $\boldsymbol{z = 0} \equiv T = \boldsymbol{T_c}$ with co-ords. $(x, y) = \left(1, \left[1 - \frac{1}{\delta}\right]\right)$
- Using this property of $f_{G\chi}(z)$, we find $\cdots$

Re-scaled order parameter

- If we have a **re-scaled** order parameter $\mathcal{M} = H^{-1/\delta} M$, given by

$$\mathcal{M}(T,H) = h_0^{-1/\delta} f_{G\chi}\big(z(T,H)\big) + \mathcal{M}_{reg}(T,H) \tag{5}$$

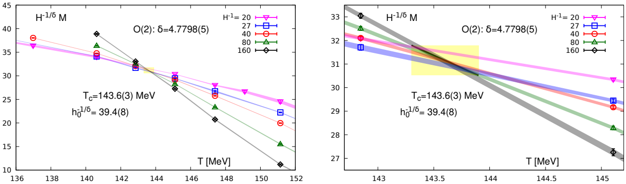
- then **at** $\underline{T = T_c}$, this reduces to

$$\mathcal{M}\,(T = T_c\,,\,H) = h_0^{-1/\delta}\,(1 - 1/\delta) + \mathcal{M}_{reg}\,(T_c\,,\,H) \tag{6}$$

This clearly indicates that in the $\underline{\mathcal{M}\text{ vs }T\text{ plot}}$ (next slide)

- For some H, if $\boldsymbol{\mathcal{M}_{reg} \ll h_0^{-1/\delta}\,(1 - 1/\delta)}$, then → these **different** H lines **intersect** at a **unique common point at** $T = T_c$.
- **Otherwise** they **shift away**, with **no common** pt. of intersection. Evidence using Lattice QCD $\cdots$

Plots of re-scaled order parameter



- **unique** crossing point for $1/H = 40, 80, 160$ data (within errors)
- clear remnant H-dependence at T_c for $1/H = 20$ (**regular** or **correction-to-scaling** contribution?).
- For $1/H = 27 \rightarrow$ it is marginal (needs better resolution in T)
- **Reproduce** $T_c, h_0^{-\frac{1}{\delta}} \xleftarrow[\text{before from}]{\text{obtained}}$ scal. fits in $T \in [140{:}146]$ MeV
- **Quantify** reg. cont. at $T_c \xrightarrow{\text{presently}} \mathcal{M}_{reg} < \mathbf{2\%}$ for $H = 1/20$.

Note $\mathcal{M}(T)$ **not** parameter-free, still depends on δ value. So, $\cdots$

The new observables

- Construct observables $\boldsymbol{R}\,(T, H_1, H_2)$, $\boldsymbol{B}\,(T, H_1, H_2)$, where

$$R\,(T, H_1, H_2) = \frac{M\,(T, H_2)}{M\,(T, H_1)}\,, \quad B\,(T, H_1, H_2) = \frac{\ln R\,(T, H_1, H_2)}{\ln\left[H_2/H_1\right]} \tag{7}$$

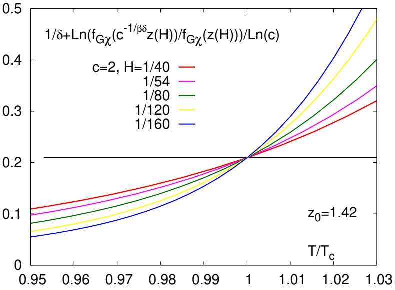
- In SR, with $\underline{M_{sing} \gg M_{reg}}$, $c = H_2/H_1$, Eqn. (6) $\rightarrow$

$$R \rightarrow R_s(t, H_1, c) = c^{1/\delta}\left(f_{G\chi}(z_2)/f_{G\chi}(z_1)\right) \tag{8}$$

$$B \rightarrow B_s(t, H_1, c) = \frac{1}{\delta} + \frac{\ln[f_{G\chi}(z_2)/f_{G\chi}(z_1)]}{\ln(c)} \tag{9}$$

- Eqns.(8),(9) $\rightarrow R\,(\boldsymbol{T_c}, H_1, c) = c^{1/\delta}$, $B\,(\boldsymbol{T_c}, H_1, c) = 1/\delta \Rightarrow$ find
- **unique** int. pt. **at** $T = T_c$ in $\underline{R\text{ - }T \text{ and } B\text{ - }T \text{ planes}}$. With
- **No input parameters** (New parameter-free feature). Like $\cdots$

B vs T plot : In ideal case



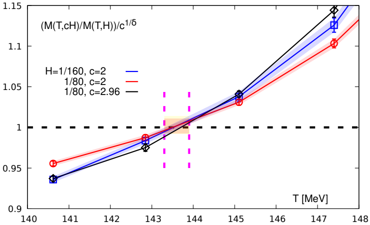
- Int. pt. **exactly at** $\left[x = 1,\ y = \frac{1}{\delta}\right] \rightarrow$ **indept.** of params. $\boldsymbol{z_0}, \beta$ ($z_0 = 1.42$ is only to compare with earlier work.). But note, one
- finds this common pt. $\xleftarrow[\text{when}]{\textbf{only}}$ $M \gg M_{reg}$, making $M \approx M_{sing}$.
- Larger $H, m_\ell \rightarrow$ more $M_{reg} \longrightarrow$ **deviations** from this point. These
- appear in Lattice QCD calc.. We cannot isolate M_{sing} only $\cdots$

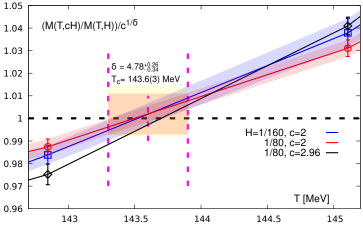
Things beforehand

For the new Lattice QCD results (will show now), we have :

- Worked on $N_\tau = 8$ lattices only
- Considered mass ratios $H^{-1} = 20, 27, 40, 80, 160$

R = Ratio of M

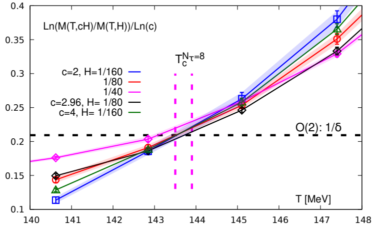


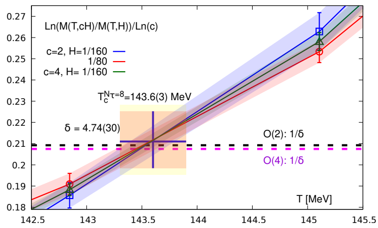


- Calculate $M(H_1), M(H_2), R$. Observe R as a function of T
- For $H \leq 1/40$ (**red** , **blue**) $\rightarrow$ intersect **at** $\boldsymbol{y = 1}$ **within** T_c [HotQCD] $\rightarrow$ agreement within T_c , $\delta_{[O(2)]}$ error bars
- ($H = 1/27, c = 2.96$) **black** line **deviate** ; intersect later beyond T_c range. Because, one gets
- **Appreciable** $\mathcal{O}(H^3)$ M_{reg} for $H = 1/\mathbf{27}$, over $1/\{40, 80, 160\}$.

Plots for $B \cdots$

B vs T plots





- For $H \leq 1/40 \rightarrow \underline{\text{unique int. pt.}} \left(T_c, \frac{1}{\delta}\right) \leftarrow$ **blue** , **red** , **green**
- Find deviations for $H = 1/\mathbf{20}\,, 1/\mathbf{27}$ (more for $1/\mathbf{20}$). However,
- Present calculations, based on existing data $\xrightarrow[\text{to}]{\text{allow}}$ estimate δ, already with **6**% accuracy
- So to do better, improve accuracy $\cdots$

Where we are and where we plan to go ?

What do we need to do ?

- We need to scan **more closely** $T \to T_c$
- Add **additional masses** to quantify regular contributions better

So, work is going on $\to$ in which

- We are generating more statistics for T close to T_c
- We are adding **more** T-data points near T_c
- We are adding **more** m_ℓ values (points). And all these $\xrightarrow[\text{that}]{\text{so}}$ we
- Understand systematics, H-dependence of $\mathcal{M}$ near T_c better
- Thereby, helping to understand if the deviations
- Come from the regular terms or the corrections-to-scaling terms

Conclusions and Outlook

- Constructed an estimator for T_c, δ based on ratios of improved order parameter evaluated at different values of light quark masses.
- At least in principle, this allows determining T_c , δ without relying on fits in the extended T, H ranges
- Within scaling region, one obtains a unique intersection point for lighter quark masses, also from lattice QCD
- This point agrees with predicted T_c results obtained from fits explicitly using δ of O(2) universality class.

- Understand systematics, behaviour of correction terms near T_c
- More precise calculations to resolve the δ's of $O(2)$ and $O(4)$ are on the way, but may be challenging
- Proceed towards continuum limit.

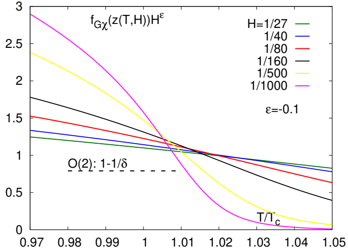
THANK YOU ALL

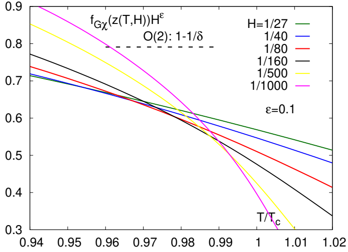
FOR

YOUR ATTENTION

BACKUP SLIDES

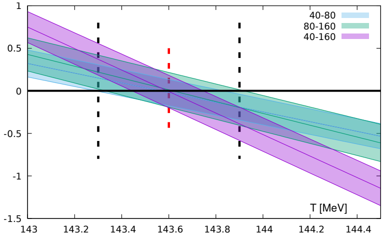
Parameter dependence in re-scaled order parameter





- No common and unique intersection point
- Depends on the value of δ used for re-scaling $\rightarrow \delta$ dependent
- Hence, parameter dependent
- Depends on if $\epsilon = \frac{1}{\delta'} - \frac{1}{\delta}$, varies as $H^{-\epsilon}$
- Determines if intersection points lie below or above $f_{G\chi}(t=0) = \left(1 - \frac{1}{\delta}\right)$

How the yellow box has been constructed



Important formulae

$$f_\chi(z) = \frac{1}{\delta} f_G(z) - \frac{z}{\beta\delta} f_G'(z) \tag{10}$$

$$f_{G\chi}(z) = \left(1 - \frac{1}{\delta}\right) f_G(z) + \frac{z}{\beta\delta} f_G'(z) \tag{11}$$

$$M = h^{1/\delta} f_{G\chi}(z) + M_{reg} \tag{12}$$

$$H^{-1/\delta} M = h_0^{-1/\delta} f_{G\chi}(z) + H^{-1/\delta} M_{reg} \tag{13}$$

Scaling variable constructions

$$t = \frac{T - T_c}{t_0\, T_c} = \frac{\tau}{t_0}\,, \qquad h = \frac{m_\ell}{h_0\, m_s} = \frac{H}{h_0}\,, \qquad \ell = \frac{\ell_0}{L} \tag{14}$$

where t_0, h_0, ℓ_0 are non-universal scaling parameters.

$$z = z_0\, z_b \;\;, \qquad z_L = z_{L,0}\, z_{L,b} \qquad \text{where}$$

$$z_0 = \frac{h_0^{1/\beta\delta}}{t_0} \;\;, \qquad z_{L,0} = h_0^{\nu/\beta\delta}\, \ell_0 \qquad \text{(fitting param.)}$$

$$z_b = \tau\, H^{-1/\beta\delta} \;\;, \qquad z_{L,b} = \frac{1}{L}\, H^{-\nu/\beta\delta}$$

Observables in scaling region

$$M_\ell = h^{\frac{1}{\delta}}\, f_G(z) + f_{sub}\,(T, H)$$
$$\chi_\ell = h_0^{-1}\, h^{\frac{1}{\delta}-1}\, f_\chi(z) + g_{sub}\,(T, H)$$
$$M = h^{\frac{1}{\delta}}\, f_{G\chi}(z) + p_{sub}\,(T, H)$$

where

$$\mathcal{O}_{sub} = \mathcal{O}_{cts} + \mathcal{O}_{reg}\,, \qquad \left\{\mathcal{O} \equiv M_\ell,\ \chi_{ell},\ M\right\}$$