What is the IR Phase Really About?

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People involved: [various stages/aspects]

Andrei Alexandru (George Washington), Peter Markoš (Comenius), Robert Mendris (Shawnee)

Beijing (Yi-Bo Yang et al), Keh-Fei Liu (Kentucky), Massimo D'Elia (Pisa), Claudio Bonanno (Madrid)

Literature: [w degrees of separation]

0-th	1-st	2-nd	3-rd
1906.08047	1502.07732	1405.2968	1807.03995
	2103.05607	1412.1777	1809.07249
	2110.04833	hep-lat/0607031	2205.11520
	2305.09459	hep-lat/0610121	2110.11266
	2404.12298	hep-lat/0703010	2207.13569
	2310.03621	0803.2744	2212.09806

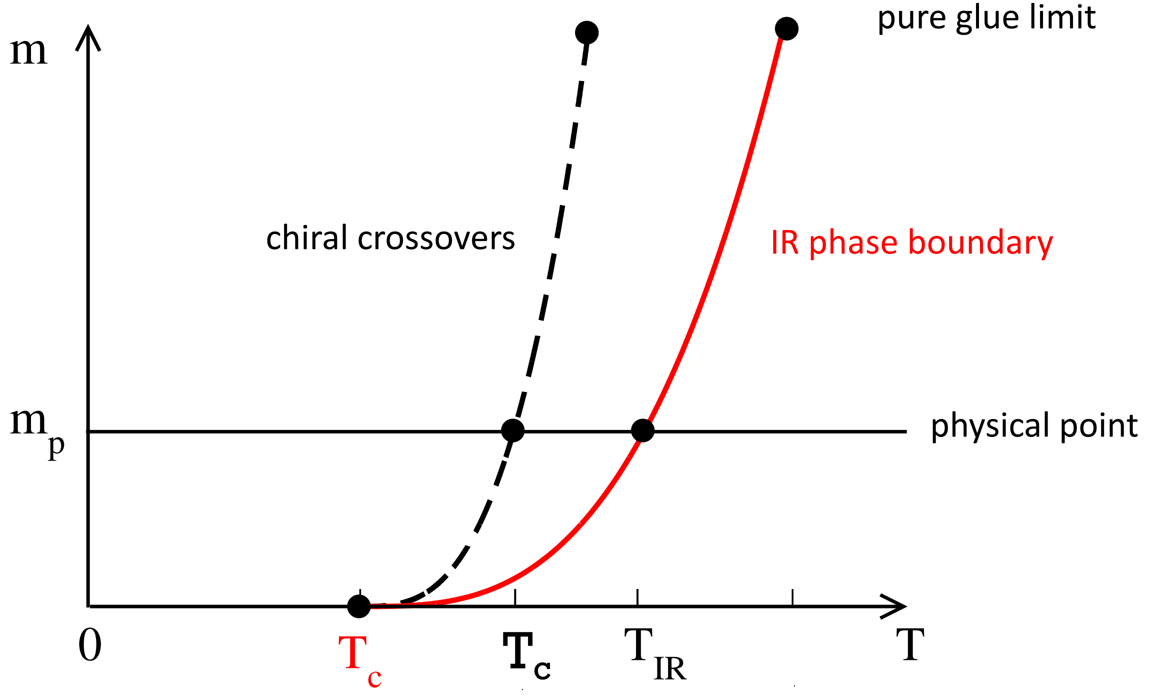
Technical credits: Dimitris Petrellis

Setup: μ_B=0

What is the IR Phase Really About?

- IR PHASE IS NOT ABOUT CHIRAL SYMMETRY [One of the rationales is to get away from dependence on chiral considerations.
- IT IS ABOUT GLUE SCALE INVARIANCE AND SEPARATION OF DOFs [Applies generally.]
- THE TWO CONCEPTS MEET IN THE CHIRAL LIMIT

E.g. N_f=2 theory:
IR phase trends known from
AA & IH 1502.07732



OUTLINE:

A. IR Phase of SU(3) Gauge Theories

B. Gluon Condensate via Spectral Density [Amusingly clarifying.] ✓

C. Scale Invariance in IR Phase More Formal: Anomaly & Stuff

D. Order Parameter for IR Phase

Have to proceed deductively regarding the IR phase. See below for inductive approach.

Original talk: https://indico.cern.ch/event/764552/contributions/3420459/attachments/1865996/3068382/WuHan_jun_2019_infra.pdf

Useful talk: https://drive.google.com/file/d/1vZ0AY0WsZAfF9iV7-Br-E_2NiwaZzRGp/view

See also a recent talk:

https://indico.cern.ch/event/1293041/contributions/5946693/attachments/2914234/5113815/Horvath_confXVI_Aug_2024_w_refs.pdf

A. SU(3) Gauge Theories w Fundamental Quarks

$$S = -\frac{1}{2g^2}\,\mathrm{tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{f=1}^{N_f} \bar{\psi}_f (D + m_f)\psi_f$$

[Euclidean formulation]

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \qquad A_\mu \in su(3)$$

[gluons]

$$D\chi \equiv \gamma_\mu(\partial_\mu + A_\mu)\chi \qquad \chi(x) \in C^{12}$$

[fundamental quarks]

Consider these at arbitrary temperature T and μ_B=0.

(1) Real-world QCD : N_f=2+1 at physical quark-masses and better

(2) Varied behaviors, including conformality

(3) For N_f < 16.5 we know how to take the continuum limit

A. IR Phase of SU(3) Gauge Theories…

AA & IH 1906.08047

$$\text{phase} \;=\; \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \rho(\lambda) \propto \lambda^p \;\;, \quad \lambda \to 0$$

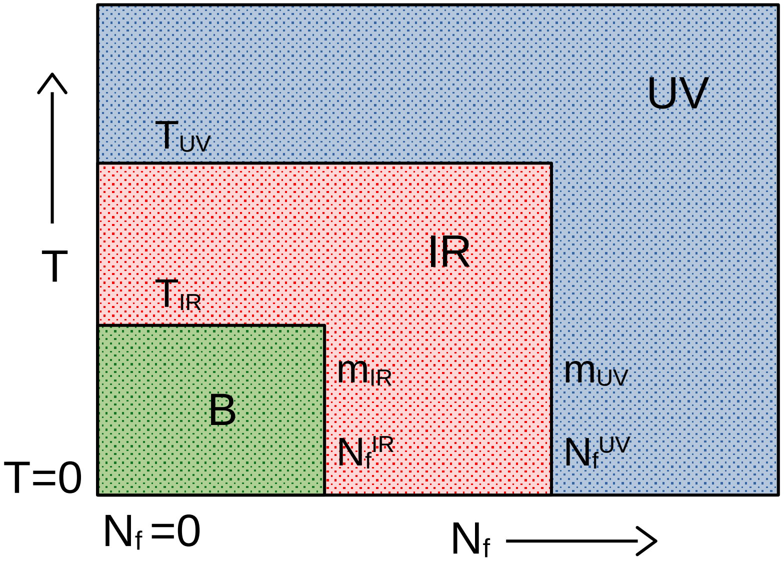
B = IR scale-broken

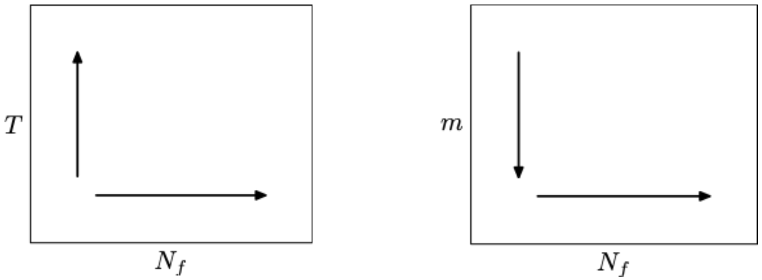
IR = IR scale-symmetric

UV = IR trivial

ρ(λ) = Dirac spectral density

SU(3) theories with fundamental quarks





Changes consistent with directions of arrows can induce transitions from B➞IR or from IR➞UV.

See also 1502.07732

- Most known detail comes from B➞IR thermal case: IR PHASE OF THERMAL QCD
- Important also B➞IR light-flavor case (T=0): IR PHASE = STRONGLY COUPLED PART OF CONFORMAL WINDOW

  1405.2968, 1412.1777, 1906.08047
- No hard evidence for traditional UV phase! 50-50?

A. IR Phase of SU(3) Gauge Theories… Important Aspects

I. IR PHASE OF THERMAL QCD

1906.08047, 2404.12298, 2305.09459

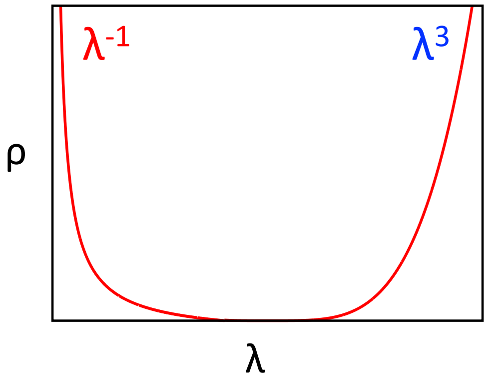
above/different from χ-crossover T_c

$T_c < T_{IR} < T < T_{UV}$

≈155 MeV   200-230 MeV   perturb

II. WHY IR?

- Power-law accumulation of DOFs in IR
  AA & IH 1906.08047
- Thermal QCD in IR phase:
  ⟶ highly unusual scales $\Lambda < 1$ MeV
  in fact, IR-bottomless
  ⟶ partial deconfinement 1502.07732



$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta = \delta(a) \rightarrow 0$

$a \rightarrow 0$?

III. WHY PHASE?

At T_{IR}:

1906.08047
2103.05607
2110.04833

(i) IR BECOMES AN AUTONOMOUS SUBSYSTEM
[IR-BULK decoupling, from1-component to 2-component system]

(ii) SCALE INVARIANT GLUE IN IR COMPONENT

(iii) NON-ANALYTICITIES APPEAR

(iv) INFINITE GLUE SCREENING LENGTHS APPEAR

A. IR Phase of SU(3) Gauge Theories… role of Anderson-like transitions

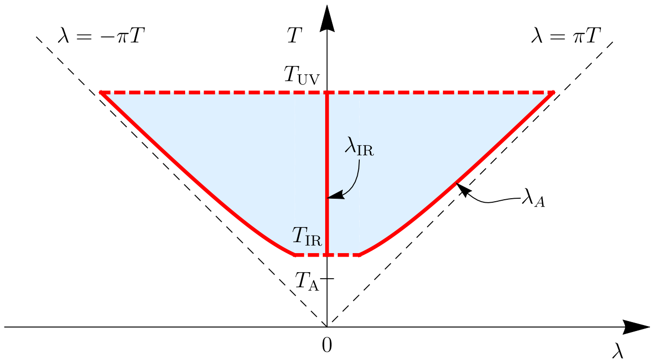
Upon transition to IR phase:

- (i) IR BECOMES AN AUTONOMOUS SUBSYSTEM [IR-BULK decoupling]
- (ii) SCALE INVARIANT GLUE IN IR COMPONENT
- (iii) NON-ANALYTICITIES APPEAR
- (iv) INFINITE GLUE SCREENING LENGTHS APPEAR

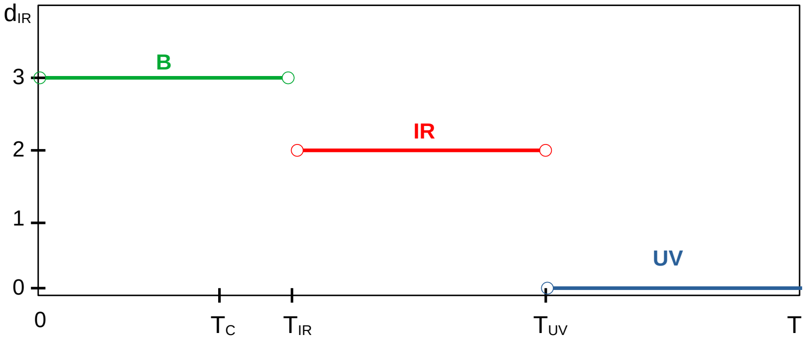
Focus on (i) and (ii) here

All elements put forward in 1906.08047 based on lattice evidence & ensuing consistency.

But associating it to Anderson-like transitions 2110.04833 and the new effective dimension theory 2103.05607, 1807.03995, 2205.11520 clarified details and interconnected them at different level!



Dirac spectral phase diagram in IR phase 2110.04833



Dimensions of near-zero modes and of $\langle F^2 \rangle_{\rm IR}$ 2103.05607, 2305.09459, 2310.03621

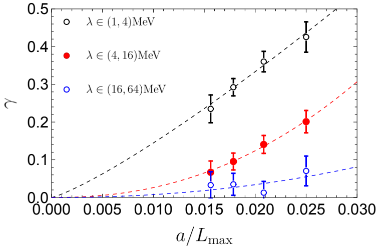
λ_A for metal-to-insulator

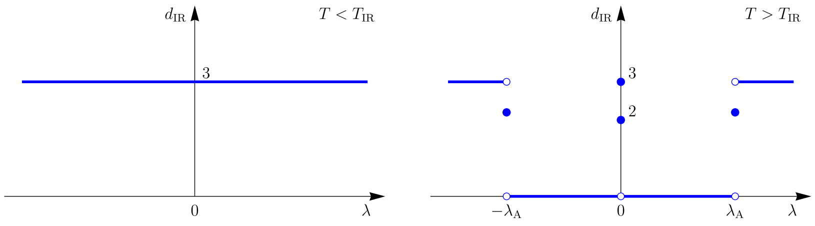
Garcia-Garcia & Osborn hep-lat/0611019

Kovacs & Pitler 1006.1205

Giordano, Kovacs & Pitler 1312.1179

$\lambda_{\rm IR}$ for metal-to-critical of 2110.04833





A. IR Phase: Temperature & QCD – DOFs & Scales

Scales fairy tale: thermal agitation erodes condensates and melts them upon T reaching T_c

DOF fairy tale: thermal agitation reduces IR DOF-s and depletes them when T reaches T_c

BUT IS THIS TRUE? Lattice perfect for unambiguous answer if lucky with scales.

Need a construct that expresses the distribution of DOFs over scales:

$$D = D[A] \qquad D\psi_\lambda = i\lambda\,\psi_\lambda \qquad \rho(\lambda, V_4) \equiv \frac{\#\,\mathrm{modes\ near}\ \lambda}{V_4\,\mathrm{d}\lambda}$$

Dirac
spectral
density

1) DISTRIBUTION OF QUARK DOFs ACROSS ENERGY-LIKE SCALES

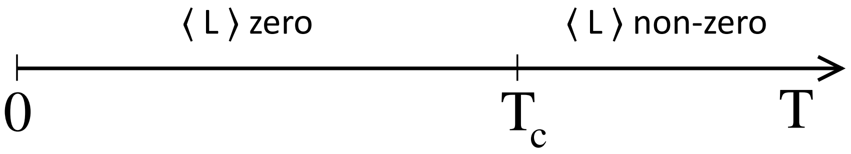
2) GAUGE-INVARIANT SCALE-DEPENDENT GLUE OPERATOR
[Quantifies contributions to F^2 from different energy-like scales.]

This should tell us truth about the stories & it does!

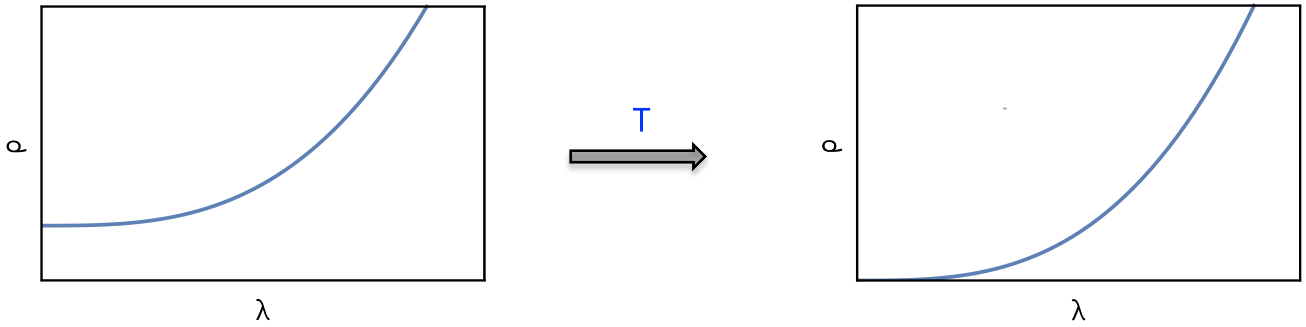
Due to 2) we say since 1502.07732: ``Give us your glue and we will tell you who you are.”

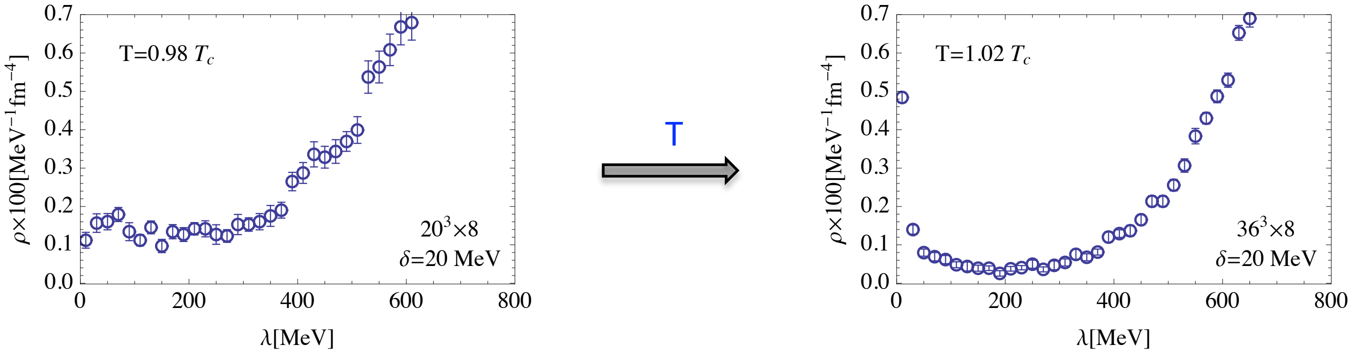
A. IR Phase: The Trigger

N_f=0 QCD



⟨ L ⟩ = Polyakov loop
1-st order transition



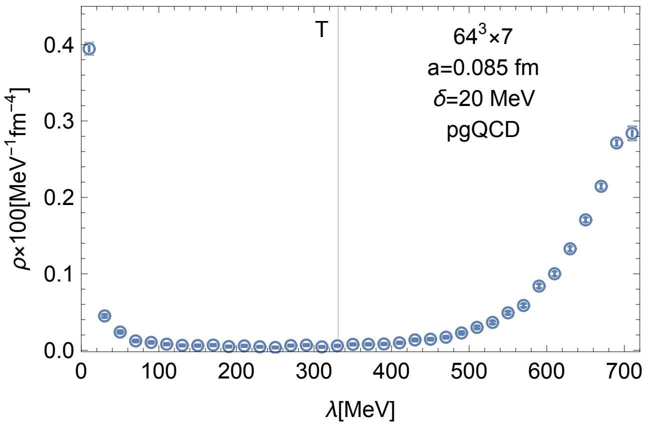


AA & IH 1502.07732

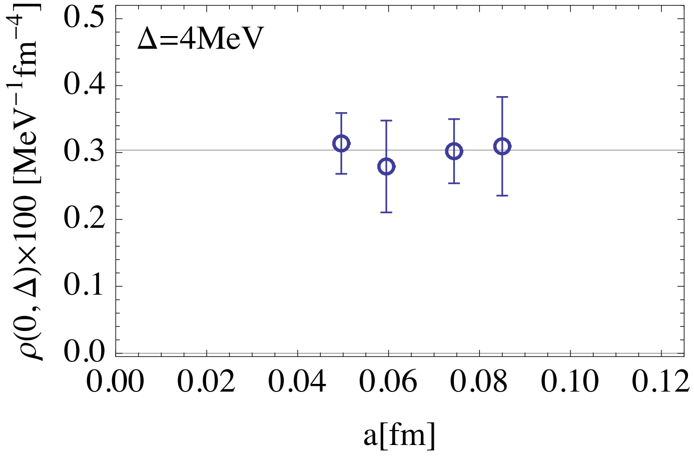
Well, perhaps some sort of an artifact? Edwards et al, hep-lat/9910041

A. IR Phase: The Real Thing?

$T=1.12\ T_c$



L=2 fm, fixed

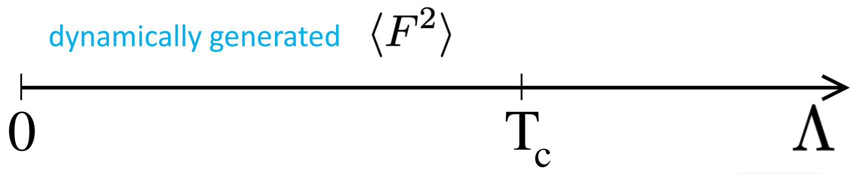


NO ARTIFACT in $N_f=0$ QCD. Strength of IR peak scales!

AA & IH 1502.07732

Stories: $N_f=0$ adaptation

Thermal agitation erodes $\langle F^2\rangle$ and melts it upon T reaching T_c



Thermal agitation reduces IR DOF-s and depletes them upon T reaching T_c. NOT TRUE

Removing scales was supposed to restore IR scale invariance trivially by removing IR DOF-s!

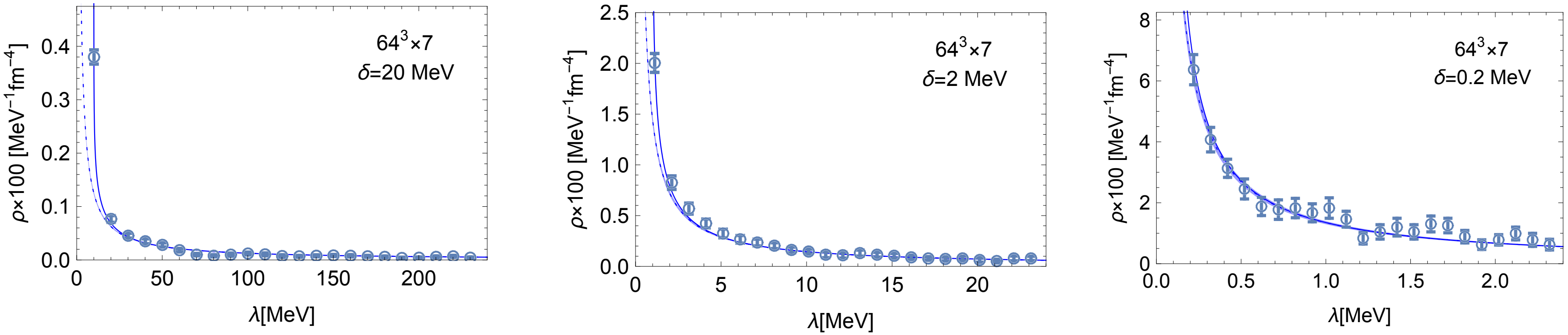
The reality is that IR DOF-s actually proliferate ☺. Thank you lattice!

Could it be that IR scale invariance is restored non-trivially???

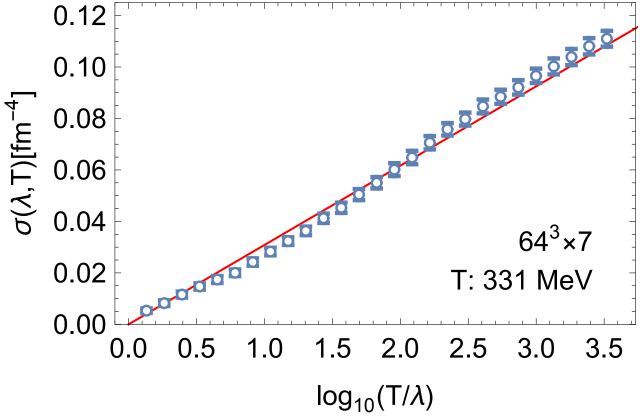
AA & IH 1906.08047

A. IR Phase: The Real Thing?

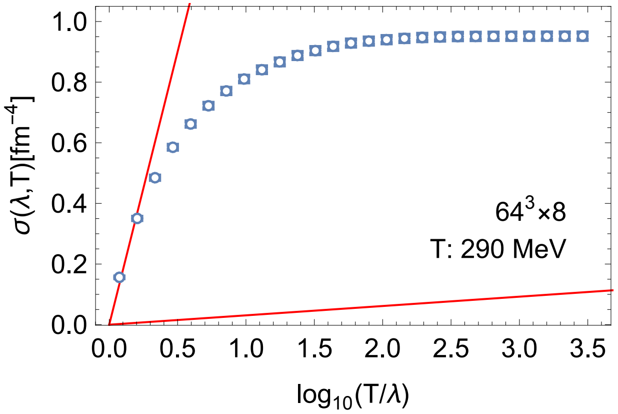
Fits to $\rho(\lambda) \propto 1/\lambda$ [$N_f=0$, $T=1.12\ T_c$] AA & IH 1906.08047



T=1.12 T_C



T=0.98 T_C



Data: (1) IR SCALE-INVARIANT DENSITY ($\lambda < T$) OVER 3 ORDERS OF MAGNITUDE IN SCALE

(2) NEGATIVE POWER-LAW ACCUMULATION OF DIRAC MODES IN IR: $\rho(\lambda) \propto \lambda^p$ $p \gtrsim -1$

Proposal: THIS REFLECTS IR SCALE-INVARIANT GLUE: <u>IR PHASE</u> [$p<0$] 1906.08047

A. IR Phase: WHAT JUST HAPPENED HERE?

T=0 classically scale invariant theory

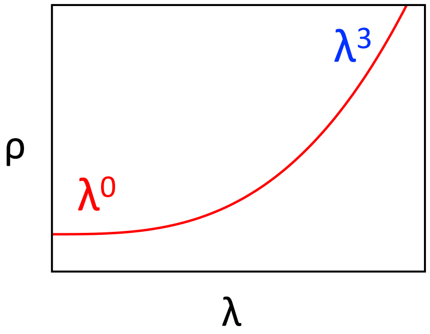
quantum fluctuations ⟹ scale anomaly

scales got generated world of hadrons etc

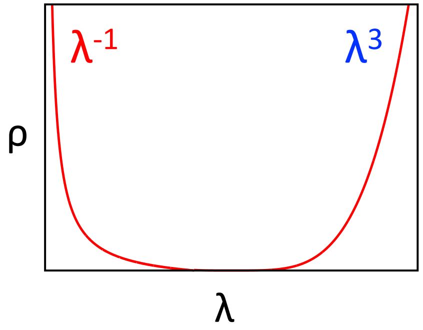
scale-broken T=0 theory

thermal fluctuations ⟹ increasing T

scale-invariant but only for $\Lambda < \Lambda_{IR} < T$



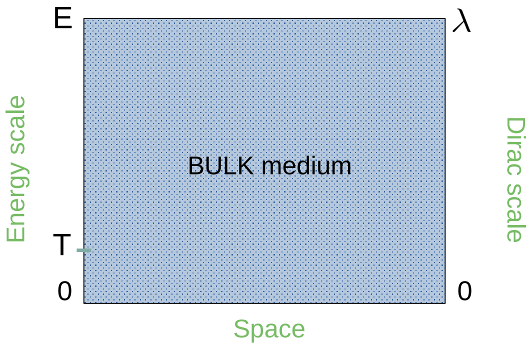
thermal agitation ⟹



$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta=\delta(a) \rightarrow 0$?

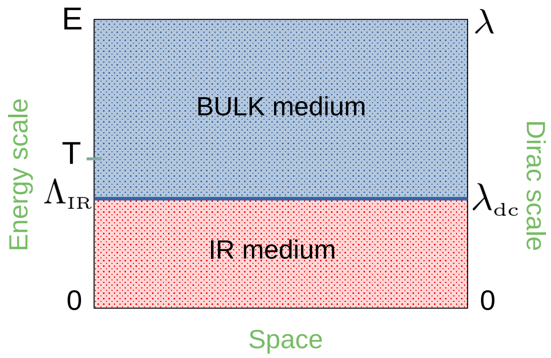
$a \rightarrow 0$



thermal agitation ⟹

IR-BULK SEPARATION

AA & IH 1906.08047

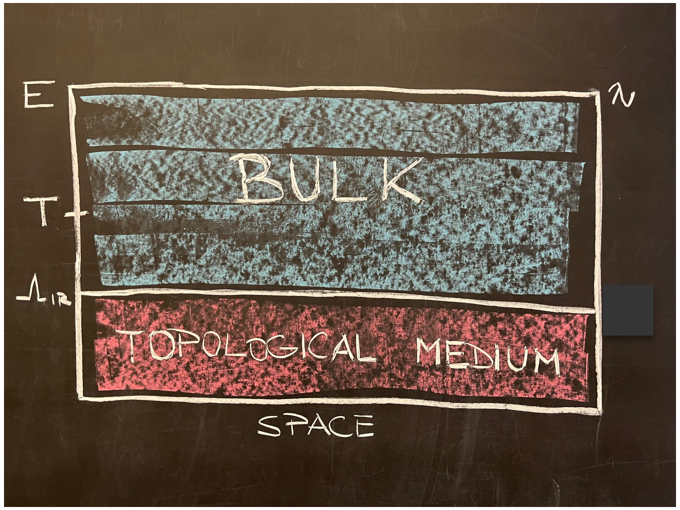


NON-INVARIANT

INVARIANT

AT $T=T_{IR}$ THERMAL QCD BECOMES 2-COMPONENT SYSTEM: IR MEDIUM AN AUTONOMOUS SUBSYSTEM

A. IR Phase: Connection to Near-Perfect Fluid of RHIC & ALICE?

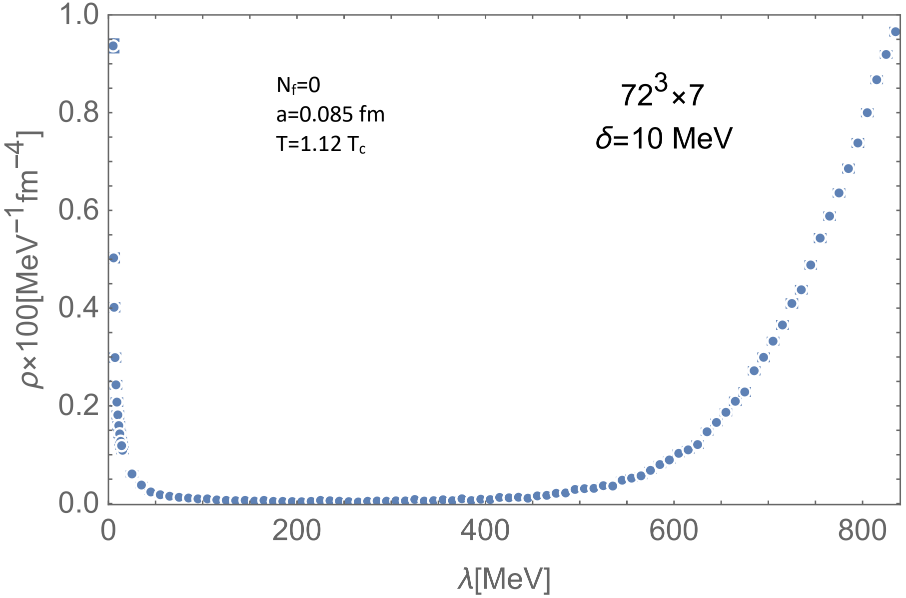


Hypothesis: IR phase describes the near-perfect fluid [RHIC, ALICE] state of matter.

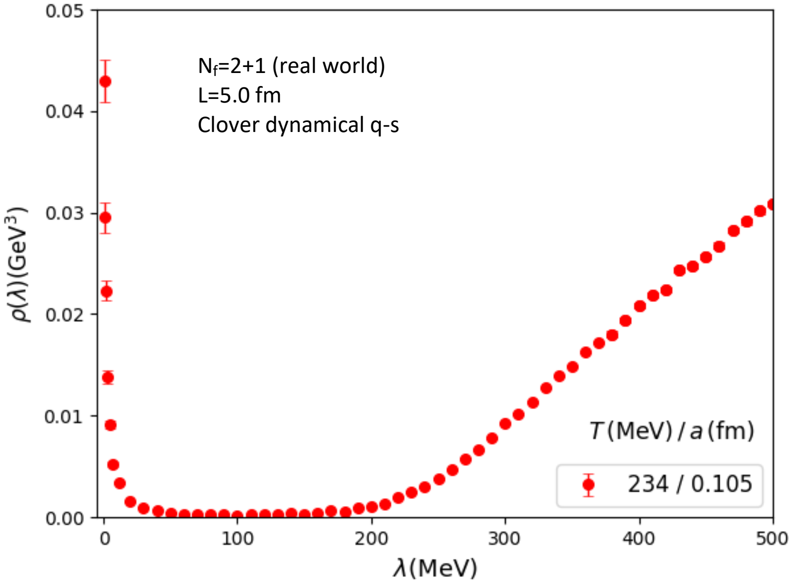
AA & IH 1906.08047

Experimental signatures on ALICE3?

AA & IH unpublished

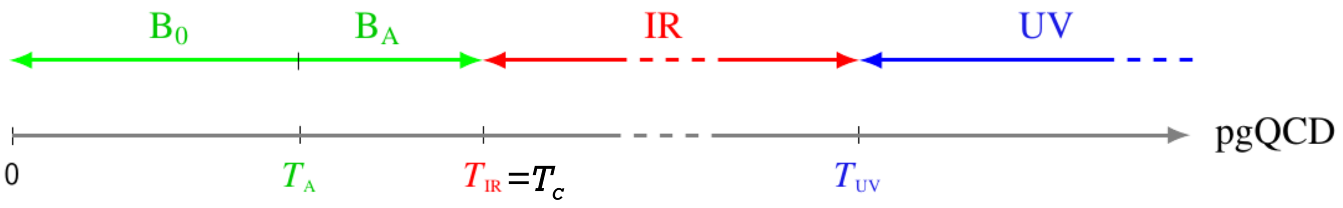


X. Meng et al 2305.09459



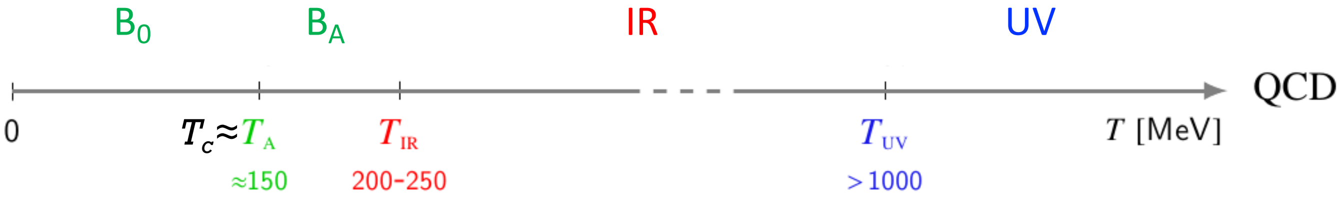
A. Phase Diagrams of Thermal QCD via IR Scale Invariance of Glue AA & IH 1906.08047

- Thermal phase diagram for N_f=0: Polyakov line transition coincides w IR-phase transition



IR BROKEN p = 0 IR SYMMETRIC p ≈ -1 IR TRIVIAL p ≫ 0

- Thermal phase diagram of N_f=2+1 ``real-world”: chiral crossover <u>below</u> IR-phase transition



STRONG COUPLING p = 0
HADRON GAS

STRONG COUPLING p ≈ -1
NEAR-PERFECT FLUID?

WEAK COUPLING p ≫ 0
PERTURBATIVE QGP

- Notes: (a) no hard evidence for existence of UV phase [Lives as likely an oversimplified but historically first picture of high-T QCD. Strict UVp: 50-50 chance!]
  (b) other transitions/crossovers in the bulk possible

B. Gluon Condensate via Dirac Spectral Density

$$\mathrm{tr}_{cs}\hat{D}_{x,x}(U) - \mathrm{tr}_{cs}\hat{D}_{x,x}(\mathbb{I}) \;=\; c_{\mathrm{S}}\, a^4\, \mathrm{tr}_c\, F_{\mu\nu}F_{\mu\nu}(x,A) \;+\; \mathcal{O}(a^6)$$

IH hep-lat/0610121, hep-lat/0607031 AA+IH+KFL arXiv:0803.2744

$\hat{D} \;\equiv\; aD$ Lattice Dirac operator: hypercubic symmetries + classical limit

$$D\psi_\lambda = \lambda\psi_\lambda \quad , \quad \lambda = \lambda_{\mathrm{R}} + i\lambda_{\mathrm{I}} \;\in\; \mathbb{C}$$

A classical continuum glue field

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \qquad A_\mu \in su(3)$$

U transcription of A onto hypercubic lattice

$\mathbb{I}$ free-field configuration

$$c_{\mathrm{S}} \neq 0 \quad \Longrightarrow \quad \text{defines} \quad F^2(x) \equiv \mathrm{tr}_c\, F_{\mu\nu}F_{\mu\nu}(x)$$

$$F^2(x,U) = \frac{1}{c_{\mathrm{S}}\, a^3}\, \mathrm{tr}_{cs}\Big[D_{x,x}(U) - D_{x,x}(\mathbb{I})\Big] \quad \longrightarrow \quad \langle\, F^2 \,\rangle = \frac{a}{c_{\mathrm{S}}}\, \frac{T}{L^3} \Big\langle\, \mathrm{Tr}\Big[D(U) - D(\mathbb{I})\Big] \,\Big\rangle$$

B. Gluon Condensate via Dirac Spectral Density

$$\langle\, \mathrm{Tr}\, D \,\rangle \;=\; ?$$

$$\rho_s(\lambda) = \frac{\#(\lambda, d\mathcal{S})}{V_4\, d\mathcal{S}} \qquad d\mathcal{S} = d\lambda_{\mathrm{R}} d\lambda_{\mathrm{I}}$$

surface spectral density

$$\langle\, \mathrm{Tr}\, D \,\rangle \;=\; \frac{L^3}{T} \int_{\mathbb{C}} d\mathcal{S}\, \rho_s(\lambda)\, \lambda$$

$$\langle\, F^2 \,\rangle = \frac{a}{c_{\mathrm{S}}} \int_{\mathbb{C}} d\mathcal{S}\, \lambda\, \rho_s(\lambda) \,+\, \mathrm{K}$$

K = theory-independent constant

$$\langle\, F^2 \,\rangle = \frac{a}{c_{\mathrm{S}}} \int_{\mathbb{C}} d\mathcal{S}\, \lambda\, \rho_s^{\mathrm{eff}}(\lambda) \qquad \rho_s^{\mathrm{eff}} \;\equiv\; \rho_s - \rho_s^0$$

Amusing new/general formula offering some insight.

B. Gluon Condensate via Dirac Spectral Density…

Overlap Dirac operators:
Neuberger, 1998

$$a\,\frac{D(\Delta)}{\Delta} = 1 + \frac{\hat{D}_W - \Delta}{\sqrt{(\hat{D}_W - \Delta)^\dagger(\hat{D}_W - \Delta)}} \qquad \Delta \in (0,2)$$

- ○ D_W is massless Wilson-Dirac operator
- ○ γ_5 – Hermiticity (complex pairs, antiparticles)
- ○ Ginsparg-Wilson relation: chiral symmetry and $\Delta(\lambda+\lambda^*) = a\,\lambda^*\lambda$

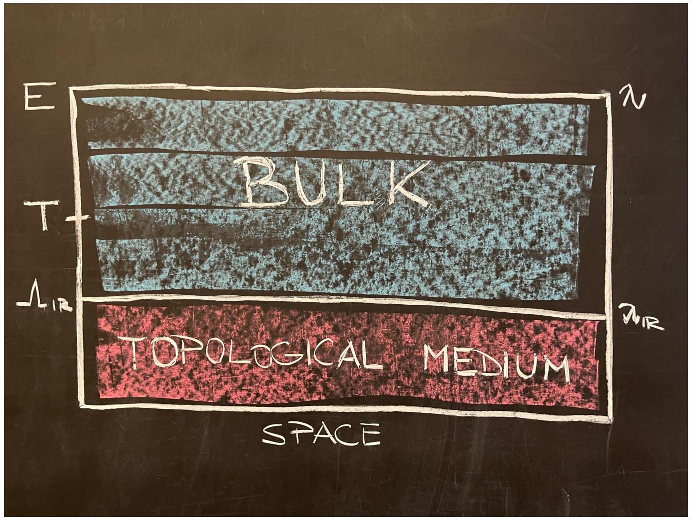
$$\rho_s(\sigma,\varphi) = \frac{\rho(\sigma)}{\sigma}\,\delta\Big(\varphi - \cos^{-1}\frac{a\sigma}{2\Delta}\Big) \qquad \sigma^2 = \lambda^*\lambda \qquad 0 \le \sigma \le \frac{2\Delta}{a}$$

$$\langle\, F^2 \,\rangle \;=\; \frac{a^2}{c_{\mathrm{S}}\Delta}\int_0^{(\frac{2\Delta}{a})^-} d\sigma\,\sigma^2\,\rho_{\mathrm{eff}}(\sigma) \;+\; T\,\frac{2\Delta}{c_{\mathrm{S}}}\,\frac{\langle n_0\rangle}{L^3}$$

$$\rho_{\mathrm{eff}}(\sigma) \;\equiv\; \rho(\sigma) - \rho_0(\sigma) \qquad\qquad n_0 \;=\; \text{number of zeromodes}$$

C. Scale Invariance and IR Phase: Anomaly & Stuff

$$\text{anomalous part } T_{\mu\mu} = \frac{\beta(g)}{2g}\langle F^2\rangle + \gamma_m(g)\, m\, \langle\bar{\psi}\psi\rangle$$



Claim of scale invariant <u>IR glue</u>:

A.A & I.H. 1906.08047

For consistent claim, its contribution to scale anomaly should vanish.

DOES IT???

$$\langle F^2 \rangle = \frac{a^2}{c_S \Delta} \int_0^{\left(\frac{2\Delta}{a}\right)^-} d\sigma\, \sigma^2 \rho_{\text{eff}}(\sigma) + T\, \frac{2\Delta}{c_S} \frac{\langle n_0 \rangle}{L^3}$$

$$\langle F^2\rangle = \langle F^2\rangle_{\text{IR}} + \langle F^2\rangle_{\text{B}} \qquad \langle F^2 \rangle_{\text{IR}} = \frac{a^2}{c_S \Delta} \int_0^{\sigma_{\text{IR}}} d\sigma\, \sigma^2 \rho(\sigma)$$

since $\sigma_{\text{IR}} < T$ (finite) IR CONTRIBUTION <u>VANISHES IN THE CONTINUUM LIMIT</u>!

C. Scale Invariance and IR Phase: Anomaly & Stuff…

$$\text{anomalous part } T_{\mu\mu} \;=\; \frac{\beta(g)}{2g}\langle F^2\rangle \;+\; \gamma_m(g)\, m\,\langle\bar{\psi}\psi\rangle$$

$$\langle F^2\rangle \;=\; \langle F^2\rangle_{\rm IR} \;+\; \langle F^2\rangle_{\rm B} \qquad\qquad \langle\, F^2\,\rangle_{\rm IR} \;=\; \frac{a^2}{c_{\rm S}\Delta}\int_0^{\sigma_{\rm IR}} d\sigma\, \sigma^2\, \rho(\sigma)$$

Crucial subtlety:

$$\langle\, F^2_{\rm IR}\,\rangle \;\longrightarrow\; 0 \quad \text{for } \; a \to 0$$

NEED BOTH!

$$\langle\, F^2_{\rm IR} F^2_{\rm B}\,\rangle_c \;\longrightarrow\; 0 \quad \text{for } \; L \to \infty$$

NO CIGAR WITHOUT DECOUPLING!

GLUE CONTRIBUTION OF IR COMPONENT TO SCALE ANOMALY VANISHES!

Formal statement if IR scale invariance in IR phase!

C. Fun with Gluon Condensate

$$\langle\, F^2 \,\rangle \;=\; \frac{a^2}{c_{\rm S}\Delta} \int_0^{\left(\frac{2\Delta}{a}\right)^-} d\sigma\, \sigma^2\, \rho_{\rm eff}(\sigma) \;+\; T\, \frac{2\Delta}{c_{\rm S}}\, \frac{\langle n_0 \rangle}{L^3}$$

Gluon condensate is a UV quantity!

[Quark condensate is a strictly IR quantity!]

$$\rho^{\rm eff}(\lambda) \;=\; \frac{c}{\lambda} + \hat{\rho}^{\rm eff}(\lambda) \qquad \longrightarrow \qquad \langle\, F^2 \,\rangle \;=\; 2\Delta\, \frac{c}{c_{\rm S}}$$

to be continued...

Analogue of Banks-Casher. Several different forms. TBA

D. Order Parameter for IR Phase

Let λ_0 be the smallest non-zero λ such that $<F^2>_{\lambda 0}$ is decoupled from $<F^2>_{2\lambda 0}$.

Then $<F^2>_{2\lambda 0}$ is an order parameter of B-IR phase transition!

Equivalent to what I said before!

IR Phase of SU(3) Gauge Theories...

AA & IH 1906.08047

$$\text{phase} \;=\; \begin{cases} \mathrm{B} & \text{if} \quad p = 0 \\ \mathrm{IR} & \text{if} \quad p < 0 \\ \mathrm{UV} & \text{if} \quad p > 0 \end{cases} \qquad \rho(\lambda) \,\propto\, \lambda^{p} \;\;,\;\; \lambda \to 0$$

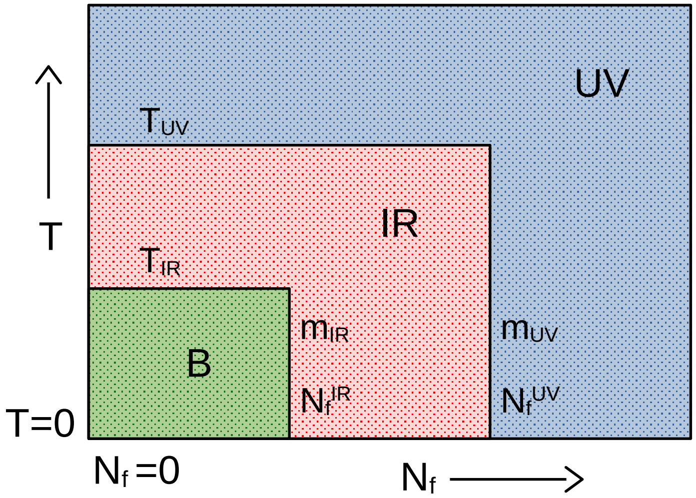
B = IR scale-broken

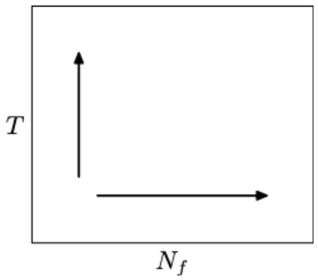
IR = IR scale-symmetric

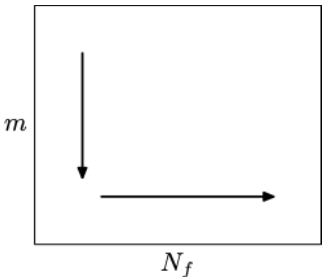
UV = IR trivial

ρ(λ) = Dirac spectral density

SU(3) theories with fundamental quarks

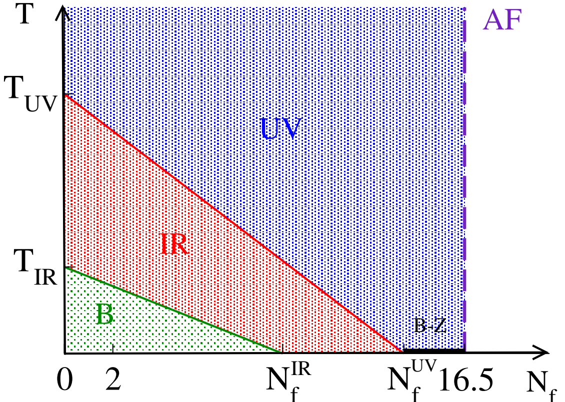






Changes consistent with directions of arrows can induce transitions from B➞IR or from IR➞UV.

See also 1502.07732



Studies of Dirac Spectra in Other Contexts

$U_A(1)$ problem and other

Dick et al 1502.06190

Kaczmarek et al 2102.06136

Aoki et al 2011.0149

Ding et al 2010.14836

Kehr et al 2304.13617

Glozman et al 2204.05083

Bonanno & Giordano 2312.02857

Kaczmarek et al 2301.11610

Kovacs & Vig 1706.03562

Rohrhofer et al 1902.03191

Cardinali et al 2107.02745

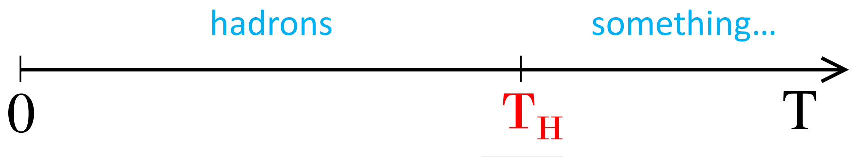
Giordano 2404.03546

Pandey et al 2407.09253

and other…

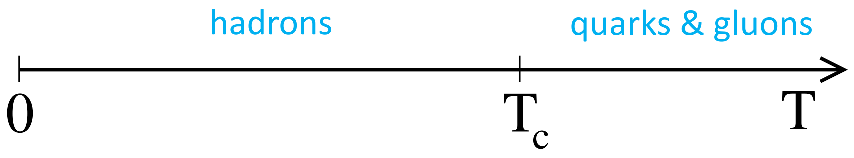
BACKUPS/DETAILS

Stories of Temperature & QCD


hadrons something…

0 T_H T

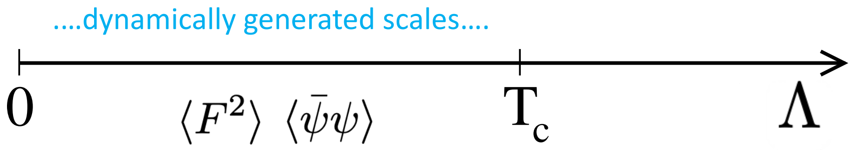
Hagedorn, 1965-71


hadrons quarks & gluons

0 T_c T

QCD, 1971-73

Effects of Temperature:


….dynamically generated scales….

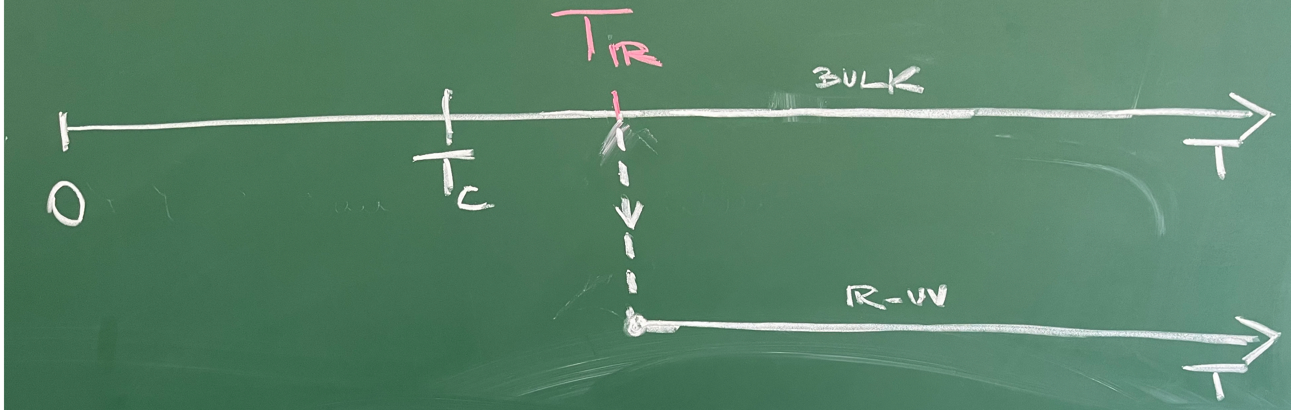
0 $\langle F^2 \rangle$ $\langle \bar{\psi}\psi \rangle$ T_c Λ

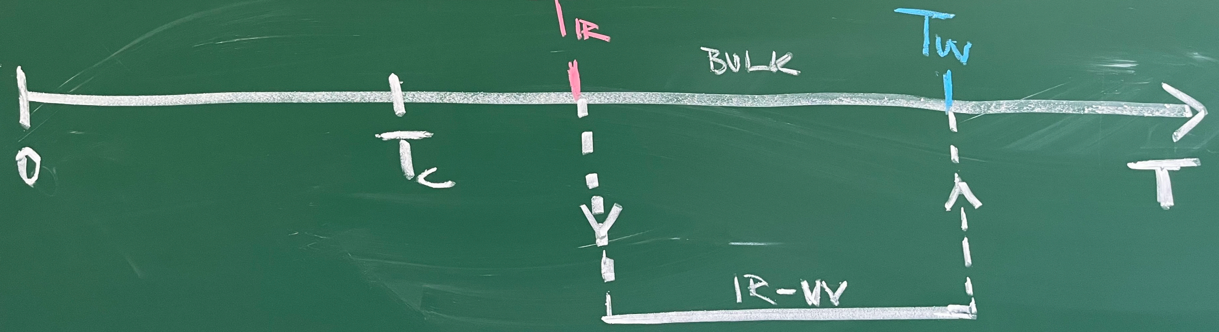
[energy scale]

Scales Story: thermal agitation erodes condensates and melts them upon T reaching T_c

DOFs Story: thermal agitation reduces IR dof-s and depletes them when T reaches T_c

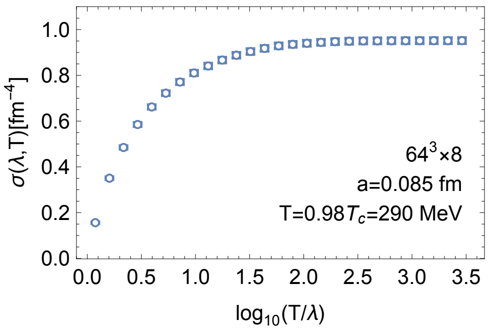
[DOFs = quark & glue]

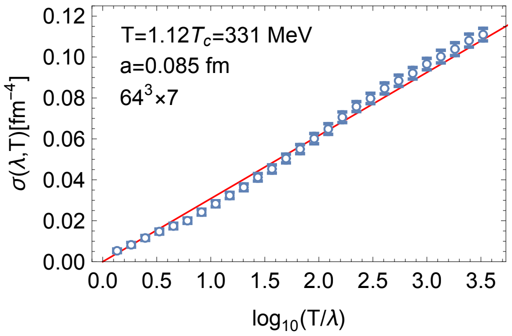

T_{IR}
BULK
0
T_c
T
IR-UV
T


T_{IR}
BULK
T_{UV}
0
T_c
T
IR-UV

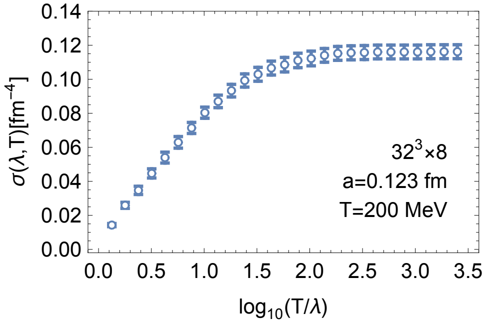
IR Phase: Real-World QCD

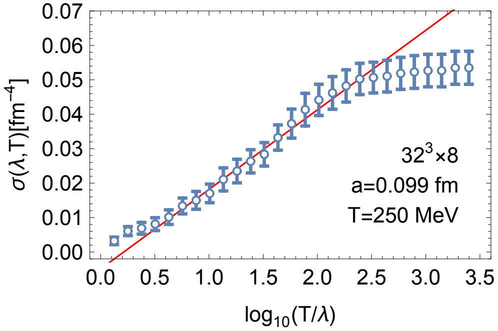
$N_f=0$





$N_f=2+1$





Real-world QCD is $N_f=2+1$ at physical quark masses of stouted staggered quarks (Wuppertal-Budapest) here.

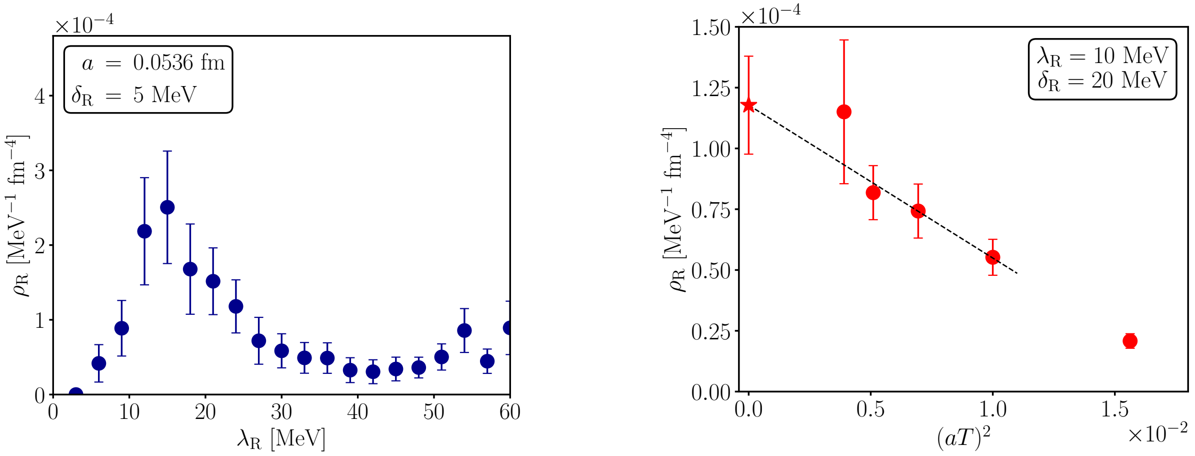
Conjecture: REAL-WORLD QCD HAS IR PHASE WITH p≈-1

$$200\,\mathrm{MeV} \;<\; T_{\mathrm{IR}} \;<\; 250\,\mathrm{MeV}$$

AA & IH 1906.08047

IR Phase: Real-World QCD…

AA, Bonanno, D`Elia, IH 2404.12298



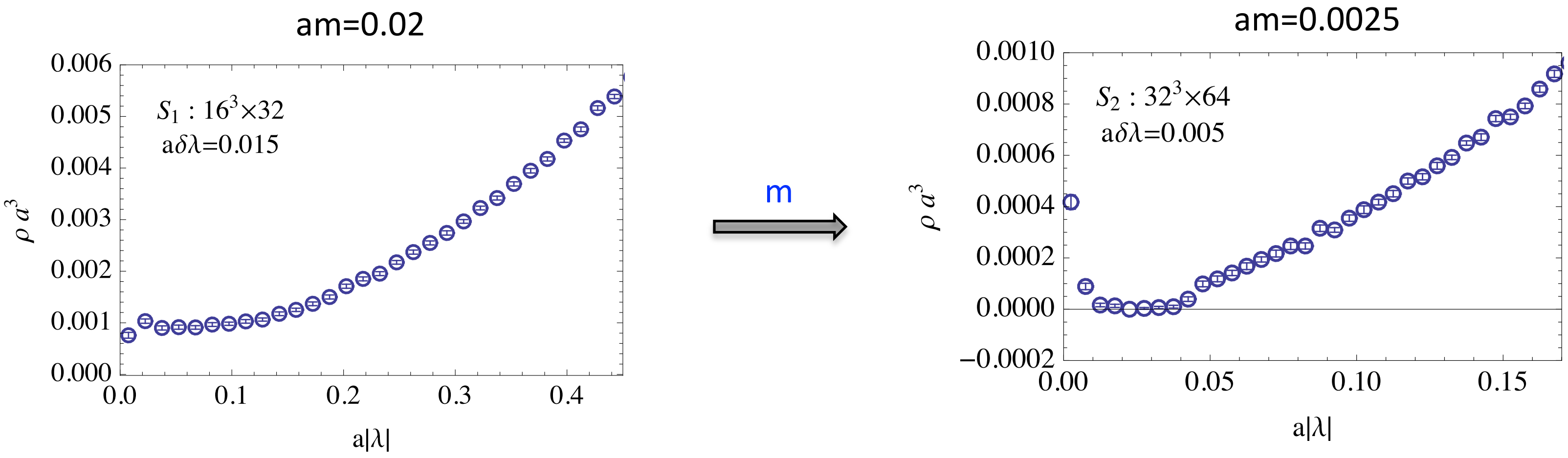
Real-world QCD is N_f=2+1 at physical quark masses of stouted staggered quarks here.

Lattice Dirac operator = stouted staggered [not overlap]

- ☐ IR structure exists in Dirac operator describing dynamical quarks
- ☐ not a lattice artifact
- ☐ IR medium is a quark-glue medium
- ☐ green light to study IR phase using overlap: correct & efficient

IR Phase: Theories with Many Flavors

N_f=12, T=0   Configss: A. Hasenfratz et al, 1207.7162   staggered with nHYP   AA & IH 1405.2968 1411.1777

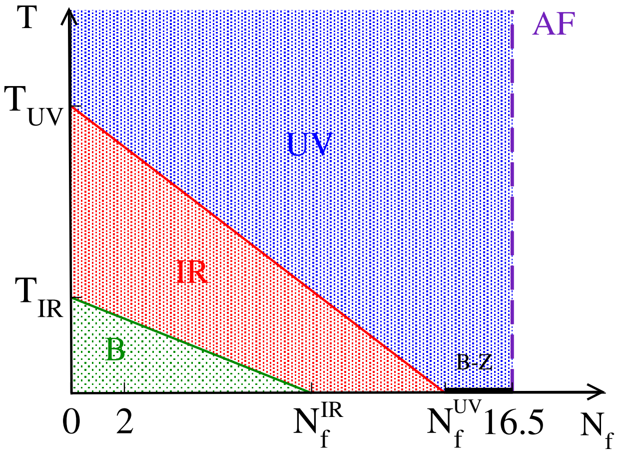


Lowering quark mass at sufficient number of flavors can generate IR phase

Conjecture: AA & IH 1906.08047

Conformal window has a strongly coupled part with p < 0.

$$N_f^c \equiv N_f^{\mathrm{IR}} < N_f < N_f^{\mathrm{UV}} \leq 16.5$$



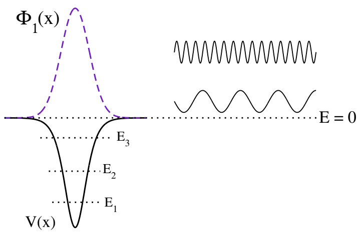
Anderson Localization & Transitions

courtesy of P.Markos

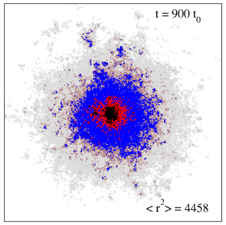
Quantum mechanics: eigenstates of quantum particle could be

bounded extended . . . and localized

[P. W. Anderson 1958]



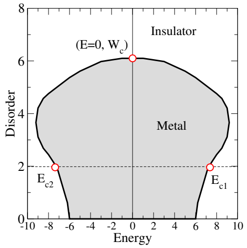
$E < 0$ $E > 0$

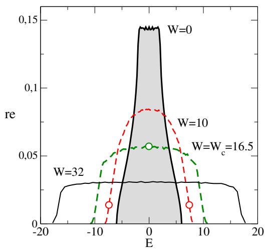


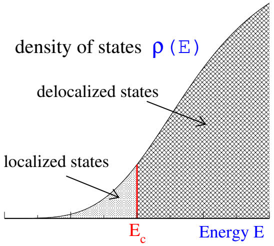
Localization is a consequence of

- disorder
- wave character of particle
- $\Phi(\vec{r}) \propto \exp\,[-r/\lambda]$

3D Anderson model: phase diagram, density of state, mobility edge





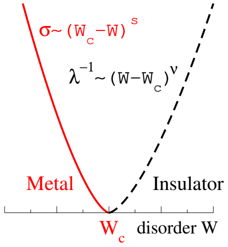


Critical exponents:

conductivity: $\sigma(E) \sim (E_c - E)^s$

localization length: $\lambda(E) \sim (E - E_c)^{-\nu}$

$s = (d-2)\nu$ . . . critical exponents.



λ is a localization length here $\lambda \longrightarrow \ell$

(1) $\ell \propto \xi$

density-density correlation length within the mode

(2) $\langle\, \psi^2_{loc}(x)\psi^2_{ext}(y)\,\rangle_c \longrightarrow 0 \quad \text{for } L \to \infty$

Anderson-like transition in thermal QCD:
Disorder W ⟶ Temperature T
Mobility edge $\lambda_A \neq 0$ invoked for understanding chiral phase transition: aka metal-to-insulator picture

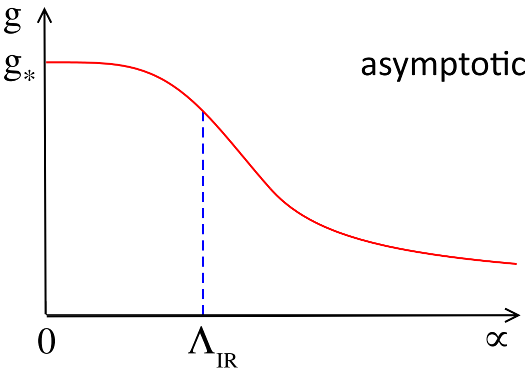
Garcia-Garcia & Osborn hep-lat/0611019

Kovacs & Pitler 1006.1205

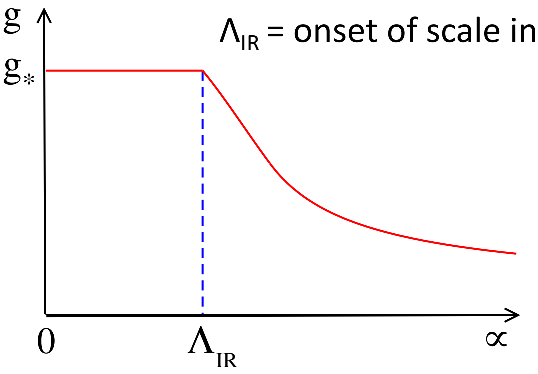
Giordano, Kovacs & Pitler 1312.1179

Shortly after AA & IH 1906.0804 we realized that λ_A is a very friendly feature to our claim and accepted it ☺.

Why is Anderson-Like λ_A Handy?



asymptotic vs exact



Λ_{IR} = onset of scale invariance

g(μ) non-analytic at Λ_{IR}

Q: Do non-analyticities exist and, if so, how do they arise?

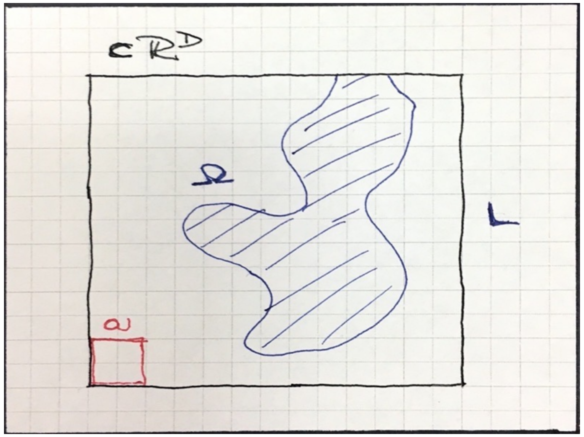
Their existence in λ-dependences would also facilitate IR-BULK decoupling!

Hint: Given the existence of λ_A and its nature, focus on spatial IR dimensions of modes.

WHAT IS IR DIMENSION OF MODES? Concept didn't exist.

IH & RM 1807.03995	effective-number theory
IH, PM and RM 2205.11520	effective-dimension theory

Effective Dimensions



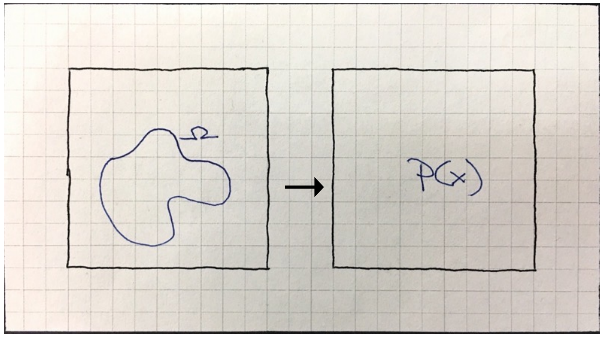
characterize fine [UV] and global [IR] features of fixed sets

all points/elements of regularized space: $N \propto (L/a)^D$

points/elements covering Ω: N_+

$$\text{UV:} \quad N_+(a,L) \propto a^{-d_{\text{UV}}(L)} \;, \quad a \to 0$$

$$\text{IR:} \quad N_+(a,L) \propto L^{d_{\text{IR}}(a)} \;, \quad L \to \infty$$



$$P(x) \quad \Longrightarrow \quad \Omega_{\text{eff}}$$

But how to proceed when instead of fixed Ω we have P(x)?

1) Count how many points $\mathcal{N} = \mathcal{N}[P] = \mathcal{N}(p_1, p_2, \ldots, p_N)$ are effectively selected by *P.*

2) Select Ω_{eff} as $\mathcal{N}$ most probable points on the lattice

3) Proceed as Minkowski/box-counting with N_+

Consistent realization of this program leads to unique effective dimensions IH, PM and RM 2205.11520

$$\mathcal{N}_\star[P] = \sum_{i=1}^{N} \mathfrak{n}_\star(Np_i) \quad , \quad \mathfrak{n}_\star(c) = \min\{c, 1\}$$

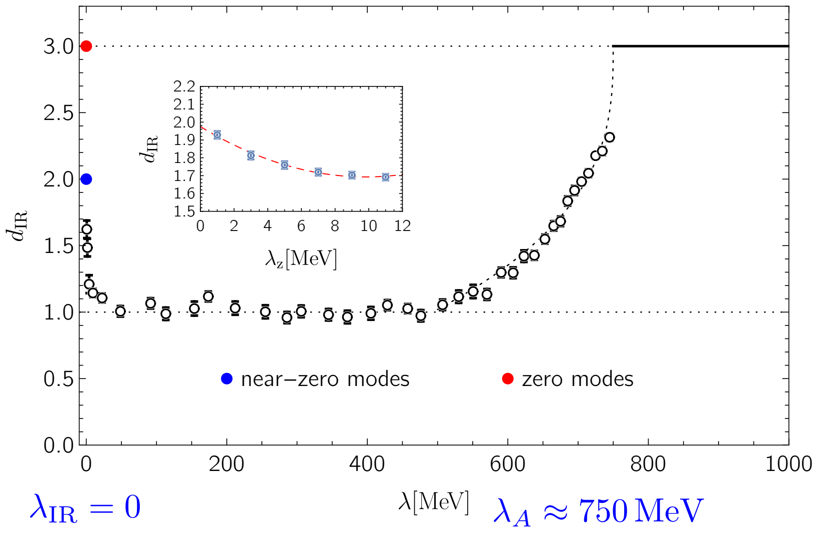
IH & RM 1807.03995

$$\text{Box:} \quad N \longrightarrow N_+ \qquad \text{Effective:} \quad N \longrightarrow \mathcal{N}_\star[P]$$

Anderson Localization in IR Phase

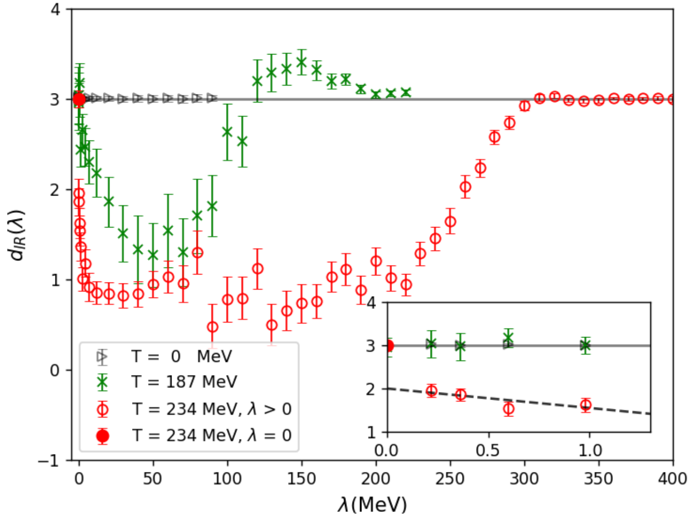
AA & IH 2103.05607

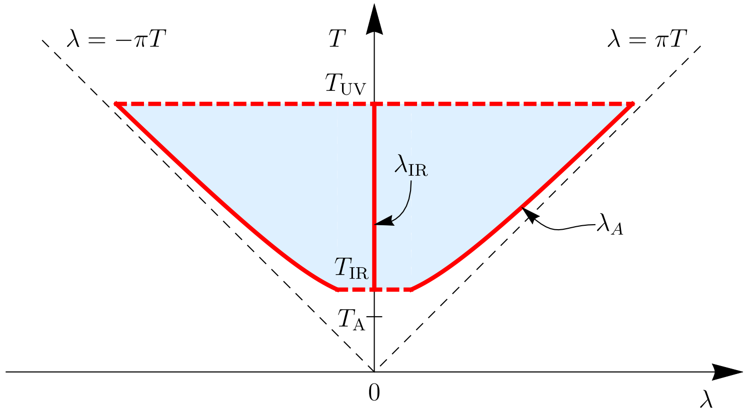
N_f=0, a=0.085 fm, T=1.12T_{IR}



X. Meng et al 2305.09459

N_f=2+1 physical point, a=0.105 fm

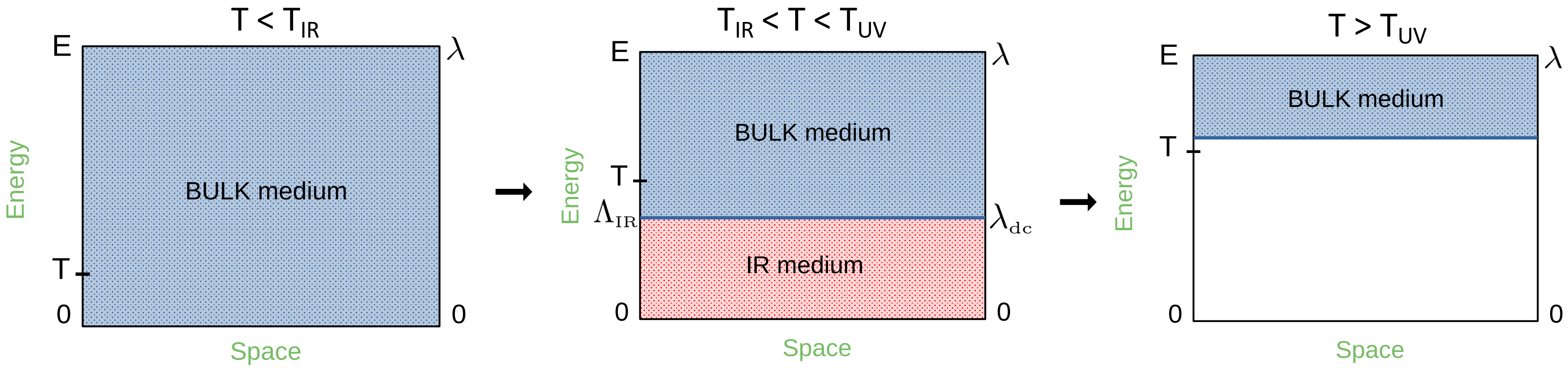




AA & IH 2110.04833

Dirac spectral phase diagram of QCD

- IR phase associated with Dirac non-analyticity
- Explains non-analytic running
- Entails IR-Bulk decoupling
- Second mobility edge λ_{IR}=0 [gives long-range physics]
- Recall that λ_A facilitates decoupling
  [see also model in Kovács 2311.04208 for support of decoupling]
- T=187 MeV is different from T=234 MeV as predicted

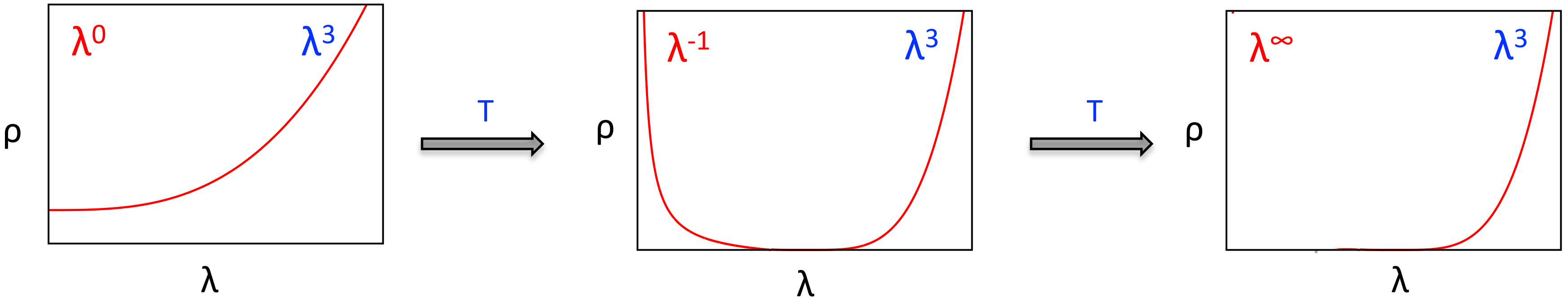

$T < T_{IR}$
E
λ
BULK medium
Energy
T
0
0
Space
$T_{IR} < T < T_{UV}$
E
λ
BULK medium
Energy
T
$\Lambda_{\rm IR}$
$\lambda_{\rm dc}$
IR medium
0
0
Space
$T > T_{UV}$
E
λ
BULK medium
T
Energy
0
0
Space

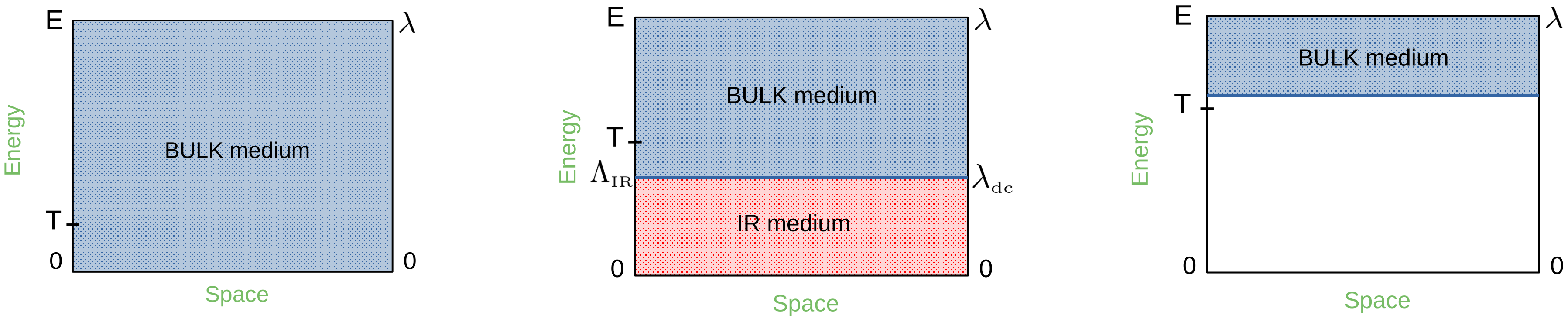
WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION

N_F=0 easier to communicate the point [same in N_f=2+1 just more awkward.]

T=0 classically scale invariant theory $\xrightarrow[\text{scale anomaly}]{\text{quantum fluctuations}}$ scales got generated world of hadrons etc

scale-broken T=0 theory $\xrightarrow[\text{increasing T}]{\text{thermal fluctuations}}$ scale-invariant but only for $\Lambda \leq \Lambda_{\mathrm{IR}} < T$

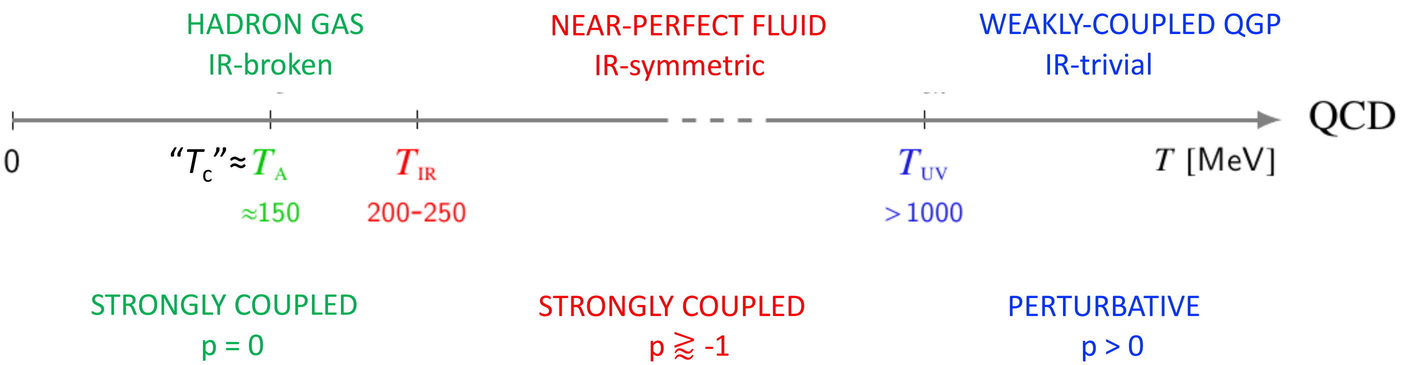




WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION…

PHASE STRUCTURE OF THERMAL QCD IN TERMS OF GLUE IR SCALE INVARIANCE

[AA & IH 1906.08047]



HADRON GAS
IR-broken

NEAR-PERFECT FLUID
IR-symmetric

WEAKLY-COUPLED QGP
IR-trivial

STRONGLY COUPLED
p = 0

STRONGLY COUPLED
$p \gtrapprox -1$

PERTURBATIVE
p > 0

$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \text{with} \qquad \rho(\lambda) \propto \lambda^p \quad \text{for} \quad \lambda \to 0$$

Original talk: https://indico.cern.ch/event/764552/contributions/3420459/attachments/1865996/3068382/WuHan_jun_2019_infra.pdf

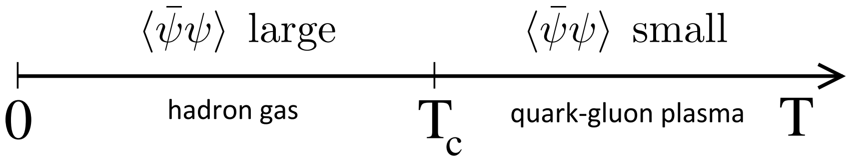
Useful talk: https://drive.google.com/file/d/1vZ0AY0WsZAfF9iV7-Br-E_2NiwaZzRGp/view

Betting on Glue

[1906.08047 , 1502.07732]

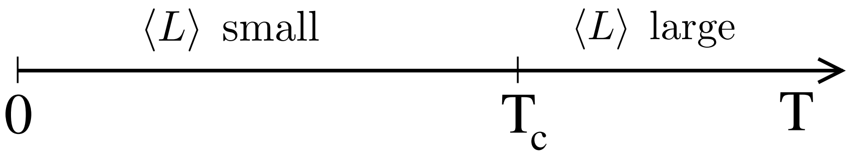
Standard approaches to phases:

quark view:



quark scalar density

glue view:



Polyakov line

Quarks won the popularity contest [$T_c \approx$ 155 MeV, crossover , Aoki et al, 2007]

NEED NEW IDEA!

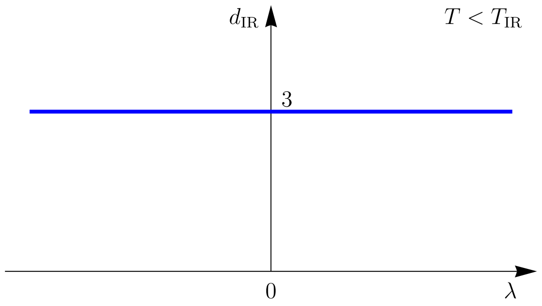
Point 1: $\langle\bar{\psi}\psi\rangle$ is cleaner because it reflects deeper IR of glue

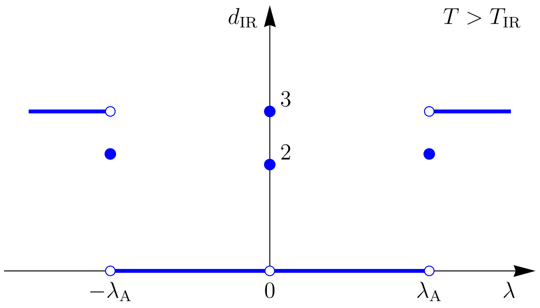
Point 2: both $\langle\bar{\psi}\psi\rangle$ and $\langle L\rangle$ are limited in terms of reflecting glue

Point 3: need glue probe that is sensitive to any scale by construction
object with explicit scale dependence

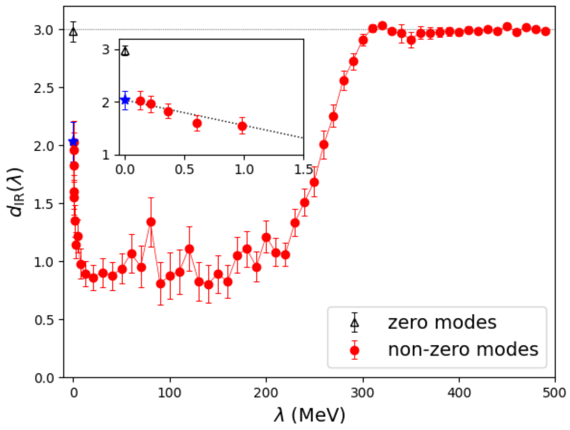
TOPOLOGICAL ORIGIN AND NON-ANALYTICITY

A.A. & I.H. 2103.05607, 2110.04833, 2310.03621, 2305.09459





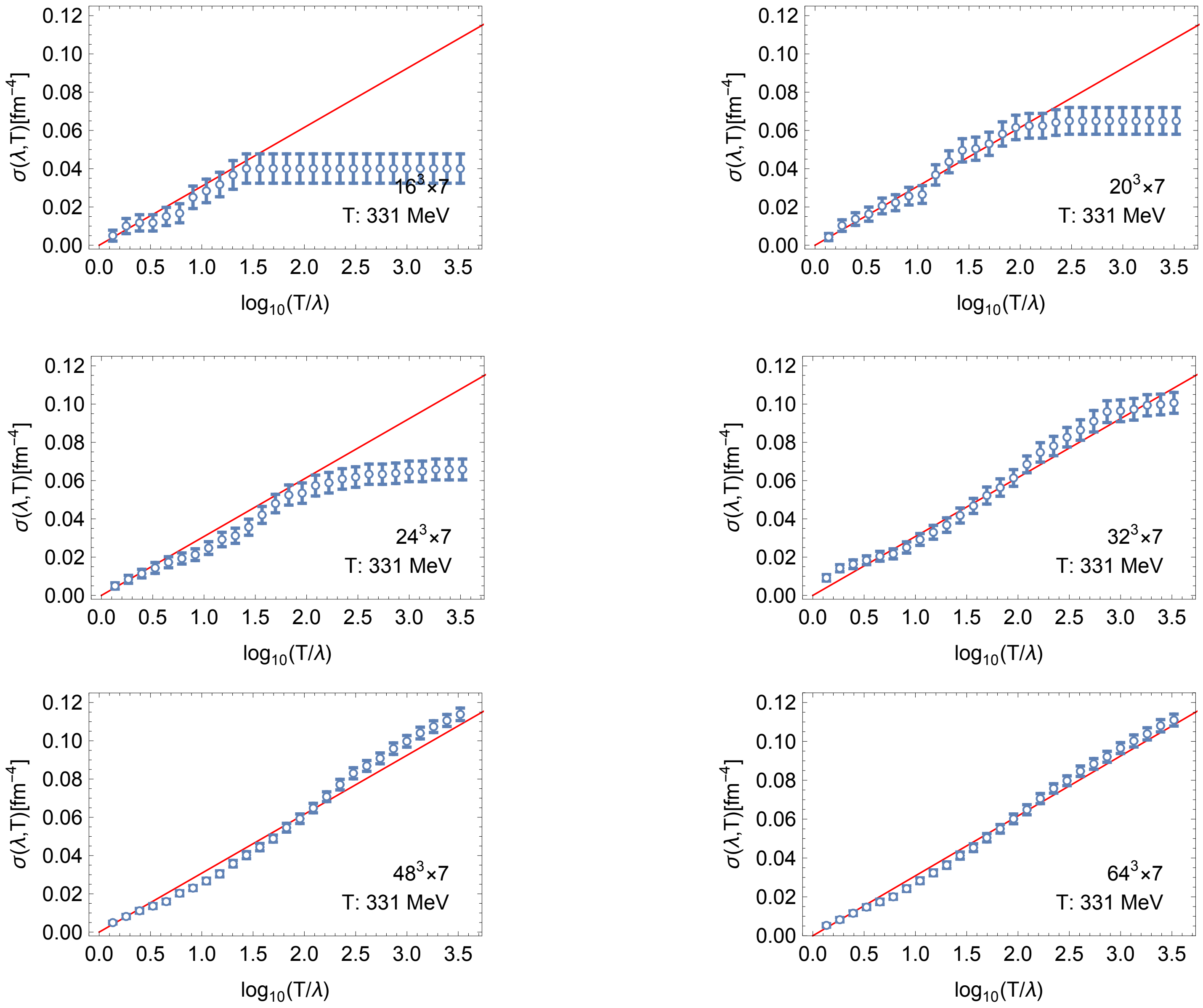
d_{IR}= effective IR dimension of modes



2305.09459

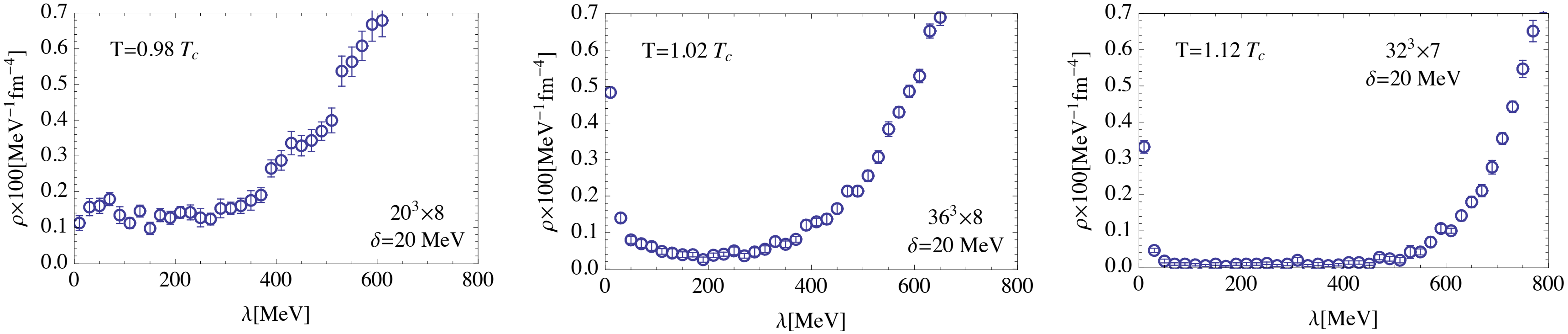
$N_f = 2 + 1$ at physical point T = 234 MeV a=0.105 fm overlap mode-density <u>glue operator</u>

Check this further & better… $\sigma(\lambda_1,\lambda_2) \equiv \int_{\lambda_1}^{\lambda_2} d\lambda\, \rho(\lambda)$ $\rho(\lambda) = c/\lambda \longrightarrow \sigma(\lambda,T) = c\ln(T/\lambda)$



- Peak in IR overlap spectrum upon crossing T$_c$ (pure glue) [Edwards, Heller, Narayanan, Kiskis, 1999]

- Our version of it [AA & IH, 1502.07732]



- knee-jerk reaction was: quenched chiral condensate may diverge in high-temperature pure glue
- knee-jerk reaction should be: what on earth is glue doing to produce this? [1502.07732]
- didn't know but went on with it, e.g., around chiral crossover we got this: