Dense QCD Matter constrained by astrophysical observation

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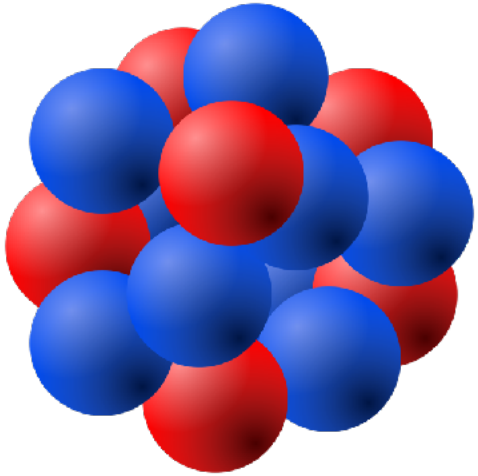
— New developments in studies of the QCD phase diagram —

Nuclear Matter

Convenient unit to measure the energy density


[Nuclei from Wikipedia]

Heavy nuclei have an almost constant density.

$$n_{\mathrm{sat}} = 0.16\ (\mathrm{nucleon})\ \mathrm{fm}^{-3}$$

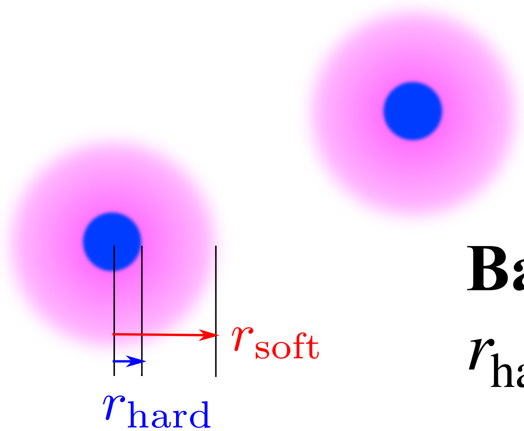
saturation density

In astrophysics, the rest-mass density is used:

$$\rho_{\mathrm{sat}} = 2.6 \times 10^{14}\ \mathrm{g\,cm}^{-3}$$

Nuclear Matter

How dense (dilute) is nuclear/quark matter?



Interaction Cloud Size

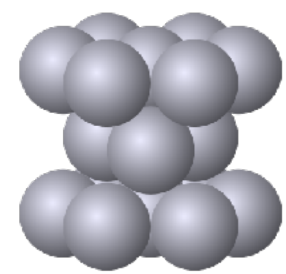
$r_{\mathrm{soft}} \sim 1/(2m_\pi) \sim 0.7\ \mathrm{fm}$

Baryon Number Distribution Size

$r_{\mathrm{hard}} \sim 0.5\ \mathrm{fm}$

Closest Packed State (hcp/fcc) Filling rate ~ 74%

$$0.74 \times \left(\frac{4\pi}{3} r_{\mathrm{hard}}^3\right)^{-1} \approx 1.4\ \mathrm{fm}^{-3} \approx 8.3\, n_{\mathrm{sat}}$$



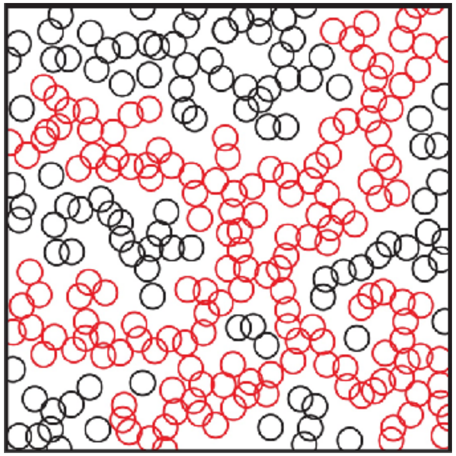
Nuclear matter cannot exist at this density!

Nuclear Matter

More realistic bound?

Percolation transition?

3D critical filling density ~ 34%



(From Wikipedia)

Interaction-mediated Percolation

$$0.34 \times \left(\frac{4\pi}{3} r_{\rm soft}^3\right)^{-1} \approx 0.24\,{\rm fm}^{-3} \approx \underline{1.5\, n_{\rm sat}}$$

Standard nuclear-physics calculations may break down at this density due to the lack of multi-body interactions.

For even more realistic arguments on quantum percolation, see: Fukushima-Kojo-Weise (2020) for more details.

Critical Energy Density

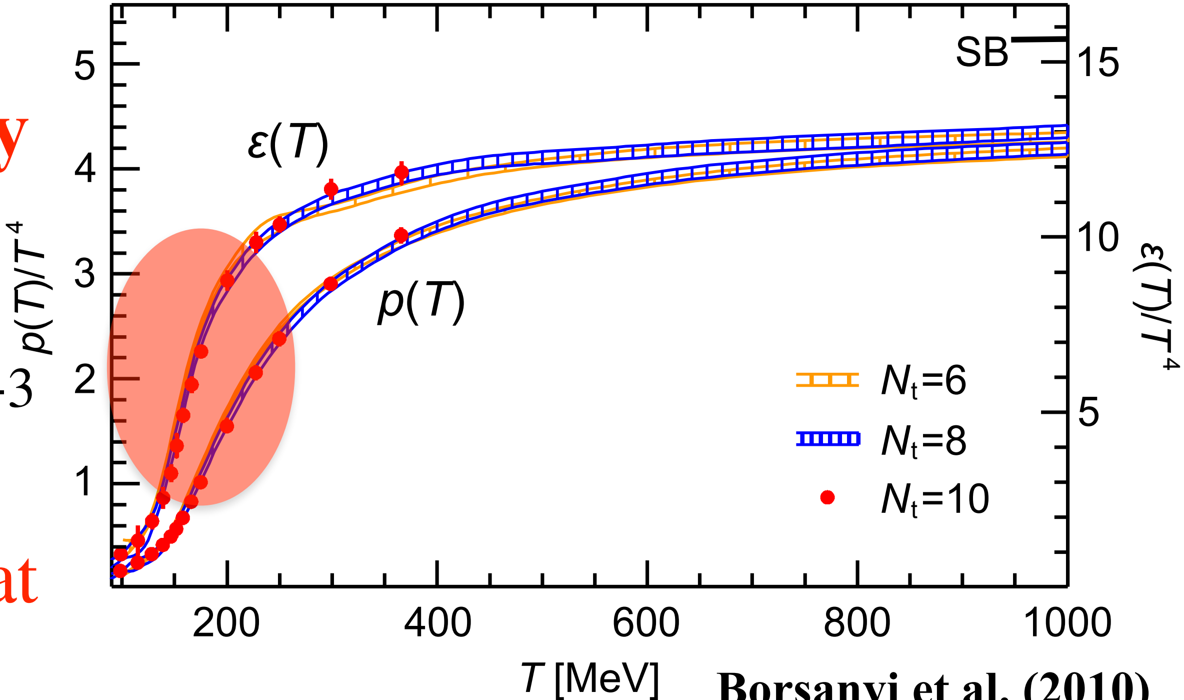
One neutron energy $E_N = M_n c^2 \simeq 939\,\text{MeV}$

Saturation energy $\varepsilon_{\text{sat}} \simeq 150\,\text{MeV fm}^{-3}$

Transition energy to a QGP

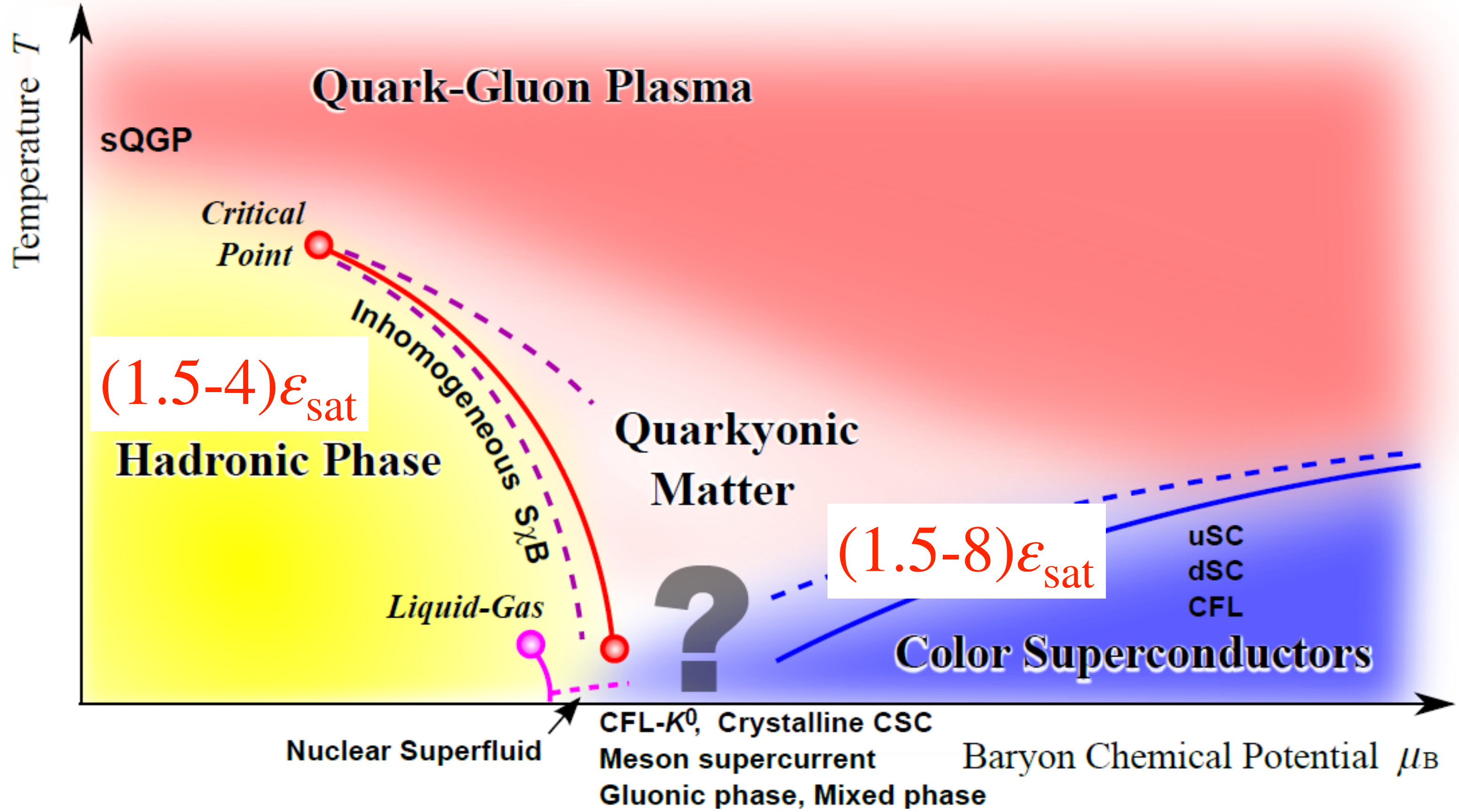
200-500 MeV fm^{-3}

$\sim 1.5 - 4 \times \varepsilon_{\text{sat}}$



Borsanyi et al. (2010)

Phase Diagram

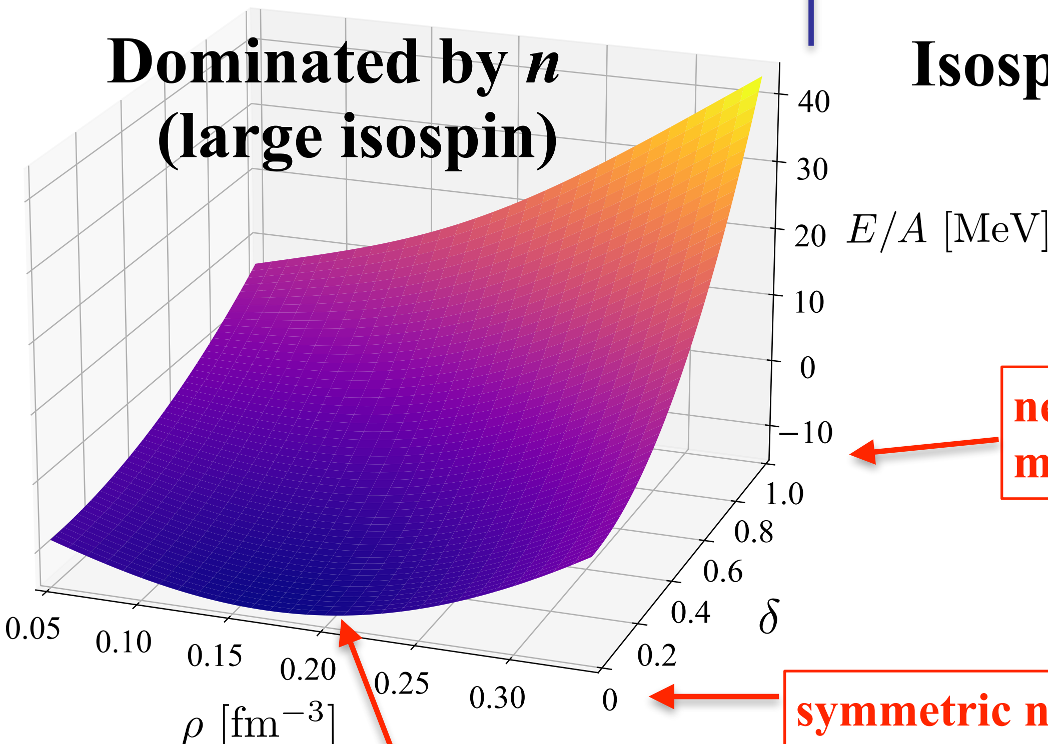


Fukushima-Hatsuda (2010); see also 50 Years of QCD Chap.7 (2023)

Nuclear vs. Neutron Matter

Neutron Star

Dominated by *n*
(large isospin)



saturation density
(1st-order transition)

Heavy-Ion Collision

Isospin fixed by *p*/*n* ratio
(small isospin)

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

neutron matter

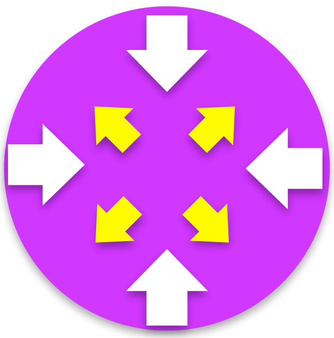
$$\rho = \rho_n + \rho_p$$

symmetric nuclear matter (HIC)

Phase structure may be different.

Neutron Star

Force Balance



Gravitational force is supported by the pressure from inside.

Hydrostatic condition for $r \sim r + dr$

$$\frac{dp(r)}{dr} = -G\frac{M(r)}{r^2}\varepsilon(r)$$

***M*(*r*) represents the integrated mass in *r*-sphere.**

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

(In Newtonian gravity)

Neutron Star

$$\frac{dp(r)}{dr} = -G\frac{M(r)\,\varepsilon(r)}{r^2}$$

Tolman-Oppenheimer-Volkoff Eq.

General Relativistic extension →

$$\frac{dp(r)}{dr} = -G\frac{M\varepsilon}{r^2}\left(1+\frac{p}{\varepsilon}\right)\left(1+\frac{4\pi r^3 p}{M}\right)\left(1-\frac{2GM}{r}\right)^{-1}$$

One condition still missing…

A relation between p and ε → **Equation of State (EOS)**

Initial

$r = 0$

$\varepsilon(r=0) = \varepsilon_c$ **free parameter**

$p(r=0) = p_c = p(\varepsilon_c)$

Final

$r = R$

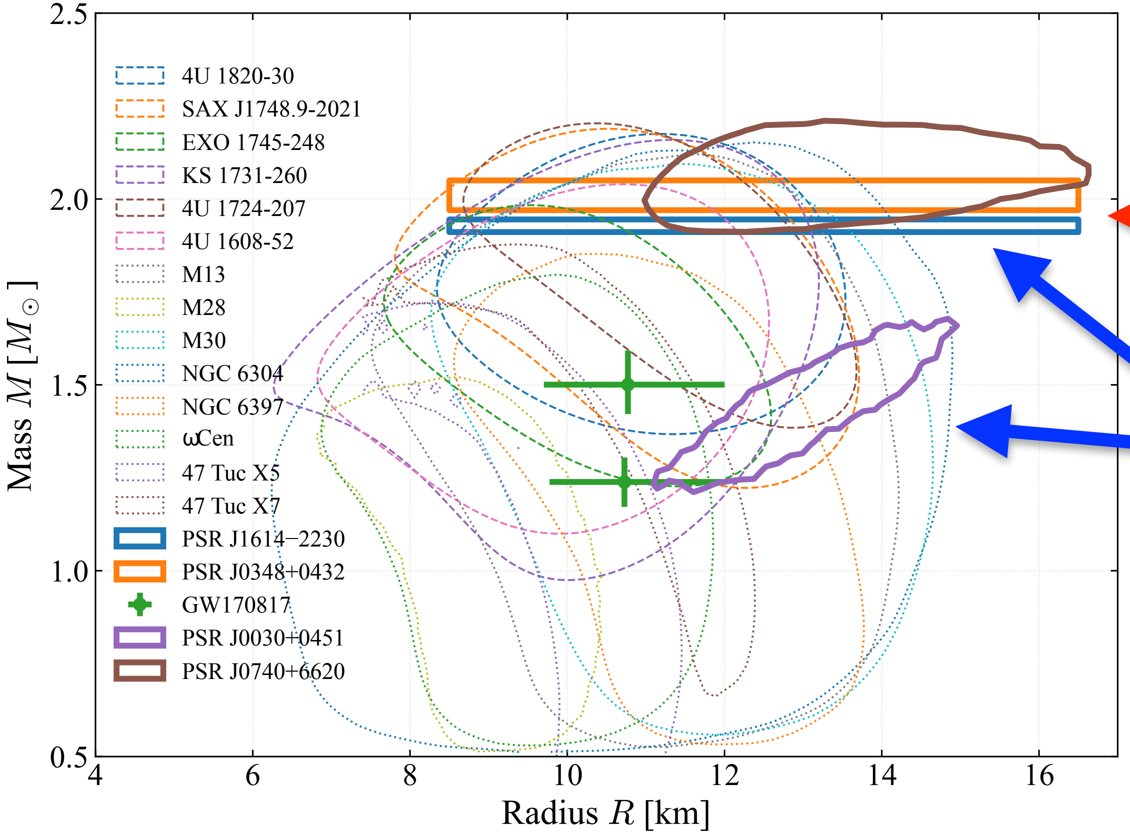
$p(r=R) = 0$

$M = \int dr 4\pi r^2 \varepsilon(r)$

Neutron Star

Compilation of the observed data (68% Credible)

Fujimoto-Fukushima-Kamata-Murase (2024)



EOS must be "stiff" enough to reach here. (Shapiro delay)

Lensing and timing (NICER) constraining the gravity strength and the radius.

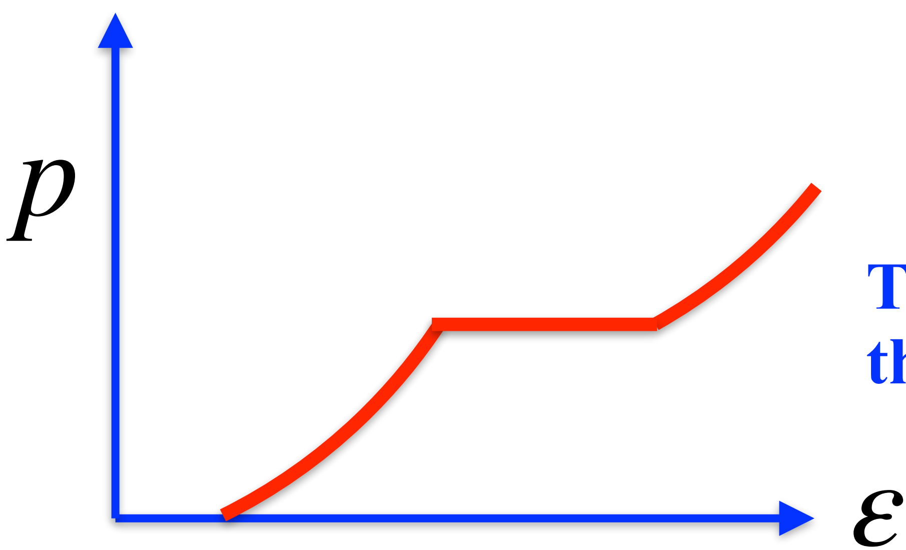
EOS Basics

Mathematically proven:

$$p = p(\varepsilon) \longleftrightarrow M = M(R)$$

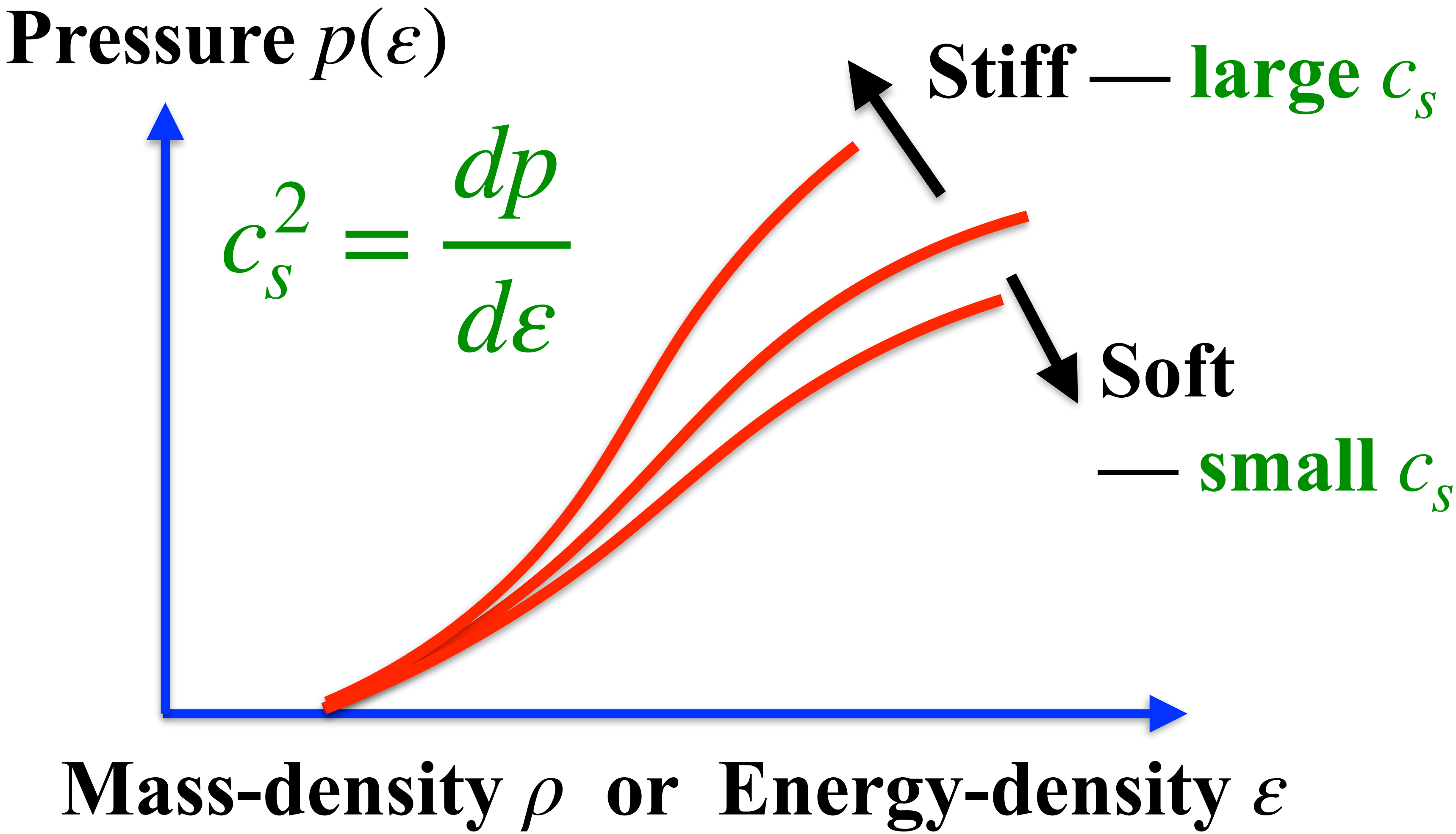
One-to-one correspondence through TOV eq.

Lindblom (1992)



This is the case even with the 1st-order phase transition.

EOS Basics



Pressure $p(\varepsilon)$
$c_s^2 = \frac{dp}{d\varepsilon}$
Stiff — large c_s
Soft
— small c_s
Mass-density ρ or Energy-density ε

EOS Basics

Structures of pQCD on ε-p

LO: $p \sim \# \mu_B^4$ **(massless case)** $\rightarrow \varepsilon = \frac{1}{3} p$

NLO: $p \sim (\# + \alpha_s \#) \mu_B^4$ $\rightarrow \varepsilon = \frac{1}{3} p$ **(unchanged!)**

N²LO: $p \sim (\# + \alpha_s \# + \alpha_s^2 \# + \# \alpha_s^2 \ln \mu_B^2 / \mu_0^2) \mu_B^4$

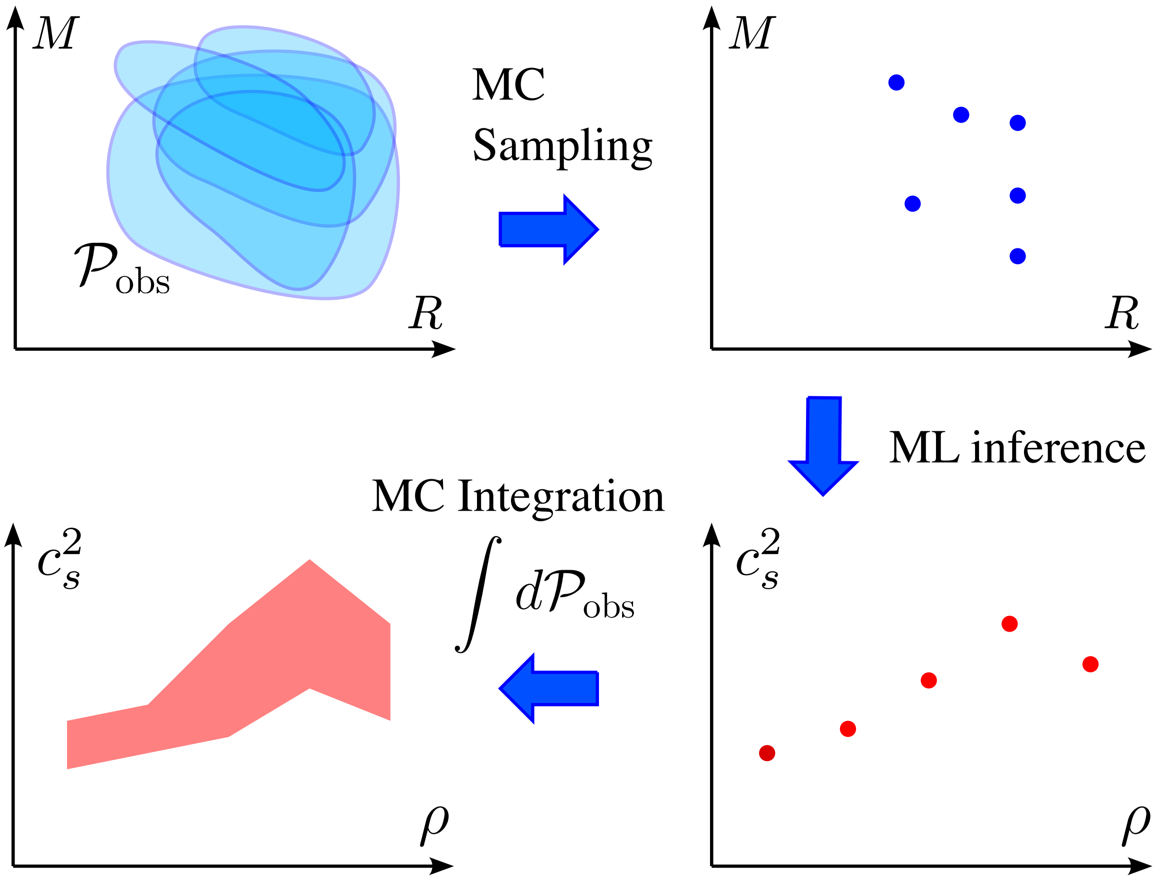
Running Coupling

Conformality broken

$\sim \ln(X^2 \mu_q^2 / \Lambda_{\overline{\mathrm{MS}}}^2)$

EOS Inference Program

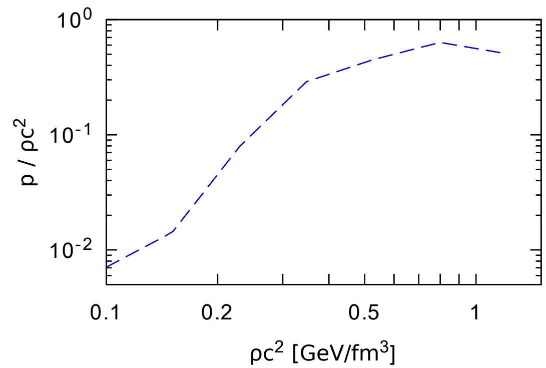
Fujimoto-Fukushima-Kamata-Murase (2018-2024)



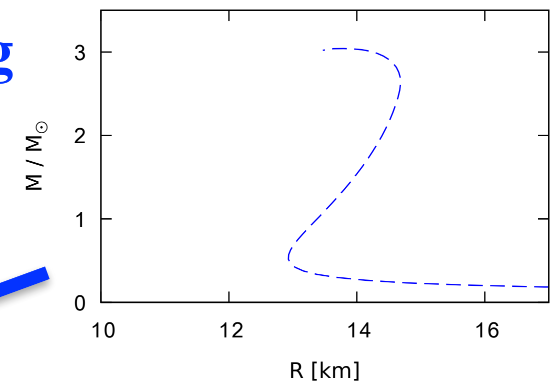
EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)

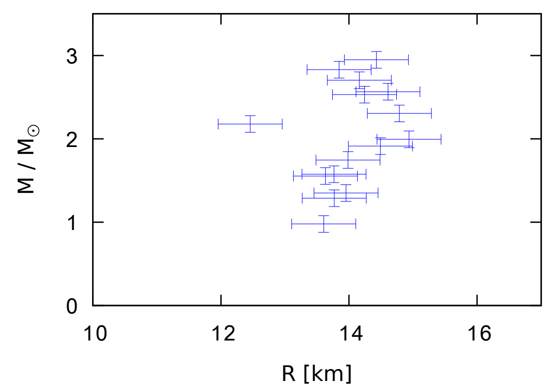
Proof of principle



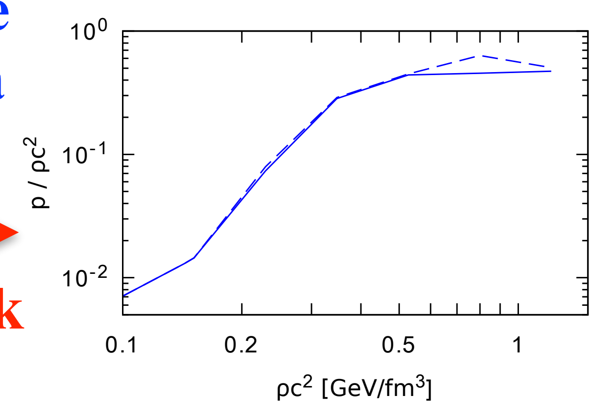
Corresponding M-R relation



Mimic the astro data



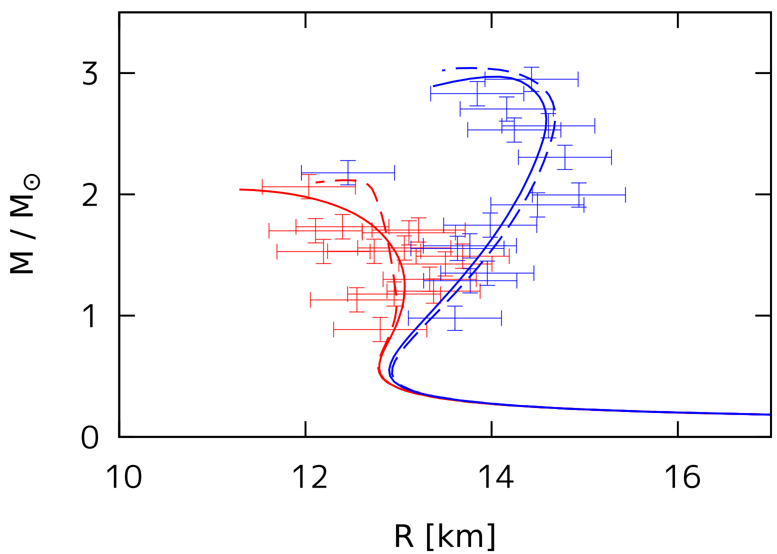
Validity check

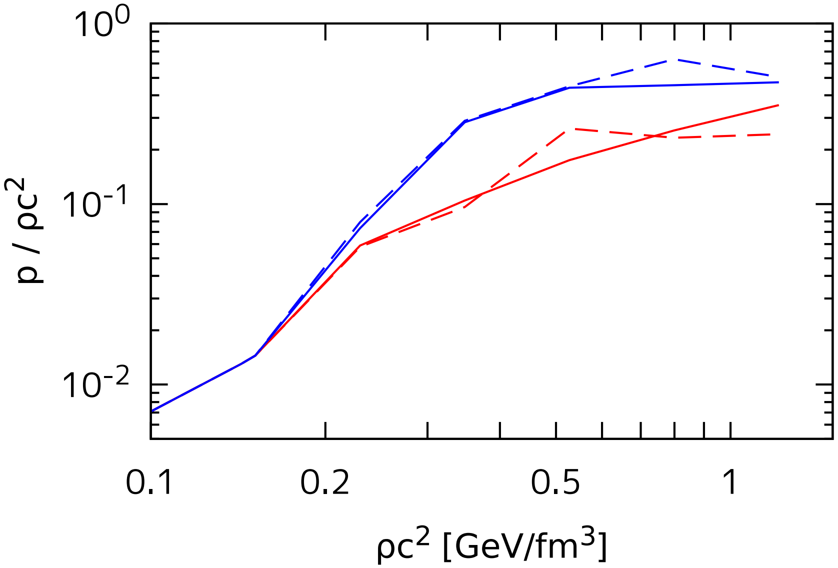


EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)

Machine learning shows amazing performance!

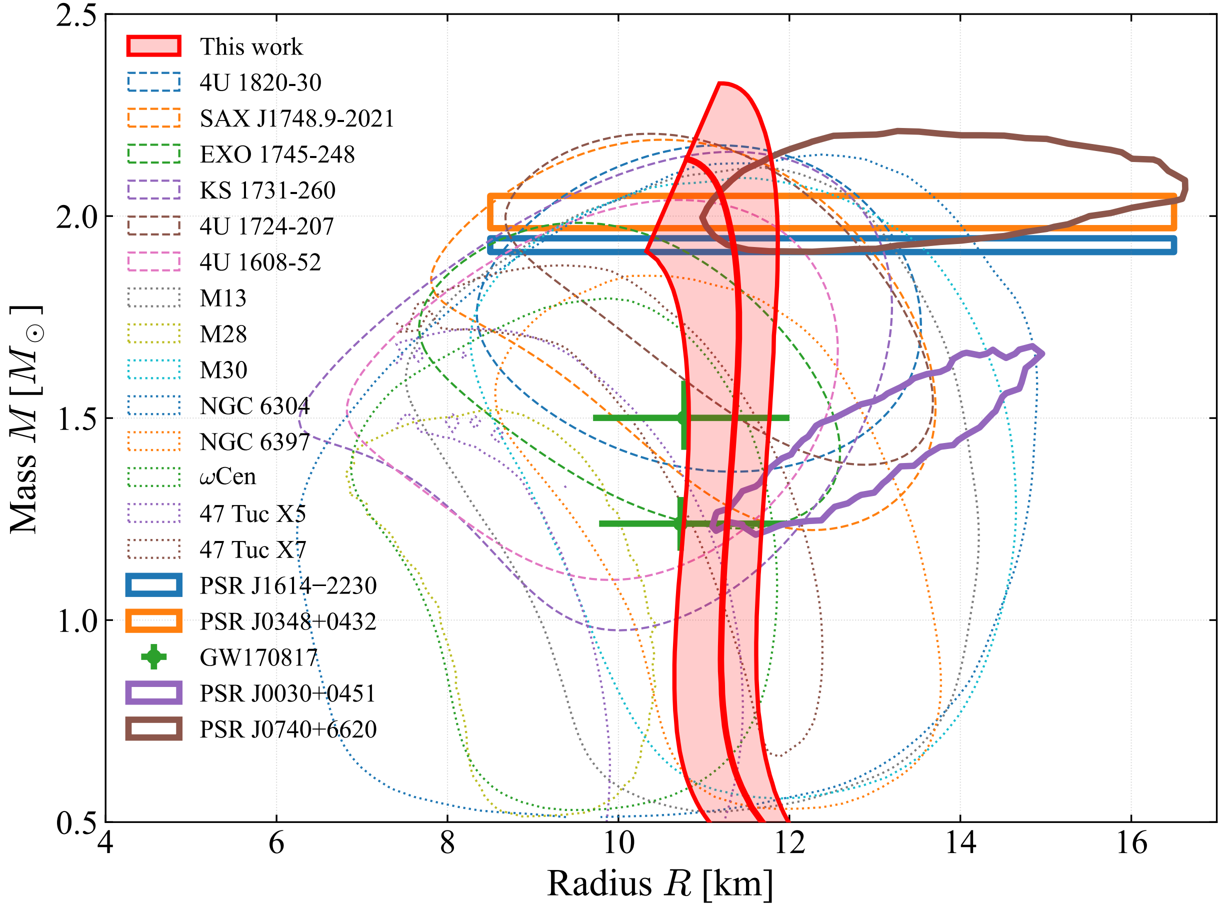




Overfitting is miraculously avoided!

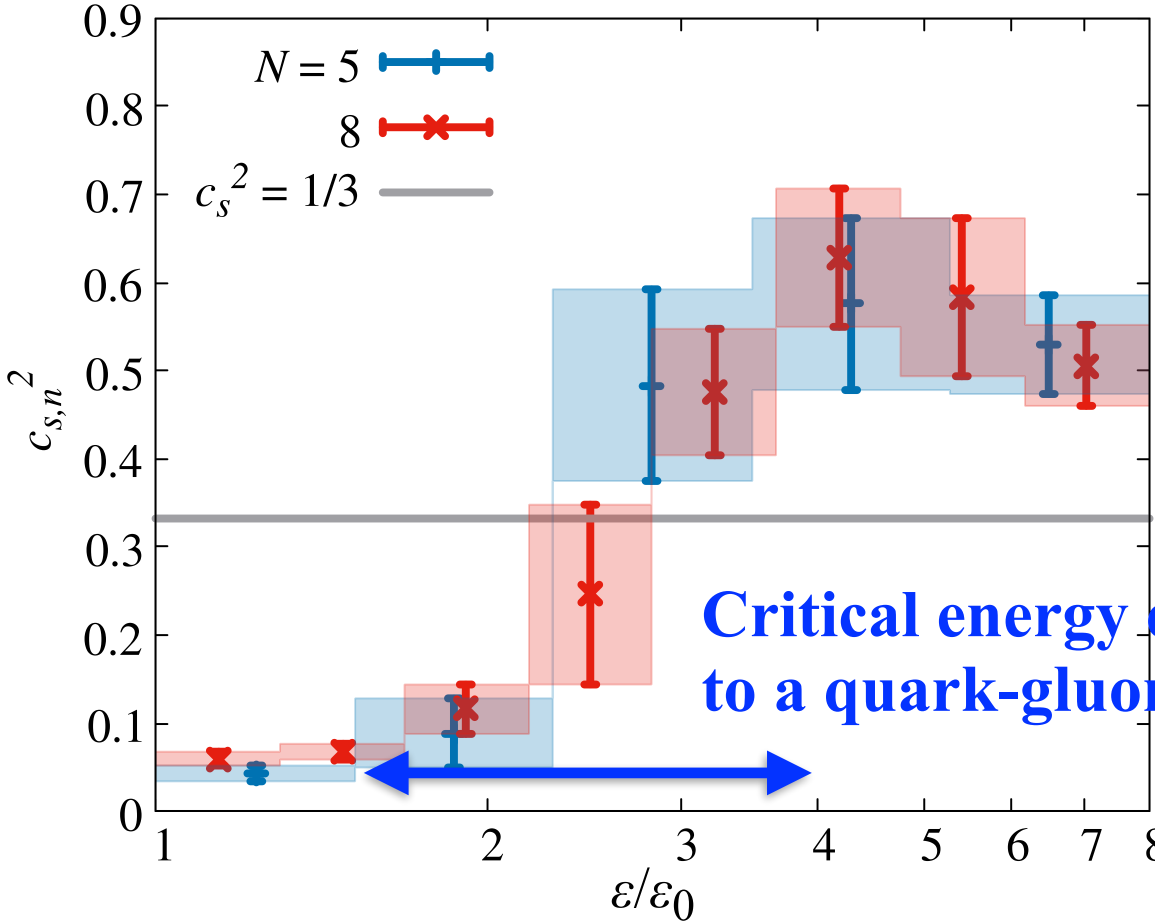
EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)



EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)



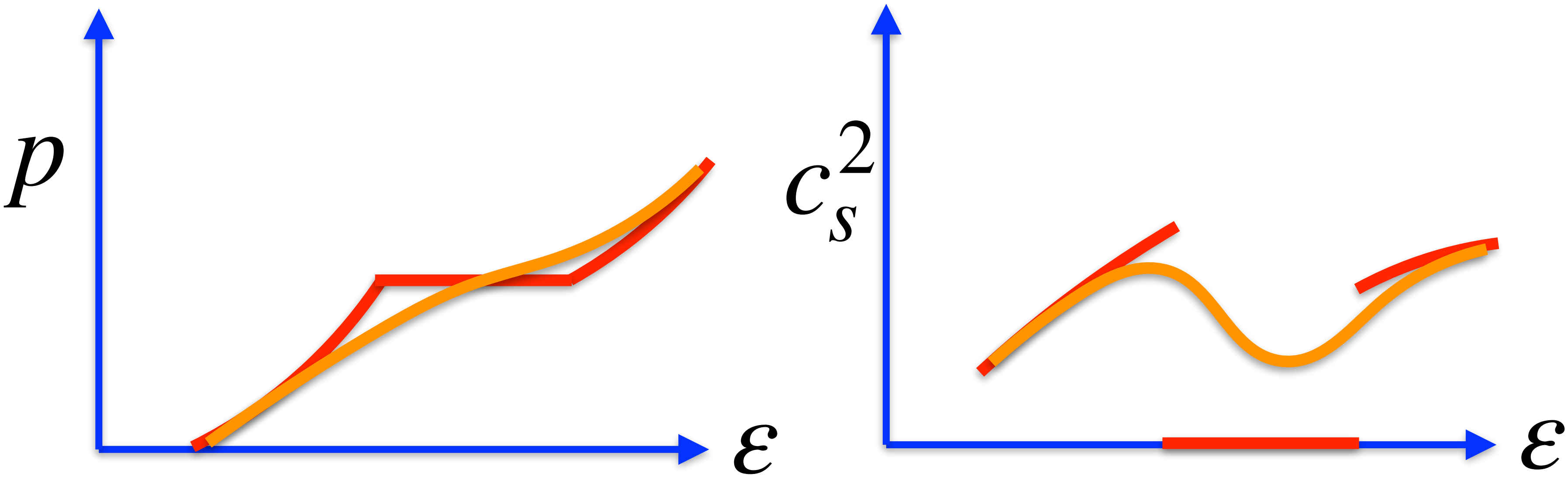
Not minimum, but maximum!?

Critical energy density to a quark-gluon plasma

$\varepsilon_0 = 150\,\mathrm{MeV\ fm^{-3}}$

Interpretation

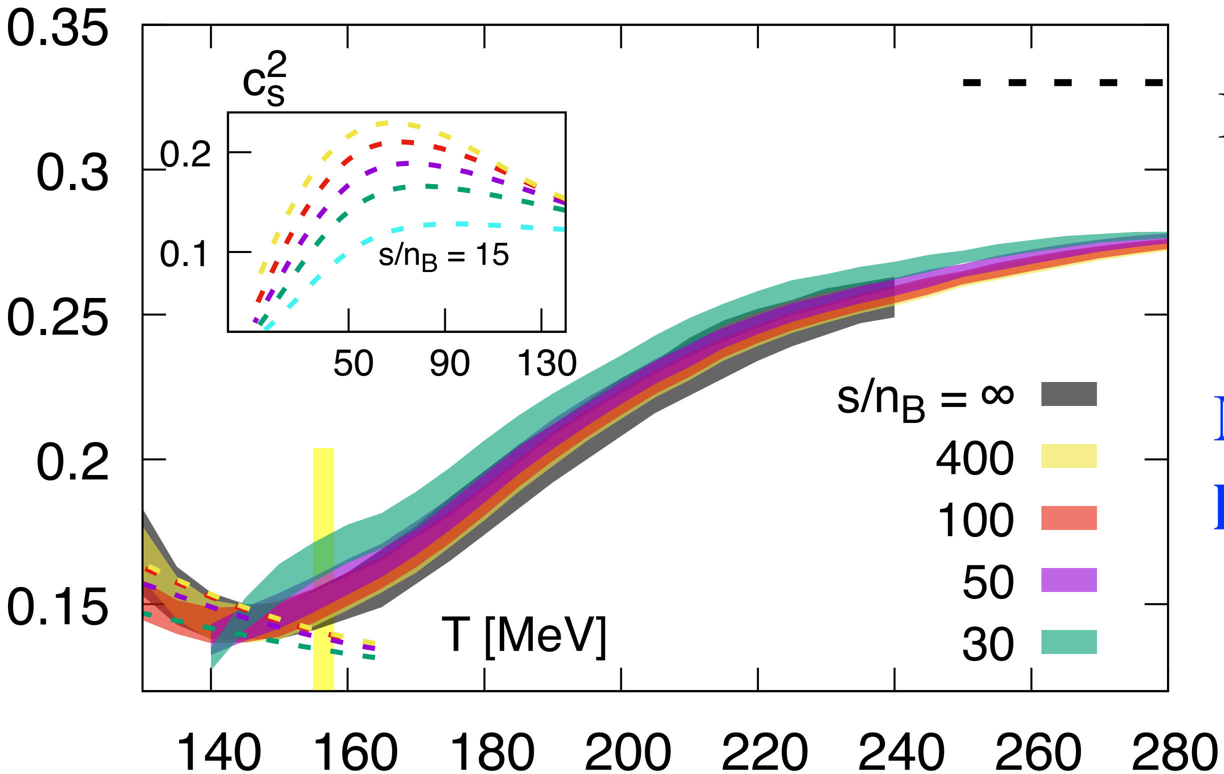
[1st-order-like EoS]



Phase transition is manifested by a minimum in the speed of sound.

Interpretation

[High-Temperature QCD — QGP Crossover]



HotQCD Collab. (2212.09043)

Minimum around phase transition

Interpretation

Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

Measure of conformality: $\Delta = \dfrac{1}{3} - \dfrac{p}{\varepsilon}$

Gavai-Gupta-Mukherjee (2004)

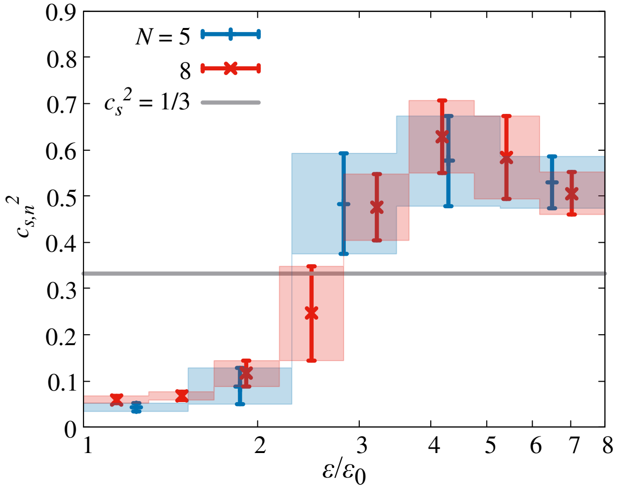
$$c_s^2 = \frac{dp}{d\varepsilon} = c_{s,\,\mathrm{deriv}}^2 + c_{s,\,\mathrm{non-deriv}}^2$$

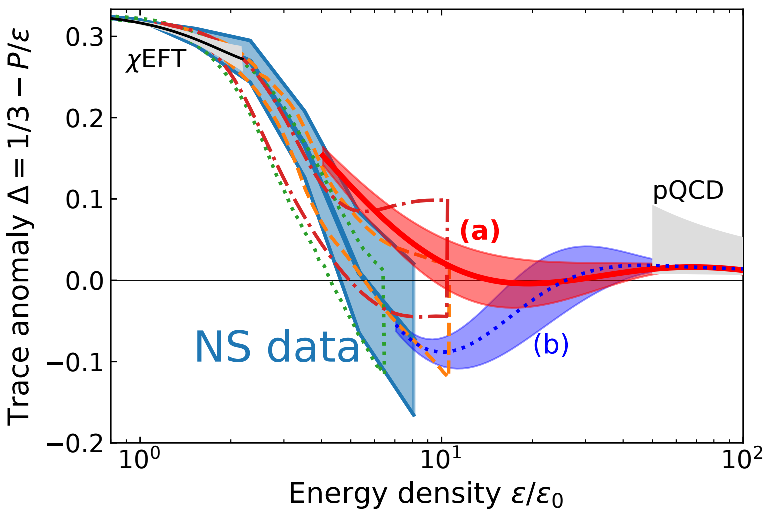
$$c_{s,\,\mathrm{deriv}}^2 = -\,\varepsilon\frac{d\Delta}{d\varepsilon} \qquad c_{s,\,\mathrm{non-deriv}}^2 = \frac{1}{3} - \Delta$$

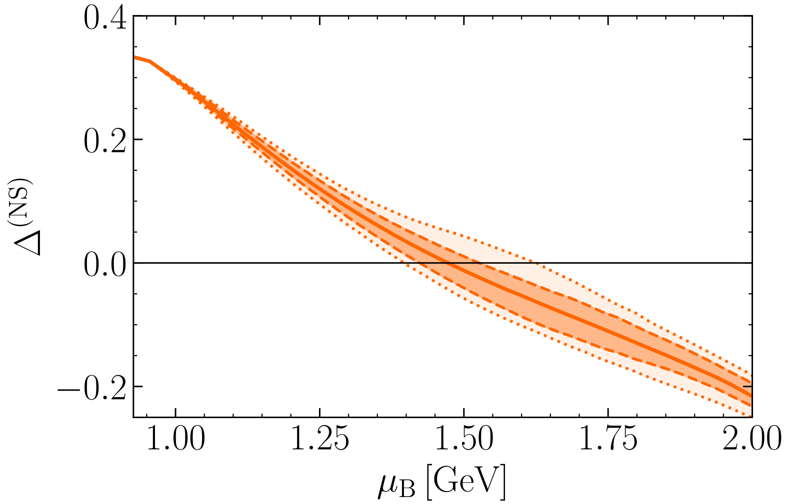
Derivative **Non-Derivative**

Dominant at high density to make a peak!

Interpretation





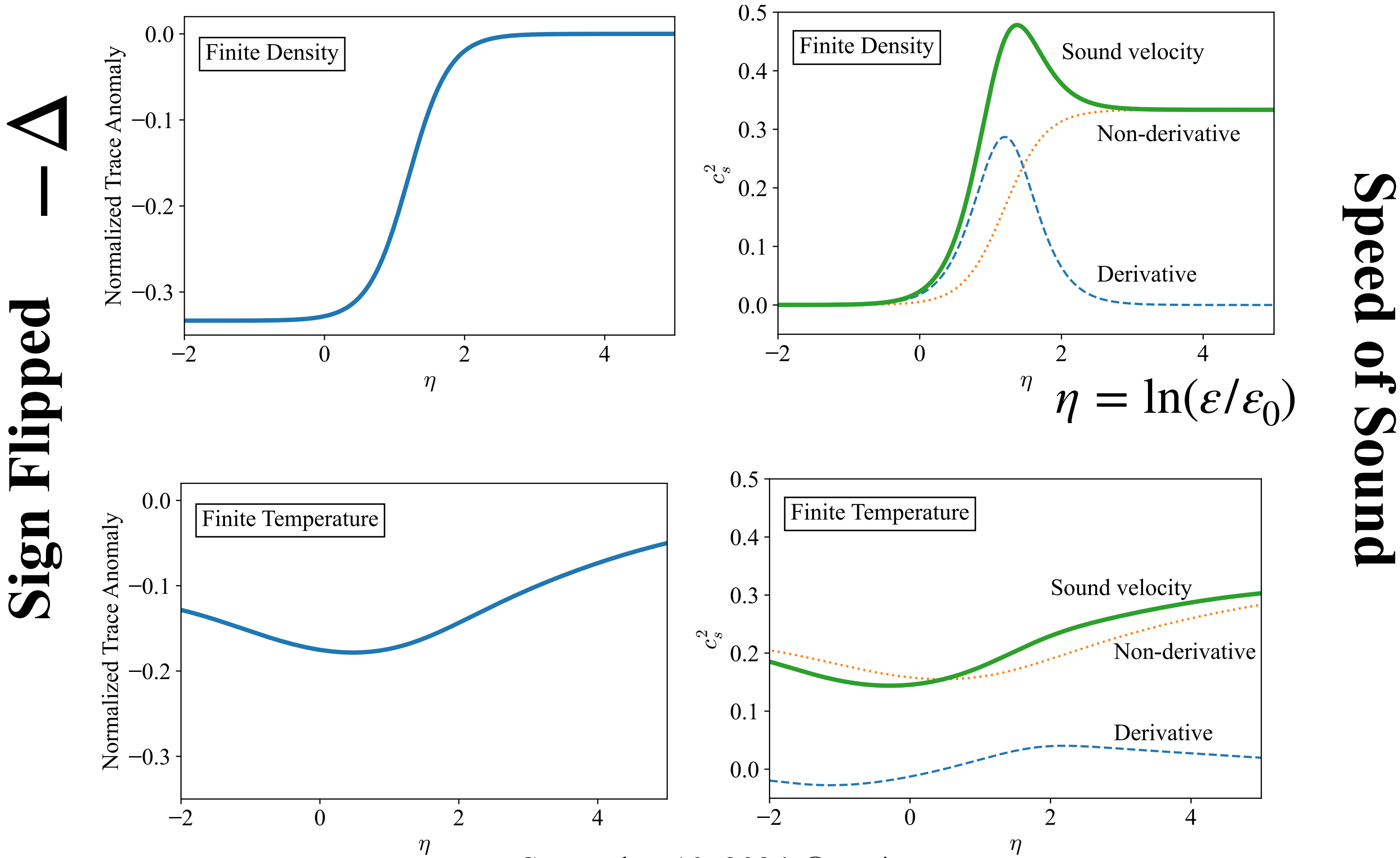


Brandes-Fukushima-Iida-Yu (2024)

Newer analysis suggests that the trace anomaly goes negative!

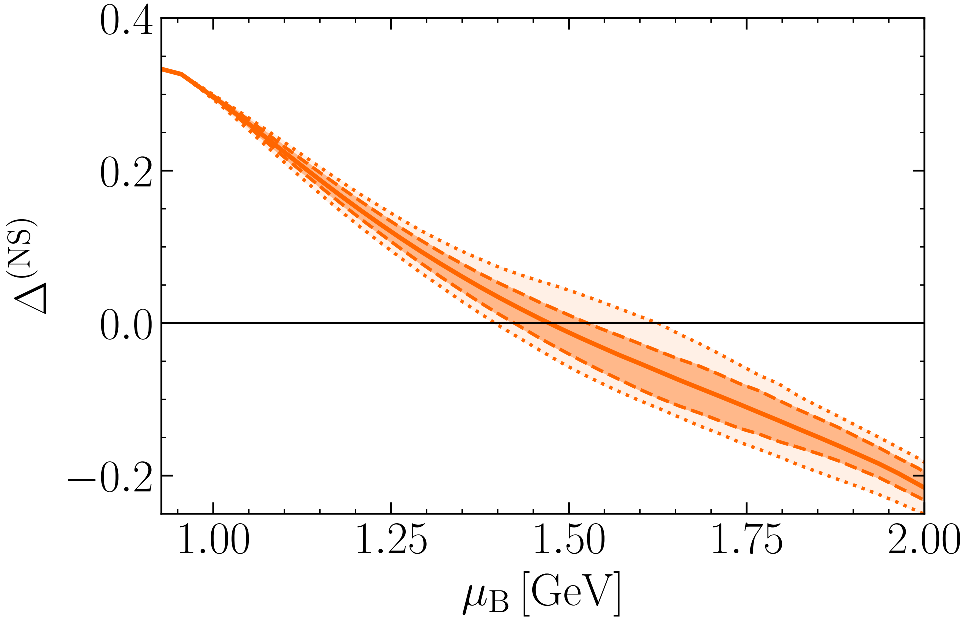
Interpretation

Derivative contribution makes a peak structure!



Interpretation

Interesting question… $\Delta < 0$???



$$\Delta \propto \varepsilon - 3p$$

$$\propto \frac{d}{d\mu}\left(\frac{p}{\mu^4}\right)$$

Thermodynamic degrees of freedom

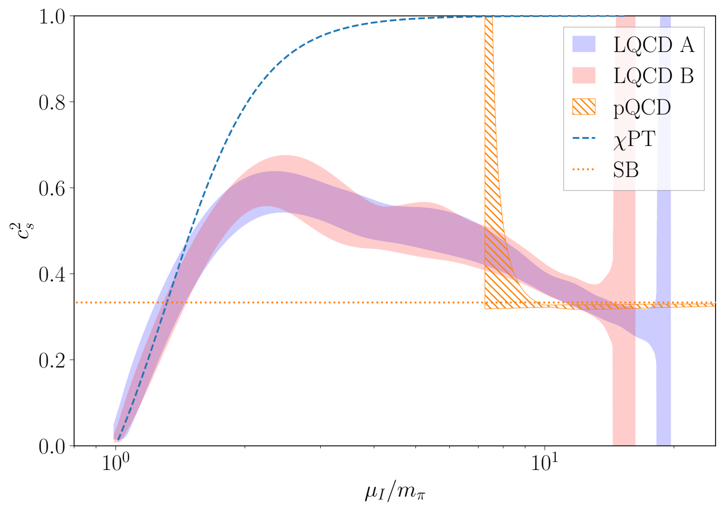
Negative trace anomaly implies the presence of "condensates"!?

Interpretation

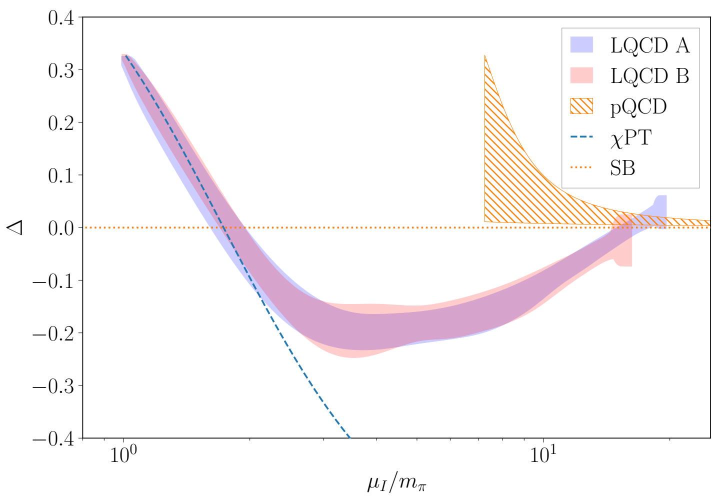
Lesson from high-isospin matter

Abbott+ (2023)

[Speed of sound peak]



[Negative trace anomaly]



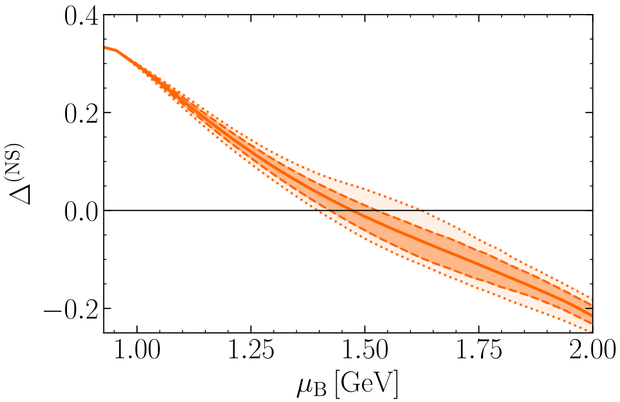
$$p_{\chi\mathrm{PT}} = \frac{f_\pi^2 \mu_I^2}{2}\left(1 - \frac{m_\pi^2}{\mu_I^2}\right)^2$$

Tree level without loop!

Interpretation

Brandes-Fukushima-Iida-Yu (2024)

Assume a general Ginzburg-Landau potential for "some" bosonic condensates to fit the NS trace anomaly behavior.



$$V(|\phi|^2) = m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4 + \frac{\eta}{6}|\phi|^6$$

Δ depends on only m^2 and $\tilde{\eta} = \eta m^2/\lambda^2$

NS data very well reproduced by

$$M_{\mathrm{diquark}} = \frac{0.033}{\tilde{\eta} - 0.075} + 0.65\ [\mathrm{GeV}] \qquad \frac{|\lambda||\phi|^2}{M^2} = \frac{\sqrt{1 + 8\tilde{\eta}(M)(\mu^2/M^2 - 1)} + 1}{2\tilde{\eta}(M)}$$

cf. Kurkela-Rajagopal-Steinhorst (2024) / Fujimoto (2024) indicating small gap…

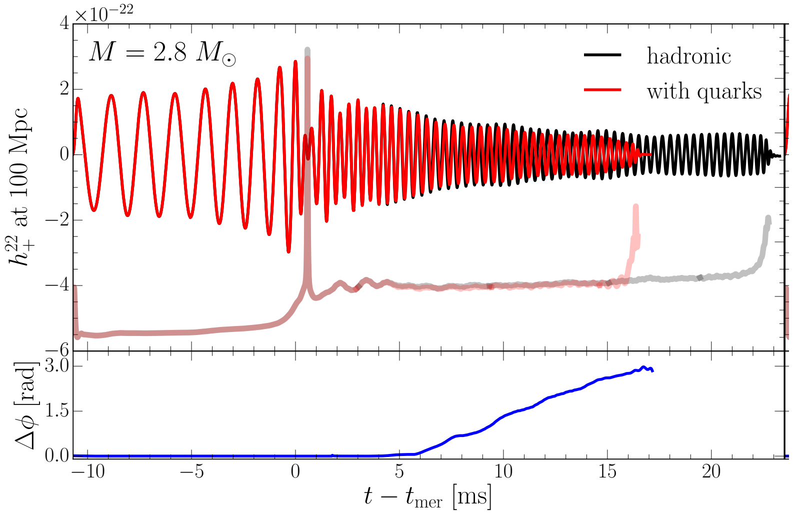
PT and GW

Can we see the phase transition with the GW signal?

Most-Papenfort-Dexheimer-Hanauske-Schramm-Stocker-Rezzolla (2018)

$\mathrm{CMF_Q}$: EOS with a strong-1st PT to Quark Matter (3~4 times n_{sat})

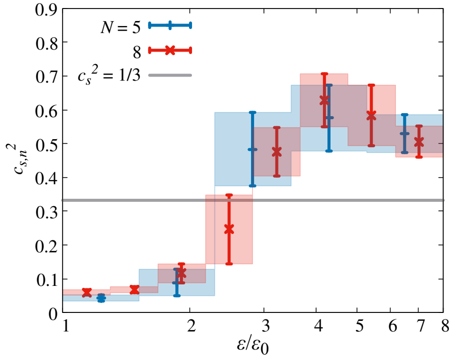
$\mathrm{CMF_H}$: EOS without quarks



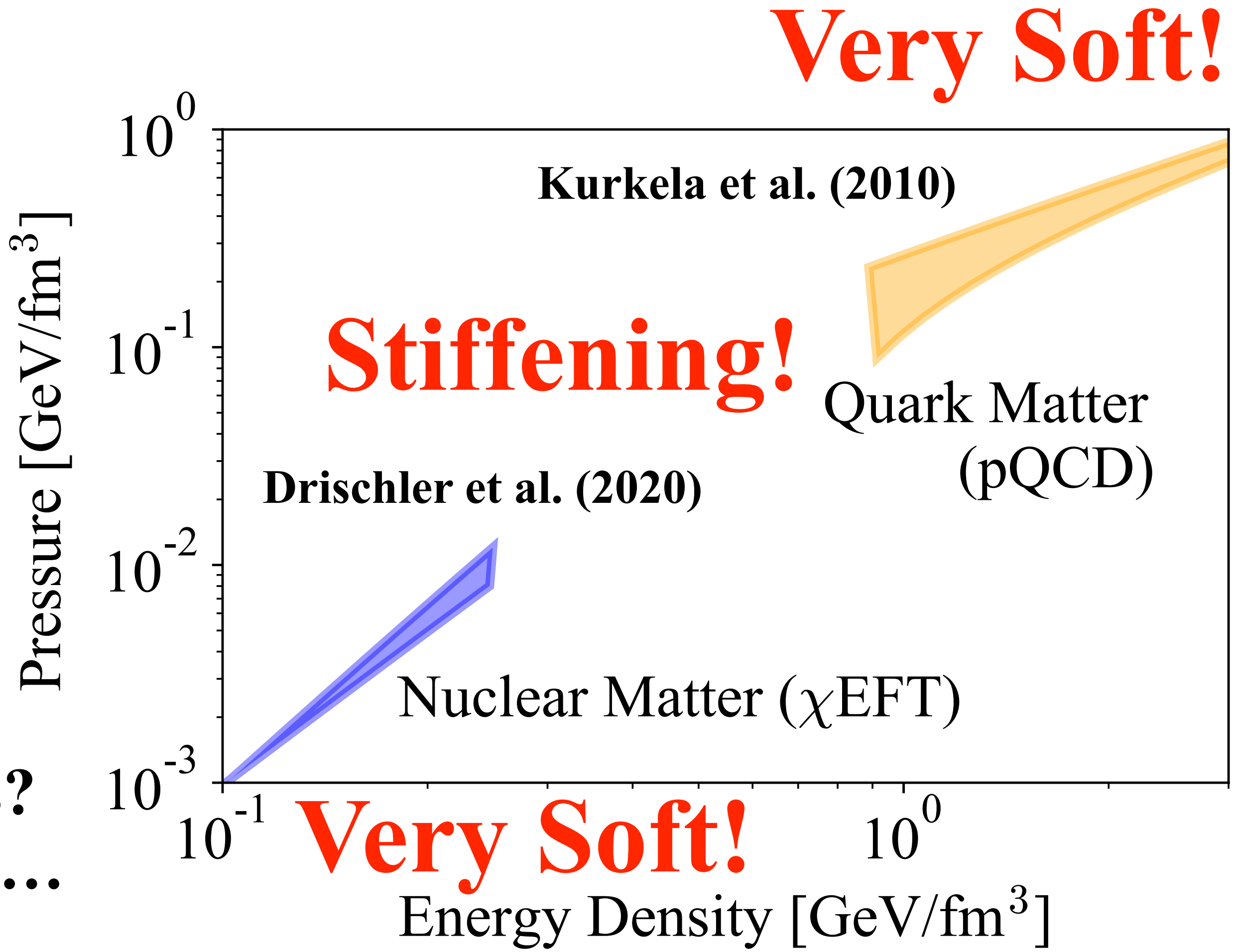
Quark matter shortens the lifetime of post-merger supramassive/hypermassive (uniform / differential) neutron star.

What if the transition is only a smooth crossover?

PT and GW



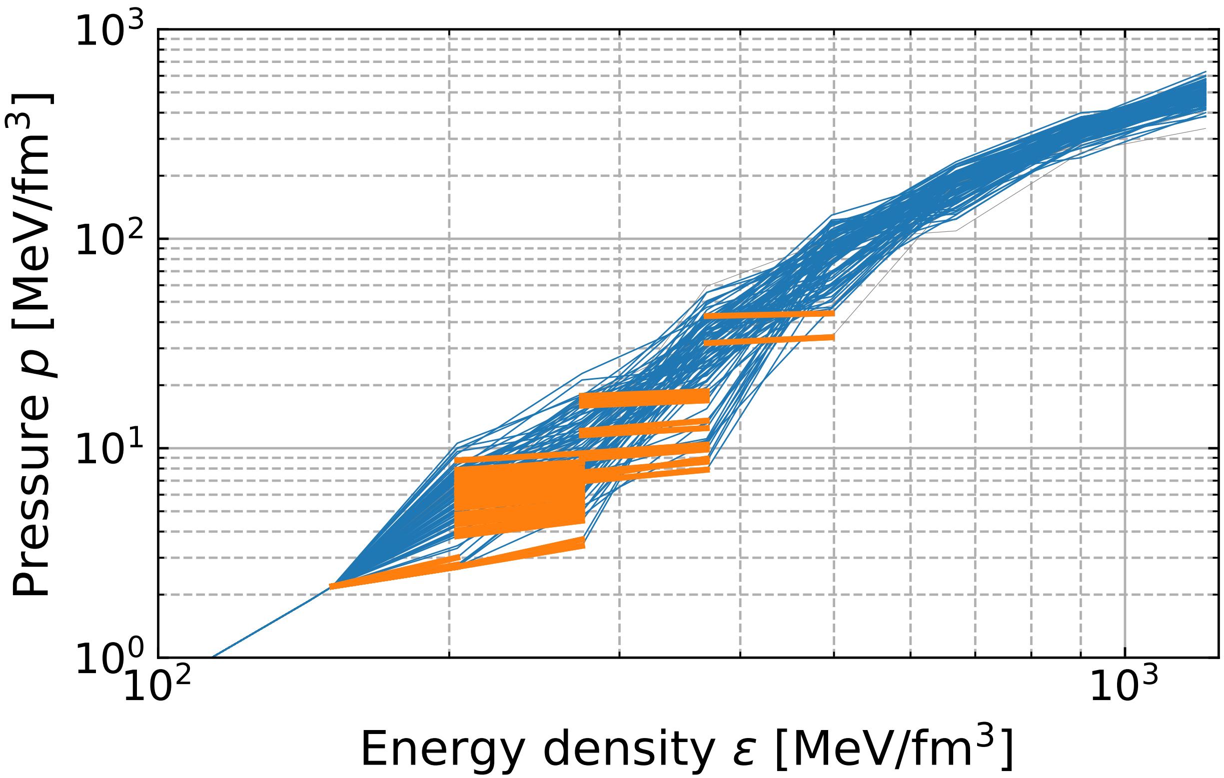
Necessary and likely behavior but how to explain it from nuclear physics? A big challenge…



PT and GW

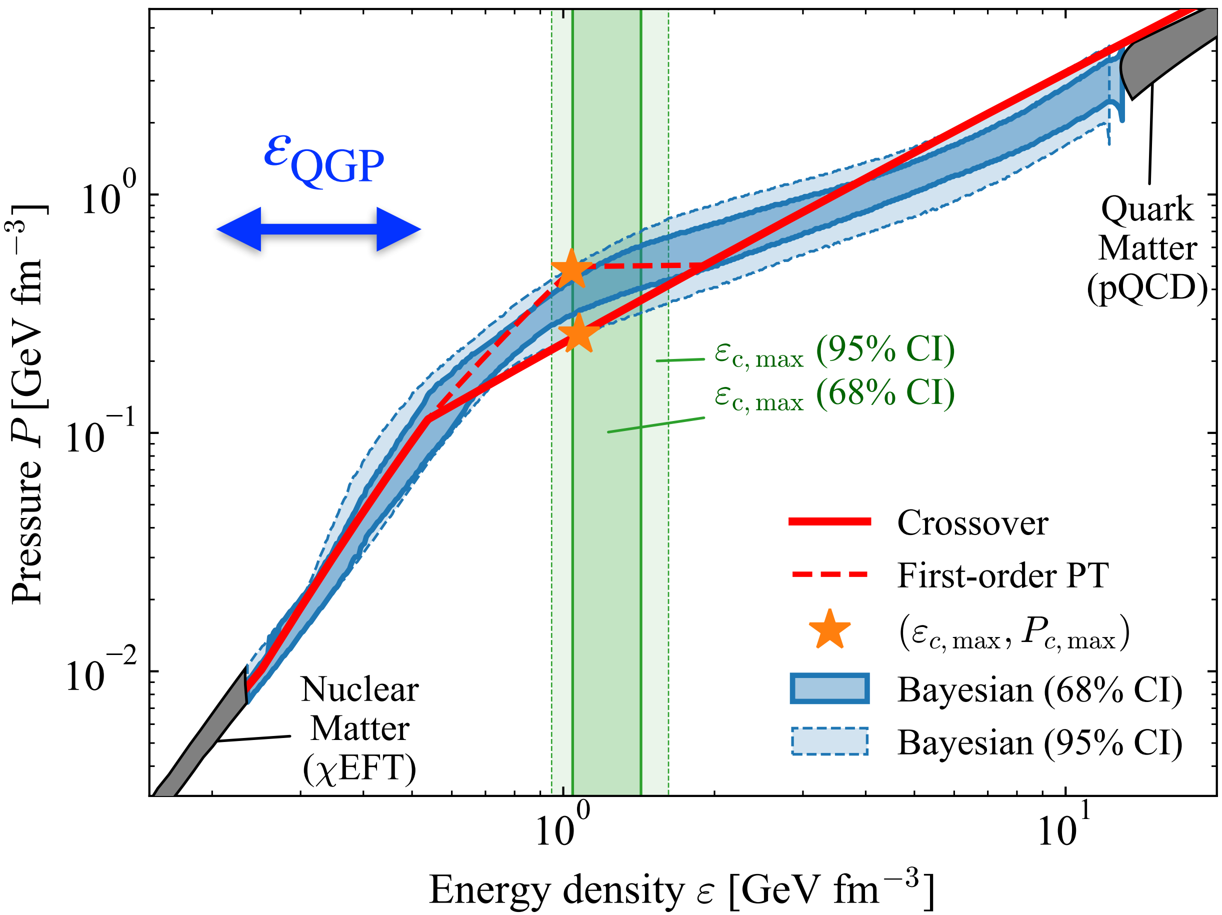
Caution: 1st-order phase transition is NOT excluded…

ML inferred EoSs: Fujimoto-Fukushima-Murase (2021)



PT and GW

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)



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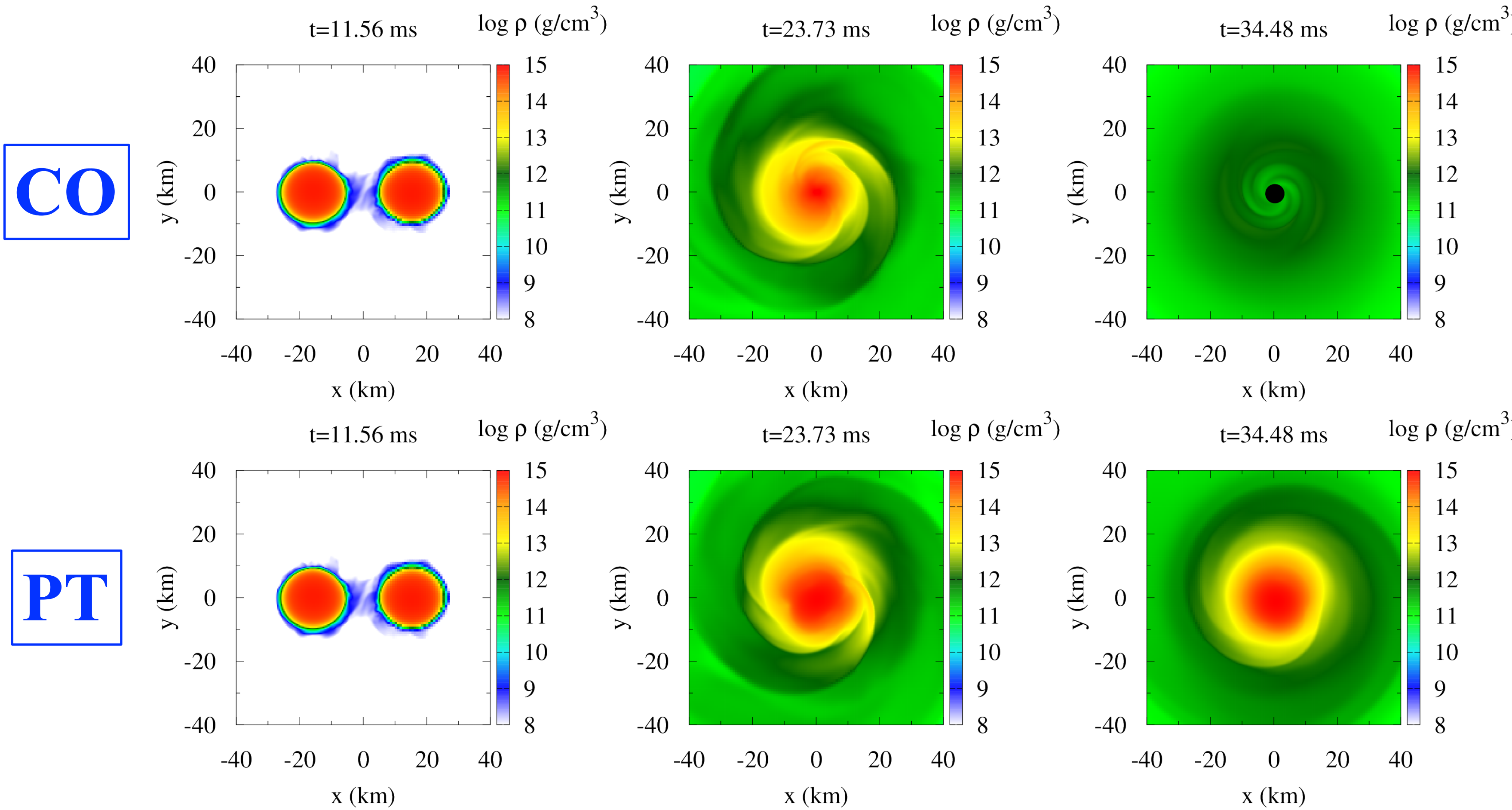

PT and GW

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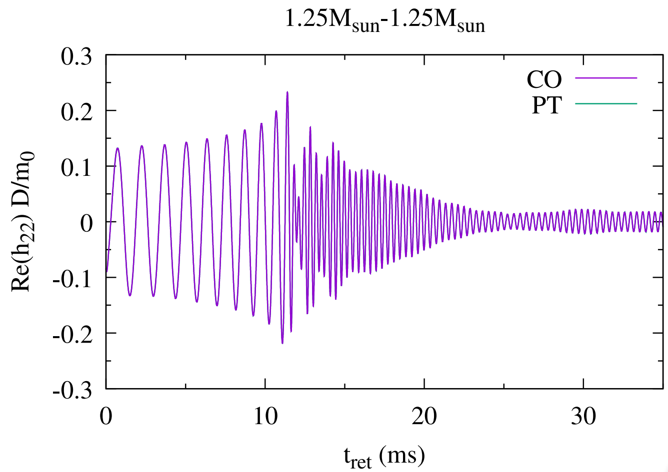
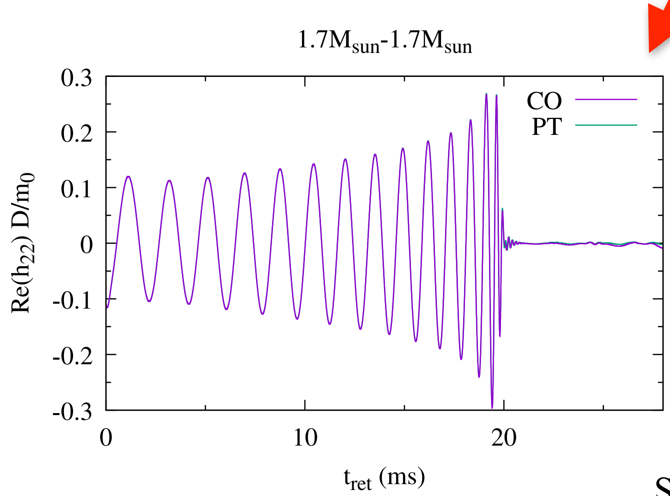
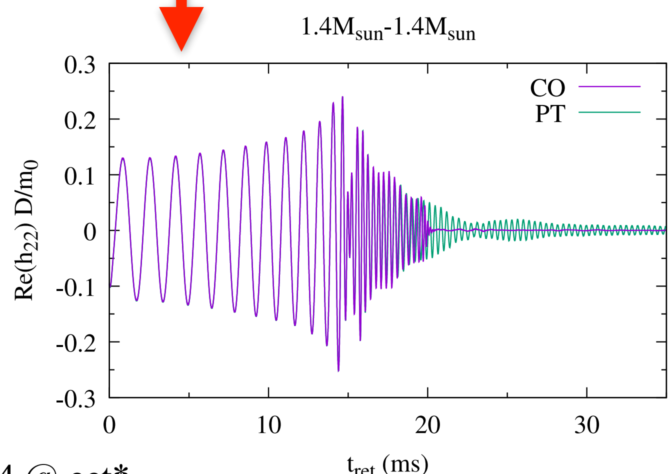


Light System (No BH)

Heavy System (Both BH)

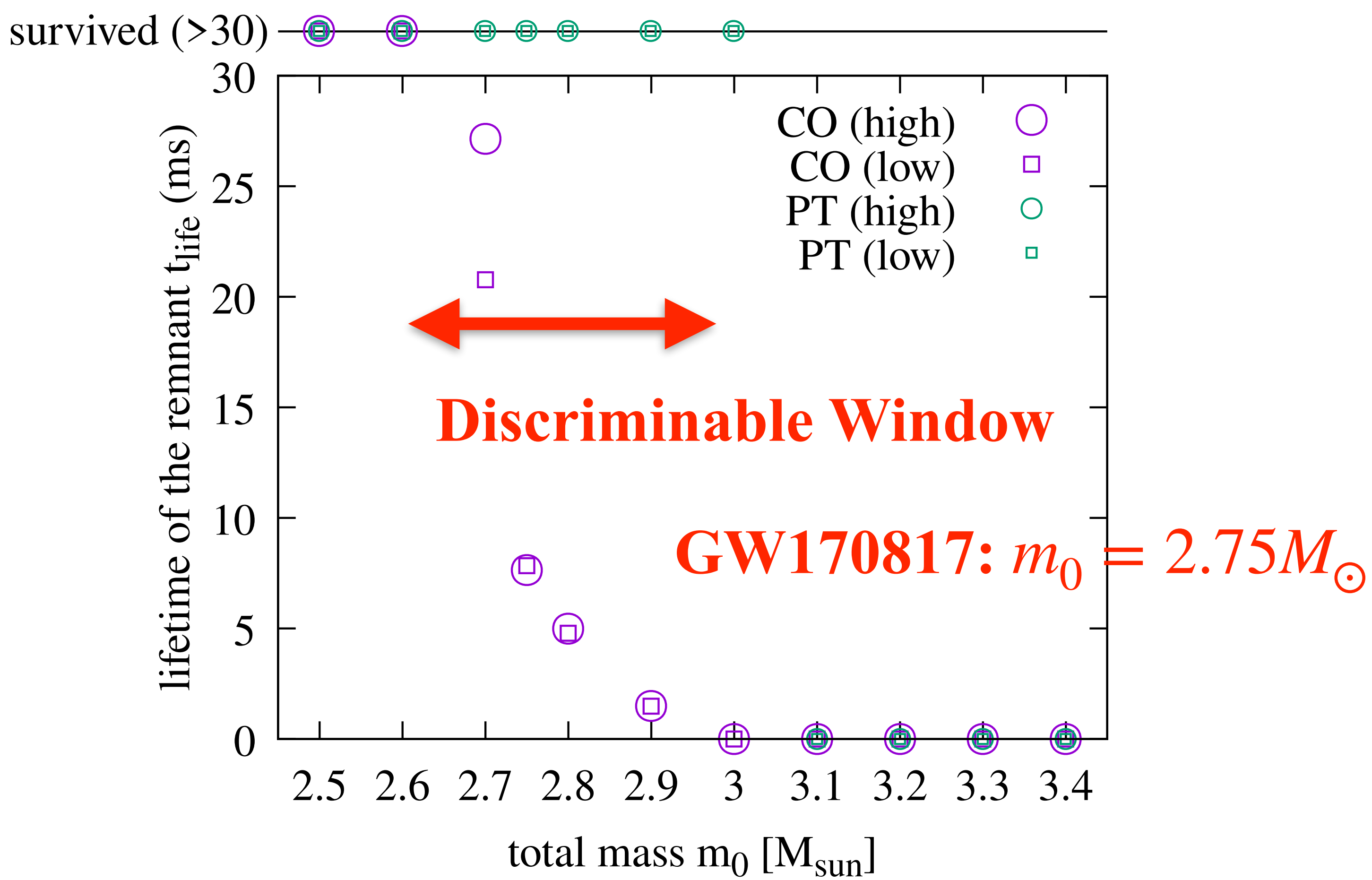
Like GW170817 (Discriminable!)





PT and GW

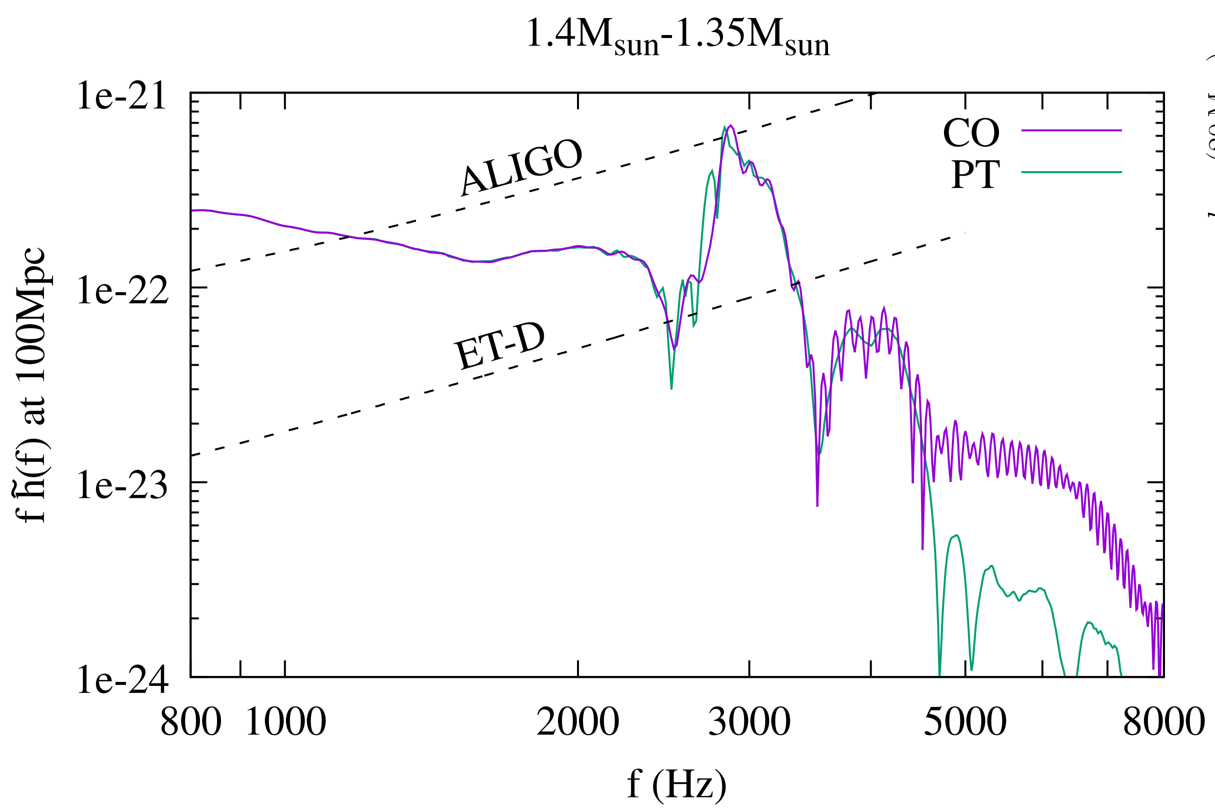
Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)

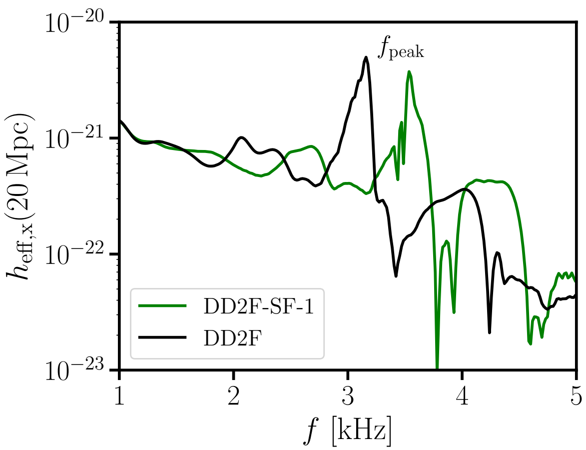


PT and GW

Post-merger stage is very challenging to see:

↓ Our crossover case… no difference?



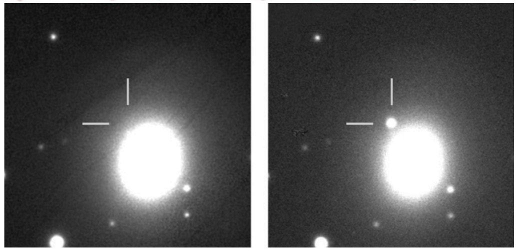


↑ Strong 1st-order PT scenario

Bausewein+ (2018)

Electromagnetic Counterpart

Kilonova brightness:
ejected mass $> 0.05 M_\odot$



AT 2017 gfo

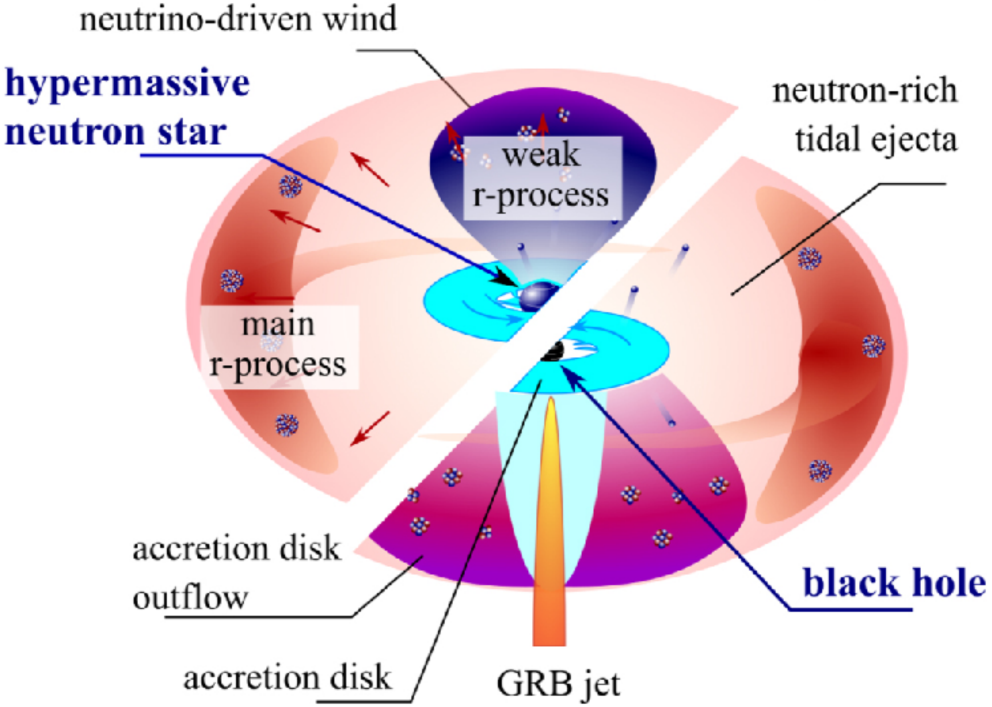
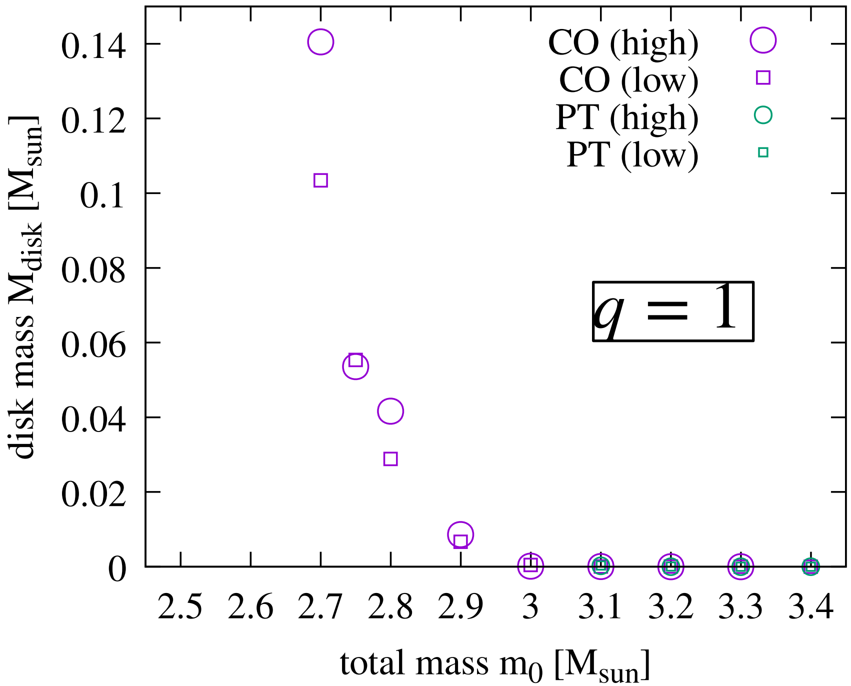
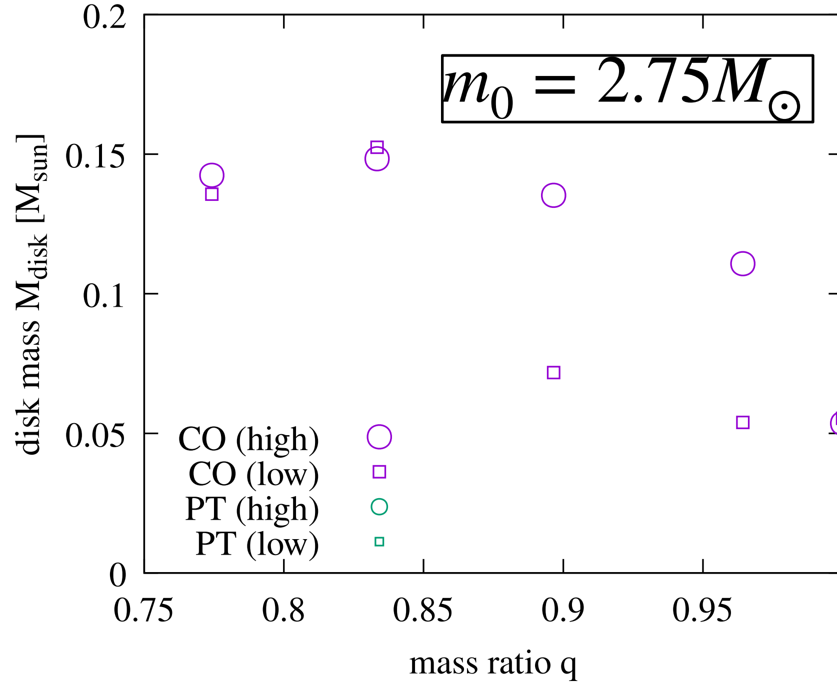


Illustration from Korobkin+ (2021)

Brightness and "color" depend on the EOS and the total mass.

Electromagnetic Counterpart





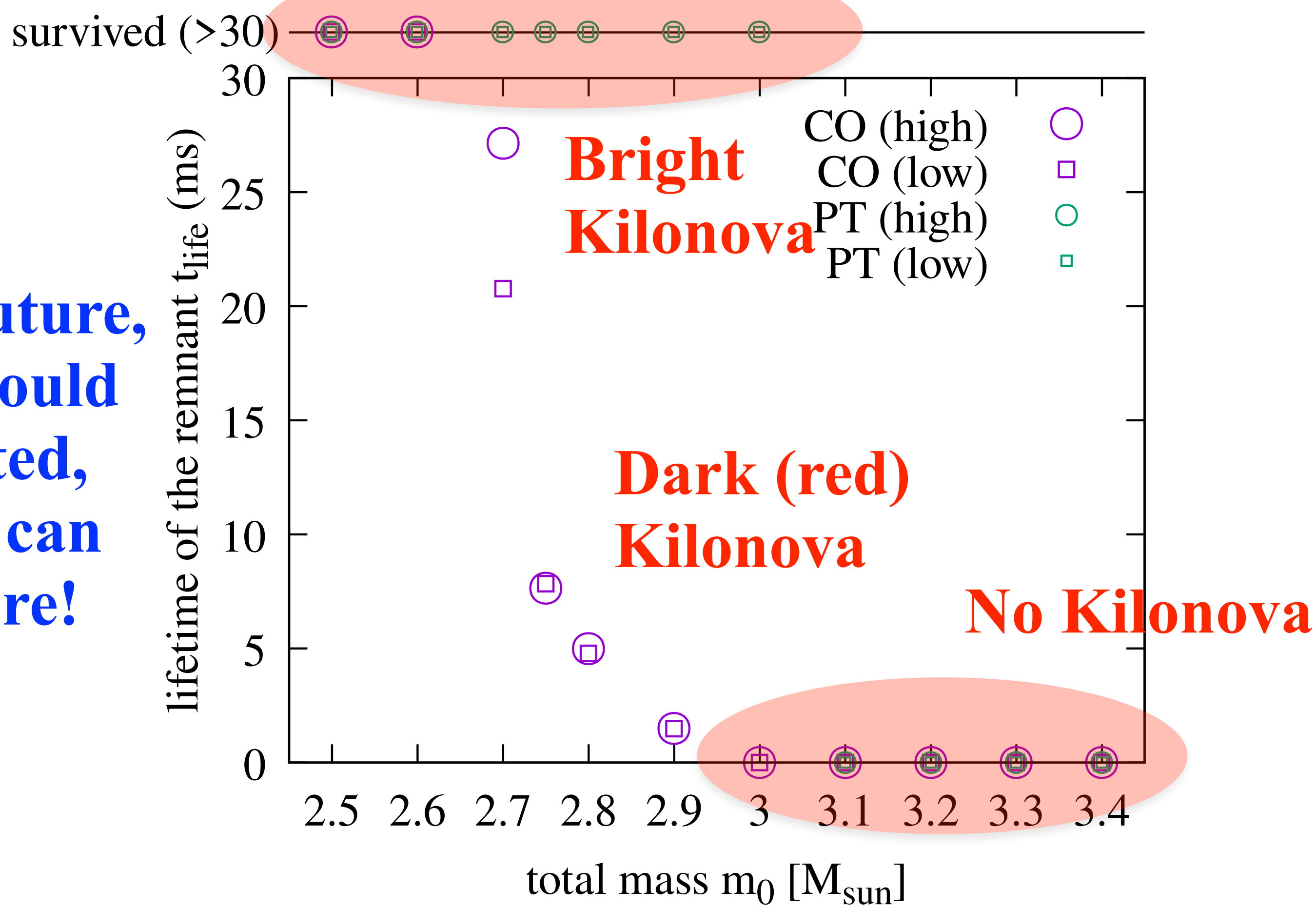
This situation (1.375+1.375) is already ruled out.

Asymmetric mass system can still be consistent.

Consistency with the kilonova tells us a lot!

Electromagnetic Counterpart

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)



In the near future, more data should be accumulated, and then, we can say much more!

Summary

- **Speed of sound at high density may increase above the conformal value. It would be interesting to confirm this by the heavy-ion collision.**
- **Trace anomaly is going negative and it implies the presence of "some" condensates. Color-super?**
- **QCD phase transition is detectable through the GW signal/kilonova even if it is a smooth crossover.**
- **GW170817 was such a fine-tuned event. Mass distribution will be determined in the near future.**