Exploring Baryon Fluctuations with Coulomb Gauge QCD

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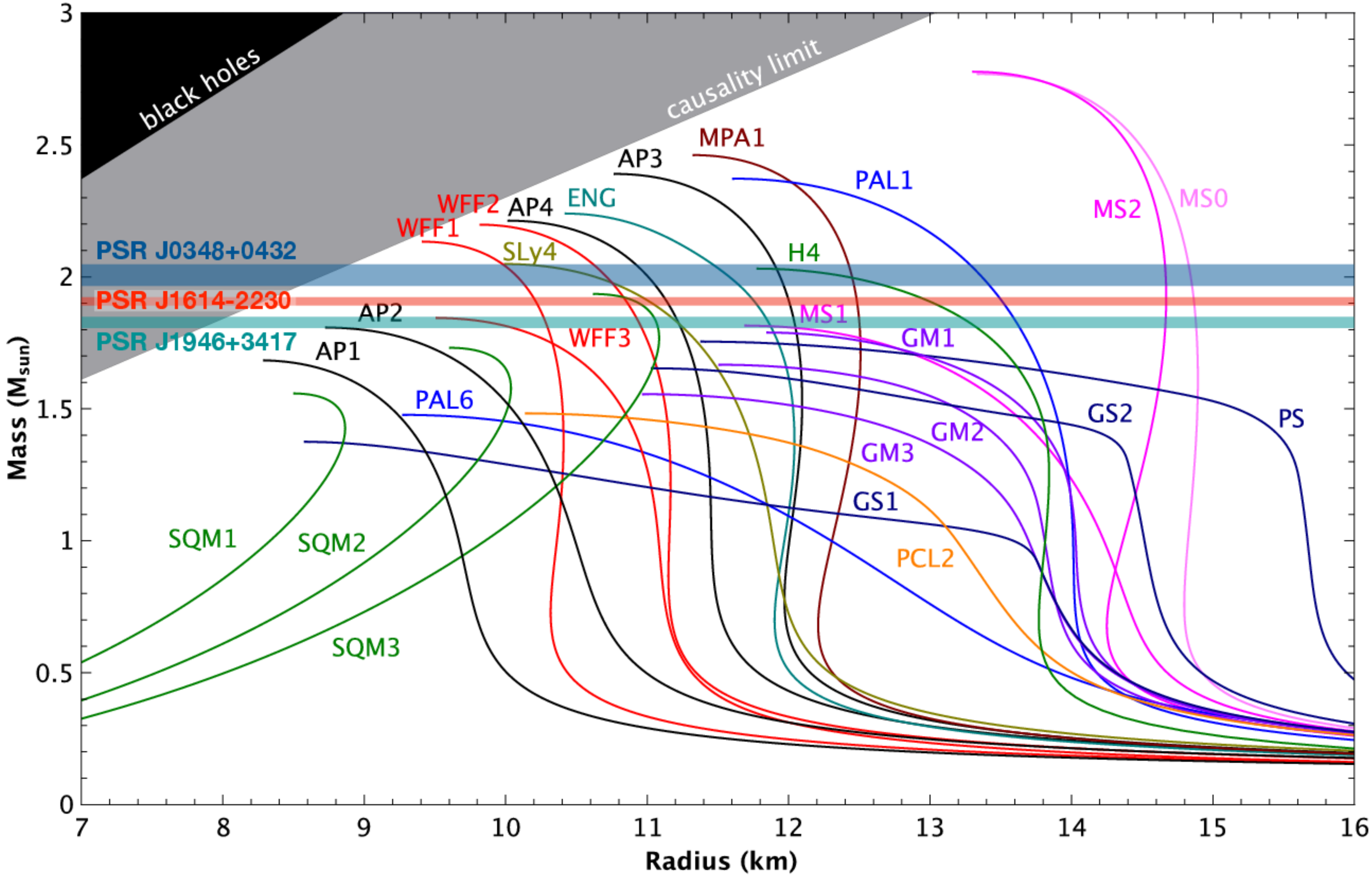
University of Wrocław
(In collaboration with Pok Man Lo, Peter Kovacs, Gyozo Kovacs)

NEW DEVELOPMENTS IN THE STUDIES OF QCD PHASE DIAGRAM, ECT*
09.09.2024 - 13.09.2024
TRENTO, ITALY

MASS : Astrophysical Constraint for high μ, low T



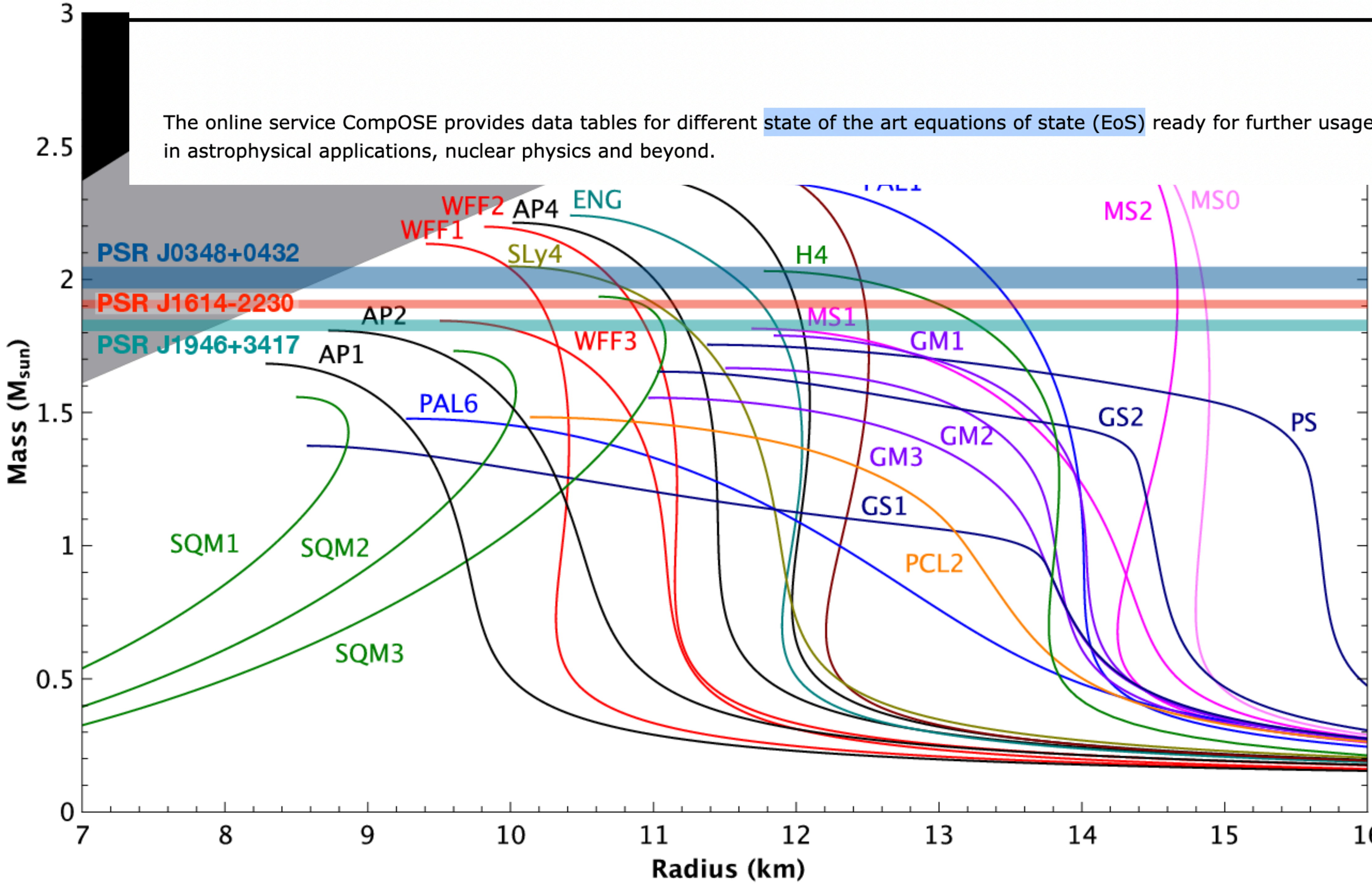
S. Typel, M. Oertel, T. Klähn, Phys. Part. Nucl. 46, 633

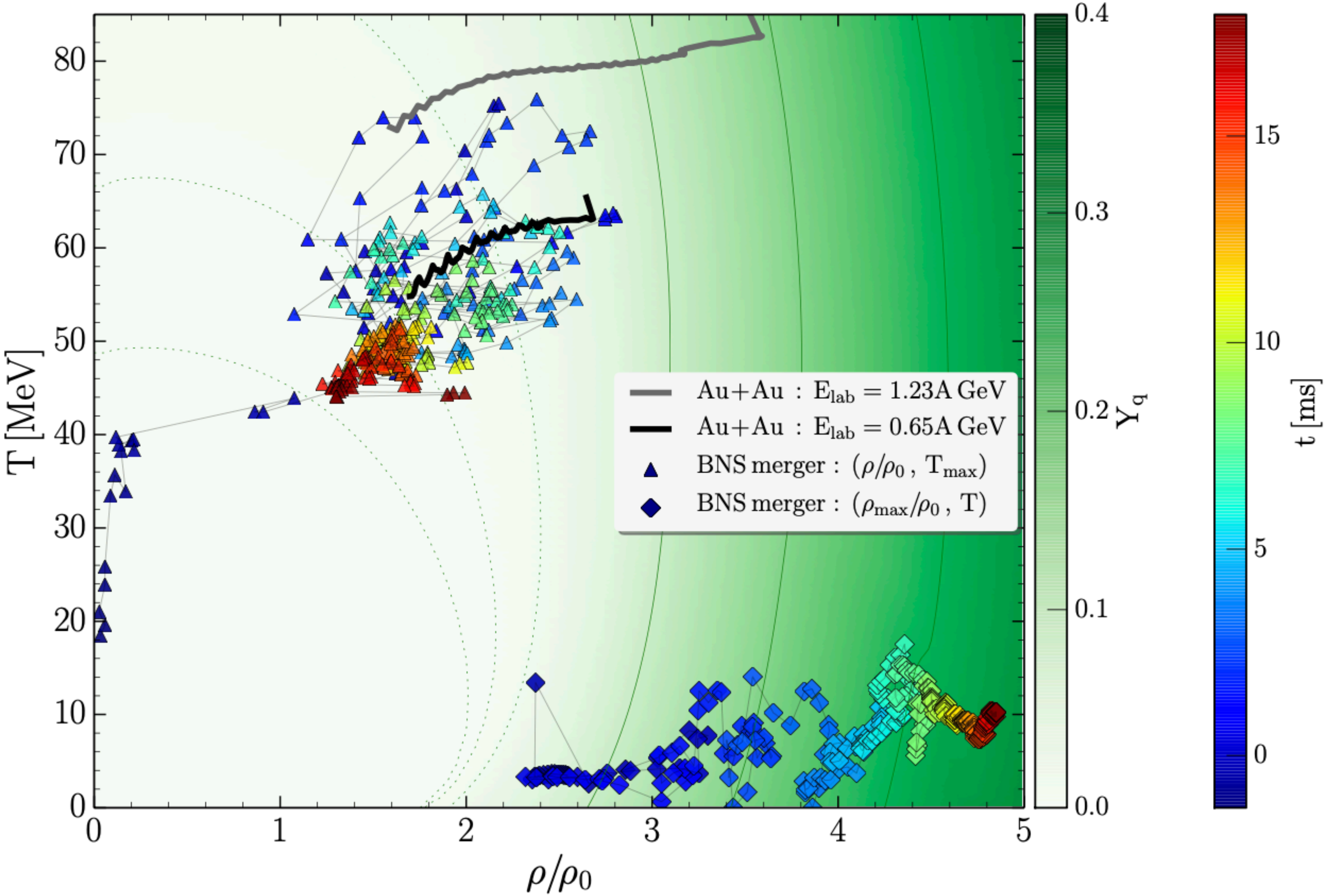
MAS T

CompOSE

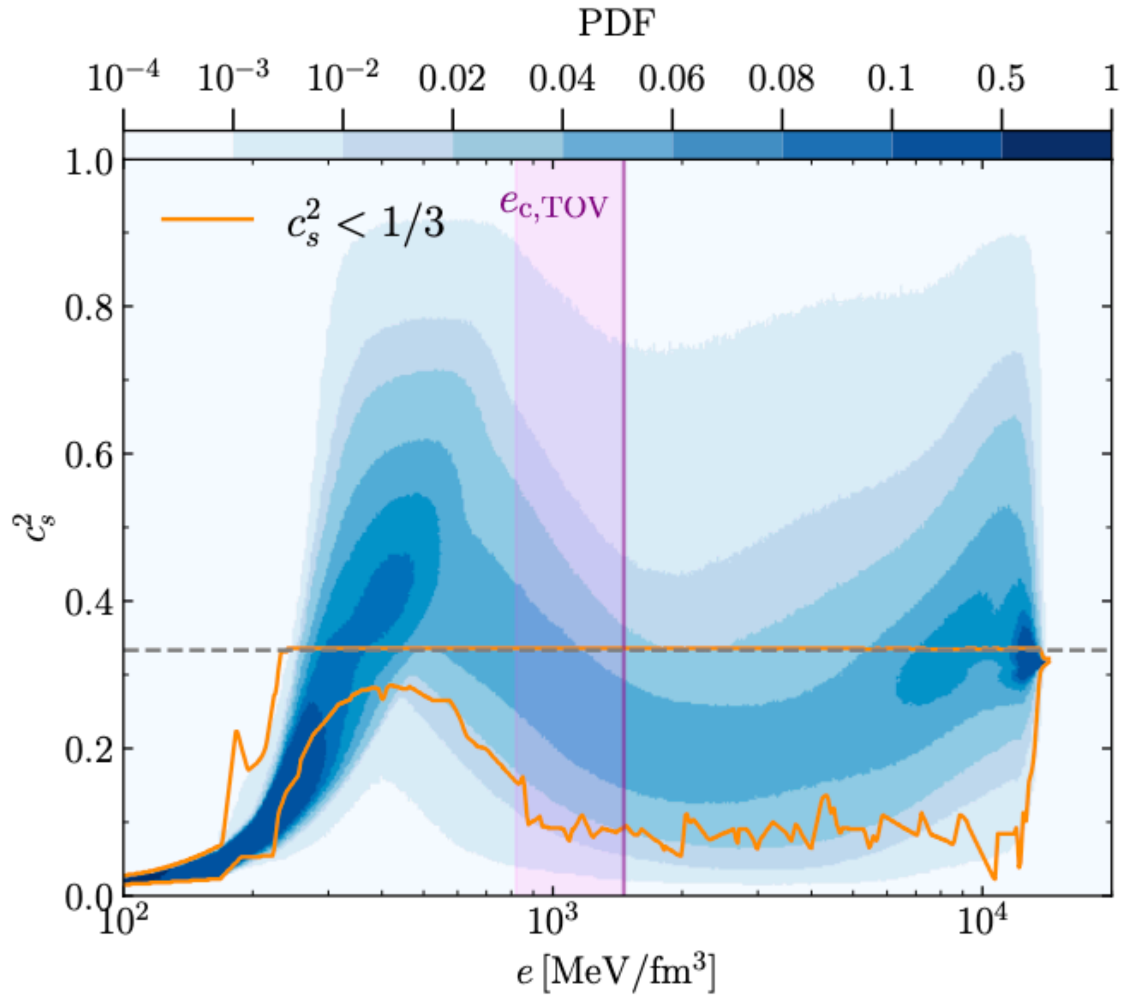
CompStar Online Supernovae Equations of State

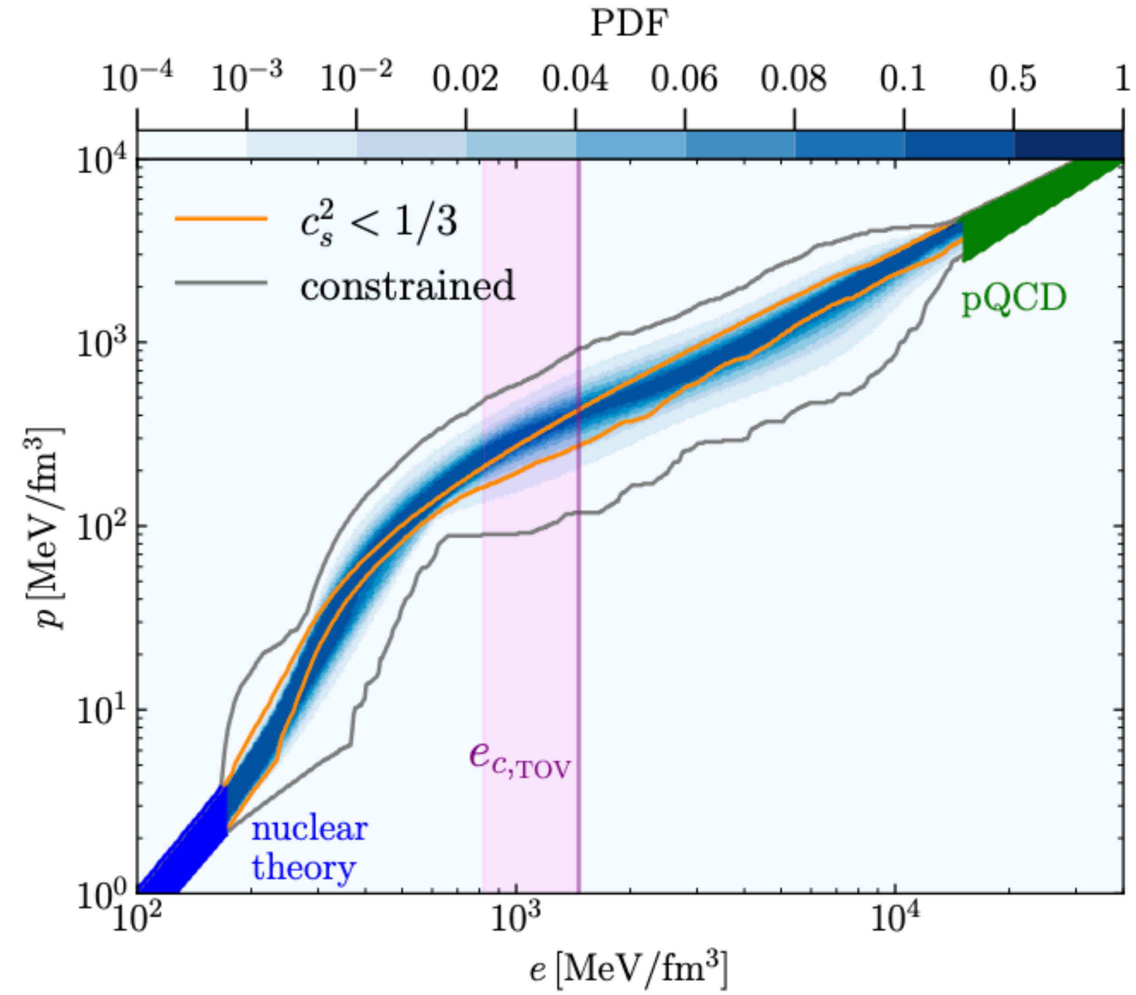
The online service CompOSE provides data tables for different state of the art equations of state (EoS) ready for further usage in astrophysical applications, nuclear physics and beyond.

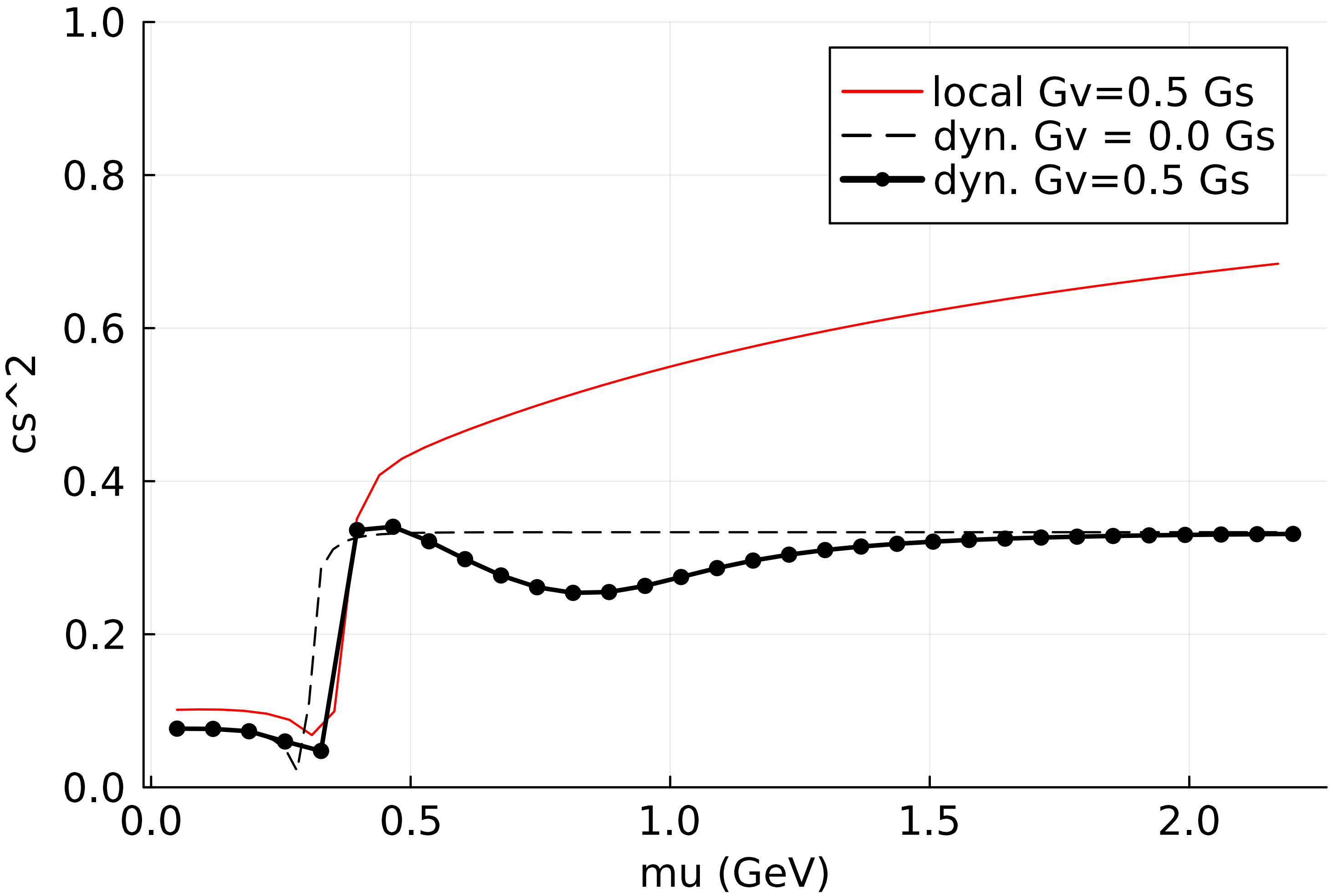




Hanauske et al., Particles 2 (2019) no. 1







NJL-like theory fails

Dynamical chiral Quark Model

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CONFINEMENT MECHANISM IN COUOMB GAUGE QCD

QED IN COULOMB GAUGE

$$\mathcal{H}_{\text{Coulomb}} = \bar{\psi}\left(-i\vec{\gamma}\cdot\nabla + m\right)\psi - g\bar{\psi}\vec{\gamma}\psi\cdot\vec{A}_{\perp} + \frac{1}{2}\vec{\Pi}_{\perp}^{2} + \frac{1}{2}\vec{B}^{2}$$

$$+\frac{1}{2}g^{2}\rho\,\frac{-1}{\nabla^{2}}\,\rho$$

where $\rho = \bar{\psi}\gamma^{0}\psi$

$$\nabla\cdot\vec{A} = 0 \rightarrow \vec{A} = \vec{A}_{\perp}$$

$$(A^{0}, \vec{A})$$

only 2 DoFs

A0 is ***NOT*** *dynamical*

Trade it w Gauss Law!

$$-\nabla^{2}A^{0} = \rho$$

=> $A^{0} = -\frac{1}{\nabla^{2}}\rho$

Potential $\frac{-1}{\nabla^{2}} \rightarrow \frac{1}{4\pi r}$

N. H. Christ and T. D. Lee
Phys. Rev. D 22, 939 (1980)

QCD IN COULOMB GAUGE

$$\mathcal{H} = -i\bar{\psi}\vec{\gamma}\cdot\nabla\psi + m\bar{\psi}\psi + \frac{1}{2}(\vec{E}^2 + \vec{B}^2) - g\bar{\psi}\,\vec{\gamma}\,T^a\cdot\vec{A}^a$$

$$+\frac{1}{2}\rho\left[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}\right]\rho$$

$$\rho^a = \bar{\psi}\gamma^0 T^a\psi + f^{\mathrm{abc}}\,A_i^b\,E_c^i$$

$$\vec{D}^{ab} = \delta^{ab}\vec{\nabla} + igT^c_{ab}\vec{A}^c$$

both quarks and gluons are color charged & confined

Potential

$$V_{ab}(x,y;\vec{A}_\perp) = \langle x,a|\left[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}\right]|y,b\rangle$$

N. H. Christ and T. D. Lee
Phys. Rev. D 22, 939 (1980)

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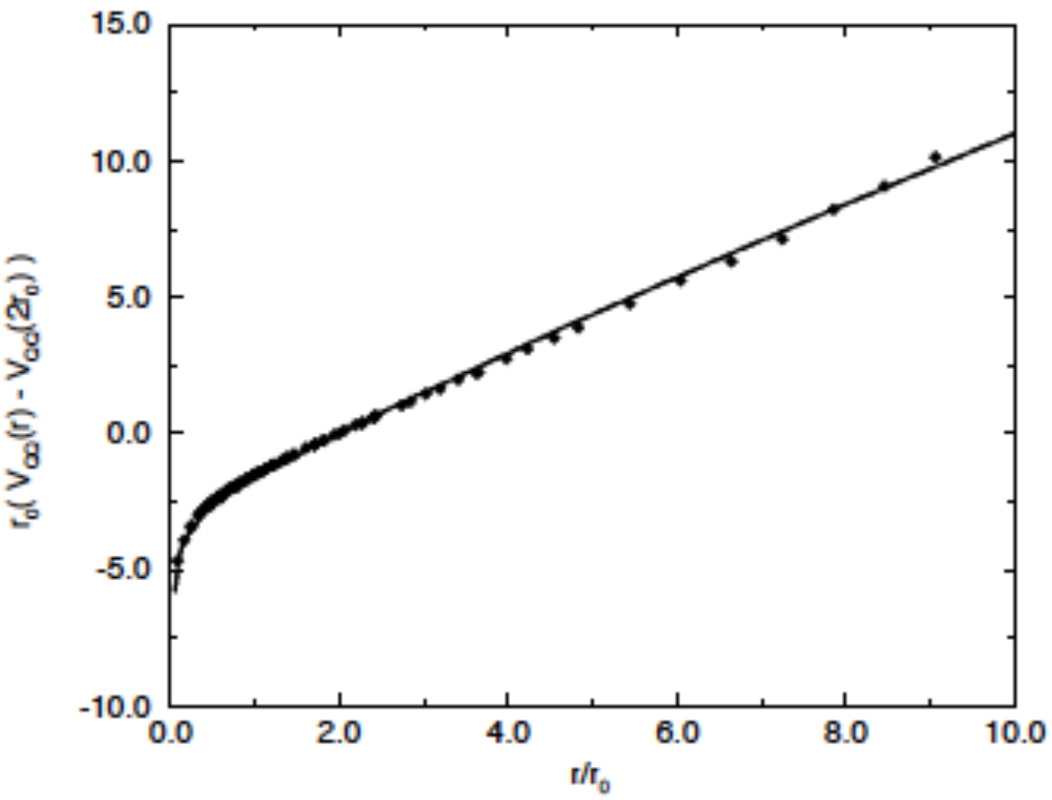
Confining Potential



A. Szczepaniak and E. Swanson
Phys. Rev. D 65, 025012 (2002)

How confinement works?

“confinement” via thermal suppression

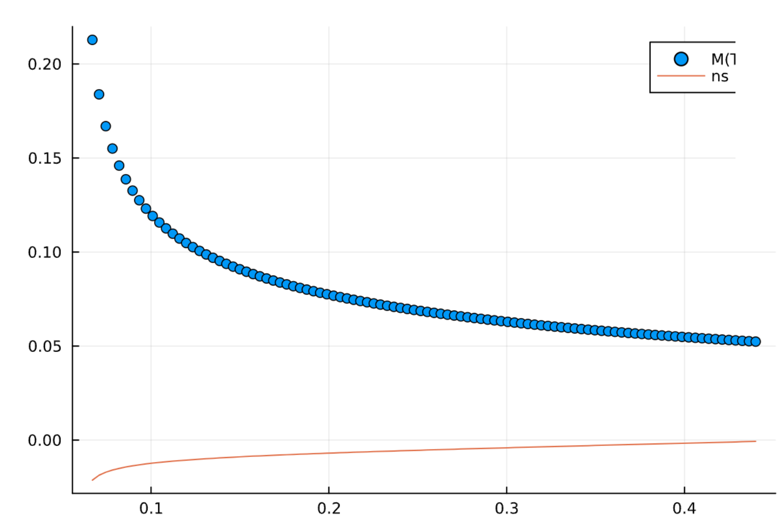
$$n_Q = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(\sqrt{p^2+M_Q^2}-\mu_Q)}+1}$$

“confinement” via thermal suppression

$$n_Q = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta\left(\sqrt{p^2+M_Q^2}-\mu_Q\right)}+1}$$

string-flip model

$$M_Q \propto 1/n^{\frac{1}{3}} \to \infty$$



PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

G. Röpke and D. Blaschke
Department of Physics, Wilhelm-Pieck-Universtiy, 2500 Rostock, German Democratic Republic

H. Schulz
Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic and The Niels Bohr Institute, 2100 Copenhagen, Denmark
(Received 16 December 1985)

G. Roepke, D. Blaschke and H. Schulz
PRD 34 11 (1986)

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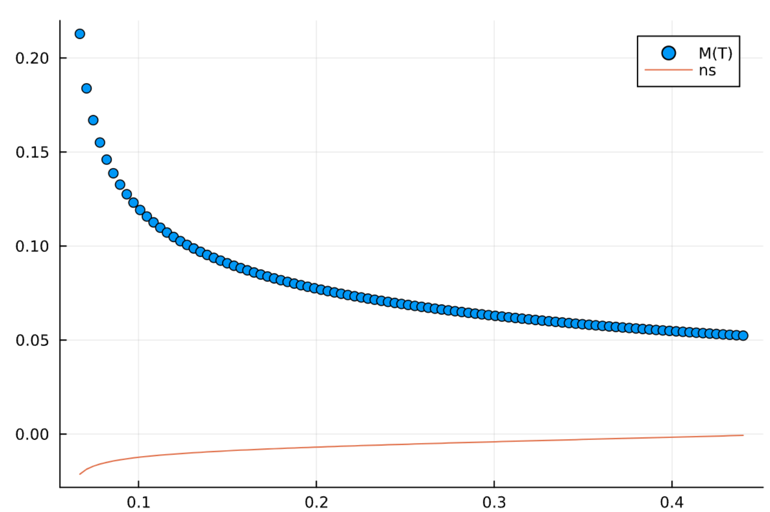


PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

BUT...

not really confinement...

mess up chiral physics in vac.

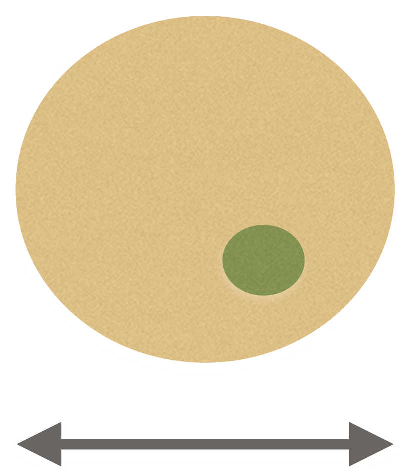
mess up hadron spectrum

"confinement" via thermal suppression

$$n_Q = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(\sqrt{p^2+M_Q^2}-\mu_Q)}+1}$$

Bag Model

$$\int \frac{d^3p}{(2\pi)^3} \to \sum_{p_n}$$



$$p_{\text{min.}} = \pi/L$$

Better model of confinement

motivates an IR scale

$$\Lambda_{\text{IR}} \approx 0.2\,\text{GeV}$$

"confinement" via thermal suppression

$$n_Q = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(\sqrt{p^2+M_Q^2}-\mu_Q)}+1}$$

Bag Model

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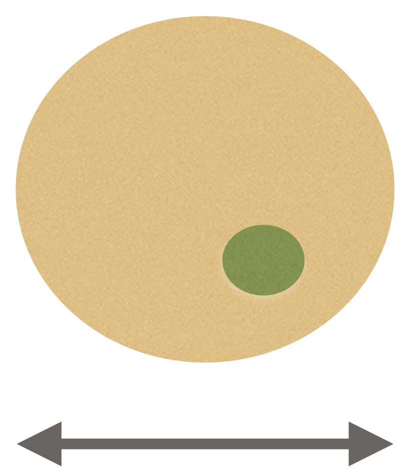
Proton Wavefunction



$$p_{\mathrm{min.}} = \pi/L$$

Better model of confinement

motivates an IR scale

$$\Lambda_{\mathrm{IR}} \approx 0.2\,\mathrm{GeV}$$

also recently

G. Krein EPJA 18 (2003)
Bicudo et. al. PRD 45 5 (1992)

K. Fukushima, T. Kojo, W. Weise
PRD 102, 096017 (2020)

“confinement” via thermal suppression

$$n_Q = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(\sqrt{p^2+M_Q^2}-\mu_Q)}+1}$$

A-conf

$$E(p) \to \tilde{E}(p) = A(p)\sqrt{p^2 + M(p)^2}$$

$A(p), M(p)$ *satisfy a coupled Dyson eqns.*

infrared enhanced!

"confinement" via thermal suppression

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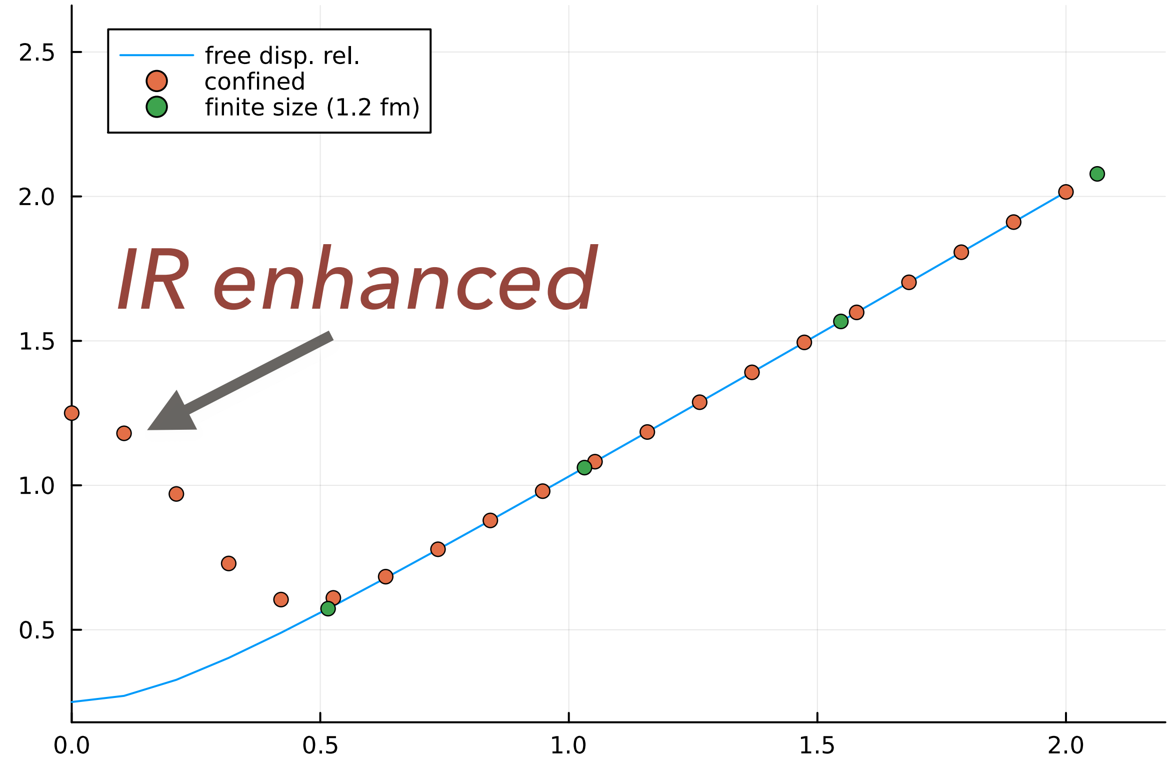


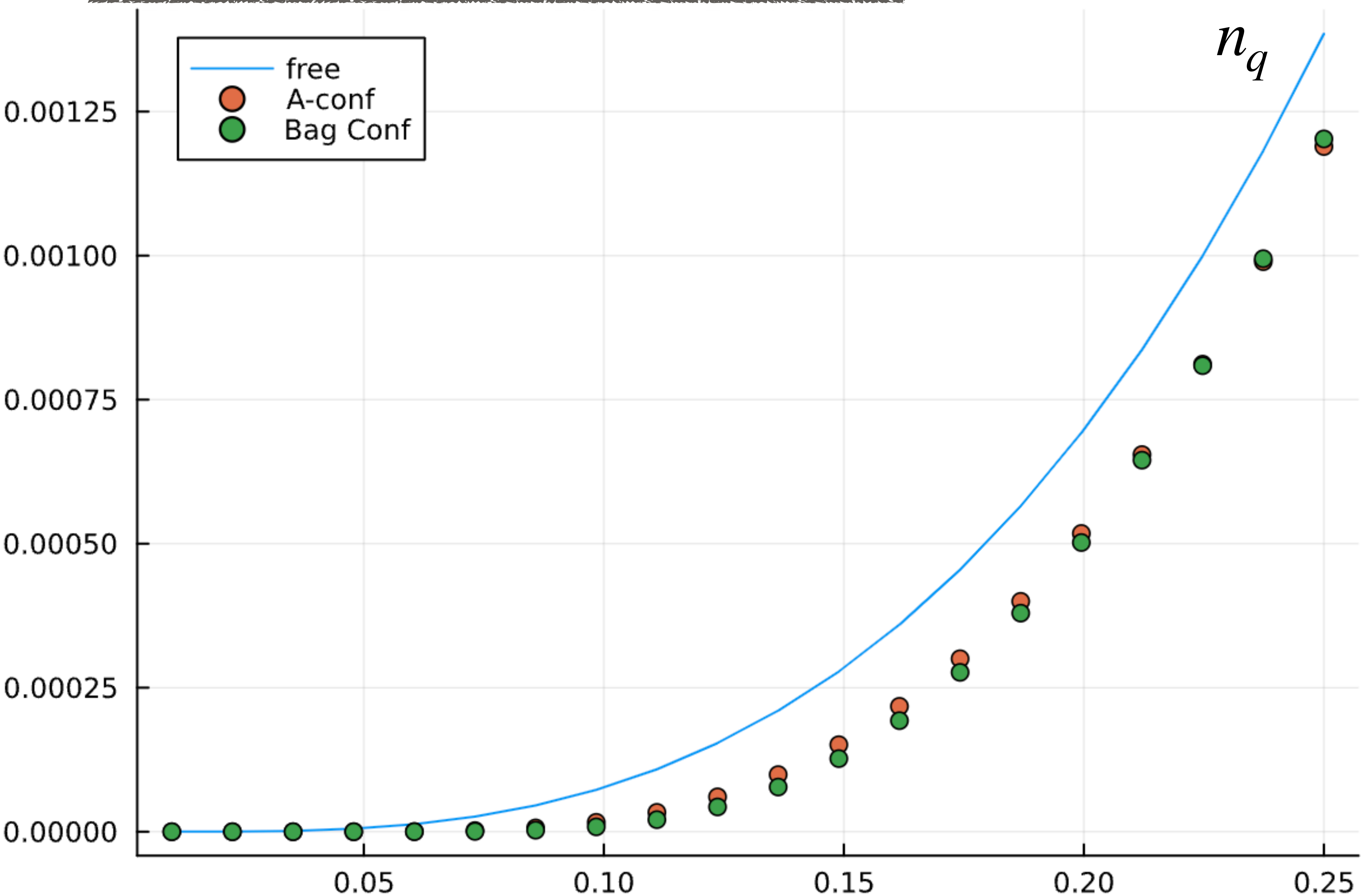
$A(p), M(p)$ satisfy a coupled Dyson eqns.

infrared enhanced!

E. Swanson and P. M. Lo
PRD 81 034030 (2010)

“c rmal suppression

effects on thermal obs.
are similar!



1

$p)^2$

coupled
n eqns.

ced!

P. M. Lo
(2010)

Confinement of Quarks

$$S^{-1}(p) = A_0(p)\, p^0 \gamma^0 - A(p)\, \vec{p} \cdot \vec{\gamma} - B(p)$$

$$\Sigma(p) \approx C_F \int \frac{d^4 q}{(2\pi)^4} V(\vec{p} - \vec{q})\, i\, \gamma^0 S(q) \gamma^0 .$$

$$V_{ab}(x, y; \vec{A}_\perp) = \langle x, a | [\frac{g}{\nabla \cdot D} (-\nabla^2) \frac{g}{\nabla \cdot D}] | y, b \rangle$$

$A(p), B(p)$ *are IR div! But*

$$M(p) = \frac{B(p)}{A(p)}$$ *is finite!*

$$\langle \bar{\psi} \psi \rangle = N_c \int \frac{d^3 q}{(2\pi)^3} \frac{-4\, B(q)}{2 \sqrt{A(q)^2 q^2 + B(q)^2}}$$

VS string-flip model:
M -> infinity
(too large sigma mass)

QUARK SDE

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E}) \right)$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{B(q)}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E}) \right)$$

$$A(p) = 1 + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{A(q)\, \vec{p} \cdot \vec{q}}{\vec{p}^2} \frac{1}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E}) \right)$$

$$\tilde{E}(p) = \sqrt{A(p)^2\, p^2 + B(p)^2}$$

$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q) - \mu'(q))} + 1}.$$

RESULTS & DISCUSSION

ASYMPTOTIC FREEDOM

Mean fields

$$\mu' = \mu - 2G_V \int \frac{d^3q}{(2\pi)^3} \, (n_F - \bar{n}_F)$$

$\searrow \propto \mu'^3$

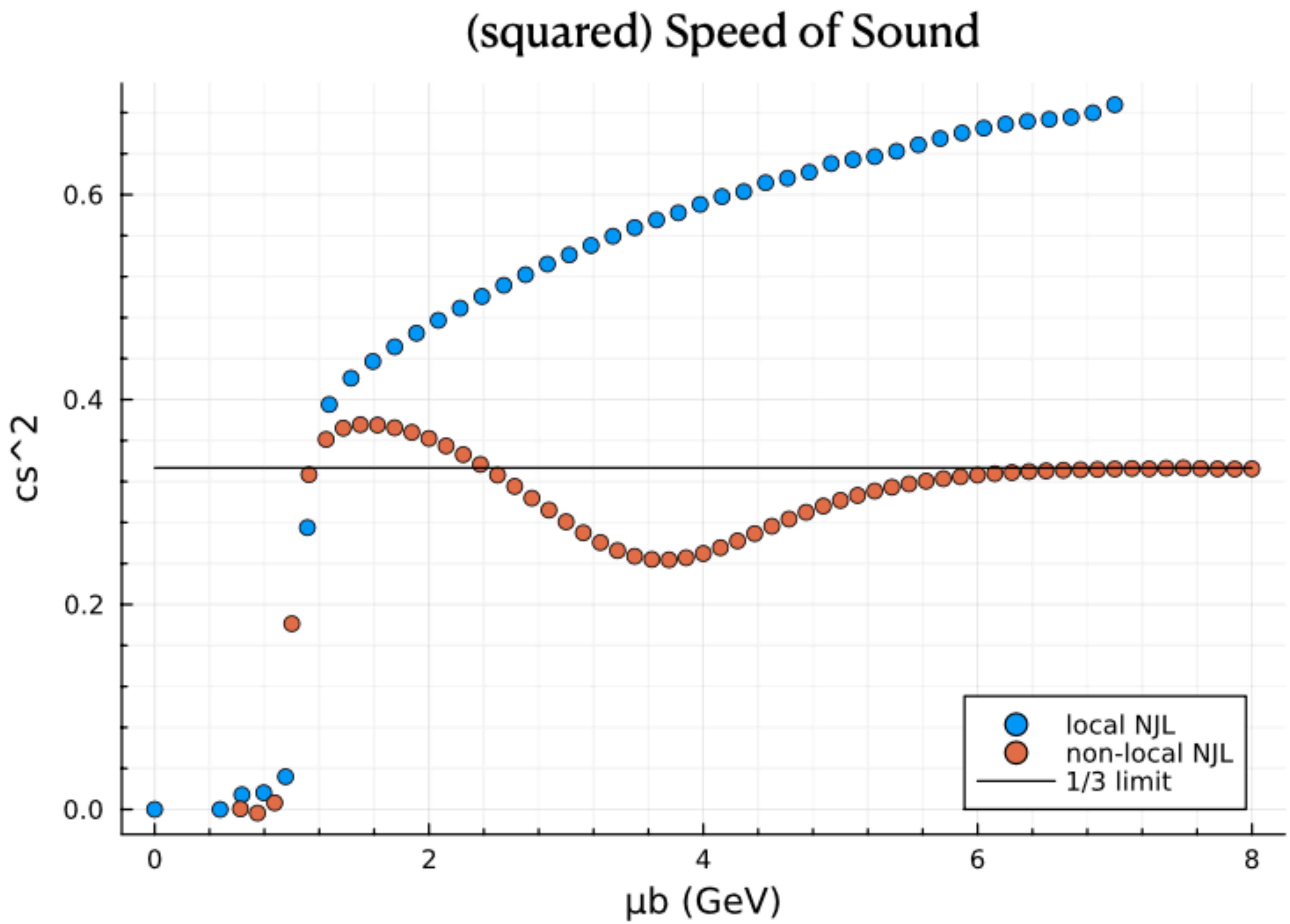
versus $\qquad \mu' \propto \mu^{\frac{1}{3}} \longrightarrow c_S^2 \to 1$

Dynamical model

$$\mu'(p) = \mu + \int \frac{d^3q}{(2\pi)^3} \frac{1}{2} V(\vec{p} - \vec{q}) \, (n_F - \bar{n}_F)$$

If V->0 as p-> Inf: $\quad \mu' \to \mu \longrightarrow c_s^2 \to \frac{1}{3}$

e.g. $\quad V(p,q) \approx V_0 \, e^{-p^2/\Lambda^2} e^{-q^2/\Lambda^2}$ (SEPARABLE APPROX.)


(squared) Speed of Sound
cs^2
0.0
0.2
0.4
0.6
0
2
4
6
8
μb (GeV)
local NJL
non-local NJL
1/3 limit

BARYON FLUCTUATIONS

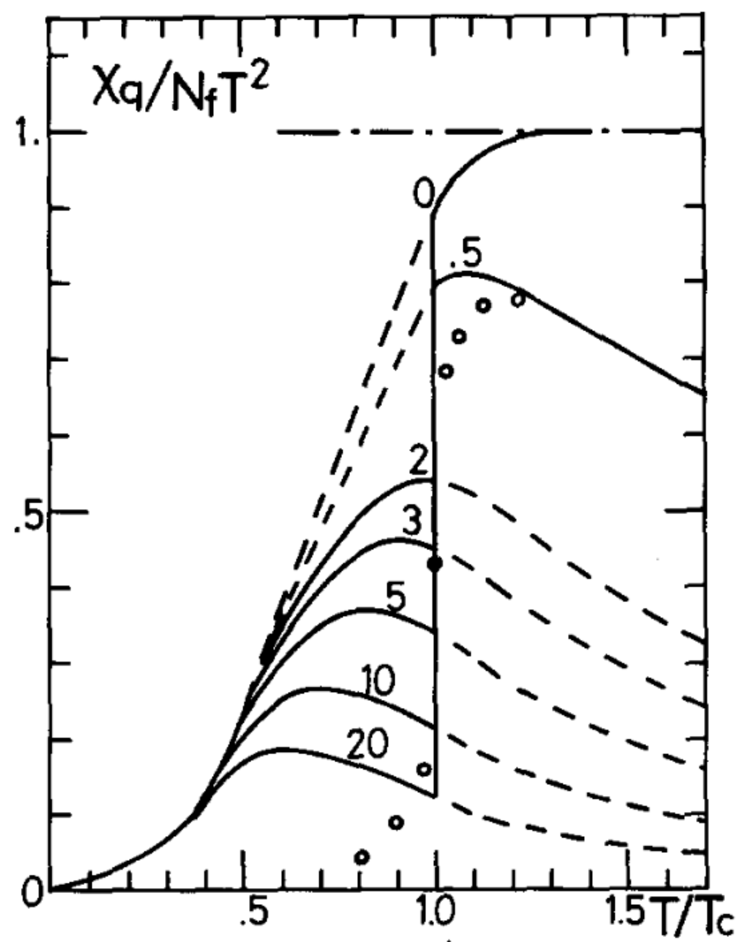


Fig. 1. The temperature dependence of the quark-number susceptibility χ_q in the unit of N_fT^2 with some of the vector coupling $g_V\Lambda^2$: $g_V\Lambda^2=0$, 0.5, 2, 3, 5, 10, 20, which are indicated with the numbers attached to the respective curves. The dash-dotted line shows the free massless case. The small circles are the lattice result on an $8^3\times4$ lattice with the quark mass $m/T=0.2$ [7] compiled in ref. [9].

$$\chi_2 = \frac{\partial n_v}{\partial \mu}$$

Physics Letters B 271 (1991) 395–402
North-Holland

PHYSICS LETTERS B

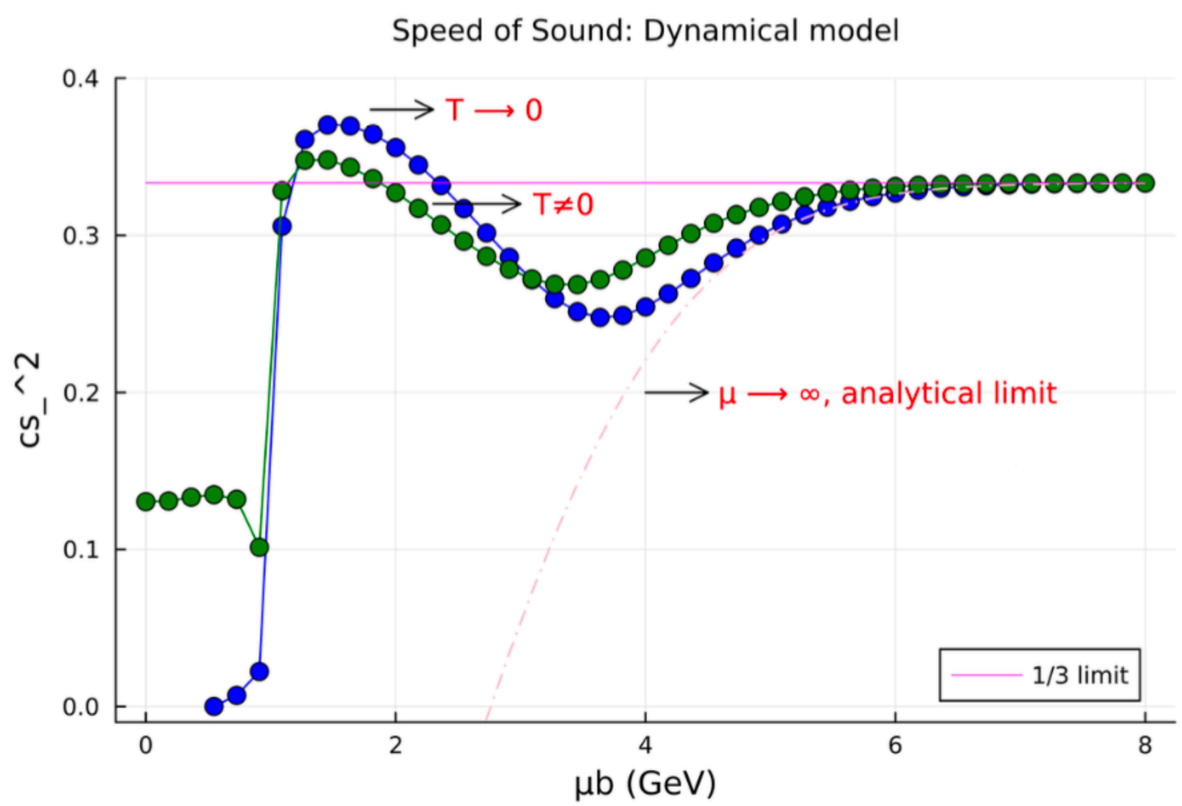
Quark-number susceptibility and fluctuations in the vector channel at high temperatures ☆

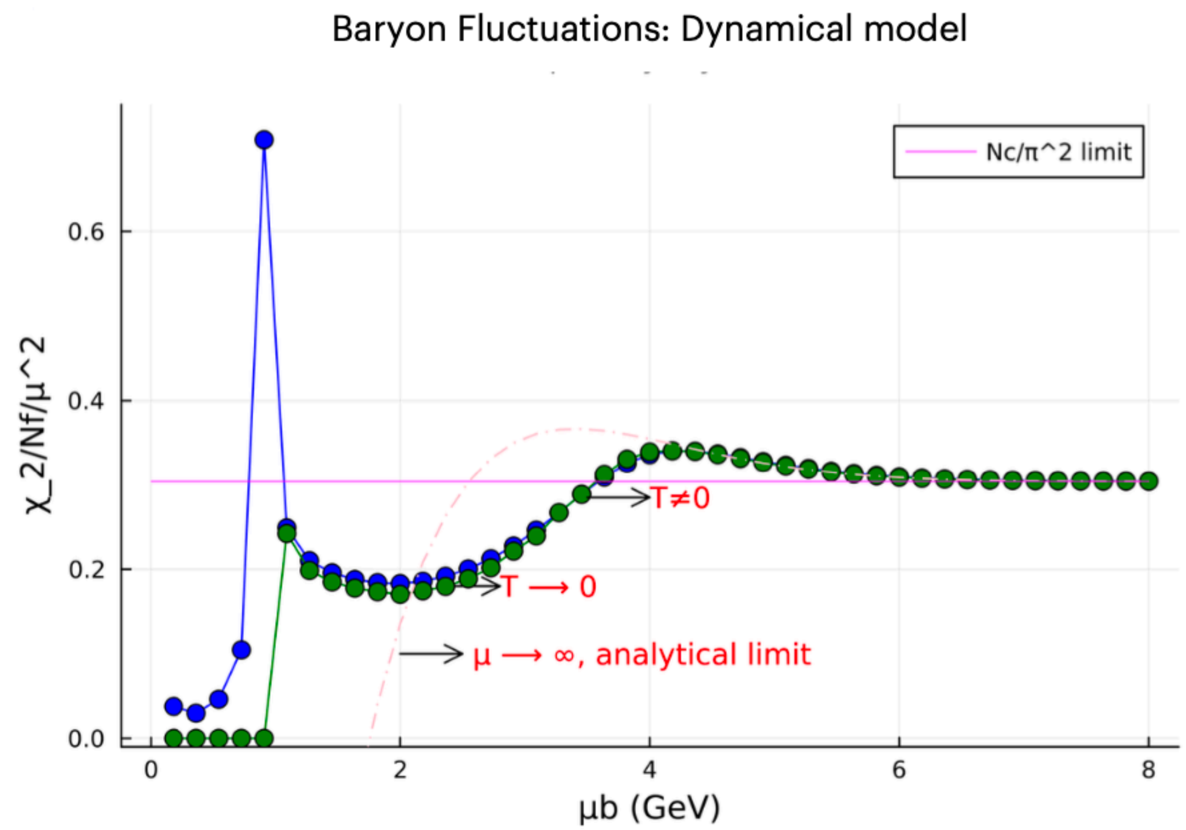
Teiji Kunihiro
Faculty of Science and Technology, Ryukoku University, Seta, Otsu-city 520-21, Japan

Received 9 July 1991; revised manuscript received 10 September 1991

The quark-number susceptibility χ_q is examined as an observable which may help to reveal the physical picture of the high-temperature phase of QCD. It is emphasized that χ_q is intimately related with the fluctuations in the vector channel of the system. It is shown that the results of the recent lattice simulations of χ_q can be understood in terms of a possible change of the interactions between quark and anti-quarks in the vector channel, and imply that the fluctuations in the vector channel is greatly suppressed in the high-temperature phase in contrast with those in the scalar and pseudo-scalar ones.

SPEED OF SOUND V/S BARYON FLUCTUATIONS

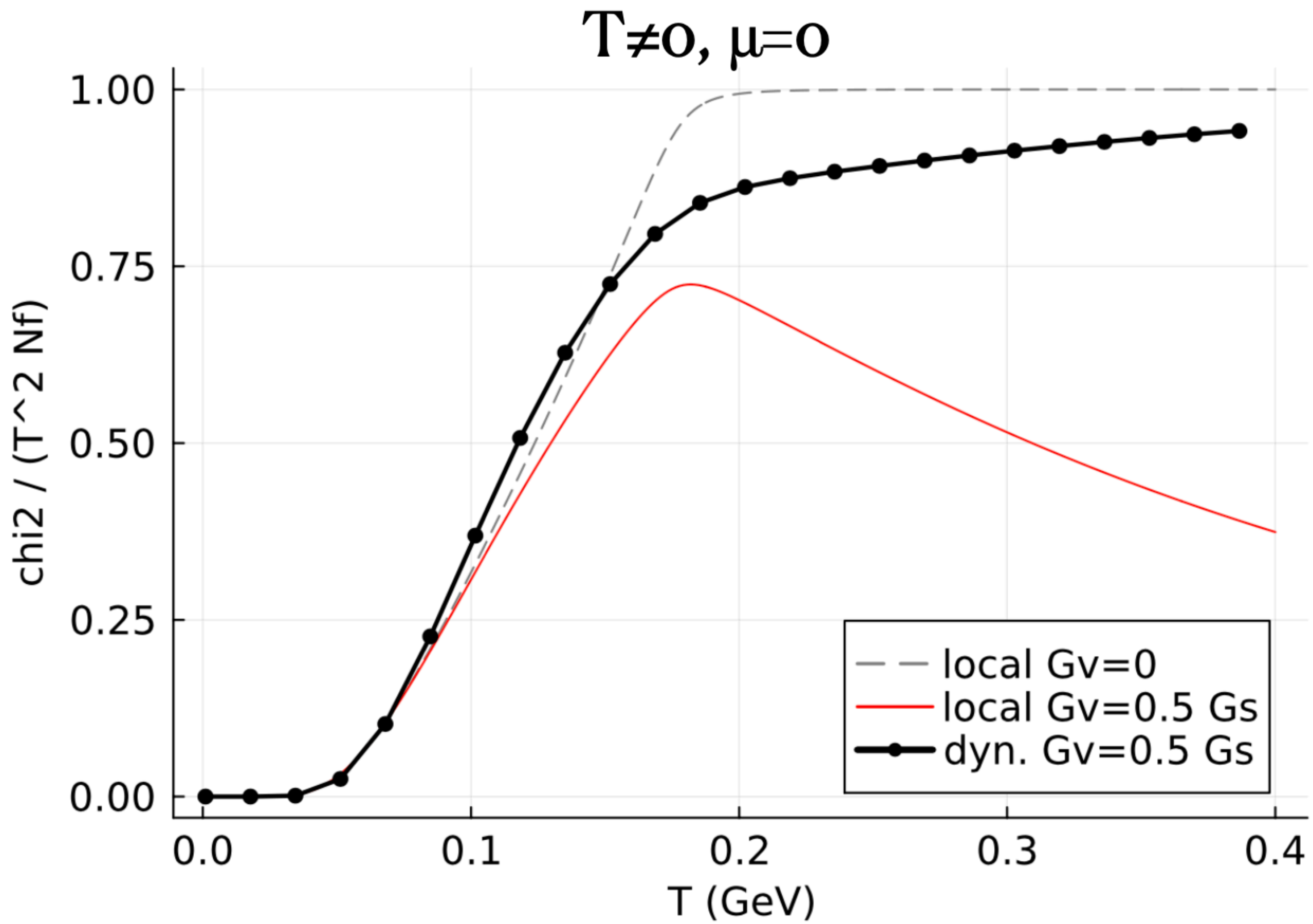




$$cs^2_{\mu\to\infty} \approx \frac{1}{3}\left[1 - \frac{\omega_\infty}{\mu}\left(1+\frac{2\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

$$\chi 2_{\mu\to\infty} \approx \frac{N_c N_f}{\pi^2}\mu^2\left[1 - \frac{2\omega_\infty}{\mu}\left(1-\frac{\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

BARYON FLUCTUATIONS: DYNAMICAL MODEL



local

$$\chi_2 = \frac{dn_v}{d\mu} \propto \frac{T^2}{1 + CT^2}$$

dynamical

$$\chi_2 = \frac{dn_v}{d\mu} \propto T^2$$

CONFINEMENT

QUARK SDE

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{B(q)}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

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$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 + B(p)^2}$$

$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q)-\mu'(q))}+1}.$$

Alkofer et al. A->1 in thermal

Conf. via:
A -> Infinity
thermal weights -> 0;
non-sense!

QUARK SDE

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

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thermal weights -> 0;
non-sense!

Alkofer et al. A->1 in thermal

$A \to 1$ *in* $n, \bar{n}$

R. Alkofer, P. A. Amundsen, K. Langfeld
Z. Phys. C 42 199-208 (1989)

QUARK SDE

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

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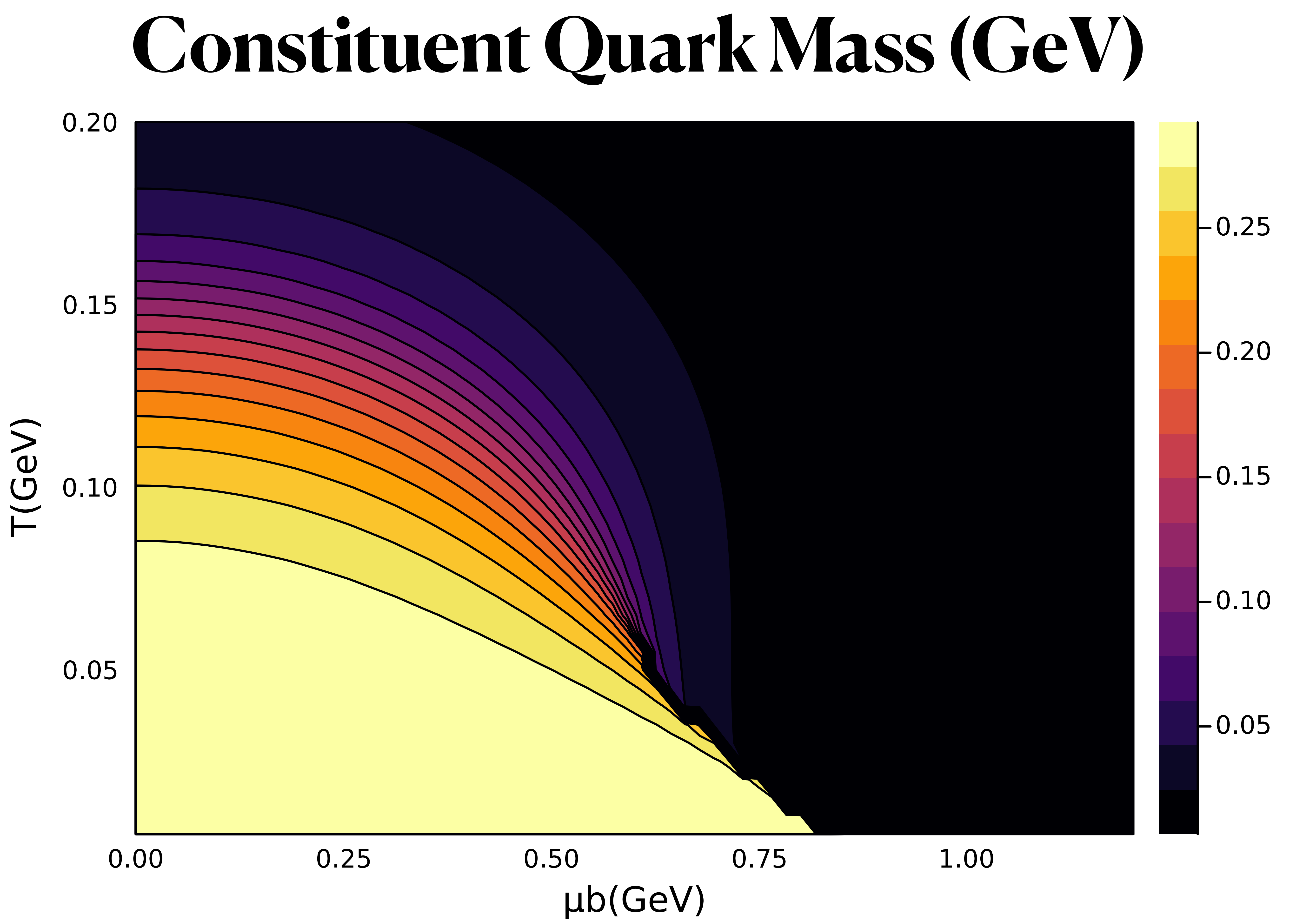
Conf. via:
A -> Infinity
thermal weights -> 0;
non-sense??
Quark Suppression


Constituent Quark Mass (GeV)
0.20
0.15
0.10
0.05
T(GeV)
0.00
0.25
0.50
0.75
1.00
μb(GeV)
0.25
0.20
0.15
0.10
0.05

HEAVY QUARK FREE ENERGY

$$|\langle L\rangle|^2 = e^{-\beta F_{Q\bar{Q}}[r\to\infty,T]}$$

Kaczmarek *et. al.*

$T < T_c$

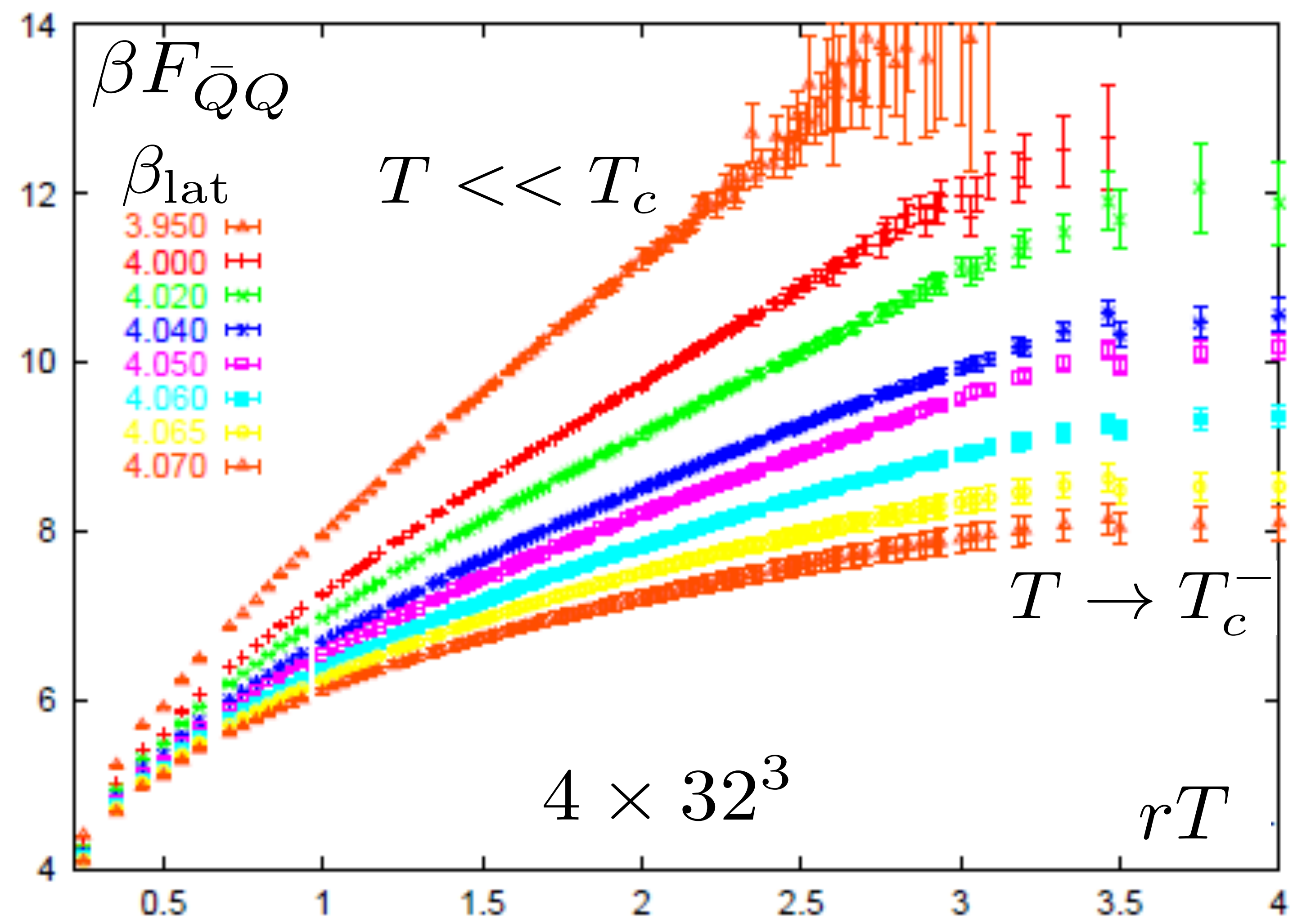
$\langle L\rangle = 0$

confined

$T > T_c$

$\langle L\rangle \neq 0$

deconfined



HEAVY QUARK FREE ENERGY

$$|\langle L\rangle|^2 = e^{-\beta F_{Q\bar{Q}}[r\to\infty,T]}$$

IR-regulated potential

$$V(r) - V(0) = \frac{-\sigma}{\Lambda_{IR}}[e^{-r\Lambda_{IR}} - 1]$$

$T < T_c$

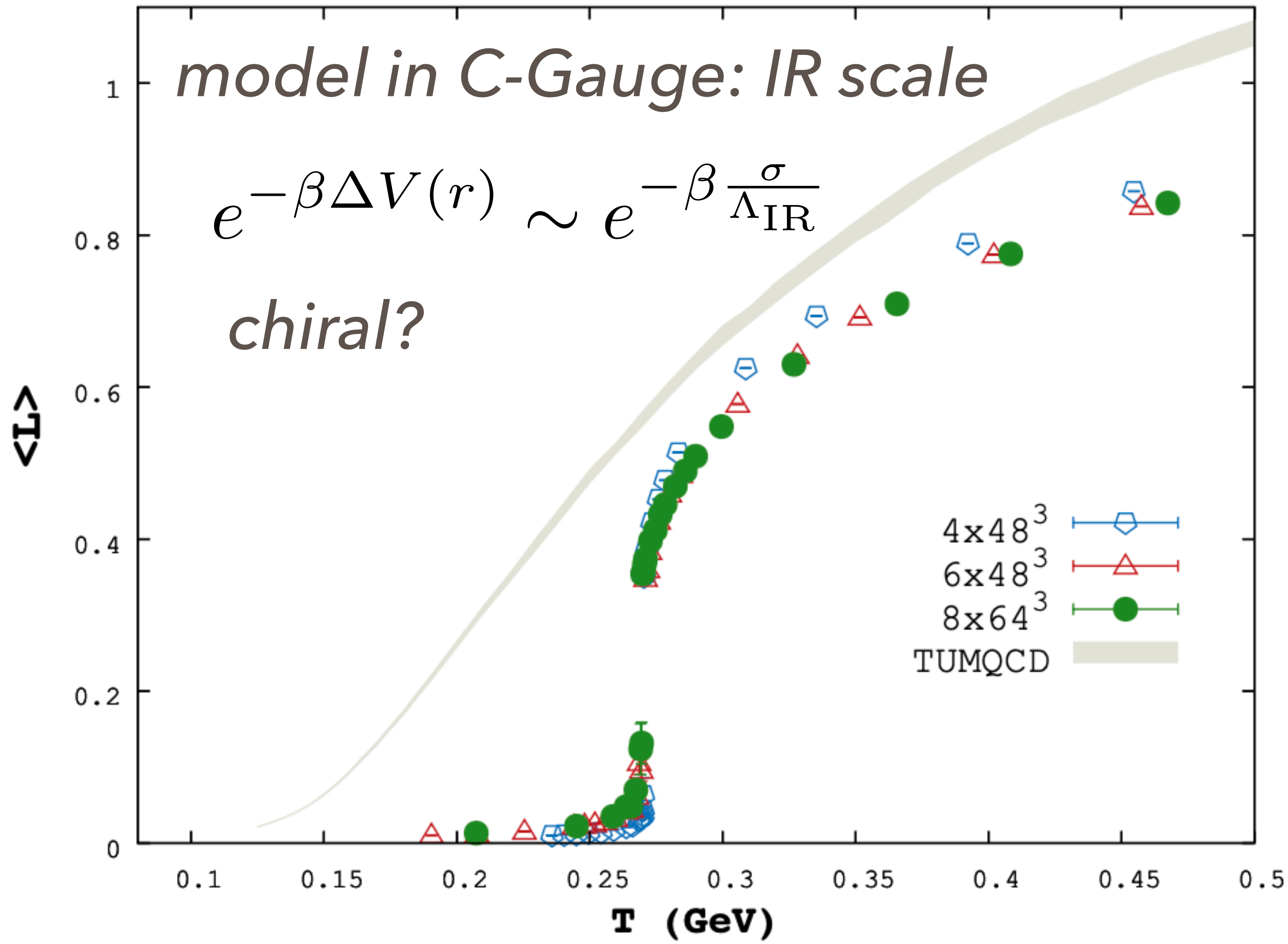
$\langle L\rangle = 0$

confined

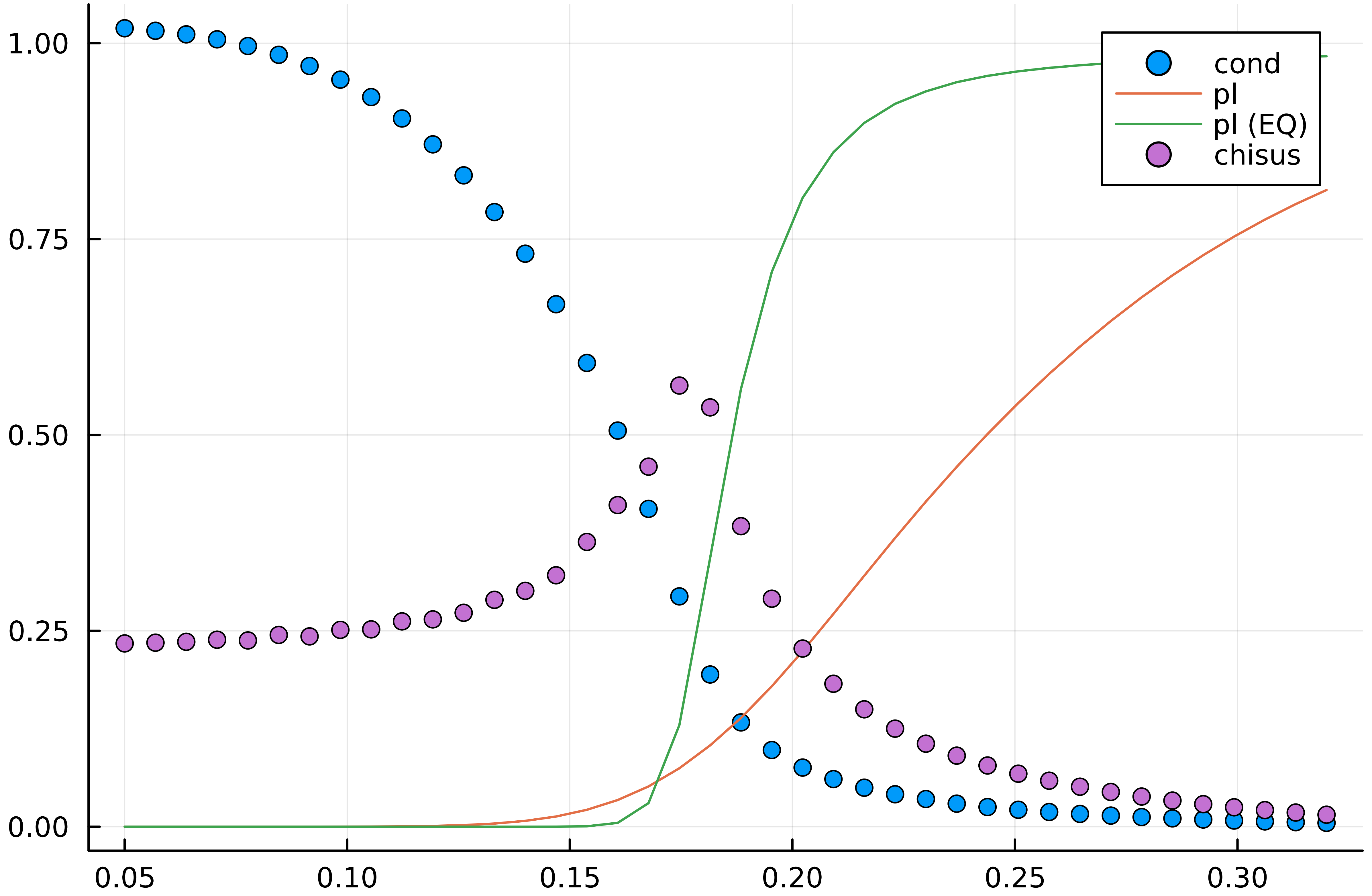
$T > T_c$

$\langle L\rangle \neq 0$

deconfined



new model *illustration*



SUMMARY & CONCLUSIONS

- Dynamical interaction —> asymptotic freedom —> cs^2 and χ_2 receive essential, quantifiable contribution (from dynamical interaction).
- Towards a first attempt to determine phase diagram of Coulomb gauge QCD
- Fixing vacuum naturally leads to a Tc ~ 155 MeV —> Alkofer approximation.
- Polyakov loop can be computed as an observable.
- To do: beyond Alkofer, include confinement in dense matter (ring - *Eur.Phys.J.A* 58 (2022) 9, 172).

Thank you for your attention.