New developments in studies of the QCD phase diagram

Strangeness neutrality and QCD phase structure from functional renormalization group

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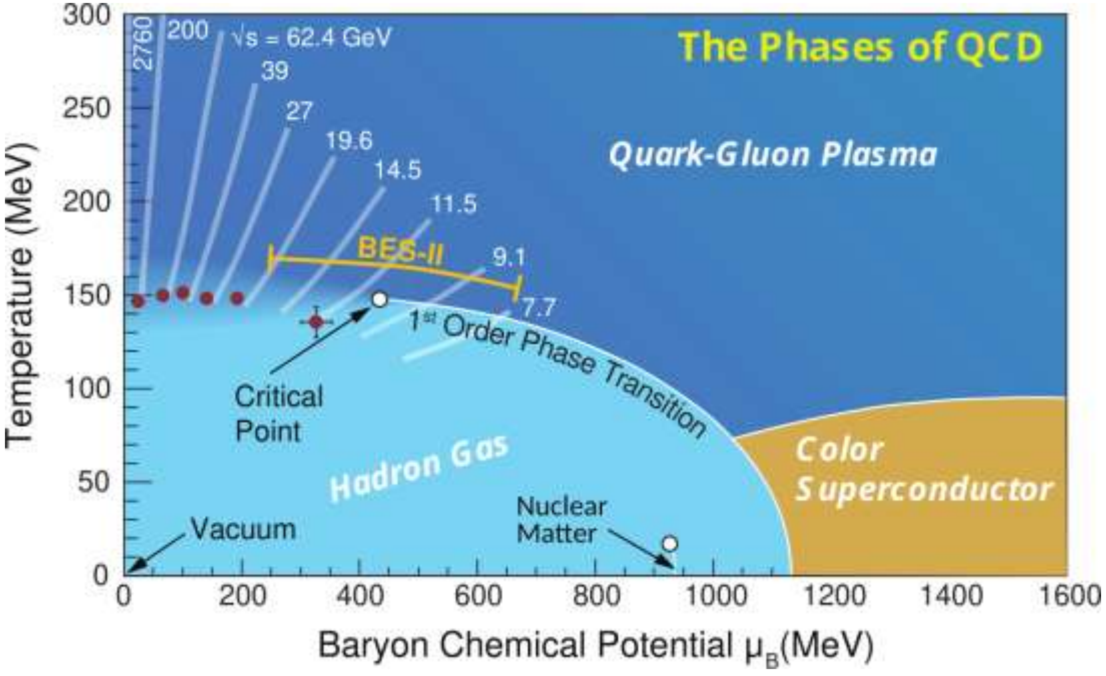
Based on: W.-j. Fu, C. Huang, J. M. Pawlowski, F. Rennecke, R. Wen, S. Yin. (2024) in preparation

fQCD collaboration: J. Braun, Y.-r. Chen, W.-j. Fu, F. Gao, F. Ihssen, A. Geissel, C. Huang,Y. Lu, J. M. Pawlowski, F. Rennecke, F. R. Sattler, B. Schallmo, J. Stoll, Y.-y. Tan, S. T¨opfel, J. Turnwald, R. Wen, J. Wessely, N. Wink, S. Yin, P.-w. Zheng and N. Zorbach, (2024).

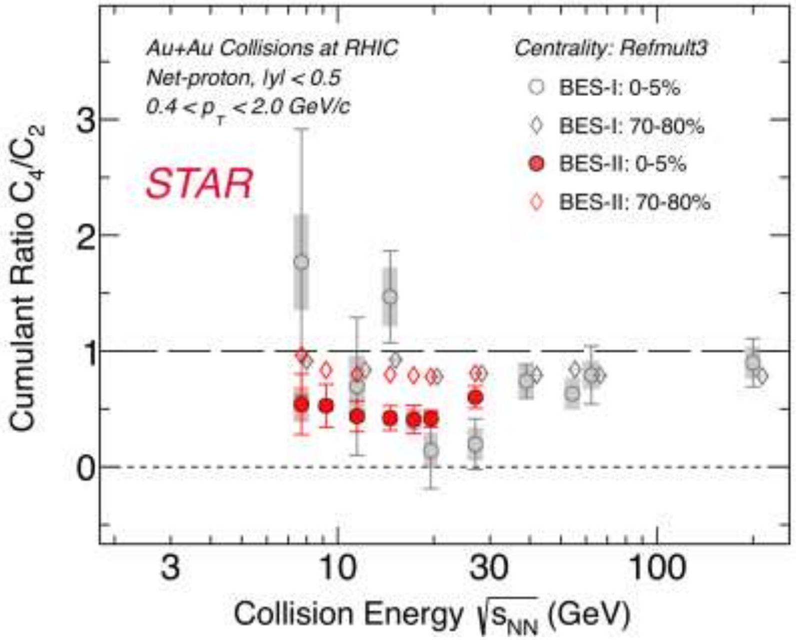
Outline

- Introduction
- The 2+1 flavor QCD Theory within FRG
- Numerical results
- Gluon propagator; quarks and mesons mass; chiral condensates
- QCD phase structure
- B-S Correlation
- Summary

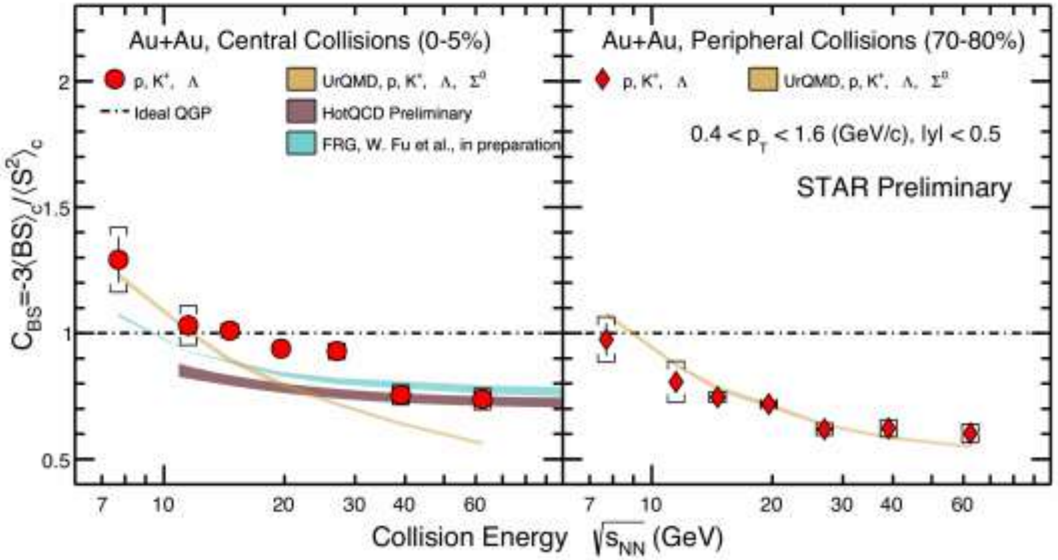
Introduction



Adam Bzdak, Shinichi Esumi et.al. *Phys.Rept.* 853 (2020) 1-87

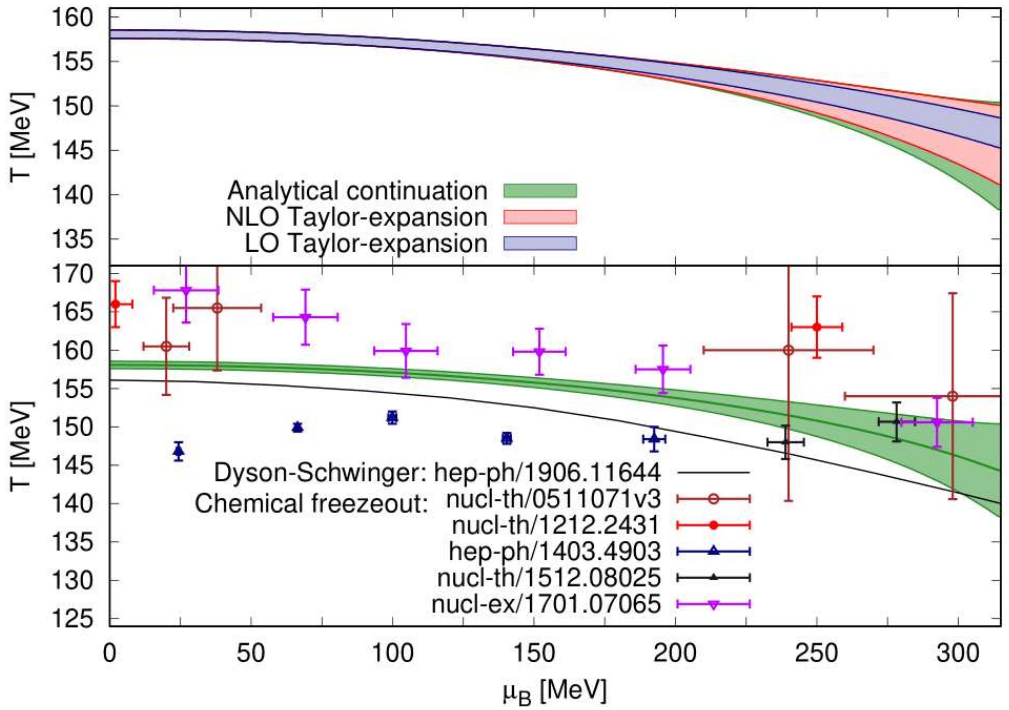


STAR Collaboration, CPOD2024

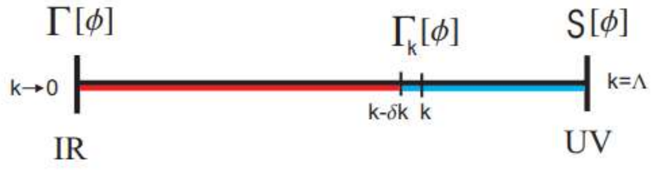


STAR Collaboration, XQCD2024

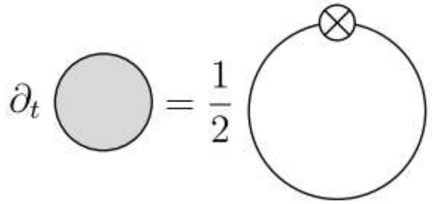
Introduction



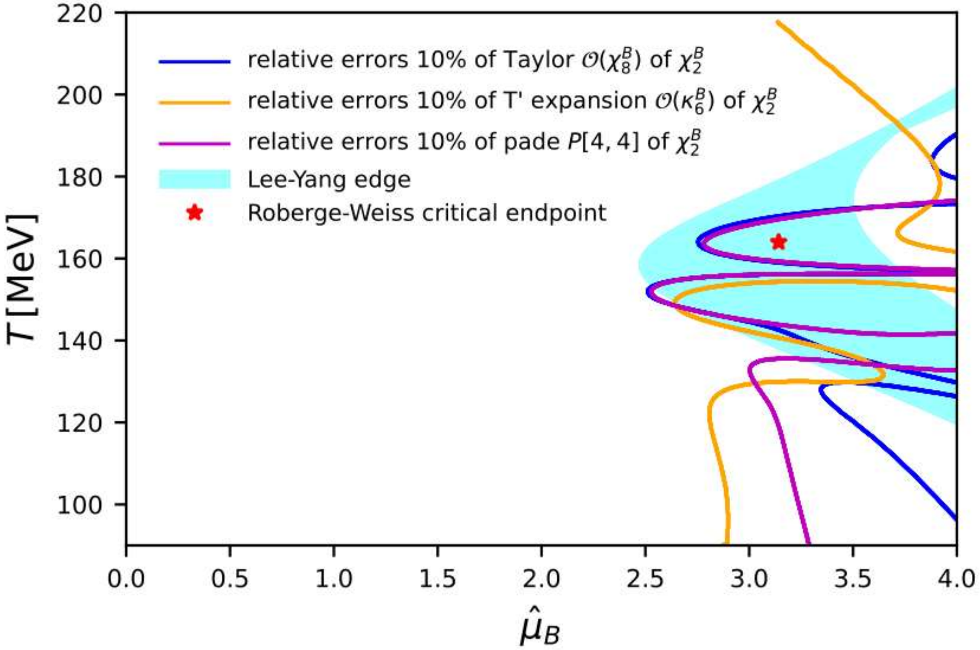
Szabolcs Borsanyi, et. al.
Phys. Rev. Lett. 125, 052001 (2020)



The Wetterich equation

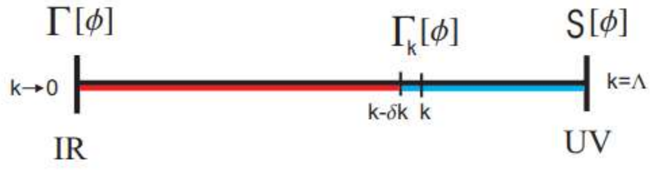


PQM model

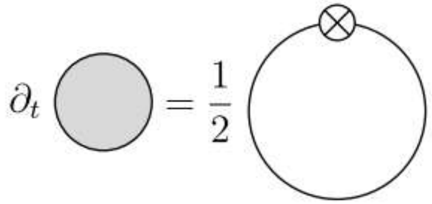


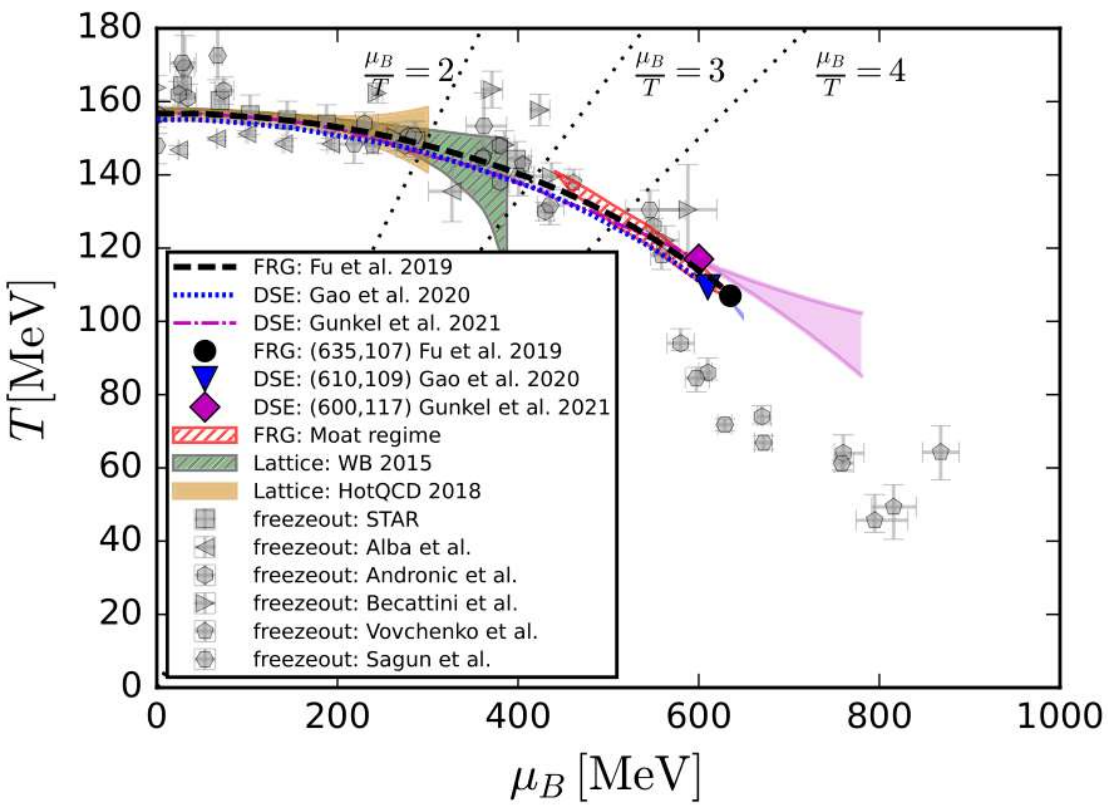
Rui Wen, Shi Yin, Wei-jie Fu. Phys.Rev.D 110 (2024), 016008

Introduction

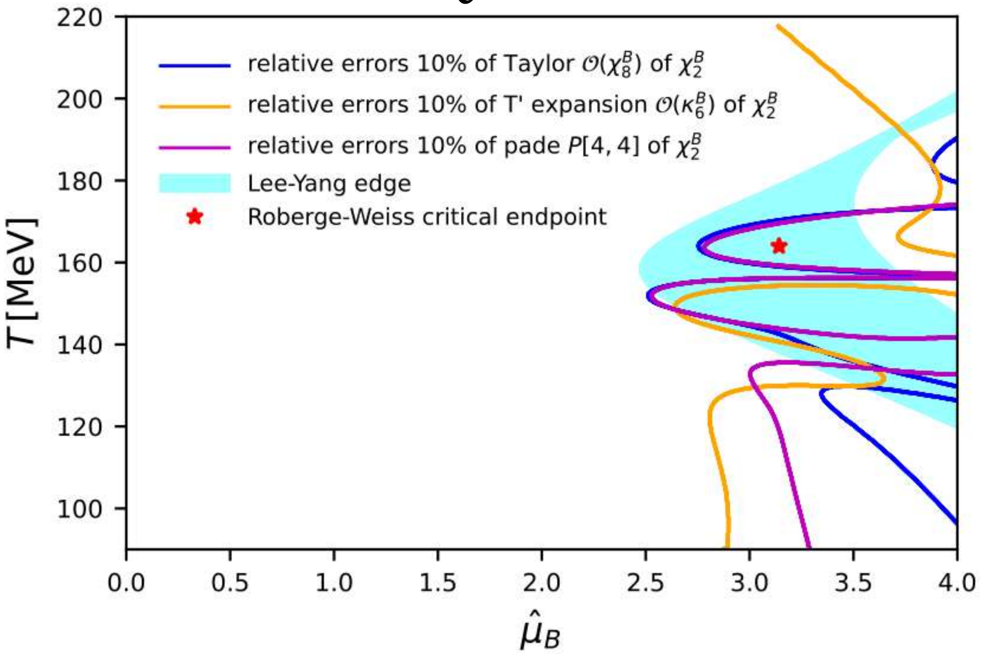


The Wetterich equation





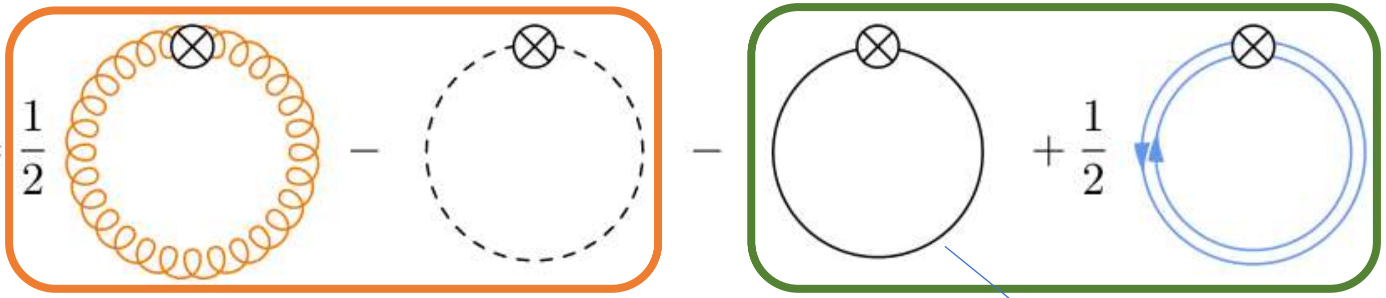
PQM model



Rui Wen, Shi Yin, Wei-jie Fu. Phys.Rev.D 110 (2024), 016008

The 2+1 flavor QCD Theory within FRG

$\partial_t \Gamma_k[\Phi] =$



$\frac{1}{2}$ (glue loop) $-$ (ghost loop) $-$ (quark loop) $+\frac{1}{2}$ (meson loop)

Glue sector

Matter sector

π, σ → π, K, η, η' a_0, σ, κ, f_0

$q = (l, l, s)^T$

$$\Gamma_k = \int_x \Big\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + Z_c\,(\partial_\mu \bar{c}^a) D^{ab}_\mu c^b + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2$$
$$+ \frac{1}{2}\int_p A^a_\mu(-p)\left(\Gamma^{(2)\,ab}_{AA\mu\nu}(p) - Z_A \Pi^\perp_{\mu\nu}\delta^{ab}p^2\right) A^b_\nu(p)$$
$$+ \bar{q}\left[Z_q\,(\gamma_\mu D_\mu - \gamma_0 \hat{\mu}) + m_s(\sigma_s)\right] q$$
$$- \lambda_q \left[(\bar{q}\,\tau^0 q)^2 + (\bar{q}\,\boldsymbol{\tau} q)^2\right] + h\,\bar{q}\,\left(\tau^0\sigma + \boldsymbol{\tau}\cdot\boldsymbol{\pi}\right) q$$
$$+ \frac{1}{2} Z_\phi\,(\partial_\mu\phi)^2 + V_k(\rho, A_0) - c_\sigma\,\sigma - \frac{1}{\sqrt{2}}\,c_{\sigma_s}\,\sigma_s \Big\},$$

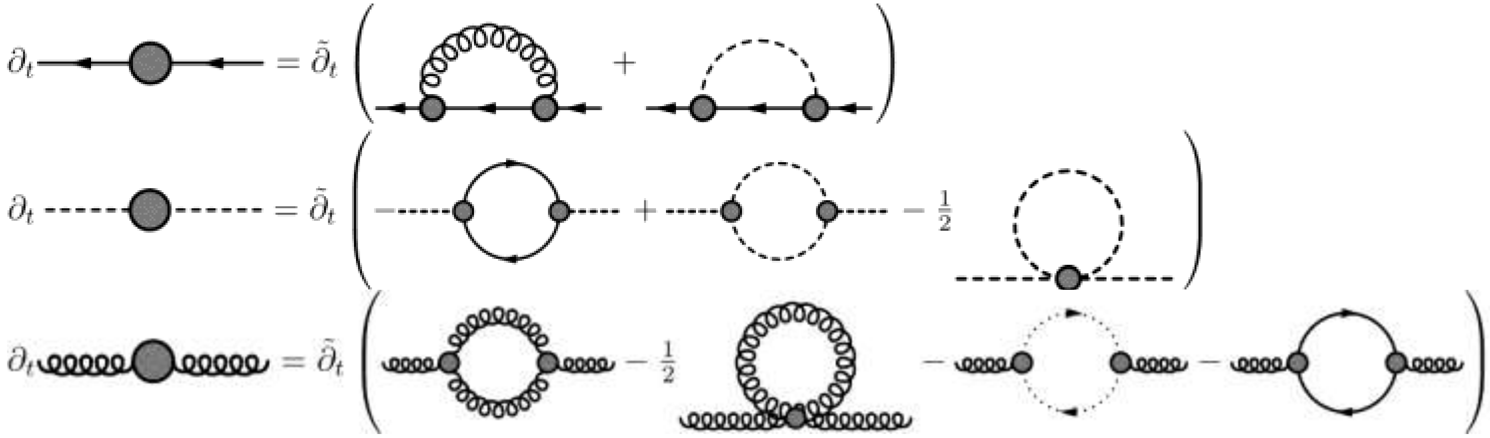
⇒

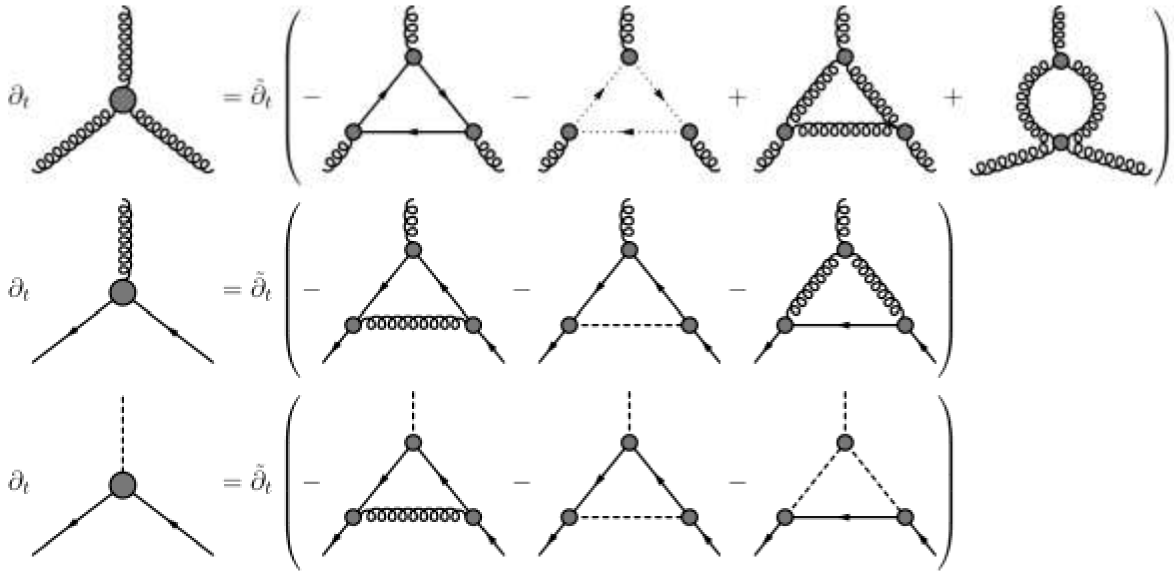
$$+ \bar{q}[Z_q(\gamma_\mu D_\mu - \gamma_0(\hat{\mu} + igA_0)]q$$
$$- \lambda_q \sum_{a=0}^{8}[(\bar{q}T_a q)^2 + (\bar{q} i\gamma_5 T_a q)^2]$$
$$+ \bar{q} h^{1/2}\cdot\Sigma_5\cdot h^{1/2} q + tr\left(Z_\Sigma^{1/2}\cdot \bar{D}_\mu\Sigma\cdot Z_\Sigma^{1/2}\cdot\bar{D}_\mu\Sigma^\dagger\right)$$
$$+ \tilde{U}_k(\Sigma, \Sigma^\dagger) + V_{glue}(L, \bar{L})\Big\},$$

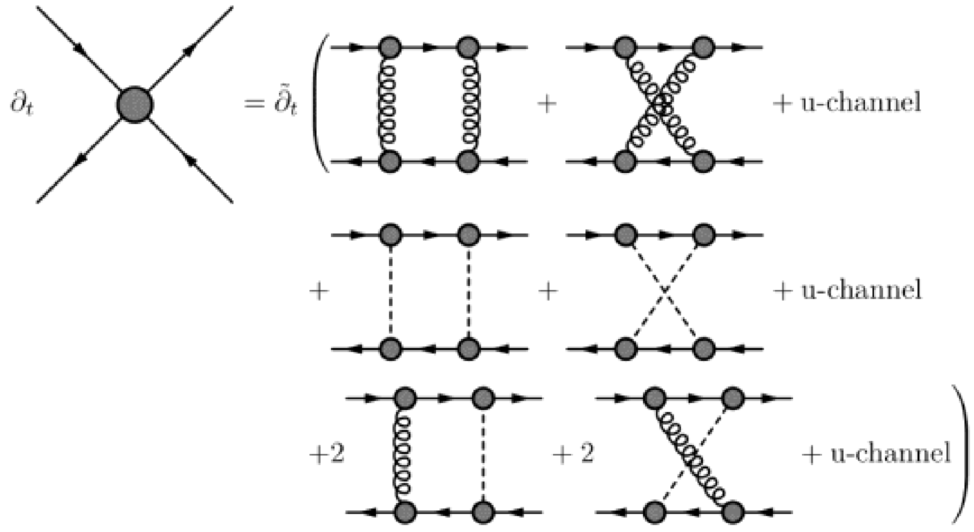
$$\tilde{U}_k(\Sigma,\Sigma^\dagger) = U_k(\{\tilde{\rho}_i\}) - c_A\xi - c_l\sigma_l - c_s\sigma_s/\sqrt{2}$$
$$\xi = \det(\Sigma) + \det(\Sigma^\dagger).$$

W-j Fu, J.M. Pawlowski, F. Rennecke, *Phys.Rev.D* 101 (2020) 5, 054032

The flow equations

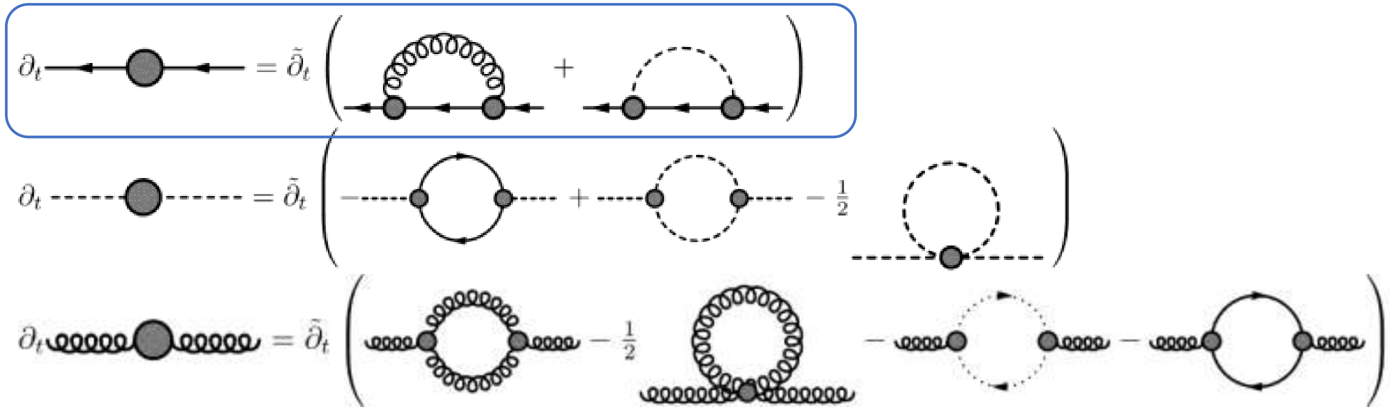




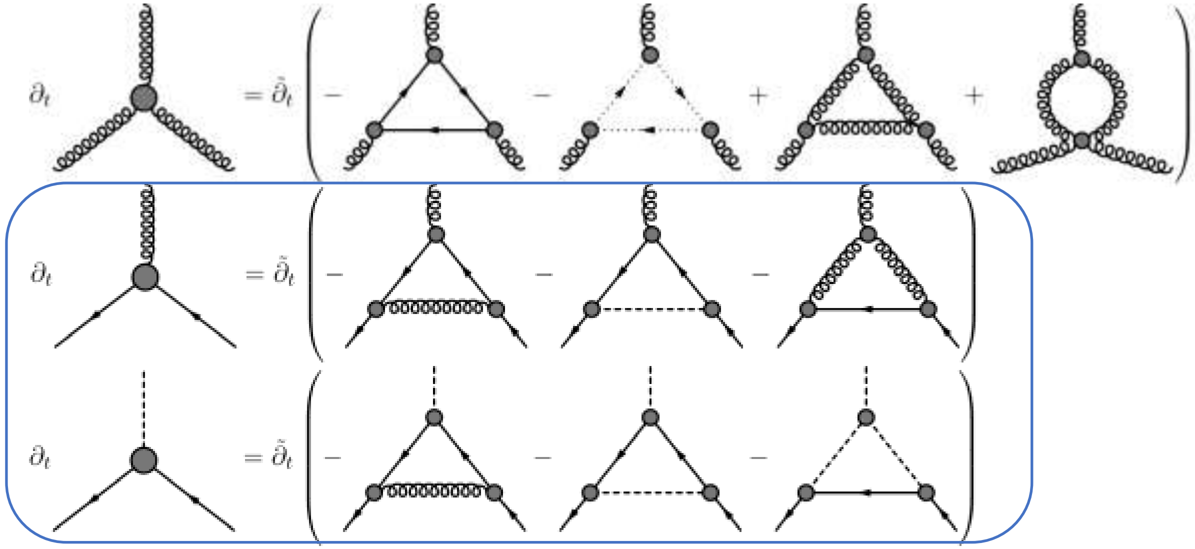


The flow equations

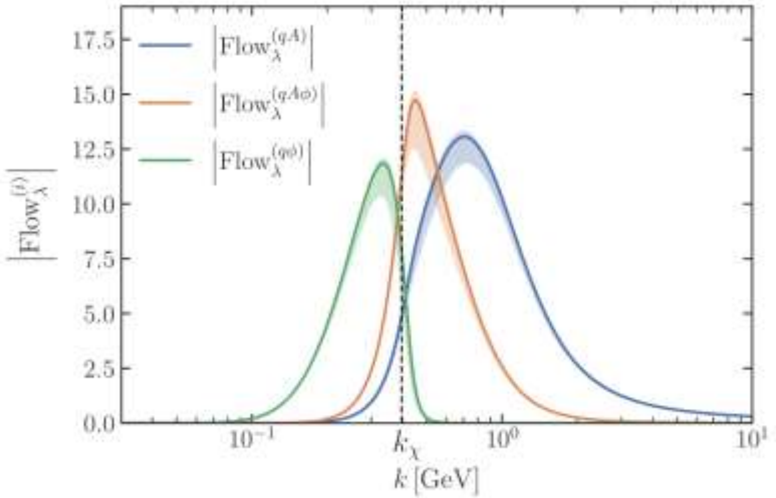
propagators :



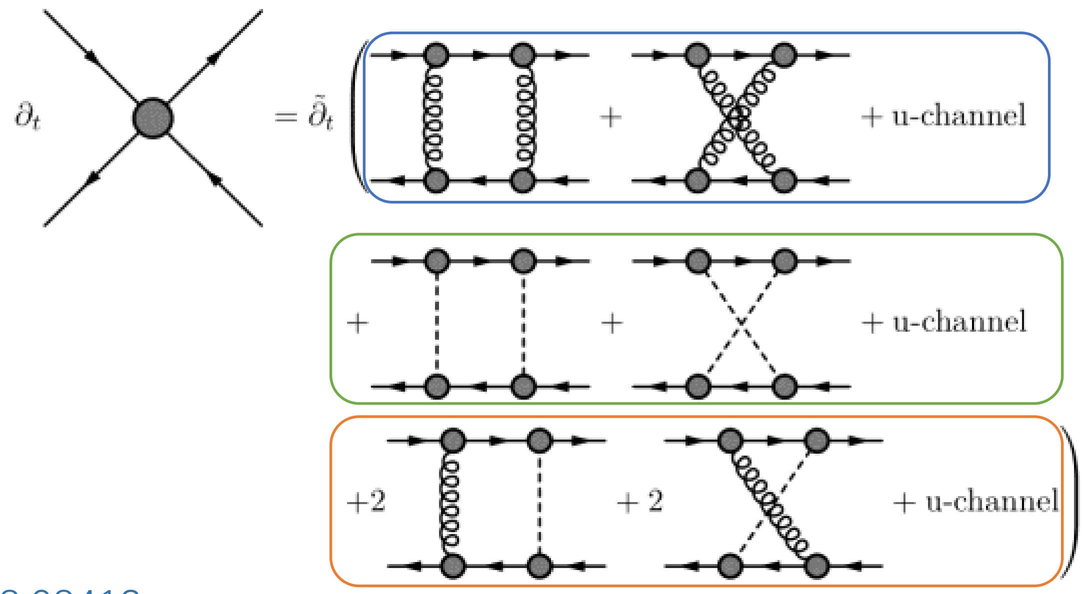
vertices:



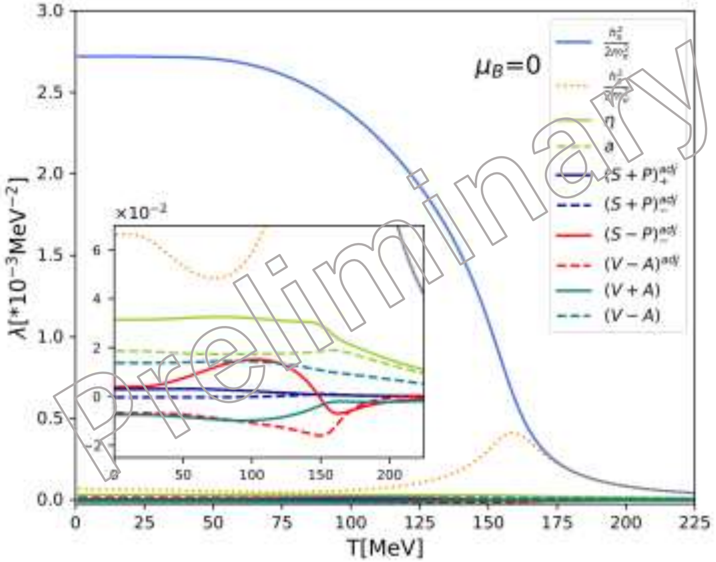
Four-quark vertex



F. Ihssen, J. M. Pawlowski, F. R. Sattler, N. Wink, arXiv:2408.08413



Fierz-complete

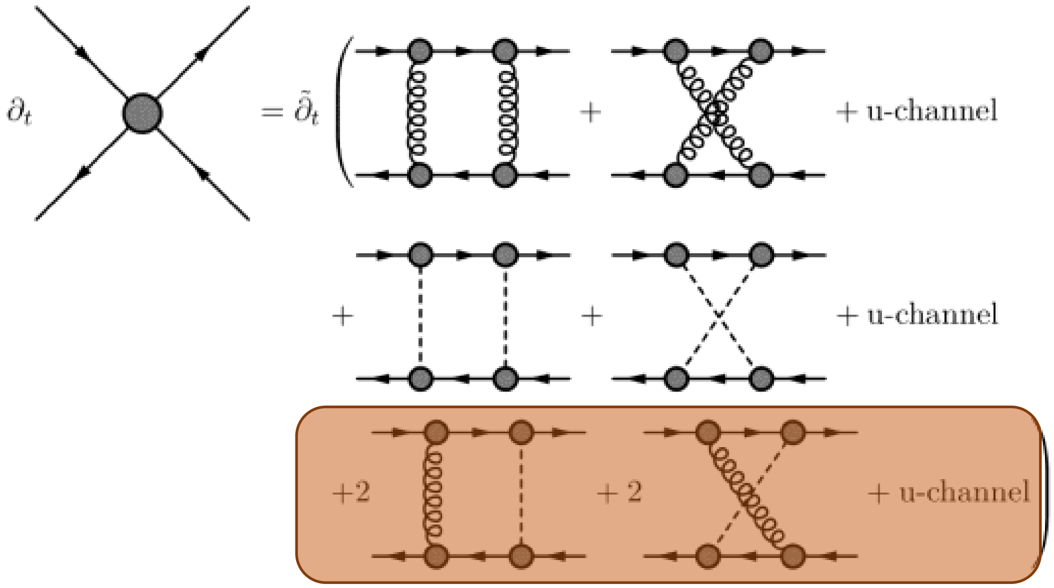


Z.-n. Wang, C. Huang, R. Wen, S. Yin, W.-j. Fu, (2024) in preparation

Ignore the mixed quark-gluon-meson box

The scalar and pseudoscalar channel

Dynamical hadronization



The scale dependent meson fields:

$$\langle \partial_t \hat{\phi}_k \rangle = [(\bar{q}\dot{\boldsymbol{A}}_k^{\frac{1}{2}} T_a \dot{\boldsymbol{A}}_k^{\frac{1}{2}} q) + (\bar{q}\dot{\boldsymbol{A}}_k^{\frac{1}{2}} i\gamma_5 T_a \dot{\boldsymbol{A}}_k^{\frac{1}{2}} q)] + \dot{B}_k \Sigma$$

$$\dot{\boldsymbol{A}}_k = \begin{pmatrix} \dot{A}_{l,k} & 0 & 0 \\ 0 & \dot{A}_{l,k} & 0 \\ 0 & 0 & \dot{A}_{s,k} \end{pmatrix}$$

The Wetterich equation with dynamical hadronization:

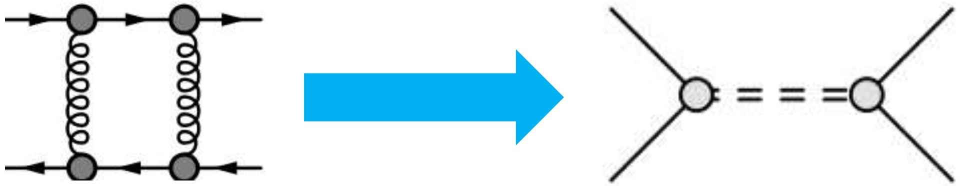
$$\partial_t \Gamma_k[\Phi] + \int \langle \partial_t \hat{\phi}_{k,i} \rangle \left(\frac{\delta \Gamma_k[\Phi]}{\delta \phi_i} + c_\sigma \delta_{i\,\sigma} \right)$$

$$= \frac{1}{2} \mathrm{Tr}\big(G_k[\Phi]\, \partial_t R_k\big) + \mathrm{Tr}\left(G_{\phi\Phi_j}[\Phi] \frac{\delta \langle \partial_t \hat{\phi}_k \rangle}{\delta \Phi_j} R_\phi \right)$$

With the fully hadronized condition:

$$\lambda_q \equiv 0, \quad \forall k$$

We get the hadronization functions :

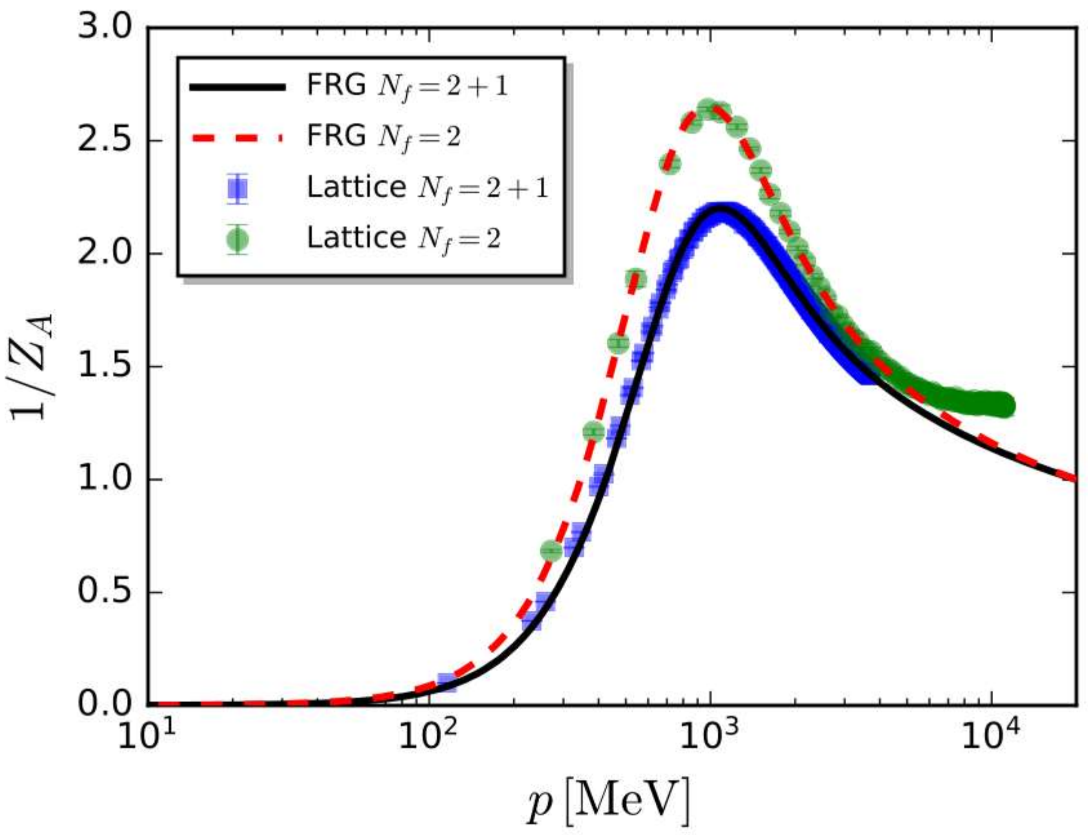


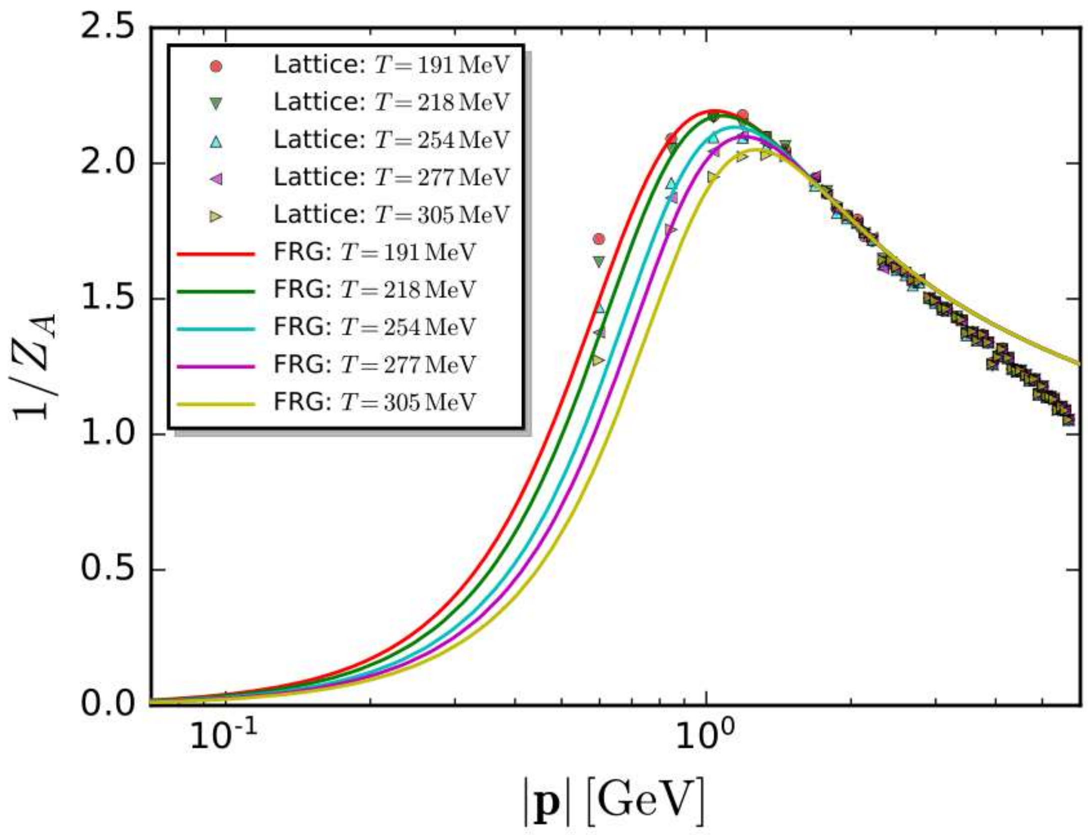
Gies, Wetterich , PRD 65 (2002) 065001; 69 (2004) 025001
Pawlowski, AP 322 (2007) 2831
Flörchinger, Wetterich, PLB 680 (2009) 371

$$\dot{A}_{l,k} = -\frac{1}{h_{l,k}} \mathrm{Flow}^{(4)}_{(\bar{q}T^L q)(\bar{q}T^L q)} \qquad \partial_t h_{l,k} = -\frac{1}{\sigma_l} \frac{\delta \tilde{U}(\Sigma)}{\delta \sigma_l} \dot{A}_{l,k} + \frac{1}{\sigma_l} \mathrm{Re}(\mathrm{Flow}^{(2)}_{(\bar{q}T^l q)}),$$

$$\dot{A}_{s,k} = -\frac{1}{h_{s,k}} \mathrm{Flow}^{(4)}_{(\bar{q}T^S q)(\bar{q}T^S q)} \qquad \partial_t h_{s,k} = -\frac{1}{\sigma_s} \frac{\delta \tilde{U}(\Sigma)}{\delta \sigma_s} \dot{A}_{s,k} + \frac{1}{\sigma_s} \mathrm{Re}(\mathrm{Flow}^{(2)}_{(\bar{q}T^s q)}).$$

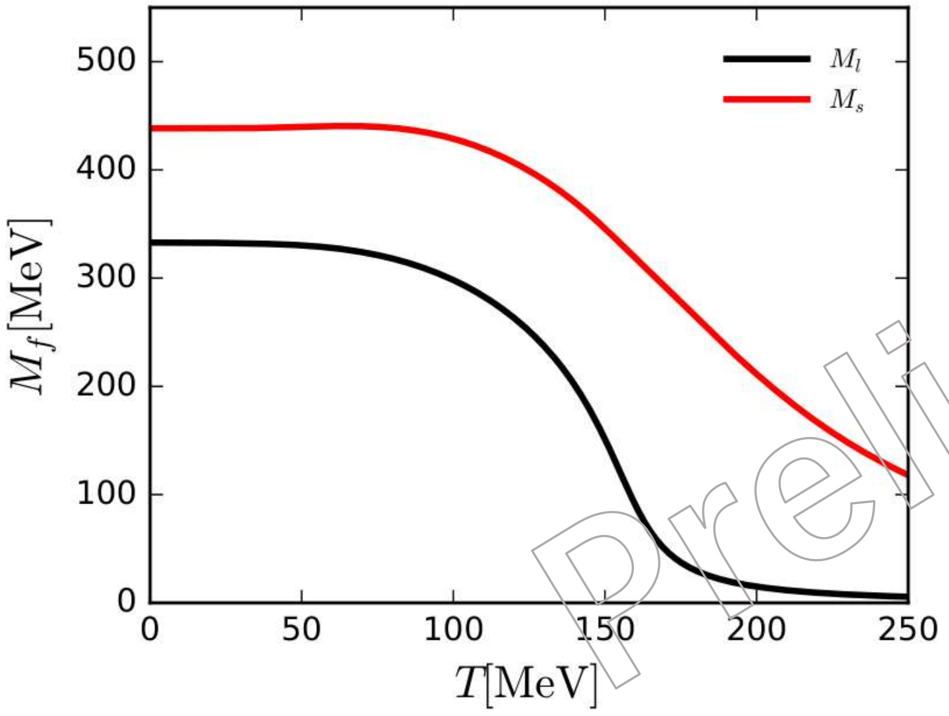
The Gluon Propagator at T=0 and Finite T

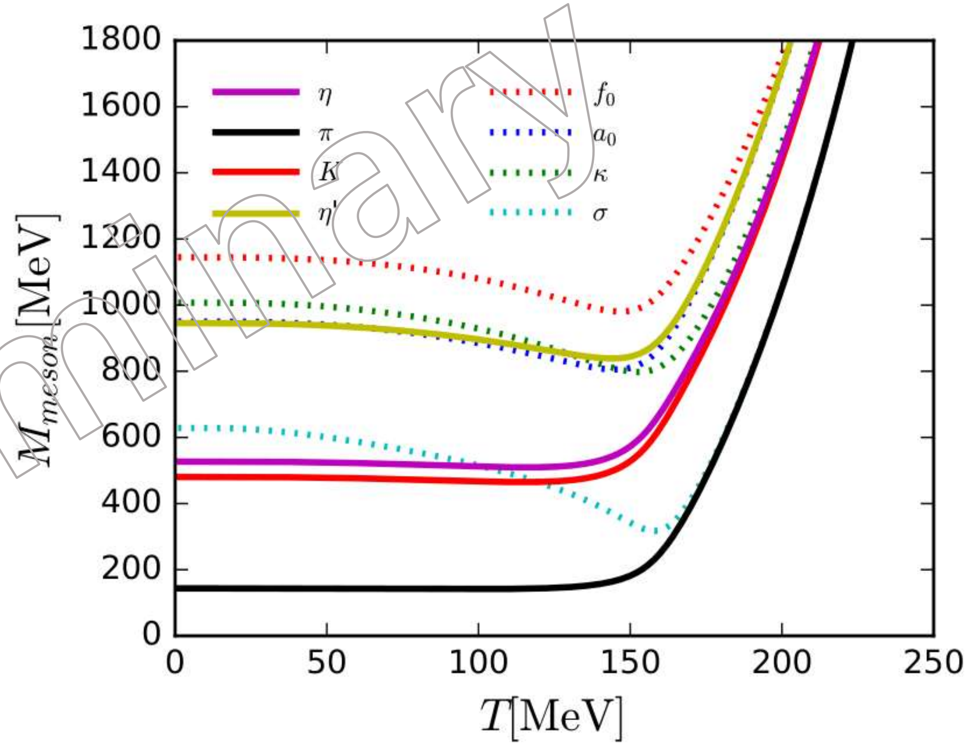




W-j Fu, J.M. Pawlowski, F. Rennecke, Phys.Rev.D 101 (2020) 5, 054032

Quarks and Mesons Masses





$\Lambda = 20\text{GeV}$
$m_u = m_d = m_l$
$m_{s,k=\Lambda}/m_{l,k=\Lambda} = 27.4$

$m_{l,k=IR} = 333$ MeV
$m_{s,k=IR} = 438$ MeV

$m_\pi = 142$ MeV
$m_K = 481$ MeV

$m_\sigma = 628$ MeV
$m_\eta = 527$ MeV
$m_{\eta'} = 946$ MeV

The mixing angles between light-strange (LS) basis and physical basis

Hessian matrix

$$H_{ij} = \frac{\partial^2 \tilde{U}_k(\Sigma, \Sigma^\dagger)}{\partial \phi_i \partial \phi_j}$$

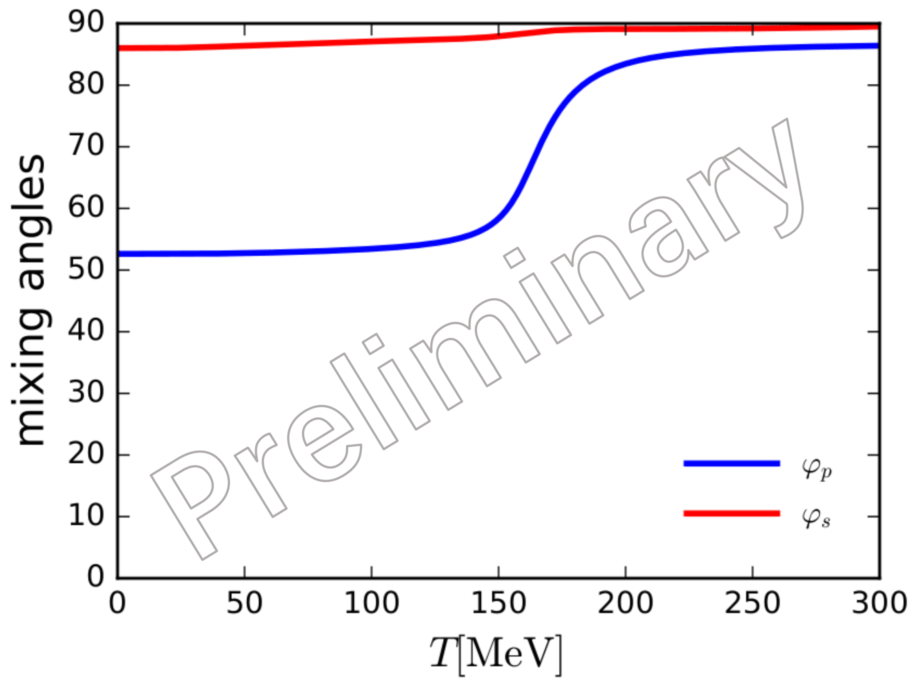
Because the nonvanishing nondiagonal element $H_{s/p,ls}$

$$\begin{pmatrix} f_0 \\ \sigma \end{pmatrix} = \begin{pmatrix} \cos\varphi_s & -\sin\varphi_s \\ \sin\varphi_s & \cos\varphi_s \end{pmatrix} \begin{pmatrix} \sigma_l \\ \sigma_s \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\varphi_p & -\sin\varphi_p \\ \sin\varphi_p & \cos\varphi_p \end{pmatrix} \begin{pmatrix} \eta_l \\ \eta_s \end{pmatrix}$$

The mixing angles

$$\varphi_{s/p} = \frac{1}{2}\arctan\left(\frac{2H_{s/p,ls}}{H_{s/p,ss} - H_{s/p,ll}}\right)$$

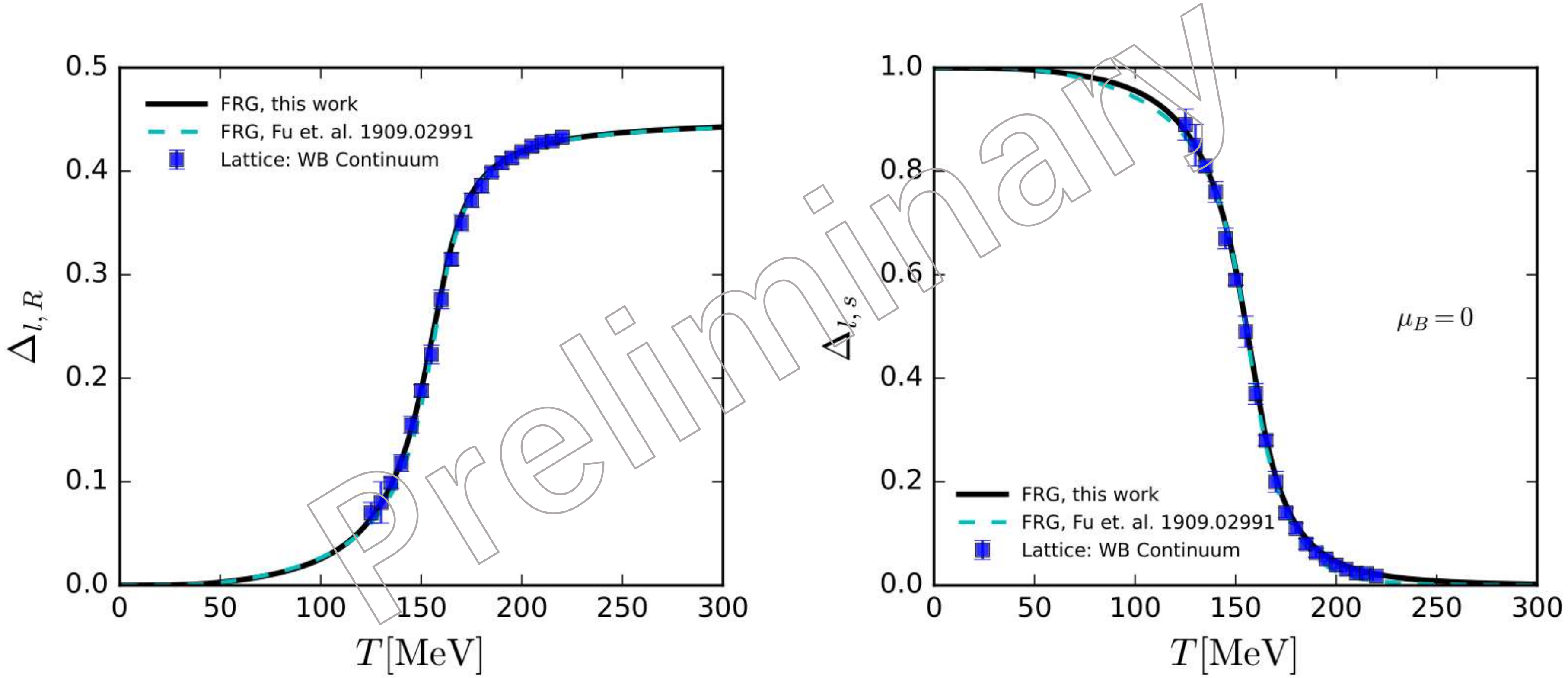


The light and reduced chiral condensate

$$\Delta_{q_i} = m_{q_i}^0 \frac{T}{\mathcal{V}} \int_x \langle \bar{q}_i(x) q_i(x) \rangle$$

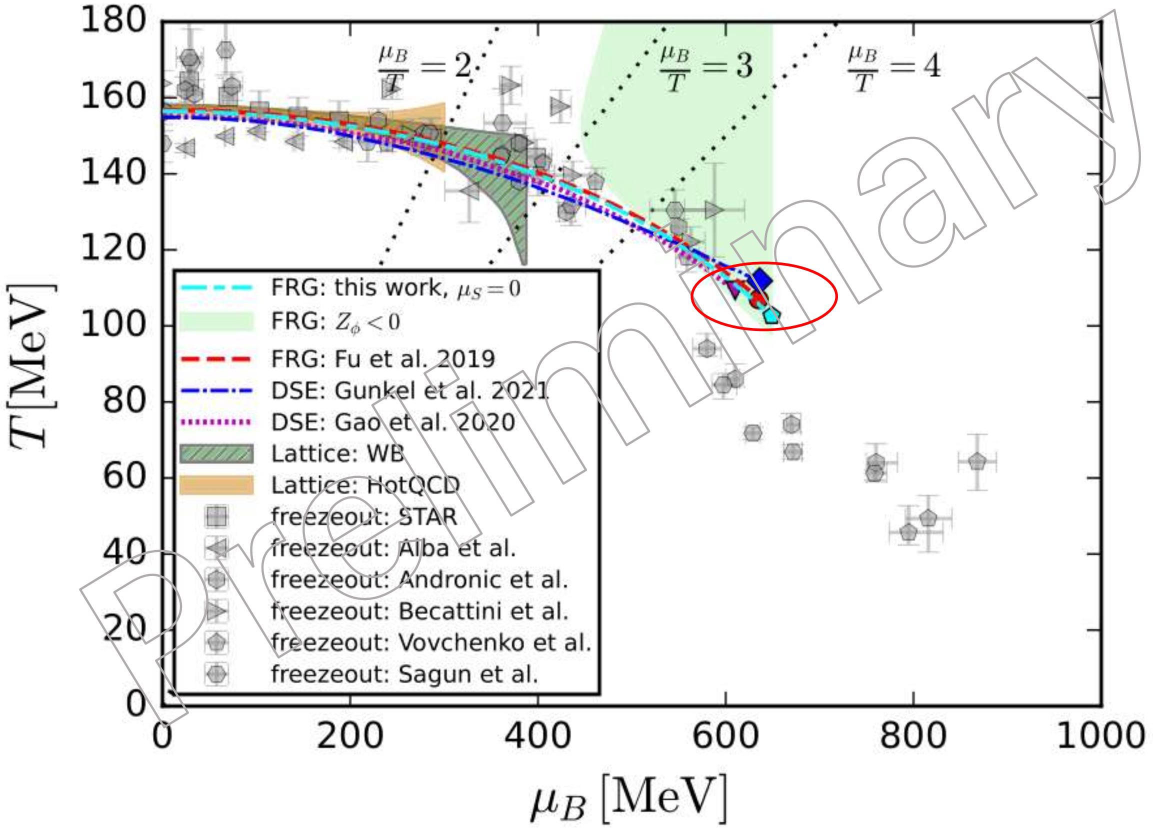
$$\Delta_{q_i,R} = \frac{1}{\mathcal{N}_R} [\Delta_{q_i}(T,\mu_q) - \Delta_{q_i}(0,0)]$$

$$\Delta_{l,s} \equiv \frac{\Delta_l(T,\mu_B) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_B)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)} = \frac{\sigma_l(T,\mu_B) - (\frac{\sqrt{2}c_l}{c_s})\sigma_s(T,\mu_B)}{\sigma_l(0,0) - (\frac{\sqrt{2}c_l}{c_s})\sigma_s(0,0)}.$$



Wuppertal-Budapest Collaboration: JHEP 09 (2010) 073

Phase diagram with $\mu_s = 0$



$T_{pc} = 156\ MeV$

$$\frac{T_c(\mu_B)}{T_c(\mu_B = 0)} = 1 - \kappa_2 \left[\frac{\mu_B}{T_c(\mu_B)}\right]^2 - \kappa_4 \left[\frac{\mu_B}{T_c(\mu_B)}\right]^4 \ldots$$

$\kappa_2(\mu_s = 0) = 0.0148(2)$

$\kappa_2 = 0.015(1)$

H. T. Ding, et.al. arXiv:2403.09390

$\kappa_2 = 0.0153 \pm 0.0018$

S. Borsanyi, et.al. Phys. Rev. Lett. 125, 052001 (2020),

$(T_{CEP} = 102MeV, \mu_B = 649MeV), \mu_s = 0$

$(T, \mu_B)_{\mathrm{CEP}} = (107, 635)\mathbf{MeV}$

fRG: W-j Fu, Pawlowski, Rennecke, PRD 101 (2020), 054032

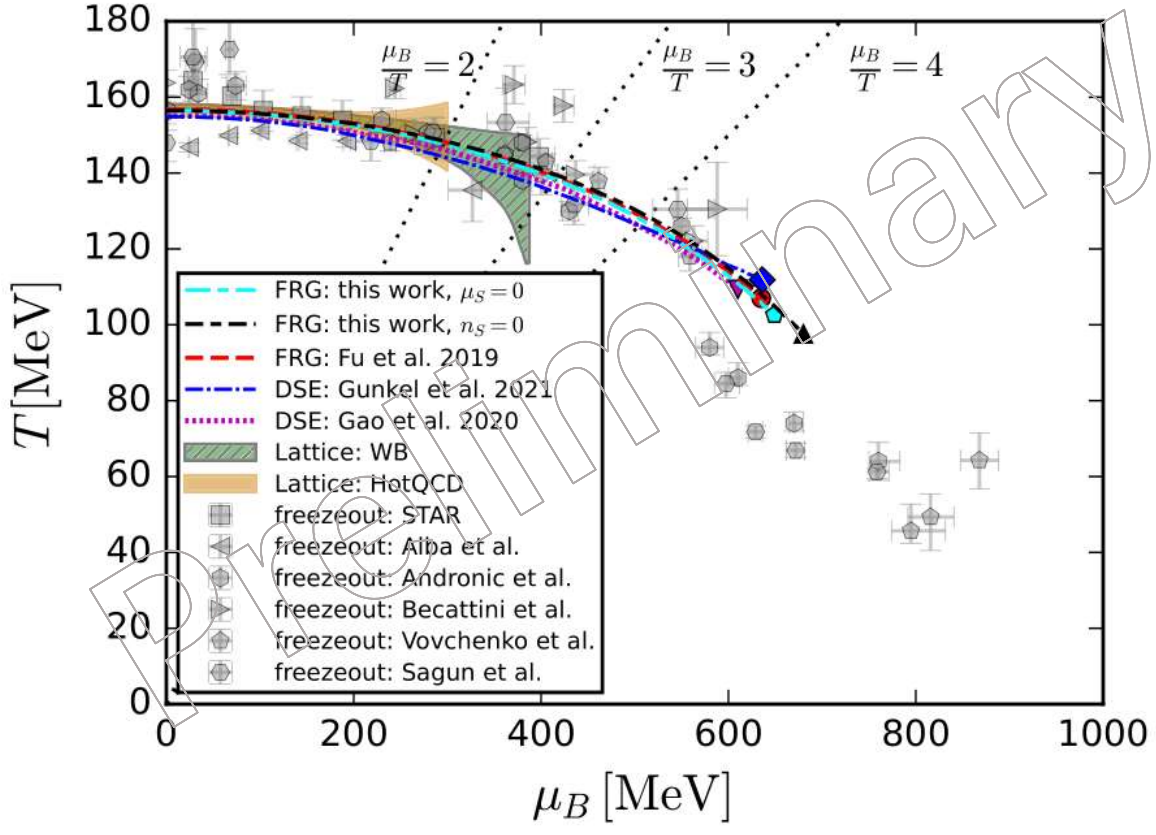
$(T, \mu_B)_{\mathrm{CEP}} = (109, 610)\mathbf{MeV}$

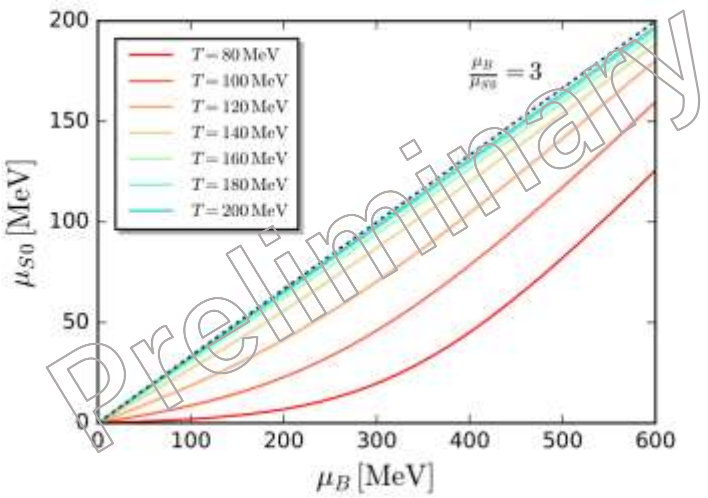
DSE (fRG): Gao, Pawlowski, PLB 820 (2021) 136584

$(T, \mu_B)_{\mathrm{CEP}} = (112, 636)\mathbf{MeV}$

DSE: Gunkel, Fischer, PRD 104 (2021) 5, 054022

Phase diagram with $\mu_s = 0$ and $n_s = 0$





$\kappa_2(\mu_s = 0) = 0.0148(2)$

$\kappa_2(n_s = 0) = 0.0134(2)$

$\kappa_2(n_s = 0)/\kappa_2(\mu_s = 0) = 0.91(2)$

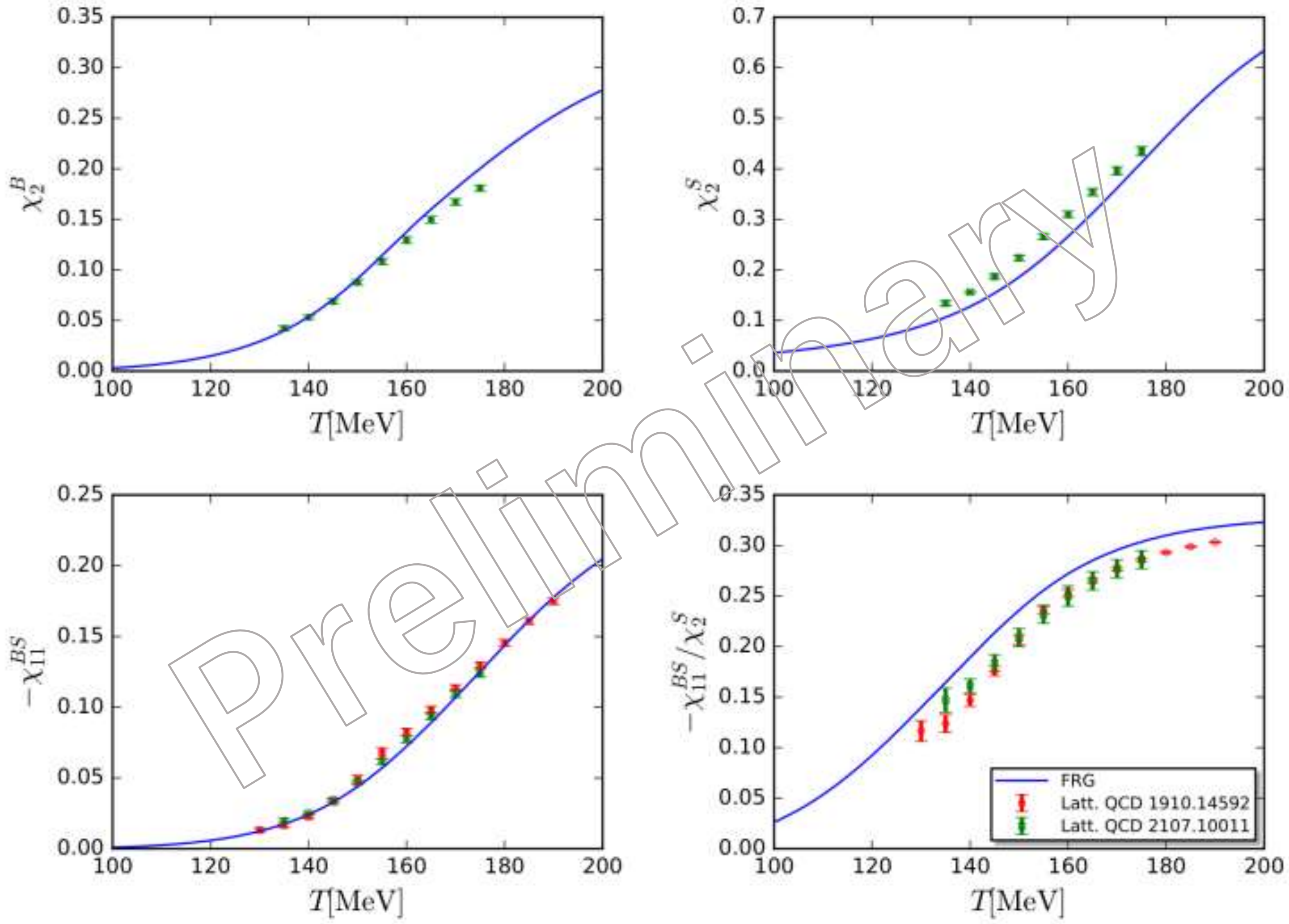
$(T_{CEP} = 102MeV, \mu_B = 649MeV), \mu_s = 0$

$(T_{CEP} = 97MeV, \mu_B = 680MeV), n_s = 0$

$\kappa_2^B(n_S = 0)/\kappa_2^B(\mu_S = 0) = 0.893(35)$

H. T. Ding, et.al. arXiv:2403.09390

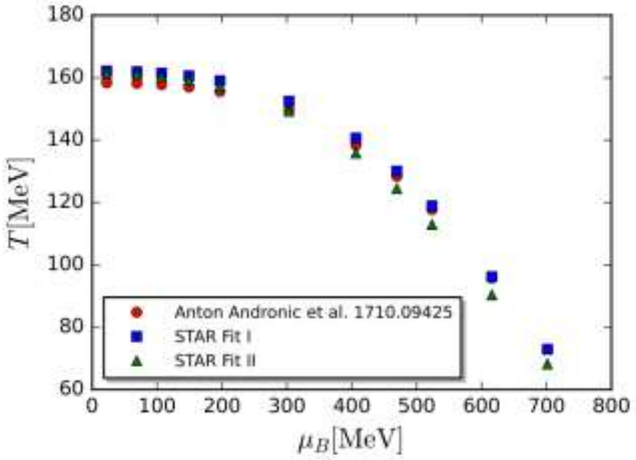
The 2nd order susceptibilities

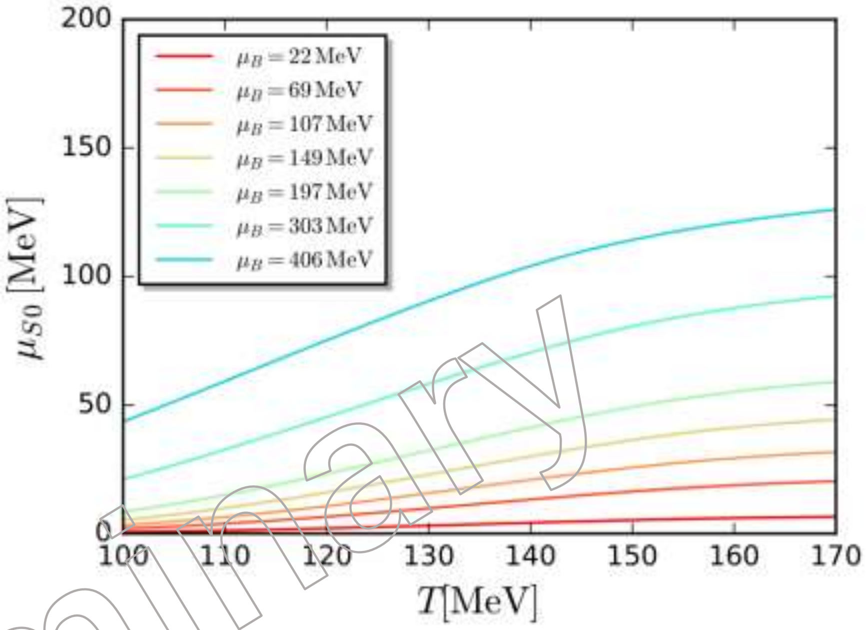


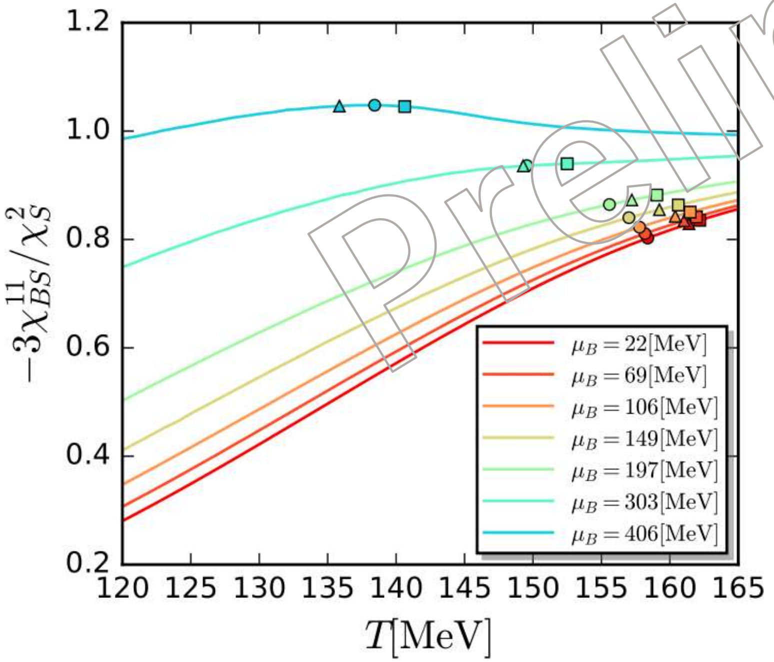
$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k}}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k} \frac{p}{T^4}$$

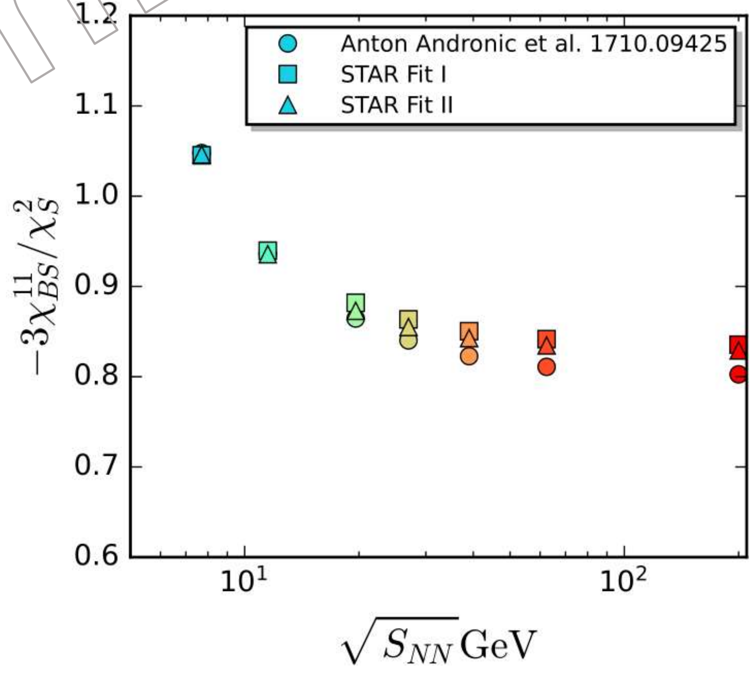
B-S Correlation

W.-j. Fu, X.f. Luo, J.M. Pawlowski F. Rennecke, R. Wen,Y. Shi Phys.Rev.D 104 (2021) 9, 094047









Summary

- We build the 2+1 flavor QCD theory within FRG approach. The light and reduced chiral condensate are in good agreement with the Lattice results.
- The curvature of the phase boundary

$$\kappa_2(n_s = 0)/\kappa_2(\mu_s = 0) = 0.91(2)$$

  and the $\mu_{B,CEP}(\mathrm{n}_S = 0)$ is slightly larger than $\mu_{B,CEP}(\mu_S = 0)$.
- The 2nd order baryon-strangeness correlation functions are monotonic changed with the collision energy.

Thanks for your attention