QCD critical point in the era of BES II

Wei-jie Fu

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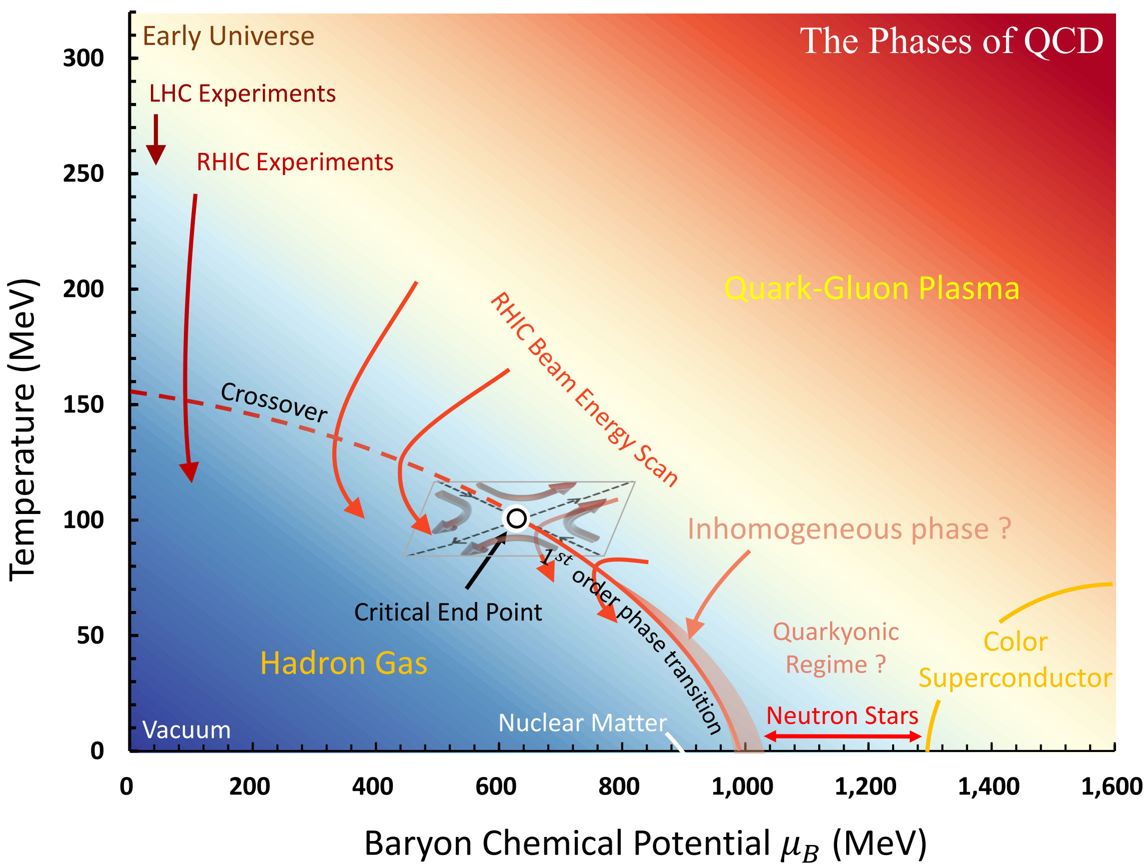
New developments in studies of the QCD phase diagram
ECT*, Trento, Sep 9-13, 2024

Based on:
WF, Xiaofeng Luo, Jan M. Pawlowski, Fabian Rennecke, Shi Yin, *Ripples of the QCD Critical Point*, arXiv: 2308.15508;
WF, Xiaofeng Luo, Jan M. Pawlowski, Fabian Rennecke, Rui Wen, Shi Yin, *Hyper-order baryon number fluctuations at finite temperature and density*, PRD 104 (2021) 094047, arXiv: 2101.06035;
Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, *Soft modes in hot QCD matter*, arXiv:2310.19853;
Yang-yang Tan, Shi Yin, Yong-rui Chen, Chuang Huang, WF, *Real-time evolution of critical modes in the QCD phase diagram*, in preparation

fQCD collaboration:
Braun, Chen, Fu, Gao, Geissel, Huang, Lu, Ihssen, Pawlowski, Rennecke, Sattler, Schallmo, Stoll, Tan, Töpfel, Turnwald, Wessely, Wen, Wink, Yin, Zheng, Zorbach

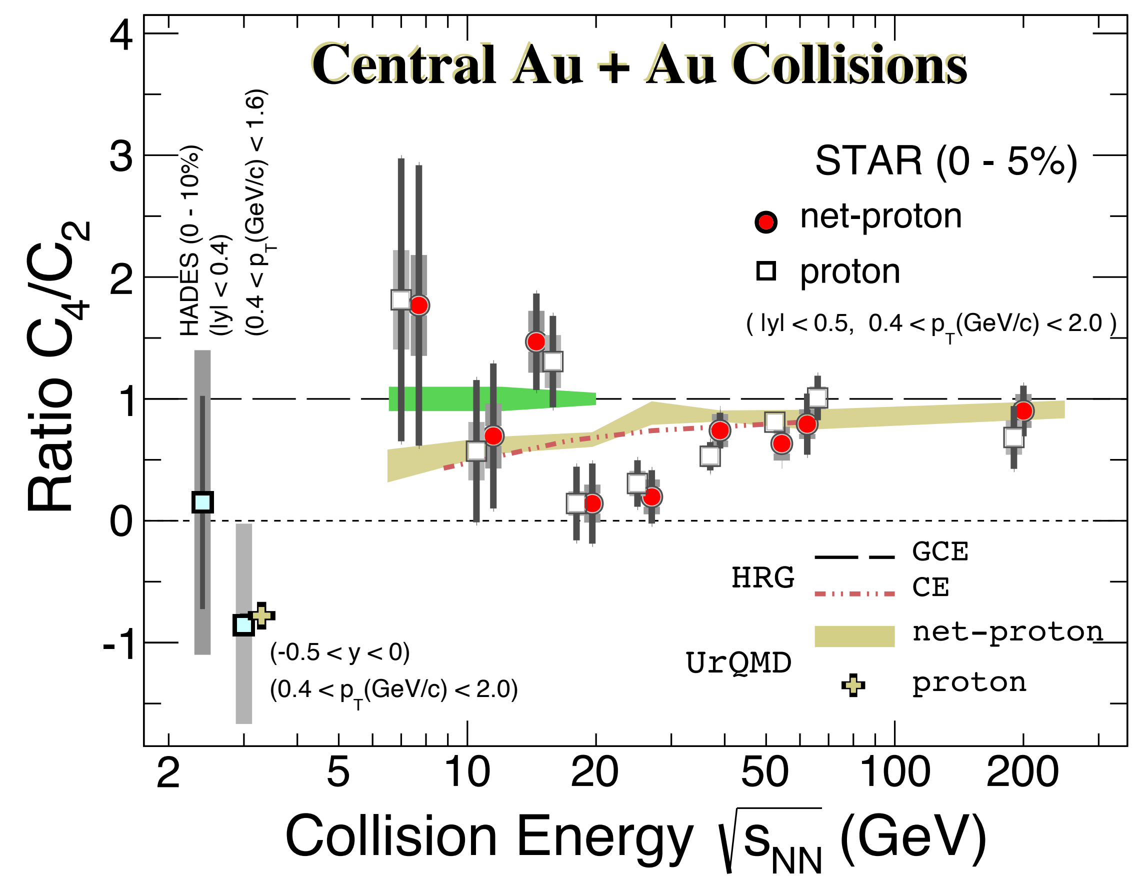
CEP in QCD phase diagram

QCD phase diagram



Non-monotonicity:
M. Stephanov, *PRL* 107 (2011) 052301

Fluctuations measured in BES-I

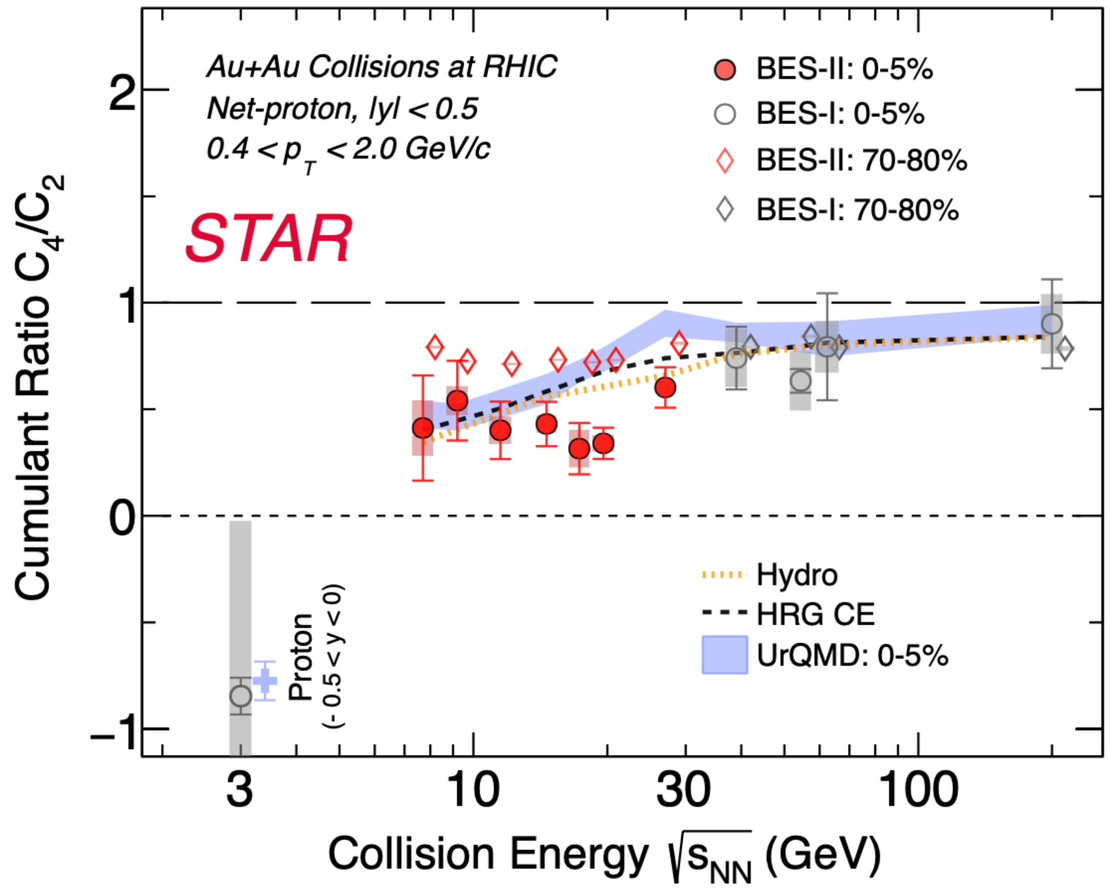


STAR:
J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301;
M. Abdallah *et al.* (STAR), *PRC* 104 (2021), 024902;
M. Abdallah *et al.* (STAR), *PRL* 128 (2022) 20, 202303

- The non-monotonicity of the kurtosis is observed with 3.1σ significance in the phase I of BES program (BES-I) at STAR

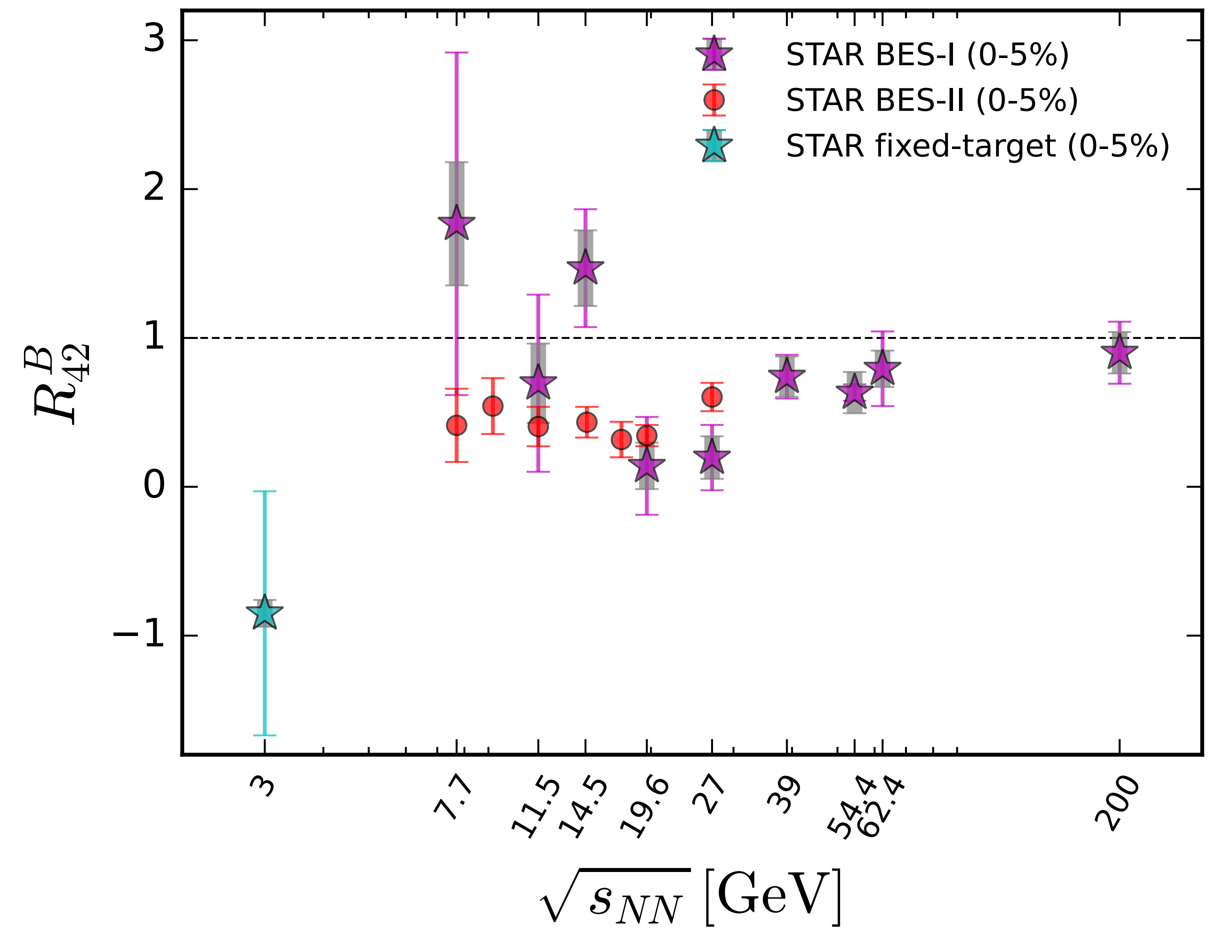
Recent results in BES-II

Net proton kurtosis



Ashish Pandav for STAR Collaboration in CPOD2024

Results in BES-I and BES-II



- The kurtosis in the energy regime of fixed-target experiments, i.e. 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, then become pivotal.
- Is there a "peak" structure?

also cf. talk by Xiaofeng Luo

Recent results in BES-II

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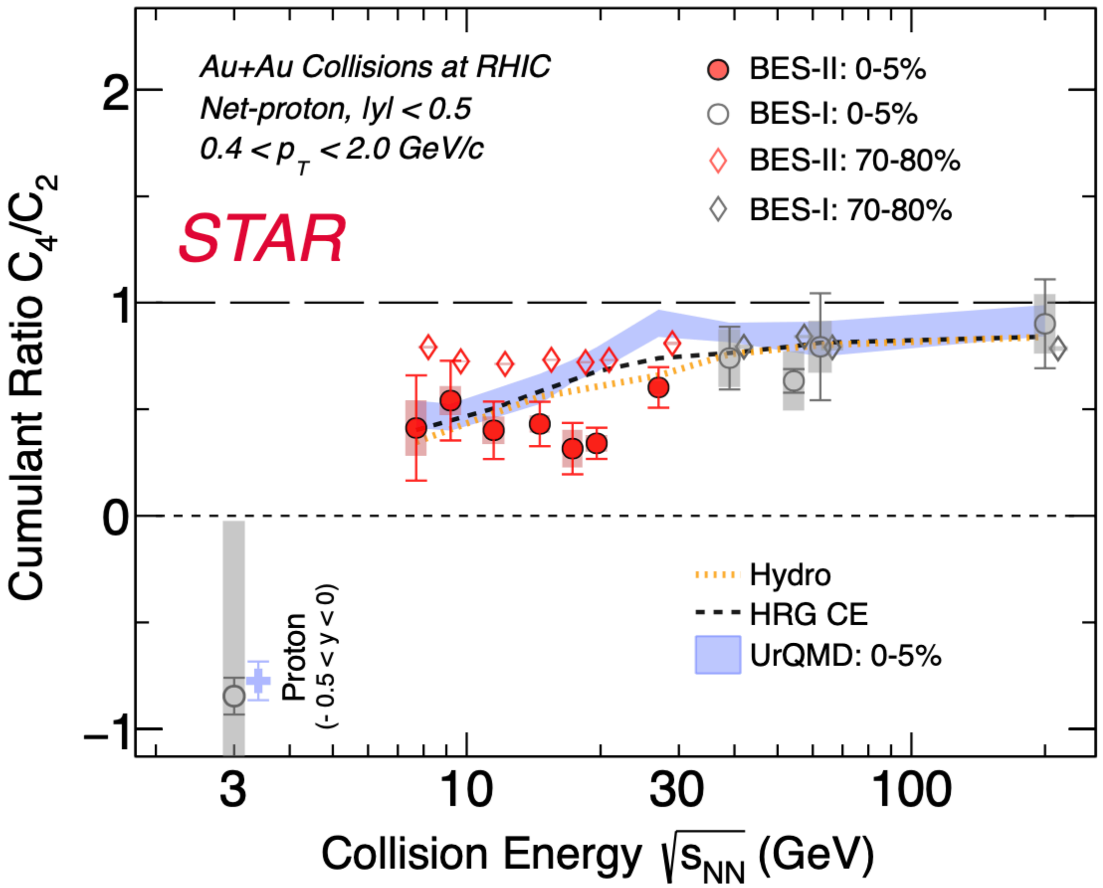

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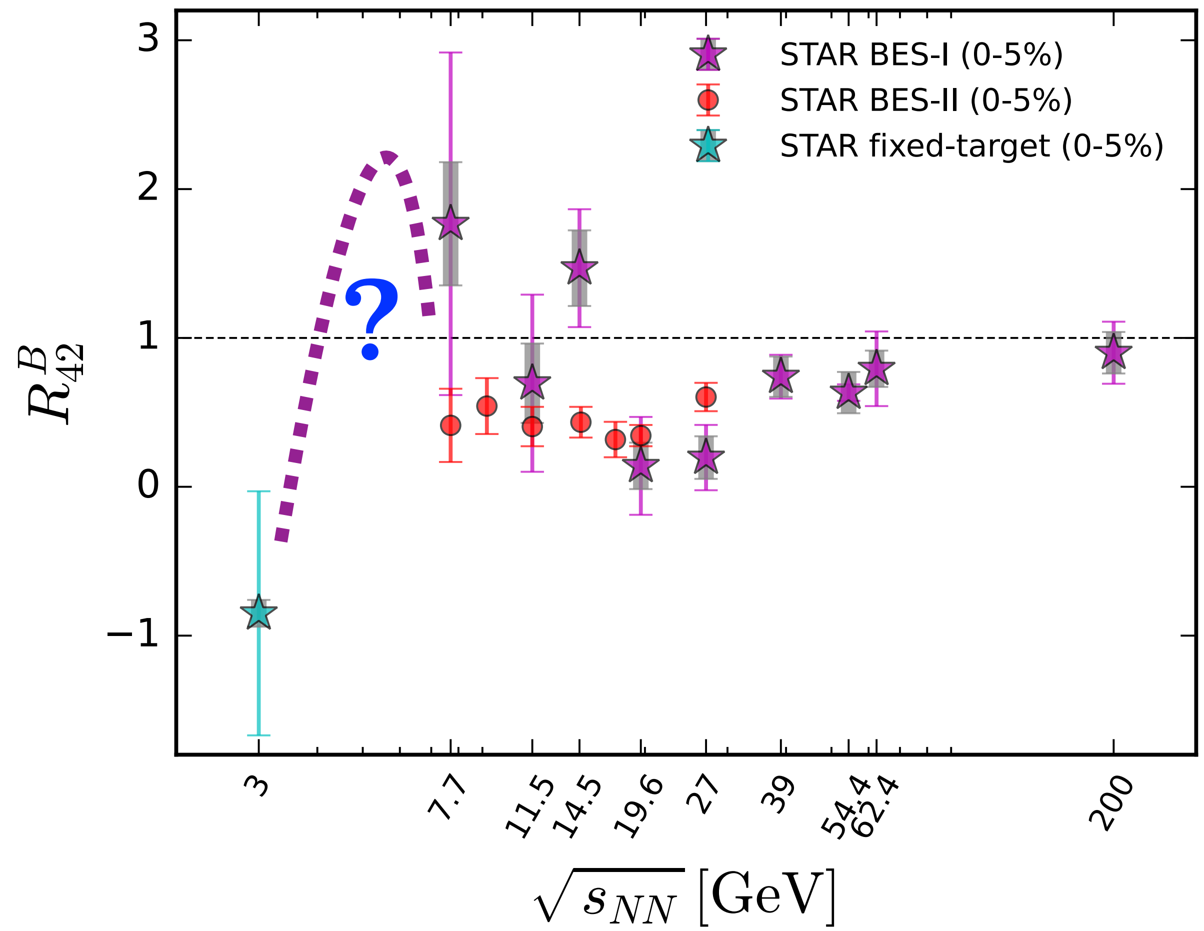
Results in BES-I and BES-II



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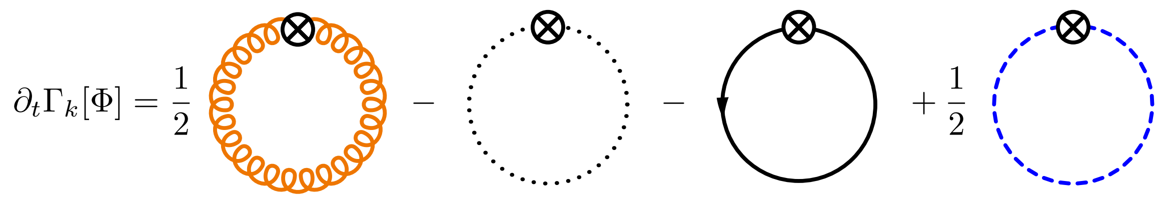
also cf. talk by Xiaofeng Luo

Outline

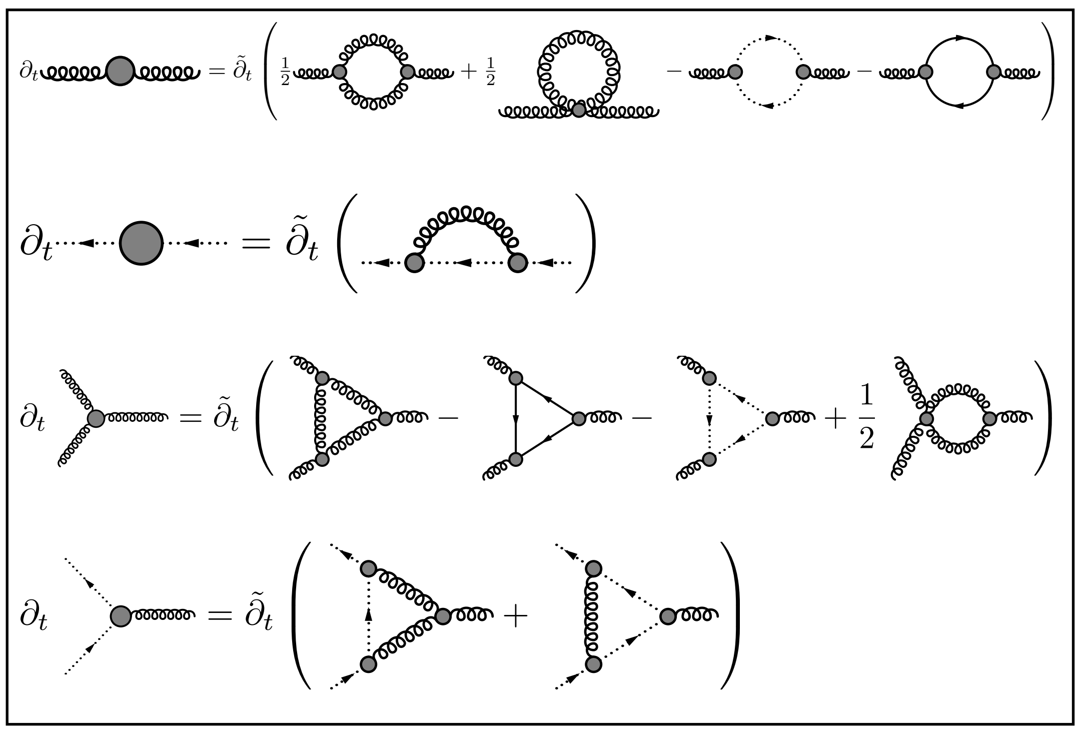
- Introduction
- Recent advance of QCD phase structure from functional QCD
- Baryon number fluctuations at high density
- Ripples of the QCD critical point
- Size of critical region near CEP
- Summary

First-principles QCD within fRG

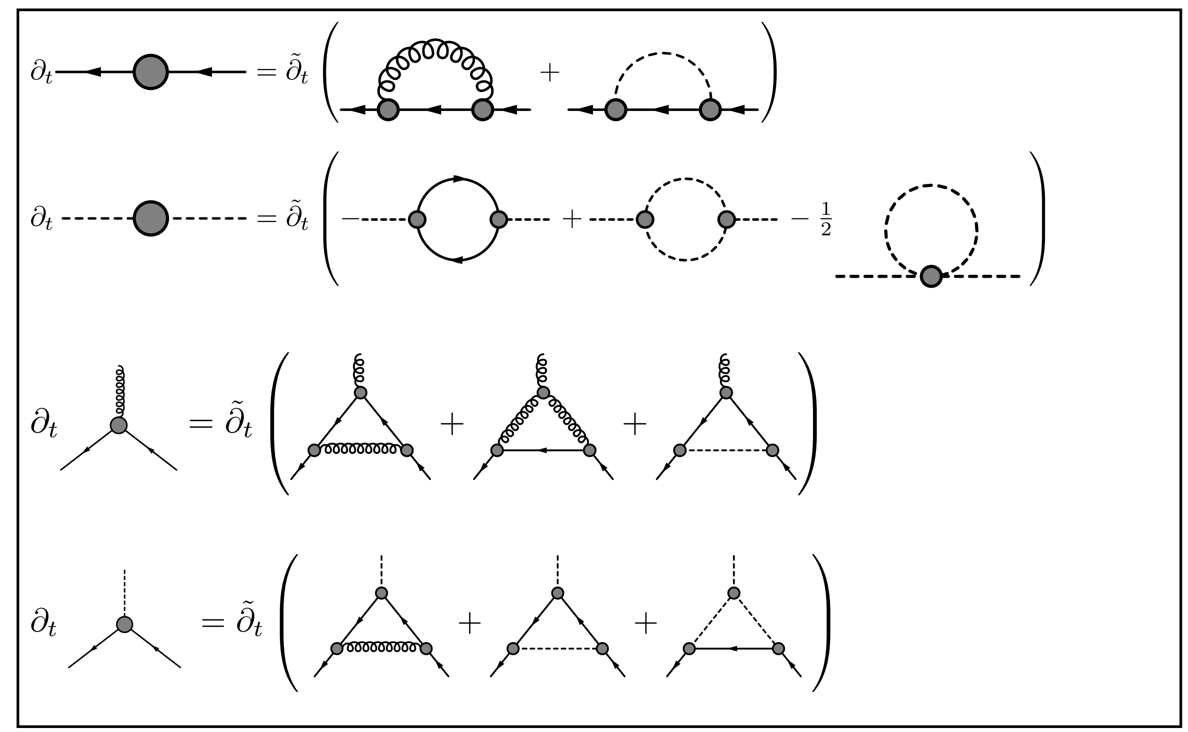
QCD flow equation:



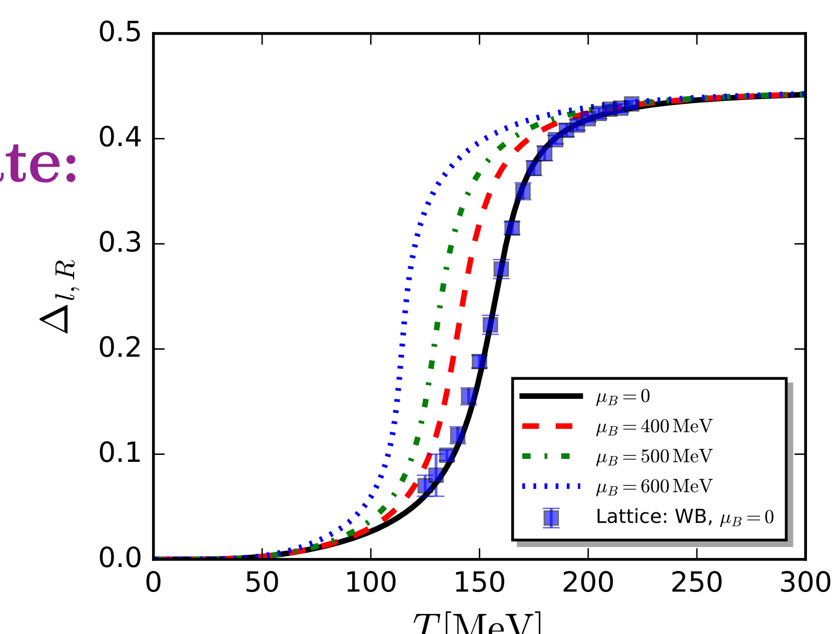
Glue sector:

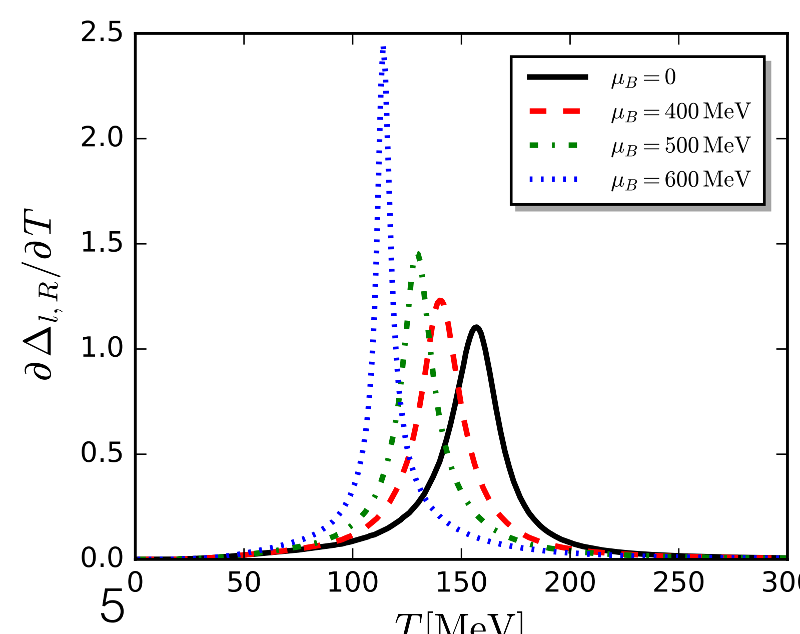


Matter sector:



quark condensate:



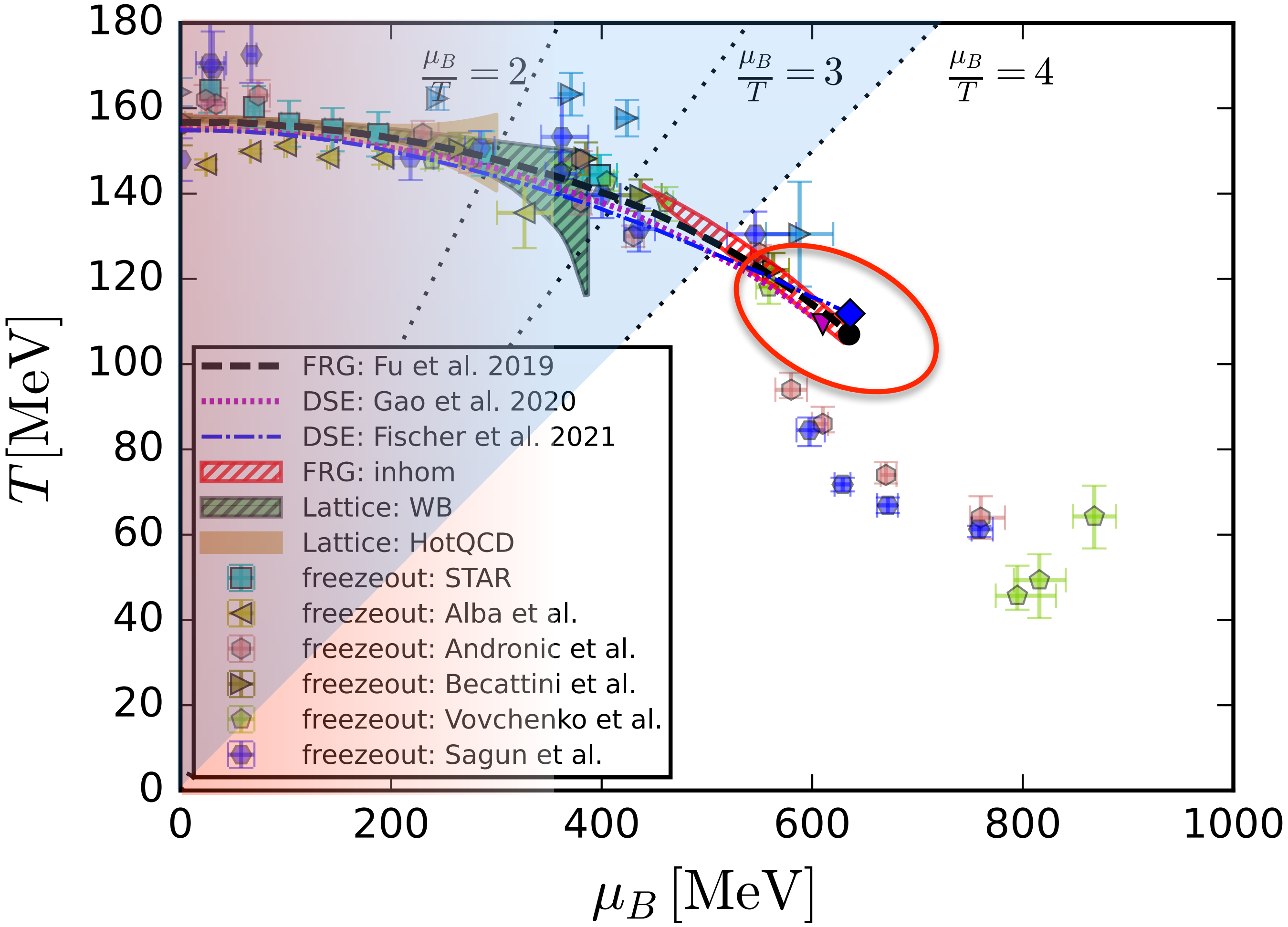


fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

Quantitative errors analysis in fRG: Ihssen, Pawlowski, Sattler, Wink, arXiv:2408.08413

CEP from first-principles functional QCD



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B.

Regime of quantitative reliability of functional QCD with $\mu_B/T \lesssim 4$.

also cf. talks by Jan M. Pawlowski and Rui Wen

Estimates of the location of CEP from first-principles functional QCD:

fRG:

● $(T, \mu_B)_{\text{CEP}} = (107, 635)\textbf{MeV}$

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

DSE:

▼ $(T, \mu_B)_{\text{CEP}} = (109, 610)\textbf{MeV}$

DSE (fRG): Gao, Pawlowski, *PLB* 820 (2021) 136584

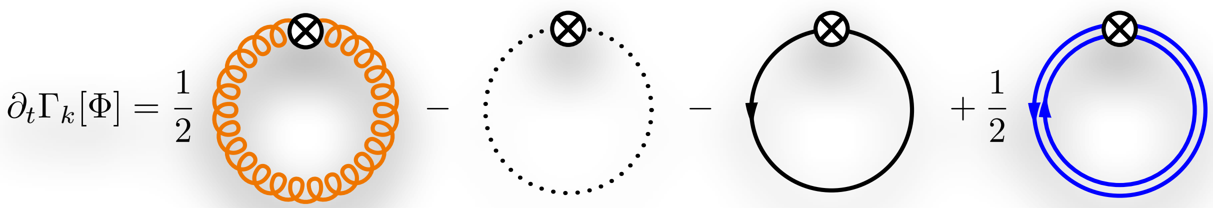
◆ $(T, \mu_B)_{\text{CEP}} = (112, 636)\textbf{MeV}$

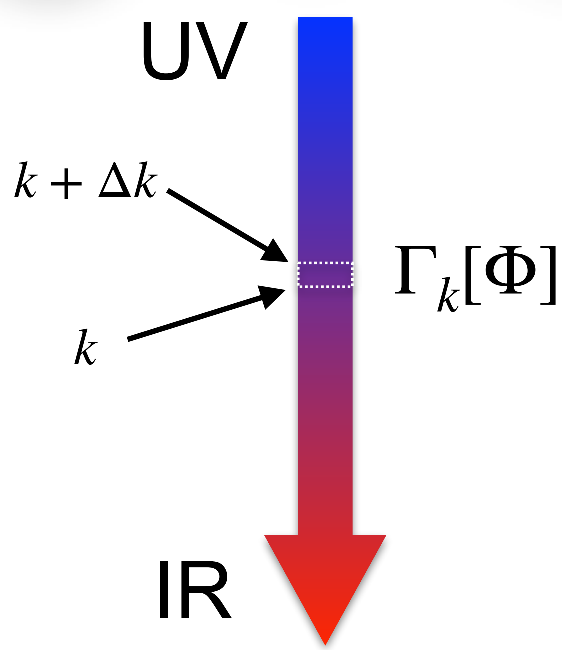
DSE: Gunkel, Fischer, *PRD* 104 (2021) 5, 054022

- No CEP observed in $\mu_B/T \lesssim 2 \sim 3$ from lattice QCD. Karsch, *PoS* CORFU2018 (2019)163
- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP: 600 MeV $\lesssim \mu_{B_{\text{CEP}}} \lesssim$ 650 MeV.

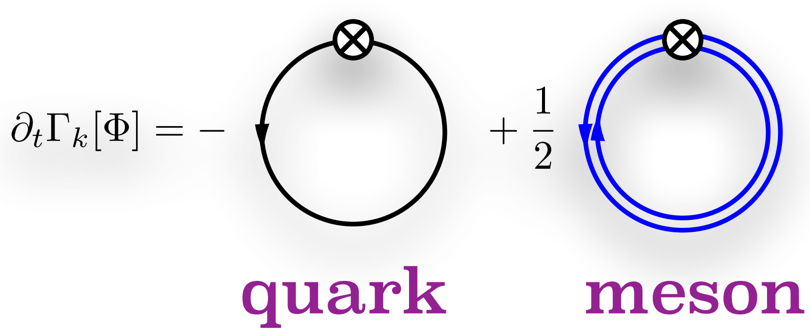
QCD-assisted LEFT

QCD flow equation:

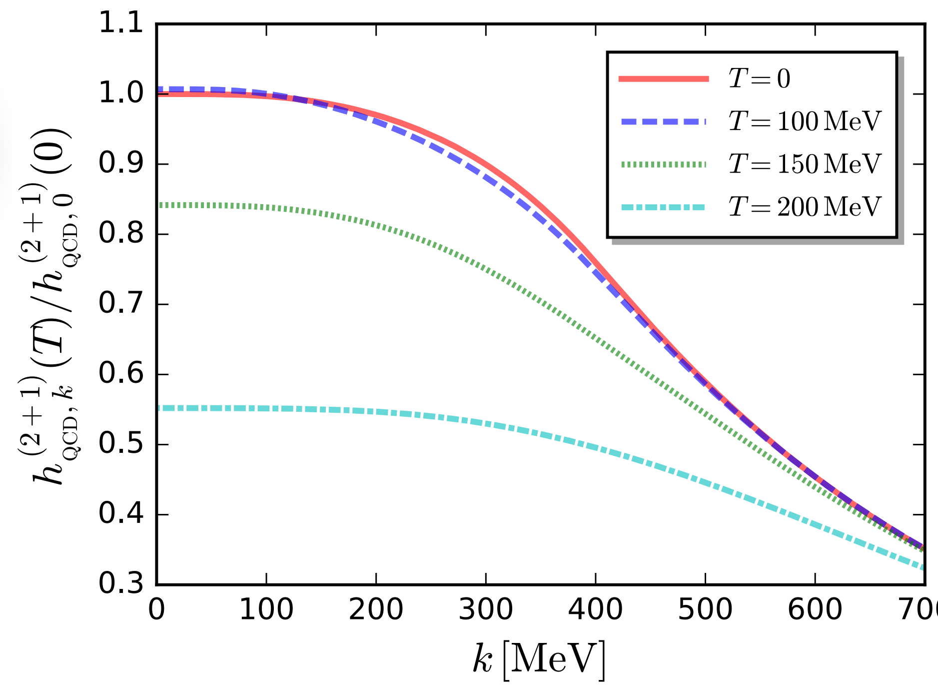




LEFT flow equation:

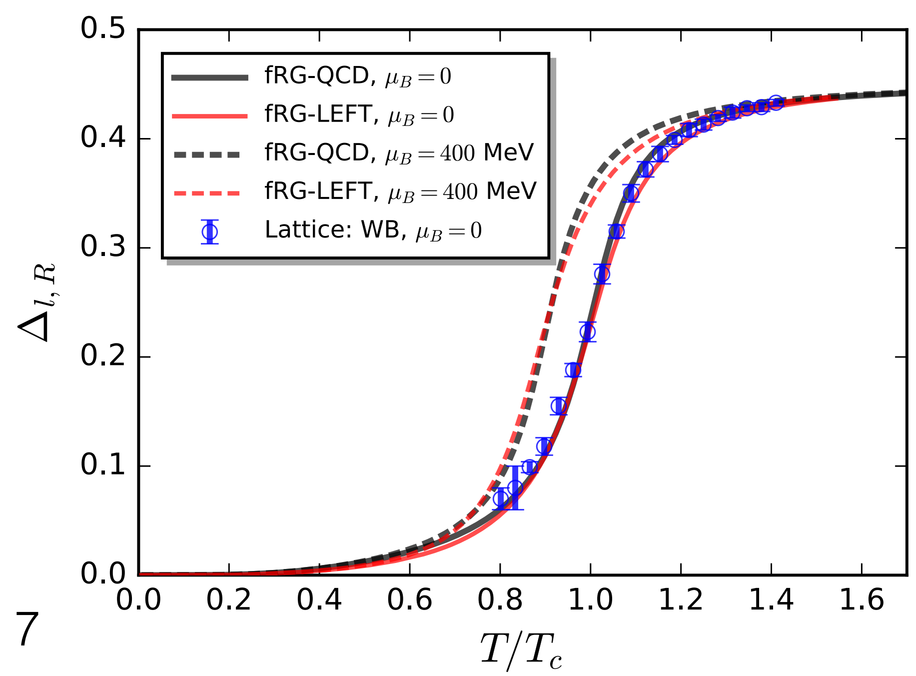


- **Yukawa couplings obtained in QCD inputted in QCD-assisted LEFT**



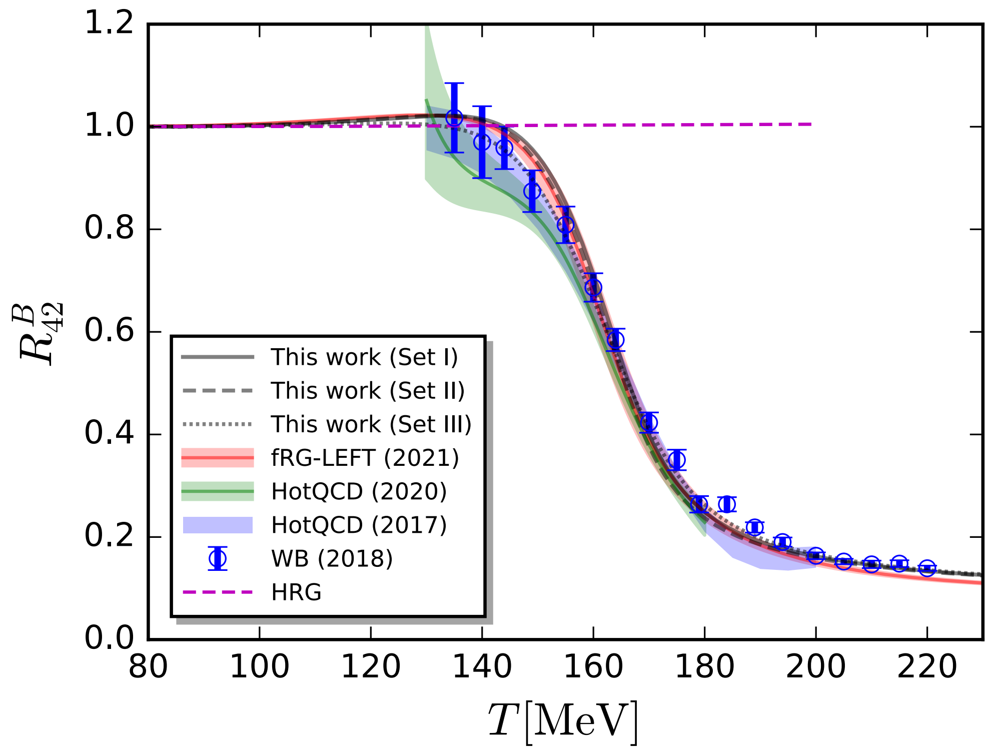
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

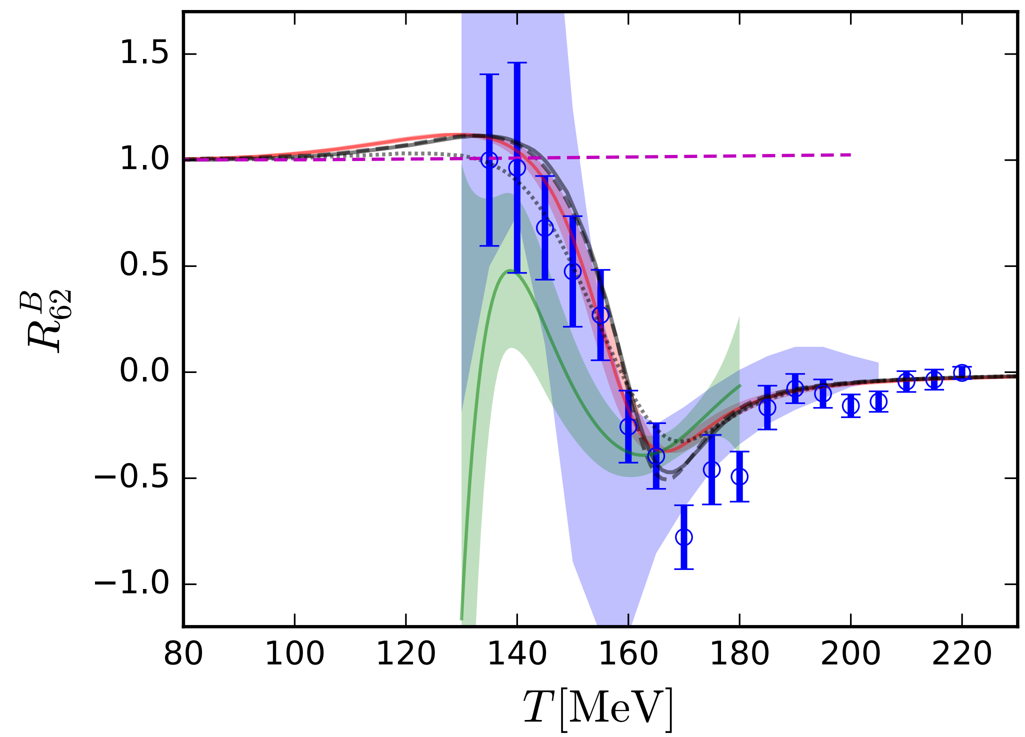
- **Chiral condensates in QCD and QCD-assisted LEFT in agreement**



WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

Baryon number fluctuations





$\mu_B = 0$

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508;
WF, Luo, Pawlowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047

HotQCD: A. Bazavov *et al.*, arXiv: *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502

WB: S. Borsanyi *et al.*, arXiv: *JHEP* 10 (2018) 205

baryon number fluctuations

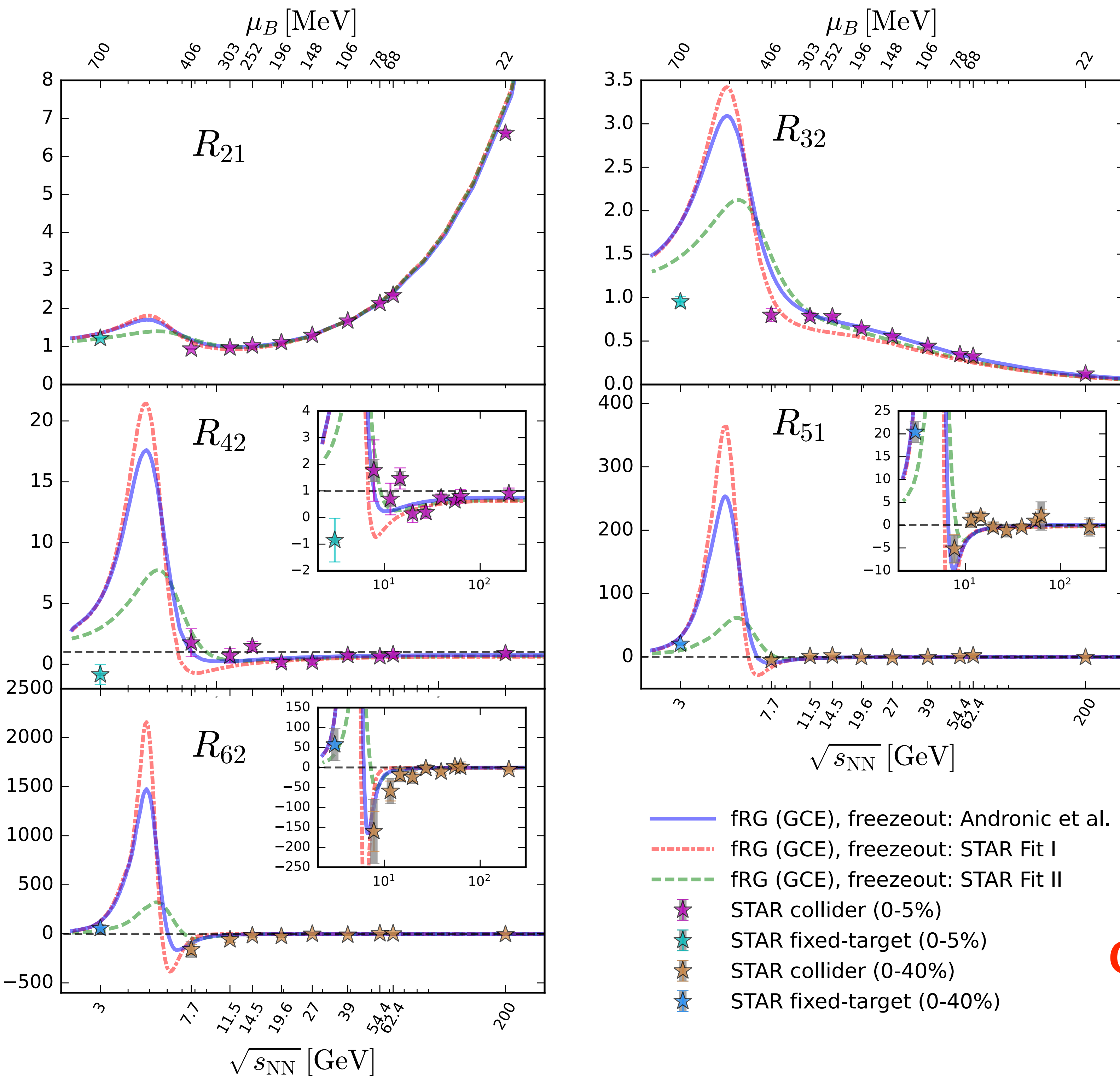
$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4} \qquad R_{nm}^B = \frac{\chi_n^B}{\chi_m^B}$$

relation to the cumulants

$$\frac{M}{VT^3} = \chi_1^B\,,\ \frac{\sigma^2}{VT^3} = \chi_2^B\,,\ S = \frac{\chi_3^B}{\chi_2^B\sigma}\,,\ \kappa = \frac{\chi_4^B}{\chi_2^B\sigma^2}\,,$$

- **In comparison to lattice results and our former results, the improved results of baryon number fluctuations at vanishing chemical potential in the QCD-assisted LEFT are convergent and consistent.**

Grand canonical fluctuations at the freeze-out



STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301;
Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303;
Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

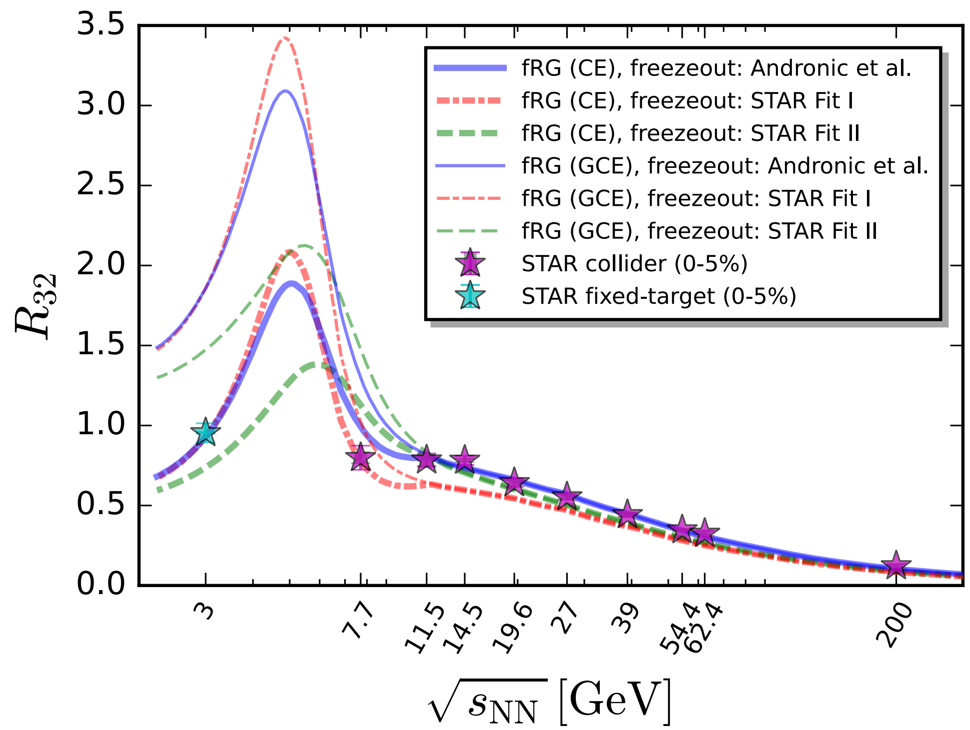
fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

- **Results in fRG are obtained in the QCD-assisted LEFT with a CEP at $(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (98{,}643)$ MeV.**
- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\mathrm{NN}}} \lesssim$ 7.7 GeV.**
- **Agreement between the theory and experiment is worsening with $\sqrt{s_{\mathrm{NN}}} \lesssim$ 11.5 GeV.**
- **Effects of global baryon number conservation in the regime of low collision energy should be taken into account.**

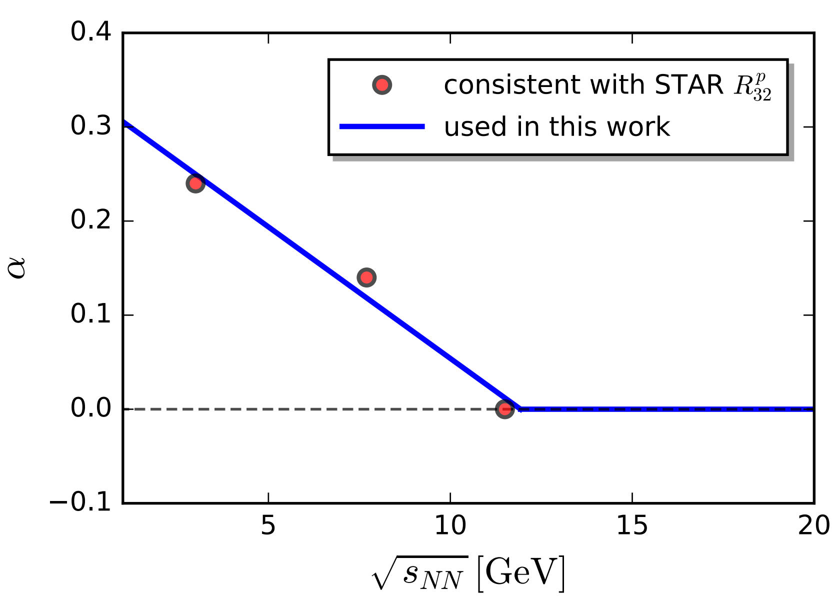
Caveat:

Fluctuations of baryon number in theory are compared with those of proton number in experiments.

Canonical corrections with SAM



- **Experimental data R_{32} is used to constrain the parameter α in the range $\sqrt{s_{\rm NN}} \lesssim 11.5$ GeV.**
- **We choose the simplest linear dependence**



$$\alpha(\bar{s}) = a\left(1 - \sqrt{\bar{s}}\right)\theta\left(1 - \bar{s}\right)$$

$$a = 0.33\,, \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{\rm NN}}}{11.9\,{\rm GeV}}$$

SAM:

- We adopt the subensemble acceptance method (SAM) to take into account the effects of global baryon number conservation:

$$\alpha = \frac{V_1}{V}$$

V_1: the subensemble volume measured in the acceptance window, V: the volume of the whole system.

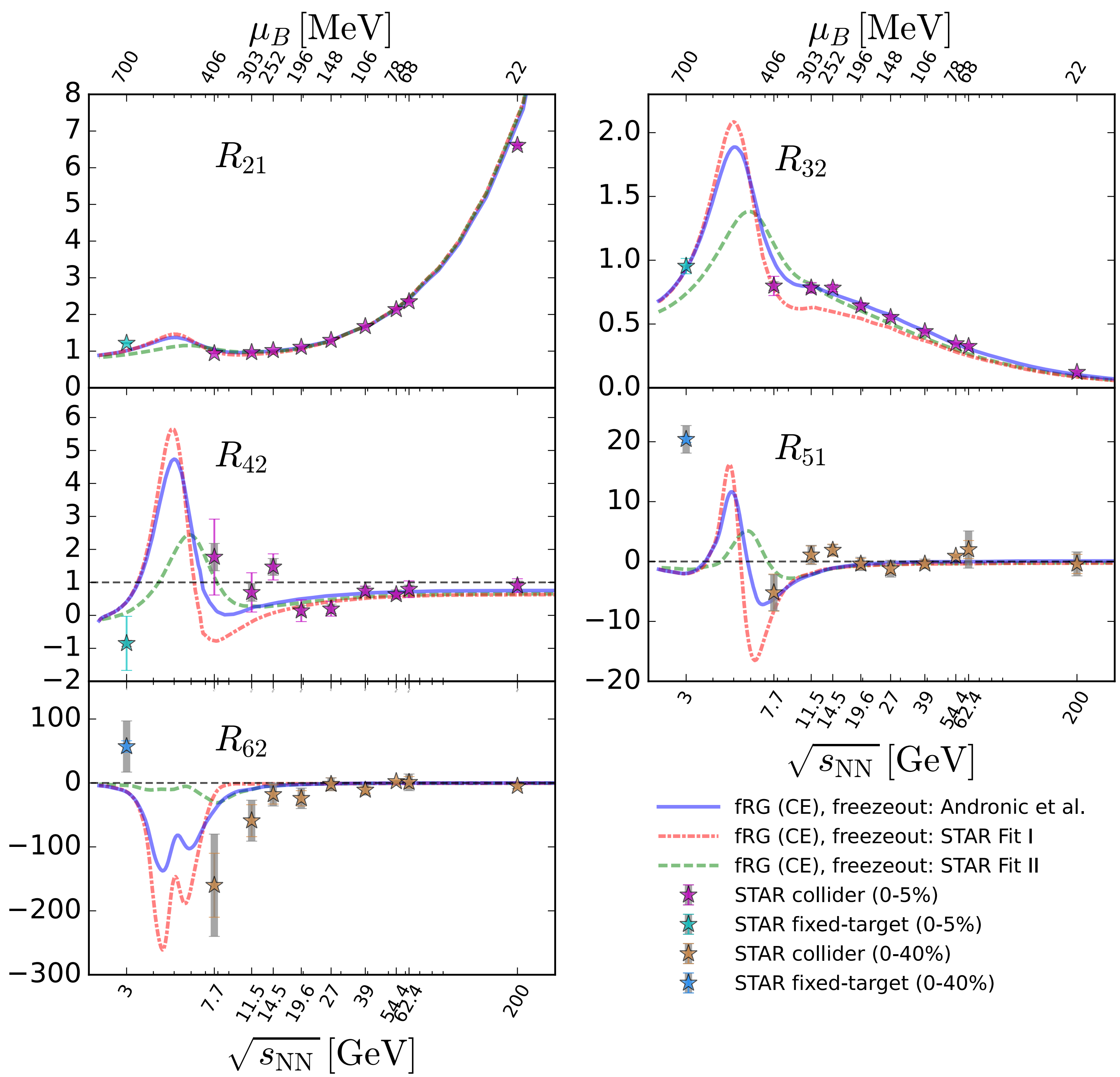
- fluctuations with canonical corrections are related to grand canonical fluctuations as follows:

$$\bar{R}_{21}^B = \beta R_{21}^B, \qquad \bar{R}_{32}^B = (1-2\alpha)R_{32}^B,$$

$$\bar{R}_{42}^B = (1-3\alpha\beta)R_{42}^B - 3\alpha\beta(R_{32}^B)^2$$

SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch , *PLB* 811 (2020) 135868

Canonical fluctuations at the freeze-out



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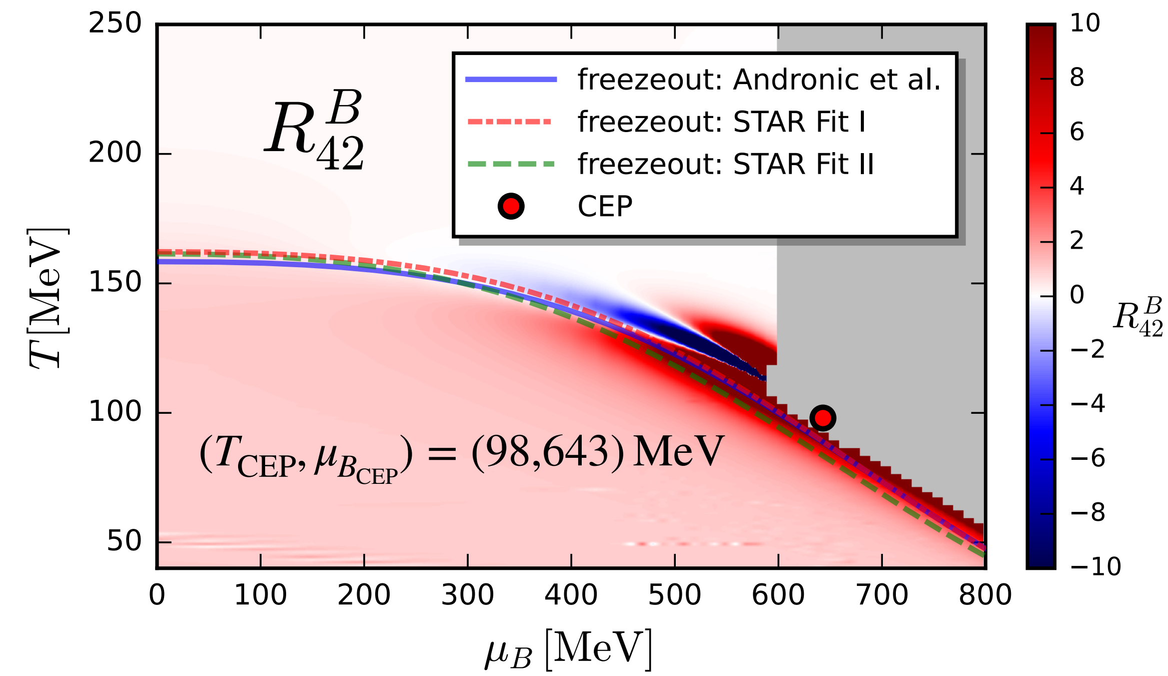
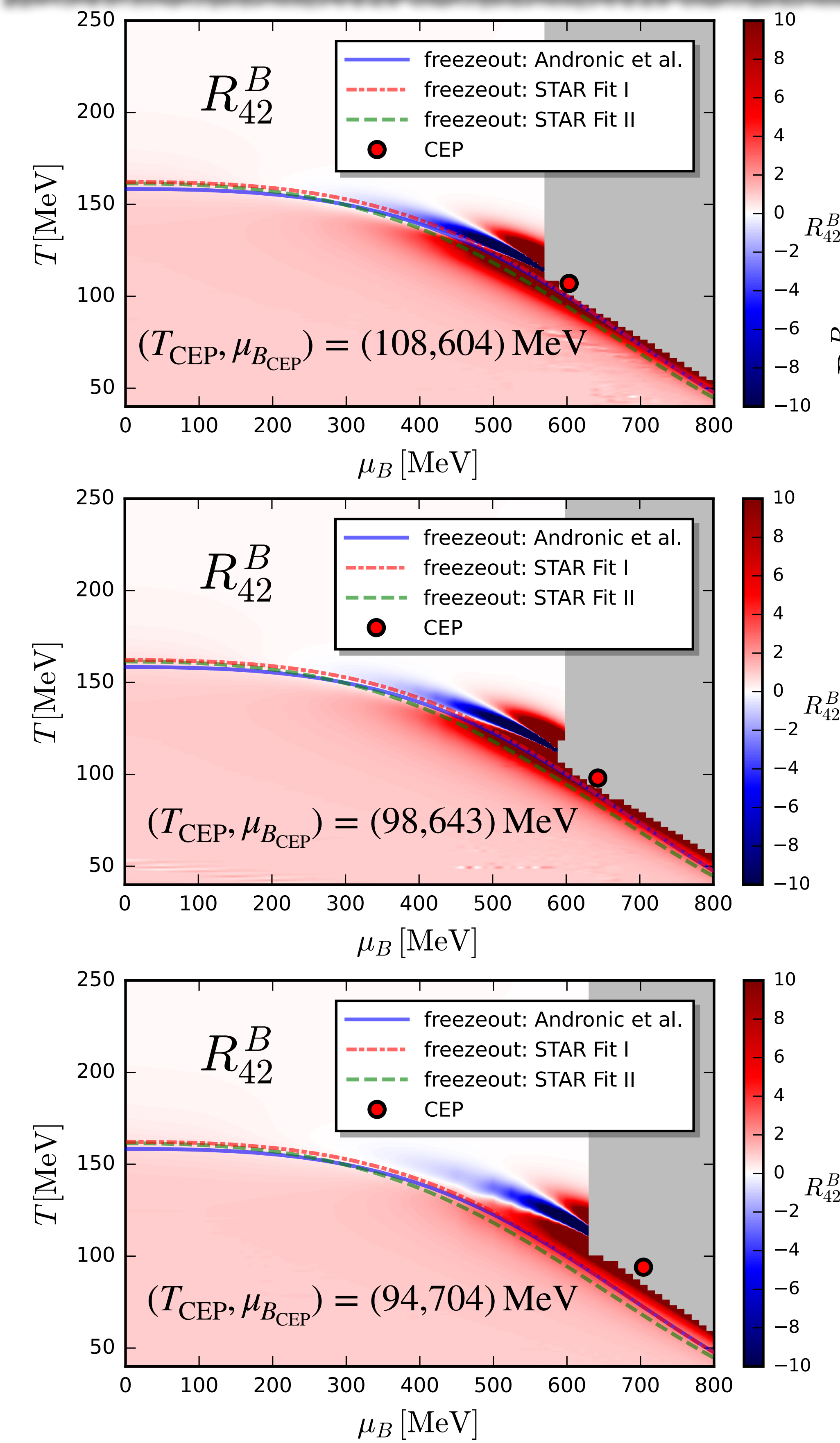
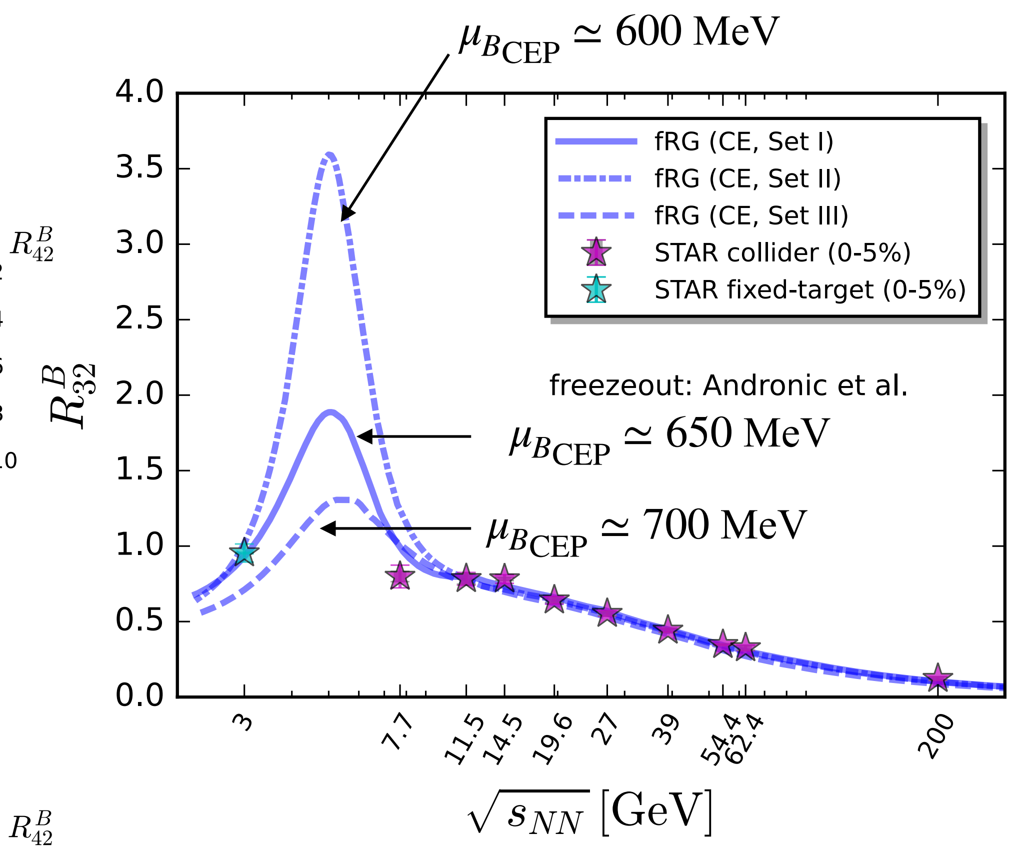
fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

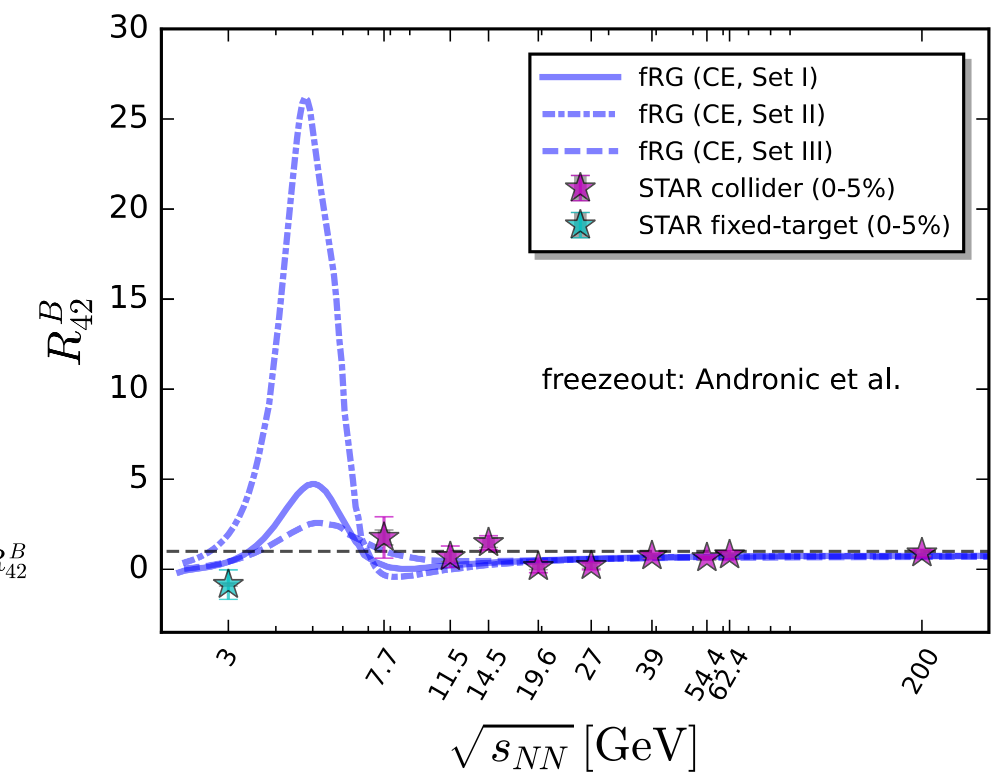


- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV.**
- **Position of peak in R_{42} is $\mu_{B_{\rm peak}} =$ 536, 541 and 486 MeV for the three freeze-out curves, significantly smaller than $\mu_{B_{\rm CEP}} = 643$ MeV.**

Dependence on the location of the CEP







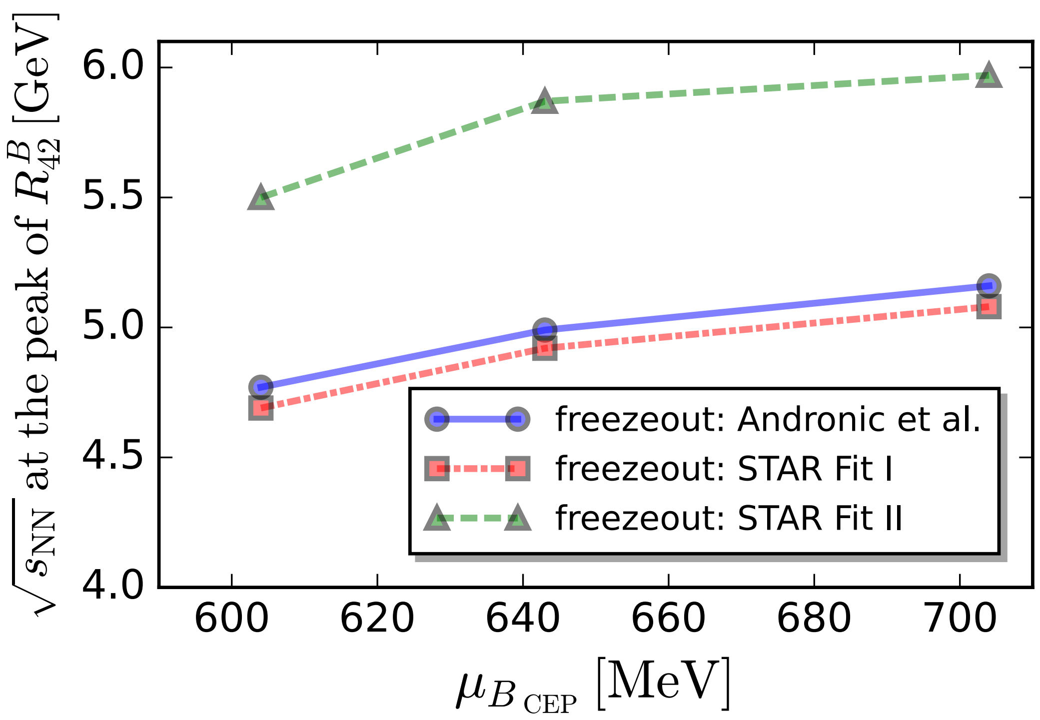
STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

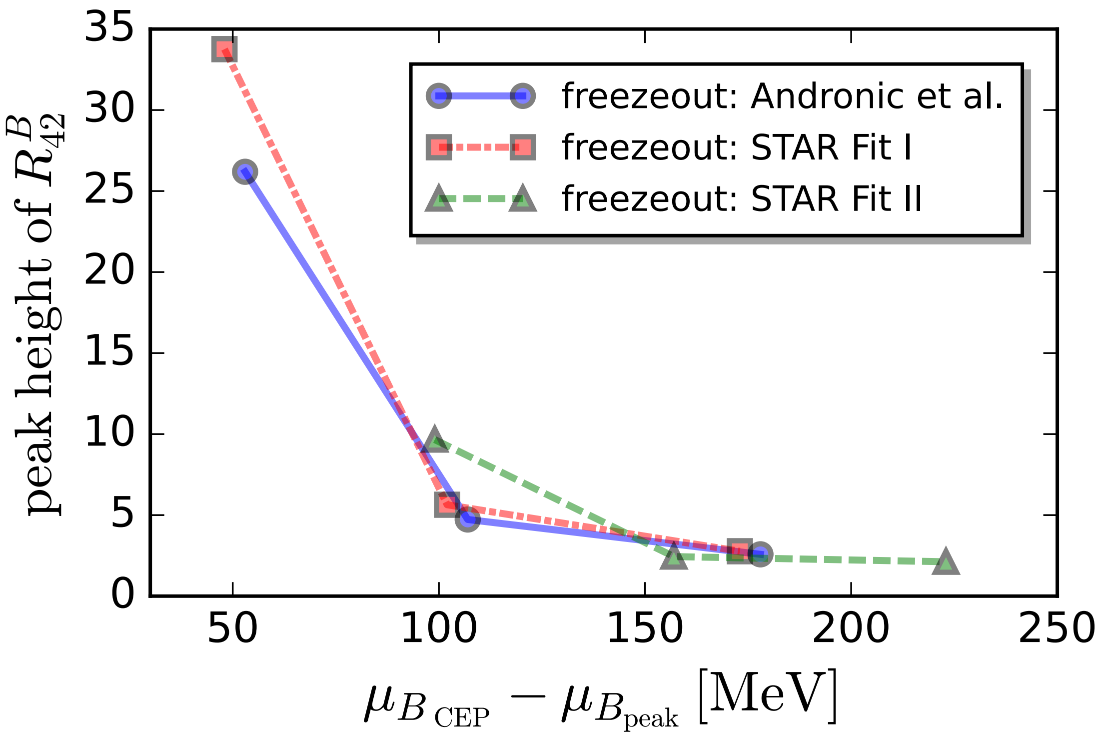
- **Position of the peak is insensitive to the location of CEP.**
- **Height of peak decreases as CEP moves towards larger μ_B.**

Ripples of the QCD critical point

Position of peak:



Height of peak:

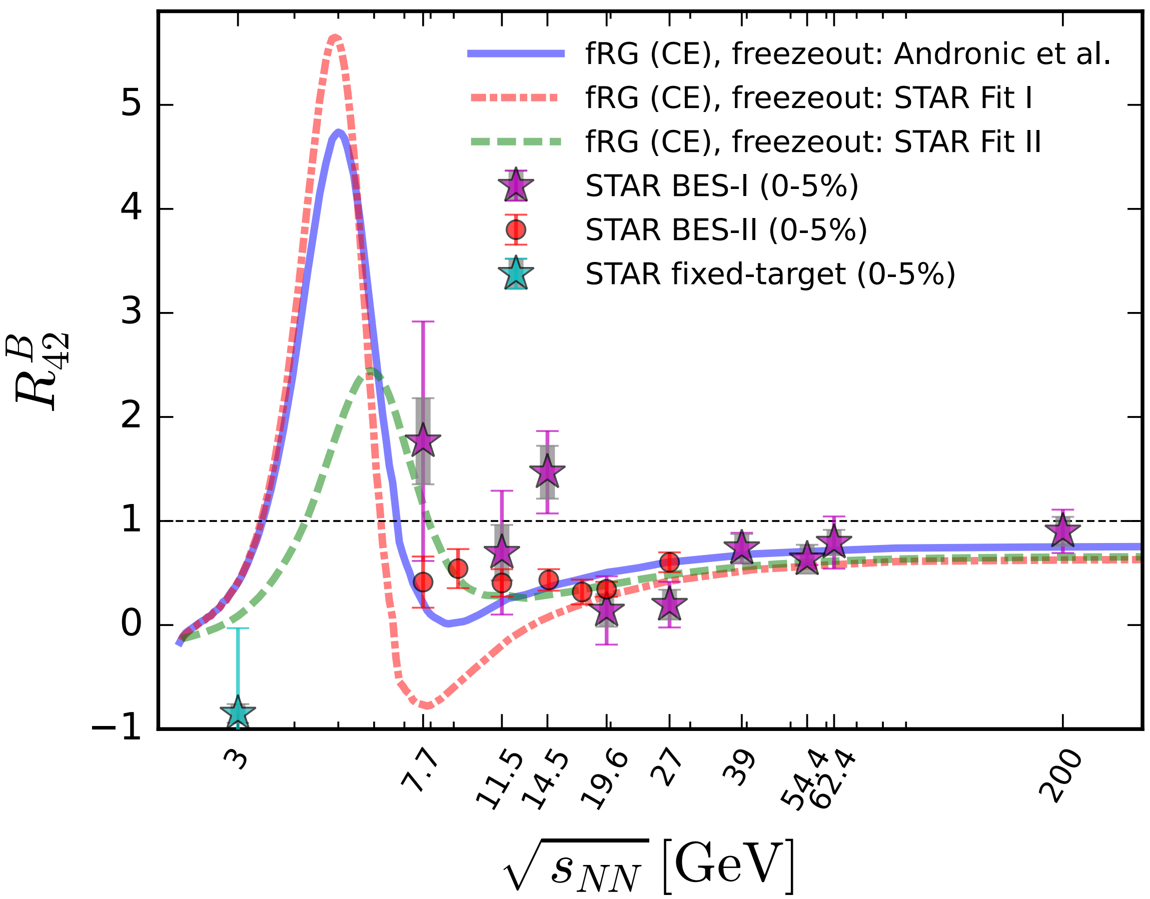


fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

- **Note that the ripples of CEP are far away from the critical region characterized by the universal scaling properties, e.g., the critical slowing down.**
- **But, the information of CEP, such as its location and properties, etc., is still encoded in the ripples.**

Comparison to BES-II

Net baryon (proton) number Kurtosis:



- In comparison to BES-I, BES-II results are better consistent with the theoretical prediction.
- Experimental results in the energy regime of fixed-target experiments, i.e. 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, are now very important!! It will finally tell us whether there is a CEP.

Magnetic equation of state

- The magnetic equation of state (EoS) is obtained via the chiral condensate:

$$\Delta_q = m_q \frac{\partial \Omega\big(T; m_q(T)\big)}{\partial m_q} = m_q \frac{T}{V}\int_x \big\langle \bar{q}(x)\, q(x) \big\rangle$$

- The chiral properties of the magnetic EoS are encoded in the magnetic susceptibility:

$$\chi_M = -\frac{\partial \bar{\Delta}_l}{\partial m_l}\,, \qquad \text{with} \qquad \bar{\Delta}_l = \frac{\Delta_l}{m_l}$$

- In the critical region, the magnetic EoS can be expressed as a universal scaling function $f_G(z)$ through

$$\bar{\Delta}_l = m_l^{1/\delta} f_G(z)$$

with

$$z = t\, m_l^{-1/\beta\delta}\,, \qquad \text{and} \qquad t = (T - T_c)/T_c$$

z is the scaling variable and t is the reduced temperature.

- The pseudo-critical temperature T_{pc}, which is defined through the peak location of χ_M, is readily obtained from the scaling function as

$$T_{pc}(m_\pi) \approx T_c + c\, m_\pi^p\,, \qquad \text{with} \qquad p = 2/(\beta\delta)$$

Critical exponent in fRG for 3d-O(4):

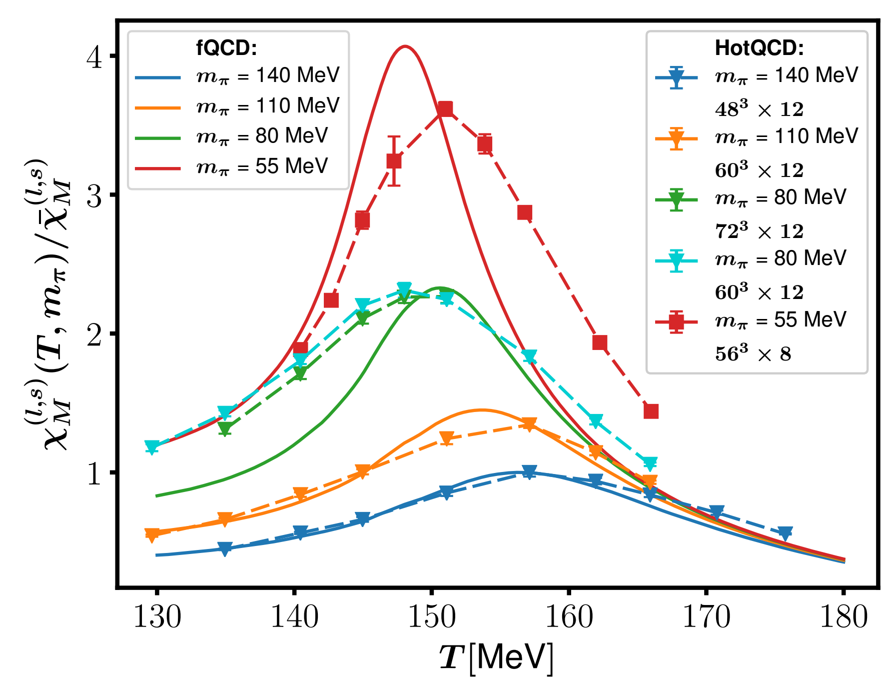
$$\beta = 0.405\,, \quad \delta = 4.784\,, \quad \theta_H = 0.272\,,$$

obtained from the fixed-point equation for the Wilson-Fisher fixed point, which leads us $p_{\rm fRG} = 1.03$

Critical exponent in mean field:

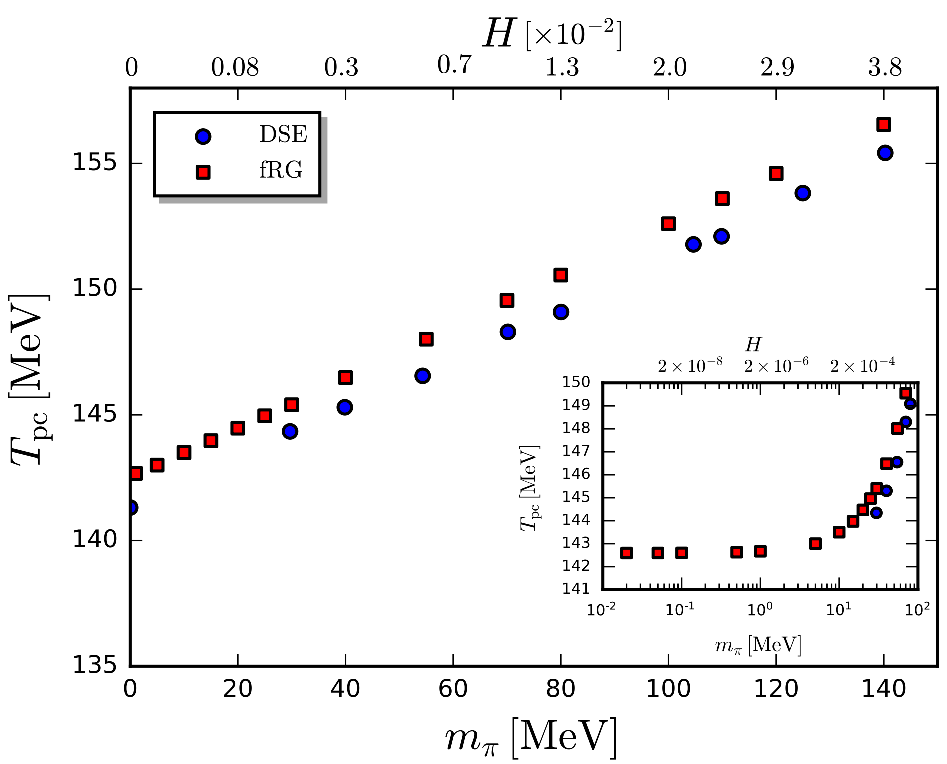
$$\beta_{\rm MF} = 1/2\,, \quad \delta_{\rm MF} = 3\,,$$

thus, one has $p_{\rm MF} = 4/3$



Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020), 056010.

Magnetic equation of state



$$T_{pc}(m_\pi) \approx T_c + c\, m_\pi^p$$

Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, arXiv:2310.19853.

Lattice (HotQCD):

$$T_c^{\text{lattice}} = 132^{+3}_{-6}\,\text{MeV},$$

Ding *et al.*, *PRL* 123 (2019) 062002.

fRG:

$$T_c^{\text{fRG}} \approx 142\,\text{MeV}\,, \qquad p_{\text{fRG}} = 1.024$$

Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020) 056010.

DSE:

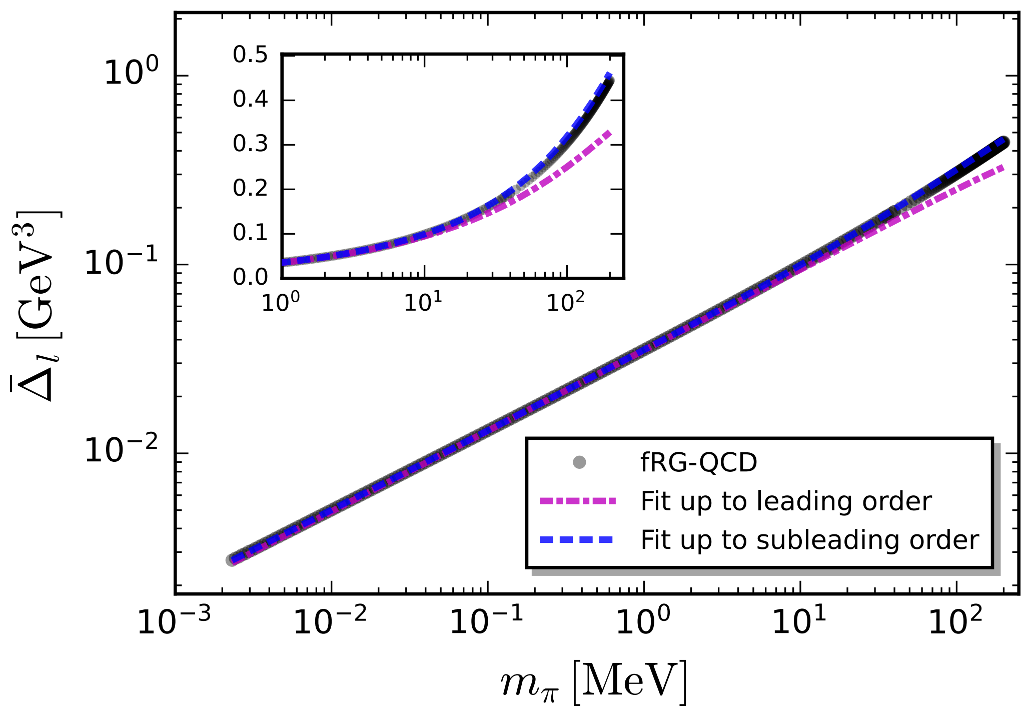
$$T_c^{\text{DSE}} \approx 141\,\text{MeV}\,, \qquad p_{\text{DSE}} = 0.9606$$

Gao, Pawlowski, *PRD* 105 (2022) 9, 094020, arXiv: 2112.01395.

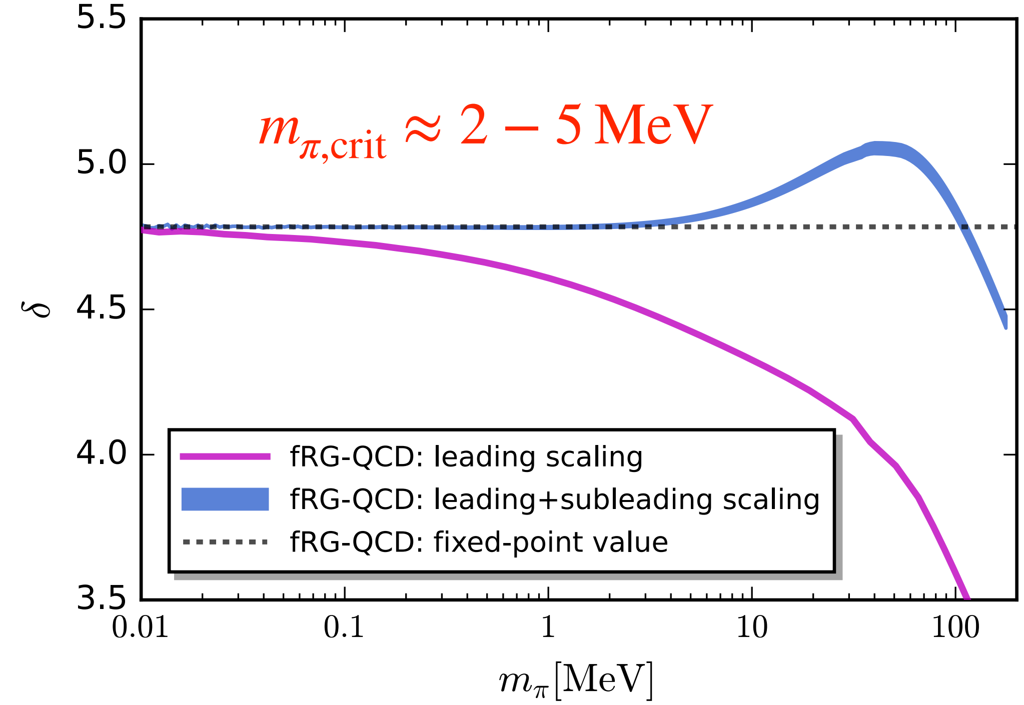
- **The almost linear dependence of the pseudo-critical temperature on the pion mass has nothing to do with the criticality.**
- **So what is the size of the critical region in QCD?**

Critical region in QCD

Scaling in the external field:

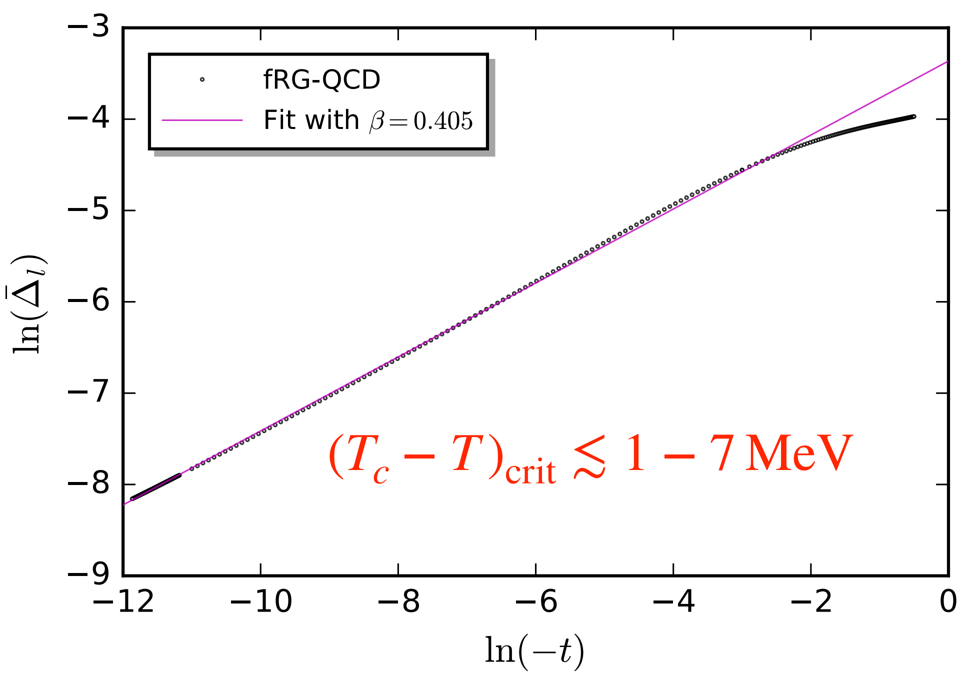


Critical exponent δ:



$$\bar{\Delta}_l^{(\mathrm{crit})}(m_\pi) = B_c\, m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H}\right]$$

Scaling in the temperature:



- **QCD at physical light quark mass is far away from the critical region.**
- **The scaling behavior is observed for the first time in the calculations of first-principles QCD.**

Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, arXiv:2310.19853.

Relaxation dynamics of the critical mode

- Langevin dynamics of the critical mode:

$$Z_\phi^{(t)}\partial_t\sigma - Z_\phi^{(i)}\partial_i^2\sigma + U'(\sigma) = \xi$$

with the correlation of the Gaussian white noise

$$\langle \xi(t,\boldsymbol{x})\xi(t',\boldsymbol{x}')\rangle = 2\,Z_\phi^{(t)}\,T\,\delta(t-t')\delta(\boldsymbol{x}-\boldsymbol{x}')$$

- Inputs from first-principles functional QCD: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Effective potential:

$$U'(\sigma) = \left.\frac{\delta\Gamma[\Phi]}{\delta\sigma}\right|_{\substack{\sigma(x)=\sigma \\ \tilde{\Phi}=\tilde{\Phi}_{\rm EoM}}}$$

Spatial wave function:

$$Z_\phi^{(i)} = \left.\frac{\partial\Gamma^{(2)}_{\sigma\sigma}(p_0,\boldsymbol{p})}{\partial\boldsymbol{p}^2}\right|_{\substack{p_0=0 \\ \boldsymbol{p}=0}}$$

Temporal wave function:

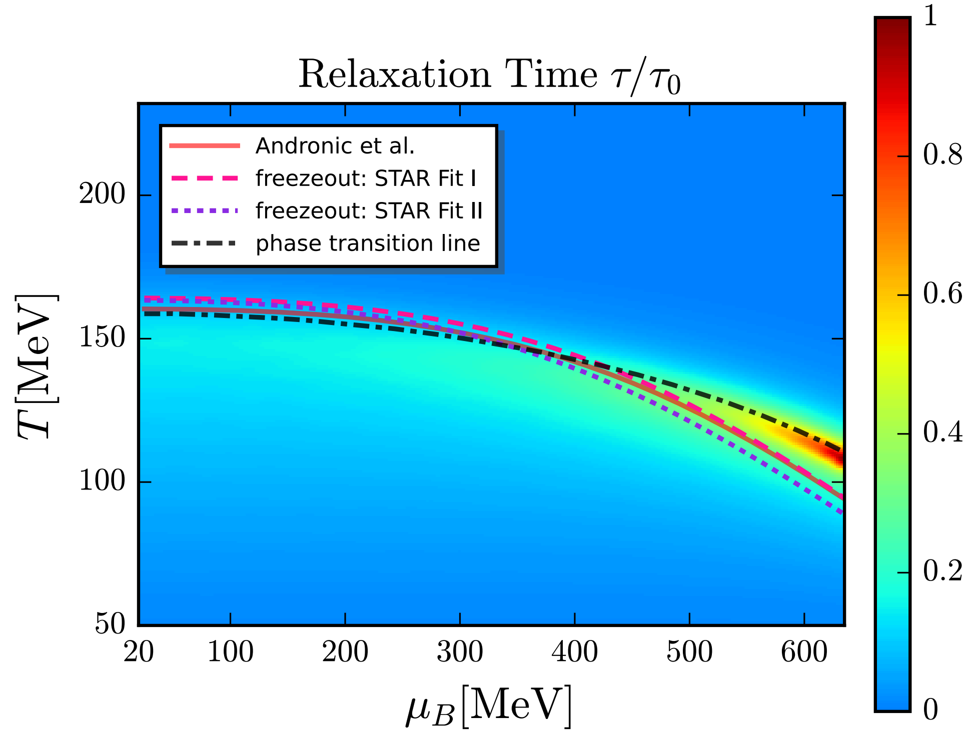
$$Z_\phi^{(t)} = \lim_{|\boldsymbol{p}|\to 0}\lim_{\omega\to 0}\frac{\partial}{\partial\omega}{\rm Im}\,\Gamma^{(2)}_{\sigma\sigma,{\rm R}}(\omega,\boldsymbol{p})$$

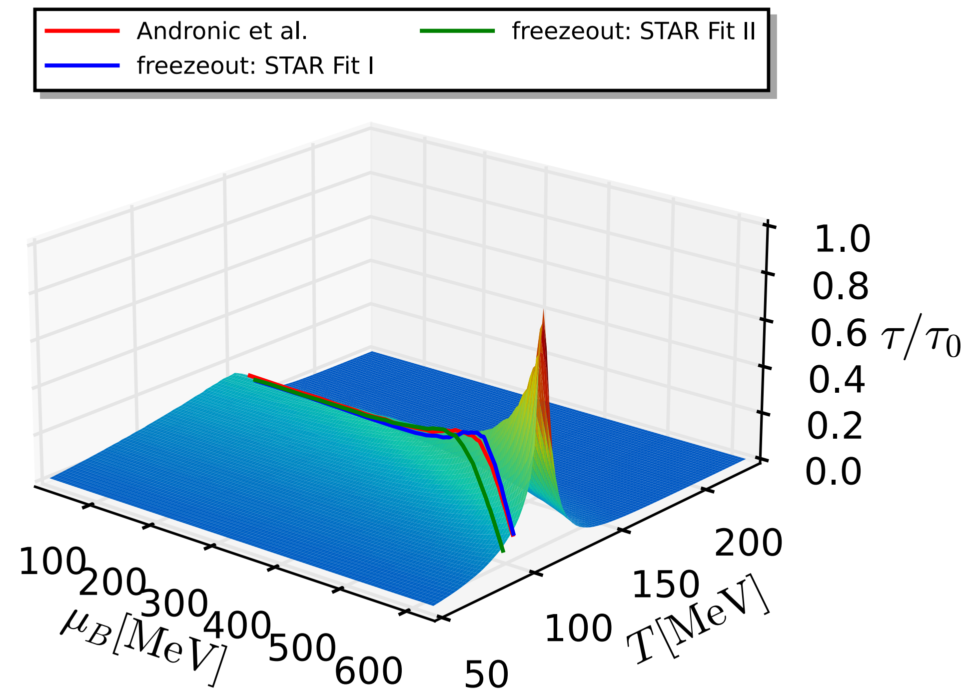
with

$$\Gamma^{(2)}_{\sigma\sigma,{\rm R}}(\omega,\boldsymbol{p}) = \lim_{\epsilon\to 0^+}\Gamma^{(2)}_{\sigma\sigma}\big(p_0=-{\rm i}(\omega+{\rm i}\epsilon),\boldsymbol{p}\big)$$

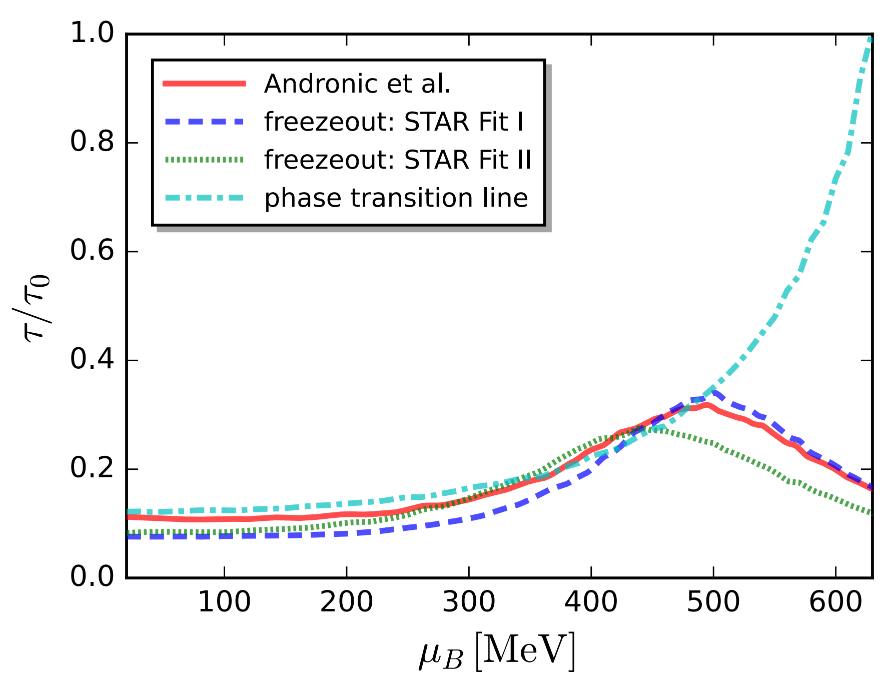
Relaxation time in QCD phase diagram

Relaxation time:





Relaxation time at the freezeout :

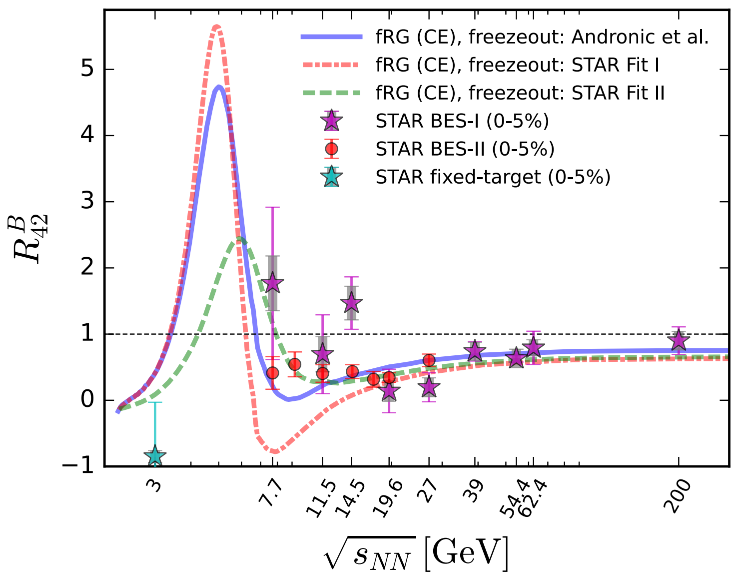


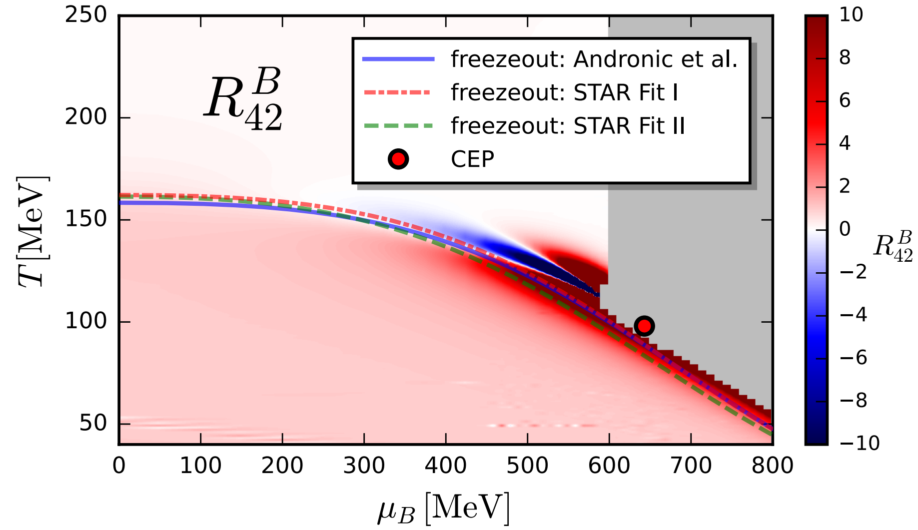
Tan, Yin, Chen, Huang, WF, in preparation

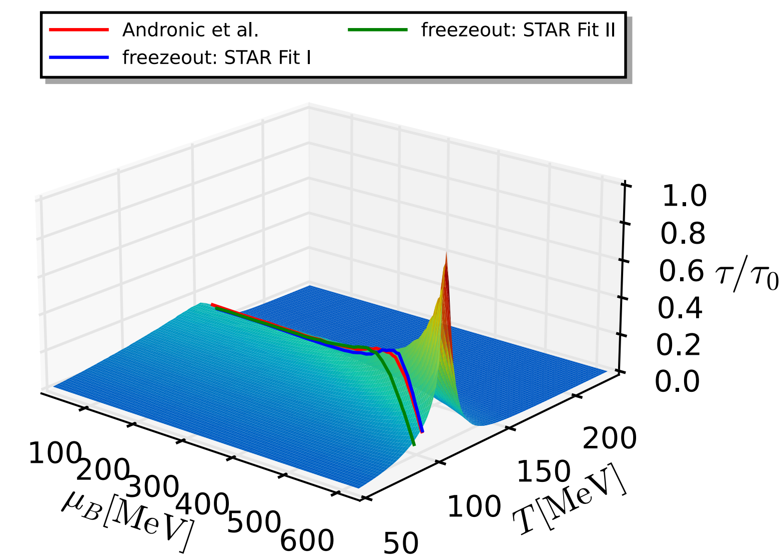
See also:
M. Bluhm *et al.*, *NPA* 982 (2019) 871

- Relaxation time drops quickly once the system is away from the critical regime.

Summary





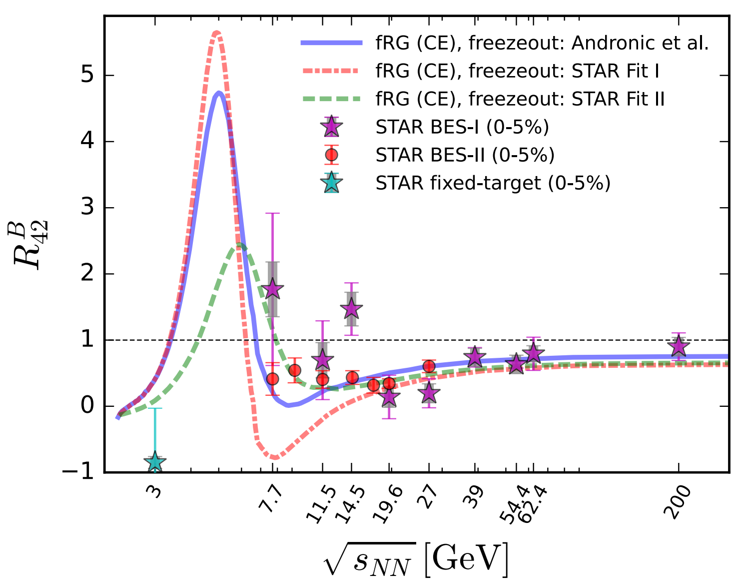


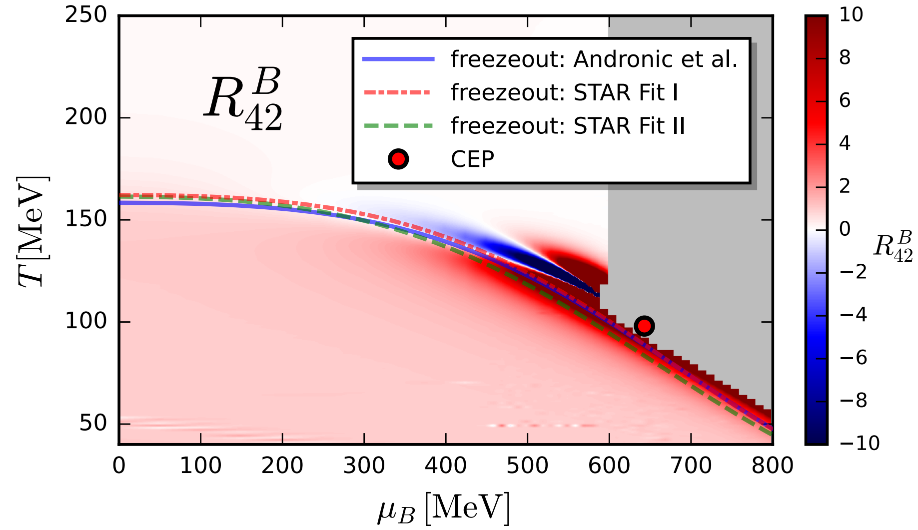
★ Recent BES-II fluctuation data indicate that, very possibly, there is no CEP with $\mu_B \lesssim 400$ MeV.

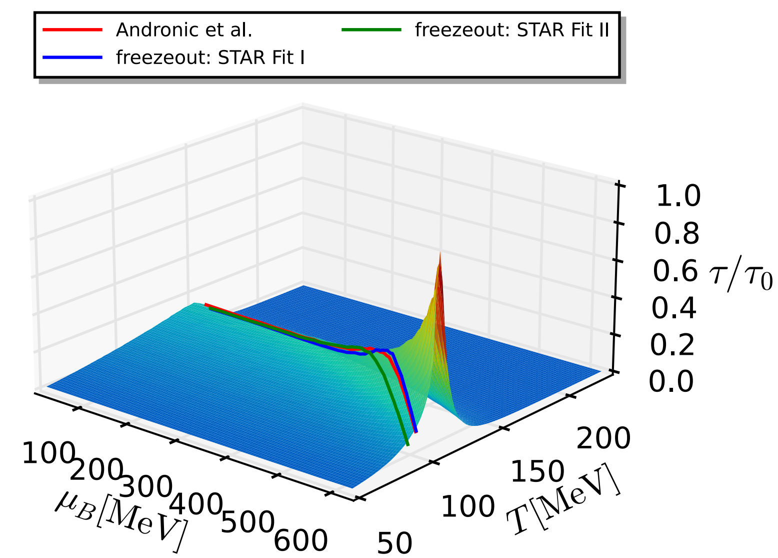
★ A prominent peak structure is predicted in baryon number fluctuations in the collision energy range of 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, which need to be confirmed in experiments in the near future.

★ The size of critical region near CEP is found to be small from first-principles functional QCD from both static and dynamic perspectives.

Summary







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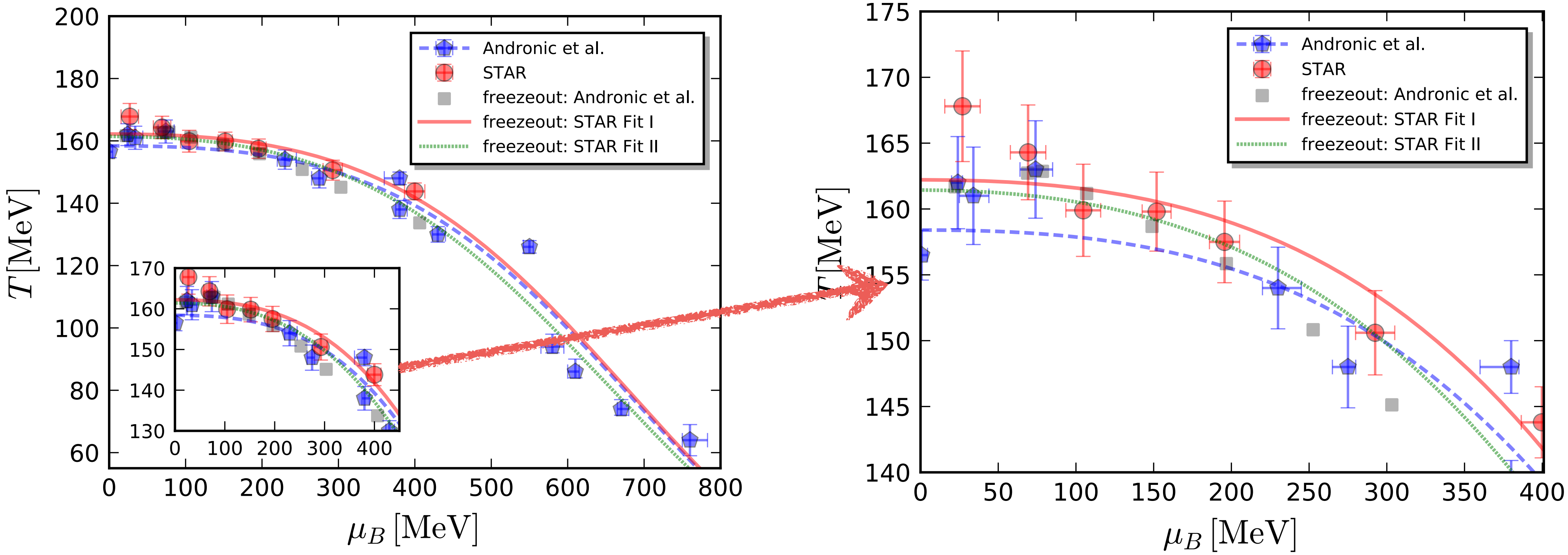
Thank you very much for your attentions!

Backup

Determination of the freeze-out curve



three freeze-out curves

1. **freeze-out: Andronic *et al.***

Andronic, Braun-Munzinger, Redlich, *Nature* 561 (2018) 7723, 321

2. **freeze-out: STAR Fit I**

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

3. **freeze-out: STAR Fit II**

$$\mu_{B_{\mathrm{CF}}} = \frac{a}{1+0.288\sqrt{s_{\mathrm{NN}}}} \,,$$

$$T_{\mathrm{CF}} = \frac{T^{(0)}_{\mathrm{CF}}}{1+\exp\left(2.60-\ln(\sqrt{s_{\mathrm{NN}}})/0.45\right)}$$

all data points

neglecting first two at low μ_B and the last one

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

Functional renormalization group

Functional integral with an IR regulator

$$Z_k[J] = \int (\mathscr{D}\hat{\Phi}) \exp\left\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \right\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

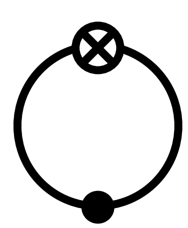
flow of the Schwinger function:

$$\partial_t W_k[J] = -\frac{1}{2}\mathrm{STr}\left[\left(\partial_t R_k\right) G_k\right] - \frac{1}{2}\Phi_a \partial_t R_k^{ab} \Phi_b$$

Legendre transformation:

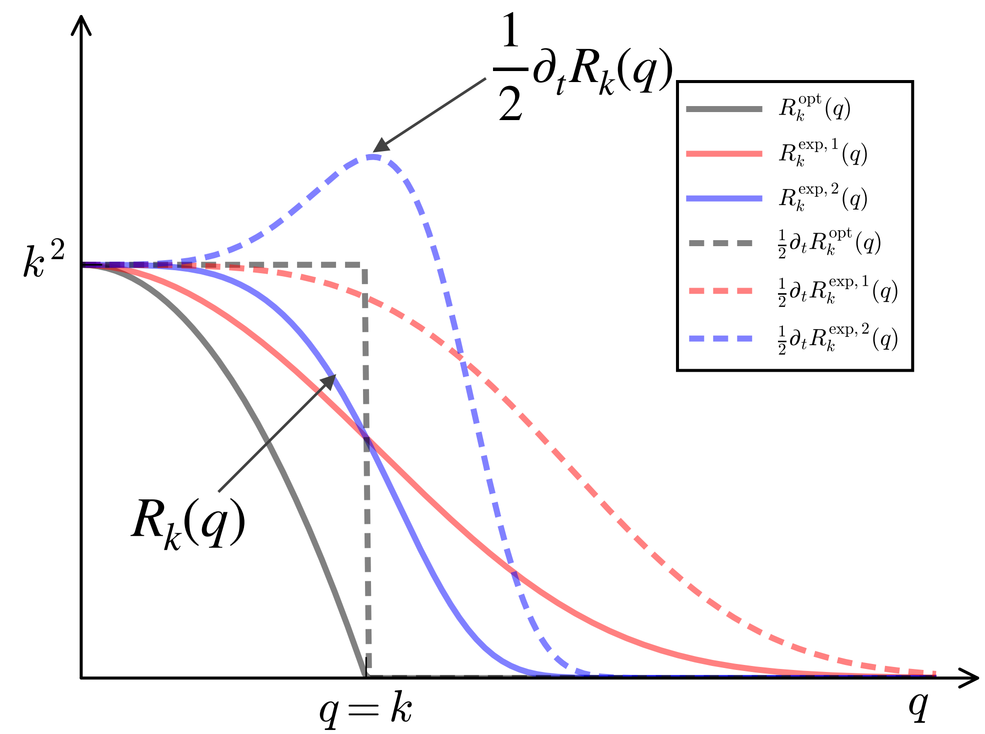
$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

flow of the effective action:

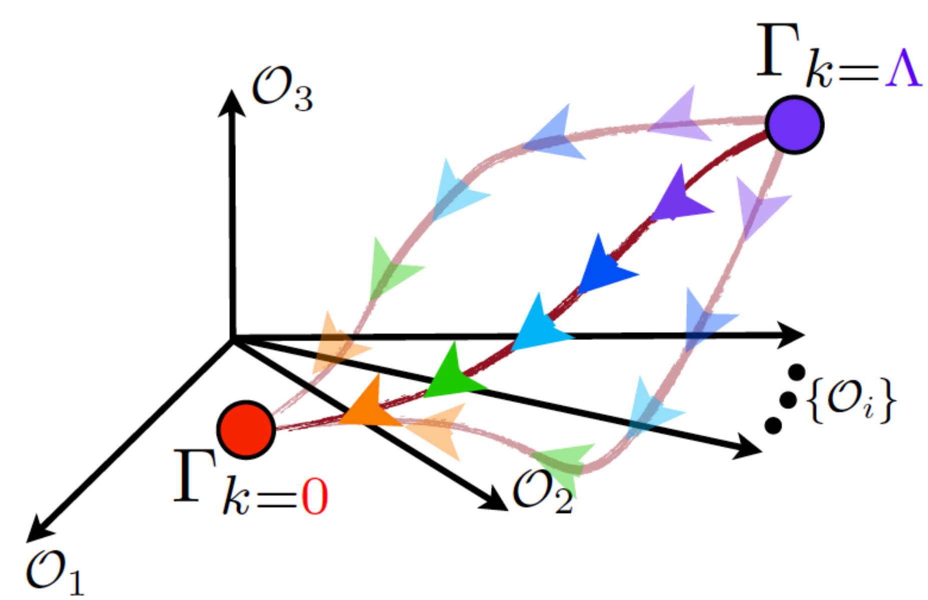
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\left[\left(\partial_t R_k\right) G_k\right] = \frac{1}{2}$$ 

Wetterich formula

C. Wetterich, *PLB*, 301 (1993) 90

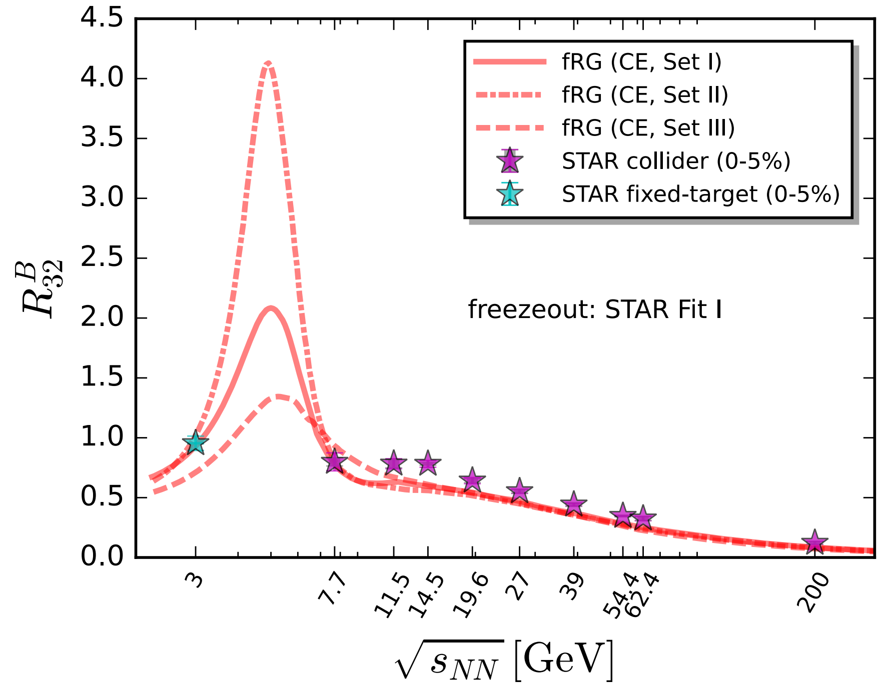


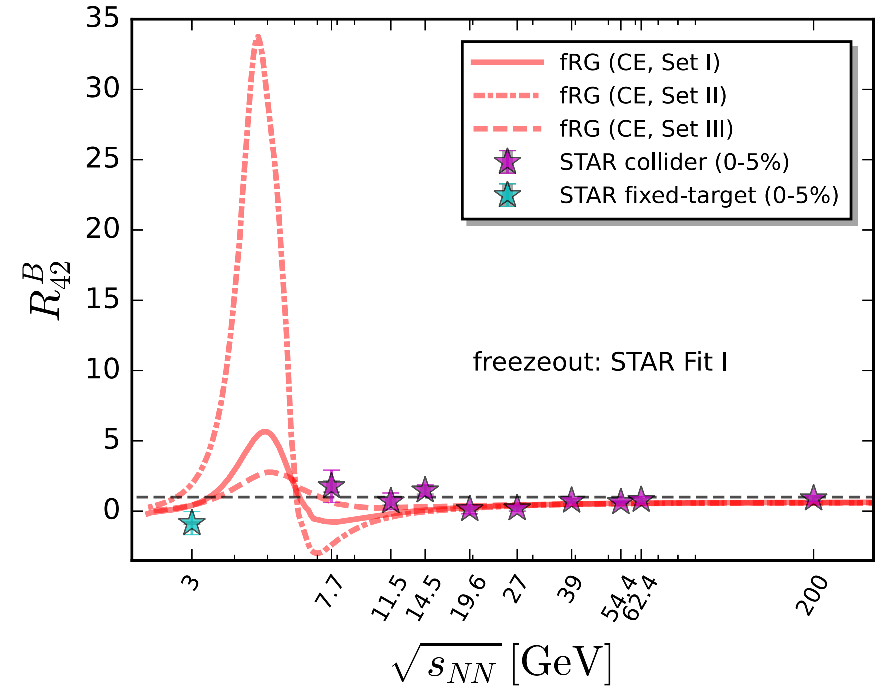
$$G_{k,ab} = \gamma^c{}_a\left(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi]\right)^{-1}_{cb},$$



Review: WF, *CTP* 74 (2022) 097304, arXiv: 2205.00468 [hep-ph]

Dependence of the location of CEP





STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508