Locating the critical end point and the phase boundary of QCD from first principle QCD

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Trento, September 9th 2024

for the fQCD collaboration

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Bundesministerium für Bildung und Forschung

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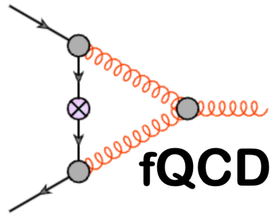
STRUCTURES CLUSTER OF EXCELLENCE

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

isoQuant SFB1225

fQCD collaboration



Dalian, Beijing, Darmstadt, Heidelberg, Gießen

Braun, Chen, Fu, Gao, Geissel, Huang, Lu, Ihssen, Pawlowski, Rennecke, Sattler, Schallmo, Stoll, Tan, Töpfel, Turnwald, Wessely, Wen, Yin, Zheng, Zorbach

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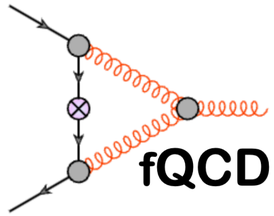

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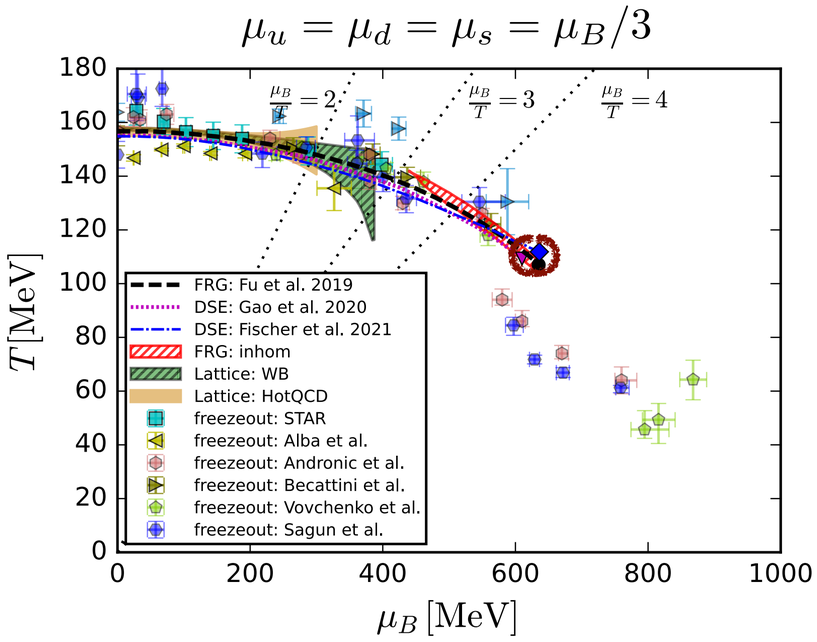
& Experiment-Theory collaboration

Phase structure of QCD and the CEP



Functional QCD: CEP estimate

fRG: **Fu, JMP, Rennecke, PRD 101 (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**

Gunkel, Fischer, PRD 104 (2021) 054022

$$(\mu_B\,,\,T)_{\mathrm{CEP}} \sim (600-650\,,\,105-115)\ \mathrm{MeV}$$

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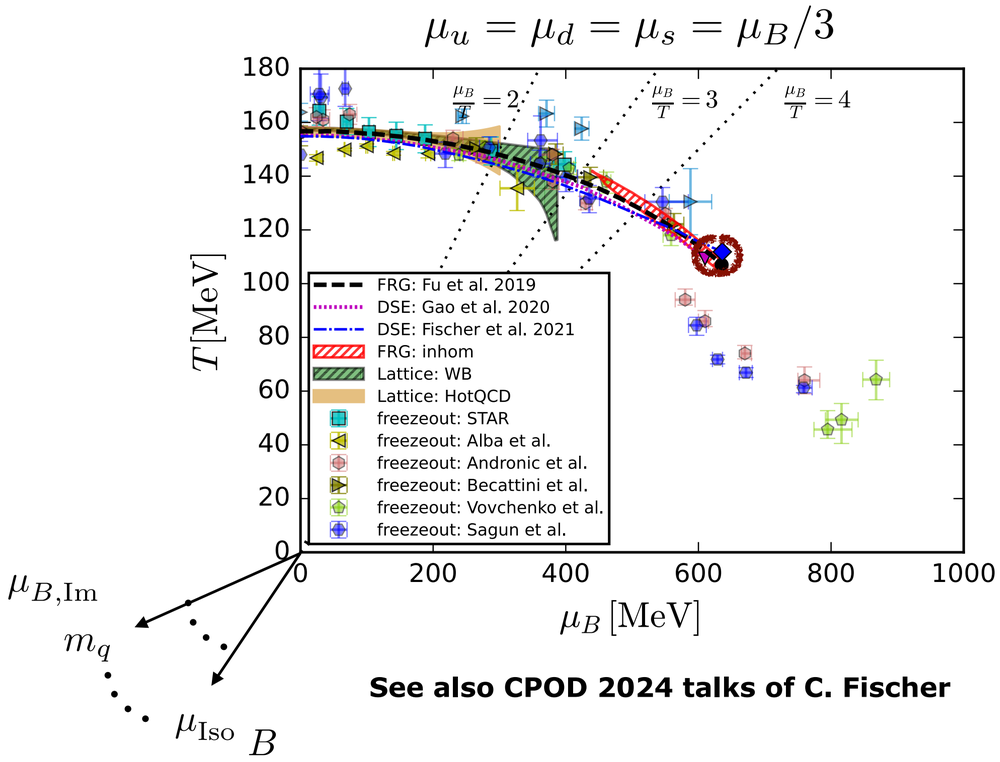

See also CPOD 2024 talks of C. Fischer

Collect all possible information/structure for physics understanding & extrapolations

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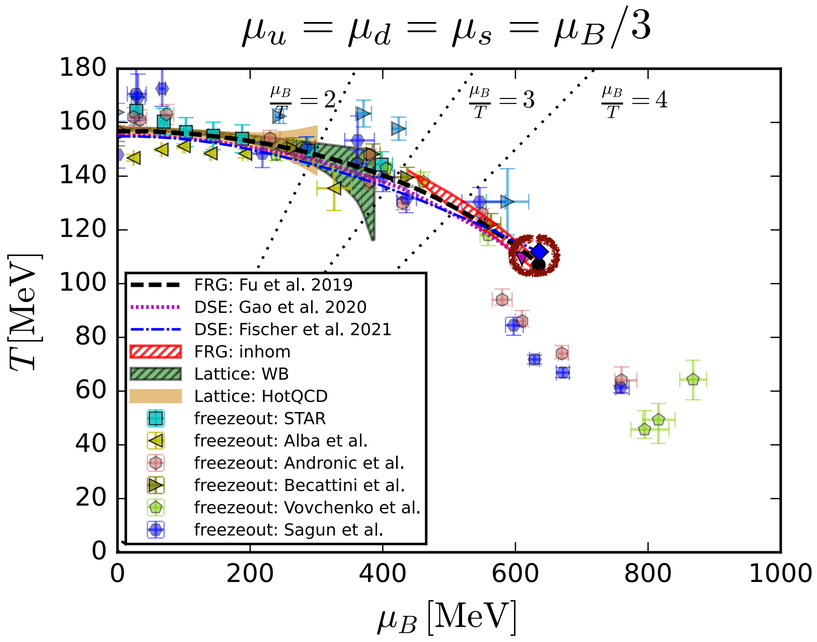

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Estimates & predictions

Requires computations in 1st principle QCD at

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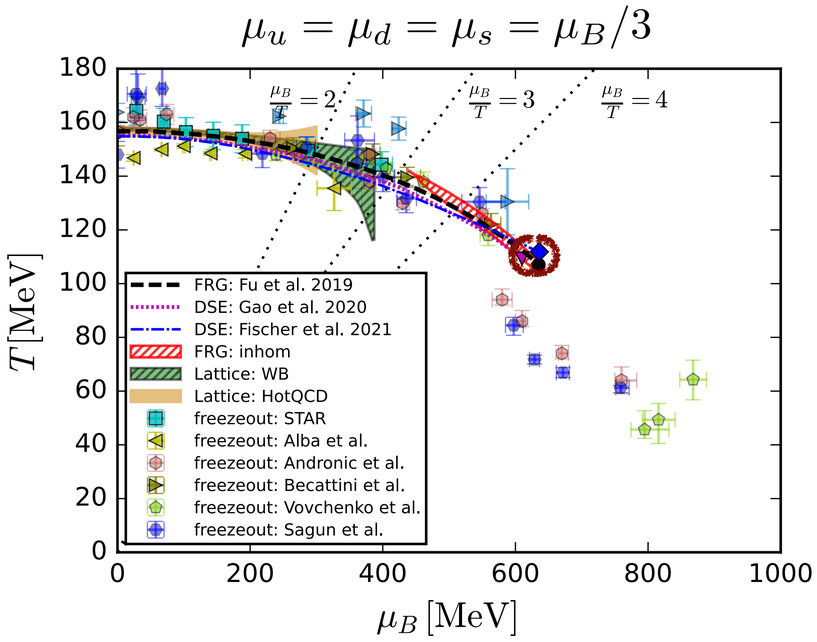

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Extrapolations for Pheno

Requires a discussion of the explicit & implicit assumptions

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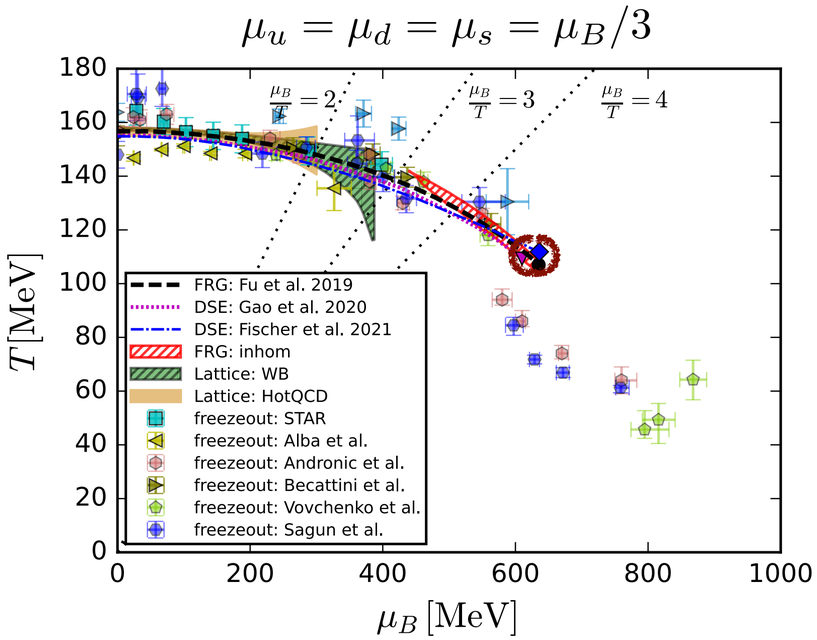

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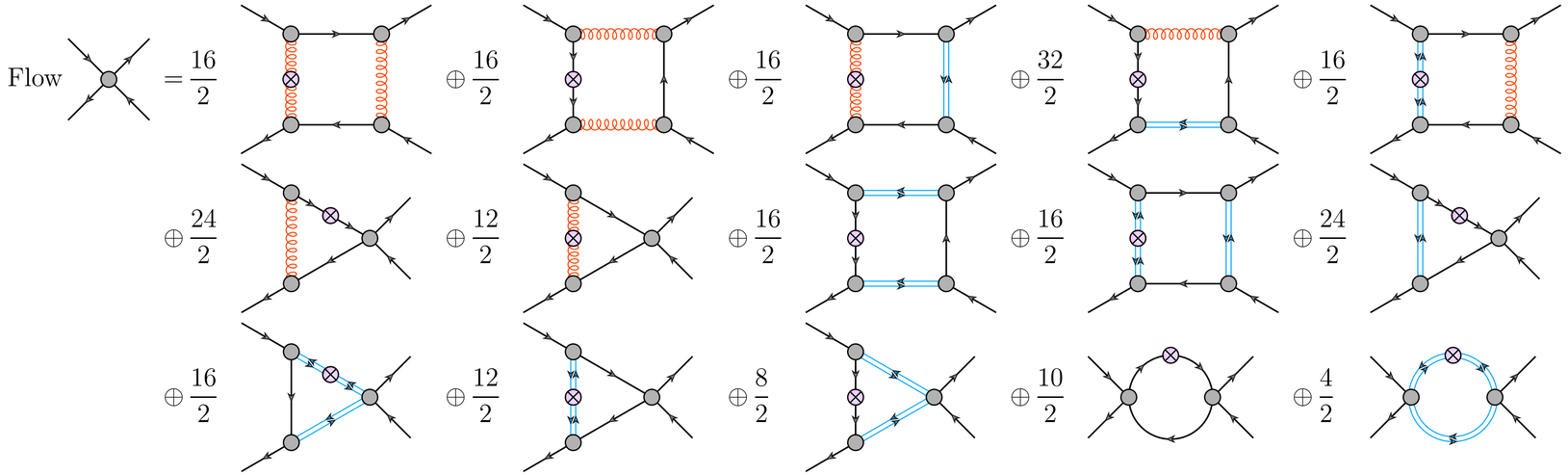
Lattice extrapolations

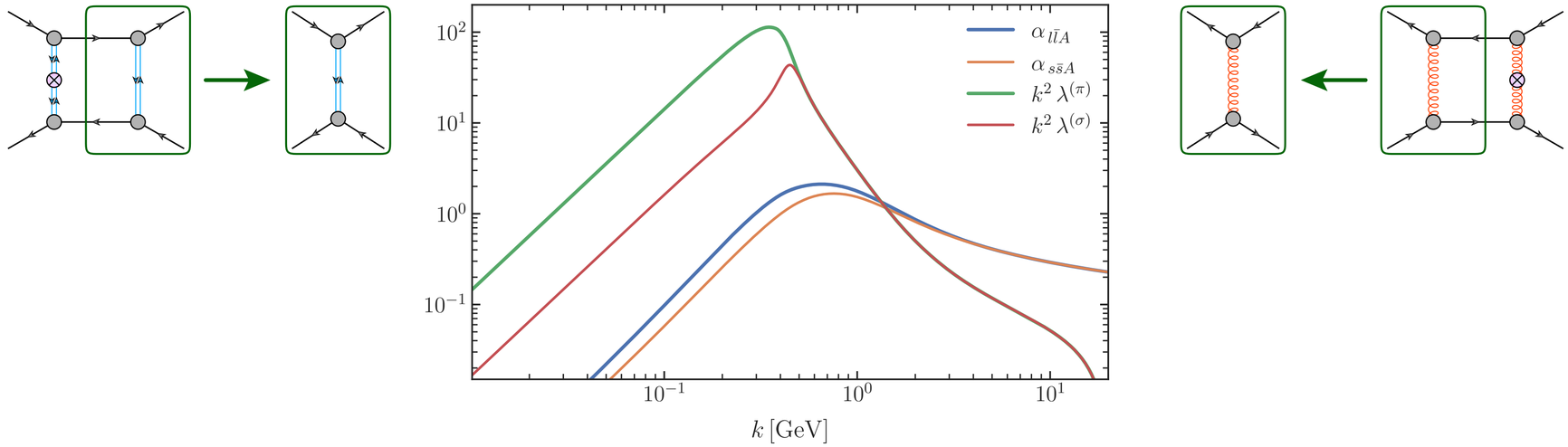
low energy effective theories: QM, NJL, PQM, PNJL, …, Holography

Phase structure from functional QCD: how to

How to: systematic error estimates & the LEGO® principle



Example: 4-quark scattering vertex



Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

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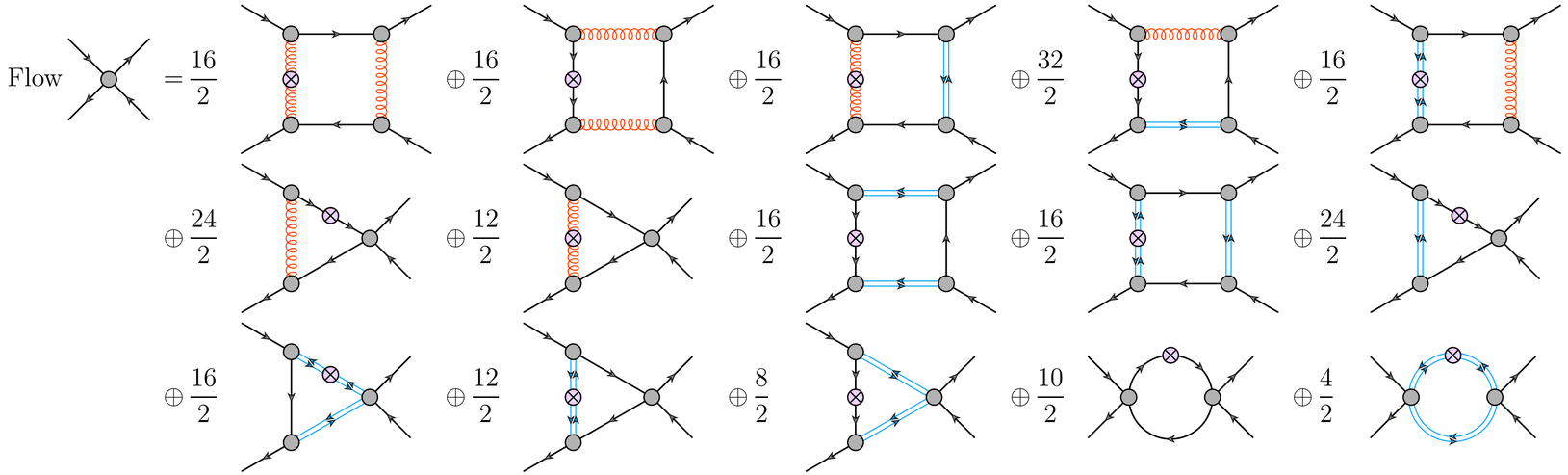

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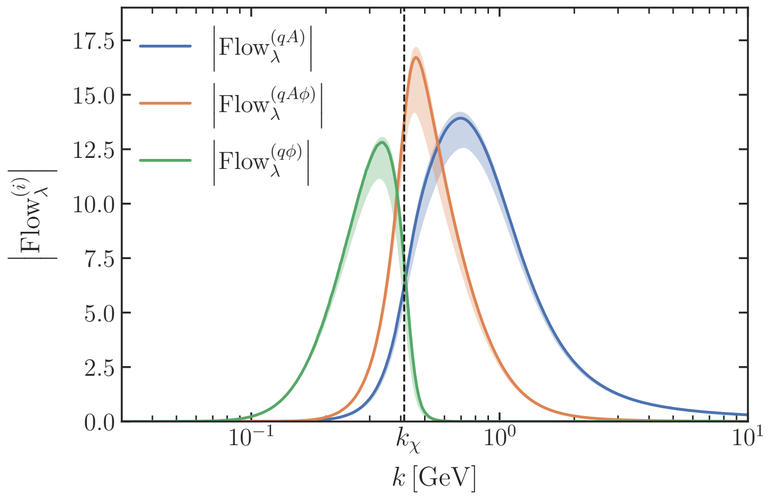

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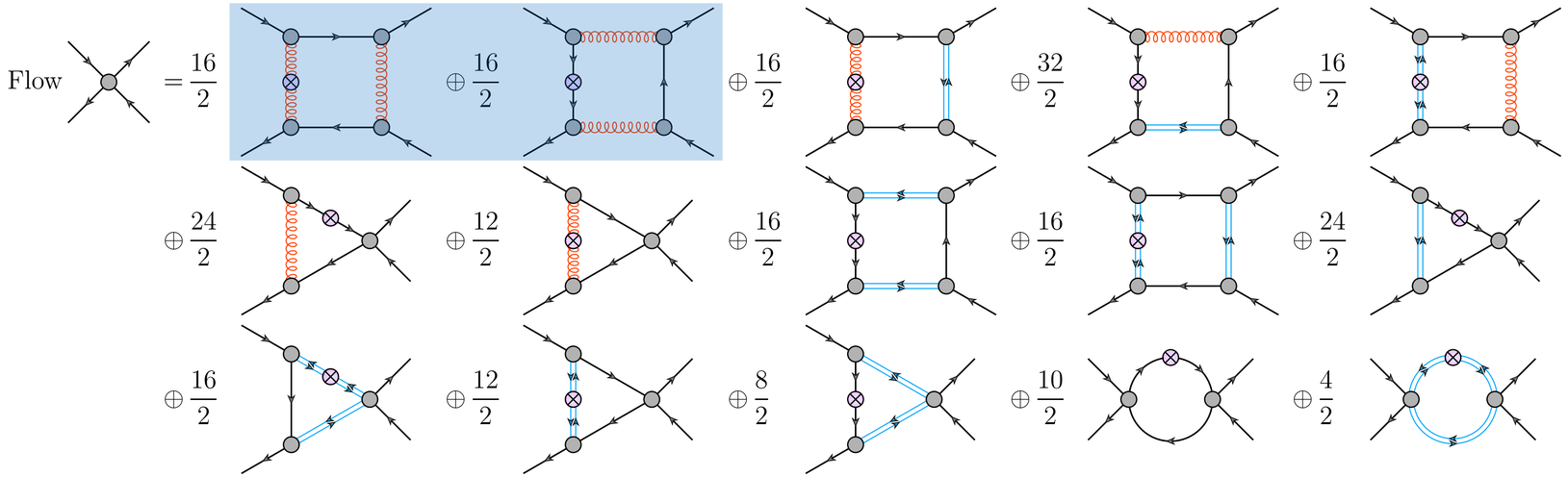

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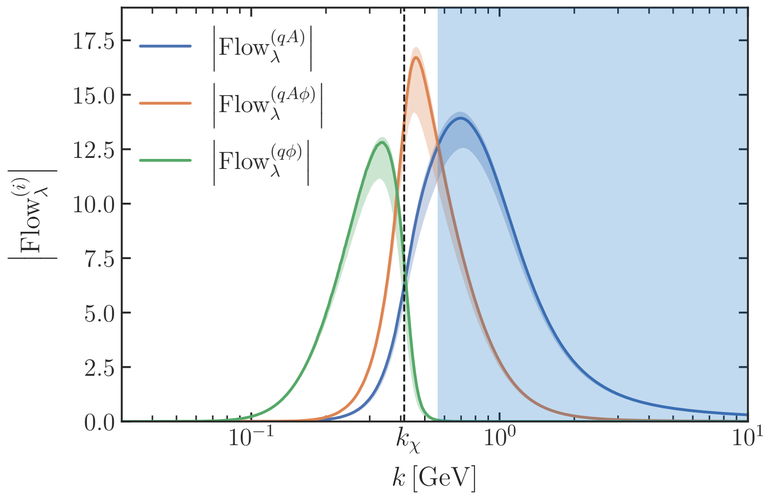

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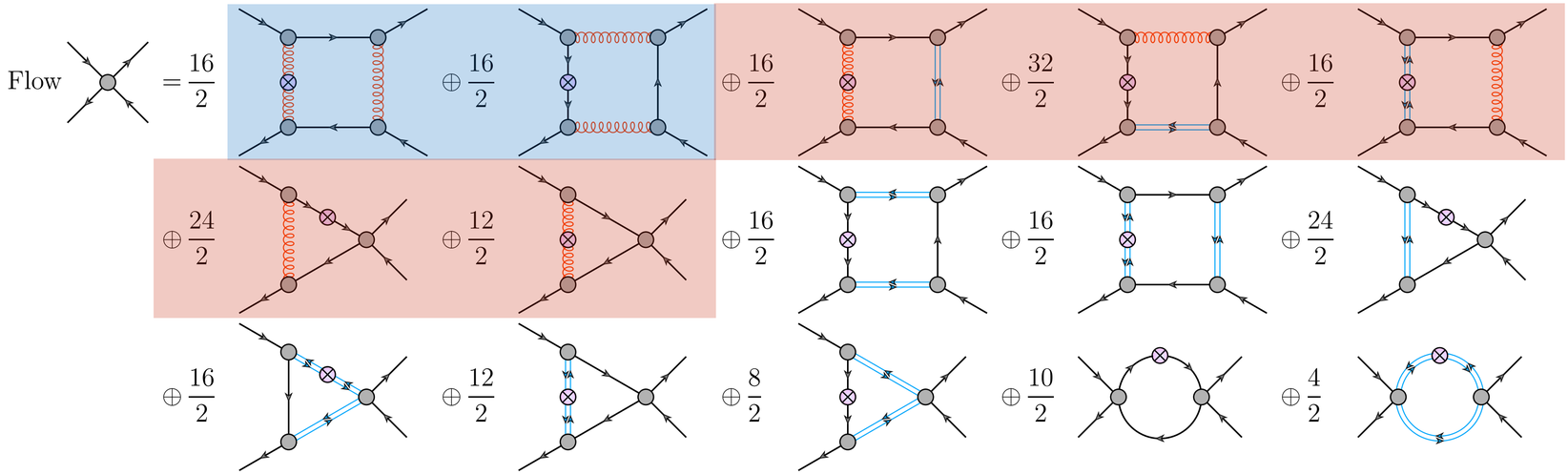

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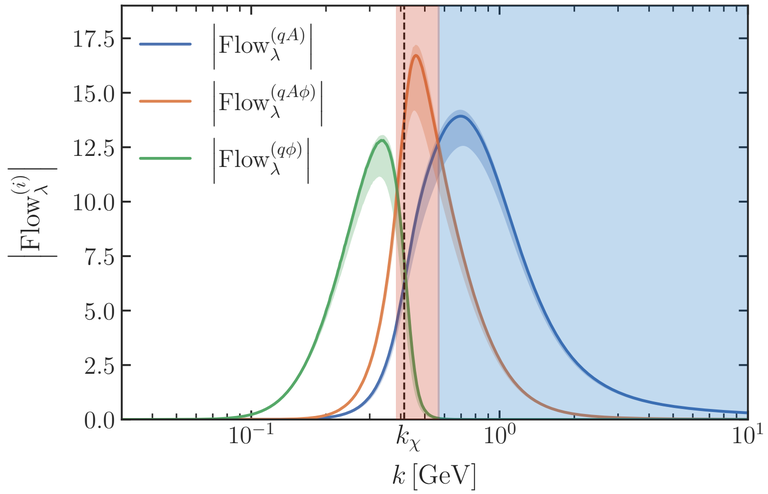

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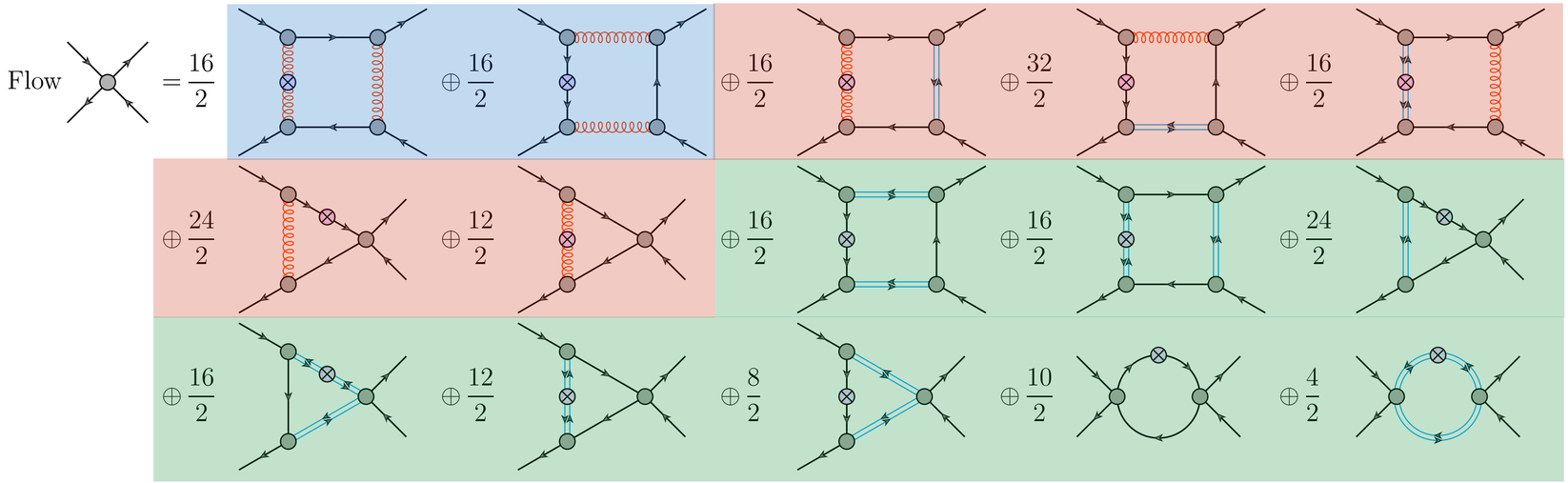

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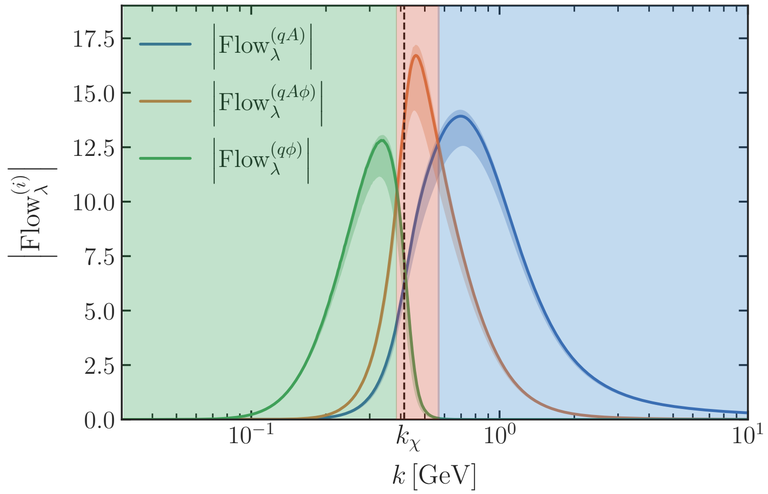

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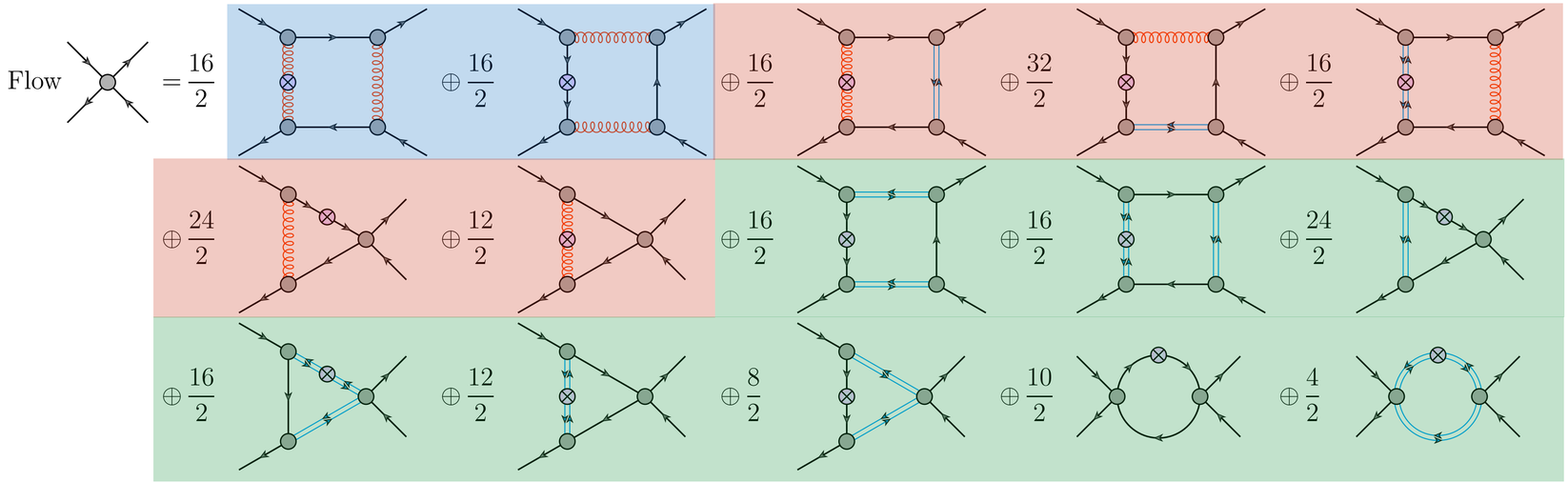
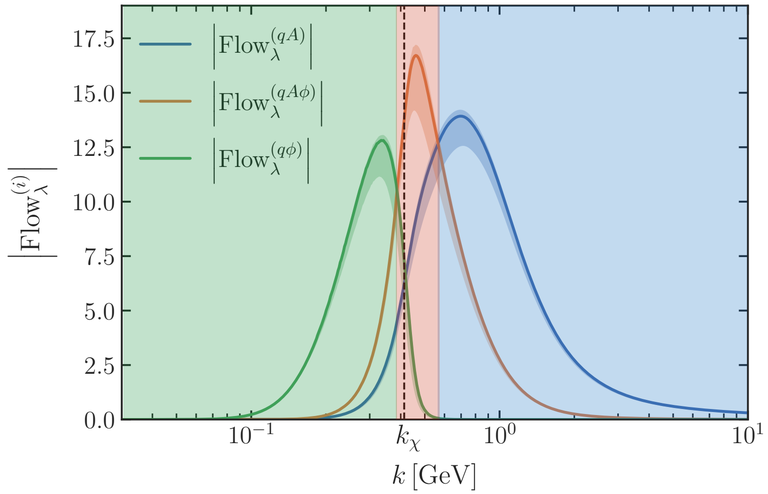


The unreasonable effectiveness of low energy effective theories



Access and combined use of error estimates from functional QCD & LEFTs

Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

How to: direct computations and the minimal point of view

Those are my interpretations,
and if you don't like them....
well, I have others

- **Self-consistent truncations to functional relations define analytic functions in μ_B , eg:**

$$\partial_t \Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B) = \mathrm{Loop}\left[\Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B),\ \Big\langle q(x)A_\mu(y)\bar{q}(z)\Big\rangle(\mu_B), \cdots;\ \mu_B\right]$$

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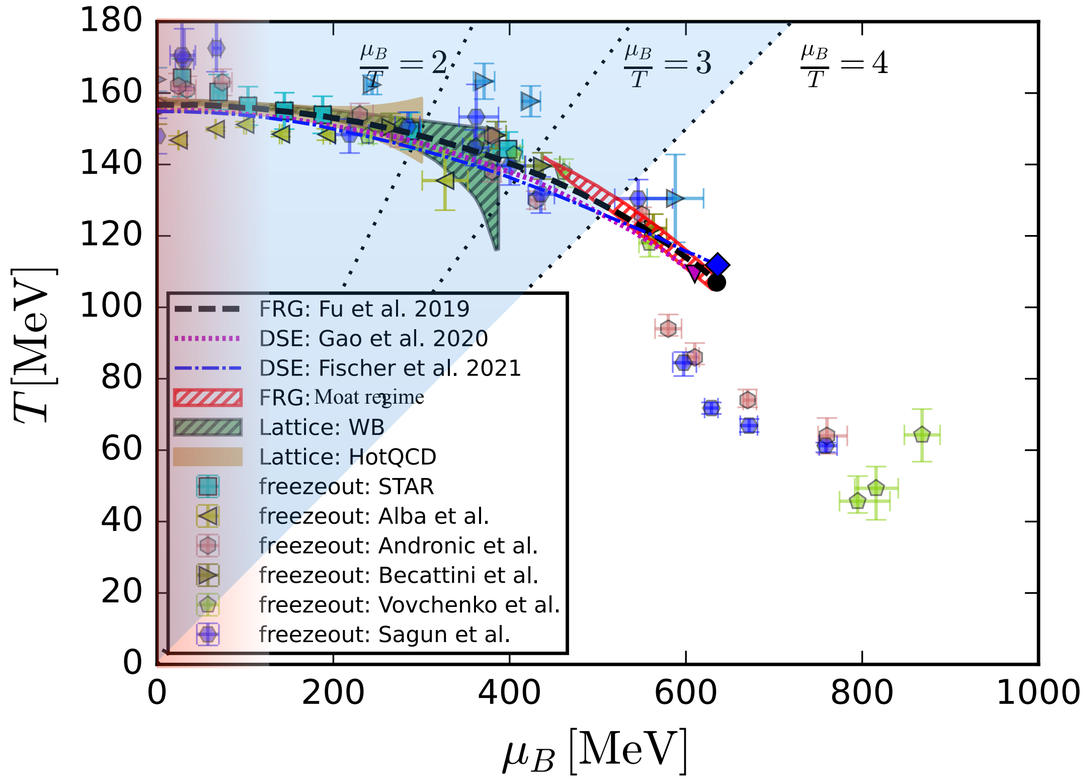
- **Consequences for functional QCD predictions at finite density**



By now the best truncations to functional QCD pass lattice benchmark tests at vanishing and small chemical potential

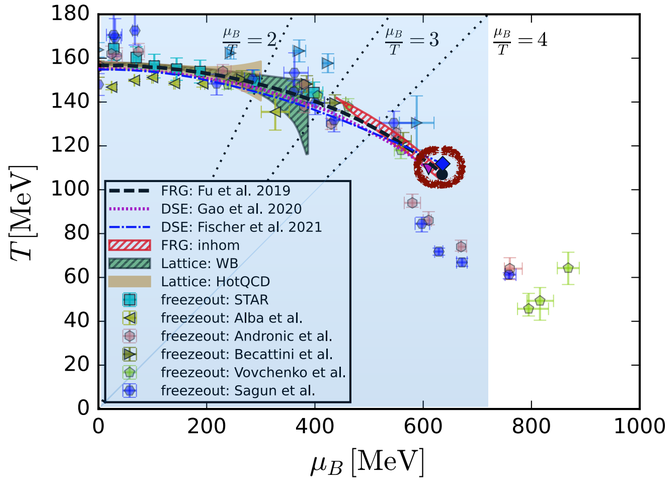
Regime of quantitative reliability of current best truncation

Unique: QCD-based analytic continuations that satisfy the lattice benchmarks at small chemical potential.

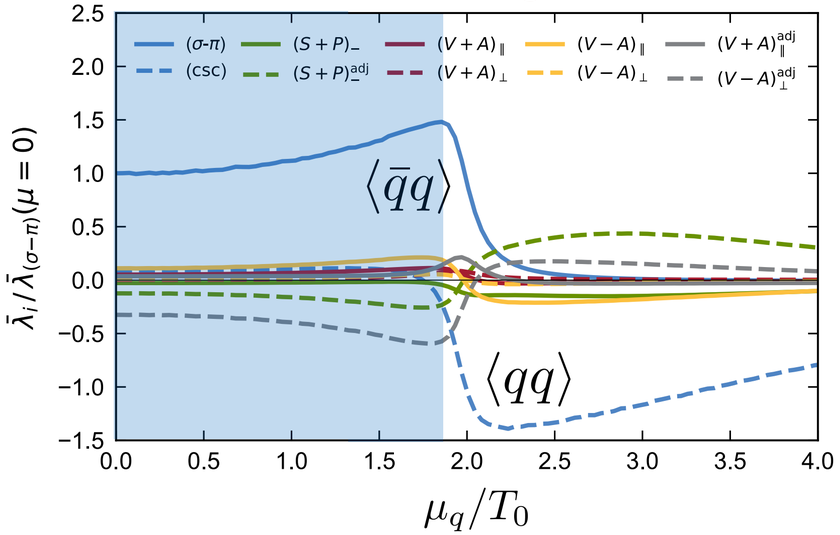
Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD + LEFTs)

Phase structure from functional QCD: Predictions & estimates

Predictions & estimates



Four-quark scattering channels



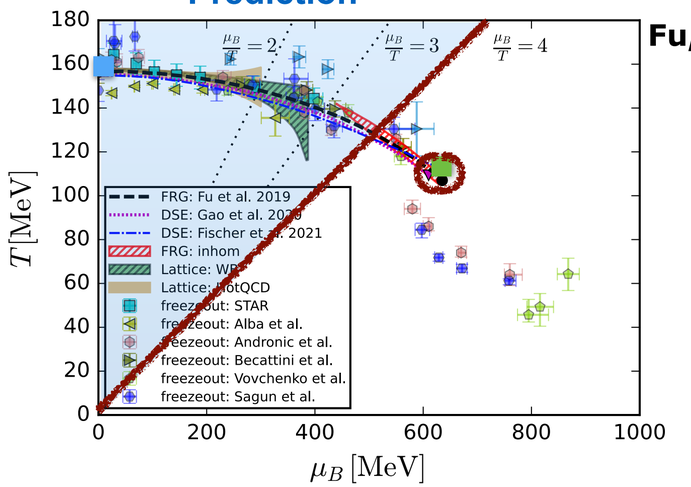
Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Dominance of scalar-pseudoscalar fluctuations

Pions & sigma mode

Predictions & estimates

Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032

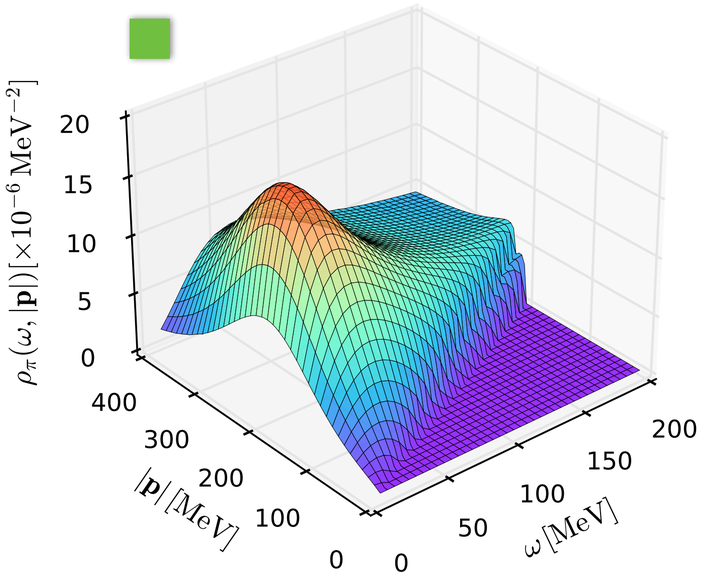
Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

see talk of Fabian Rennecke

Regime of quantitative reliability of current best truncation

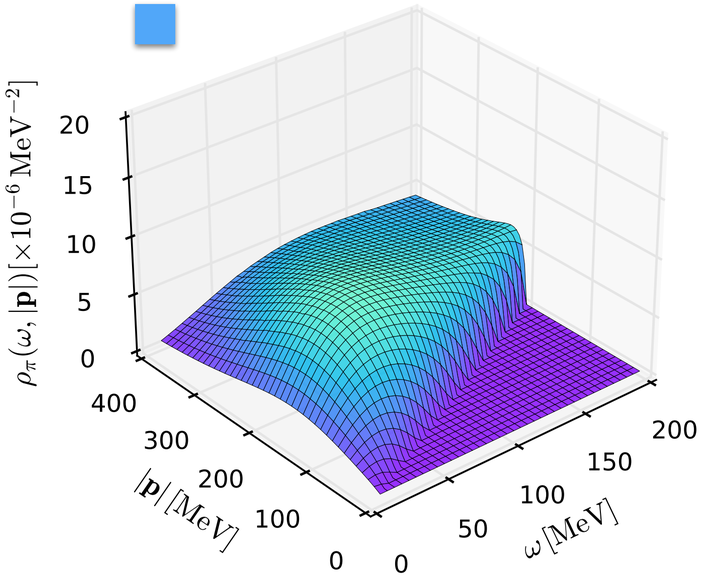
T=114 MeV & μ_B =630 MeV



Pion spectral functions

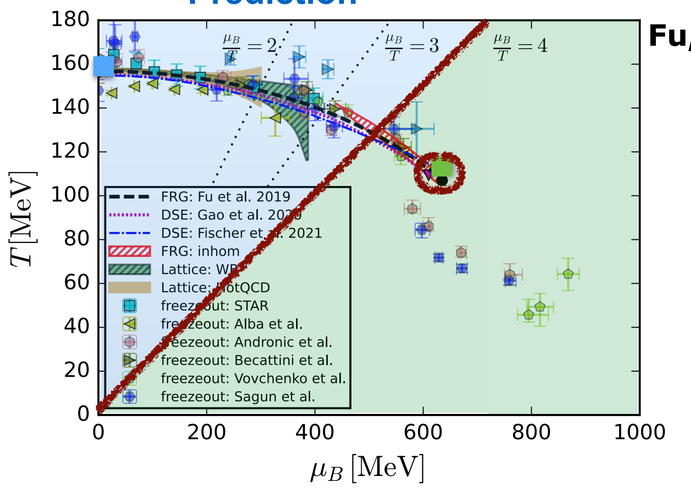
Fu, JMP, Pisarski, Rennecke, Wen, Yin, in prep

T=160 MeV & μ_B =0 MeV



Predictions & estimates

Prediction



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Estimate

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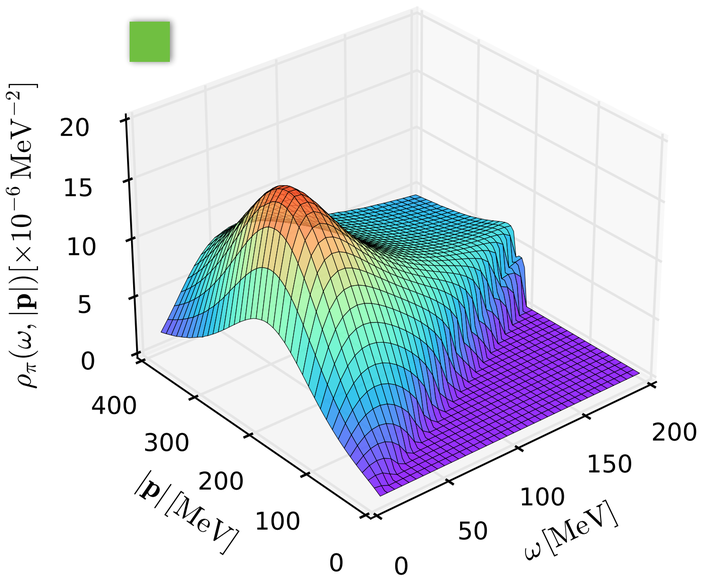

Moat regime is not captured quantitatively

Pion spectral functions

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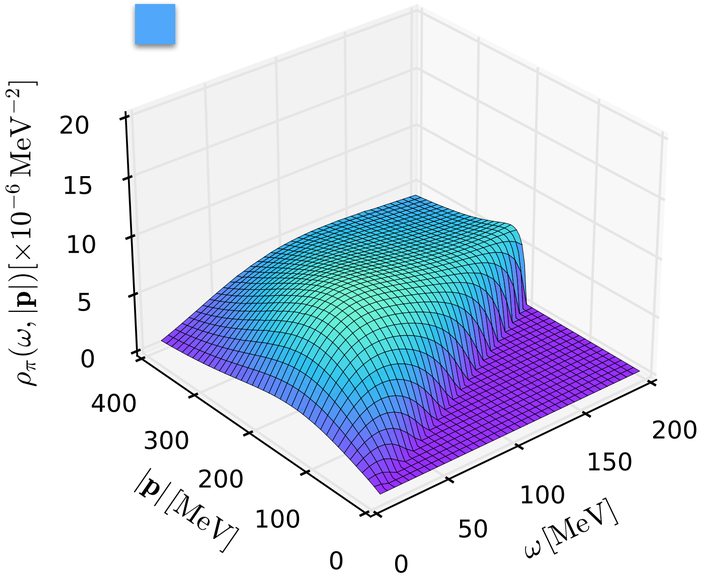
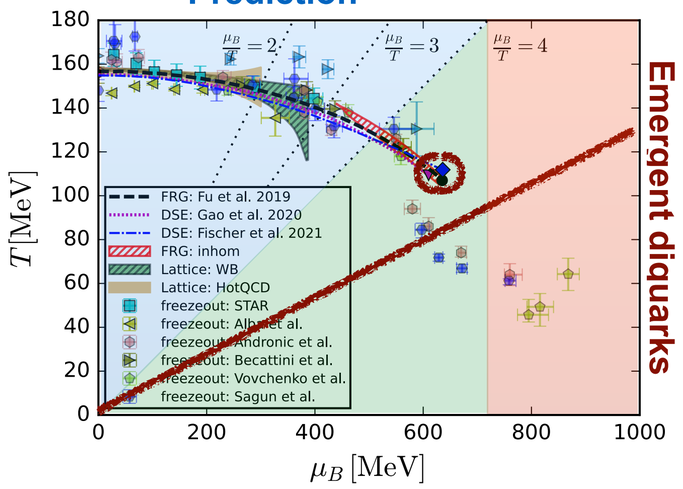


Predictions & estimates

Emergent diquarks

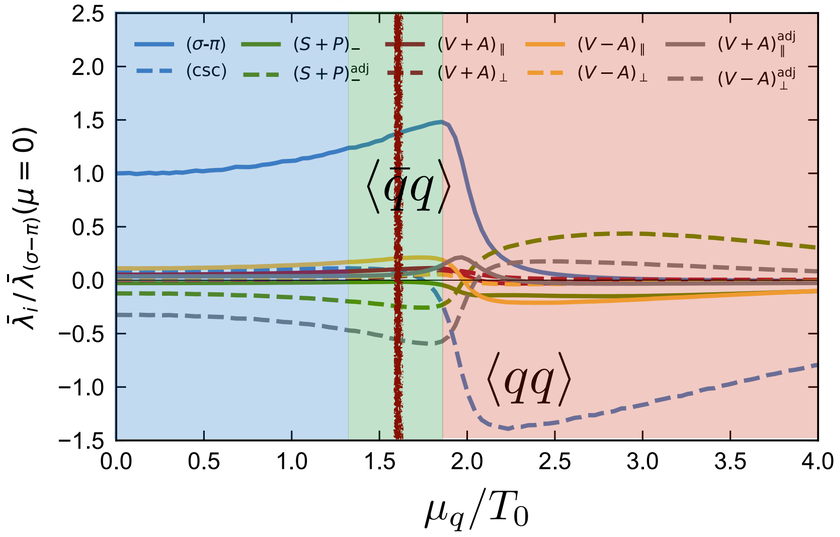
Prediction



Estimate

Regime of quantitative reliability of current best truncation

Emergent diquarks (fRG)

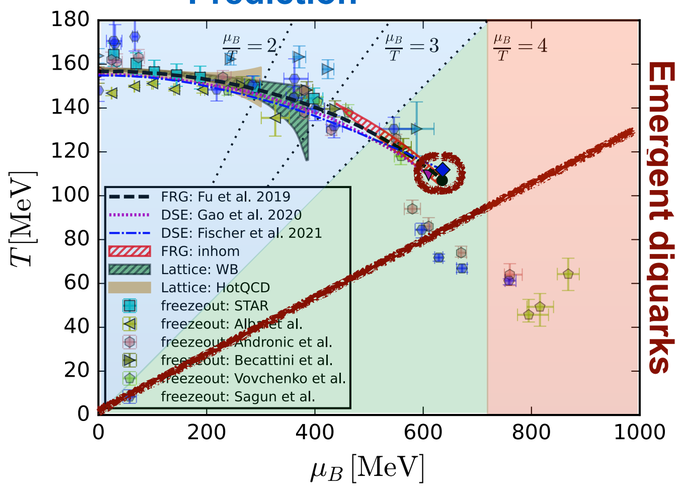


Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Predictions & estimates

Emergent diquarks

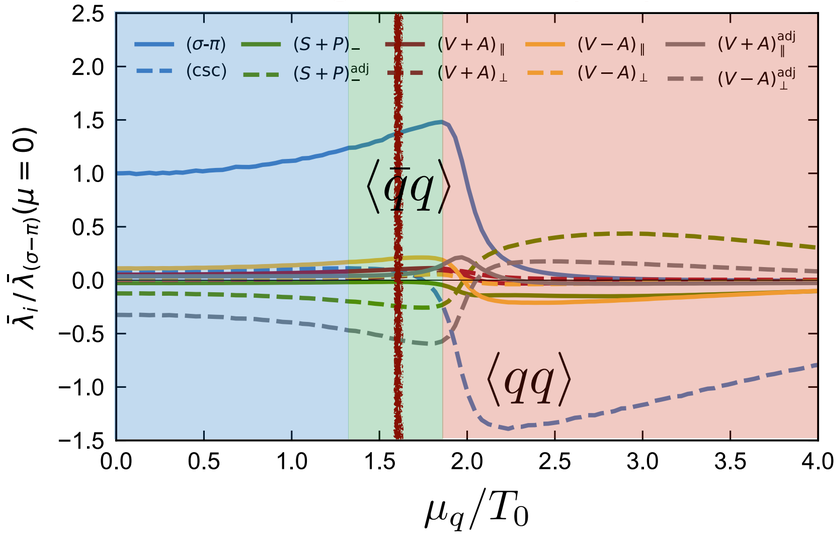
Prediction



Estimate

Regime of quantitative reliability of current best truncation

Emergent diquarks (fRG)



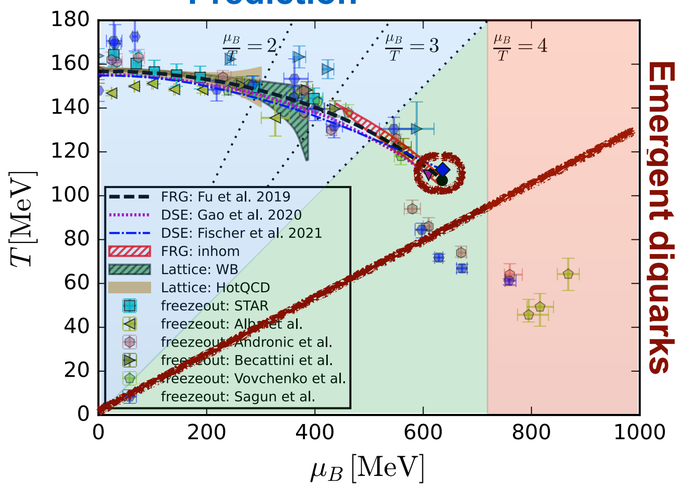
Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Emergent diquarks are not captured by extrapolations

Predictions & estimates

Emergent diquarks

Prediction



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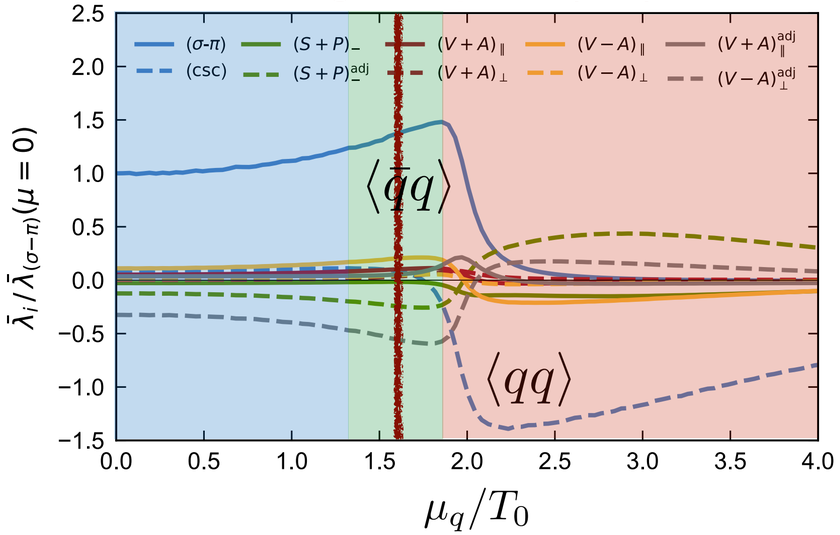
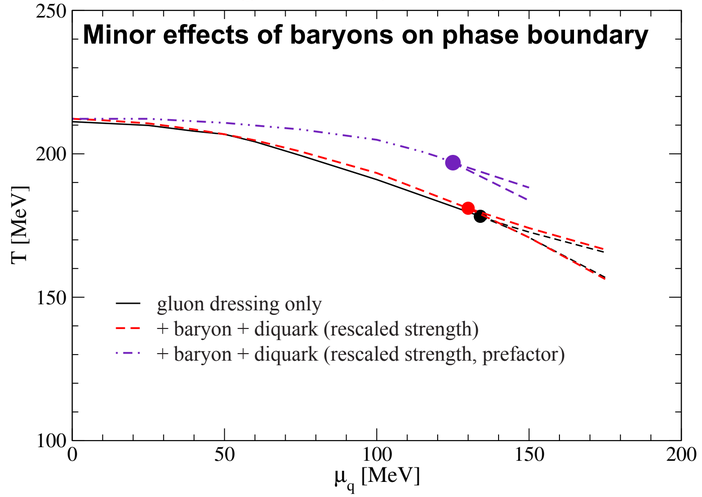


Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Emergent baryons (DSE)

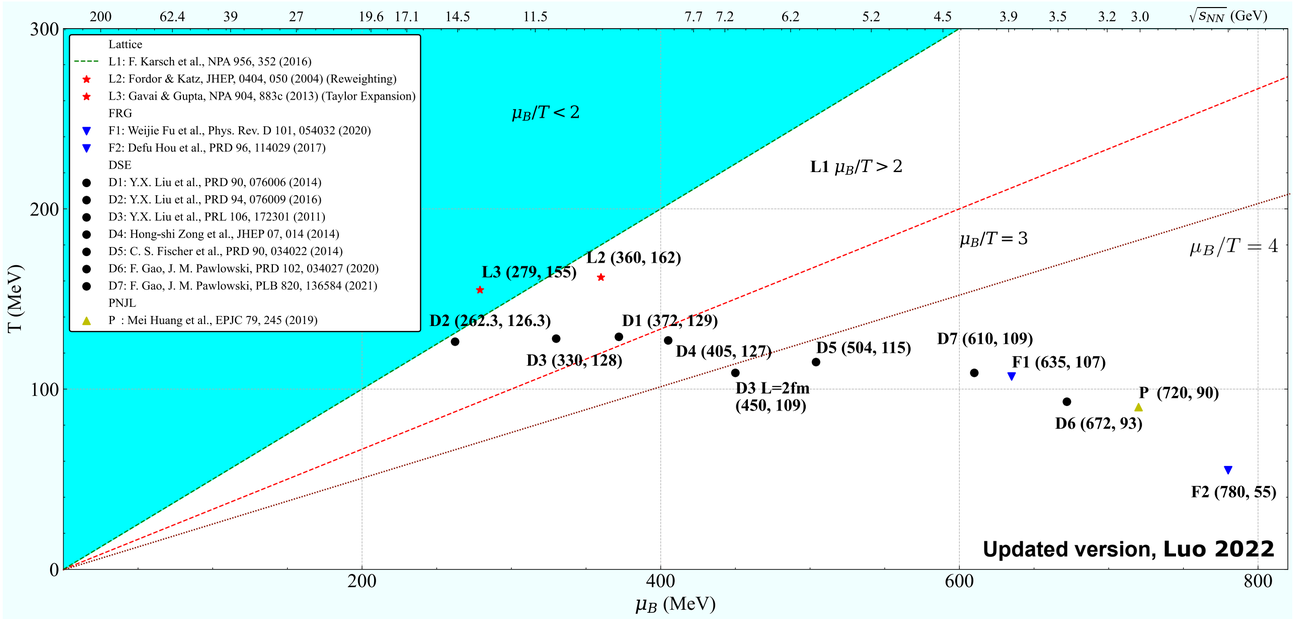


Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

CEP is standing for 'regime with new physics'

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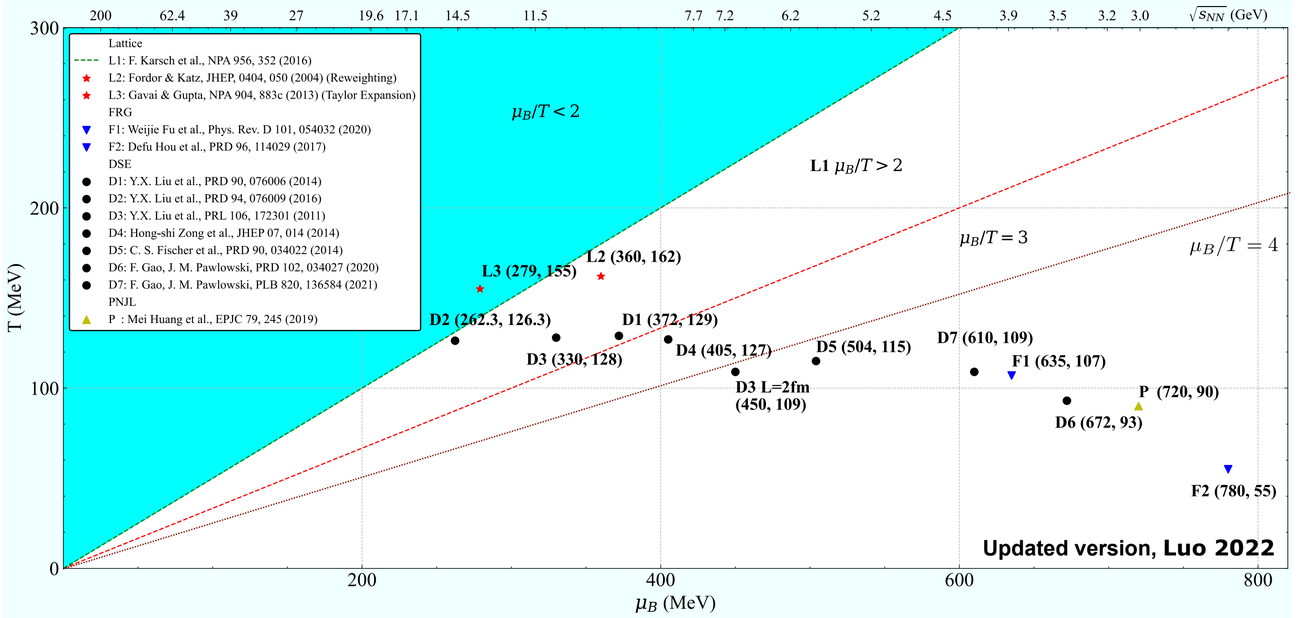

Common folklore since ~2004 ⟷ Large uncertainties for the estimation of CP location.

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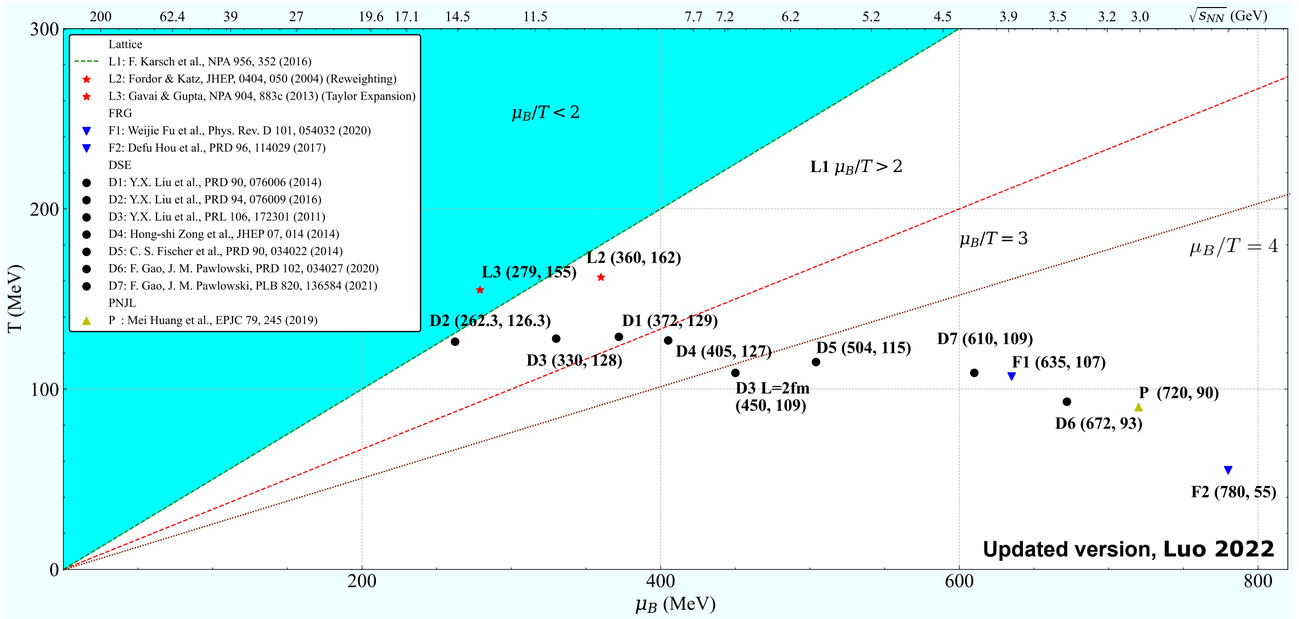

Large uncertainties for the estimation of CP location.

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

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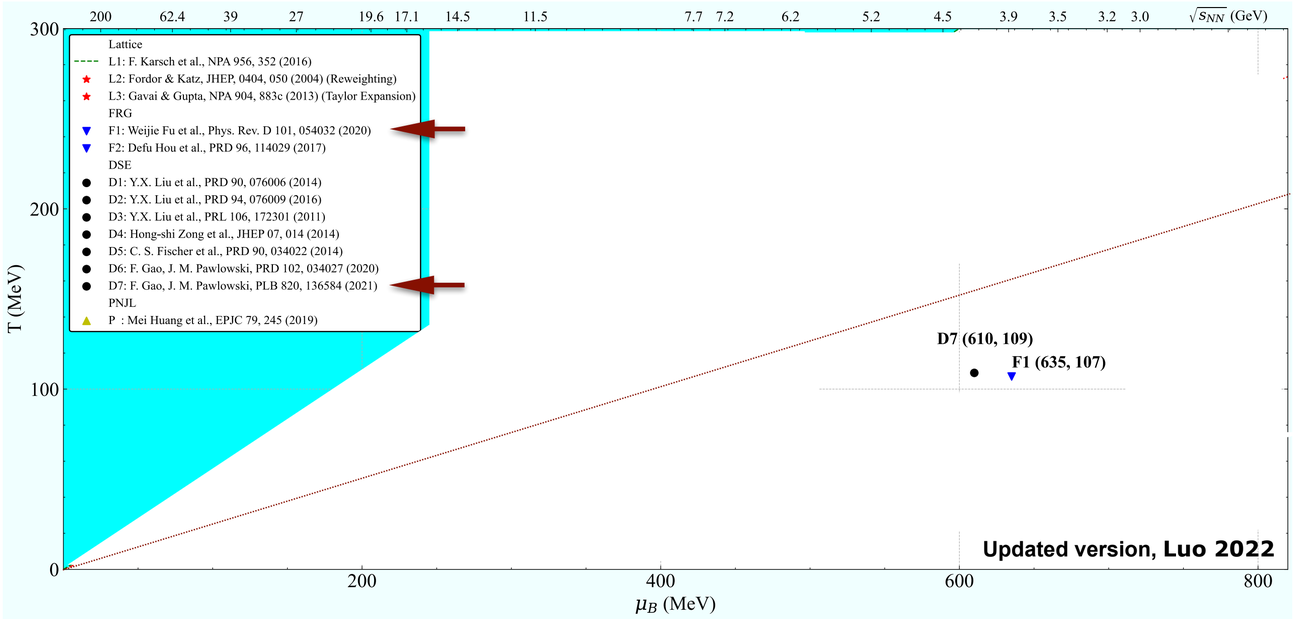
Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)

Functional QCD



Large uncertainties for the estimation of CP location.

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

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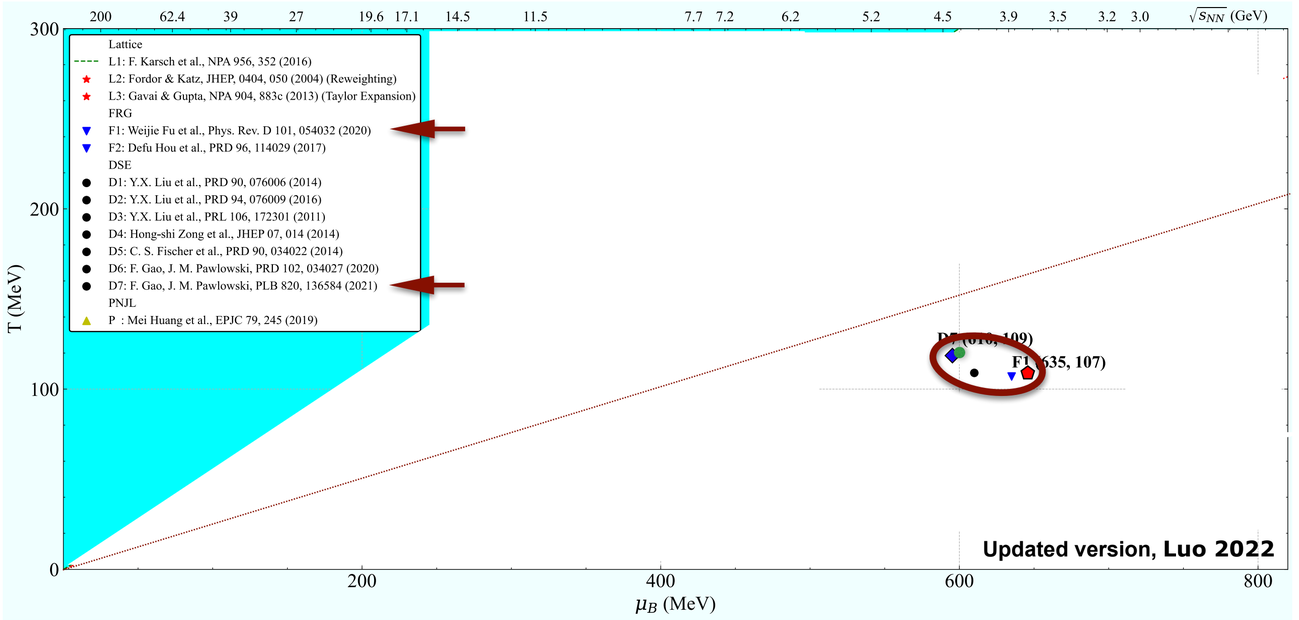

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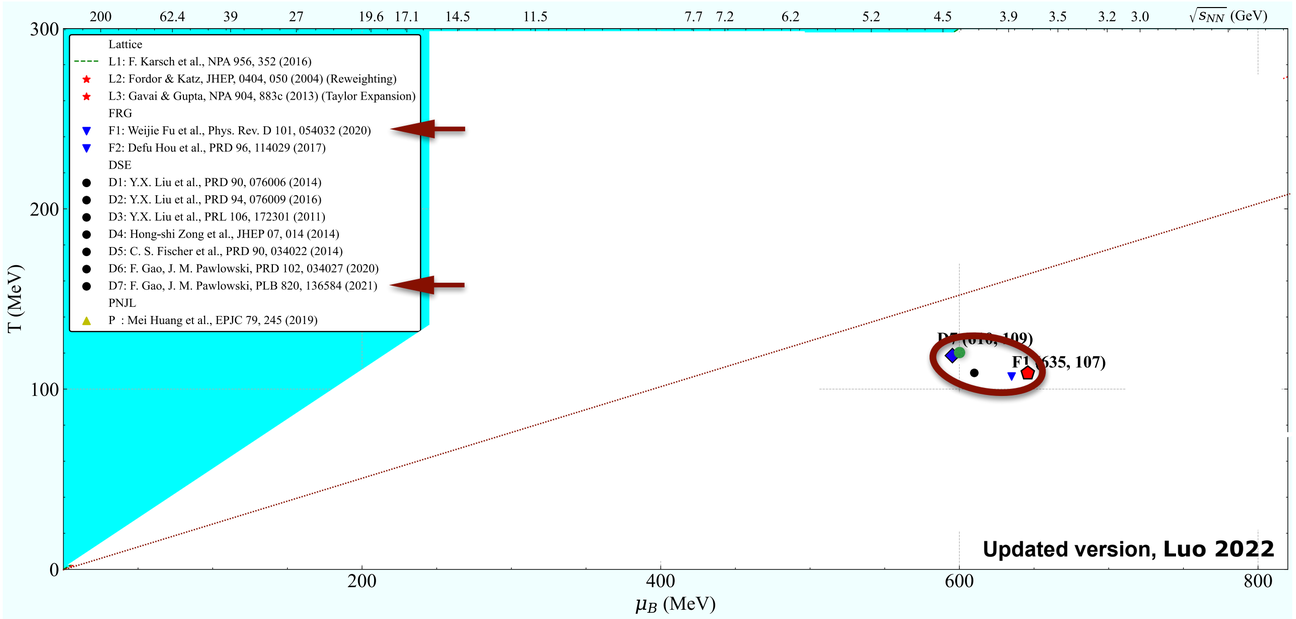

Still small uncertainties for the estimation of CP location **/Onset of new phases**

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RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

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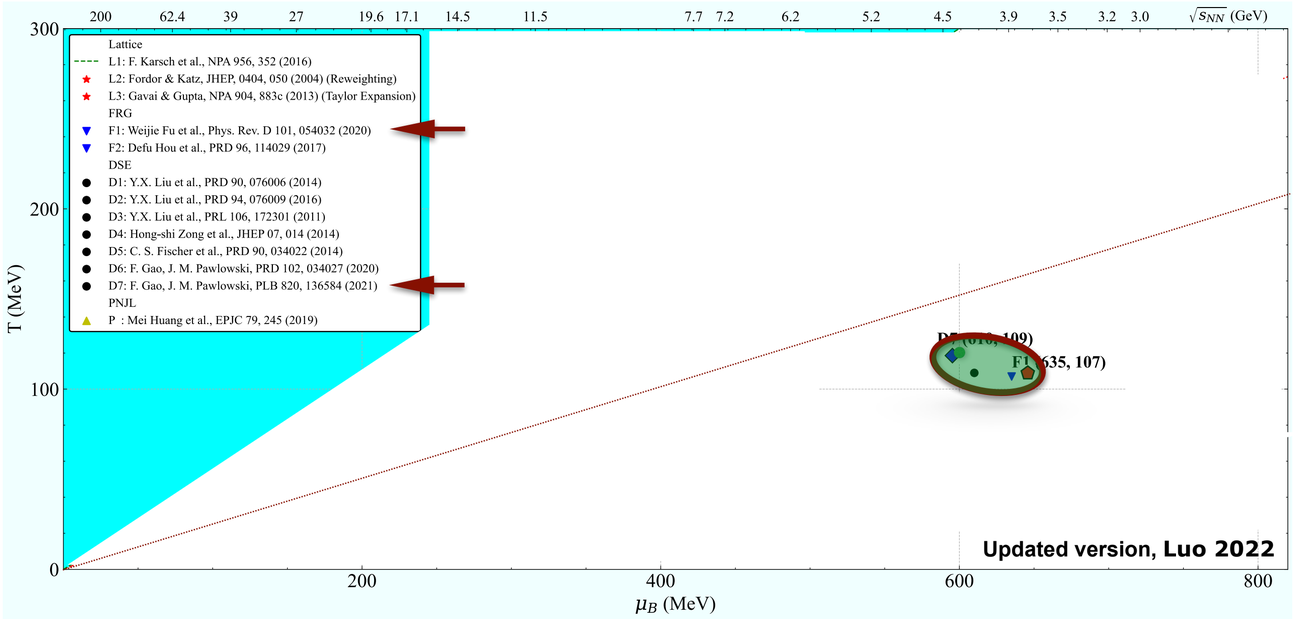
Extrapolations

Lattice extrapolations: Basar, PRC 110 (2024) 015203
Bielefeld-Parma, arXiv:2405.10196

⋮

Holographic models: Hippert, Grefa, Manning, Noronha, Noronha-Hostler, Portillo Vazquez, Ratti, Rougemont, Trujillo, arXiv: 2309.00579

⋮



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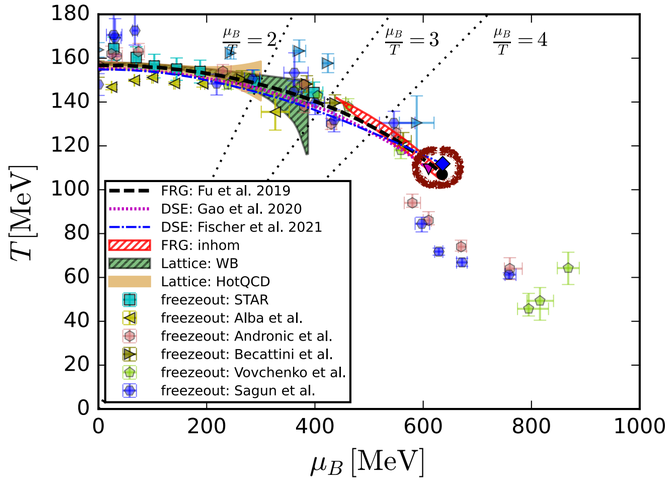
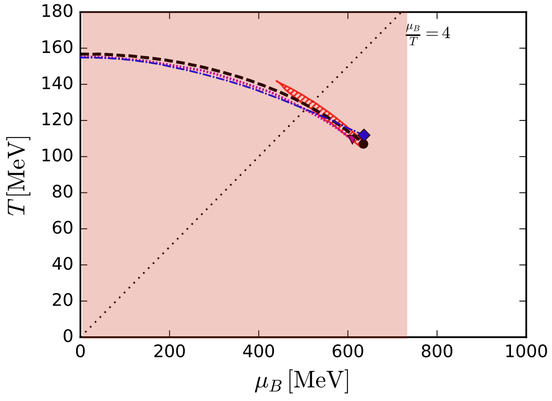


Scenario I

Scaling

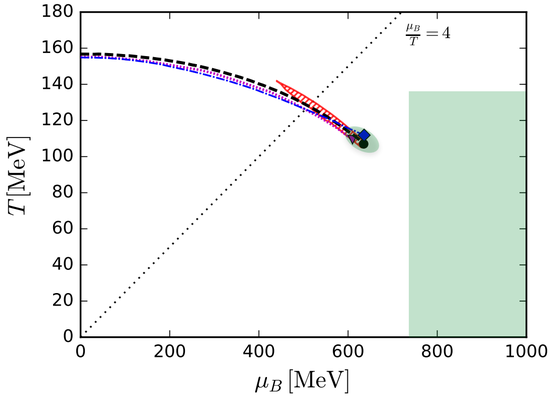


Extrapolations for Pheno

✓

Scenario II

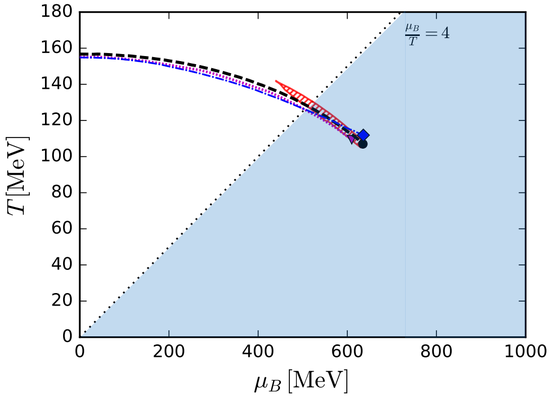
Minimal scaling and new phases



✓

Scenario III

Scaling and/or new phases



✗

Predictions, estimates & extrapolations and how to use them

Out by the LEGO ®principle

Fu, JMP, Rennecke, PRD 101 (2020) 054032

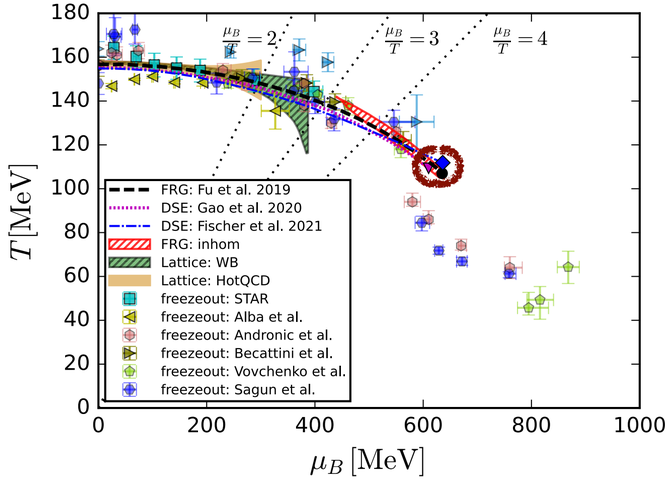
+

Size of scaling regime in LEFTs

Schaefer, Wambach, PRD 75 (2007) 085015

Braun, Klein, Piasecki, EPJC 71 (2011) 1576

⋮



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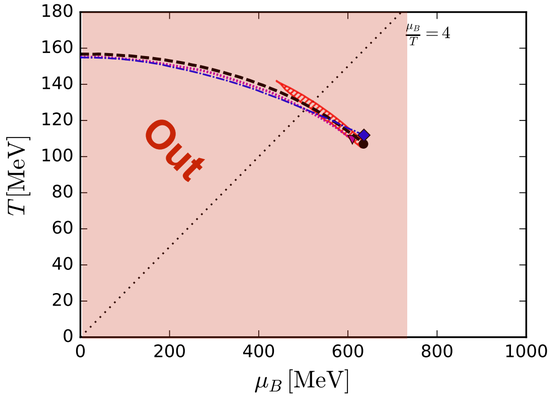
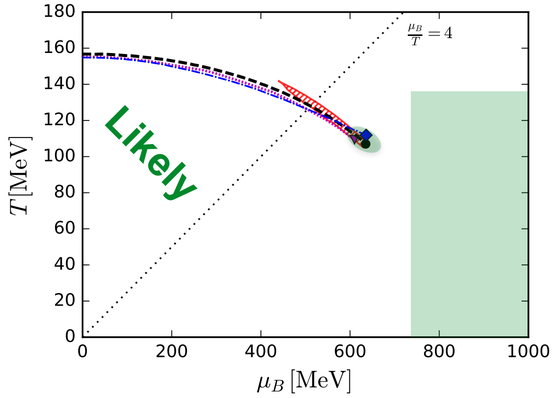


Scenario II

Minimal scaling and new phases



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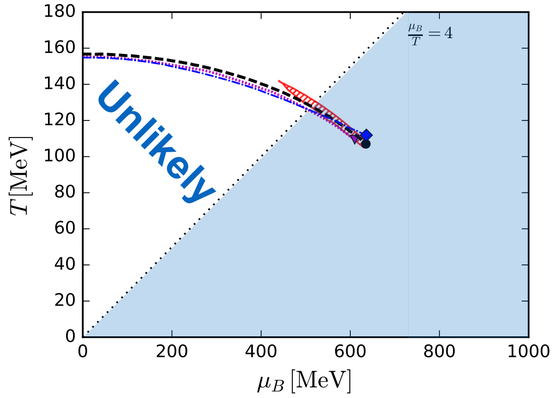


Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

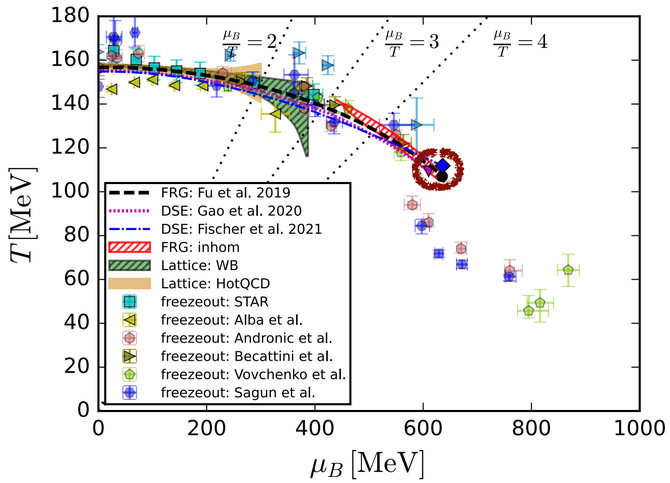
Gao, JMP, PRD 105 (2022) 094020

Soft modes in hot QCD matter: Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, arXiv:2310.19853

+ many results in dynamical low energy effective theories

LEGO® principle

Predictions, estimates & extrapolations and how to use them



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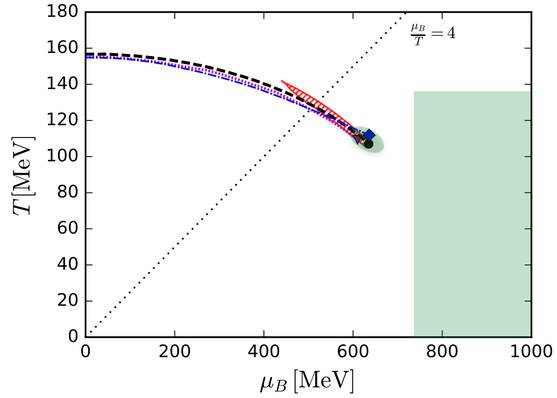
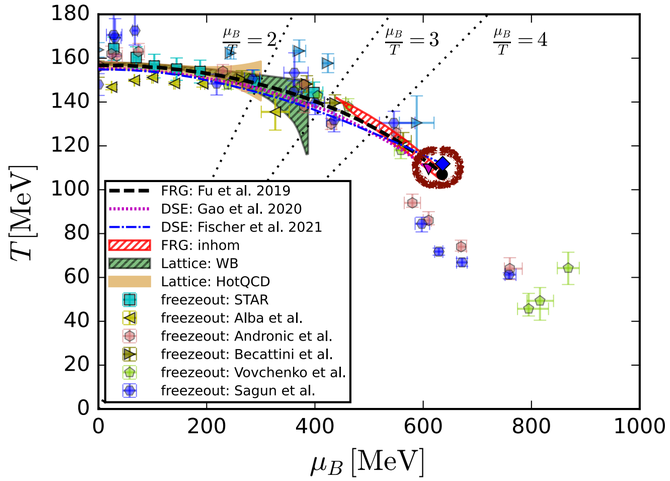


Extrapolations for Pheno

✓

Predictions, estimates & extrapolations and how to use them



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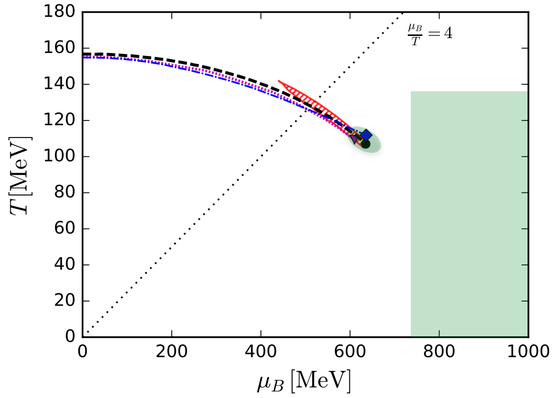
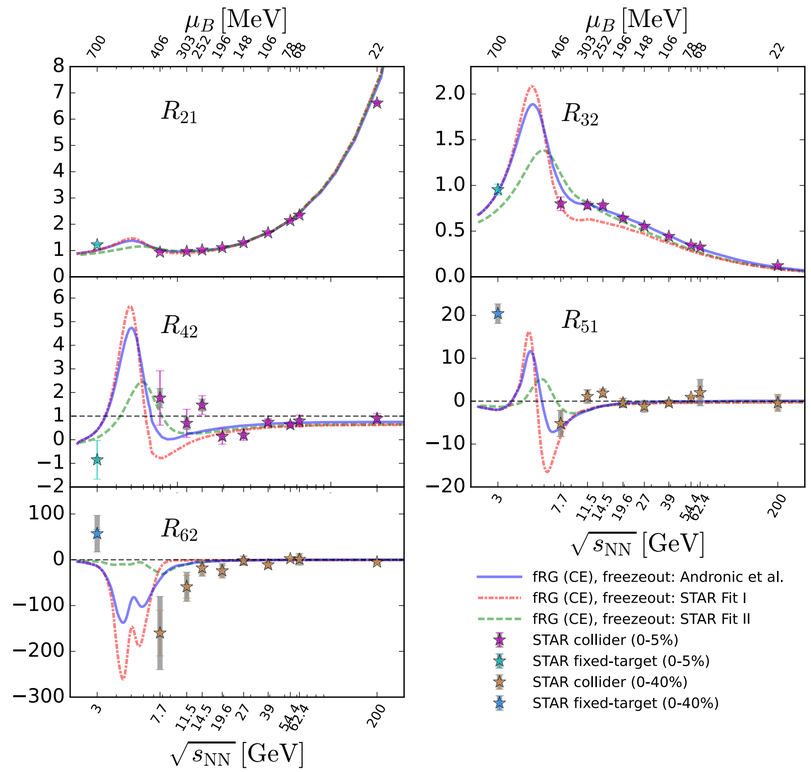


Ripples of the critical end point

baryon & proton number fluctuations



see talk of Wei-jie Fu

Extrapolations for Pheno

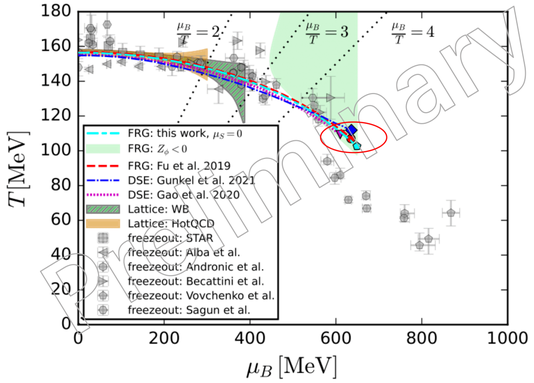
✓

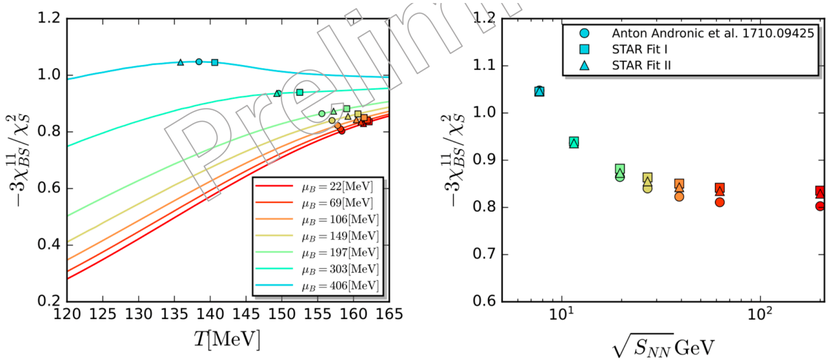
Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

Predictions, estimates & extrapolations and how to use them

Strangeness neutrality

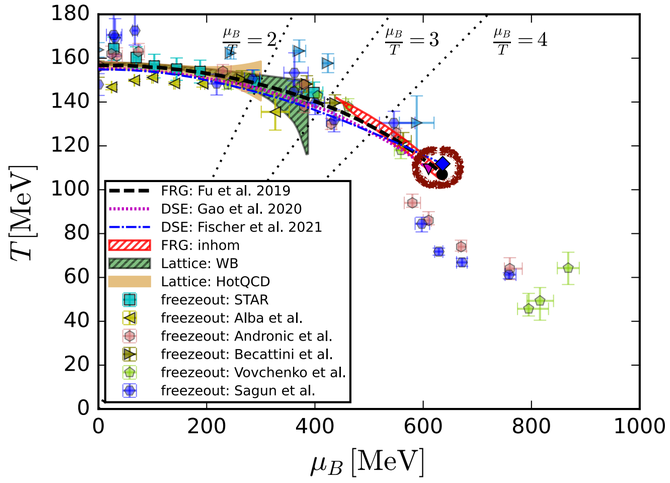
baryon & proton number fluctuations





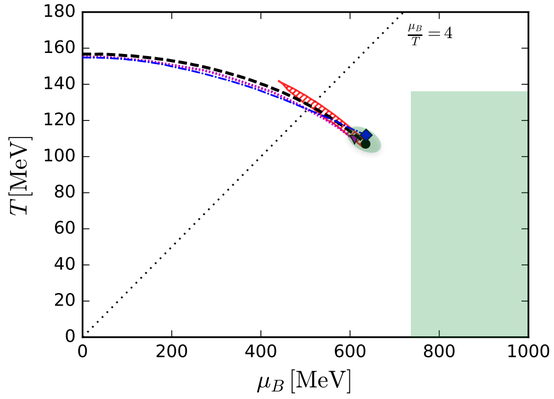
Preliminary

see talk of Rui Wen



Scenario II

Minimal scaling and new phases



Fu, Huang, JMP, Rennecke, Wen, Yin, in preparation

Ripples of the critical end point

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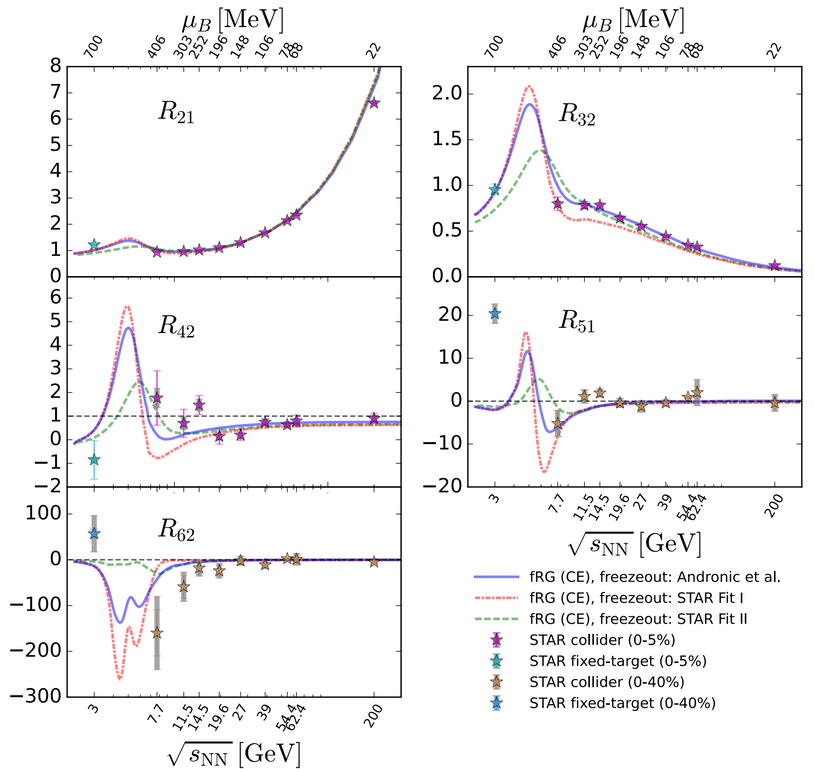

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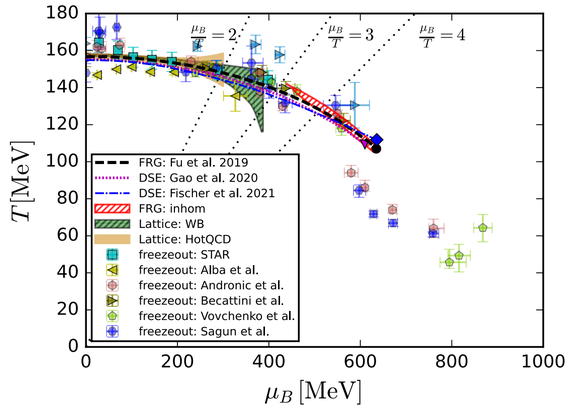
Summary

fQCD

- **Functional QCD provides direct 1st principle results for the phase structure at finite density**
  - **Predictions:** $\frac{\mu_B}{T} \lesssim 4$
  - **Estimates:** $4 \lesssim \frac{\mu_B}{T} \lesssim 8$



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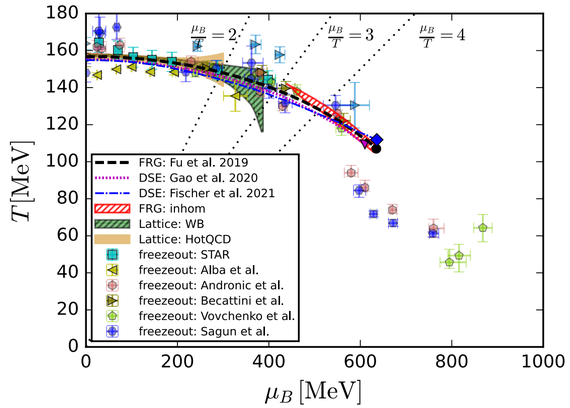

- **The results support the use of extrapolation approaches for phenomenological applications**
  - **Explanation for the convergence of CEP locations of extrapolation approaches**
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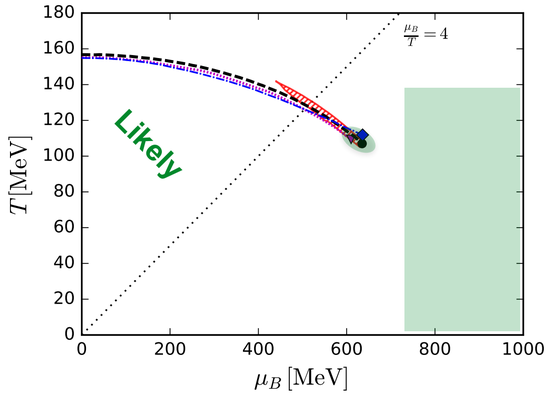
Minimal scaling and new phases



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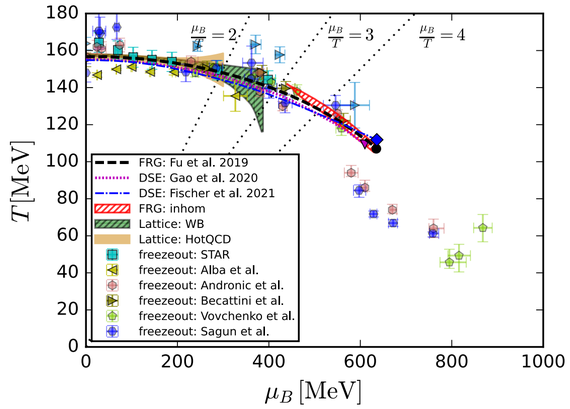

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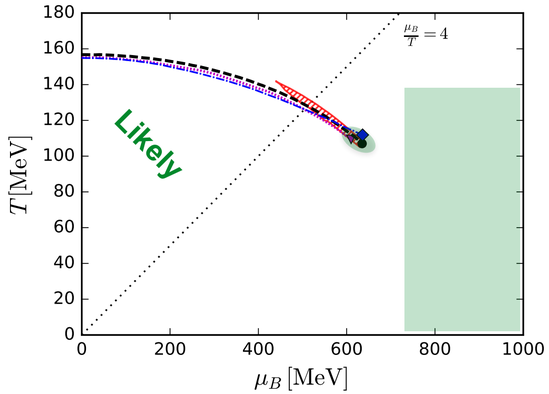


- **Systematic error estimates with the LEGO® principle**

CEP Estimate → CEP Prediction

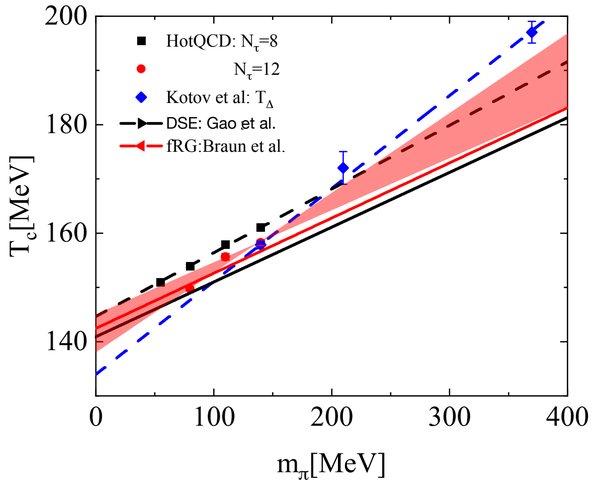
Stay tuned

Diquarks/baryons: ✓ **Density channel/mode:** (✓) **Moat/inhomogeneous regime:** ((✓))

Backup Material

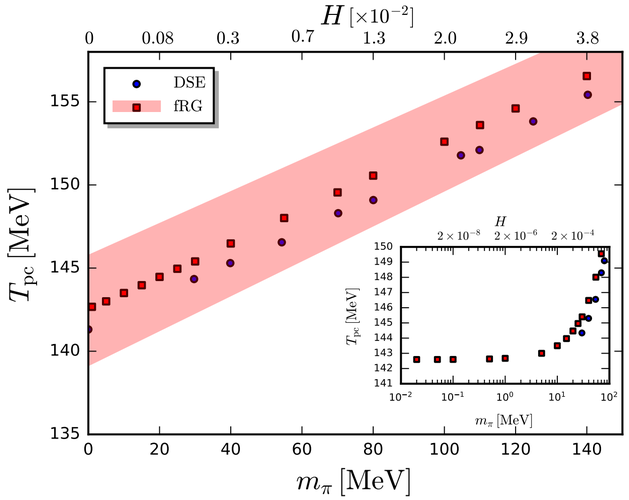
Chiral dynamics & soft modes

To be (critical) or not (to be)

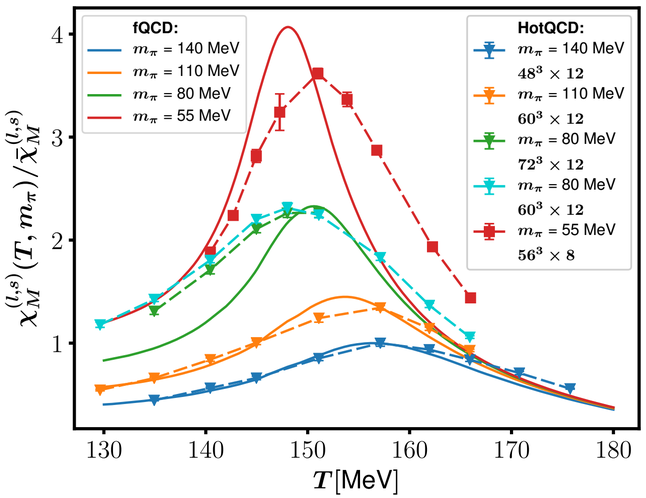


Chiral transition temperature

$$H = \frac{m_l}{m_s}$$



Magnetic susceptibility



Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

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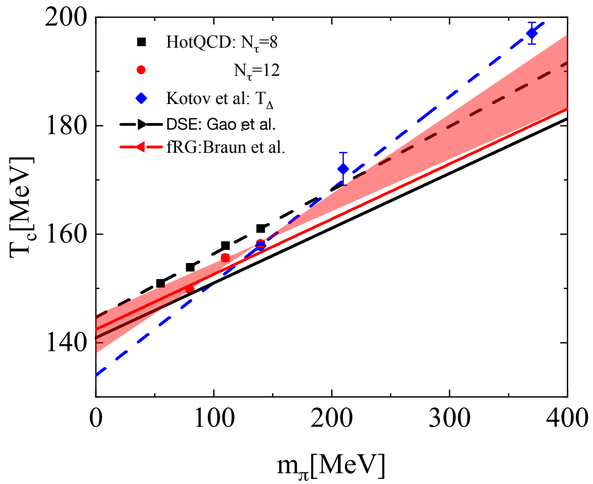

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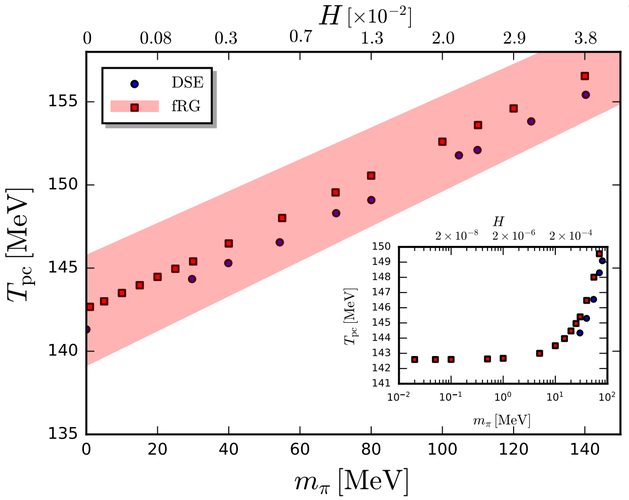
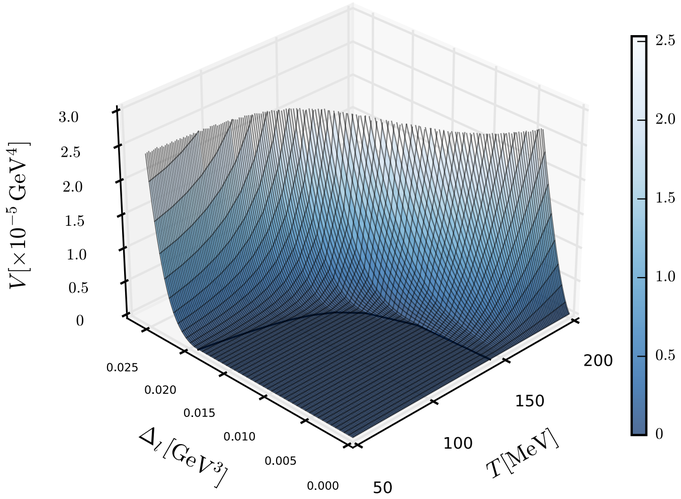


Order parameter potential & scaling

$$V_\chi \approx \Delta_l^n \quad \Longleftrightarrow \quad \Delta_l(H) \propto H^{\frac{1}{n-1}}$$



$$V_\chi^{(\mathrm{fRG})} \approx V_\chi^{(\mathrm{DSE})}$$

(Critical) exponent: $\frac{1}{\delta} = \frac{1}{n-1}$

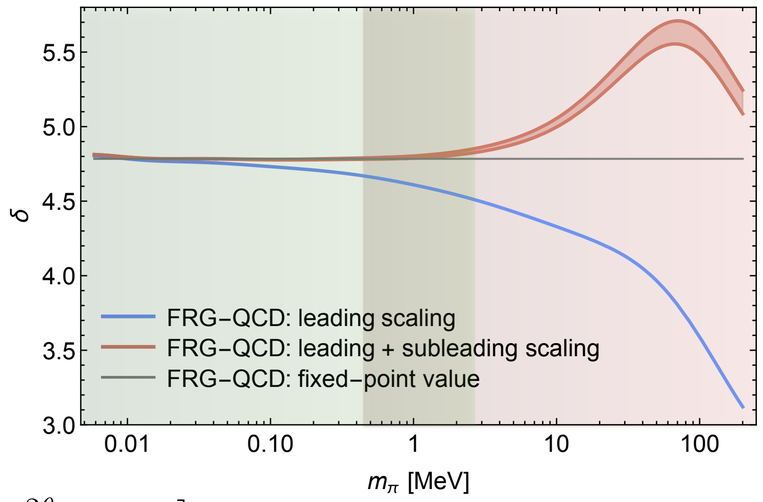
Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



Critical O(4) scaling

'chiral scaling'

Trivial $\Delta_l^{1+\delta}$ scaling

QM: Chen, Wen, WF, PRD 104 (2021) 054009

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

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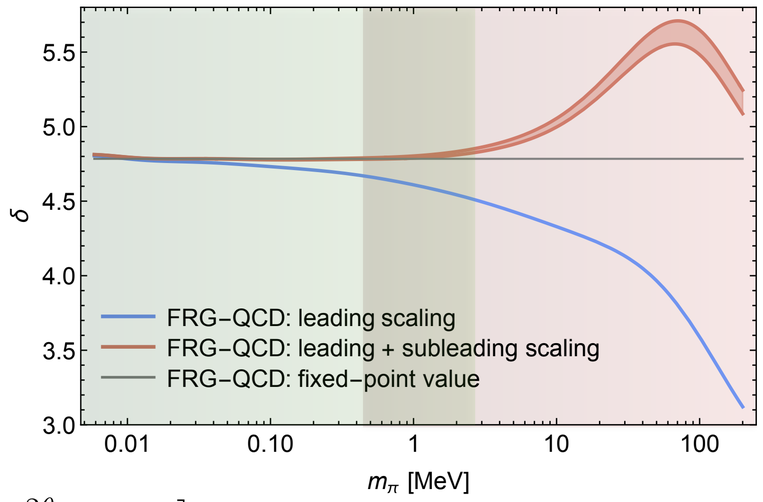

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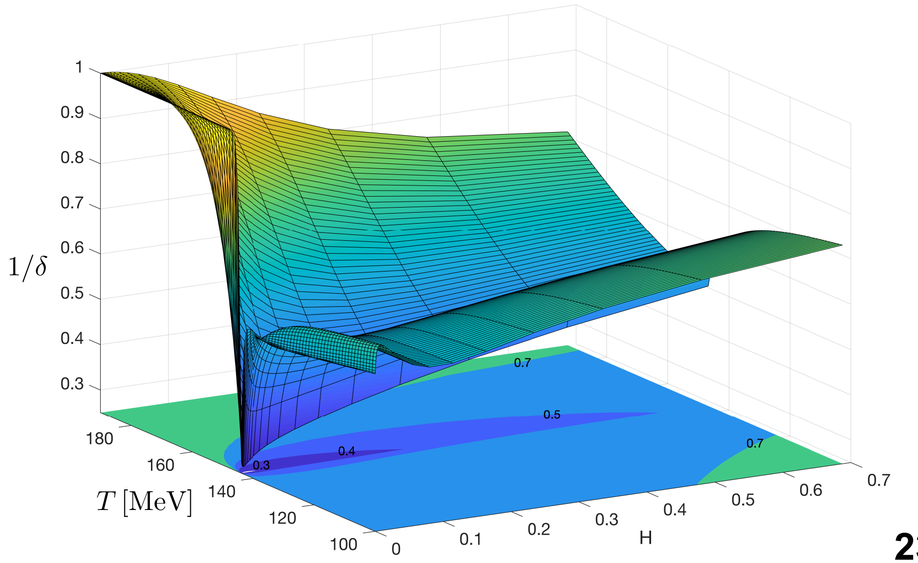

Small chiral scaling regime

↓

Small critical regime around pot. CEP

Chiral dynamics & quasi-massless modes

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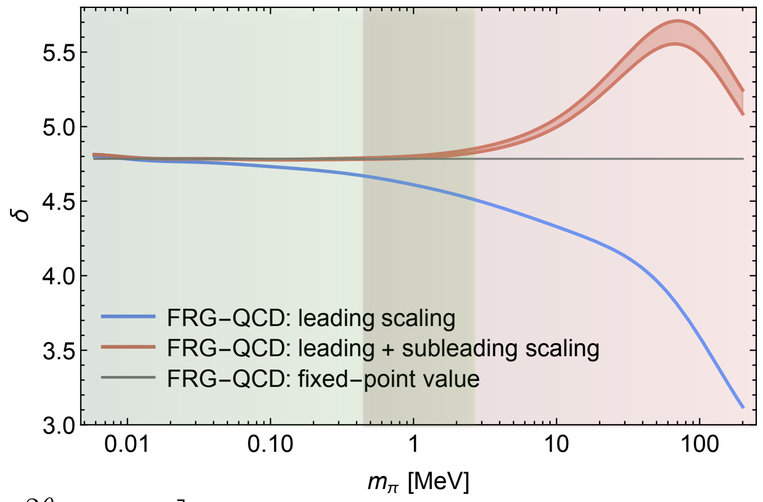

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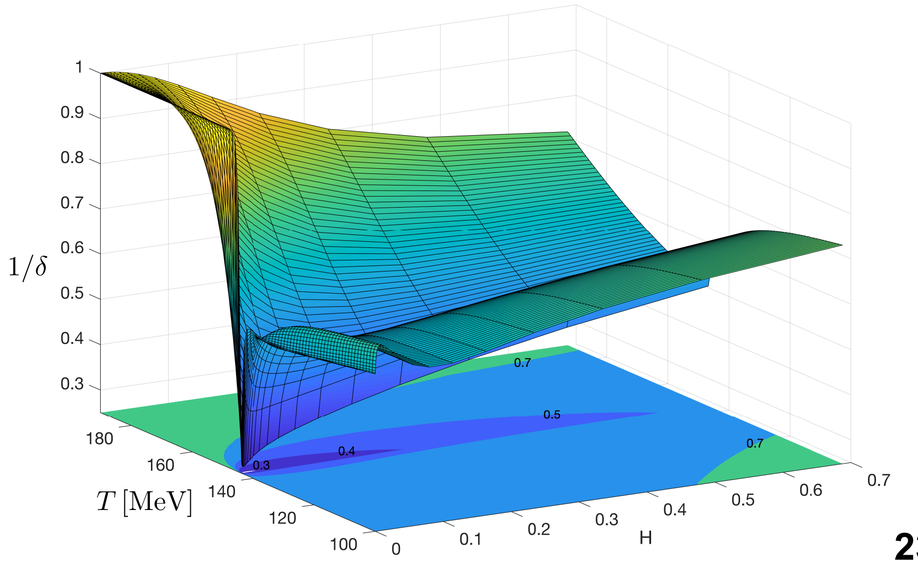
$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853



Small chiral scaling regime

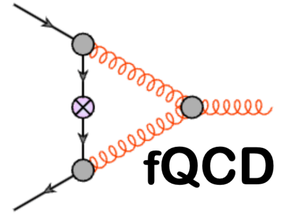
↓

Small critical regime around pot. CEP

!!Great News!!

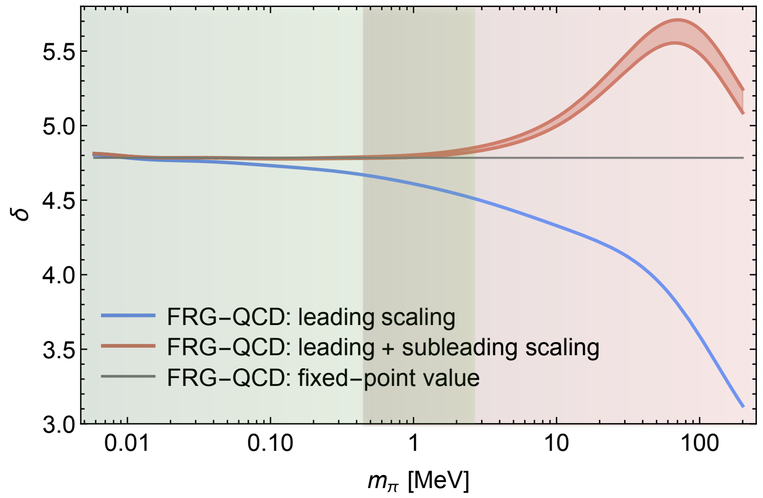
Location of CEP/New phase accessible via combination of precision measurements & computations

Chiral dynamics & quasi-massless modes



Critical O(4) scaling

Scaling coefficient as function of the pion mass



'chiral scaling'

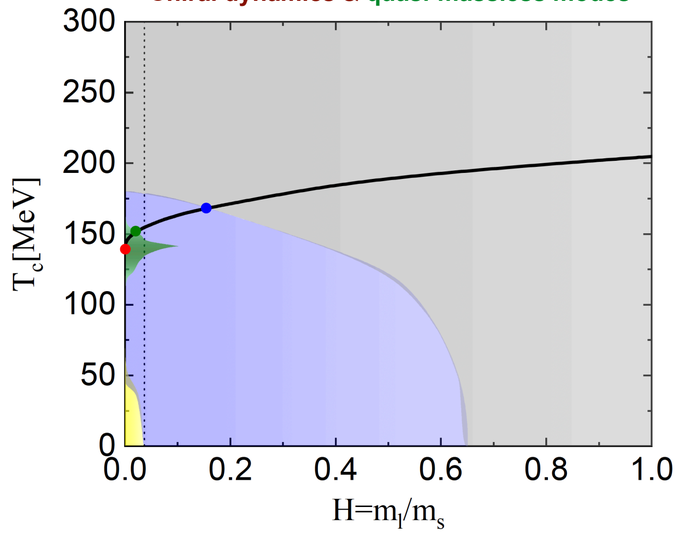
Trivial $\Delta_l^{1+\delta}$ scaling

fQCD collaboration, in preparation

QM: Chen, Wen, WF, PRD 104 (2021) 054009

Critical scaling

Chiral dynamics & quasi-massless modes



Gao, JMP, PRD 105 (2022) 094020

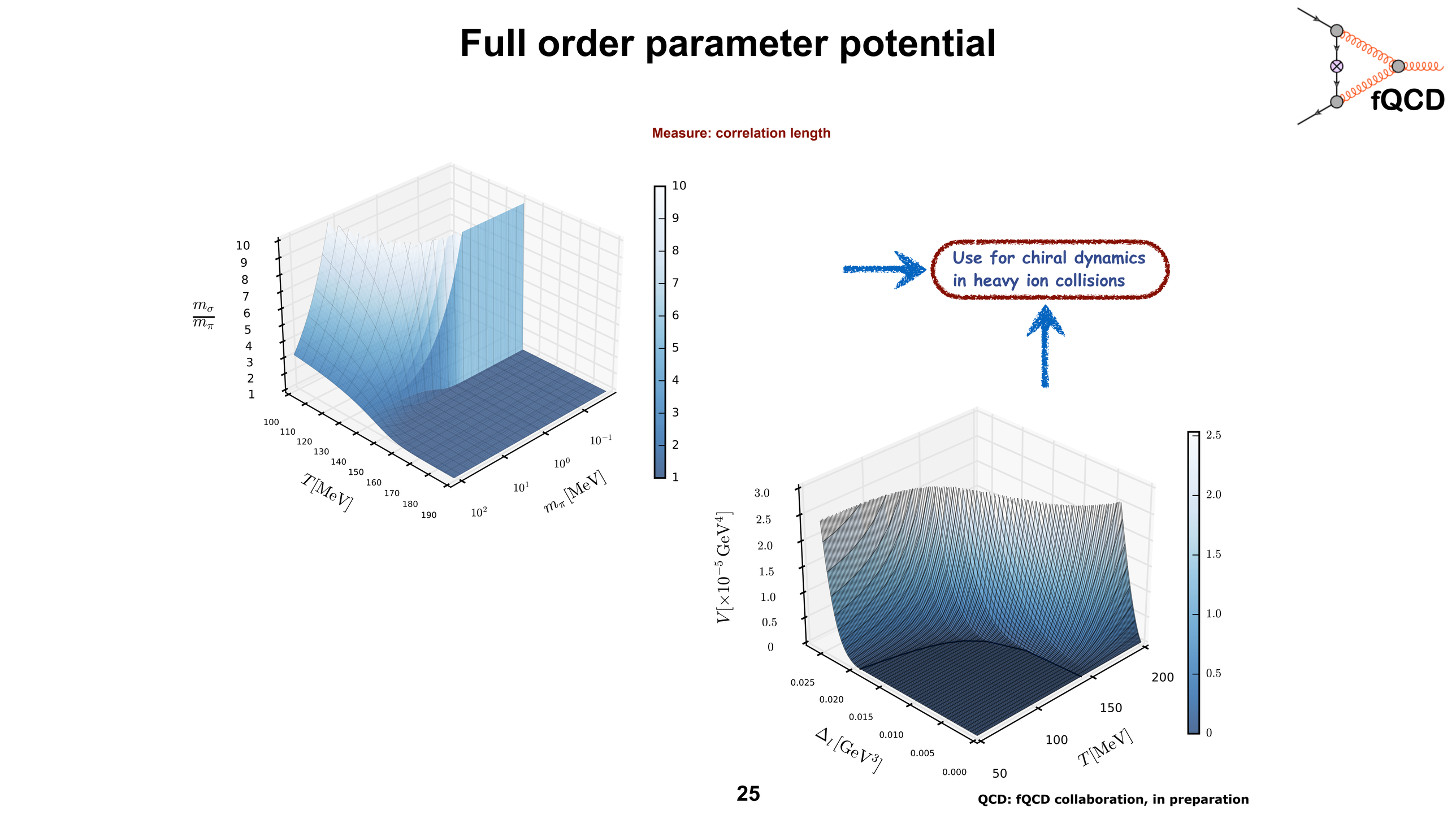
'Non-critical chiral scaling'

Far away from the critical regime for $m_\pi \gtrsim 1\,\mathrm{MeV}$

$$\Delta_l(T,H) \approx \Delta_{l,\chi}(0)\left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H\right)$$

$$V_\chi(\Delta_l) \propto \quad \Delta_l^6 \quad \Delta_l^4 \quad \Delta_l^2$$

Full order parameter potential



Measure: correlation length

Use for chiral dynamics
in heavy ion collisions

QCD: fQCD collaboration, in preparation