Modified power counting and the importance of three-body forces

ECT* workshop

The nuclear interaction: post-modern developments

Chieh-Jen (Jerry) Yang

August 19, 2024

nuclear physics

The origin of coupling constants

Absolute theory or god-given-like fundamental theory
(Everything can be derived, none or very little fitting)

In reality/practice:
Some portion of details in the mother theory are **integrated out**, absorbed and encoded in the low energy constants (LECs).

EFT viewpoints:
That‘s call renormalization, and it’s OK. But it will then be of importance to check whether the results after renormalization satisfies the renormalization group requirement.

The Nuclear Force Problem: Is the Never-Ending Story Coming to an End?

R. Machleidt

Department of Physics, University of Idaho, Moscow, Idaho, U.S.A.

Table 1. Seven Decades of Struggle: The Theory of Nuclear Forces

1935	**Yukawa: Meson Theory**
1950's	*The "Pion Theories"* One-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions ≡ multi-pion resonances: $\sigma, \rho, \omega, \ldots$ The One-Boson-Exchange Model
1970's	Refine meson theory: Sophisticated 2π exchange models (Stony Brook, Paris, Bonn)
1980's	Nuclear physicists discover **QCD** Quark Cluster Models
1990's and beyond	Nuclear physicists discover **EFT** Weinberg, van Kolck **Back to Meson Theory!** *But, with Chiral Symmetry*

Guessing, build empirical models or potentials (**encoded** in coupling & mass constants, etc).

Breakthrough 1:
But QCD not calculable to nuclei

Breakthrough 2:

Through Effective Field Theory

Recent struggles (post-modern)

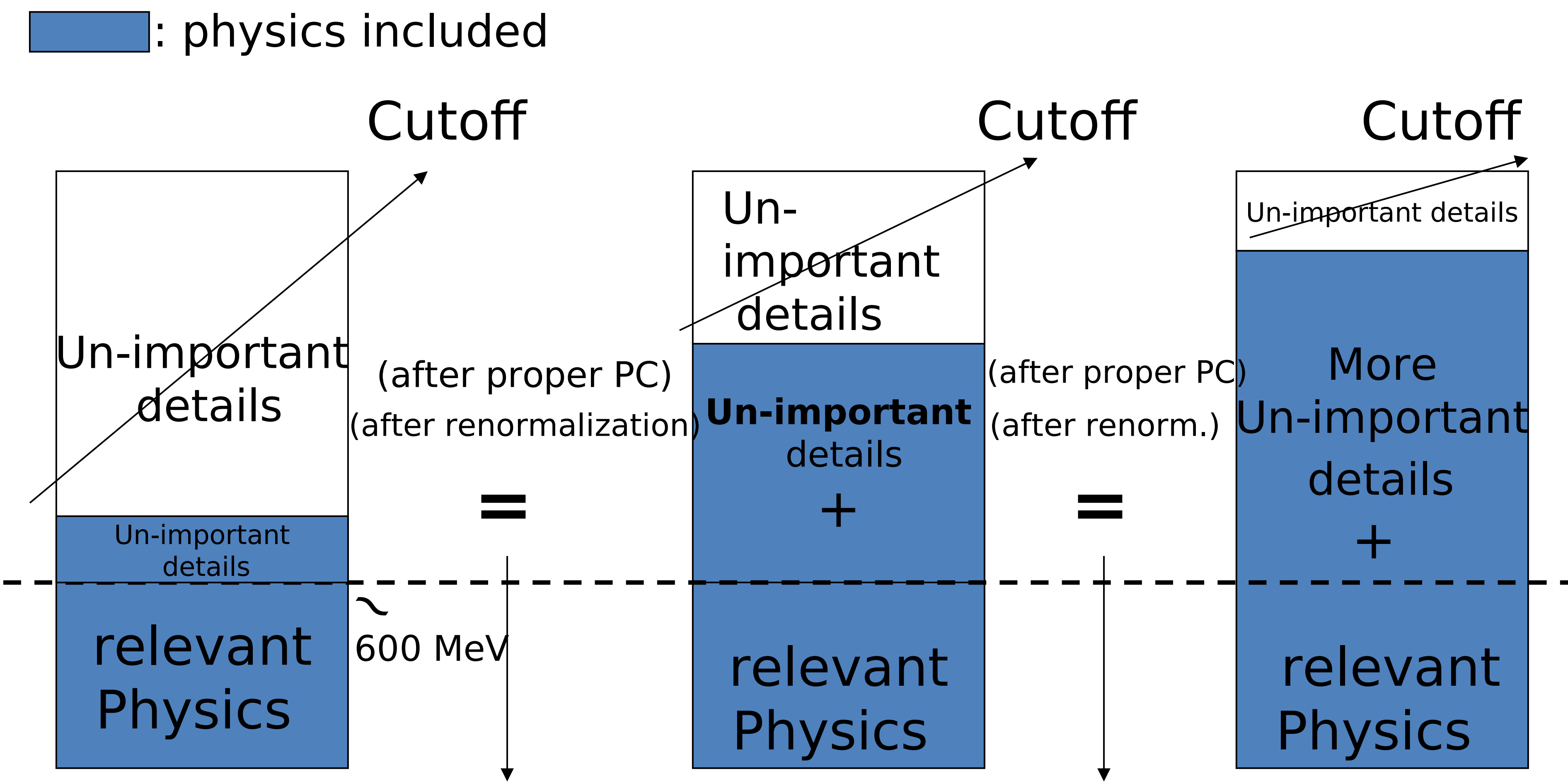
- The original proposal (WPC) is to **iterate all** chiral potentials **truncated** up to a certain order **non-perturbatively** (Weinberg 90’, van Kolck, Epelbaum, Machleidt, etc.).

1996’s, First problem: Once the pion-exchange is iterated, there’s no way to properly renormalize the divergence caused by varying the pion mass. KSW, Nucl.Phys.B 478 (1996) 629-659.

2005’s, Second problem: Even without varying m_π, there’s still RG-issues (especially if Λ>600 MeV). Nogga, Timmermans, van Kolck, PRC 72 (2005) 054006

2020’s, 3rd problem (this applies to perturbative PC as well): The importance of many-body forces can grow with the number of nucleons. C.J. Yang, A. Ekström, C. Forssén, G. Hagen, G. Rupak, U. van Kolck, EPJA 59 (2023) 10, 233

Second problem (RG-related)



***Only source of error**: given by the high order terms.

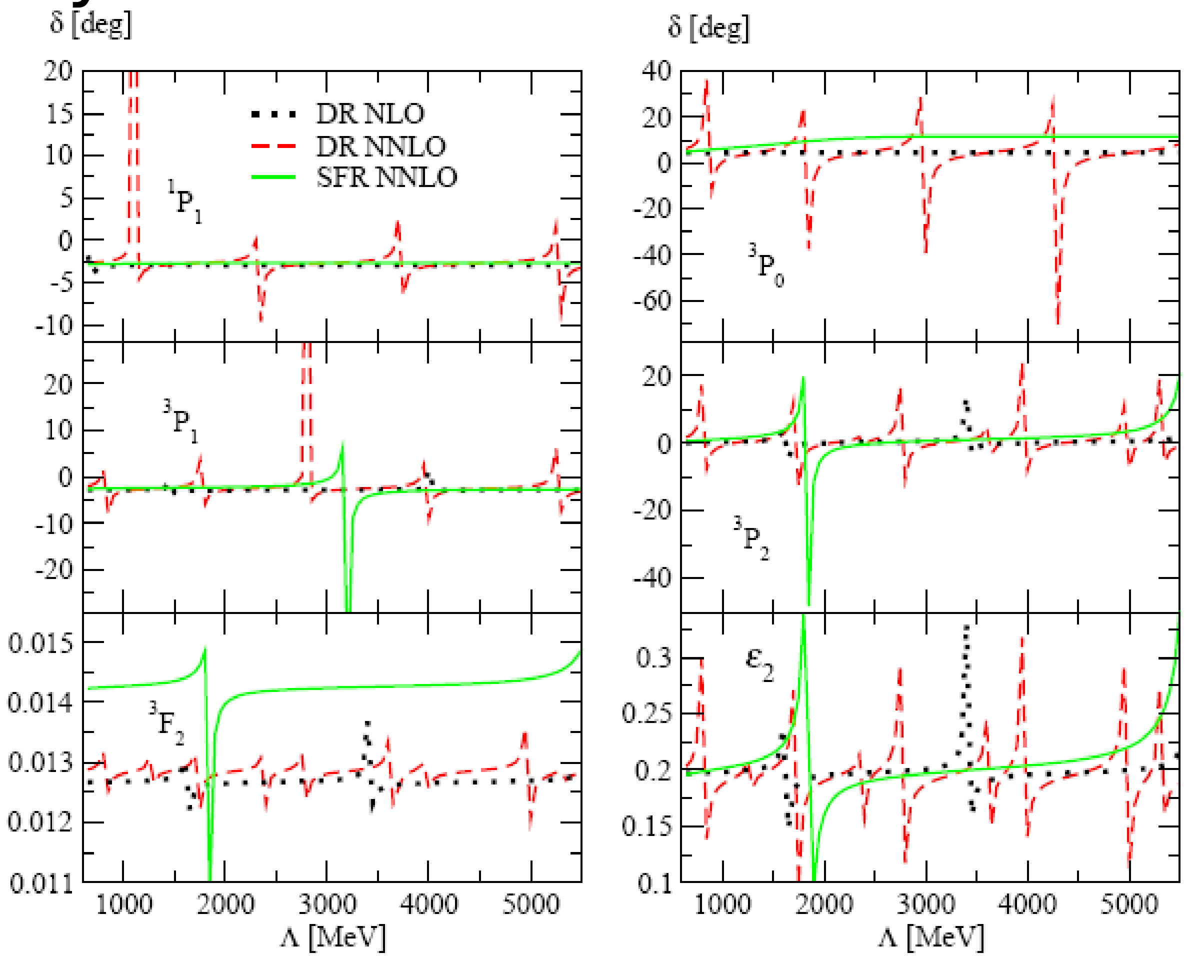
If not so, → **the power counting isn't completely correct!**

(un-important are not really unimportant)

Problems of WPC

WPC is wrong at LO ! (Nogga, Timmermans, van Kolck, *PRC 72 (2005) 054006*)

- Beyond LO: (Yang, Elster, Phillips (2008-2010))



Same story for:
N3LO WPC

Ch. Zeoli, R. Machleidt and D. R. Entem, Few-body syst., 54, 12, 2191 (2012)

In short, WPC might be WPP (pragmatic proposal)
(many in-debate issues, but not the topic today)
More details/opinions could be found in:

Few Body Syst. 62 (2021) 4, 85 and Few Body Syst. 63, no.2, 44 (2022)

Nuclear Effective Field Theories: Reverberations of the early days

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July 27, 2021

What Can Possibly Go Wrong?

Harald. W. Grießhammer

Received: date / Accepted: date

Abstract A lot.

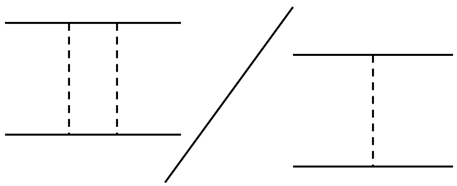
New power counting

Decided by RG

Long & Yang, (2010-2012)

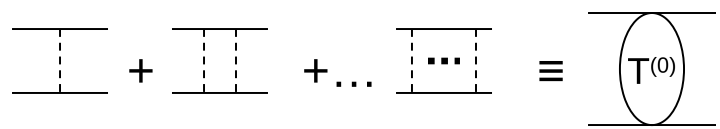
LO: Still iterate to all order (at least for $l<2$).

Reason: van Kolck, Bedaque,… etc.



$$\sim \frac{g_A^2 M}{8\pi f_\pi^2} Q$$

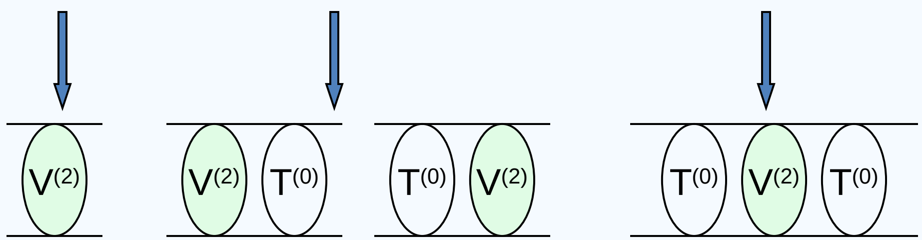
Thus, $O(Q^0)$:



Start at NLO, do perturbation. ($T = T^{(0)}+T^{(1)}+T^{(2)}+T^{(3)}+\dots$)

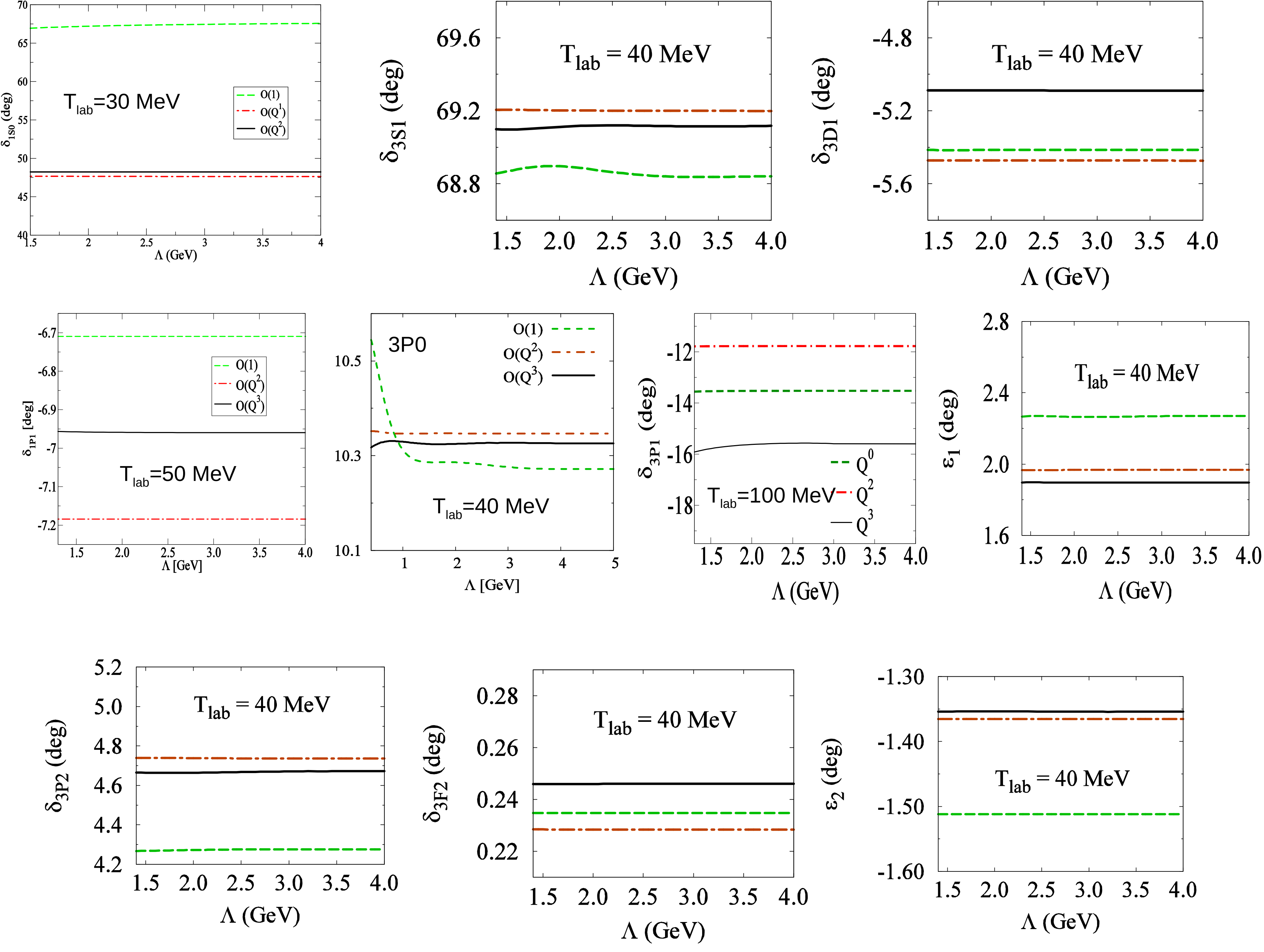
If $V^{(1)}$ is absent:

$$T^{(2)} = V^{(2)} + 2V^{(2)}GT^{(0)} + T^{(0)}GV^{(2)}GT^{(0)}.$$



$$G \equiv \frac{2M_N}{\pi}\int_0^\Lambda \frac{p^2\,dp}{p_0^2 - p^2 + i\varepsilon}$$

$$T^{(3)} = V^{(3)} + 2V^{(3)}GT^{(0)} + T^{(0)}GV^{(3)}GT^{(0)}.$$



T_lab=30 MeV
O(1)
O(Q^1)
O(Q^2)
δ_1S0 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3S1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3D1 (deg)
Λ (GeV)
T_lab=50 MeV
O(1)
O(Q^2)
O(Q^3)
δ_1P1 [deg]
Λ [GeV]
3P0
O(1)
O(Q^2)
O(Q^3)
T_lab=40 MeV
Λ [GeV]
T_lab=100 MeV
Q^0
Q^2
Q^3
δ_3P1 (deg)
Λ (GeV)
T_lab = 40 MeV
ε_1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3P2 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3F2 (deg)
Λ (GeV)
T_lab = 40 MeV
ε_2 (deg)
Λ (GeV)

So far so good, but...

The issue of “exceptional zero”

PHYSICAL REVIEW C **107**, 034001 (2023)

“Renormalization-group-invariant effective field theory” for few-nucleon systems is cutoff dependent

A. M. Gasparyan * and E. Epelbaum †
Ruhr-Universität Bochum, Fakultät für Physik und Astronomie, Institut für Theoretische Physik II, D-44780 Bochum, Germany

(Received 10 November 2022; accepted 10 March 2023; published 28 March 2023)

We consider nucleon-nucleon scattering using the formulation of chiral effective field theory which is claimed to be renormalization group invariant. The cornerstone of this framework is the existence of a well-defined infinite-cutoff limit for the scattering amplitude at each order of the expansion, which should not depend on a particular regulator form. Focusing on the $^3P_0$ partial wave as a representative example, we show that this requirement can in general not be fulfilled beyond the leading order, in spite of the perturbative treatment of subleading contributions to the amplitude. Several previous studies along these lines, including the next-to-leading order calculation by B. Long and C. J. Yang [Phys. Rev. C **84**, 057001 (2011)] and a toy model example with singular long-range potentials by B. Long and U. van Kolck [Ann. Phys. **323**, 1304 (2008)], are critically reviewed and scrutinized in detail.

DOI: 10.1103/PhysRevC.107.034001

RG-issue at $O(Q^2)$ of long & Yang

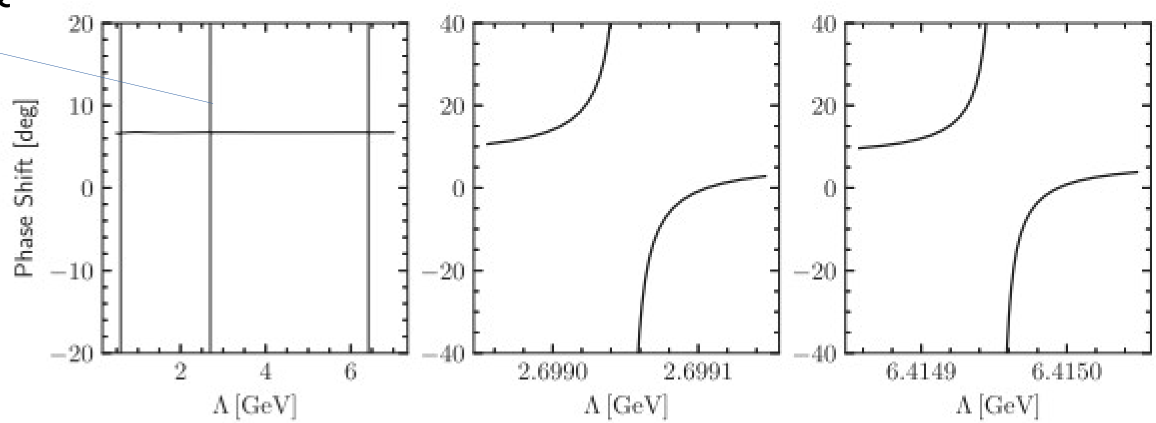


FIG. 5. Cutoff dependence of $^3P_0$ phase shift calculated at the fixed laboratory energy of $T_{\text{lab}} = 130$ MeV using the approach of Ref. [30] at NLO. The middle and right panels show zoomed regions in the vicinity of two exceptional cutoffs.

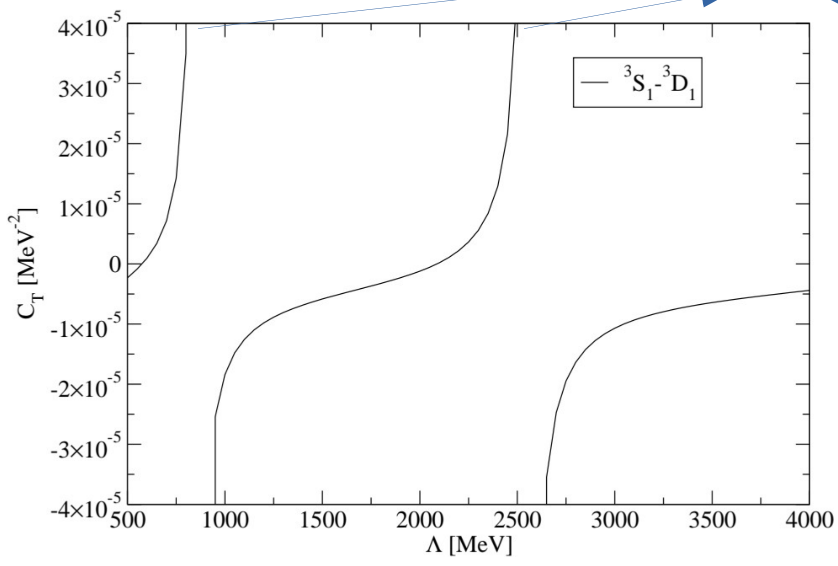
Origin of the issue

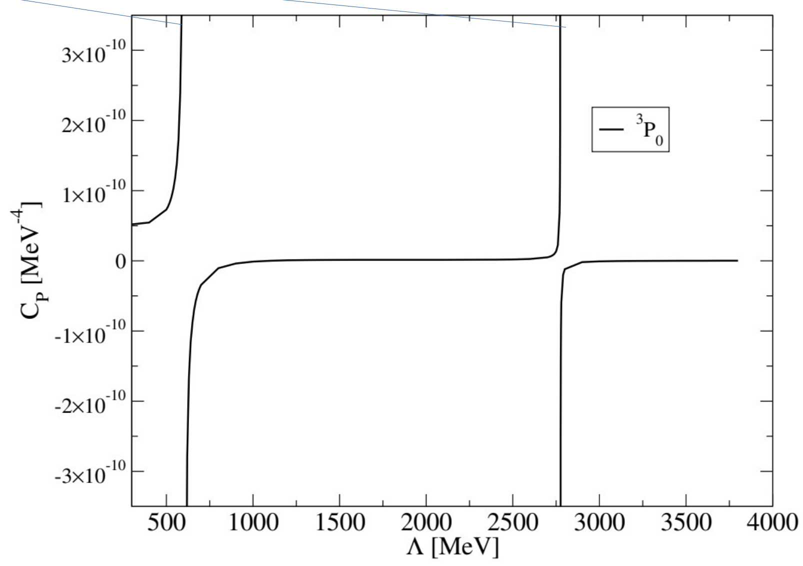
- LECs at LO (non-per. treatment) could have limit-cycle running.
- At LO, this is ok, even exactly at Λ_e where $c(\Lambda_e)=\infty$. Because: (non-per) = (matrix diagonalization), which guarantee that **each eigenvalue $<\Phi_{LO,i}|H_{LO}|\Phi_{LO,i}>=E_i$ is finite**.

∵ <KE> and $<V_{LO}>$ are finite, => $c(\Lambda_e)<\Phi_{LO,i}|\hat{O}_{ct}|\Phi_{LO,i}>$=finite for all i.

∞ 0

You could perform subtraction to get δ at Λ_e.





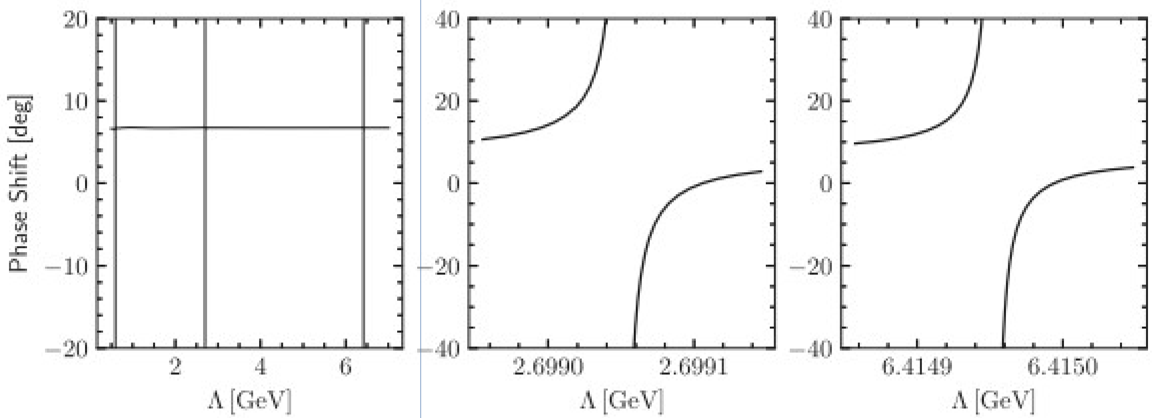
However, the same **won't hold** for NLO or higher-orders, if **DWBA** is adopted.

Origin of the issue

- At NLO (or higher), additional CT enters, but unlike LO, where $c(\Lambda_e)<\Phi_{LO,i}|\hat{O}_{LO,ct}|\Phi_{LO,i}>$=finite for all i, the DWBA correction $d(\Lambda_*)<\Phi_{LO,i}|\hat{O}_{NLO,ct}|\Phi_{LO,i}>\neq$finite for all i (as we are not protected by the eigenvalue feature).

  => At a certain i* (correspond to E*), $<\Phi_{LO,i}|\hat{O}_{NLO,ct}|\Phi_{LO,i}>=0$, but for other i it's not!

- This means, if one **choose to renormalize** at E=E*, one faces the choice of using d→ ∞, in order to have a non-zero NLO correction. But then observable at other E blow up. On the other hand, using d≠∞ will make this CT have zero contribution (not good either).



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Allowed to choose anywhere below M_{hi}

In practice (on Long & Yang)

- At NLO (or higher), additional CT enters, but unlike LO, where $c(\Lambda_e)<\Phi_{LO,i}|\hat{O}_{LO,ct}|\Phi_{LO,i}>$=finite for all i, the DWBA correction $d(\Lambda_*)<\Phi_{LO,i}|\hat{O}_{NLO,ct}|\Phi_{LO,i}>\neq$finite for all i (as we are not protected by the eigenvalue feature).

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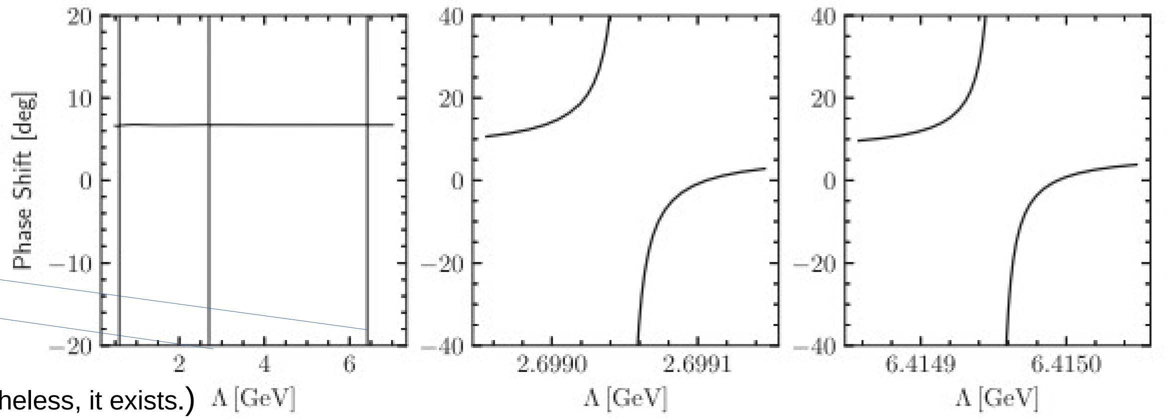
Problematic Λ_*,
where $E^*\in p_{tye}$
(Extremely narrow, nevertheless, it exists.)



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Conditions of the breakdown (for the above case$_{\text{Long\&Yang}}$):

1. $\hat{O}_{NLO,ct}\neq\hat{O}_{LO,ct}$
2. Adopt Λ very close (>4 significant digits the same) to those problematic Λ_*.
3. Choose to renormalize **exactly** at E* (or exactly on a set of particular E_i, if number of LECs≥2).

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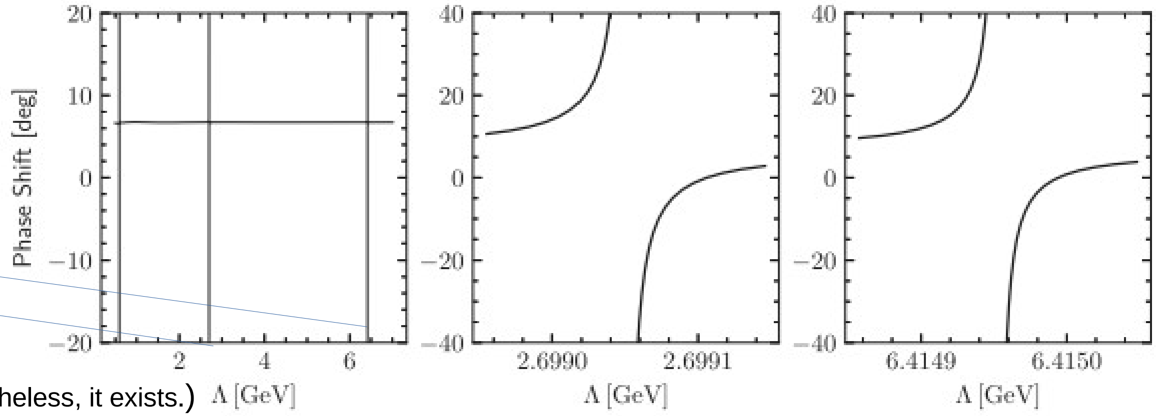

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3. Choose to renormalize **exactly** at E* (... particular E_i, if number of LECs≥2).

Key word!

Origin of the issue

- At NLO (or higher), additional CT enters, but unlike LO, where $c(\Lambda_e)\langle\Phi_{LO,i}|\hat{O}_{LO,ct}|\Phi_{LO,i}\rangle$=finite for all i, the DWBA correction $d(\Lambda_*)\langle\Phi_{LO,i}|\hat{O}_{NLO,ct}|\Phi_{LO,i}\rangle\neq$finite for all i (as we are not protected by the eigenvalue feature).

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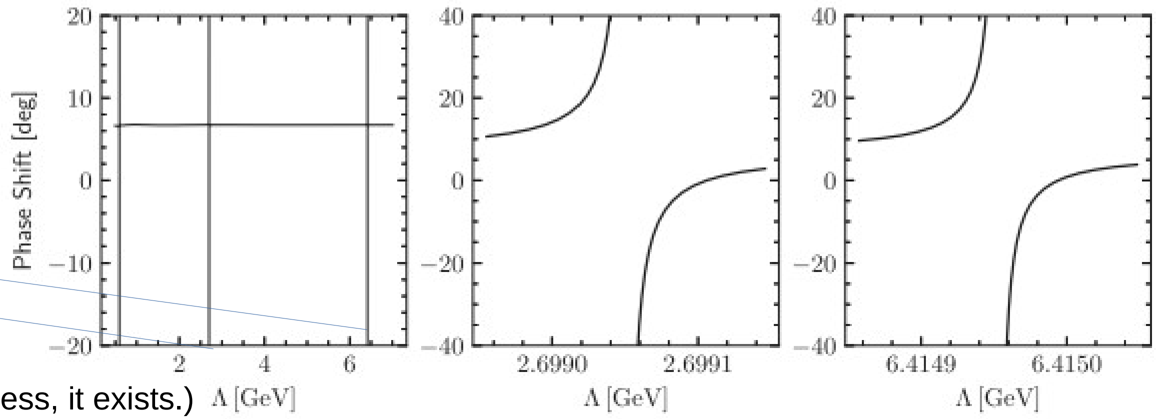

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However, the issue occurs only when one treats those incomplete, truncated amplitudes exactly or beyond the degree to which they should be trusted.

Root of the problem (**nothing to do with PC**, but a general feature of perturbative corrections)

The above has taken $\langle\Phi_{LO,i}|$ (and therefore the **NLO matrix element**) too **exact.**

Under EFT, it should always be accompanied by an uncertainty $\sim O(p/M_{hi})^n$.

Under EFT principles, one should always associate the result with an uncertainty which is adequate to its EFT order.

$$O_n(M_{lo};\Lambda;M_{hi})=\sum_i^n \left(\frac{M_{lo}}{M_{hi}}\right)^i \wp_i(M_{lo};M_{hi})+\Re_n(\Lambda;M_{lo};M_{hi})\left(\frac{M_{lo}}{M_{hi}}\right)^{n+1}$$

Trustable part   uncertainty

You are allowed to choose to fit anywhere below M_{hi}, but shouldn't ignore the EFT uncertainty associated with the observable you renormalize to. In other words, you shouldn't ask what will happen if you choose to renormalize exactly at E*, if your result doesn't have this accuracy! One way to accommodate this is to encode its effect into a more general form of contact terms, or, a slight change on the regulator.

1. $$f_R(\Lambda)\rightarrow F_R=xf_a(\Lambda)+(1-x)f_b(\Lambda)$$

Choose two regulators have only slight difference

$0\leq x\leq 1$, x *accounts for uncertainty, not an LEC!*

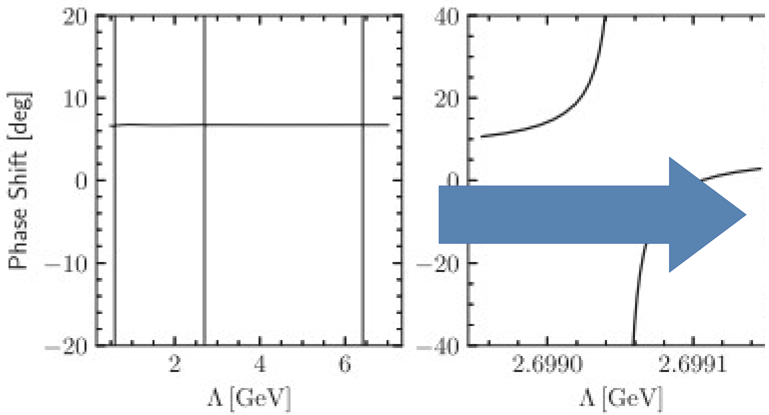
2. *Requirement: for* $0\leq x\leq 1$, *the variation of* $\left|\langle\phi|(V_{NLO}F_R)|\phi\rangle_i\right|\leq \Re_n(M_{lo};\Lambda;M_{hi})\left(\frac{M_{lo}}{M_{hi}}\right)^{n+1}$ *holds for all* $p_i\leq M_{hi}$.

⇓ If yes

Then you are allow to adjust x to whatever value ∈[0,1], and see if this avoid the aforementioned issue. => E.g., if the original issue occurs at x=1 with f_a, see if x=0.5 it still persists

For PC of Long & Yang

- Adopting $xf_a(\Lambda)+(1-x)f_b(\Lambda+\Lambda/1000)$ (or: f_a sharp cutoff, f_b as a super-gaussian) solves the issue.



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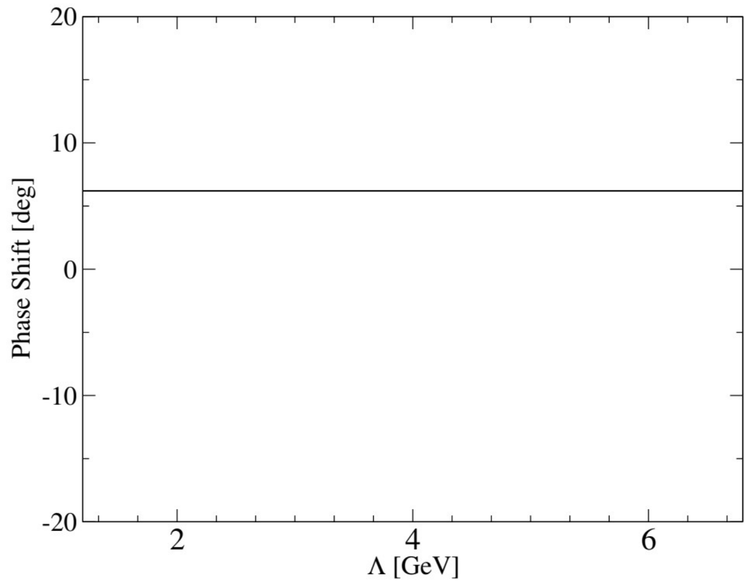


C.-J. Yang et al, in preparation.

Or

Rui Peng, Bingwei Long, Fu-Rong Xu, arXiv: 2407.08342 [nucl-th]

For the toy model

$\Lambda_{NLO}=2\Lambda_{LO}$ or $\Lambda_{LO}/2$ of Gasparyan & Epelbaum

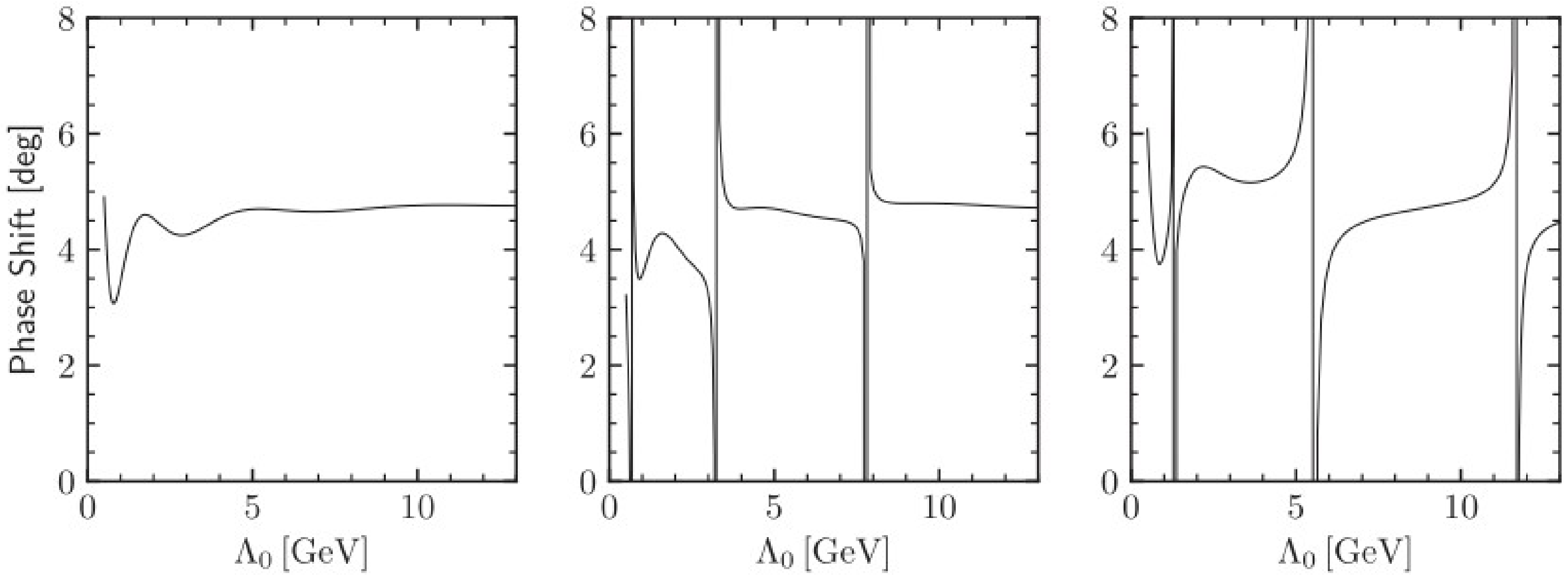
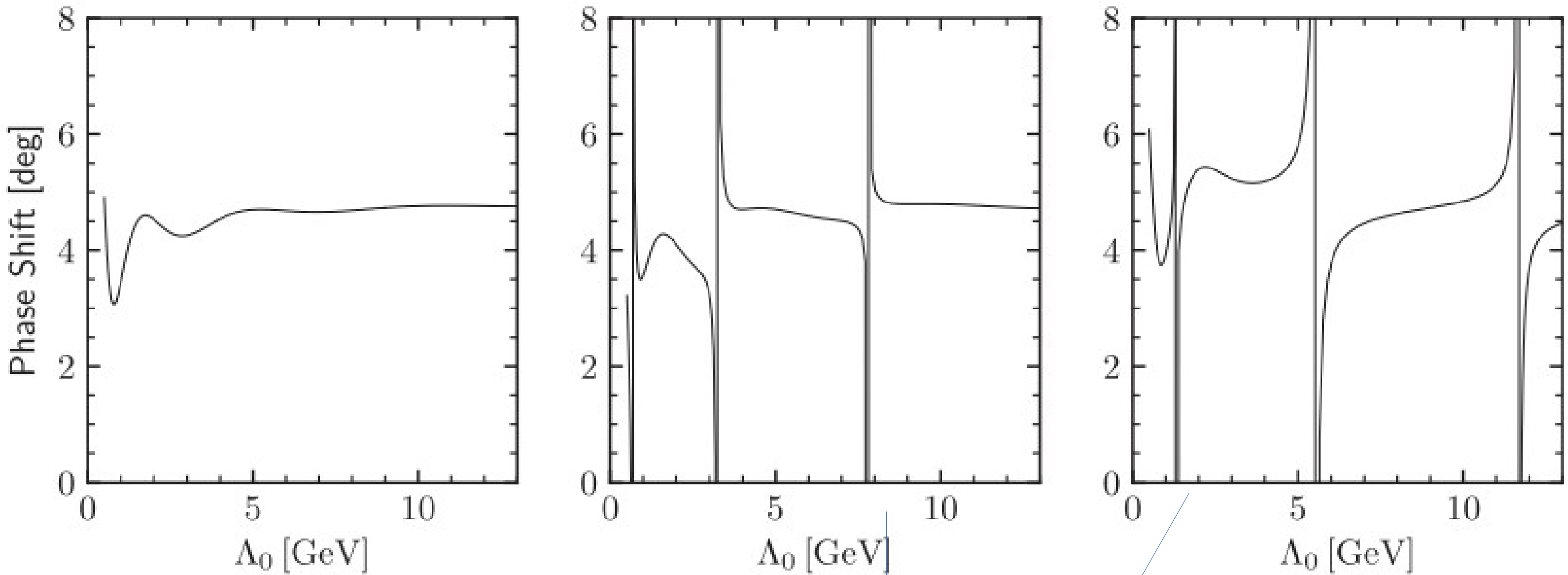


FIG. 2. The $^3P_0$ phase shift at the fixed laboratory energy of T_{lab} = 130 MeV calculated in the simplified model at NLO as a function of the cutoff for $\Lambda_2 = \Lambda_0$ (left panel), $\Lambda_2 = 2\Lambda_0$ (middle panel), and $\Lambda_2 = \Lambda_0/2$ (right panel).

This is equivalent to imposing $F_R=xf_a+(1-x)f_b$, where $f_b=f_a(2\Lambda)$ or $f_b=f_a(\Lambda/2)$.

$$\text{The variation of } |\langle\phi|(V_{NLO}F_R)|\phi\rangle_i| (\text{for } 0\le x\le 1) \gg \Re_n(M_{lo};\Lambda;M_{hi})\left(\frac{M_{lo}}{M_{hi}}\right)^{n+1} > 200\% \times |\langle H_{LO}\rangle_i|.$$

For the toy model $\Lambda_{NLO}=2\Lambda_{LO}$ or $\Lambda_{LO}/2$ of Gasparyan & Epelbaum



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This means, the problem **cannot** be cured by taking uncertainty into account.

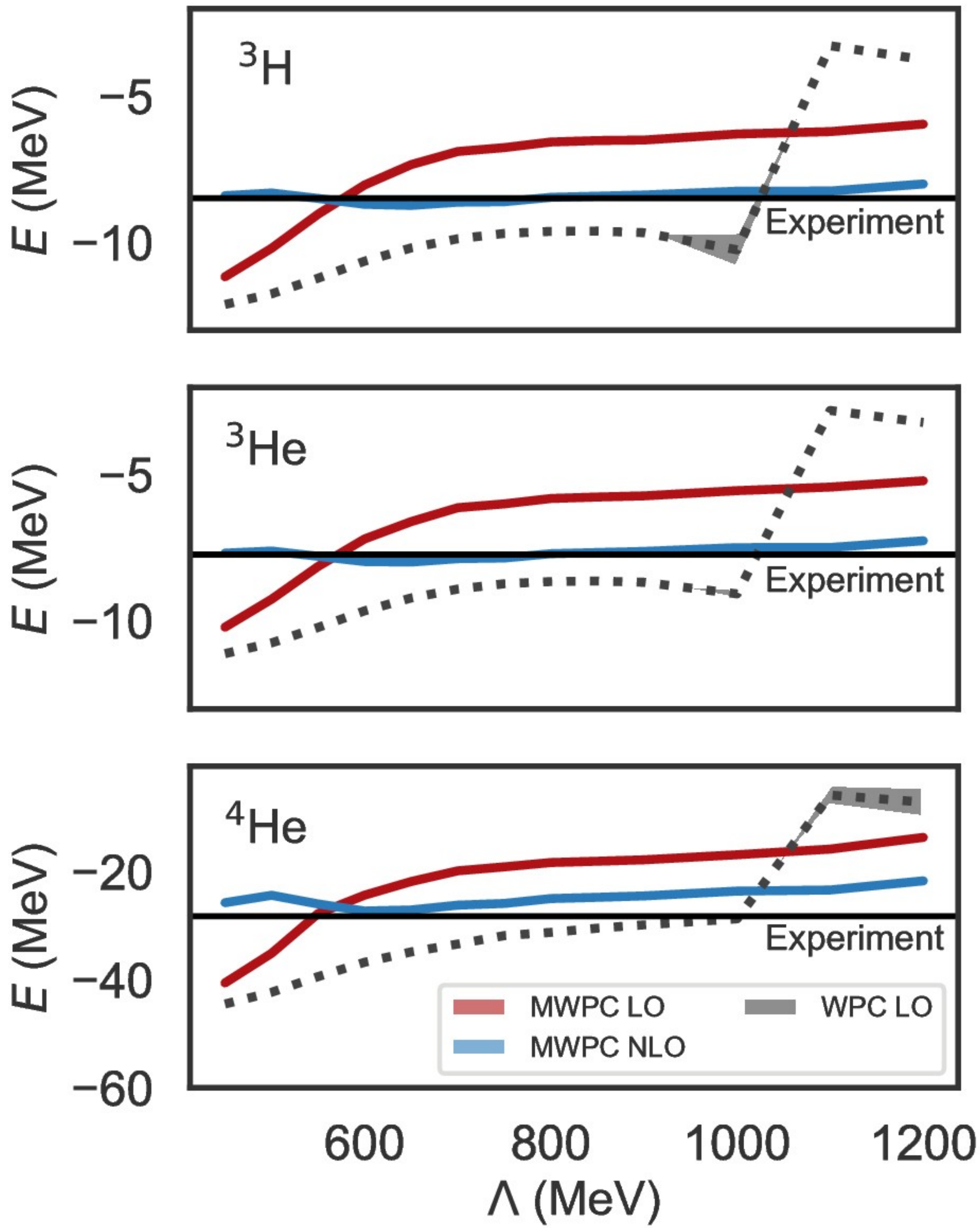
→ It's a real problem.

In other words, DWBA-based PC really doesn't work for the prescribed potentials.

Next check → apply new PC to A>2 systems (via ab-initio calculations)

Let's start from light systems: where 3NFs are small

Use only 2NF up to next-to leading order, do ^{3}H, ^{3}He, ^{4}He



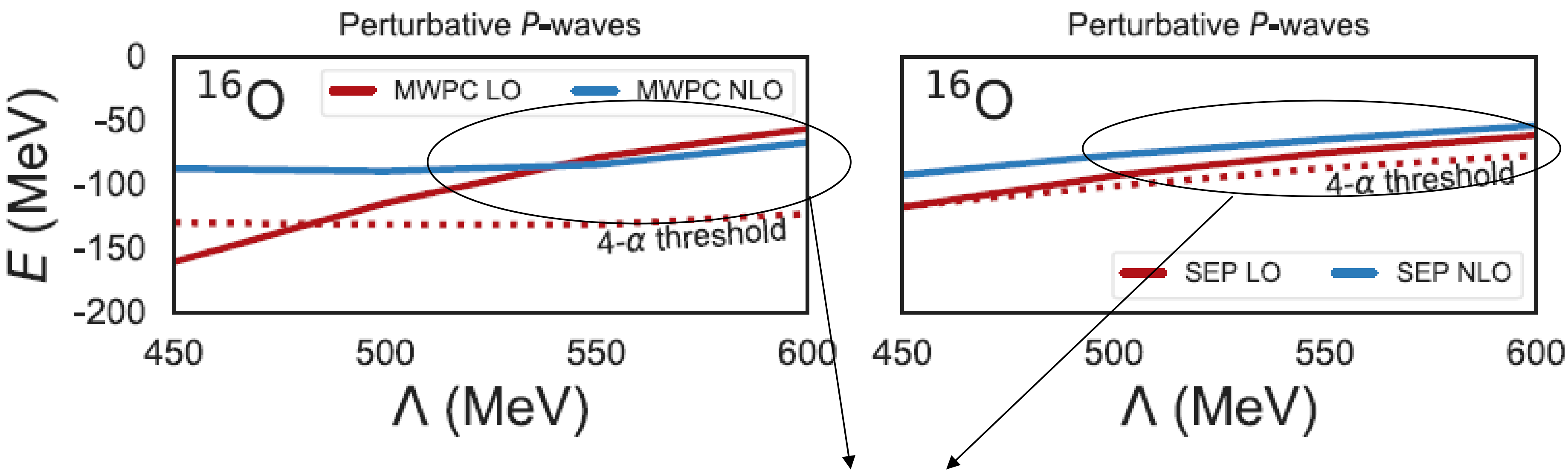
Conclusion:
2NFs up to NLO works
for A≤4 systems.

C.-J. Yang, A. Ekstrom, C. Forssen, G. Hagen,
PRC 103 (2021) 5, 054304.

For A up to 3 see also:
Nogga et al, PRC 72 (2005), 054006
Song et al, PRC 96 (2017), 024002.

So far so good, let continue to A>4

^{16}O results (LO, NN only)



^{16}O non-physical !

MWPC:

At LO, Nogga, Timmerman, van Kolck PC
(*Phys.Rev.C 72 (2005) 054006)*

NLO, plus Long & Yang PC
(*Phys.Rev.C* 86 (2012) 024001)

SEP: NN 1s0 adopts dibaryon field
(Phys.Rev.C 97 (2018) 2, 024001)

Perturbative P-waves: PC by S. Wu & B. Long (*Phys.Rev.C* 99 (2019) 2, 024003)

Wrong ^{16}O pole

The same NN interaction generates ^{16}O with the *wrong* pole structure (not stable w.r.t. 4α decay) at LO. Also, deformed state becomes deeper than spherical state.

Same thing for PC improved with auxiliary dibaryon fields, Weinberg counting and pionless EFT.

M. S. Sánchez, C.-J. Yang, Bingwei Long, U. van Kolck, Phys.Rev. C97 (2018) no.2, 024001.

In fact, nobody got ^{16}O right at LO yet!

- We have exhausted all possibilities (dibaryon, perturbative P-waves, different fitting of LECs) we could think of in the NN sector.

What to do then (to restore the correct pole)?

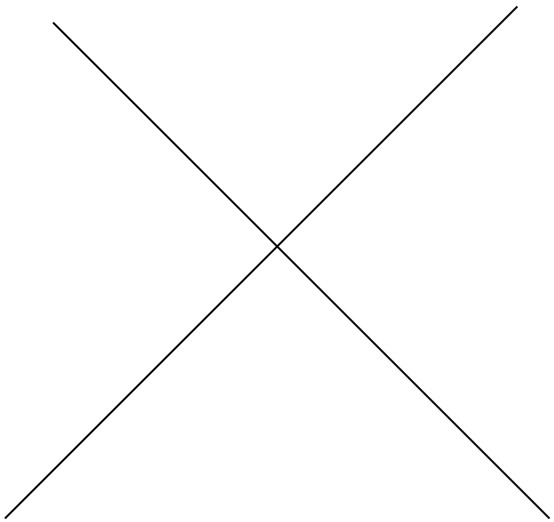
- "Improved action" applied to LO.

L. Contessi, M. Schäfer, U. van Kolck, Phys.Rev.A 109 (2024) 2, 022814
L. Contessi, M. Pavon Valderrama, and U. van Kolck, arXiv:2403.16596 [nucl-th]

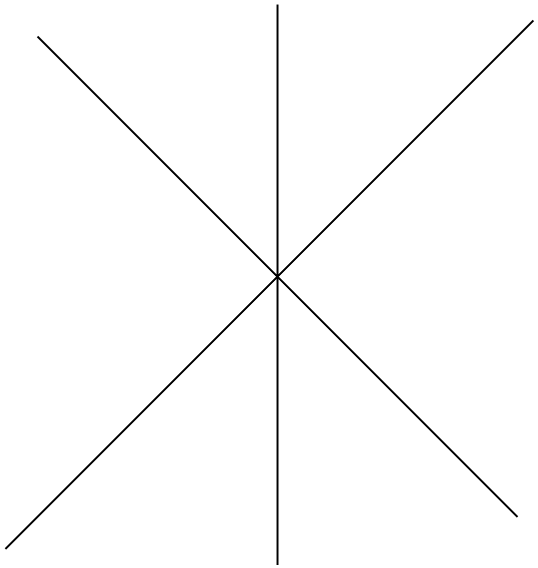
- Seek if other ingredients should belong to LO is missing. PC works on NN and few-body level, but fails for A>10 →rethink the importance of 3NF.

C.J. Yang, A. Ekström, C. Forssén, G. Hagen, G. Rupak, U. van Kolck, Eur.Phys.J.A 59 (2023) 10, 233

Naïve dimensional analysis (NDA)



2 nucleon force



3 nucleon force

$$\text{3NFs/2NFs} \approx \frac{N^{+}N}{f_\pi^{2} M_{hi}} \approx \frac{\rho_0}{93^2 \cdot 500} \approx 0.28$$

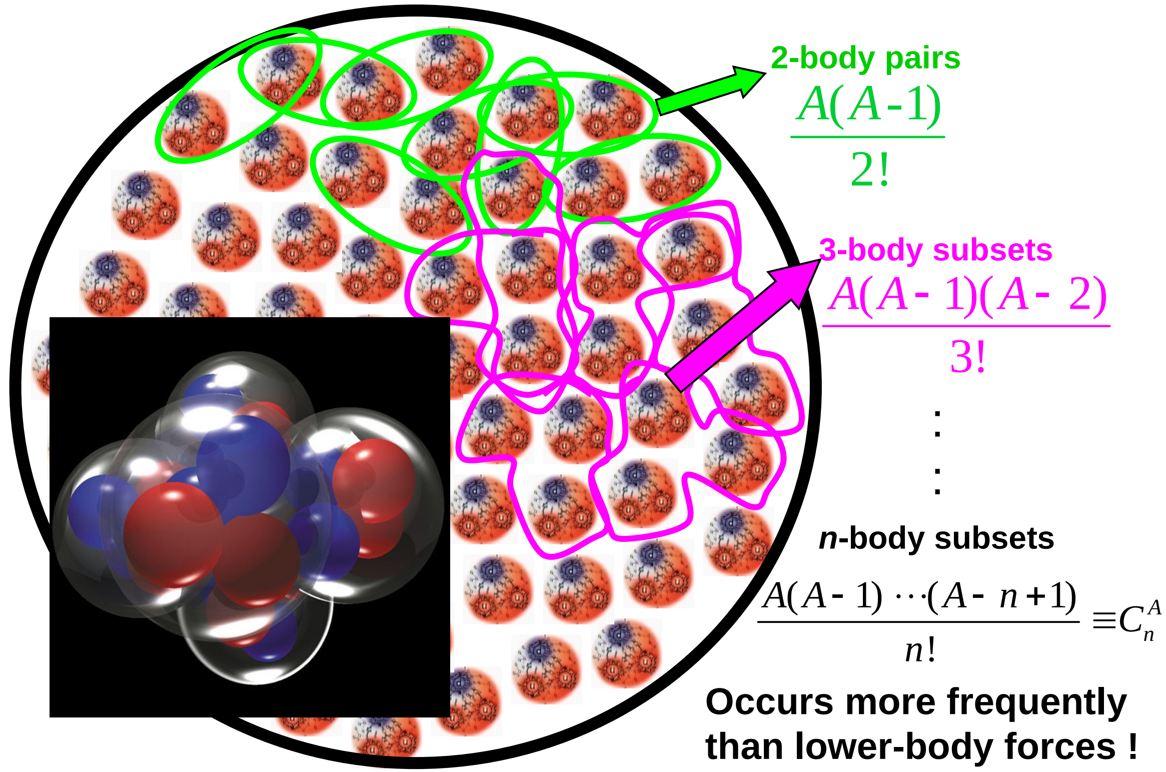
Under NDA:

3^{+}-body forces are less important, which means they should appear later, i.e., accompanied with higher-order (e.g.,NNLO in Δ-less) 2nfs.

However, NDA doesn't take A into account!

Many-body forces in complex systems

- Some of many-body couplings are genuine and unknown, i.e., cannot be derived from NN couplings.
- They are estimated to be weaker by naïve dimension analysis (NDA).
- However, their importance can grow in a large system.



A	number of doublets $\frac{A(A-1)}{2}$	number of triplets $\frac{A(A-1)(A-2)}{6}$
3	3	1
4	6	4
5	10	10
6	15	20

C.-J. Yang, Eur.Phys.J.A 56 (2020) 3, 96

“A choose n” enhancements

$$C_n^A = \frac{A(A-1)(A-2)...(A-n+1)}{n!}$$

- In a self-bound system, the above enhancement won’t be fully counted. For example, an n-body subset will have nearly zero contribution if its constituents span a distance much larger than the range of the n-body forces.→ density saturates, not $\to\infty$.

- On the other hand, those small contributions could still add up to become sizable, due to the fact that there are many of them.

- Thus, the growth of n-body forces in large systems depends on multiple factors such as the **range** and the **form** of interactions, the mass of particles, etc., ➔ **Require actual ab-initio calculations to check the PC.**

Estimations

- Combine NDA and "A choose n":

Combine both:

Approx. with nuclear saturation density

$$\frac{C_n^A F_n}{C_m^A F_m} = \frac{A-m}{n}\left(\frac{\rho_0}{f_\pi^2 M_{\mathrm{hi}}}\right)^{n-m} = \frac{A-m}{n}\left(\frac{142(\mathrm{MeV})}{M_{\mathrm{hi}}}\right)^{n-m}$$

~1

NN and NNN becomes the same important starting from **A=13-26** (M_{hi}=500-1000 MeV)

*****NNN and NNNN becomes the same important starting from A=17-34.

*****5^+-body force is more suppressed (s≥1), only equal to NNNN after A>500.

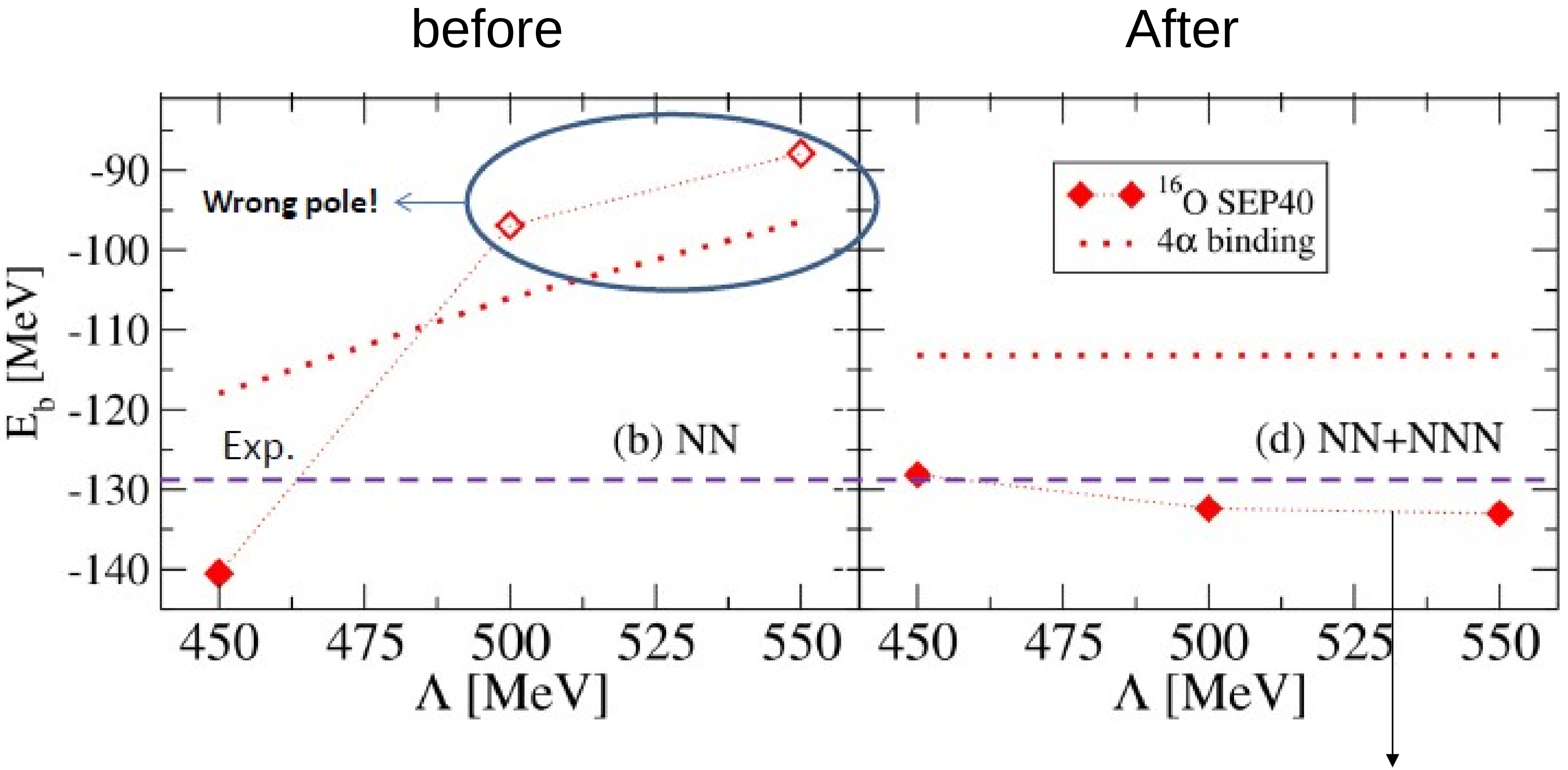
As nuclear forces are short-range, the enhancement can be weaker.

NNN will be LO for A>13

^{16}O has A=16!

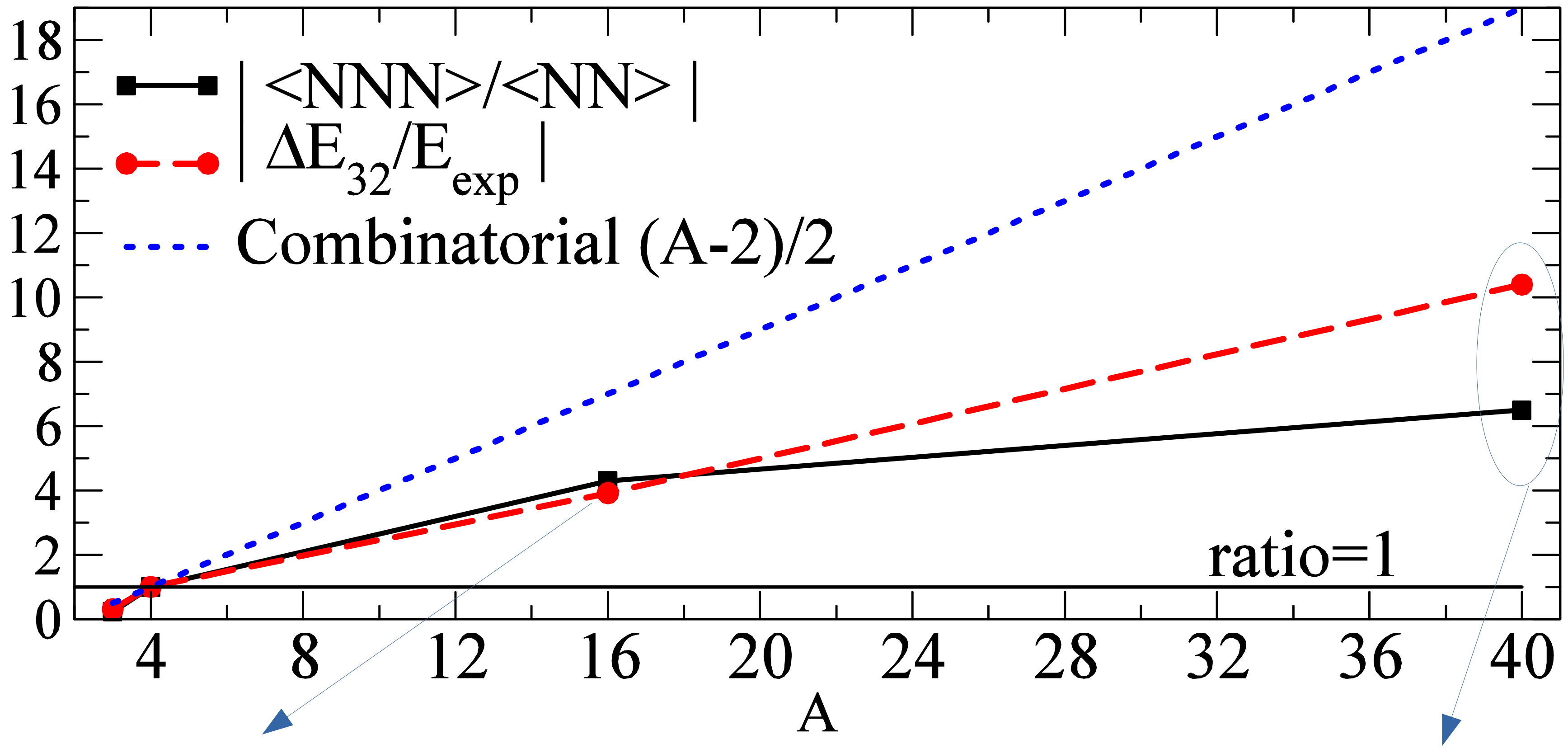
=> Already need NNN at LO

With 3NFs’ size limited to be NNLO on A≤4 systems



Problem solved! ^{16}O great already at LO!

Real Growth (accounting all effects) of 3NF/2NF with A



(Number of particles in the nuclei)

~5 times (promoted at least 1 order ~1/3)

6~10 times

Opposite opinions (from various resources)

1. Double count the combinatorial factor?

P. Navrátil, G. P. Kamuntavičius, and B. R. Barrett, Phys. Rev. C 61, 044001(2000).

(A2) $$\left\langle \sum_{i<j=1}^{A} V_{ij} \right\rangle = \frac{1}{2} A(A-1) \langle V(\sqrt{2}\,\vec{\eta}_{A-1}) \rangle$$

Total from NN=(combinatorial factor)*(V_{NN})

One should arrange (power count) on this!

Arranged by NDA

(A4) $$\left\langle \sum_{i<j<k=1}^{A} V_{ijk} \right\rangle = \frac{1}{6} A(A-1)(A-2) \langle V(\vec{\vartheta}_{A-2}, \vec{\eta}_{A-1}) \rangle$$

Total from NNN=(combinatorial factor)*(V_{NNN})

So, No double counting!

2. Nucleons only interact with nearby nucleons (i.e., the factor is there, but is weaken to a negligible degree)

=> Model space to converge ab-initio ≉ Hartree-Fock → The impact of not nearby interaction in nuclear binding will be ≳ the size of |(converged result) – (HF)|.

=> Compare **the same weakening** in NN to NNN (i.e., weakening also applies to NN).

=> The growing of NNN does stop at saturation (A≈56), with the exception of extreme conditions (e.g., the core of a neutron star).

3. Not enough evidence (e.g., **Bayesian** analysis on WPC does not see such a need).

=> So far it also says **WPC is o.k.** on almost everything (if Λ is restricted).

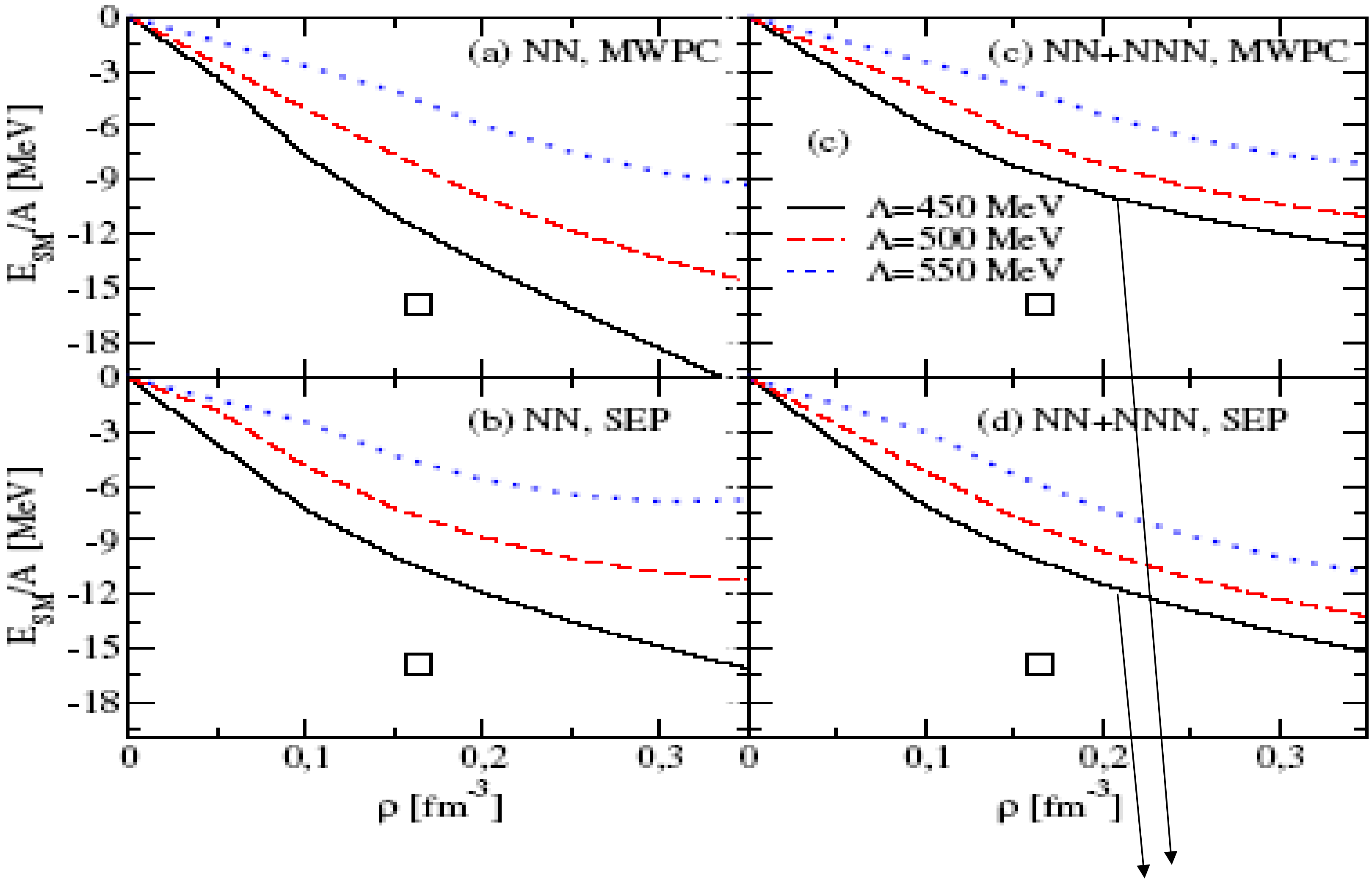
=> The wrong pole at LO without NNN **only shows up when Λ>500 MeV**.

This suggests:

3NFs are LO at least for A≥16

NNNN at LO for larger A?

No-go test by nuclear matter (EoS)



Conclusion: NN+NNN seems **no enough** !

Summary

Why modified PC?

- Because it provides solutions/improvements of conceptual problem of WPC (allow RG to be o.k., or aka, a systematical control of the uncertainty).

Why A-dep PC?

- The combinatorial enhancement becomes important for A>10. This makes the promotion of many-body forces (NNN and NNNN) ***necessary***!

I don’t like it either, but sometimes the correct way happens to be the painful way.

A few thought-provoking questions

1. Are we going back to (EFT-inspired) models→i.e., build whatever describes data? → The error might be controlled (and even reduced at higher-orders to some degree) by a carefully chosen Λ + fitting procedure + Bayesian analysis?

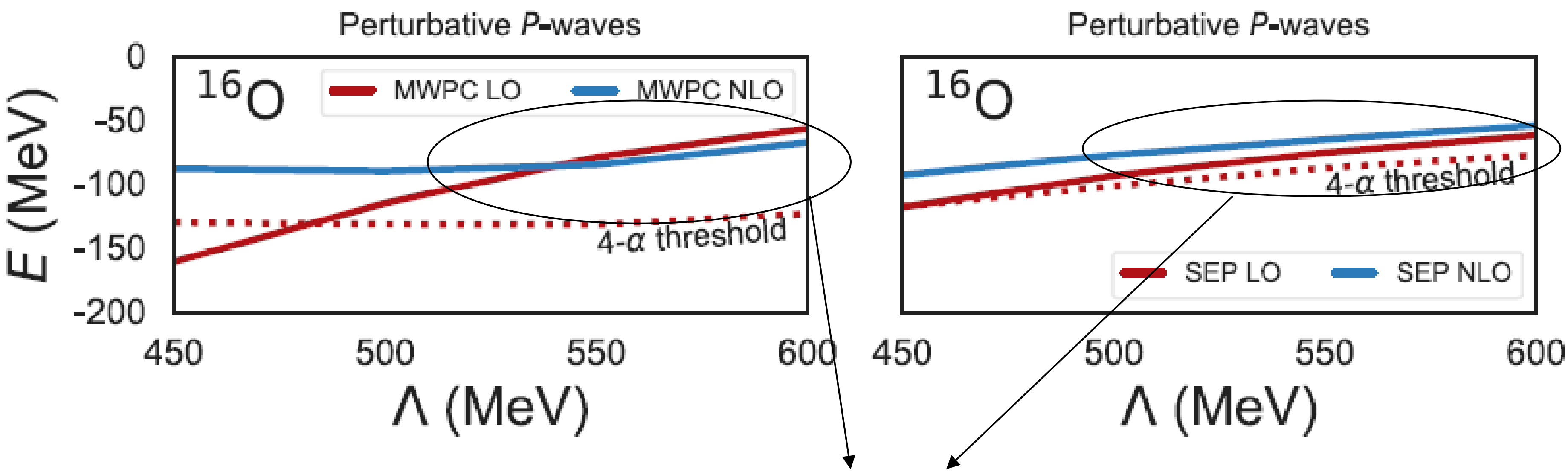
Or, we insist to do the truly EFT-based approach (there might be more things to learn with try & error)?

2. Can WPC (and it's rel. version) solve Ay puzzle?

3. Any doubt on 'the importance of many-body forces' and it's dependence on the number of nucleons?

Thank you!

^{16}O results (LO, NN only)



^{16}O non-physical !

MWPC:

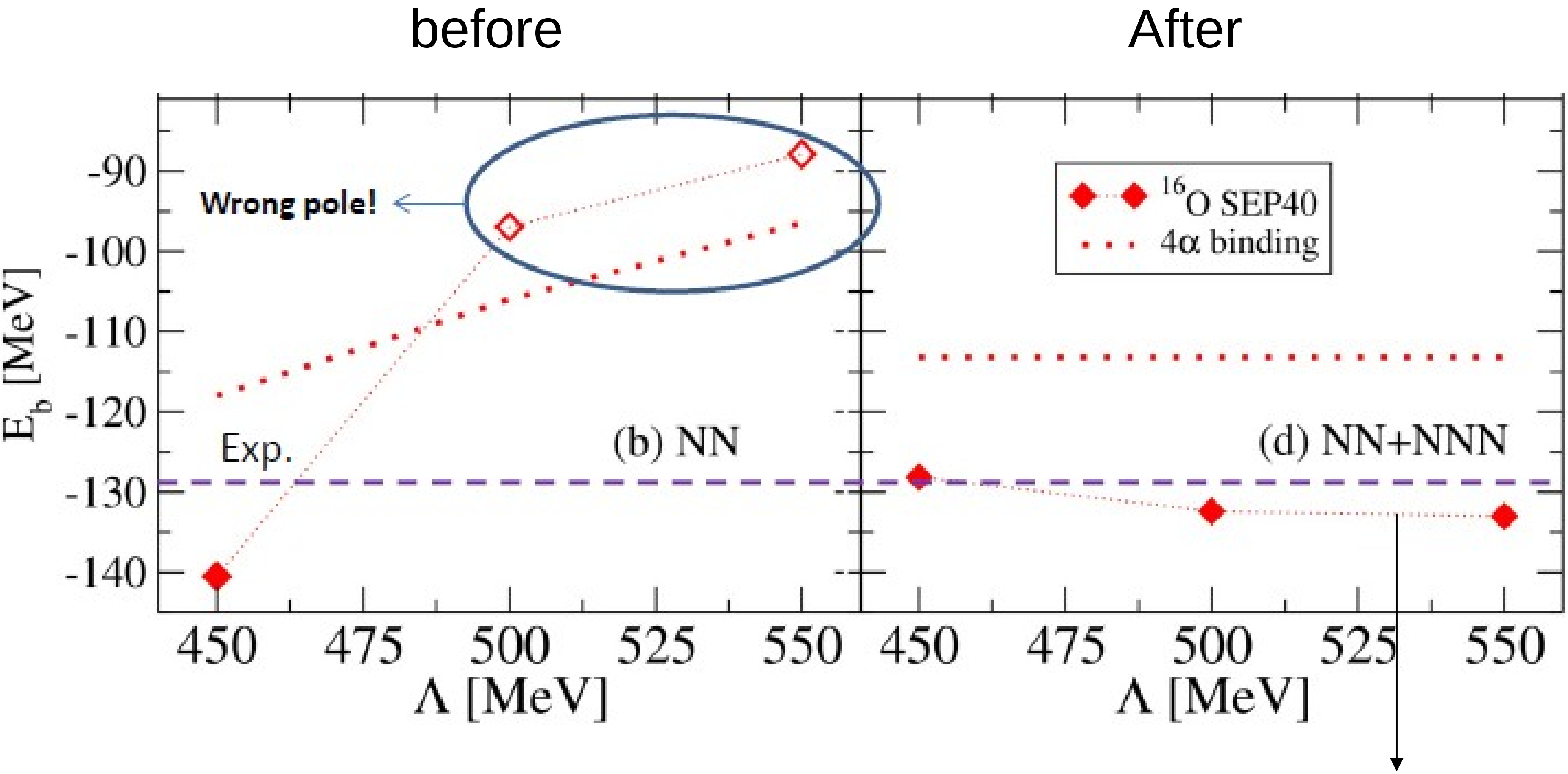
At LO, Nogga, Timmerman, van Kolck PC
(*Phys.Rev.C 72 (2005) 054006)*

NLO, plus Long & Yang PC
(*Phys.Rev.C* 86 (2012) 024001)

SEP: NN 1s0 adopts dibaryon field
(Phys.Rev.C 97 (2018) 2, 024001)

Perturbative P-waves: PC by S. Wu & B. Long (*Phys.Rev.C* 99 (2019) 2, 024003)

With 3NFs' size limited to be NNLO on A≤4 systems



Problem solved! ^{16}O great already at LO!