Few-nucleon scattering in $\not\pi$EFT : status, long-term plan, and challenges

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20th August 2024
ECT* Trento

Nuclear interaction behind the looking glass

Nuclear interaction :

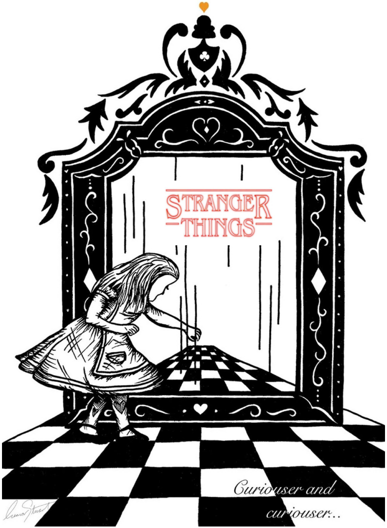
Two-body
(*NN* scattering, $^2\mathrm{H}$)

Three-body
(*Nd* scattering, $^3\mathrm{H}$, $^3\mathrm{He}$, ...)

Bound state properties
($^4\mathrm{He}$, ...)

$\longrightarrow$
Precise few-body methods
$\longrightarrow$

Few-body $A > 3$ continuum
(scattering, reactions, ...)



Nuclear few-body continuum

Analyzing power A_y in low-energy N-d and p-$^3\mathrm{He}$ elastic scattering

(A. Margaryan et al. Phys. Rev. C 93 (2016) 054001; L. Girlanda, Phys. Rev. C 99 (2019) 054003)

Isoscalar monopole resonance of $^4\mathrm{He}$ (the first $^4\mathrm{He}$ excited state)

(S. Bacca et al., Phys. Rev. Lett. 110 (2013) 042503; S. Kegel et al. Phys. Rev. Lett. 130 (2023) 152502)

→ discrepancy between experimental and theoretically predicted monopole transition form factor

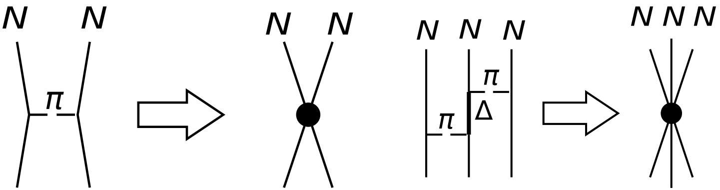
Splitting between $^2P_{3/2}$ and $^2P_{1/2}$ partial waves in $^4\mathrm{He} + \mathrm{n}$

(R. Lazauskas, Phys. Rev C 97 (2018) 044002; A. M. Shirokov et al.,Phys. Rev. C 98 (2018) 044624)

Big Bang Nucleosynthesis reactions

→ astrophysical S-factors of $d(d,p)^3\mathrm{H}$ and $d(d,n)^3\mathrm{He}$ reactions at very low energies (100 keV)

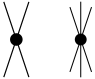
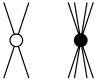
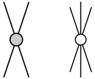
$\not\pi$EFT - basic idea



Baryonic EFT :

$\rightarrow$ no pionic degrees of freedom

$\not{\pi}$EFT

LO		$\delta(\mathbf{r}_{12}),\ \delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})$
NLO		$\overleftarrow{\nabla}^2_{\mathbf{r}_{12}}\delta(\mathbf{r}_{12}) + \delta(\mathbf{r}_{12})\overrightarrow{\nabla}^2_{\mathbf{r}_{12}},\ \delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})\delta(\mathbf{r}_{34})$
N^2LO		$S-D$ tensor ($T=0$), momentum dep. 3-body
N^3LO	...	$(\nabla_{\mathbf{r}_1}\cdot\nabla_{\mathbf{r}_2})\delta(\mathbf{r}_{12})$, LS, tensor ($T=1$), more 4-body ?

(H.-W. Hammer, Sebastian König, and U. van Kolck, Rev. Mod. Phys. 92 (2020) 025004)

$\not\pi$EFT

- breakdown scale $M = m_\pi$, estimate of typical momentum $Q(^4\mathrm{He}) \approx 115\mathrm{MeV}$
- nuclear pionless EFT has large truncation error at LO
  $\rightarrow$ however, it seems to works well in few-body physics

Regularization/Renormalization

$$C\ \delta(\mathbf{r}_{ij}) \rightarrow C(\lambda)\left(\frac{\lambda}{2\sqrt{\pi}}\right)^3 \mathrm{e}^{\frac{-\lambda^2 r_{ij}^2}{4}}$$

$$D\ \delta(\mathbf{r}_{ij})\delta(\mathbf{r}_{jk}) \rightarrow D(\lambda)\left(\frac{\lambda}{2\sqrt{\pi}}\right)^6 \mathrm{e}^{\frac{-\lambda^2 (r_{ij}^2+r_{jk}^2)}{4}}$$

- $C(\lambda), D(\lambda)$ are low energy constants (LECs) tuned to reproduce two-body resp. three-body observables for each λ
- required (RG invariance for $\lambda >> M$)
  $\rightarrow$ all observable will become λ independent when $\lambda \rightarrow \infty$

$$O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \dots$$

$\not\pi$EFT potential at LO and NLO

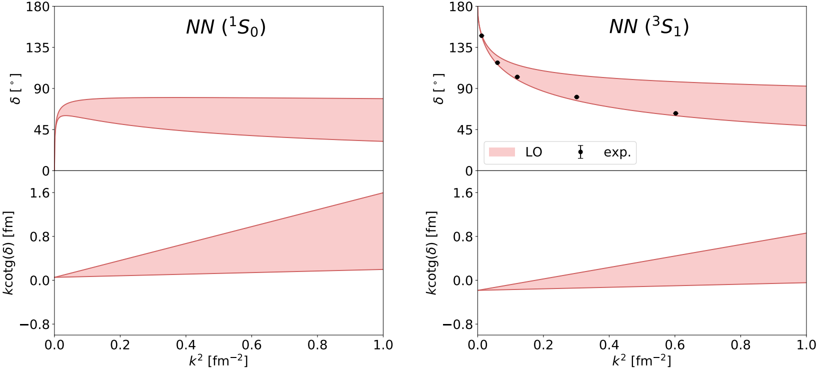
Leading order potential (3 LECs) :

$$V_\lambda^{(\mathrm{LO})} = \sum_{i<j}\left[C_0^{(0)}(\lambda)P_{ij}^{T=1,S=0} + C_1^{(0)}(\lambda)P_{ij}^{T=0,S=1}\right]e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$
$$+ D_0^{(0)}(\lambda)\sum_{i<j<k}\mathcal{Q}_{ijk}^{T=1/2,S=1/2}\sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+\mathbf{r}_{jk}^2)}$$

Next-to-leading order potential (6 LECs) :

$$V_\lambda^{(\mathrm{NLO})} = \sum_{i<j}\left[C_0^{(1)}(\lambda)P_{ij}^{T=1,S=0} + C_1^{(1)}(\lambda)P_{ij}^{T=0,S=1}\right]e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$
$$+ \sum_{i<j}\left[C_2^{(1)}(\lambda)P_{ij}^{T=1,S=0} + C_3^{(1)}(\lambda)P_{ij}^{T=0,S=1}\right](\mathbf{k}^2+\mathbf{q}^2)e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$
$$+ D_0^{(1)}(\lambda)\sum_{i<j<k}\mathcal{Q}_{ijk}^{T=1/2,S=1/2}\sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+.\mathbf{r}_{jk}^2)}$$
$$+ E_0^{(1)}(\lambda)\sum_{i<j<k<l}\mathcal{Q}_{ijkl}^{T=0,S=0}e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+\mathbf{r}_{ik}^2+\mathbf{r}_{il}^2+\mathbf{r}_{jk}^2+\mathbf{r}_{jl}^2+\mathbf{r}_{kl}^2)}$$

LO $\not\pi$EFT



Leading order (LO) :

(exp. constraints)

a^0_{NN} (a^0_{nn}) = −18.95(40) fm

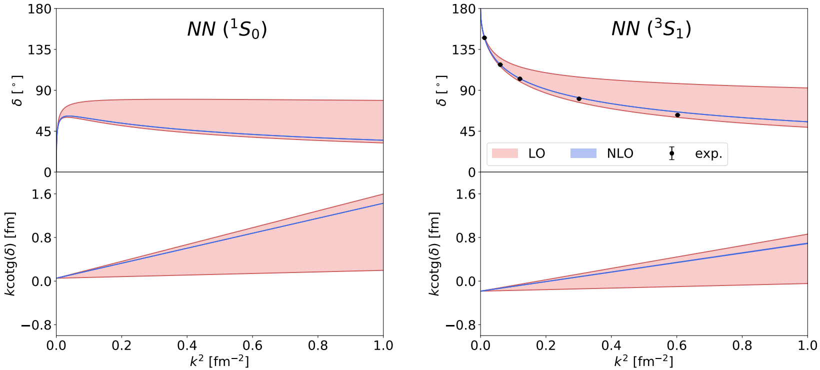
a^1_{NN} (a^1_{np}) = 5.419(7) fm

$B(^3\mathrm{H}) = 8.482$ MeV

Effective range expansion :

$$k\mathrm{cotg}(\delta) = -\frac{1}{a} + \frac{1}{2} r k^2 + \ldots$$

NLO π̸EFT



Leading order (LO) :

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$$a^1_{NN}\ (a^1_{np}) = 5.419(7)\ \text{fm}$$

$$B(^3\mathrm{H}) = 8.482\ \text{MeV}$$

Next-to-leading order (NLO) :

(exp. constraints)

$$r^0_{NN}\ (r^0_{nn}) = 2.75(11)\ \text{fm}$$

$$r^1_{NN}\ (r^1_{np}) = 1.753(8)\ \text{fm}$$

$$B(^4\mathrm{He}) = 28.296\ \text{MeV}$$

Various applications ...

Analysis of LQCD data (LO; A $\leq$ 16)

(N. Barnea et al., Phys. Rev. Lett. 114 (2015) 052501)
(M. Eliyahu, Phys. Rev. C 102 (2020) 044003)
(W. Detmold et al., Phys. Rev. D 103 (2021) 074503)
(R. Yaron et al., Phys. Rev. D 106 (2022) 014511)

(T. Attia-Weiss et al., Phys. Rev. D 109 (2024) 114515)

Hypernuclei (LO; A $\leq$ 6)

(L. Contessi et al., Phys. Rev. Lett. 121 (2018) 102502)
(L. Contessi et al., Phys. Lett. B 797 (2019) 134893)
(F. Hildenbrand et al., Phys. Rev. C 102 (2020) 064002)
(M. Schäfer et al. Phys. Lett. B 808 (2020) 135614)
M. Schäfer et al., Phys. Rev. C 103 (2021) 025204)
(M. Schäfer et al., Phys. Rev. C 105 (2022) 015202)

(M. Schäfer et al., Phys. Rev. C 106 (2022) L031001)

Atomic systems (LO & NLO; A $\leq$ 6)

(B. Bazak et al., Phys. Rev. A 94 (2016) 052502)

(B. Bazak et al. Phys. Rev. Lett. 122 (2019) 143001)

η-nuclei (LO)

(N. Barnea et al., Phys. Lett. B 771 (2017) 297)

$\rightarrow$ see Nir's talk for LQCD and Hypernuclei and Lorenzo's talk for many-body

NLO $\not\pi$EFT

Where we stand ?

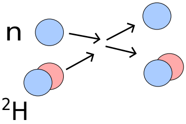
- NLO $\not\pi$EFT using 6 experimental constraints
  (a, r) of $NN(^1S_0)$ and $NN(^3S_1)$, $B(^3\mathrm{H})$, $B(^4\mathrm{He})$
- perturbative NLO using potentials
  (easily extended to 2, 3, 4, 5, 6, ...-body systems)

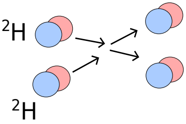
What we want to study ?

- convergence of all $\not\pi$EFT NLO predictions with λ
- comparison with experimental results

$\longrightarrow$ **no more $A \leq 5$ nuclear bound states** to test our theory $\longrightarrow$ **few-body scattering**

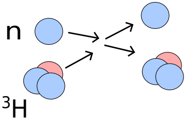
$\rightarrow$ perturbative NLO $\not\pi$EFT predictions at 4- and 5-body level

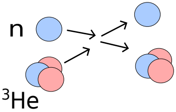




Few-Body scattering

(elastic; no Coulomb interaction)





Few-body scattering

Precise calculation of few-body scattering is rather challenging task !

1) Faddeev equations ($A = 3$)
2) Faddeev-Yakubovski equations ($A = 4, 5$)
3) RGM
4) Continuum Hyperspherical-Harmonics
5) HORST method (HO basis)
6) finite volume methods (periodic boundary condition, HO trap)

$\rightarrow$ **even more difficult once we require precise perturbative results**

In this work, we use a Harmonic oscillator trap.

BERW formula

$\rightarrow$ assumption of short-range potential with range $R << b_{\mathrm{HO}} = \sqrt{\frac{1}{\mu\omega}}$

Bush formula

$$-\sqrt{4\mu\omega}\,\frac{\Gamma\left(3/4-\epsilon_n/2\omega\right)}{\Gamma\left(1/4-\epsilon_n/2\omega\right)} = k\,\mathrm{cotg}(\delta), \qquad k=\sqrt{2\mu\epsilon_n}$$

(A. Suzuki, Phys. Rev. A 80 (2009) 033601, T. Bush Found. of Phys. 28 (1998) 4)

LO $\not\pi$EFT calculations:

$$H(\omega) = T_k + V_\lambda^{(\mathrm{LO})} + V_{\mathrm{HO}}(\omega) \longrightarrow H(\omega)\psi_n = \epsilon_n\psi_n \overset{\mathrm{BERW}}{\overbrace{\longrightarrow}} k\,\mathrm{cotg}(\delta)$$

NLO $\not\pi$EFT calculations:

$$\epsilon_n^{(\mathrm{NLO})} = \epsilon_n + \langle\psi_n|\,V_\lambda^{(\mathrm{NLO})}\,|\psi_n\rangle \overset{\mathrm{BERW}}{\overbrace{\longrightarrow}} k\,\mathrm{cotg}(\delta^{(\mathrm{NLO})})$$

Stochastic Variational Method

(K. Varga et al., NPA571 (1994) 447, K. Varga, Y. Suzuki, PRC52 (1995) 2885)

$$H\psi = E\psi, \qquad \psi = \sum_{i=0}^{N} c_i \varphi^i$$

Basis states

- antisymmetrized correlated Gaussians (assuming L=0)

$$\varphi^i_{SM_S TM_T}(\mathbf{x}, A_i) = \mathcal{A}\{G_{A_i}(\mathbf{x})\chi_{SM_S}\eta_{TM_T}\}, \quad G_{A_i}(\mathbf{x}) = e^{-\frac{1}{2}\mathbf{x}A_i\mathbf{x}}$$

- Jacobi coordinates $\mathbf{x}$, A_i symmetric positive definite matrix of $\frac{N(N-1)}{2}$ real parameters, spin χ_{SM_S} and isospin η_{TM_T} parts

optimization of variational basis in a **random trial and error procedure**

Universality

Universal fermionic relations (STM, Petrov, Deltuva, ...)
Atom-Dimer scattering

$$\frac{a_{ad}}{a_{aa}} = 1.1791 + 0.553\frac{r_{aa}}{a_{aa}}; \ \frac{r_{ad}}{a_{aa}} = -0.038 + 1.04\frac{r_{aa}}{a_{aa}}$$

Dimer-Dimer scattering

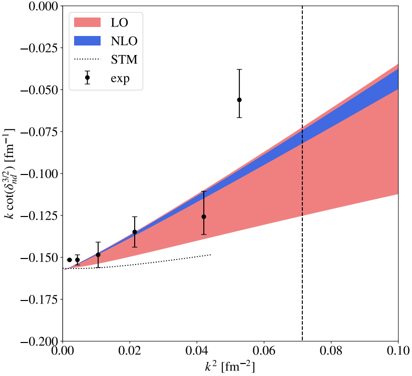
$$\frac{a_{dd}}{a_{aa}} = 0.5986 + 0.105\frac{r_{aa}}{a_{aa}}; \ \frac{r_{dd}}{a_{aa}} = 0.133 + 0.51\frac{r_{aa}}{a_{aa}}$$

These results are reproduced for spin-saturated systems:

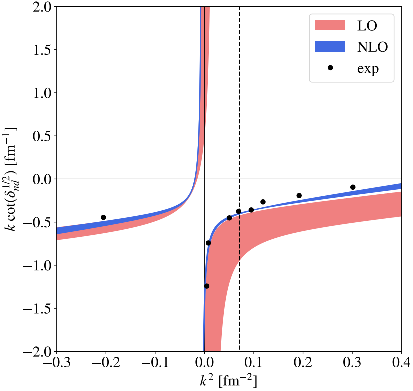
- Neutron-Deuteron $S = 3/2$ scattering
- Deuteron-Deuteron $S = 2$ scattering

$n + d$ $(S = 3/2, T = 1/2)$ and $(S = 1/2, T = 1/2)$ scattering

$(\mathbf{S} = 3/2, \mathbf{T} = 1/2)$



$(\mathbf{S} = 1/2, \mathbf{T} = 1/2)$



$n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering

- four different 4-body channels $(S = 0, T = 1)$, $(S = 0, T = 0)$, $(S = 1, T = 1)$, and $(S = 1, T = 0)$
- no isospin breaking terms, our approach does not distinguish between different 4-body T_z

For $n +{}^3\mathrm{H}$ $(T_z = -1)$ **:**

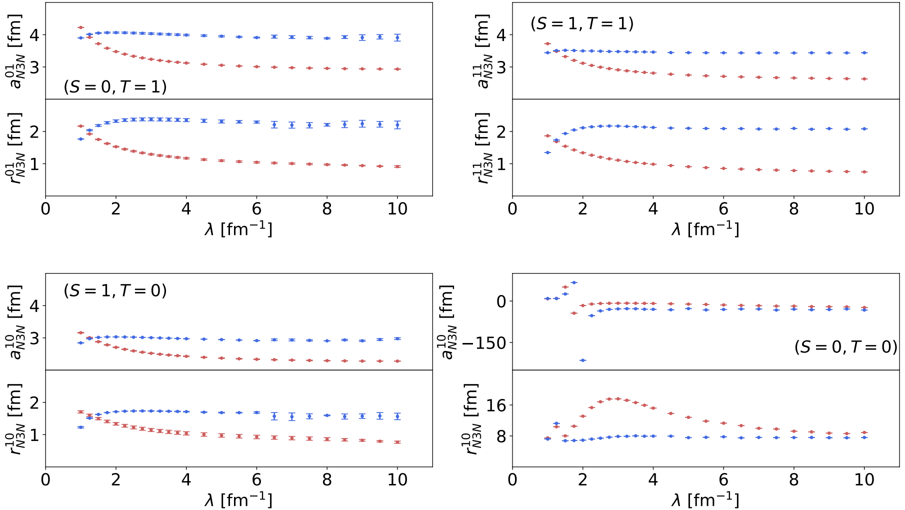
$$S = 0 \longrightarrow (S = 0, T = 1)$$
$$S = 1 \longrightarrow (S = 1, T = 1)$$

For $n +{}^3\mathrm{He}$ $(T_z = 0)$ **:**

$$S = 0 \longrightarrow (S = 0, T = 0) + (S = 0, T = 1) \quad {}^4\mathrm{He}(0_2^+) \ \text{resonance}$$
$$S = 1 \longrightarrow (S = 1, T = 0) + (S = 1, T = 1)$$

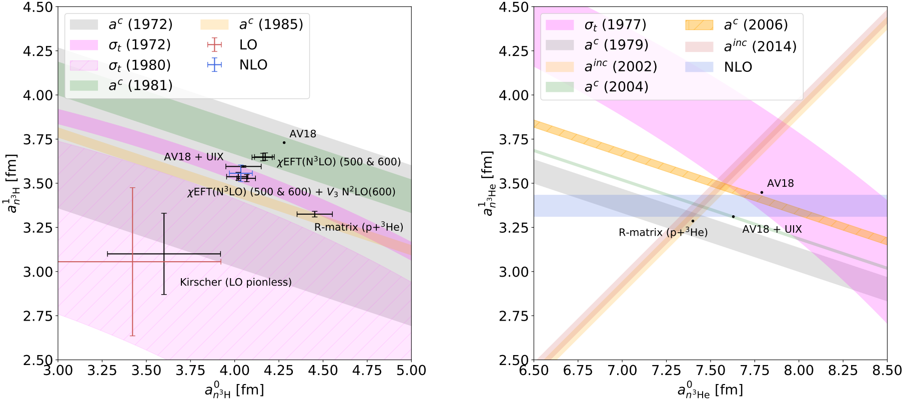
- for $n +{}^3\mathrm{He}$ scattering we must include two different isospin channels

$n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering



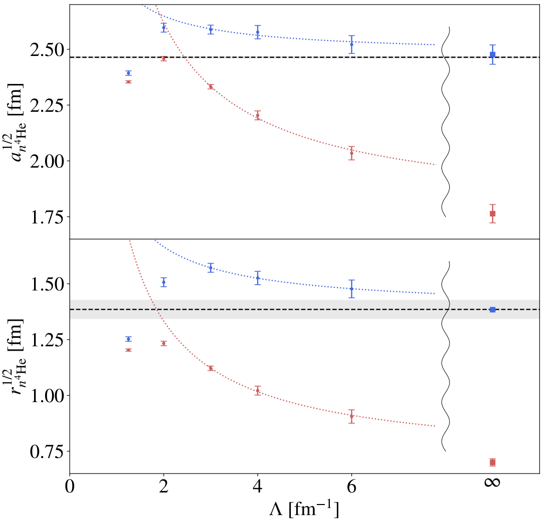
- four-body force needed only in $(S = 0, T = 0)$ channel

Experiment & Theory : $n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering lengths

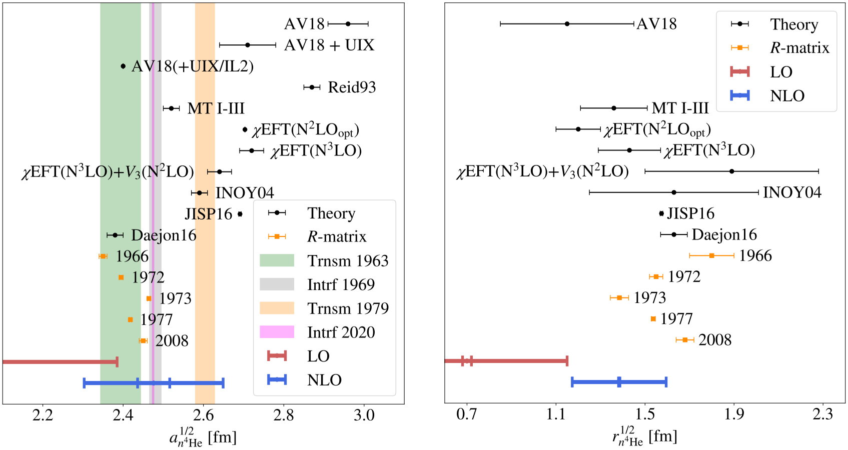


(Phys. Rev. C 42 (1990) 438; Phys. Rev. C 102 (2020) 034007; Few-Body Syst. 34 (2004) 105; Phys. Lett B 721 (2013) 355; Phys. Rev. C 68(R) (2003) 021002)

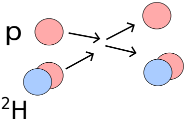
$n + {}^4\mathrm{He}$ scattering

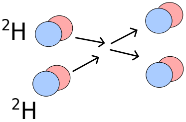


$n+{}^4$He scattering



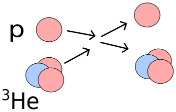
For references to all theoretical results, see (Phys. Lett. B 844 (2023) 138078).

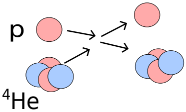




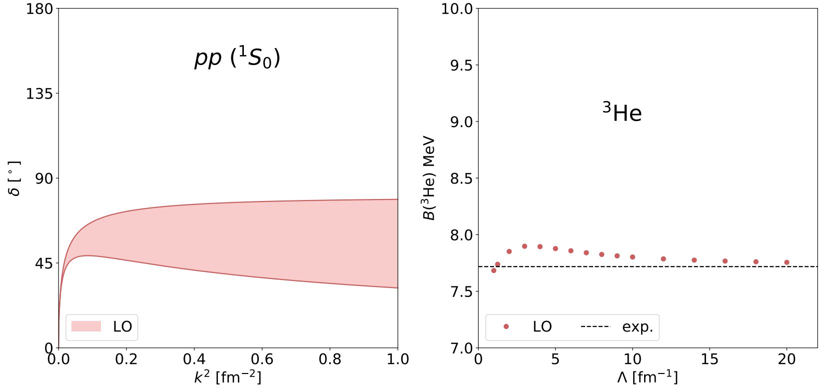
Few-Body scattering with nonperturbative Coulomb

(preliminary)





LO $\not{\pi}$EFT with nonperturbative Coulomb (preliminary)



Leading order (LO) :

$a_{nn/pn}(^1S_0) = \ -18.95(40)$ fm

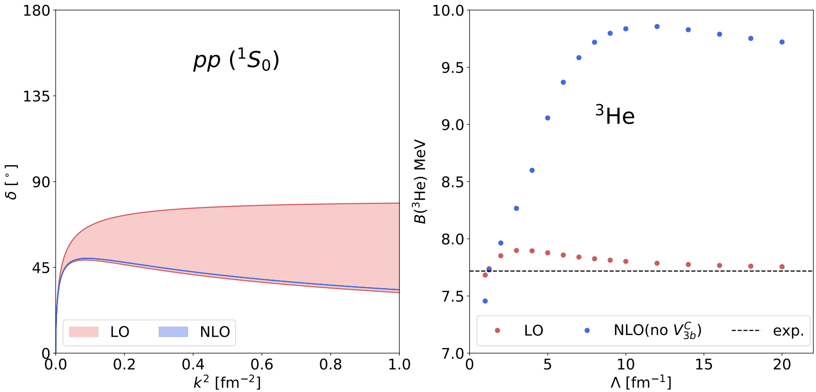
$B(^2\mathrm{H})(^3S_1) = 2.2246$ MeV

$B(^3\mathrm{H}) = 8.482$ MeV

$e^2/(\hbar c)^2 = 1.44$ MeV.fm

$a_{pp}(^1S_0) = \ -7.8063(26)$ fm

NLO $\not{\pi}$EFT with nonperturbative Coulomb (preliminary)



(new 3-body at NLO - J. Vanasse *et al.*, Phys. Rev. C 89 (2014) 064003)

Next-to-leading order (NLO) :

$r_{nn/np}(^1S_0) = 2.75(11)$ fm

$r_{pn}(^3S_1) = 1.764$ fm

$r_{pp}(^1S_0) = 2.794(14)$ fm

$B(^3\mathrm{He}) = 7.718$ MeV

$B(^4\mathrm{He}) = 28.296$ MeV

BERW formula for charged projectile and target (preliminary)

Quantization condition for the HO trap energy spectrum

$$\det\left[\cot\left(\delta(E)\right) - \mathcal{F}^{\mathrm{HO}}(E)\right] = 0$$

$$\mathcal{F}^{\mathrm{HO}}(E) = \left(2\mu k^{2l+1} C_l^2(\eta)\right)^{-1} \lim_{r,r'\to 0} \frac{\Re\left[G_l^{C}(r,r';E)\right] - G_l^{C,\,\omega}(r,r';E)}{(rr')^l}$$

(P. Guo, Phys. Rev. C 103 (2021) 064611; H. Zhang *et al.* Phys. Lett. B 850 (2024) 138490)

$C_l(\eta) = \frac{2^l}{(2l+1)!}\left|\Gamma\left(l+1+i\eta\right)\right| e^{-\frac{\pi}{2}\eta}$; $\eta = q_1 q_2 \mu / k$

$G_l^{C}(r,r';E)$ ... Coulomb Green function
$G_l^{C,\,\omega}(r,r';E)$ ... Coulomb + HO potential Green function

BERW formula for charged projectile and target (preliminary)

$\rightarrow$ developed new code to which connects $(E, \omega) \leftrightarrow (\delta, \omega)$

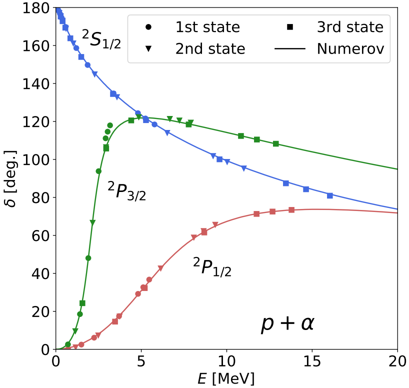
Limit approximation:

$$\frac{\Re\left[G_l^C(r_{\min}, r_{\min}, E)\right] - G_l^{C,\omega}(r_{\min}, r_{\min}, E)}{(r_{\min})^{2l}}$$

Numerical solution of Dyson equations:

$$G_l^C(r, r') = G_l^{\text{free}}(r, r') +$$
$$+ \int_0^{+\infty} \mathrm{d}r''\ G_l^{\text{free}}(r, r'')\ r''^2 V_C(r'')\ G_l^C(r'', r')$$

$$G_l^{C,\ \omega}(r, r') = G_l^{\omega}(r, r') +$$
$$+ \int_0^{+\infty} \mathrm{d}r''\ G_l^{\omega}(r, r'')\ r''^2 V_C(r'')\ G_l^{C,\ \omega}(r'', r')$$

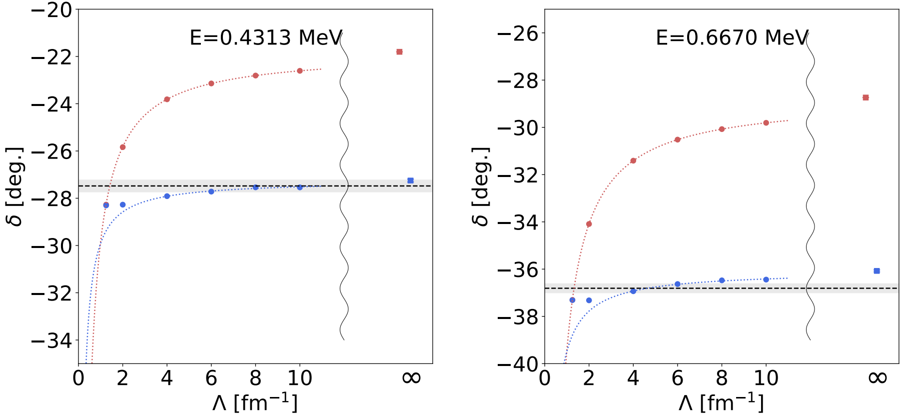


$\rightarrow$ code tested on s- and p-wave $p + \alpha$ scattering

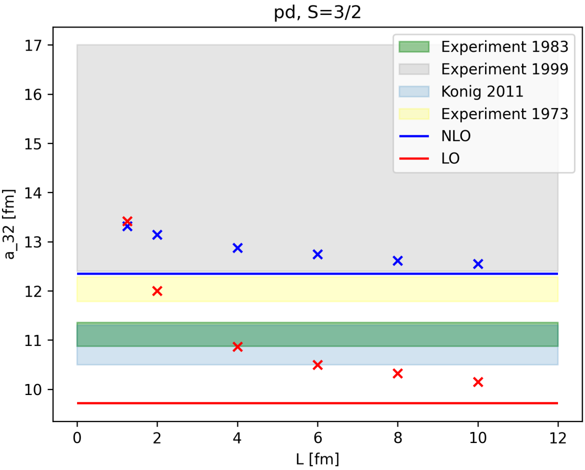
s-wave $p + d$ spin-quartet scattering (preliminary)

- experimental data from *pd* phase-shift analysis

  (M. H. Wood *et al.*, Phys. Rev. C 65 (2002) 0334002)



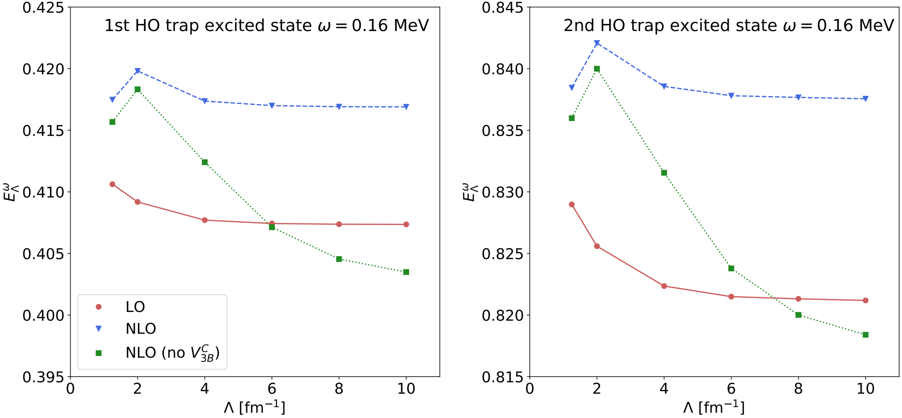
s-wave $p + d$ spin-quartet scattering (preliminary)



(T.C. Black *et al.*, Phys. Lett. B 471 (1999) 103; E. Huttel *et al.*, Nucl. Phys. A 406 (1983) 443; J. Arvieux, Nucl. Phys. A 221 (1973) 253; S. König and H.-W. Hammer, Phys. Rev. C 90 (2014) 034005)

s-wave $p + d$ spin-doublet scattering (preliminary)

stabilization of *pd* spin-doublet HO trap energy spectrum at NLO



$+$ work on $d + d$, $p+{}^3$He, and $p+{}^4$He scattering is underway

Summary & Outlook

- constructed $\not\pi$EFT potential with nonperturbative Coulomb interaction up to NLO
- tests on calculations of 2-, 3-, 4-, and 5-body elastic scattering

Next steps :

- scattering in higher partial waves
- inelastic scattering, nuclear reactions
- $^4\mathrm{He}(0^+)$ resonance, binding of $^6\mathrm{Li}$ and $^6\mathrm{He}$ at higher orders
- higher $\not\pi$EFT orders ($\mathrm{N^2LO}$, $\mathrm{N^3LO}$,...)

Questions

1. How to assess cutoff convergence/divergence in few- and many-body calculations?
2. How is the typical momentum Q size affected by few- and many-body dynamics in different nuclear observables?
3. If we could choose one few- or many-body nuclear observable where one could do a fully perturbative $\not\pi$EFT study up to very high orders, which would it be?