Effective field theory with resonant P-wave interaction

Songlin LYU

Università degli studi della Campania Luigi Vanvitelli

The nuclear interaction: post-modern developments

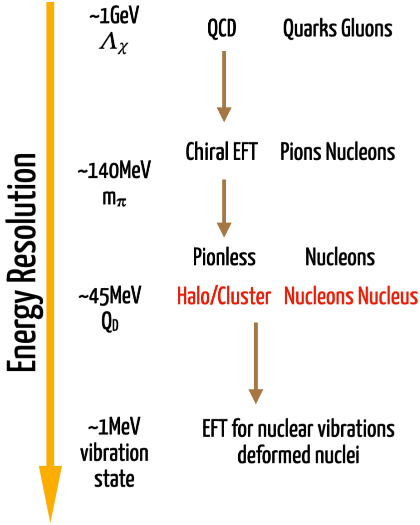
Aug. 19-23 2024

ECT*, Trento, Italy

Qingfeng Li, Songlin Lyu, Chen Ji, Bingwei Long, Phys. Rev. C 108, 024002

Università degli Studi della Campania *Luigi Vanvitelli*

Nuclear Effective field theories(EFTs)



Degrees of freedom

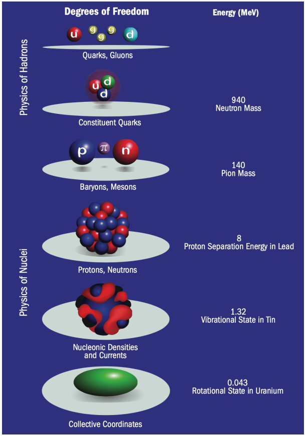


Fig.: Bertsch, Dean, Nazarewicz (2007)

Halo EFT

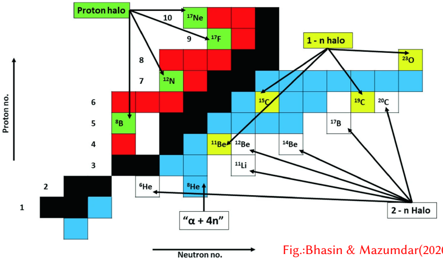


Fig.:Bhasin & Mazumdar(2020)

- Halo nuclei
  - S-wave: $^{11}\mathrm{Li}(2n + {}^{9}\mathrm{Li})$
  - P-wave: $^{6}\mathrm{He}(2n + {}^{4}\mathrm{He})$
  - D-wave: $^{15}\mathrm{C}(n + {}^{14}\mathrm{C})$
- Astrophysical reactions
- Large sizes, continuum effects difficult for direct ab initio

- Degrees of freedom
  - core + valance nucleons
- Separation of scales
  - M_{hi}: break clusters apart
  - M_{lo}: remove halo nucleons
- Controlled expansion in $M_{\mathrm{lo}}/M_{\mathrm{hi}}$

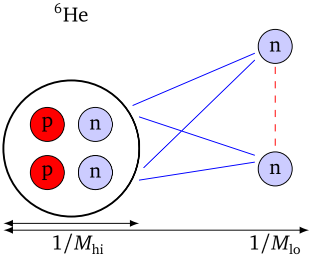


Fig: H-W Hammer, C Ji, D R Phillips (2017)

EFT for P-wave resonance

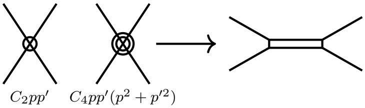
- EFT for P-wave resonance is not easy
  amplitude with poles: nonpertubative

- Dimeron auxiliary field solution

  - energy dependent potential

$$\frac{y^2 pp'}{E+\Delta}$$

  - unphysical bound state pole
    state with negative probability



Wigner bound

Bertulani, Hammer, van Kolck NPA '02, Bedaque, Hammer, van Kolck PLB '03

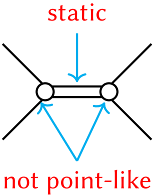
- Formulations without auxiliary dimeron fields

E. Epelbaum, et al., Few Body Syst. 62, 51 (2021)

Non-local P-wave EFT

- Non-local potential

$$V^{(0)}(p', p) = -\frac{2\pi}{\mu} \frac{\lambda p' p}{\sqrt{p'^2 + 2\mu\Delta}\sqrt{p^2 + 2\mu\Delta}}$$



  - static auxiliary field: momentum dependent
  - momentum-dependent form factor
  - two parameters at LO

- Not just another model

$$\mathcal{L}(x) = \frac{1}{2} n^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) n + \alpha^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_\alpha} \right) \alpha + \frac{\mu}{\lambda} \Psi^\dagger \Psi + \left[\Psi_a^\dagger(x) \int d^3r \mathcal{F}(r)\, n^T \left(x_0, \vec{x} + \frac{4}{5}\vec{r} \right) \vec{T}_a \cdot \hat{r}\, \alpha \left(x_0, \vec{x} - \frac{\vec{r}}{5} \right) + h.c. \right]$$

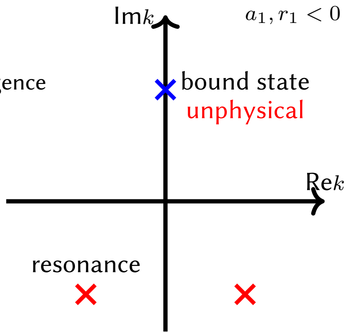
  - order-by-order convergence
  - recover effective range expansion(ERE)
  - no unphysical bound state
    crucial for many-body calculations

Non-local potential: Leading order

- Leading order(LO) two-body amplitude(ERE)

$$T^{(0)} = \frac{2\pi}{\mu} \frac{k^2}{-1/a_1 + r_1 k^2/2 - ik^3}$$

- Cubic in k: three poles $\rightarrow$ spurious bound state
- Energy-dependent form
  - unphysical bound state with negative norm
  - perturbative ik^3 C.J., Elster, Phillips, PRC ’ 14
    no spurious pole, smaller radius of convergence
  - spurious pole subtraction
    change a_1, r_1 Rotureau, van Kolck, FBS ’ 13
- Momentum-dependent form
  a pole on the positive imaginary axis
  not a bound state



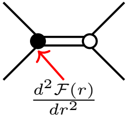
Bertulani, Hammer, van Kolck NPA ’02; Bedaque, Hammer, van Kolck PLB ’03

Non-local potential: Higher orders

$$V_\lambda^{(1)}(p',p) = -\frac{2\pi}{\mu}\frac{p'p}{\sqrt{p'^2+2\mu\Delta^{(0)}}\sqrt{p^2+2\mu\Delta^{(0)}}} \times \left\{\lambda^{(1)} - \lambda^{(0)}\mu\Delta^{(1)}\left(\frac{1}{p'^2+2\mu\Delta^{(0)}}+\frac{1}{p^2+2\mu\Delta^{(0)}}\right)\right\}$$

- NLO Potential

$$V_{g_2}^{(1)}(p',p) = \frac{2\pi}{\mu}\,\frac{g_2}{2}\,\frac{p'p\left(p'^2+p^2\right)}{\sqrt{p'^2+\gamma^2}\sqrt{p^2+\gamma^2}}$$



- Higher orders as perturbations

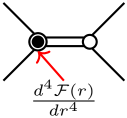
$$T^{(1)} = (1+T^{(0)}G_0)V^{(1)}(G_0T^{(0)}+1)$$

- NLO amplitude: generalized shape parameter: $P_1 k^4$

$$T^{(1)}(k) = -\frac{\mu}{2\pi}\frac{\left[T^{(0)}(k)\right]^2}{k^2}\left[\frac{\lambda_R^{(1)}}{\lambda_R^2}\gamma^2 - 2\mu\Delta^{(1)}\left(\lambda_R^{-1}+\frac{3}{2}\gamma\right) - \frac{g_{2R}}{\lambda_R}\gamma^5 + \left(\frac{\lambda_R^{(1)}}{\lambda_R^2} - \frac{g_{2R}}{\lambda_R}\gamma^2\left(\lambda_R^{-1}+\gamma\right)\right)k^2 - \frac{g_{2R}}{\lambda_R^2}k^4\right]$$

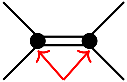
Non-local potential: Higher orders

- N²LO Potential



$$V^{(2)}_{g_4}(p',p) = \frac{2\pi}{\mu}\,\frac{g_4}{2}\,\frac{p'p\,(p'^4+p^4)}{\sqrt{p'^2+\gamma^2}\sqrt{p^2+\gamma^2}}$$

$$V^{(2)}_{g_2\Psi g_2}(p',p) = -\frac{2\pi}{\mu}\,\frac{g_2^2}{4\lambda}\,\frac{p'p\,(p'^2p^2)}{\sqrt{p'^2+\gamma^2}\sqrt{p^2+\gamma^2}}$$



$$T^{(2)} = (1+T^{(0)}G_0)(V^{(2)}+V^{(1)}G_0V^{(1)}+V^{(1)}G_0T^{(0)}G_0V^{(1)})(G_0T^{(0)}+1)$$

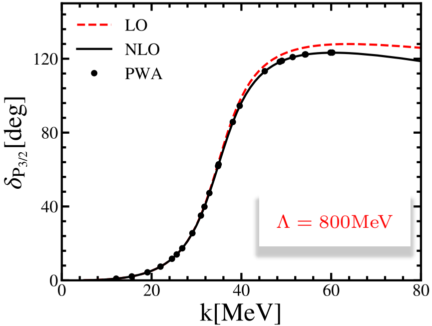
- NNLO: higher-order ERE term P_2k^6

$$(\lambda^{-2}g_4+\lambda^{-3}g_2^2)k^6$$

- Renormalization verified analytically up to NNLO
  enough counterterms to absorb divergence

neutron-alpha scattering

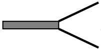
- $^5\mathrm{He}$: shallow P-wave resonances at $k_\pm = (\pm 34.5 - i6.2)$ MeV
  $n-\alpha$ scattering is dominated by the $P_{3/2}$ resonance state
- Fitting to empirical values of $n-\alpha$ elastic scattering phase shift



- Rapid order-by-order convergence
  sufficient accuracy achieved at NLO

n-n-alpha at leading order

- ^{6}He: Borromean states (none of the two-body subsystems are bound)
- Leading order
  - $n-\alpha$ P-wave interaction
  - $^1S_0$ nn interaction



$\sim iC_{\phi n}$

$$\mathcal{L}_{nn} = -C_{\phi n}\left(\phi^\dagger n_\delta S^\delta n_{-\delta} + h.c.\right) + \sigma\phi^\dagger\phi$$

  - $nn\alpha$ three-body interaction (N^{2}LO in NDA)

$\sim ih$

$$\mathcal{L}_{nn\alpha} = -h\left(G_0^{ab}T_a^{is}\Psi_b \overleftrightarrow{\partial_i} n_s\right)^\dagger \left(G_0^{cd}T_c^{jr}\Psi_d \overleftrightarrow{\partial_j} n_r\right)$$

- Three-body force at LO

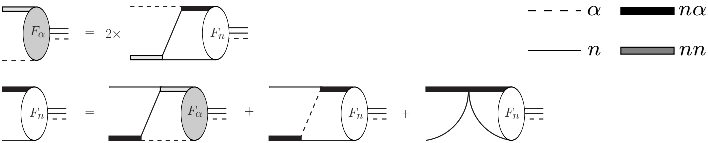
  to eliminate the cutoff dependence

  only one promotion is needed for $\frac{3}{2}^{-1}$ channel

Few Body Syst. (2017), Emil Ryberg, Christian Forssén, Lucas Platter

Faddeev equation

coupled-channel Faddeev equations



- two kernels

  $X_{n\alpha}$: exchange neutron X_{nn}: exchange α

- dressed nn and $n\alpha$ propagators

Faddeev components:
F_α: α as spectator
F_n: neutron as spectator

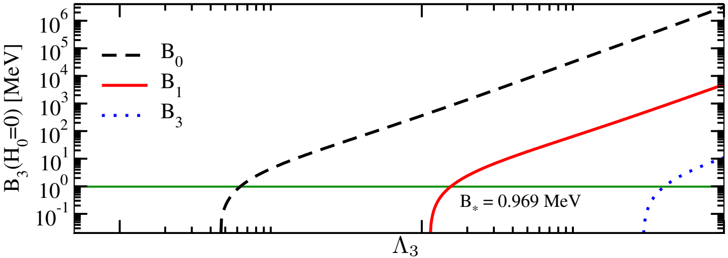
$$F_\alpha(q) = 8\pi \int_0^{\Lambda_3} q'^2 dq'\, X_{n\alpha}(q', q; B_3)\, D_{n\alpha}(\kappa_1)\, F_n(q'),$$

$$F_n(q) = 4\pi \int_0^{\Lambda_3} q'^2 dq'\, X_{n\alpha}(q, q'; B_3)\, D_{nn}(\kappa_0)\, F_\alpha(q')$$
$$+ 4\pi \int_0^{\Lambda_3} q'^2 dq' \left[X_{nn}(q, q'; B_3) + \frac{qq'}{\Lambda_3^4} H_0(\Lambda_3) \right] D_{n\alpha}(\kappa_1)\, F_n(q')$$

C. Ji, C. Elster, and D. R. Phillips, Phys. Rev. C 90, 044004 (2014)

n-n-alpha at leading order

B_3 with $H_0 = 0$

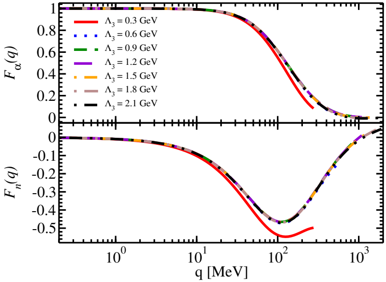


- $H_0 = 0$: three-body system unbounded from below
- Three-body force at LO
  short-range force to prevent three-body system from collapsing

J. Rotureau, U. van Kolck, Few-Body Syst (2013) 54:725

- H_0: fit to ^{6}He binding energy B_*

Faddeev component



- Rapid convergence with momentum cutoff
- Ready to observable calculations
  ^{6}He structure properties: charge/matter radius, S_{2n}
  more valance neutrons
- P-wave halo with different cores: ^{11}Be($n-{}^{10}$Be), ^{8}Li($n-{}^7$Li)

Summary and Outlook

- A momentum-dependent non-local potential to develop an EFT for shallow P-wave resonances
- $n - \alpha$ scattering and ^{6}He studied
- Halo EFT + many-body methods:
  - How effectively can we describe $^{6-10}$He by combining Halo EFT and many-body methods?

    K. Fossez, J. Rotureau, W. Nazarewicz, PRC, 2018; J. Rotureau, U. van Kolck, Few-Body Syst (2013) 54:725
  - How good can we incorporate LO three-body forces in many-body calculations?

Thanks ...

... to my collaborators:

Qingfeng Li, Chen Ji, Bingwei Long

... and for your attention!

Backups

Power counting

- Non-local formula

$$\gamma \sim M_{\rm hi} \quad \lambda_R^{-1} \sim M_{\rm hi} \quad \eta = \gamma + \lambda_R^{-1} \sim M_{\rm lo}^2/M_{\rm hi}$$

$$1/a_1 = -\gamma^2\eta \sim M_{\rm lo}^2 M_{\rm hi} \quad r_1/2 = \lambda^{-1} \sim M_{\rm hi}$$

  one parameter fine-tuned

- ik^3 resummation

$$1/a_1 \sim M_{\rm lo}^3 \quad r_1 \sim M_{\rm lo}$$

  two-parameters fine-tuned

- Perturbative ik^3

$$1/a_1 \sim M_{\rm lo}^2 M_{\rm hi} \quad r_1 \sim M_{\rm hi}$$

  one parameter fine-tuned

Lagrangian

- Lagrangian

$$\mathcal{L}(x) = \frac{1}{2} n^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) n + \alpha^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_\alpha} \right) \alpha + \frac{\mu}{\lambda} \Psi^\dagger \Psi$$

$$+ \left[\Psi_a^\dagger(x) \int d^3 r \mathcal{F}(r)\, n^T \left(x_0, \vec{x} + \frac{4}{5}\vec{r} \right) \vec{T}_a \cdot \hat{r}\, \alpha \left(x_0, \vec{x} - \frac{\vec{r}}{5} \right) + h.c. \right] + \cdots ,$$

$$\mathcal{L}^{(1)}(x) = \frac{g_2}{2\lambda} \left[\Psi_a^\dagger(x) \int d^3 r\, n^T \left(x_0, \vec{x} + \frac{4\vec{r}}{5} \right) \vec{T}_a \cdot \hat{r} \frac{d^2 \mathcal{F}(r)}{dr^2} \alpha \left(x_0, \vec{x} - \frac{\vec{r}}{5} \right) + h.c. \right] ,$$

$$\mathcal{F}(r) = \frac{d}{dr} \int \frac{d^3 p}{(2\pi)^3} \frac{e^{-i\vec{p}\cdot\vec{r}}}{\sqrt{\vec{p}^{\,2} + 2\mu\Delta}} .$$

- LS equation

$$T_l(p', p; k) = V_l(p', p) + \frac{\mu}{\pi^2} \int^\Lambda \mathrm{d}q\, q^2\, V_l(p', q) \frac{T_l(q, p; k)}{k^2 - q^2 + i0}$$

Poles

Type	η	Pole position $(k_\pm)$
bound / virtual	$\eta < 0$	$\frac{i}{2}\left[\lvert\eta\rvert \pm \sqrt{\lvert\eta\rvert(4\gamma + \lvert\eta\rvert)}\right]$
resonance	$0 < \eta$	$\frac{1}{2}\left[\pm\sqrt{\eta\left(3\gamma + \lvert\lambda_R^{-1}\rvert\right)} - i\eta\right]$
	$0 < \eta < 4\gamma$	$\frac{1}{2}\left(\pm\sqrt{4\eta\gamma - \eta^2} - i\eta\right)$
virtual	$4\gamma < \eta$	$-\frac{i}{2}\left(\eta \pm \sqrt{\eta^2 - 4\eta\gamma}\right)$

Table: Categories of pole positions of $\tau(k)$ according to the value of η.

$$T^{(0)}(p', p; k) = -\frac{2\pi}{\mu}\frac{p'}{\sqrt{p'^2 + \gamma^2}}\frac{ik - \gamma}{k^2 + i\eta k - \eta\gamma}\frac{p}{\sqrt{p^2 + \gamma^2}} \qquad (1)$$

$$T(p', p; k) \to \frac{p'}{\sqrt{p'^2 + \gamma^2}}\frac{R(B)}{k^2/2\mu + B}\frac{p}{\sqrt{p^2 + \gamma^2}} + \text{finite terms} \qquad (2)$$