What Can Possibly Go Wrong?

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Institute for Nuclear Studies
The George Washington University, DC, USA

1. The Nuclear Interaction: Post-Modern Developments?
2. Live Up To Your Promises!: Fundamentals
3. Live Up To Your Promises!: Testing Assumptions & Against Nature
4. “Mistakes Have Been Made, As All Can See And I Admit It”

6 Essential Ideas in 25 Years:

Beane/Bedaque/Savage/van Kolck NPA 700 (2002) 377: PC
Nogga/Timmermans/van Kolck PRC 72 (2005) 054006: PC
Schindler/Phillips Ann. Phys. 324 (2009) 682 & 2051: Bayes
Vanasse PRC 88 (2013) 044001: Perturbation Theory
BUQEYE PRC 92 (2015) 024005: Bayes
hg FewB. Sys. 63 (2022) 44 [2111.00930]: Brilliance

What Can Possibly Go Wrong? – *A Lot.*

An Objective, Unbiased Review of the Past 25 Years

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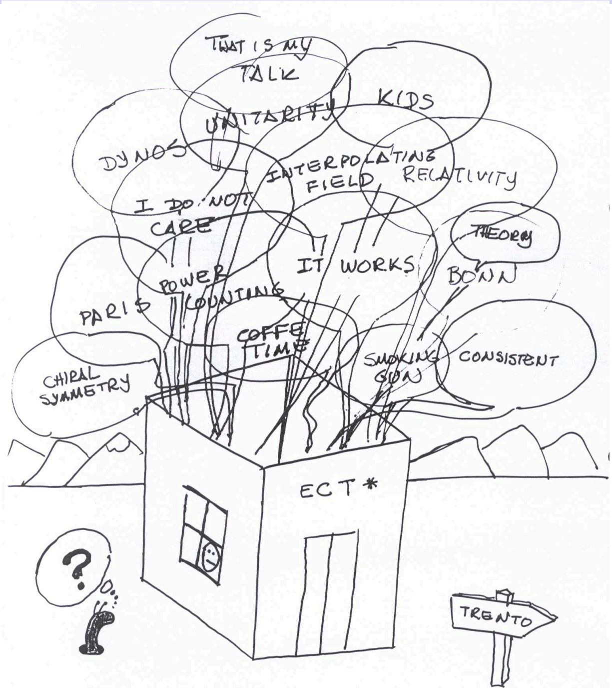
1. The Nuclear Interaction: Post-Modern Developments?

(a) 28 June – 9 July 1999: Those Were The Good Old Days



M. Robilotta: Impression of the Workshop on Nuclear Forces at the ECT*, Trento 1999

Jim Friar's closing: "*In 1-to-2 years, we will all be using χPT-designed products.*"

(b) What Are Our Goals?

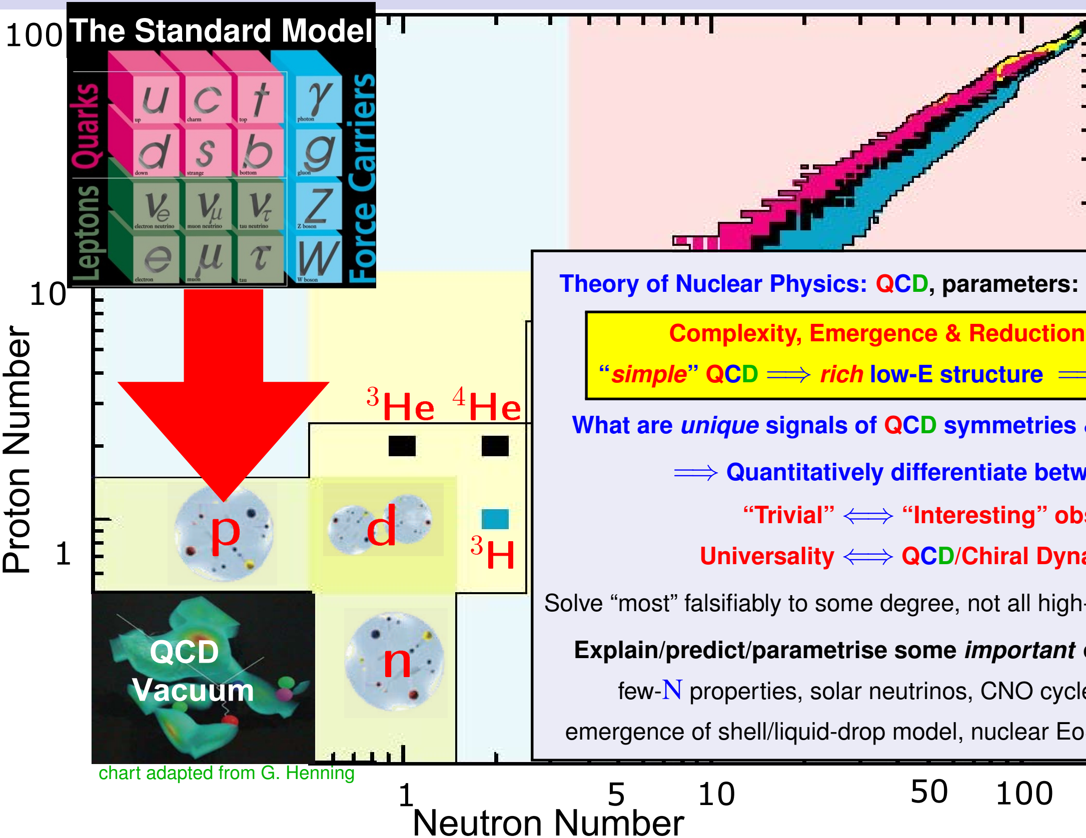


chart adapted from G. Henning

Theory of Nuclear Physics: QCD, parameters: $\overline{\Lambda}_{\text{QCD}}, m_\mathrm{u}, m_\mathrm{d}$

Complexity, Emergence & Reductionism:

"*simple*" QCD ⟹ *rich* low-E structure ⟹ *patterns*

What are *unique* signals of QCD symmetries & dynamics?

⟹ Quantitatively differentiate between:

"Trivial" ⟺ "Interesting" observables

Universality ⟺ QCD/Chiral Dynamics

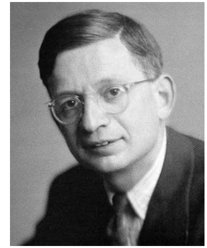
Solve "most" falsifiably to some degree, not all high-precision details.

Explain/predict/parametrise some *important* observables:

few-N properties, solar neutrinos, CNO cycle, $0\nu\beta\beta$,

emergence of shell/liquid-drop model, nuclear EoS for astro, . . .

The Trouble With Nuclear Physics



In fact the trouble in the recent past has been a surfeit of different *models* [of the nucleus], each of them successful in explaining the behavior of nuclei *in some situations*, and each in *apparent contradiction with other successful models* or with our ideas about nuclear forces.

Rudolph E. Peierls: "The Atomic Nucleus", *Scientific American* **200 (1959)**, no. 1, p. 75; emphasis added

Model: Precise description tailored to one task (process/...). – *No "fail"* but "tuning".

Theory: Comprehensive, prescriptive, predictive, accurate, Explain-All-To-Some-Degree. – *Can fail.*

Totalitarian Principle/Swiss Basic Law/ Weinberg's "Folk Theorem": Throw In the Kitchen Sink

As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you're simply writing down the most general theory you could possibly write down.

Original: Weinberg: Physica 96A (1979) 327 – here 1997 version

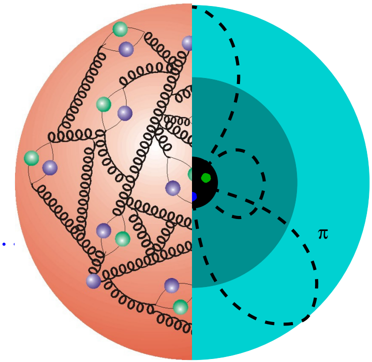
"EFT $=$ Symmetries $+$ Parametrisation of Ignorance"?? WHAT CAN POSSIBLY GO WRONG???

2. Live Up To Your Promises!: Fundamentals

(a) Chiral Effective Field Theory of Nuclear Physics

At low energies, quarks & gluons rearrange into new,
effective **low-energy degrees of freedom: Nucleons, Pions, $\Delta(1232)$.**

$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2\, \pi^a \pi^a + \dots$$
$$+ N^\dagger [\mathrm{i}\, D_0 + \frac{\vec{D}^2}{2M} + \frac{g_A}{2 f_\pi}\, \vec{\sigma}\cdot\vec{D}\pi + \dots] N + C_0 \left(N^\dagger N\right)^2 + H_0 \left(N^\dagger N\right)^3 + \dots$$



Correct long-range $+$ symmetries: Chiral SSB, gauge, iso-spin,…

$\Longrightarrow$ ***Write most general Interaction Lagrangean permitted.***

Short-range: ignorance into *minimal* parameter-set at given order.

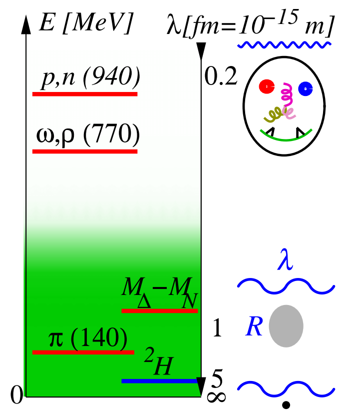
Coefficients from experiment or QCD or…

"The Power Counting":

Systematic ordering in $Q = \dfrac{\text{typ. momentum } \sim m_\pi}{\text{breakdown scale } \sim 1 \text{ GeV}} \approx \dfrac{1}{5\dots7}$.

Controlled approximation: model-independent, error-estimate.

Space for improvement.



$\Longrightarrow$ **Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD**

(b) EFTs Can Go Wrong: Check Fundamental Building Blocks – Hidden Or Not

Expand observables as $\mathcal{O} = c_0 + c_1\, Q^1 + c_2\, Q^2 + \ldots$

with $Q = \dfrac{\text{typ. momentum } p_{\text{typ.}}}{\text{breakdown scale } \overline{\Lambda}_{\text{EFT}}} < 1.$

– Incorrect usage: $p_{\text{typ.}} \nearrow \overline{\Lambda}_{\text{EFT}} \Longrightarrow Q \not\ll 1$?

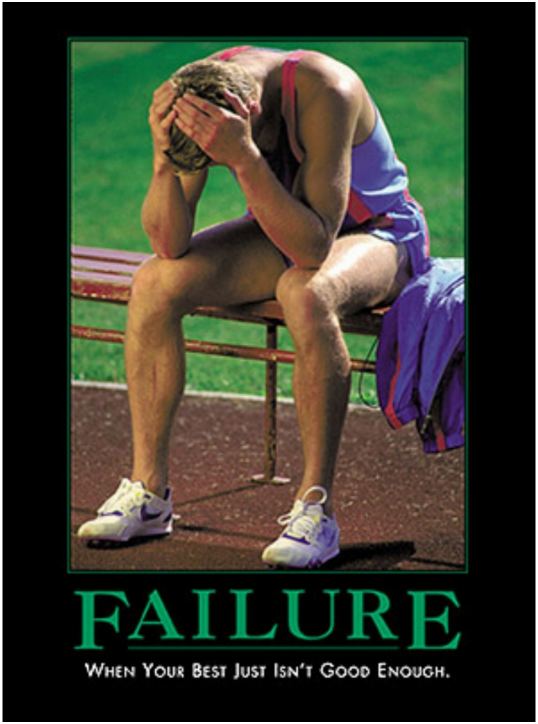
$\Longrightarrow$ Each EFT points to a more-fundamental EFT.

"EFTs carry seed of own destruction." D. R. Phillips

The Assumptions:

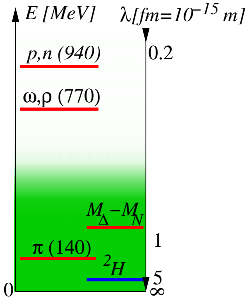
- ***Workable* Separation of Scales**
- ***Appropriate* Degrees of Freedom**
- ***Relevant* Symmetries at These D.o.F's and Scales**
- ***Consistent* Ordering Scheme**

Different choices lead to different EFTs, even with same symmetries and degrees of freedom.

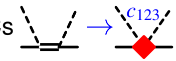


(b) EFTs Can Go Wrong: Check Fundamental Building Blocks – Hidden Or Not

Workable Separation of Scales with *Appropriate* Degrees of Freedom



The Elephant in the Room: χ**EFT at** $k \gtrsim 200$ MeV **without** $\Delta(1232)$ **inconsistent.**

Breakdown $\overline{\Lambda}_{\not\Delta} \approx M_\Delta - M_N \approx 300$ MeV with "anomalously large" LECs 

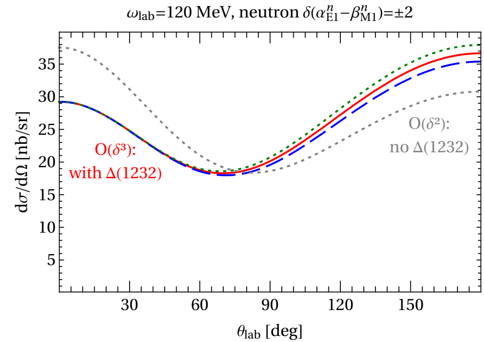
Often not considered but available.

UvK 1993, Kaiser 2000-, Krebs/...2007/8, Piarulli/Navarro Pérez/Amaro/...2016,...

Impacts softness of Nuclear EoS, Compton (here: ^{3}He),....

$\Longrightarrow$ Must justify when left out!

Rarely, symmetries forbid $\Delta(1232)$ contributions in low orders.



For $p_{\rm typ} \gtrsim \Delta_M - \Gamma/2 \approx 200$ MeV, Δ resonance dominates: resum, coupled channels.

"It's hard" is no sufficient excuse.

The Ghost: Correlated 2π Exchange $f_0(500) \approx ([400\dots550] - [200\dots350]\mathrm{i})$ MeV

Ideas: Donoghue PLB643 (2006) 165, Mishra/Ekström/Hagen/Papenbrock/Platter PRC 106 (2022) 024004

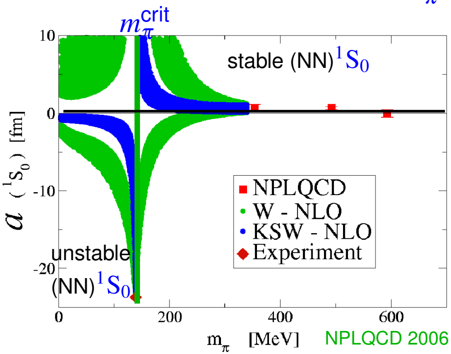
Mild E, k dependence captured in CTs? $\Longleftrightarrow$ "*Come on, we can do without that Sigma crap.*" Trento 1999

(b) EFTs Can Go Wrong: Check Fundamental Building Blocks – Hidden Or Not

Relevant Symmetries at These D.o.F's and Scales: Chiral, isospin, gauge, Lorentz,...

Overlooked Symmetries? Signals: "accidential" correlations/fine tuning.

χEFT cannot explain anom. scatt. lengths/ shallow binding: Worlds with $a \lesssim \frac{1}{m_\pi}$!

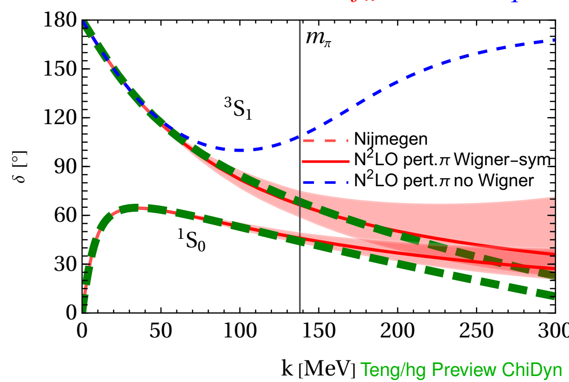


NPLQCD 2006

Unitarity Limit $a \to \infty$ has more symmetries: Scale invariance, Wigner-$\mathrm{SU}(4)$ spin-isospin. $\Longrightarrow$ Perturb about it?!

Natural in EFT($\not\pi$) König/hg/Hammer/van Kolck PRL 2017- Kievsky/Viviani/... 2018-

Broken by f_π, m_π, OπE structure: $-\frac{g_A^2}{4f_\pi^2}\,\vec{\tau}_1\cdot\vec{\tau}_2\,\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^2+m_\pi^2}$



Teng/hg Preview ChiDyn Mon 14:30

Hypothesis (at least for perturbative pions) Teng/hg MSc thesis, in preparation 2024:

Wigner-$\mathrm{SU}(4)$ symmetry-breaking part of One-Pion Exchange is *super-perturbative*, i.e. does *not enter before* N^{3}LO (suppressed by at least one more order)!

(b) EFTs Can Go Wrong: Check Fundamental Building Blocks – Hidden Or Not

Consistent Ordering Scheme: Operational Instructions, Not St. Stephen's Prescription

Requirements:

– Insensitive to short-distance details: Cutoff/RG invariant order-by-order, up to higher-order effects.

– No frequent fine tuning between different orders. – But *if so*, need remedies: Reorder PC? Missing symmetry?

– Successful *A Posteriori* check of assumptions: cutoff invariance, expansion parameter, breakdown scale, . . .

(b) EFTs Can Go Wrong: Check Fundamental Building Blocks – Hidden Or Not

Unnatural Scales Obscure the Chiral Power-Counting

Weinberg 1991, van Kolck 1992-;
cf. hg [1511.00490] [2111.00930]

Convergence to Nature **tests assumptions. –** ***After*** **theoretical consistency & uncertainties determined.**

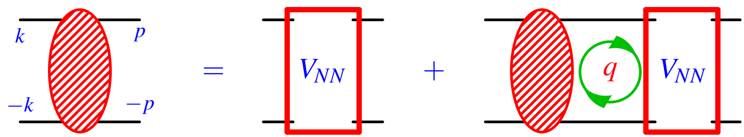
Humans abhor failure, but if an EFT fails, ***"you have learned a lot"*** **UvK Saclay 2017.**

Phenomenology: Non-relativistic system with shallow (real/virtual) bound-state. $\Longrightarrow$ **LO** ***non-perturbative.***

$\Longrightarrow$ EFT not uniquely determined by symmetries and degrees of freedom, even if consistent.

$\Longrightarrow$ **May converge, but not to Nature:** chose another consistent ordering scheme (e.g. NN perturbative).

$T_{NN}(E \sim \frac{p^2,k^2}{M}) \sim Q^{-1}$



$\sim Q^{-1}$

$$T_{\text{LO}} = V_{\text{LO}} + V_{\text{LO}}\, G^{\text{nonrel.}}_{NN}\, T_{\text{LO}}$$

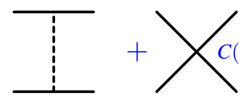
$$Q^m \qquad Q^m \qquad Q^{2m+3-2} \overset{!}{=} Q^m \quad \Longrightarrow m = -1$$

$\Longrightarrow$ LO must be Q^{-1} because of phenomenology, but what is LO? Some Choices:

$\longrightarrow$ EFT($\not\pi$): mom.-indep. contact, consistent

$\longrightarrow$ Perturbative Pions (KSW): consistent, RG-inv. ✓ but converges for $p_{\text{typ}} \lesssim 200\ \text{MeV}$?

$\longrightarrow$ Weinberg's Pragmatic Proposal WPP 1990-92: OπE+S-wave CTs :



$C(^1\mathrm{S}_0, ^3\mathrm{S}_1)$

Consistency check **of Weinberg's proposal: Observables cut-off dependent at LO?**

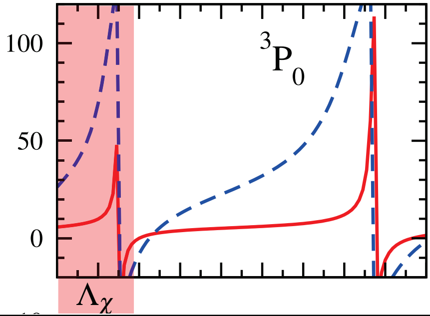
Need 4 *new*, momentum-dependent LECs for low attractive triplets: $^3\mathrm{P}_{0,2}$, $^3\mathrm{D}_{2,3}$.

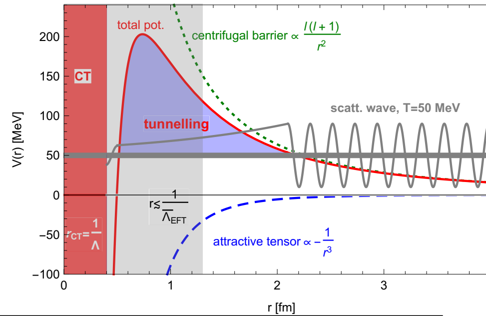
Low attractive P/D-wave triplets: Weinberg predicts zero LECs at LO (momentum-independence).

phase-shift δ(cut-off Λ):

- (solid) $E_{\mathrm{lab}} = 10$ MeV
- (dashed) 50 MeV
- (dotted) 100 MeV
- (dash-dotted) 190 MeV

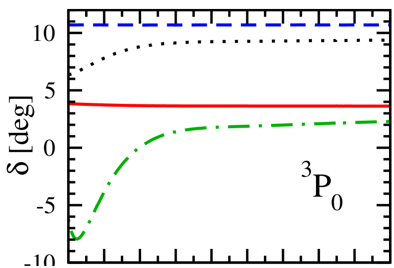
$$V(r) \propto -\frac{\#}{r^3} + \frac{l(l+1)}{r^2}$$





Without CT cutoff-dependent even for $\Lambda \approx \overline{\Lambda}_\chi$. $\Longrightarrow$ Short-distance missing. $\Longrightarrow$ *Not renormalised.*

Λ-Dependence With Counter Term

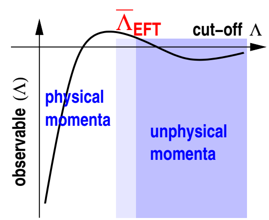


"Cutoff Democracy":

Any cutoff $\Lambda \gtrsim \overline{\Lambda}_{\mathrm{EFT}}$ is equally valid as it probes momenta beyond range where description makes sense.

Cutoff Λ unphysical

Breakdown Scale $\overline{\Lambda}_{\mathrm{EFT}}$ physical



There is always a well-known solution

to every human problem ——

neat, plausible,

H. L. Mencken

There is always a well-known solution

to every human problem ——

neat, plausible, and wrong.

H. L. Mencken

(d) "Mene, Tekel, Upharsin": Weinberg's Pragmatic Proposal

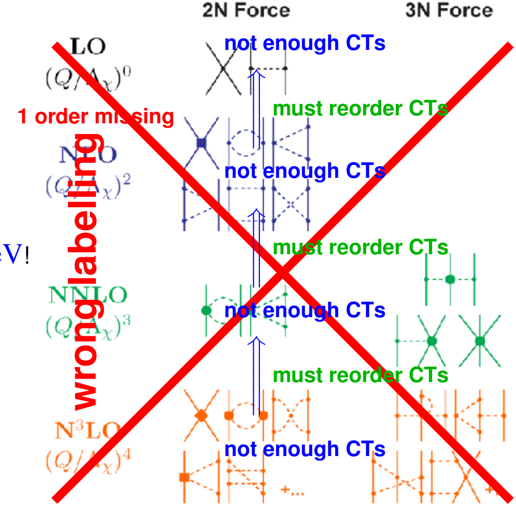
Pragmatic, widely used ("Everybody Does It").
But *conceptually inconsistent*:

– Not renormalised in low partial waves with attractive tensor.
Nogga/Timmermans/van Kolck 2005

– ${}^1\mathrm{S}_0$: m_π^2-dependence of CT and divergence do not match.

$V_C \to \frac{m_\pi^2}{\pi^2\, r}$ but $C_0 \sim m_\pi^0$ bites you for $m_\pi \neq 140\ \mathrm{MeV}$!

Kaplan/Savage/Wise 1996, Beane/Bedaque/Savage/van Kolck 2002



underlying plot by Epelbaum, butchered by hg

Not Just LO Reg/Ren Problem: ricochets through orders.

$\Longrightarrow$ WPP underestimates number of CTs per order.

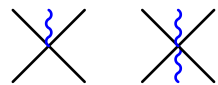
$\Longrightarrow$ WPP at alleged order Q^n not as accurate as thought:

Accurate only to lower order $Q^{n-1,2,3,\dots}$.

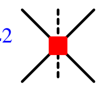
Fitting may obscure the problem. . .

Not Just NN Problem: 2N currents promoted. Phillips/Valderrama 2015

$\Longrightarrow$ Gauged & gauge-invariant currents earlier, e.g. at LO $\vec{p}\cdot\vec{p}\,' C_P \to$



$\Longrightarrow$ Chiral-gauged NN currents earlier: $D_2 m_\pi^2$ in LO ${}^1\mathrm{S}_0 \stackrel{\chi\mathrm{sym}}{\longrightarrow} D_2 \pi^2$



LO in πNucleus.

We may be unable to say whose PC is right, but we have evidence whose is wrong. WPP is; it's *In-Effective*.

Still, use it pragmatically to develop numerics & first glimpses at final theory – with *caveat on systematics*!

(e) NN χEFT Power Counting Comparison

prepared for Orsay Workshop by Grießhammer 7.3.2013
based on and approved by the authors in private communications

Derived with explicit & implicit assumptions; contentious issue.
Proposed order Q^n at which counter-term enters *differs*. $\Longrightarrow$ Predict *different* accuracy, # of parameters.

order	**Weinberg** (Pragm. Prop.) PLB**251** (1990) 288 etc.	**Birse** PRC**74** (2006) 014003 etc.	**Pavon Valderrama et al.** PRC**74** (2006) 054001 etc.	**Long/Yang** PRC**86**(2012) 024001 etc.
Q^{-1}	LO of $^1S_0$, $^3S_1$, OPE	LO of $^1S_0$, $^3S_1$, OPE; plus $^3D_1$, $^3SD_1$	LO of $^1S_0$, $^3S_1$, OPE; plus $^3P_{0,2}$, $^3D_2$	LO of $^1S_0$, $^3S_1$, OPE; plus $^3P_{0,2}$
$Q^{-\frac{1}{2}}$	none	LO of $^3P_{0,1,2}$, $^3PF_2$, $^3F_2$, $^3D_2$	LO of $^3SD_1$, $^3D_1$, $^3PF_2$, $^3F_2$	none
Q^0	none	NLO of $^1S_0$	NLO of $^1S_0$	NLO of $^1S_0$
$Q^{\frac{1}{2}}$	none	NLO of $^3S_1$, $^3D_1$, $^3SD_1$	none	none
Q^1	LO of $^3SD_1$, $^1P_1$, $^3P_{0,1,2}$; NLO of $^1S_0$, $^3S_1$	none	none	LO of $^3SD_1$, $^1P_1$, $^3P_1$, $^3PF_2$; NLO of $^3S_1$, $^3P_0$, $^3P_2$; N^{2}LO of $^1S_0$
# at Q^{-1}	2	4	5	4
# at Q^0	$+0$	$+7$	$+5$	$+1$
# at Q^1	$+7$	$+3$	$+0$	$+8$
total at Q^1	9	14	10	13

With same χ^2/d.o.f., the *self-consistent* proposal with least parameters *wins*: minimum information bias.
Still, use any *pragmatically* to develop numerics & first glimpses at final theory – with caveat on systematics!

Weinberg's Pragmatic Proposal: Brilliant First Step – not the Last Word.

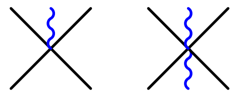
Taught us to think for ourselves, not to hermeneutically interpret Sacred Texts.

The efficient person gets the job done right. The effective person gets the right job done.

(f) EFT and Information Theory: Lossless Compression vs. Data Reproduction

Number of parameters at Q^1 for some attractive partial waves:

wave	**Weinberg** (Pragm. Prop.) PLB**251** (1990) 288 etc.	**Birse** PRC**74** (2006) 014003 etc.	**Pavon Valderrama et al.** PRC**74** (2006) 054001 etc.	**Long/Yang** PRC**86**(2012) 024001 etc.
$^3P_2$-$^3F_2$	1, very small	3 of similar size	3 of different orders	2 of different orders
$^3P_0$	1, very small	1 just below LO	1 non-perturbative (LO)	2 of different orders

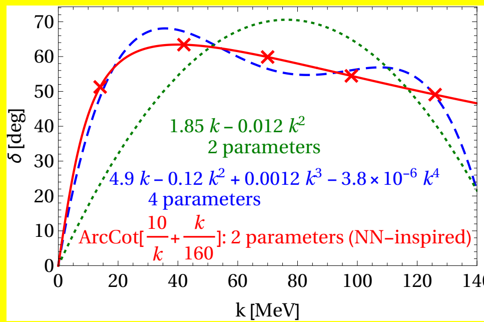
Predict different importance also for gauge currents: $\vec{p}\cdot\vec{p}\,'C_P \rightarrow$ 

Information-Theory Aspect of the EFT Promise:

Encode information about unresolved short-range at given resolution and *at given order* into smallest number of independent CTs:

minimal set of parameters for *lossless compression*.

$\Longrightarrow$ **Falsifiability; robust predictions to uncover new Physics, Alternative Worlds, hidden symmetries (unitarity, large-N_c** Schindler/Springer 2018-**,...).**



The Three Big Lies of Nuclear Theory

Nuclear Power is Safe.

They have Weapons of Mass Destruction.

My Power-Counting is Systematic.

(h) Not An Ivory Tower Exercise: Beyond-SM for $0\nu\beta\beta$, EDM and Dark Matter

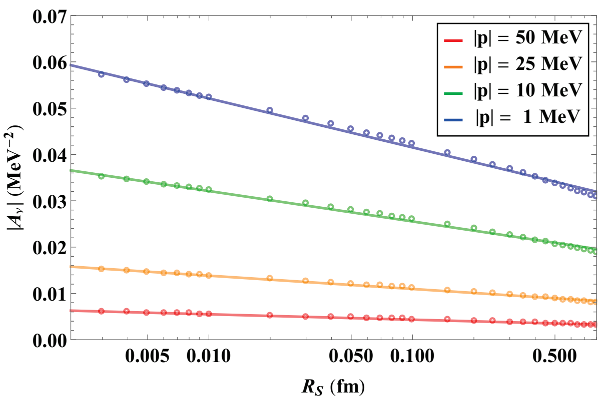
$\Longrightarrow$ "Unexpected" $2\mathrm{N}$ currents to absorb cutoff-dependence/restore RG-invariance & symmetries.

LO In Nuclear Matrix Elements for Dark Matter Detection: Hoferichter/Klos/Schwenk PLB 746 (2015) 410 [1503.04811] de Vries/Köber/Nogga/Shain [2310.11343]

NLO In Nuclear Matrix Elements for Strong-CP Violation: de Vries/Gnech/Shain PRC 103 (2021) 012501 [2007.04927]

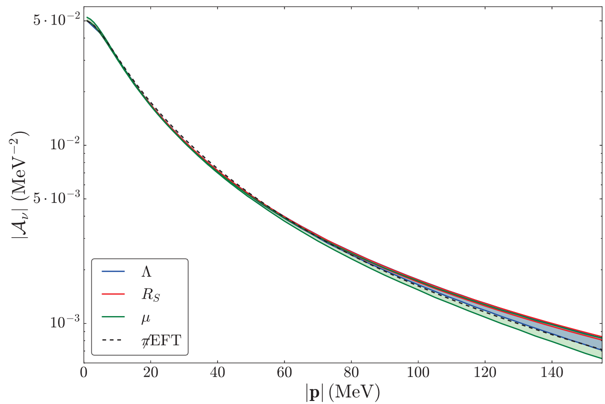
LO In Nuclear Matrix Elements for Neutrinoless Double-Beta Decay Detection:

Cirigliano/Dekens/de Vries/Graesser/Mereghetti/Pastore/van Kolck PRL120 (2018) 202001 [1802.10097]



RG/cutoff-dependence at fixed k without CT

(Weinberg's Pragmatic Proposal)



RG/cutoff-corridor with k with CT

Multi-million-$ stakes! Community acknowledgment: Snowmass 2021 White Paper, INT & ECT* Workshops.

(a) (Dis)Agreement Significant Only When All Error Sources Explored

Editorial PRA 83 (2011) 040001

1999 Workshop: "χ^2 afficiados" Machleidt $\longleftrightarrow$ "Real Theorists have error bars." Timmermans

Cockroach arguments: "systematic, consistent, power counting" – but no actual tests & theory uncertainties.

PHYSICAL REVIEW A **83**, 040001 (2011)

Editorial: Uncertainty Estimates

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers

accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation; for example, in scattering processes involving complex systems. The comparison with experiment itself provides a test of our theoretical understanding. However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made. Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and especially under the following circumstances:

1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
3. If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.

Scientific Method: Quantitative results with corridor of theoretical uncertainties for *falsifiable predictions*.

Need procedure which is established, economical, reproducible: room to argue about "error on the error".

"Double-Blind" Theory Errors: Assess with pretense of no/very limited data.

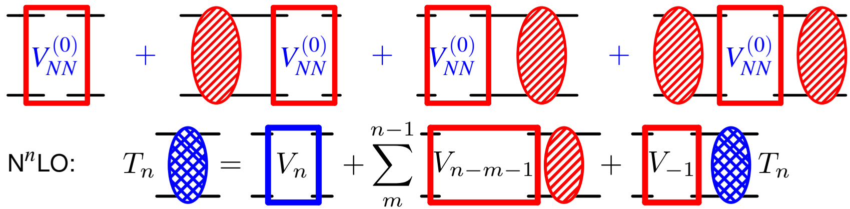
(b) Do Contributions to *Observables* Decrease With Increasing Order?

$\Longrightarrow$ Find radius of convergence $k \lesssim \overline{\Lambda}_{\text{EFT}}$, systematically estimate truncation error (Bayes) – and *only then* compare to data: beware of confirmation bias.

Corrections in $Q \ll 1$ by "strict perturbation" about LO (Distorted-Wave Born; efficient way Vanasse 1305.0283):
$\Longrightarrow$ Power-counting of amplitudes (observables); simple, no resummation artefacts.

NLO:

N^{n}LO:



Use/Develop More Strict-Perturbation Methods! cf. hg notes Trento 2018-21;

$\rightarrow$ Oliver Thim; $\Delta\mathcal{O}_0 = \dfrac{\mathcal{O}[V_{-1}+\epsilon\, V_0] - \mathcal{O}[V_{-1}]}{\epsilon}$ with $\epsilon \to 0$ Shi/.../Long/... PRC 106 (2022) 015505; [2205.02000] ...

Contrast to Popular "Partially-Resummed Perturbation" Weinberg 1990

Power-count V_{NN} & iterate $\Longrightarrow T = \dfrac{V_{\text{LO}} + V_{\text{NLO}} + \dots}{1 - (V_{\text{LO}} + V_{\text{NLO}} + \dots)\, G_{NN}}$.

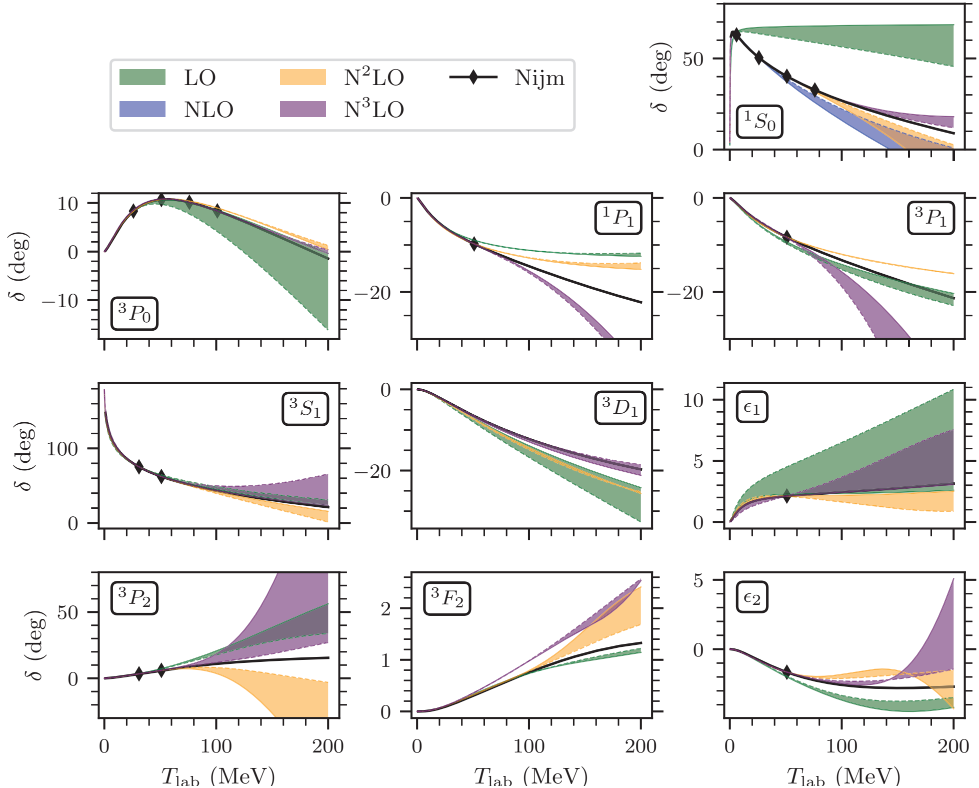
$\Longrightarrow$ Obscures PC in observables, unphysical poles around $\overline{\Lambda}_{\text{EFT}}$: artefacts, wrong causal structure.

$\Longrightarrow$ Limited to small cutoff variation range $\Lambda \approx \overline{\Lambda}_{\text{EFT}} \pm 20\%$, implementation & numerics more difficult.

Works *under assumption that* expansion indeed small, e.g. $1 + x + x^2 + x^3 - \dfrac{1}{1-x} = x^4 + \mathcal{O}(x^5)$ *if* $|x| < 1$.
But resummed version loses control over convergence test & over which interactions needed for Λ-independence (UV changed!). May still provide some guidance/insight – but beware of missed CTs!

NN in Long/Yang PC to N^{2}LO χEFT ($\not\Delta$) by Strict Perturbation

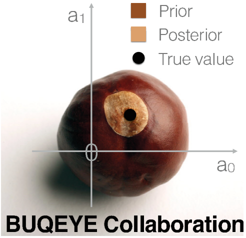
Thim/Ekström/Forssén PRC 109 (2024) 064001
[2402.15325] → Oliver Thim's talk



Order-by-order convergence ✓— Uncertainty corridors from $\Lambda \in [500; 2500]$ MeV

(c) Quantifying Theory Truncation Uncertainties: "This Is The (Bayesian) Way"

N^{k-1}LO Observable as series: $\mathcal{O}^{(k)} = c_0 + c_1 Q^1 + c_2 Q^2 + \cdots +$ unknown $\times Q^k$. $\Longrightarrow \mathcal{O}^{(k)} \pm Q^k \max\{|c_i|\}$?



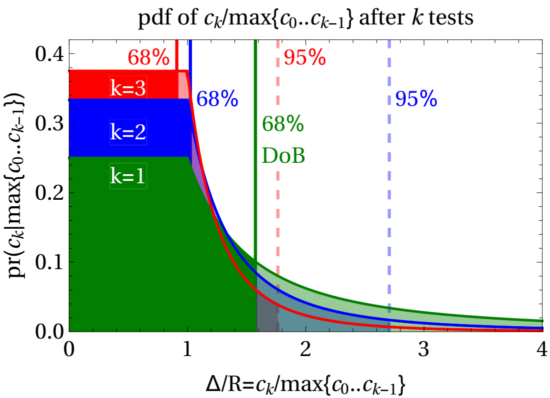
No infinite sampling pool; data fixed; more data changes confidence.

Call upon the Reverend Bayes for probabilistic interpretation!

New information increases level of confidence.

$\Longrightarrow$ **Smaller corrections, more reliable uncertainties.**

Priors clearly state your assumptions – including *Naturalness.*



Priors: all c_n "equally likely", "any upper bound" $\bar{c}$.

order	in $\pm c_{\max}$	$\Delta^{(k)}(68\%)$	$\Delta^{(k)}(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\, c_{\max}$	$11 c_{\max} = 7\, \Delta_{68}^{(1)}$
NLO	$\frac{2}{3} = 66.7\%$	$1.0\, c_{\max}$	$2.7 c_{\max} = 2.6\, \Delta_{68}^{(2)}$
N^{2}LO	$\frac{3}{4} = 75\%$	$0.9\, c_{\max}$	$1.8 c_{\max} = 1.9\, \Delta_{68}^{(3)}$
N^{k-1}LO, k terms	$\frac{k}{k+1}$	$0.68 \frac{k+1}{k} c_{\max}\, (k \geq 2)$	
Gauß	68.27%	$1.0\, c_{\max}$	$2.0\, \Delta_{68}^{(k)}$

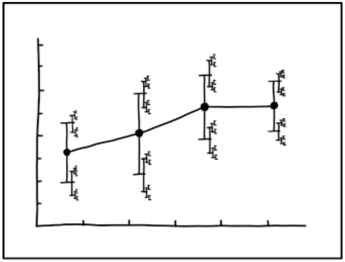
Laplace's Law of Succession: $\mathrm{pdf}(c_k > c_{\max}) = \dfrac{1}{k+1}$.

$\Longrightarrow$ **Quantified theory truncation uncertainties as** 68% ***DoB interval*** $[\mathcal{O}^{(k)} - \Delta_{68}^{(k)}; \mathcal{O}^{(k)} + \Delta_{68}]$.

Posterior pdf *not Gauß'ian:* Plateau & "fat" (power-law) tail.

(d) Extensive Use of Bayesian Statistics: Bayesian Uncertainty Quantification

max-criterion: standard lore "Since Time Immemorial" (before 6 July 1189) – Bayesian quantification e.g. Cacciari/Houdeau [1105.5152]
Nuclear: Schindler/Phillips 2009 →BUQEYE [1506.01343]+[1511.03618]+. . . , cf. hg/JMcG/DRP [1511.01952] → BAND (NSF) 2021-



I DON'T KNOW HOW TO PROPAGATE ERROR CORRECTLY, SO I JUST PUT ERROR BARS ON ALL MY ERROR BARS.
xkcd 13.02.2019

Test stability under reasonable variations of assumptions.

Reasonable people can reasonably disagree about reasonable assumptions, but no reasonable discussion without disclosing them.

- **Robust Estimates of Theory Truncation Errors & Correlations**
- **Quantitative Checks of Convergence Pattern, Q, $\overline{\Lambda}_{\text{EFT}}$, Naturalness**
- **Experimental Design:** Which future data have likely biggest impact?
- **Model Mixing:** Extrapolate theories valid for different scales & environments.

Annual ISNET workshops/conferences

ISNET Phys. G **42** no. 3 (2015)

J. Phys. G **46** no. 10 (2019)

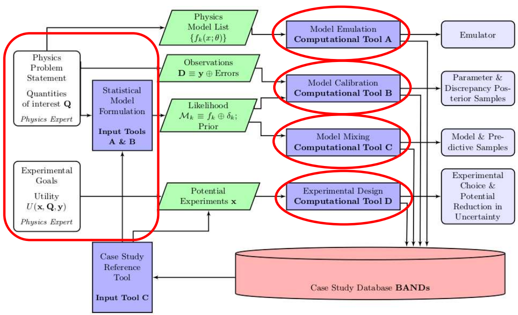
Front. Phys. Res. Topics (2022)

Open Source Software Suites:

buqeye.github.io

bandframework.github.io

Goal: *Facilitate principled Uncertainty Quantification in Nuclear Physics*



BAND
Bayesian Analysis of Nuclear Dynamics

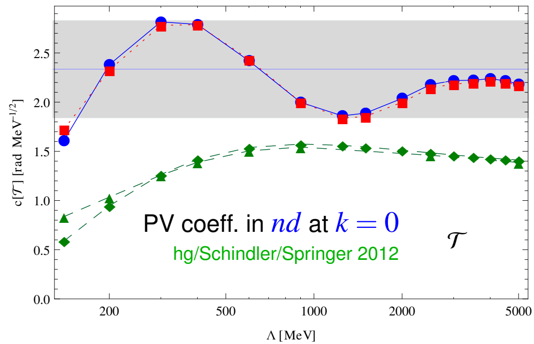
An NSF CSSI Framework
(5 years until 2026)
Look to https://bandframework.github.io/ for papers, talks, and software!
v0.3 released 10/23

Trust only theorists who show effort to estimate theory/truncation errors – or apologise when not.

(e) Some Ways to Check Consistency & Estimate Theoretical Uncertainties

- **Order-by-order Bayes, but cannot verify PC consistency.**

Laplace's Sunrise vs. Kepler: extraneous info



- **Cutoff democracy**: Any cutoff $\Lambda \gtrsim \overline{\Lambda}_\chi$ equally valid since probes momenta beyond EFT range.

  $\Longrightarrow$ Corridor mapped by Λ *in wide range* should "typically" decrease order-by-order (but often under-estimates).

  Quantify: "k-dep. RG flow of observable with Λ at N^{n}LO"

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} \propto \left(\frac{k,p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1}$$

  for any two cut-offs $\Lambda_1,\Lambda_2 \gtrsim \overline{\Lambda}_{\text{EFT}}$ hg [1511.00490].

- Order-by-order less dependence on Renormalisation Scheme/cutoff function...
- Order-by-order less dependence on particular low-E data taken for LECs. (Z-param. vs. ERE; fit to a vs. B,...)

  Optimal fit region to avoid fine-tuning: $\frac{1}{a} \ll k, p_{\text{typ}} \sim m_\pi, \Delta \ll \overline{\Lambda}_\chi \sim 700\ \text{MeV}$

- Include selected higher-order RG- & gauge-invariant effects: *This does not increase accuracy.*

Choose most conservative/worst-case error for final estimate! Clearly state your choice!

How to combine all this information on uncertainties? Aim for statistical interpretation!

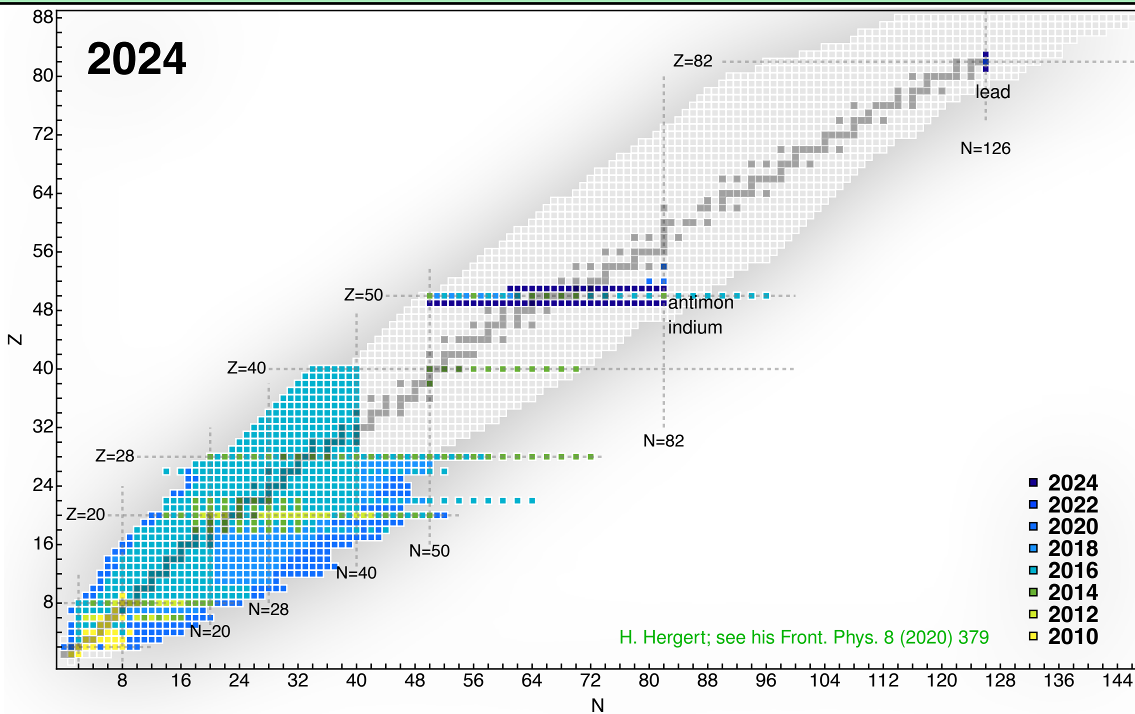
In combination, they increase our confidence in uncertainties of results.

"Double-Blind": First comprehensively explore uncertainties – then test convergence to Nature!

(f) Numerical Progress: The Nuclear Chart In the *Ab-Initio* High-Accuracy Era

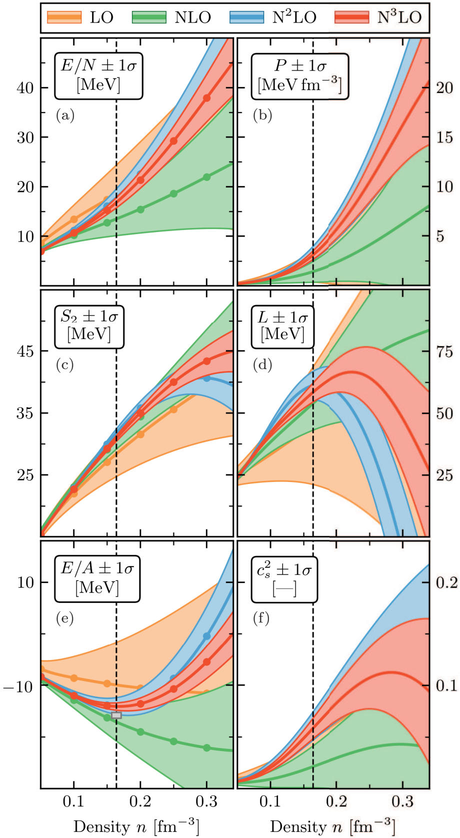
***Ab Initio*: method to reliably extrapolate, in a controlled and systematic way, to regions outside the ones used for inferring the model parameters. [. . .] a systematically improvable approach for quantitatively describing nuclei using the finest resolution scale possible while maximizing its predictive capabilities.**

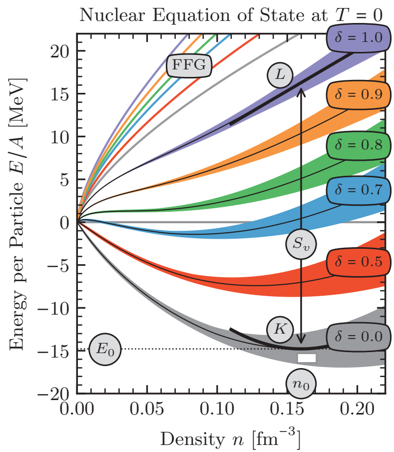
Ekström/Forssén/Hagen/Jansen/Jiang/Papenbrock Front. Phys. 11 (2023) 1129094 [2212.11064]

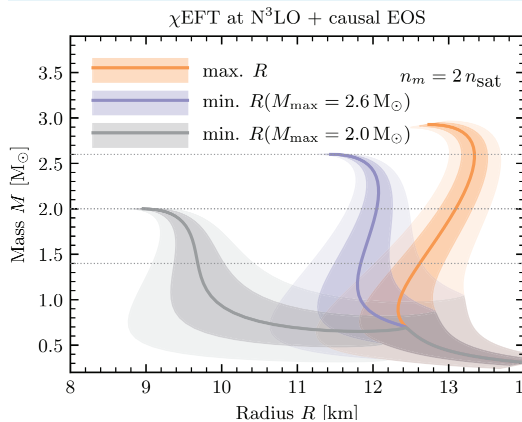


(g) A Goal: Nuclear Equations of State & Neutron Stars

Drischler et al PRL 125 (2024) 202702
Ann Rev Nucl Part Phys 71 (2021) 40







Remain Elusive:

– Order-by-order convergence;
usually not strict pert.

– Saturation point.

$\Longrightarrow$ Fine tuning??

Correct d.o.f.'s??

$\Delta(1232)$, "dressed" N, π

Correct ordering scheme?

$\Longrightarrow$ Need new ideas!

Combinatorics promoting few-N int. in Nuclei?

Yang/Ekström/Forssén/Hagen/Rupak/van Kolck EPJA 59 (2023!!) 23 [2109.13303]

4. “Mistakes Have Been Made, As All Can See And I Admit It” Ulysses S. Grant 5 Dec 1876

We have not quite followed through on EFT’s promises.

– EFTs are *much more* than “symmetries + parametrisation of ignorance”: power-counting implies choices.

Different consistent Ordering Schemes for same d.o.f.’s & symmetries may converge, but not to Nature.

– In **non-perturbative (at LO) EFT**, finding a consistent power-counting is *non-trivial*.

$\Longrightarrow$ **Much debate; Weinberg’s WPP clearly inconsistent:** not RG-invariant/renormalisable, unexpected CTs,. . .

– **Promises:** Information in minimal set of unknowns, at given resolution/order: “hidden” symmetries, predictions.

Predictions & assumptions are quantitative, falsifiable, momentum-dependent:

symmetries, constituents, scale separation, convergence pattern, breakdown scale, naturalness,. . .

Bayes helps testing but not silver bullet: cannot verify consistency.

Results must have reproducible, defensible assessment of theoretical uncertainties & assumptions.

Only after that, contrast with Nature! $\Longrightarrow$ **Error Bars for Nuclear Theory!**

Goal: World Domination by Uncertainty Quantification.

Can Unitarity Limit shed light on preference of anomalously shallow states?

Explain emergence of patterns in complexity, some important observables, not Linoleum-314.

Toss inconsistent EFTs. Articles without assessment of theory uncertainties should become unpublishable.

Discussion Points: Rôle of Unitarity Limit – Perturbative-Beyond-LO Methods – Test Assumptions & Consistency

Falls es hier jemanden gibt, den ich noch nicht beleidigt habe, den bitte ich um Entschuldigung.

[If there is anyone here whom I have not yet insulted, I beg his pardon.]

allegedly Johannes Brahms at rehearsal or leaving party

(a) Organisers' Workshop Objectives and Scientific Goals

(1) Have Chiral EFT-inspired potentials fully replaced phenomenological approaches?

– Yes, except to test numerical methods: benchmark AV18+UIX as challenging "hard-core" potential.

(2) What are the limitations of these potentials, and how can they be improved?

– Hanging on to Weinberg's Pragmatic Proposal. Often no $\Delta(1232)$. Move from nonperturbative methods to corrections in strict perturbation.

(3) Are chiral potentials converging appropriately, and is leading-order physics adequately captured?

– Studies ongoing – LO likely to change as momentum and number of nucleons increase.

(4) What is the role and scope of power counting?

– Mandatory, ensuring predictability and falsifiability – not yet settled.

(5) How significant is relativity in these models?

– <u>Kinematic</u> relativity order-by-order. Resummed only needed around "particular" kinematic points (precision threshold Physics) – and at $p_{\text{typ}} \gtrsim \overline{\Lambda}_\chi$ where χEFT is not supposed to work.

(6) Do we fully understand the dynamical implications of QCD?

– No: Nature's choice of anomalously shallow bound states still a mystery.

(7) What are the prospects of EFTs (Pionless, Halo/Cluster, and Chiral) for light and heavier nuclei?

– Unitarity! Needs to be investigated. Use degrees of freedom at appropriate scale.

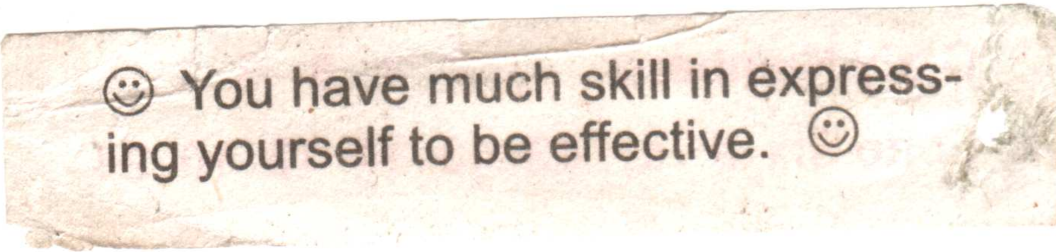
(8) How have simpler EFTs, such as pionless and halo/cluster EFTs, influenced Chiral EFT?

– Unitarity!

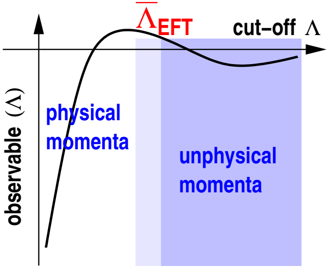
(9) How do EFTs help us to quantify uncertainties?

– Bayesian Uncertainty Quantification via order-by-order improvement; Residual Cutoff Dependence – only after that consider convergence to data.

(10) . . .


☺ You have much skill in expressing yourself to be effective. ☺

(a) Quantitative Predictions of Your PC: Advantage of Cut-Offs



Observable $\mathcal{O}(k)$ at momentum k, order Q^n in EFT, breakdown $\overline{\Lambda}_{\text{EFT}} \lesssim$ cut-off Λ:

$$\mathcal{O}_n(k;\Lambda) = \underbrace{\sum_i^n \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^i \mathcal{O}_i(k, p_{\text{typ.}})}_{\text{renormalised, } \Lambda\text{-indep.}} + \underbrace{\mathcal{C}(\Lambda; k, p_{\text{typ}}, \overline{\Lambda}_{\text{EFT}}) \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1}}_{\text{residual } \Lambda\text{-dependence}}$$

residual Λ-dependence
parametrically small
$\mathcal{C}$ "of natural size"

$\Longrightarrow$ **Difference between *any two* cut-offs:** $$\frac{\mathcal{O}_n(k;\Lambda_1) - \mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1) - \mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

Isolate breakdown scale $\overline{\Lambda}_{\text{EFT}}$, order n by double-ln plot of "derivative of observable w. r. t. cut-off".

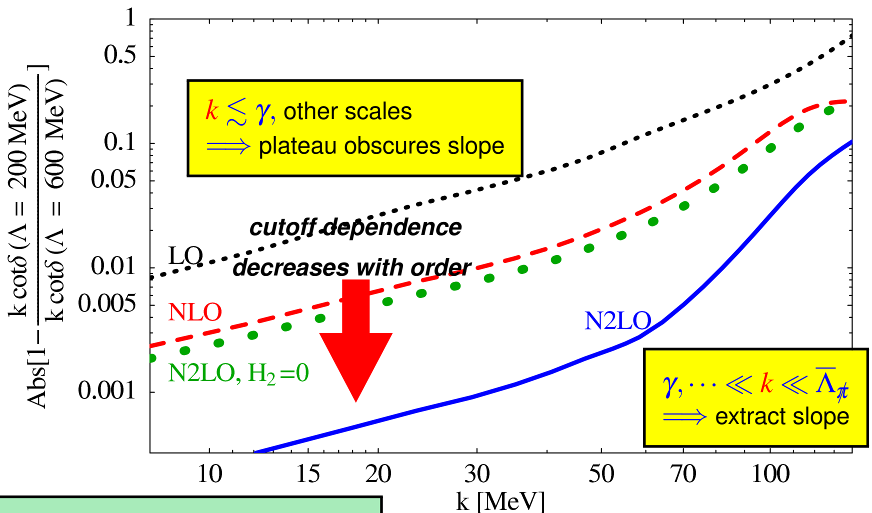
Ideally, *no resort to Data!* – Test consistency: Does numerics match predicted convergence pattern?

***After that*, quantitative test of EFT assumptions *against data*.**

Renormalisation Group Evolution: $\Lambda_1 \to \Lambda_2 \Longrightarrow \frac{\Lambda}{\mathcal{O}} \frac{d\mathcal{O}}{d\Lambda} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \frac{d \ln \mathcal{C}(\Lambda)}{d \ln \Lambda} \to 0$ if exact RGE.

Residual Λ-dependence should "usually" decrease parametrically order-by-order.

Complication: Several intrinsic low-energy scales in few-N EFT:
scattering momentum k, m_π, inverse NN scatt. lengths $\gamma(^3\text{S}_1) \approx 45$ MeV, $\gamma(^1\text{S}_0) \approx 8$ MeV,...



$$\left| 1 - \frac{k \cot\delta(\Lambda = 200\ \text{MeV})}{k \cot\delta(\Lambda = \infty)} \right| \sim \underbrace{\left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\not\pi}} \right)^{n+1}}_{Q^{n+1}}$$

	LO	NLO	N^2LO	N^2LO without H_2
$n+1$ fitted	~ 1.9	2.9	4.8	3.1
$n+1$ predicted	2	3	4	**not renormalised**

$\Longrightarrow$ Fit to $k \in [70; 100 \dots 130]$ MeV $\gg \gamma, \dots$: H_2 **is indeed N^2LO.**

Slope confirms Power Counting; estimates $\overline{\Lambda}_{\not\pi} \approx 140$ MeV.

(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\Lambda_1) - \mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1) - \mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

– Estimate k-dependence of expansion parameter $Q(k) = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)$

$\Longrightarrow$ Lower limit of residual theoretical uncertainties.

– ***"Window of Opportunity"***: Fit is most transparent for $p_{\text{typ}} \ll k \ll \overline{\Lambda}_{\text{EFT}}$.

– Any two cutoffs Λ_1, Λ_2 – Numerical leverage?! Cutoff $\Lambda \to \infty$ not necessary.

– Order n, $\overline{\Lambda}_{\text{EFT}}$ regulator independent. – But not $\mathcal{C}$: flexible regulator…

$\Longrightarrow$ Test robustness: cutoff range & schemes, fit window,…

– Non-integer powers, non-analyticities: $n+1 \to n + \text{Re}[\alpha]$ with $n \notin \mathbb{Z}$, $\text{Re}[\alpha] > 0$.

Some Limitations:

– Cannot see LECs which do *not absorb cutoff-dependence*.

– Can be numerically indecisive (e.g. small coefficients).

Test is necessary but not sufficient for consistency.

(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1)-\mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

What observable to choose?: Avoid Accidental Zeroes $\mathcal{O}(\Lambda_1)-\mathcal{O}(\Lambda_2)=0$ & Infinities $\mathcal{O}(\Lambda)=0$.

Best if unconstrained: Isolate dynamics!

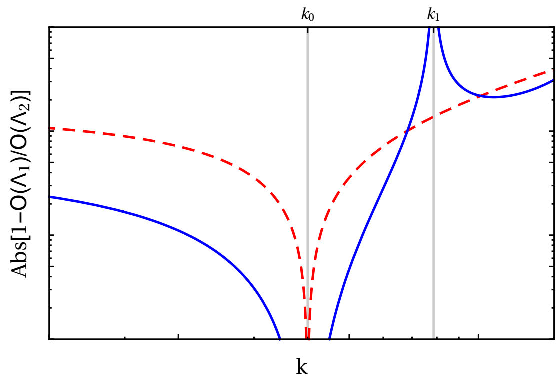
e.g. $k^{2l+1}\cot\delta_l(k)$ for lth scattering wave.

Not $\delta_l(k)$: $\delta_l(k\to 0)\propto k^{2l+1}$: constrained.

Best if same sign for all $k \lesssim \overline{\Lambda}_{\text{EFT}} \implies$ Peruse Λ_1, Λ_2.

If LECs need fitting, the fit for $k \lesssim p_{\text{typ}}$.

Slope may still emerge for $k \nearrow \overline{\Lambda}_{\text{EFT}}$; larger LEC fit error.



Goal: Test Self-Consistency, not Convergence to Data. $\implies$ *"RG Plots"* with minimal resort to experiment.

These are *not* "Lepage plots" which compare to data nucl-th/9706029. – EFT may converge but not to data.

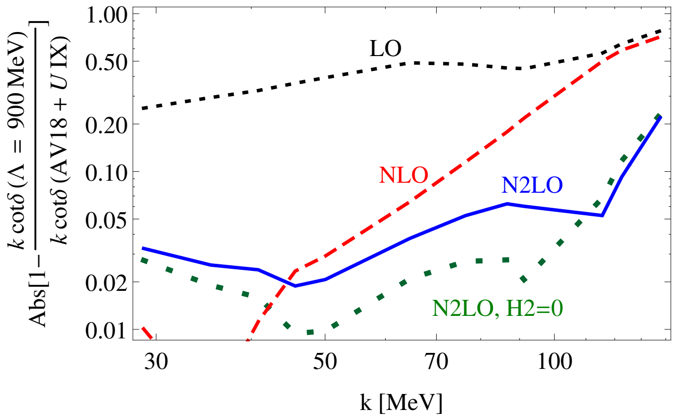
(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1)-\mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

These Are *Not* "Lepage-Plots" $\dfrac{\mathcal{O}_n(k;\Lambda)-\mathcal{O}(\textbf{data})}{\mathcal{O}(\textbf{data})}$.

Lepage: nucl-th/9706029; Steele/Furnstahl: nucl-th/9802069; ...

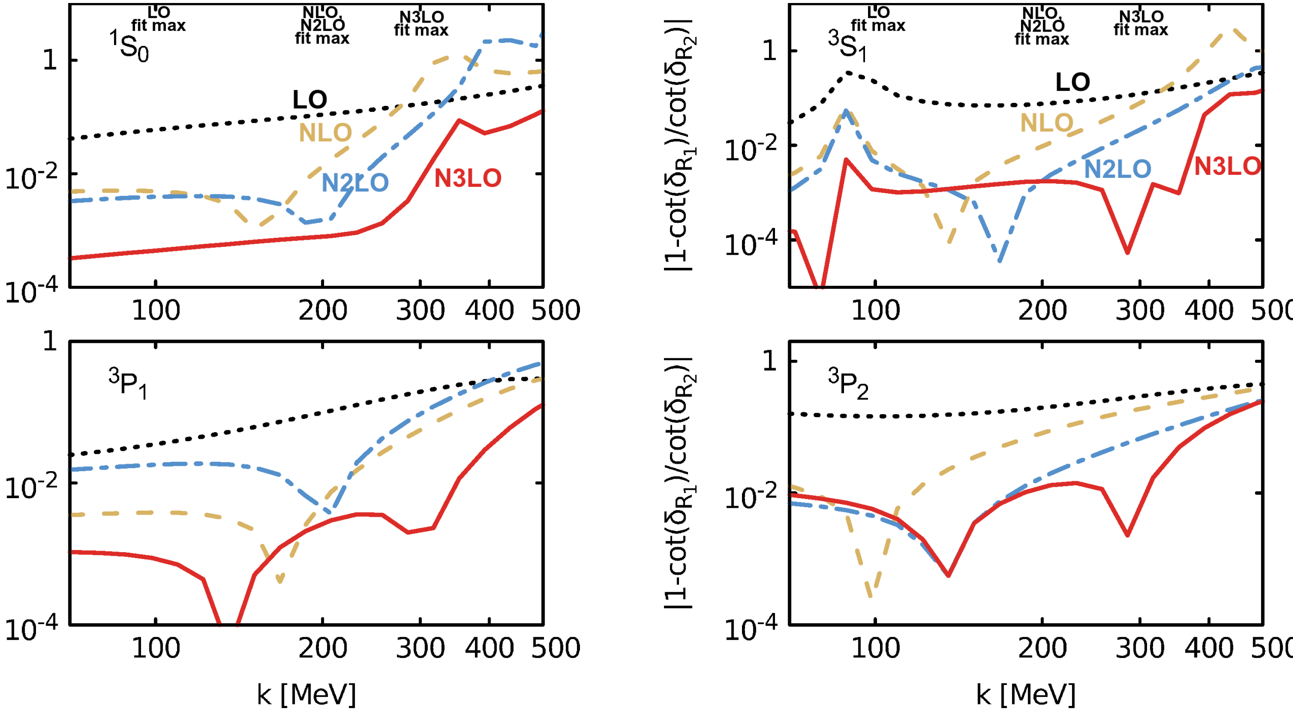
"Lepage" needs data/pseudo-data. $\Longrightarrow$ No consistency test; not double-blind; compromise predictive power.



EFT may converge by itself, but not to data. – Example χEFT without dynamical $\Delta(1232)$ at $k \sim 300$ MeV.

(d) Case of Interest: NN in χEFT: Fitting Parameters Obscures Slopes

Weinberg's Hunch is wrong, but nobody else published: Plot stolen from **Epelbaum/Krebs/Meißner EPJA51 (2015) 5, 53**.



Inconclusive: Breakdown $400 - 500$ MeV, fit- & slope-regions not clearly separated.

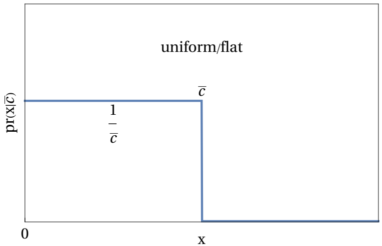
$k \gtrsim 200$ MeV, but no $\Delta(1232)$ degree of freedom.

Coupled channels; NLO & N^{2}LO parallel? Slopes?

Prior Choice: What is “Natural Size”? (SCOTUS: I Know It When I see It.)

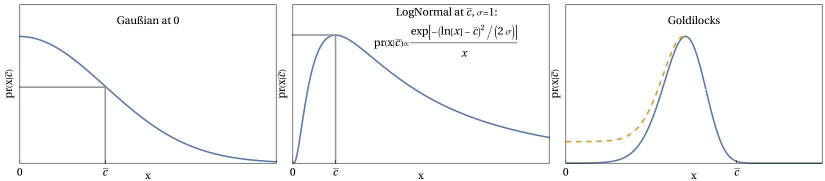
Observable $\mathcal{O} = c_0 + c_1 Q^1 + c_2 Q^2 +$ unknown $\times Q^3$: *assumed* $Q \approx 0.4$ *& “naturally-sized coefficients”* c_i.

Buqeye [1511.03618]: Bayesian technology to extract value of Q from (many) observables, with degree of belief.



Uniform ~~“Least informative/-ed”~~:

characterised by 1 number: $\bar{c}$.



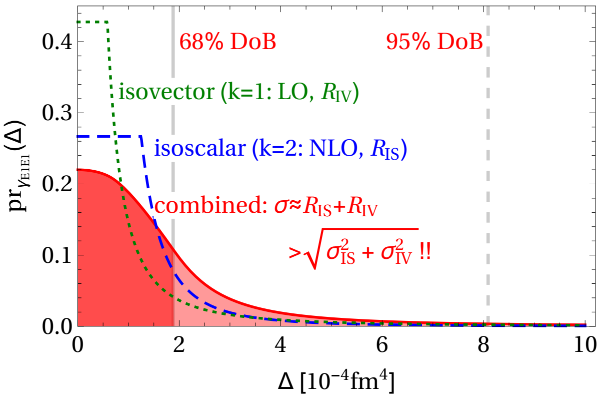
“More informed choices”: more complicated structures, more thought,

more parameters: $\bar{c}$, typ. size, spread,. . .

Buqeye: When $k \geq 2$ orders known, DoBs with different assumptions about $\bar{c}$, c_n vary by $\lesssim \pm 20\%$ for some “reasonable priors”.

For few data/large errors/precision results,
Bayes usually leads to bigger uncertainties than frequentist, and tails usually not Gaußian.
Uncomfortable/Inconvenient? Bayes spells out assumptions, kills false sense of security.

(e) More Bayes Comments



Posterior pdf *not Gauß'ian:*

Plateau & power-law tail.

$\Longrightarrow$ **Do not add in quadrature for convolution**

(more like linear).

Bayes provides well-defined *procedure*!

Example: χEFT predicts nucleon polarisabilities

hg/JMcG/DRP [1511.01952]

e.g. $\gamma_{M1M1} = [2.2 \pm 0.5_{\text{stat/indir.}} \pm 0.6_{\text{th}}] \times 10^{-4}\ \mathrm{fm}^4$

MAMI 2015: $[3.2 \pm 0.9_{\text{stat}}] \times 10^{-4}\ \mathrm{fm}^4$

Bayes in EFTs also used to estimate:

BUQEYE Furnstahl/Phillips/...
[1506.01343], [1511.03618],...

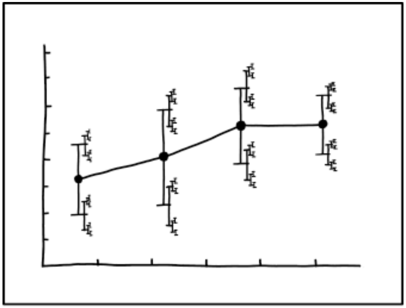
- **k-dependent $Q(k)$ estimate from (many) observables ($Q \approx 0.4$✓);**
- **breakdown scale $\overline{\Lambda}_{\text{EFT}}$;**
- **momentum-dependent data-weighting for LEC fitting/extraction;**
- **build LEC hierarchy into fit;**
- **"model quality"$\equiv$ correctness of EFT assumptions,...**

$\Longrightarrow$ **Quantitative theoretical uncertainties make EFT falsifiable:**

Economical, reproducible procedure: argue about "error on error".

"The aim is to estimate the uncertainty, not to state the exact amount[...]"

PRA Editorial 2011



I DON'T KNOW HOW TO PROPAGATE ERROR CORRECTLY, SO I JUST PUT ERROR BARS ON ALL MY ERROR BARS.

(f) Truncation Errors For Functions: Gaußian Process $\mathcal{GP}$

Energy- & angle-dependent Observable

$$\mathcal{O}(\omega,\theta) = c_0(\omega,\theta) + c_2(\omega,\theta)\ \delta^2(p_{\text{typ}}) + c_3(\omega,\theta)\ \delta^3(p_{\text{typ}}) + c_4(\omega,\theta)\ \delta^4(p_{\text{typ}}) + \dots$$

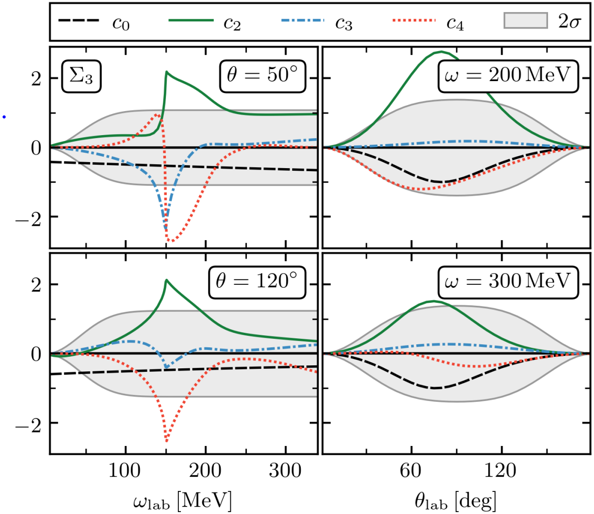
Complications:

- For some $\mathcal{O}$bs, $c_n \equiv 0$ at $\omega = 0$ or $\theta = 0$ or π.
- $\delta(p_{\text{typ}}) \approx \sqrt{\frac{m_\pi+\omega}{2\Lambda_\chi}}$ changes with ω.
- Relative importance of $\Delta(1232)$ changes with ω.
- Structure at pion cusp. $\Longrightarrow$ Skip in $\mathcal{GP}$.

Coefficient functions appear reasonable:

bounded, neither grow nor shrink with order ✓.

Find DoBs per (ω,θ)? ↯ Close-by strongly / Far-away weakly correlated.

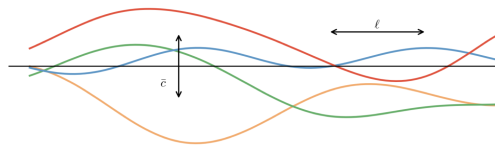


example Σ_3: θ fixed ω fixed

$\Longrightarrow$ **Hypothesis:** $c_n(\omega,\theta)$ as independent draws of **Gaußian Process** $\mathcal{GP}$, i.e. Gaußian at each (ω,θ) with

translation-inv. correlation

$$\langle c_n(\omega_1,\theta_1), c_n(\omega_2,\theta_2)\rangle = \bar{c}^2\ \exp - \left[\frac{(\omega_1-\omega_2)^2}{2\ell_\omega^2} + \frac{(\theta_1-\theta_2)^2}{2\ell_\theta^2}\right]$$



mean $\bar{c}$ (prior: $\chi^{-2}(1,1)$) and correlation lengths $(\ell_\omega,\ell_\theta)$ (prior: uniform) same for all orders n, depend on $\mathcal{O}$bs.

Training: $\bar{c},\ell_\omega,\ell_\theta$ for each $\mathcal{O}$ from known $\{c_n\}$'s. $\Longrightarrow$ typical correlation lengths $\ell_\omega \sim 50$ MeV, $\ell_\theta \sim 45°$

(f) Truncation Errors For Functions: Gaußian Process $\mathcal{GP}$

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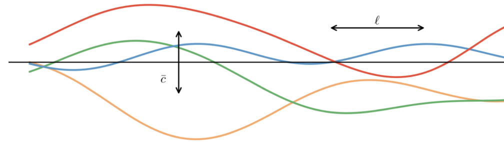

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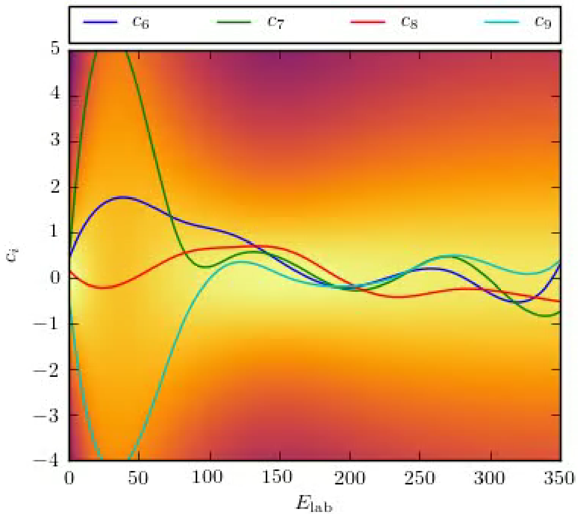
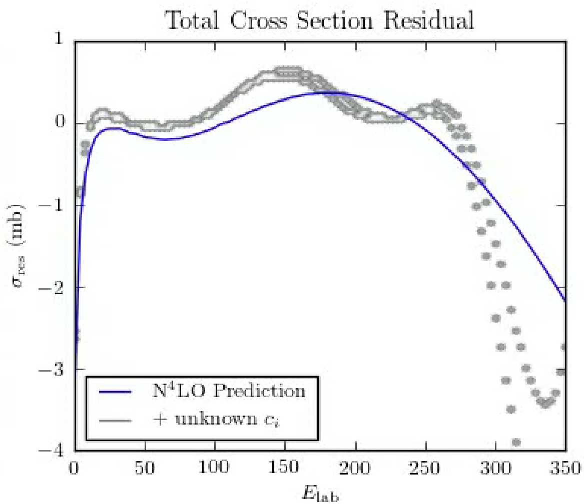
$\Longrightarrow$ **Truncation Error from range of unknown c_n's:** random functions with fixed correlation

buqeye.github.io, see PRC 100 (2019) 044001 [1904.10581]





OPTIMAL IMPACT MACHINE: Maximise benefits – minimise cost (time, money, workforce, data not taken).

Input: (1) **Present polarisability errors** $\Delta\alpha\beta\gamma$ (th & exp, some correlated) – values $\alpha\beta\gamma$ irrelevant.

(2) χ**EFT Predictions** with truncation errors via $\mathcal{GP}$, increasing as $\omega \nearrow$. posterior predictive distr.

(3) **New Data Position** $\overrightarrow{\omega\theta}$: We took 1 energy with 5 angles (exp. constraints) – values $y(\overrightarrow{\omega\theta})$ irrelevant.

(4) **New Data Quality**: **"Doable ($)"**: cross sections to $\pm 4\%$, asymmetries to ± 0.06 (absolute).

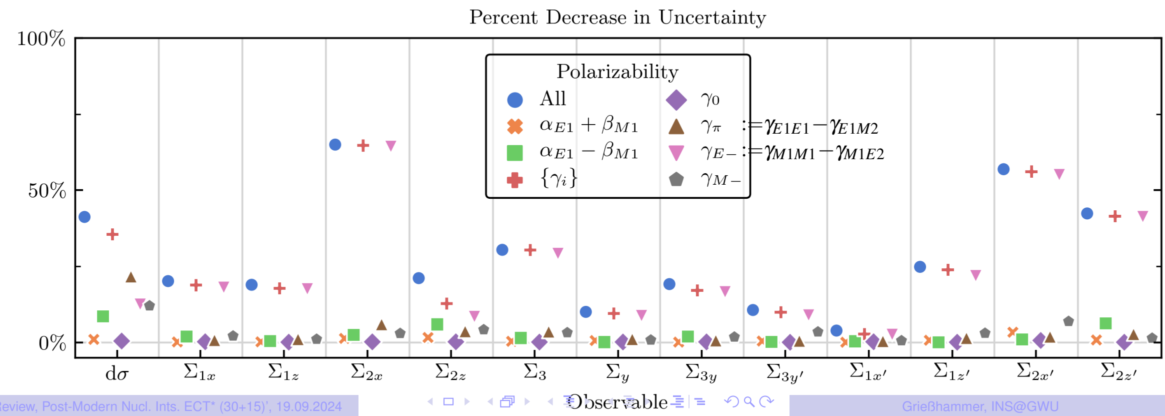
(3+4) $=$ **Expert Elicitation:** Could also add direct penalties for cost, beamtime, event rate,...

here pragmatic: impact of existing data via fits of $\alpha\beta\gamma$; choose uniform exp. constraints.

Utility Gain: What new data at points $\overrightarrow{\omega\theta}$ with results y guessed from theory (with errors) gives likely biggest

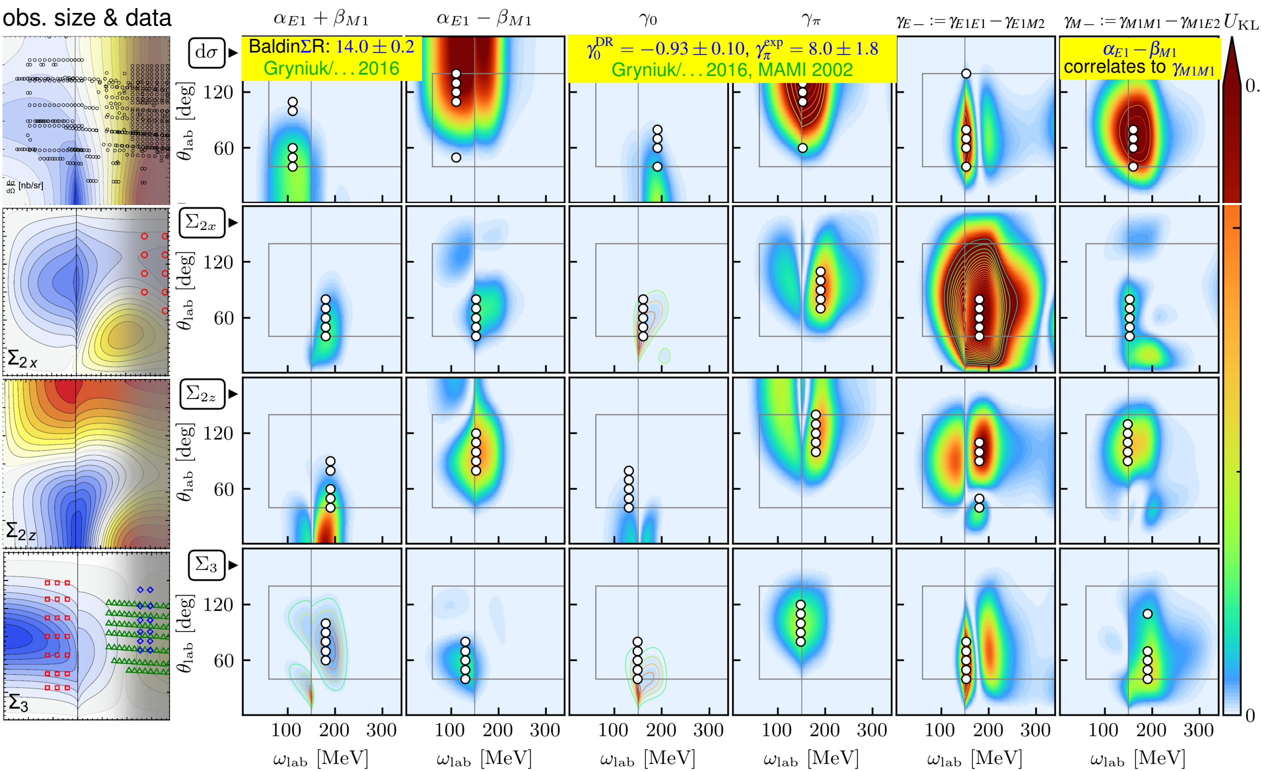
$$U_{\text{KL}} = \underbrace{\int \mathrm{d}y\, \mathrm{pr}(y|\overrightarrow{\omega\theta}) \int \mathrm{d}(\alpha\beta\gamma)}_{\text{data } y \text{ \& central } \alpha\beta\gamma \text{ marginalised}} \underbrace{\mathrm{pr}(\alpha\beta\gamma|y,\overrightarrow{\omega\theta}) \ln \frac{\mathrm{pr}(\alpha\beta\gamma|y,\overrightarrow{\omega\theta})}{\mathrm{pr}(\alpha\beta\gamma)}}_{\text{Shannon information gain}} \underset{\downarrow}{\approx} \ln\left\langle \frac{\text{error's hypervolume before}}{\text{error's hypervolume after data}} \right\rangle_{\text{avg}}$$

linearisation works very well



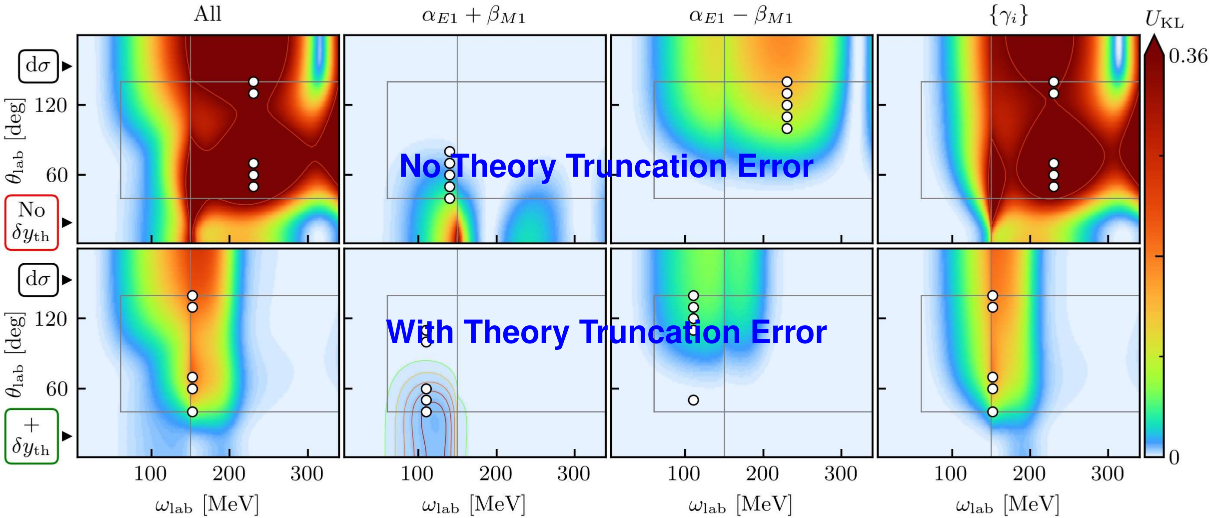
(g) Bayesian Posterior Shrinkage by Intelligent Design

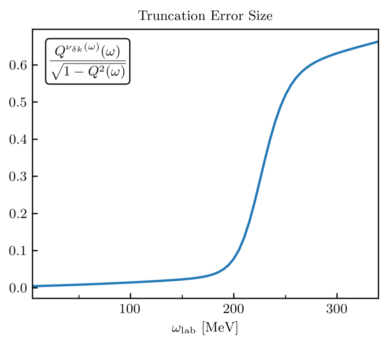
Proton: Which 5 future angles have biggest impact on a particular polarisability?



$\Longrightarrow$ **Focus on** dσ(100 MeV): $\alpha_{E1}-\beta_{M1}$, dσ(160 MeV): γ_{M-}; Σ_{2x}(170 MeV): γ_{E-} – ***not*** **beam asym.** Σ_3

(g) Bayesian Posterior Shrinkage by Intelligent Design





$$\mathcal{O} = c_0 + c_2 Q^2 + c_4 Q^3 + c_4 Q^4 + \ldots$$

$$Q = \sqrt{\frac{p_{\rm typ} \sim (\omega \sim m_\pi \nearrow \Delta_M)}{\bar{\Lambda}_\chi}}$$

Forgetting EFT Truncation Error

Over-Estimates Signal (scale changed!)

Over-Emphasises Resonance Region!

$\Longrightarrow$ **Wrong data point decision!**

(h) Statistical Interpretation of the Max-Criterion: A Simple Example

I take this table of πN scattering parameters in χEFT with effective $\Delta(1232)$ degrees of freedom from a talk by Jacobo Ruiz de Elvira. Here, I am not interested in the Physics, but use it as series $c_i = c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$ in a small expansion parameter.

parameter $[\mathrm{GeV}^{-1}]$	LO total	NLO total	N^{2}LO total	expansion $= c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$	perturbative expansion $\epsilon \approx 0.4$ (guess)
c_1	-0.69	-1.24	-1.11	$= -0.69 + 0.55 - 0.13$	$= -0.69 + 1.38\epsilon^1 - 0.81\epsilon^2$
c_2	$+0.81$	$+1.13$	$+1.28$	$= +0.81 - 0.32 - 0.15$	$= +0.81 - 0.80\epsilon^1 - 0.94\epsilon^2$
c_3	-0.45	-2.75	-2.04	$= -0.45 + 2.30 - 0.71$	$= -0.45 + 5.75\epsilon^1 - 4.44\epsilon^2$
c_4	$+0.64$	$+1.58$	$+2.07$	$= +0.64 - 0.94 - 0.49$	$= +0.64 - 2.35\epsilon^1 - 3.06\epsilon^2$

Now pick the largest absolute coefficient to estimate typical size of next-order correction $c_{i(n+1)} = c_{i3}$ in our case:

Max-Criterion: $c_{i(n+1)} \lesssim \max\limits_{n\in\{0;1;2\}} \{|c_{in}|\} =: R$ **is labelled as red in the table.**

Multiply that number with ϵ^3 to finally get a corridor of uncertainty/typical size of the ϵ^3 contribution.

For c_1: $\max\limits_{n\in\{0;1;2\}} \{|-0.69|;|1.38|;|-0.81|\} = 1.38 \implies$ error $\pm 1.38 \times (\epsilon = 0.4)^3 \approx 0.09 \implies c_1 = -0.69 \pm 0.09$.

Similar: $c_2 = 1.28 \pm 0.06$, $c_3 = -2.04 \pm 0.37$, $c_4 = 2.07 \pm 0.20$ (round significant figures conservatively).

But what's the statistical interpretation? $\implies$ Next slide!

Notes: **(1)** Provide a theoretical error *estimate* that is *reproducible*. You can then discuss with others who have different opinions. No estimate, no discussion possible. – **(2)** Sometimes, one discards the LO$\rightarrow$NLO correction if it's anomalously large. That is a "prior information" you need to disclose as "bias" of your estimate. – **(3)** Coefficients c_{in} appear "more natural" for c_1 and c_2 than for c_4 – c_4 not that well-converging? – **(4)** The uncertainty estimate is agnostic about the Physics details. Somebody just handed me a table. – **(5)** If you are not happy with the input "$\epsilon \approx 0.4$", pick another number. BUQEYE [1511.03618] developed the Bayesian technology to extract degrees of belief on what value of the expansion parameter the series suggests. – **(6)** The c_i are not observables, but they are renormalised couplings which – according to Renormalisation – should follow a perturbative expansion.

(h) Statistical Interpretation of the Max-Criterion: A Simple Example

The Bayesian interpretation of the max-criterion on the next slide will provide probability distribution (pdf)/degree-of-belief functions using a “reasonable” set of assumptions (“priors”) which give nice, analytic expressions. That’s one choice of assumptions, but other reasonable assumptions provide very similar pdf’s see BUQEYE: [1506.01343], [1511.03618],. . . .

But before that, let’s do something intuitive which gives the same statistical likeliness interpretation of the max-criterion as the Bayesian one. The Bayesian analysis formalises the example and provides actual pdf’s.

Estimating a Largest Number: Given a finite set of (finite, positive) numbers in an urn. You get to draw one number at a time.

Your mission, should you choose to accept it: Guess the largest number in the urn from a limited number of drawings.

For c_1, we first draw $c_{10} = 0.69$. I would say it’s “natural” to guess that there is a 1-in-$2 = 50\%$ chance that the next number is lower. But there is also a pretty good chance that if it is higher, then its distribution up there is not Gauß’ian but with a stronger tail.

Next, we draw $c_{11} = 1.38$ which is larger. So I revise my largest-number projection to $R = 1.38$, but I also get more confident that this may be pretty high (if not he highest already). After all, I already found one number which is lower, namely $c_{10} = 0.69$. With 2 pieces of information (0.69 and 1.38), it’s “natural” that the 3rd drawing has a 2-in-3 or $2/3$ chance to be lower.

Next, we draw $c_{12} = 0.81 < R$. Looking at my set of 3 numbers, I am even more confident that $R = c_{11} = 1.38$ is the largest number, with 3-in-4 or 75% confidence. For c_1, evil forces interfere and we have no more drawings to draw information from.

But if we could reach into the urn k times and look at the collected k results, every time revising our max-estimate, it’s “natural” to assign a $100\% \times k/(k+1)$ confidence that I have actually gotten the largest number R.

The Bayesian procedure on the next slide provides the same result. Read the BUQEYE papers for details and formulae!

In our example, we had $k = 3$ terms (drawings) for c_1. So the confidence that $R = 1.38$ is indeed the highest number is $3/4 = 75\%$, which is larger than $p(1\sigma) \approx 68\%$. For a 1σ corridor, I reasonably assume that the numbers are equi-distributed between 0 and the maximum R. Then, the 68%-error corridor is set by $\pm 68\% \times (k+1)/k \times R$ amongst the known numbers.

Now, I multiply that number with 3 powers of the expansion parameter $\epsilon \approx 0.4$ (estimate N^{3}LO terms!) (but see **Note (5)** on the previous slide): $\pm 1.38 \times (68\%/75\%) \times 0.4^3 = \pm 0.08$ is a good uncertainty estimate for a traditional 68% confidence region.

I also get a feeling that the probabilities outside the interval $[0;R]$ may not be Gauß’ian-distributed. Bayes will confirm that

(i) A Brief History of the "Max" Criterion

Estimating EFT Theory Truncation Error by progression of series is standard lore, since "time immemorial".

Example:

> To arrive at the quoted theory error on our results, we note that we perform an $\mathcal{O}(e^2\delta^3)$ fit in a framework in which the polarisabilities first enter at $\mathcal{O}(e^2\delta^2)$. We would expect corrections to be of order $\delta^2 \sim 16\%$ of the lowest-order result, or $\delta \sim 40\%$ of the shift between the LO and NLO results; taking $(\alpha_{E1}^{(\mathrm{p})} + \beta_{M1}^{(\mathrm{p})})/2 \approx 7$ to set the scale for the first approach gives 1.1, while taking the shifts in the values of $\alpha_{E1}^{(\mathrm{p})}$ and $\beta_{M1}^{(\mathrm{p})}$ from third order to fourth order to be ≈ 2 gives 0.8 in the second approach. In view of the similarity between our third- and fourth-order results (see later), the stability under
>
> PPNP67 **(2012)** 875 sect. 4.4 – used as widely accepted 4 years before EKM

The EKM Invention: $|c_{n+1}| \leq R\delta^{n+1}$ **with** $R = \max_i\{c_i\}$ [1412.0142] – **But What Does It *Mean*?**

Statistical Interpretation: Cacciari/Houdeau [1105.5152]; BUQEYE [1506.10343]+[1511.01952]