# Testing chiral EFT interactions and currents: the muon capture on deuteron

## The nuclear interaction: post-modern developments
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# Introduction

$$\mu^- + d \to n + n + \nu_\mu$$

- **Only measurable electroweak current process that involves two nucleons**

- **Intrinsically related to the proton-proton capture**

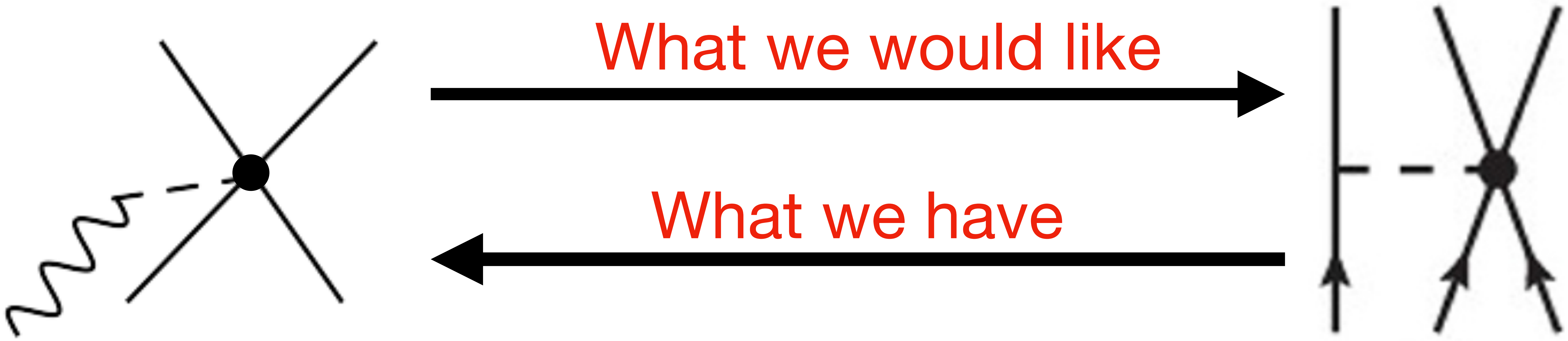
- **Experimentally explored at PSI with the MuSun experiment**

**[A. Gnech, L.E. Marcucci and M. Viviani, Phys. Rev. C 109, 035502 (2024)]**

# Why is this important for Chiral EFT?

- Measurements at PSI: MuSun goal is to reach 1.5% precision on the total capture rate

**Can we extract the LEC $c_D$ from this process with the current experimental sensitivity?**




- Independent extraction of $c_D$ in the two-body sector.

# Muon capture rate

## Differential capture rate

$$\Gamma(E) = \frac{G_V^2}{\pi} |\psi(0)|^2 \int d\cos\theta \, f(\theta, E) \sum_{[s]} \left| M_{[s]}(\theta, E) \right|^2$$

- $\theta$ and $E$ are the angle and the energy of the emitted neutron in the Lab frame
- The total capture rate (measured by MuSun) is obtained integrating on $E$

# Goals and methodology

**Analysis of the theoretical uncertainties using the most modern approaches based on chiral EFT**

**Sources of uncertainties**

- LECs uncertainties (only currents) → Standard Error Propagation
- Chiral EFT currents truncation
- Chiral EFT interaction truncation

} Bayesian Analysis

- Model dependence → Varying interaction
- Impact of power counting → Bochum/JLab-Pisa

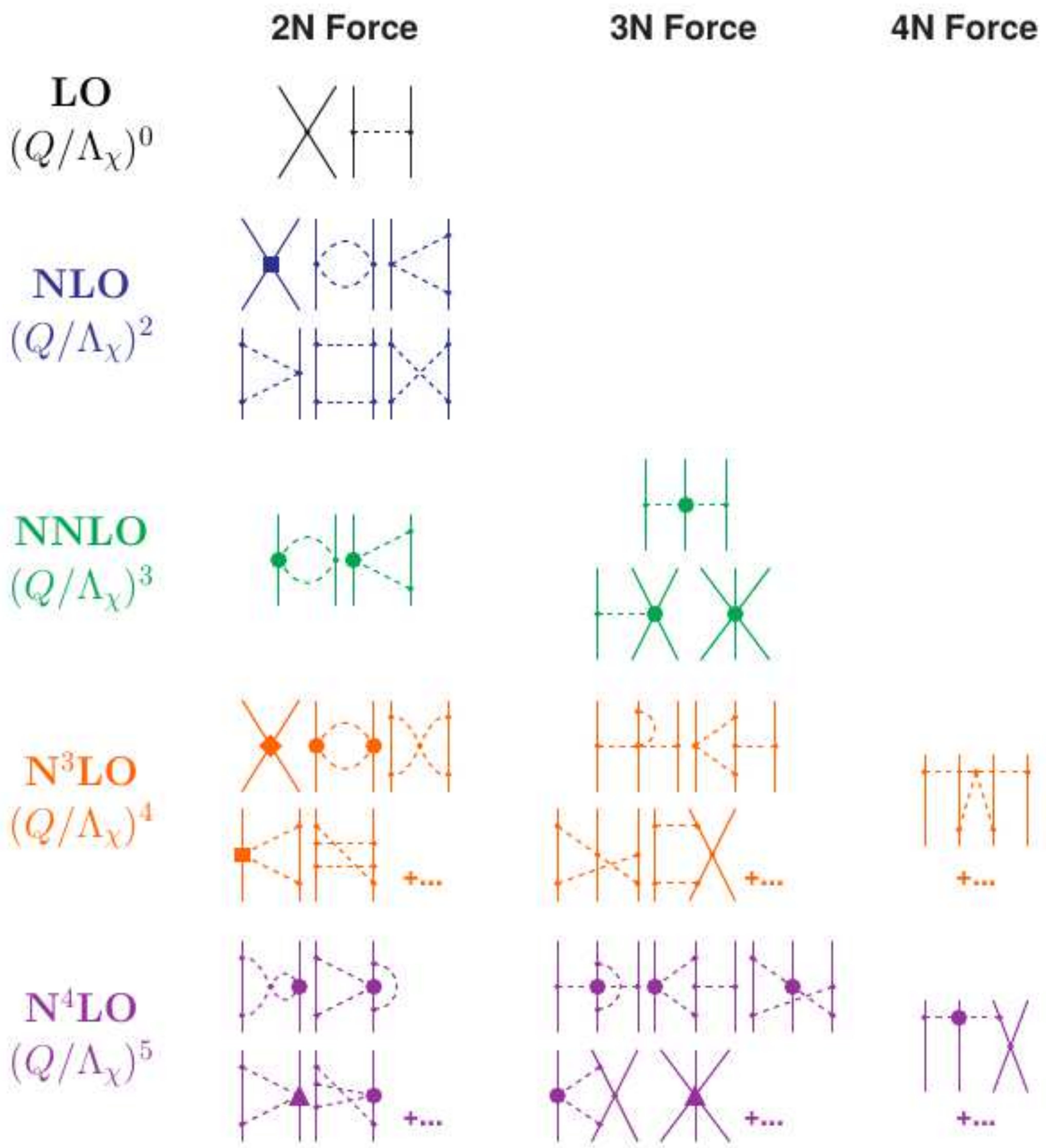
[see also Bonilla et al., Phys. Rev. C 107, 065502 (2023)]

# Interaction models

- Two nuclear interactions: Norfolk [PRC 94, 054007 (2016)] and EMN [PRC 96, 024004 (2017)]

**EMN: non-local, delta-less, three-models**
**NV: local, delta-full,four models**

**Three body-forces fitted to $^{3}H$ + $^{3}H$ beta-decay**



# Current models

- Currents from [PRC 80, 034004 (2009)] and [PRC 99, 034005 (2019)]

**Two power counting**

**BOCHUM***

for example [EPJA 56, 234 (2020)]

| Oper. | LO ($Q^{-3}$) | NLO ($Q^{-2}$) | N2LO ($Q^{-1}$) | N3LO ($Q^{0}$) |
|---|---|---|---|---|
| $\rho(A)$ | — | — | 1b(NR)<br>OPE | — |
| $\mathbf{j}(A)$ | 1b(NR) | — | OPE-$\Delta^*$<br>[1b(RC)] | CT($d_R$)<br>OPE |
| $\rho(V)$ | 1b(NR) | — | [1b(RC)] | [OPE(RC)] |
| $\mathbf{j}(V)$ | — | — | 1b(NR)<br>OPE | OPE-$\Delta^*$<br>[1b(RC)] |

| Oper. | LO ($Q^{-3}$) | NLO ($Q^{-2}$) | N2LO ($Q^{-1}$) | N3LO ($Q^{0}$) |
|---|---|---|---|---|
| $\rho(A)$ | — | 1b(NR) | OPE | — |
| $\mathbf{j}(A)$ | 1b(NR) | — | 1b(RC)<br>OPE-$\Delta^*$ | CT($d_R$)<br>OPE |
| $\rho(V)$ | 1b(NR) | — | 1b(RC) | OPE(RC) |
| $\mathbf{j}(V)$ | — | 1b(NR) | OPE | 1b(RC)<br>OPE-$\Delta^*$ |

**JLAB-PISA**

# Uncertainties on the LECs

- Main source is the nucleon axial form factor (as Ref. [PRC107, 065502 (2023)])

$$g_A(q^2) = g_A \left(1 - \frac{1}{6} r_A^2 q^2\right) \longrightarrow r_A^2 = 0.46 \pm 0.16\,\mathrm{fm}^2 \text{ [1]}$$

- LECs fitted on the electroweak processes (negligible)
  - Axial currents: $z_0$ fitted on tritium beta-decay
  - Vector currents: fitted on EM observables

- Other LECs appearing in the nuclear interactions and currents (partially considered in model dependence)

[1] Rep. Prog. Phys. **81**, 096301 (2018)

# Chiral EFT truncation errors

$$\Gamma_k(p) = \Gamma_{ref}(p) \sum_{n=0}^{k} c_n(p) Q^n(p)$$

Reference scale

Power expansion factor

$$Q(p) = \frac{1}{\Lambda_b} \frac{p^8 + m_\pi^8}{p^7 + m_\pi^7}$$

$\Lambda_b \sim 600$ MeV

$$c_n(p) \sim \mathcal{GP}\left[\mu, \bar{c}^2 r\left(p, \bar{p}; \ell\right)\right]$$

Emulated using a Gaussian process

Hyperparameters:

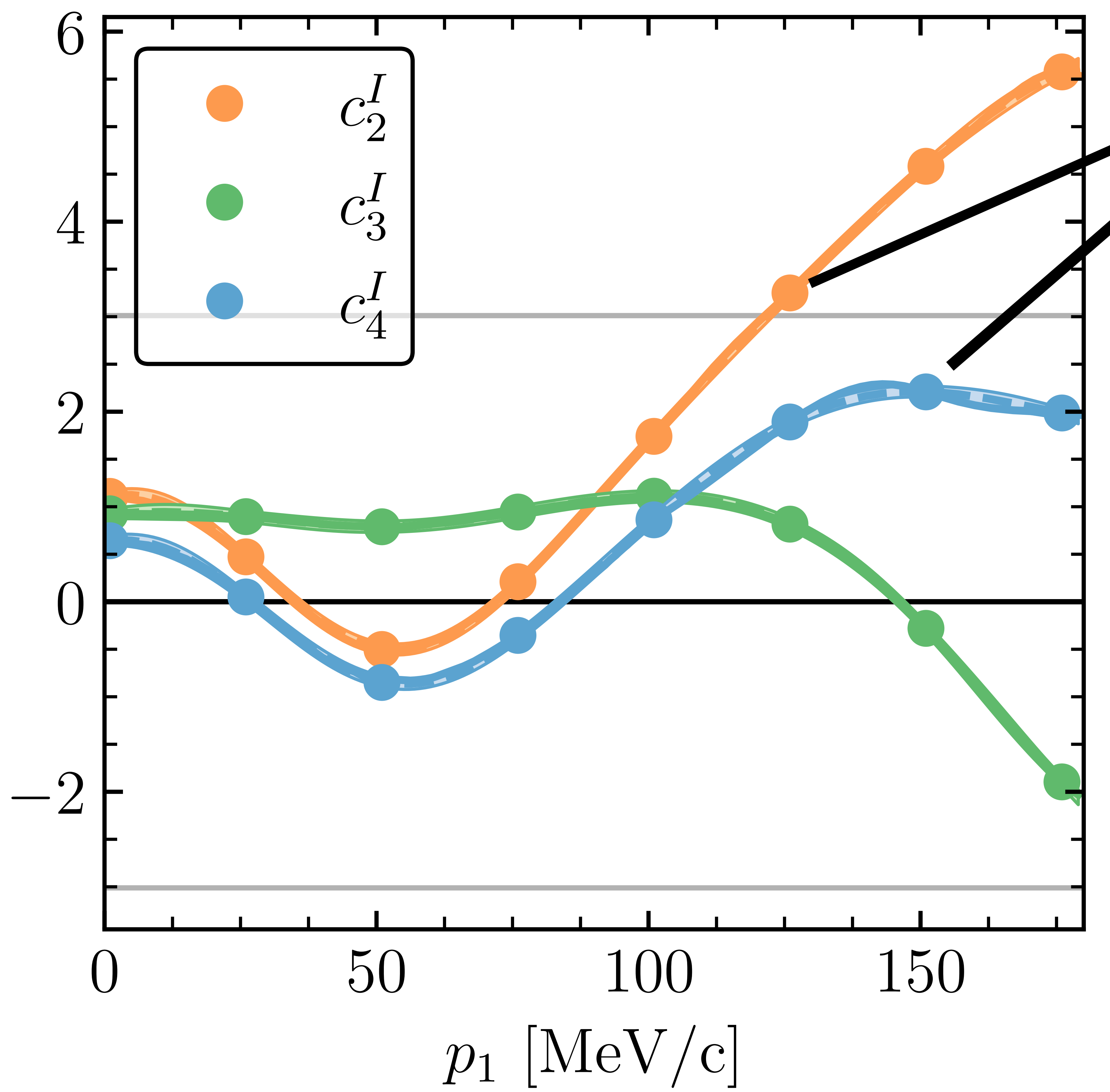
Prior distribution that are modified on the training data set based on **Bayes' Theorem**

$$\delta\Gamma_k(p) = \Gamma_{ref}(p) \sum_{n=k+1}^{\infty} c_n(p) Q^n(p)$$

Truncation error: what we want!

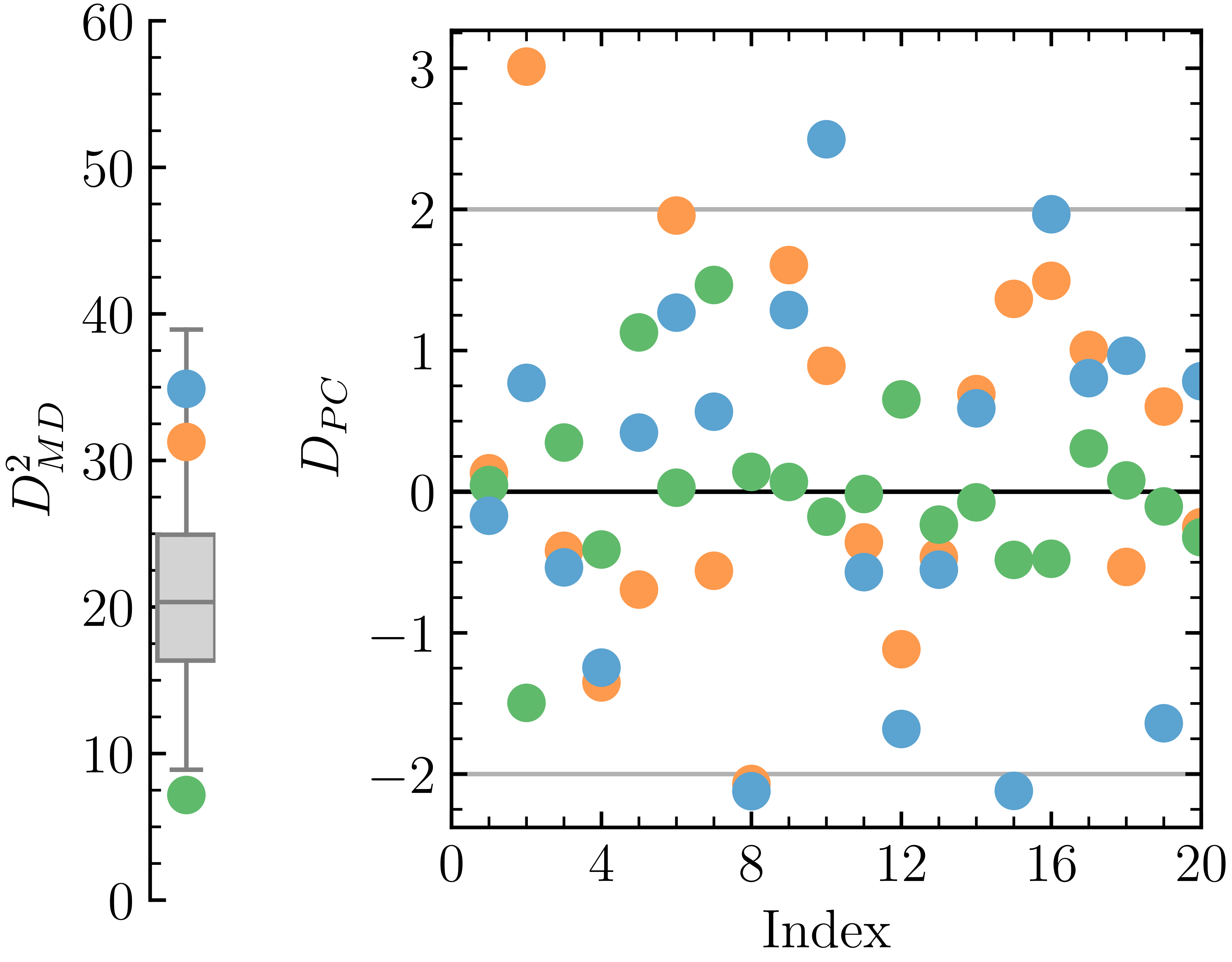
J.A. Melendez *et al.*, Phys. Rev. C **100**, 044001 (2019) (Thanks to BUQEYE Coll.)

# Emulation and testing (interaction)




Training points for the emulator

Test on selected points




Results for the EMN550 interaction
from NLO to N3LO (fixed current order)

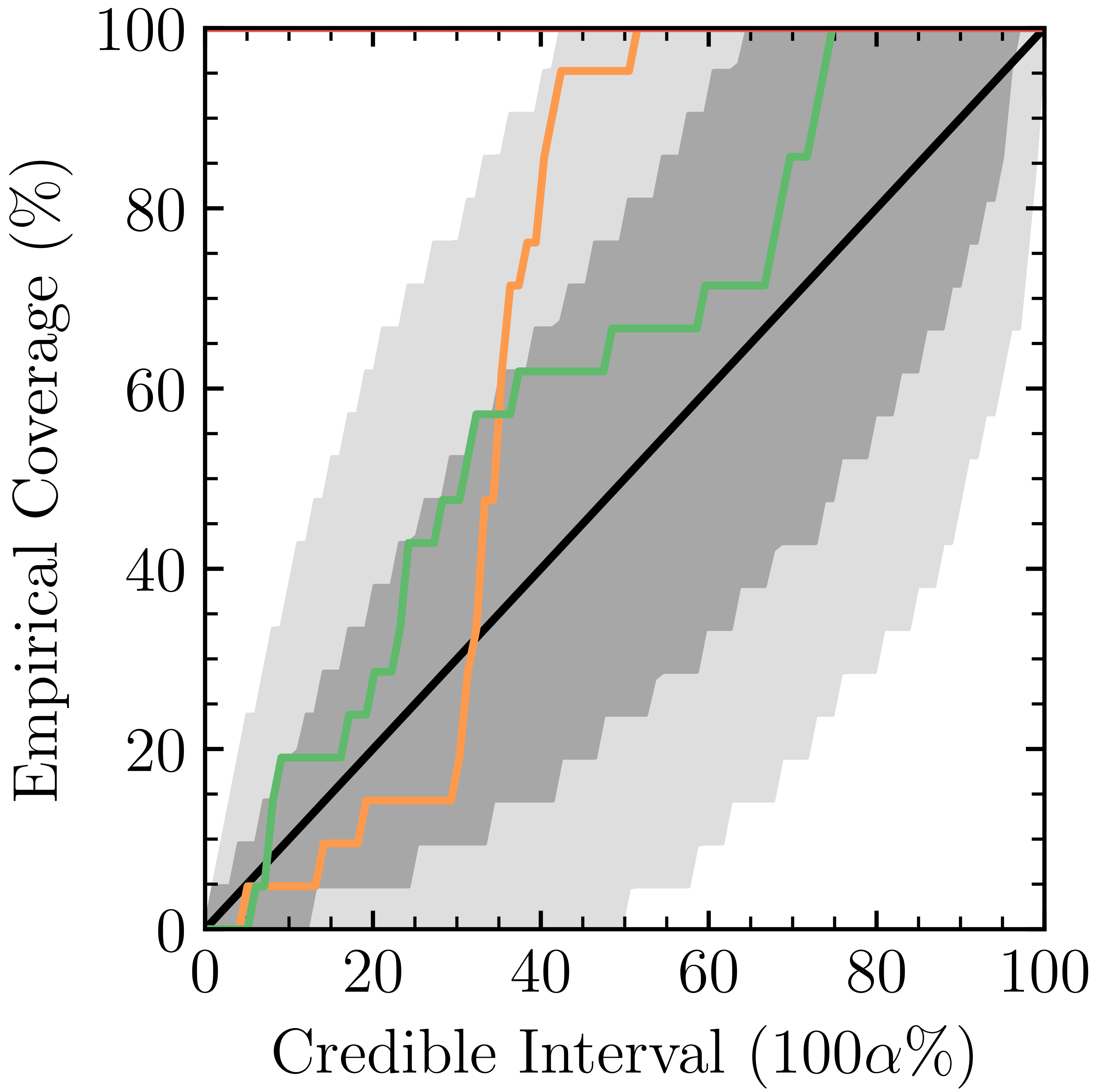
# Order by order compatibility

Credible interval diagnostic:
compatibility of the truncation error
with the next order

$$\Delta\Gamma_1 = \Gamma_{NLO} - \Gamma_{N2LO} \text{ vs } \delta\Gamma_1$$

$$\Delta\Gamma_2 = \Gamma_{N2LO} - \Gamma_{N3LO} \text{ vs } \delta\Gamma_2$$

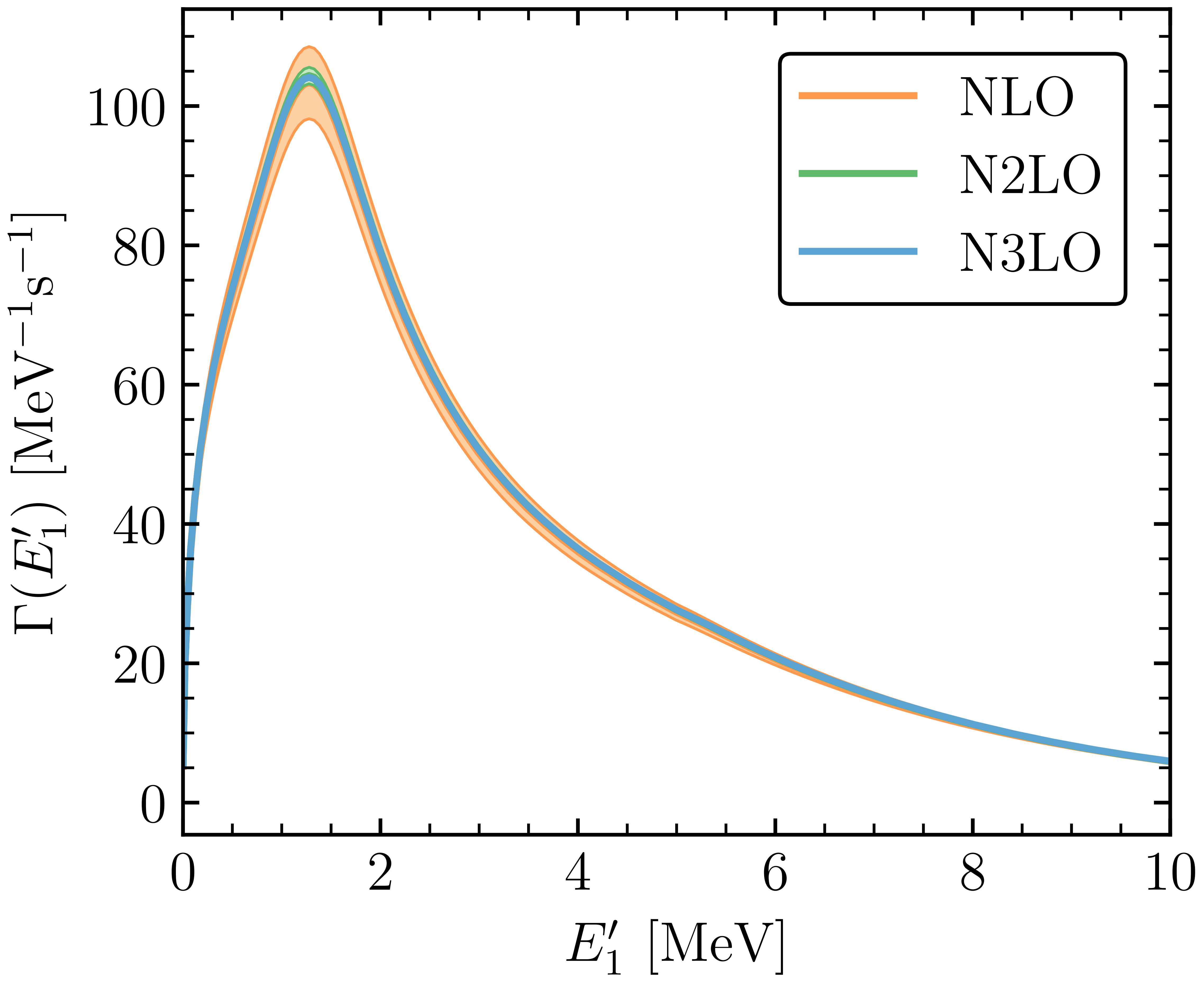
Confidence level band 68% and 95%




Results for the EMN550 interaction

# Results for the differential capture rate

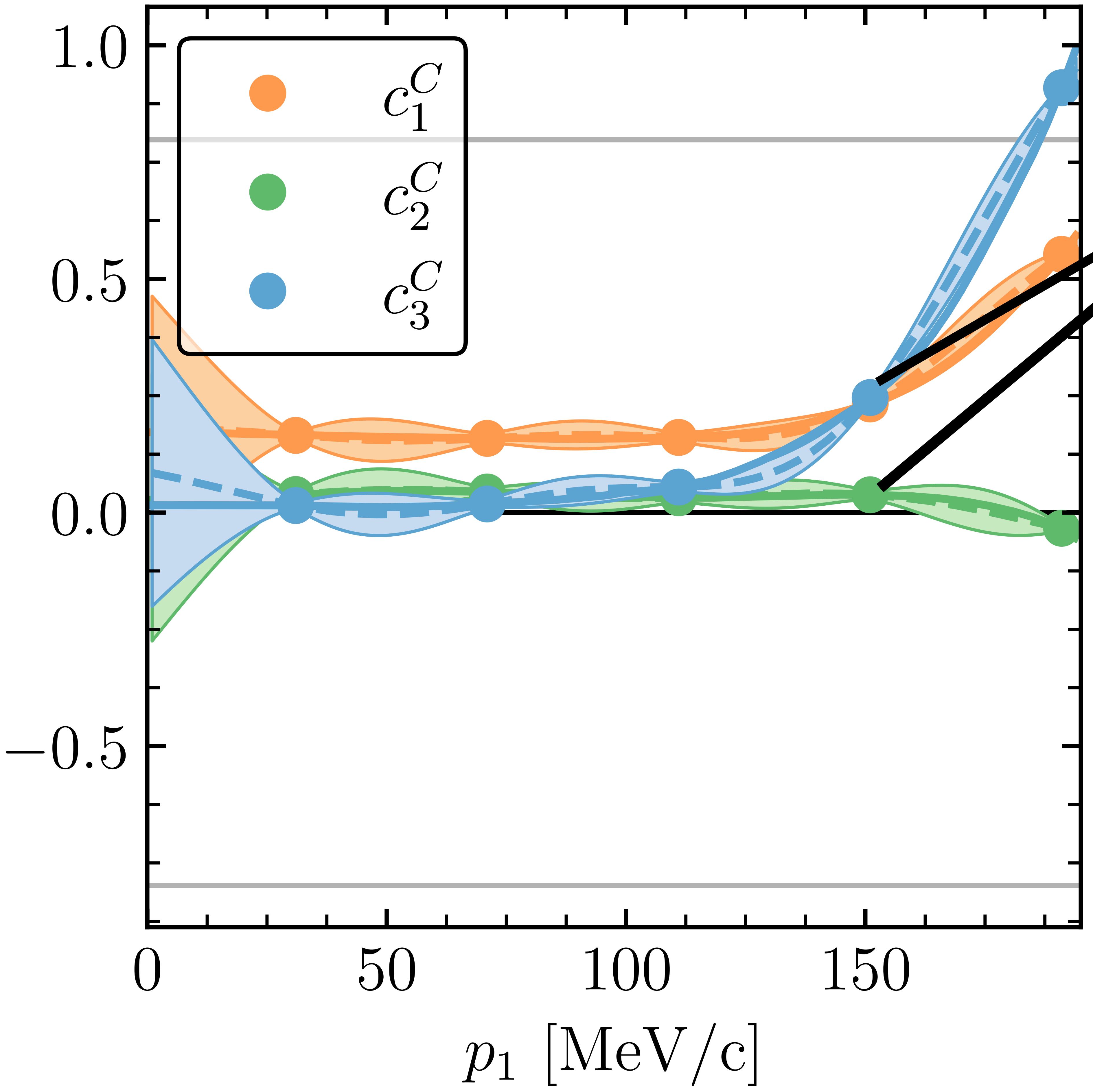
Interaction truncation errors
NLO-N3LO for EMN550




Notes:

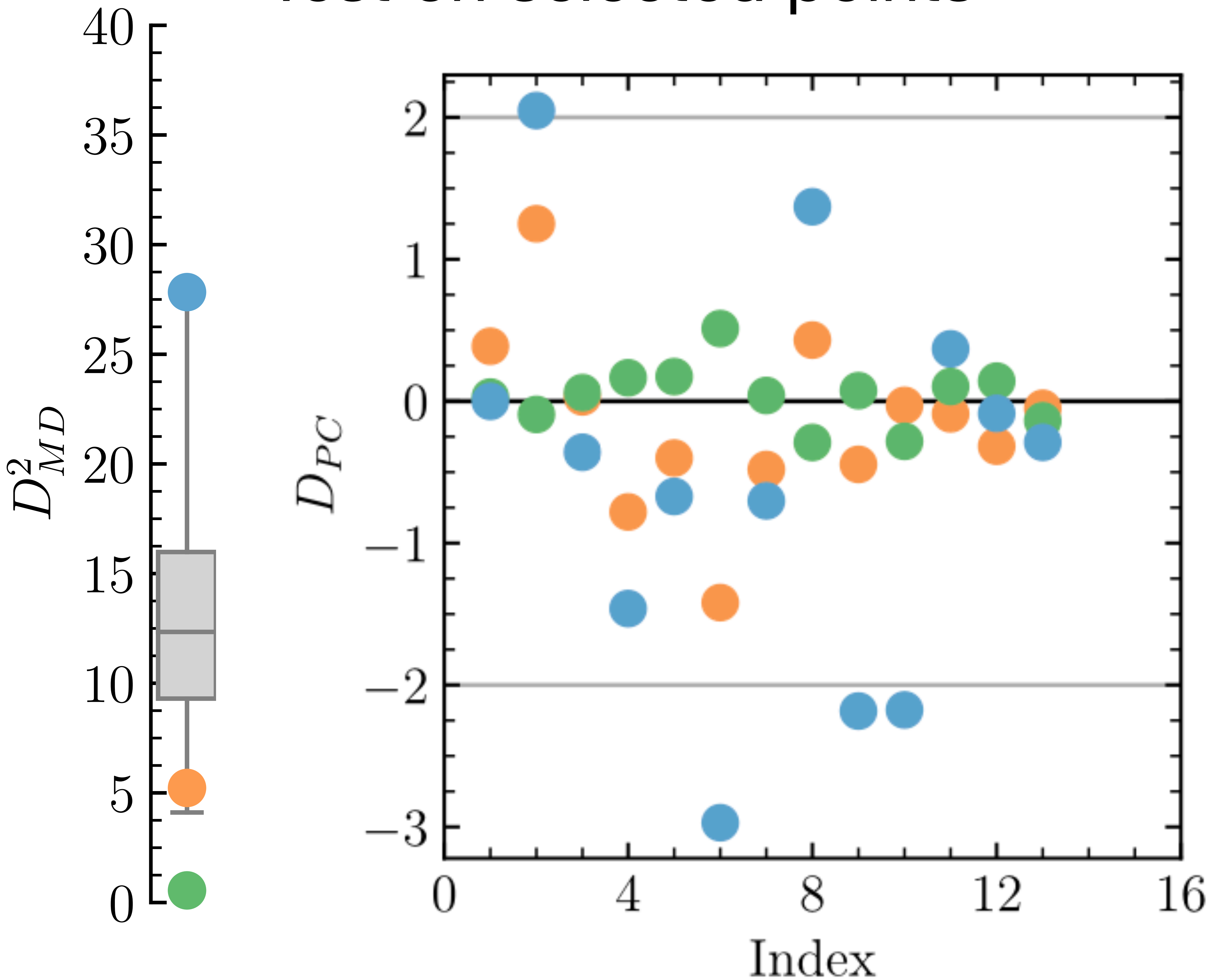
- Analysis limited to the EMN interactions
- Analysis performed at fixed current order

# Emulation and testing (currents)




Training points for the emulator

Test on selected points




Results for the EMN550 interaction
from NLO to N3LO (fixed inter. order)

# Order by order compatibility

Credible interval diagnostic

Bochum power counting:

$\Delta\Gamma_1 = \Gamma_{N2LO} - \Gamma_{N3LO}$ vs $\delta\Gamma_2$

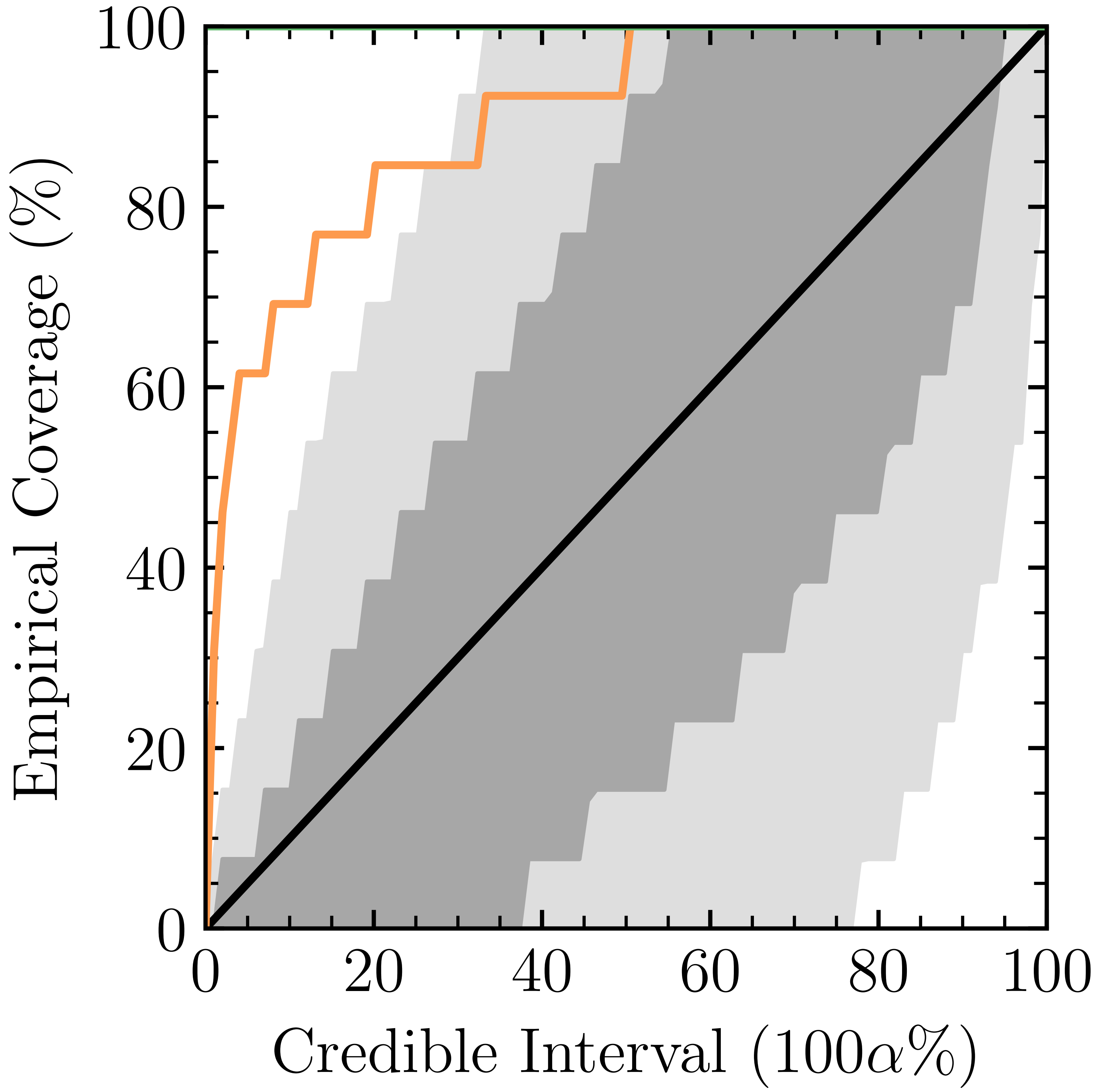
JLab-Pisa power counting:

$\Delta\Gamma_1 = \Gamma_{NLO} - \Gamma_{N2LO}$ vs $\delta\Gamma_1$

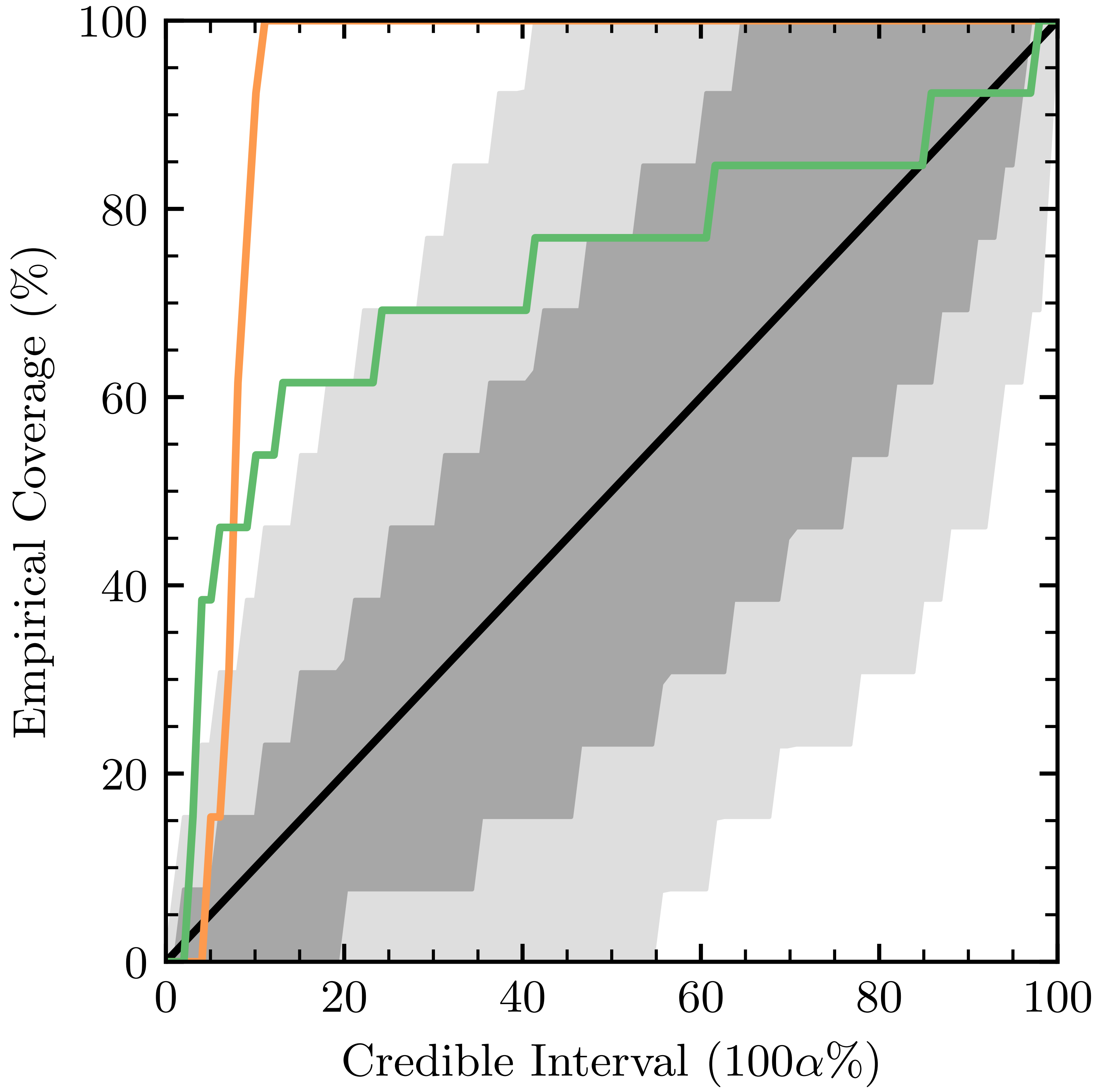
$\Delta\Gamma_2 = \Gamma_{N2LO} - \Gamma_{N3LO}$ vs $\delta\Gamma_2$

Confidence level band 68% and 95%

Bochum power counting

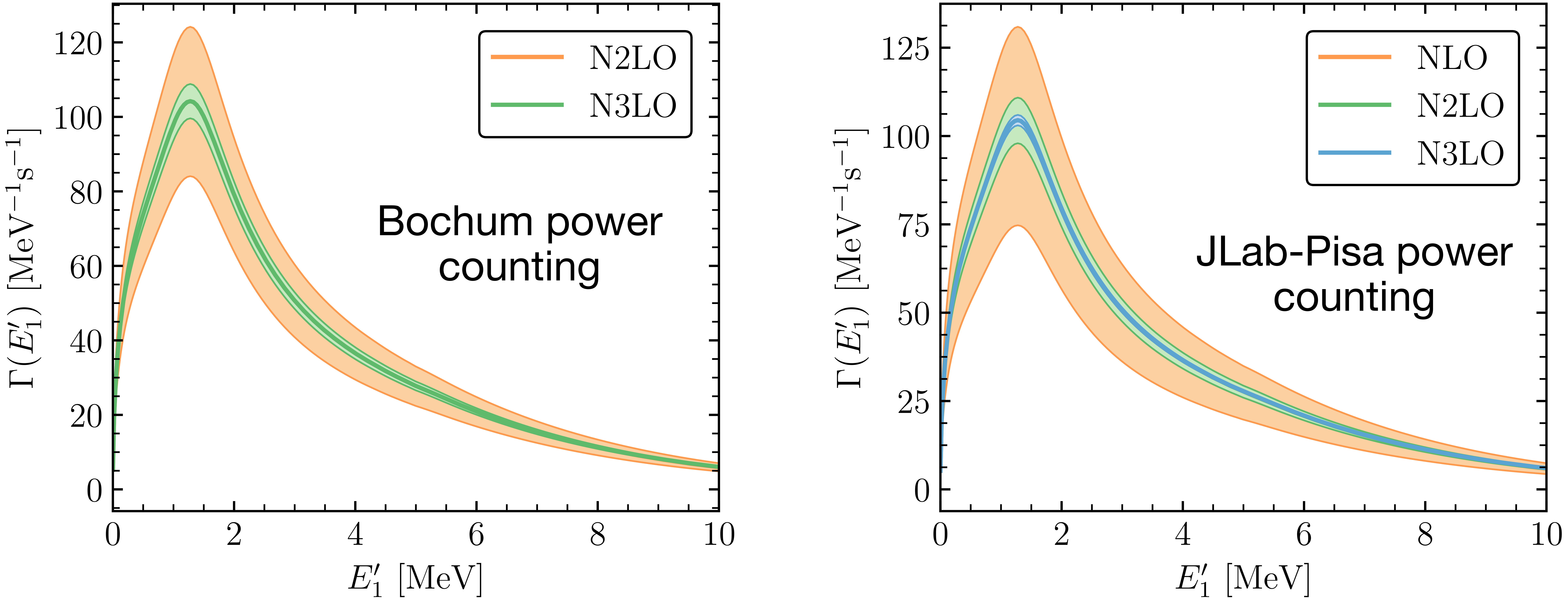



JLab-Pisa power counting



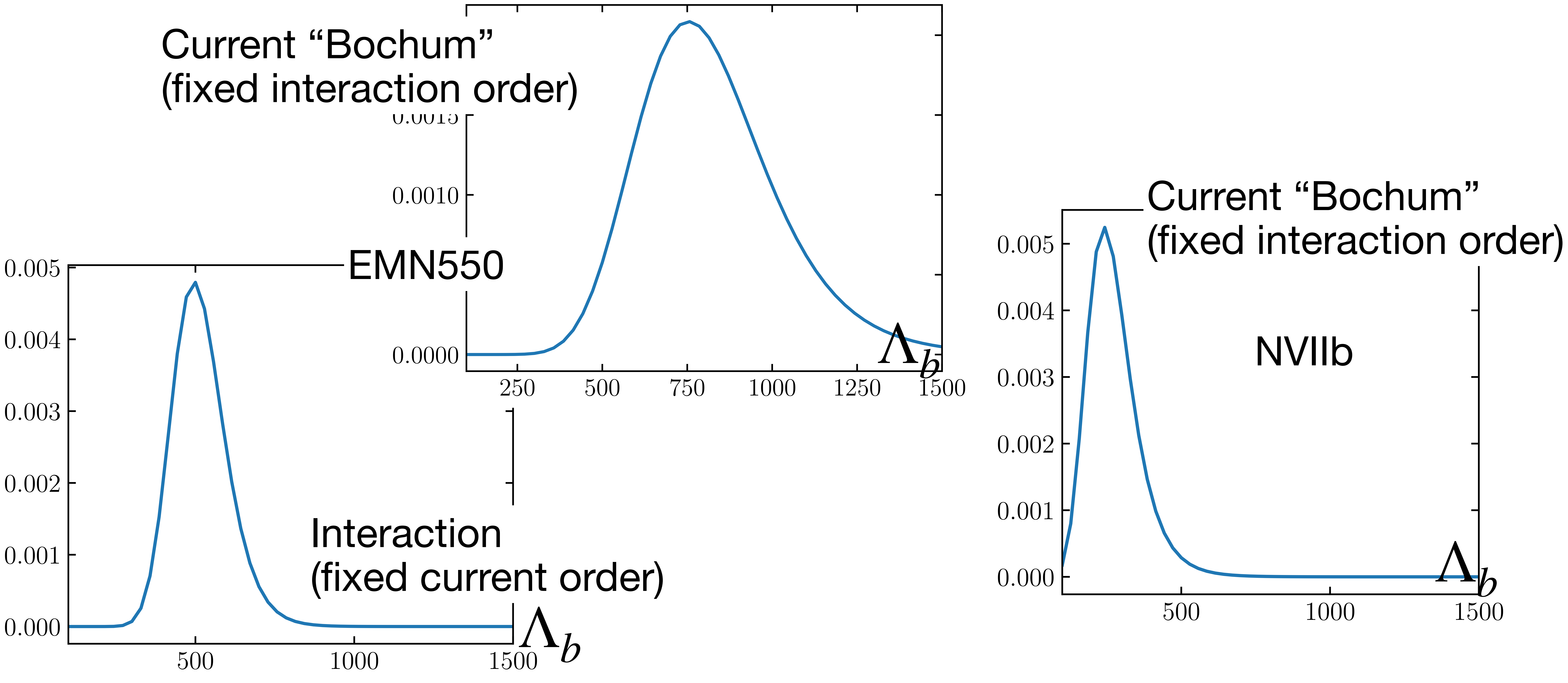

Results for the EMN550 interaction

# Results for the differential capture rate

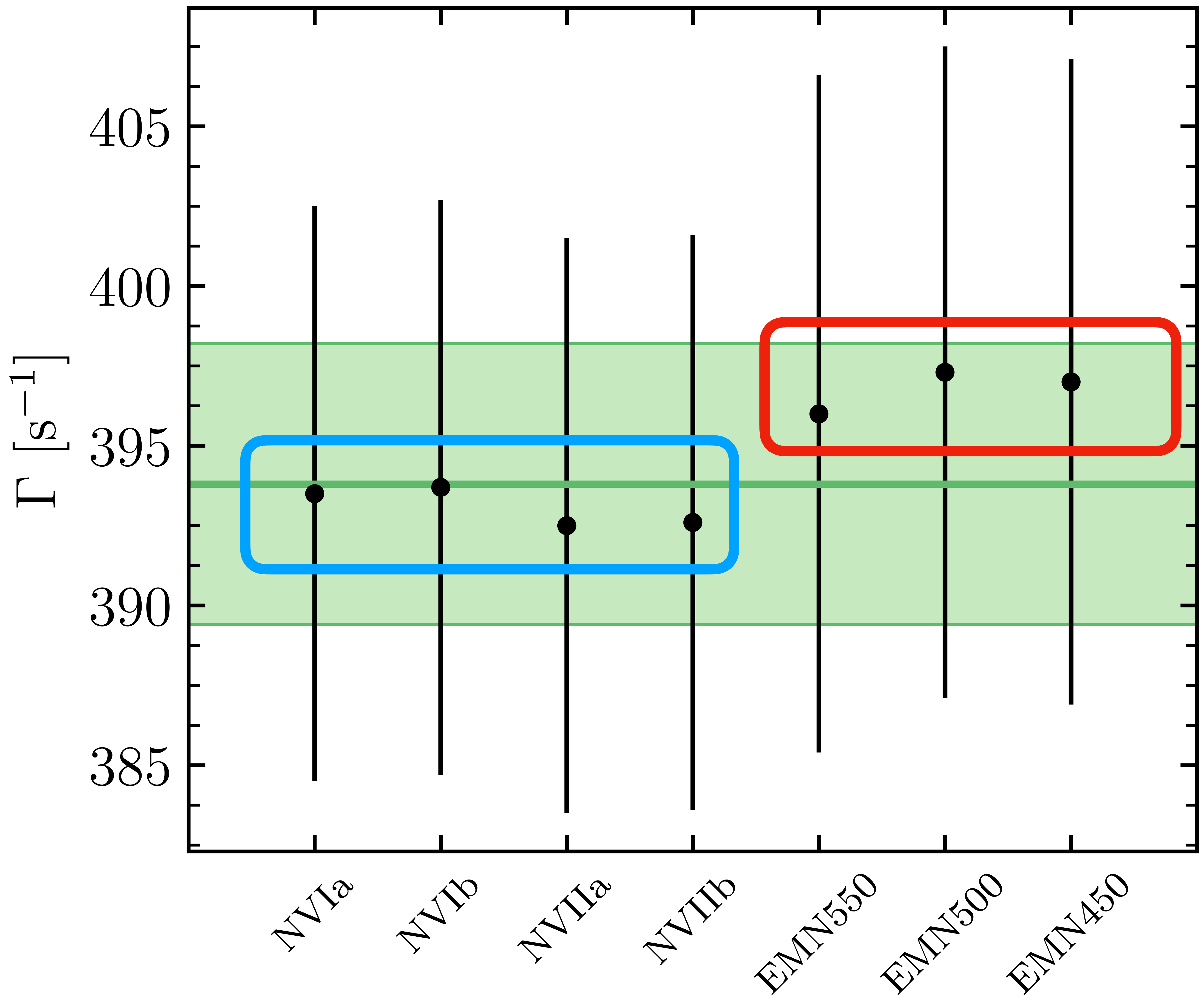



Analysis performed at fixed interaction order (N3LO EMN550)

# Breaking down scales



# Muon capture model comparison



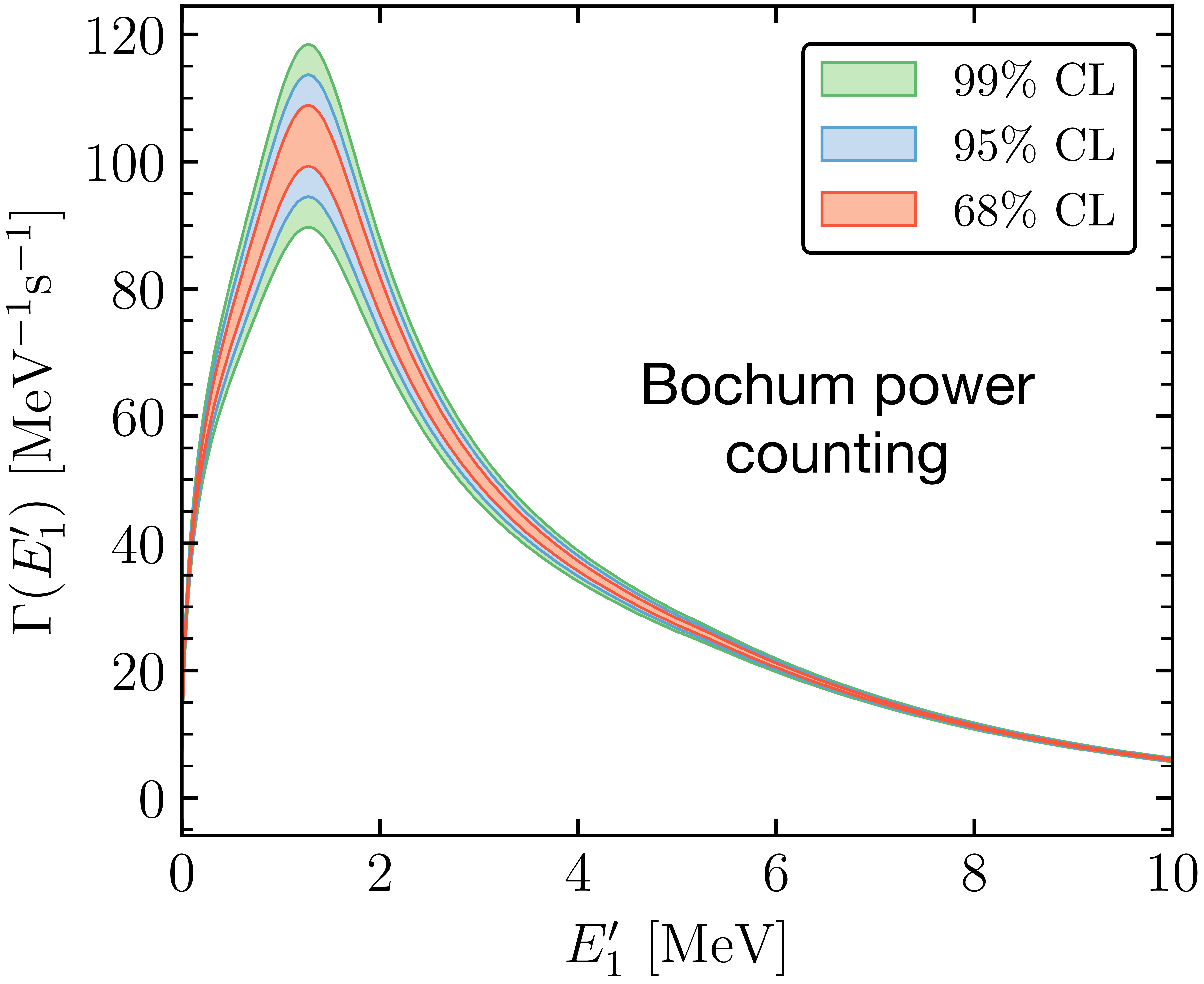

**NV** results systematically lower than **EMN** results

**Effect of the Delta?**

# Model averaging

**Assumptions**

- Theoretical errors are completely correlated $\sigma_{tot} = \sigma_C + \sigma_I + \sigma_{LEC}$
- Model combination from PRD 103, 114502 (2021)




Final differential capture rate at N3LO

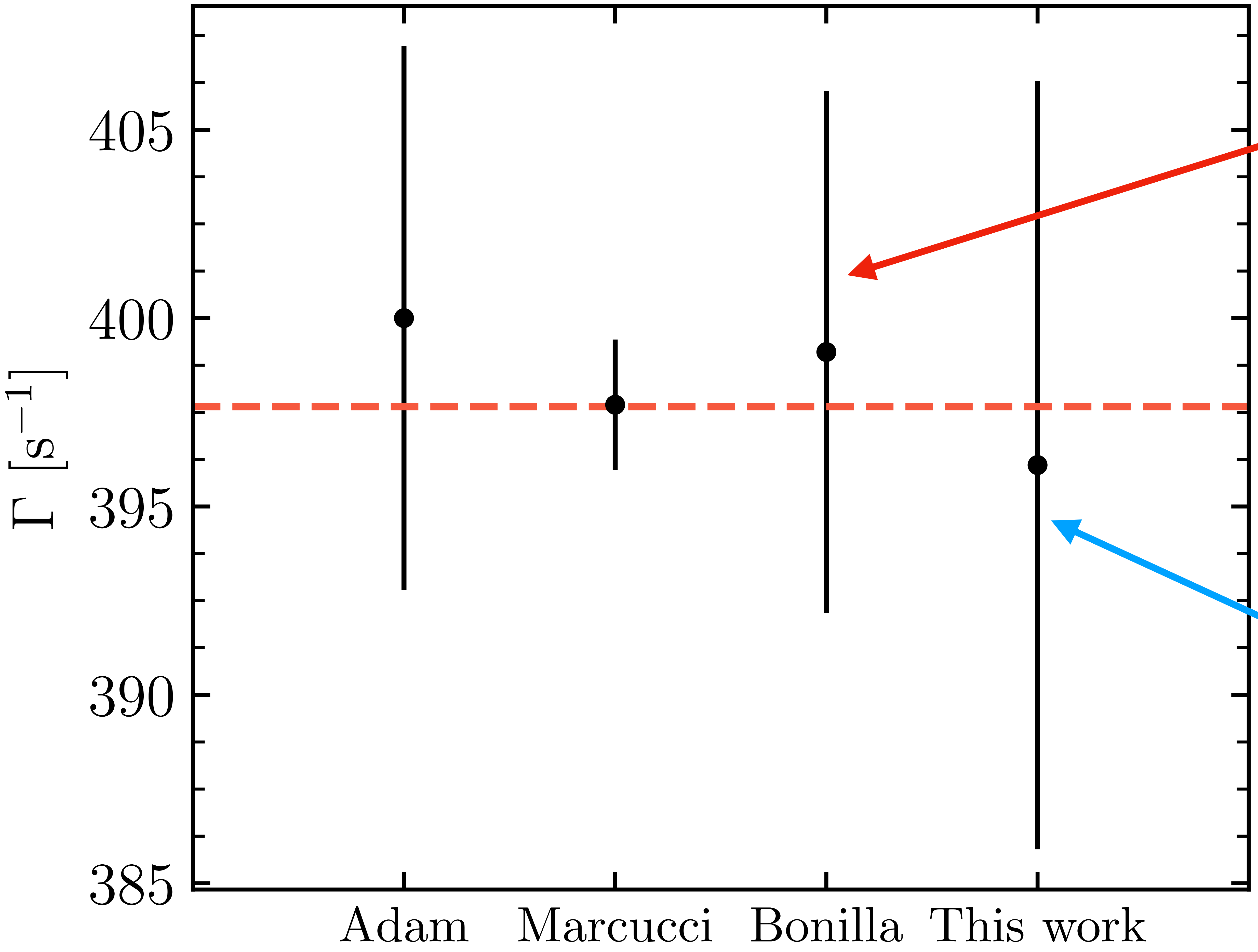
# Results summary

| Uncertainty source | BPC | JPPC |
|---|---|---|
| $r_A^2$ | 5.6(30.8%) | 5.4(75.2%) |
| Other current LECs | negligible | |
| $\chi$EFT truncation - currents | 8.1(65.2%) | 2.4(14.6%) |
| $\chi$EFT truncation - interactions | 0.5(0.3%) | 0.5(0.7%) |
| Model dependence | 1.9(3.7%) | 1.9(9.5%) |

$$\Gamma_{th}(BPC) = (395 \pm 10)\,\mathrm{s}^{-1} \quad (68\,\%\,\mathrm{CL})$$

$$\Gamma_{th}(JPPC) = (395 \pm 6)\,\mathrm{s}^{-1} \quad (68\,\%\,\mathrm{CL})$$

# Total capture rate: comparison with literature




[PRC107, 065502 (2023)]

Includes LECs error, truncation error and model dependence

Our recommended result

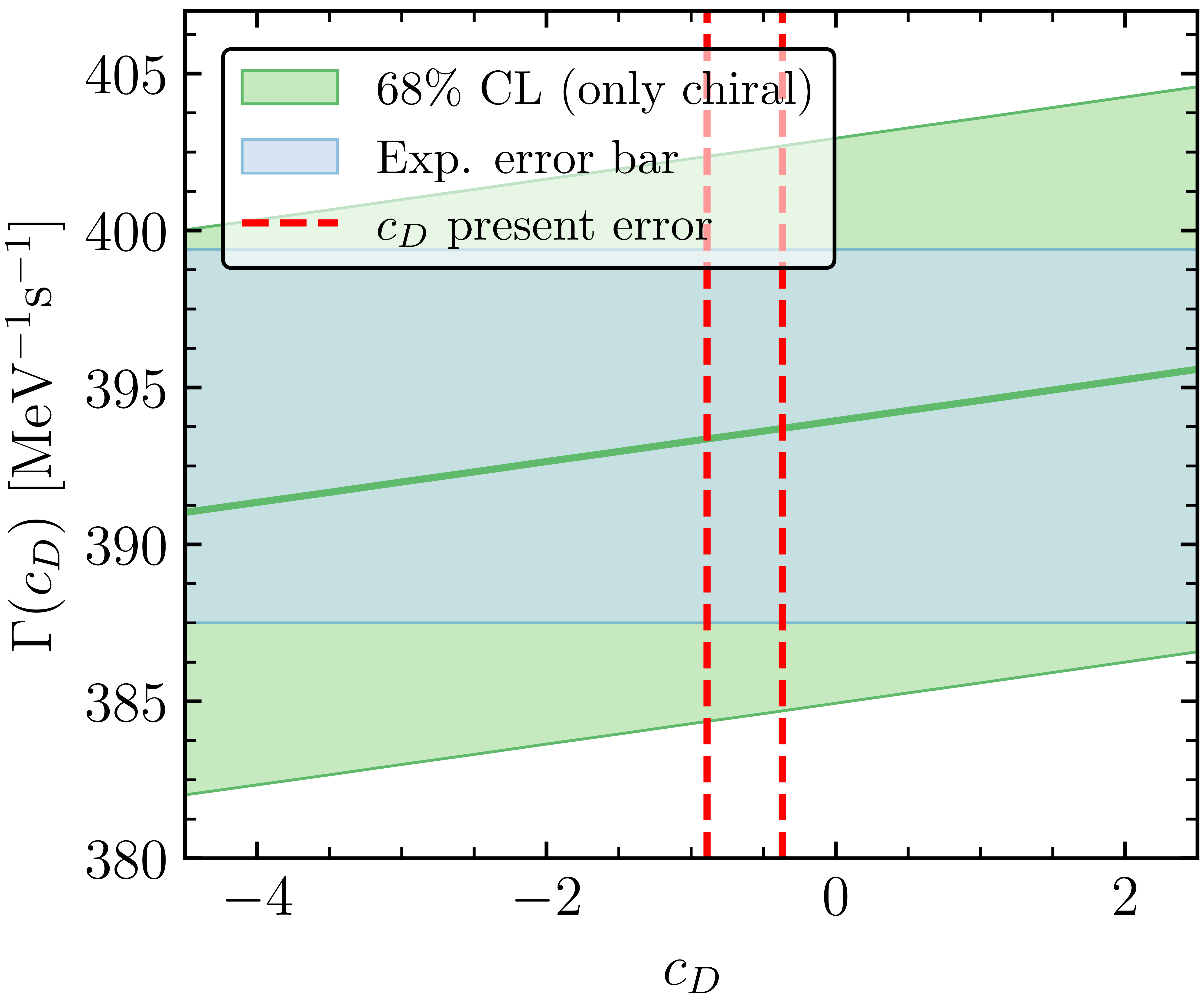
Bochum power counting

# Extracting $c_D$

**Can we obtain a more precise value of $c_D$ with the present experimental and theoretical errors?**

**The sensitivity to $c_D$ is very mild!**

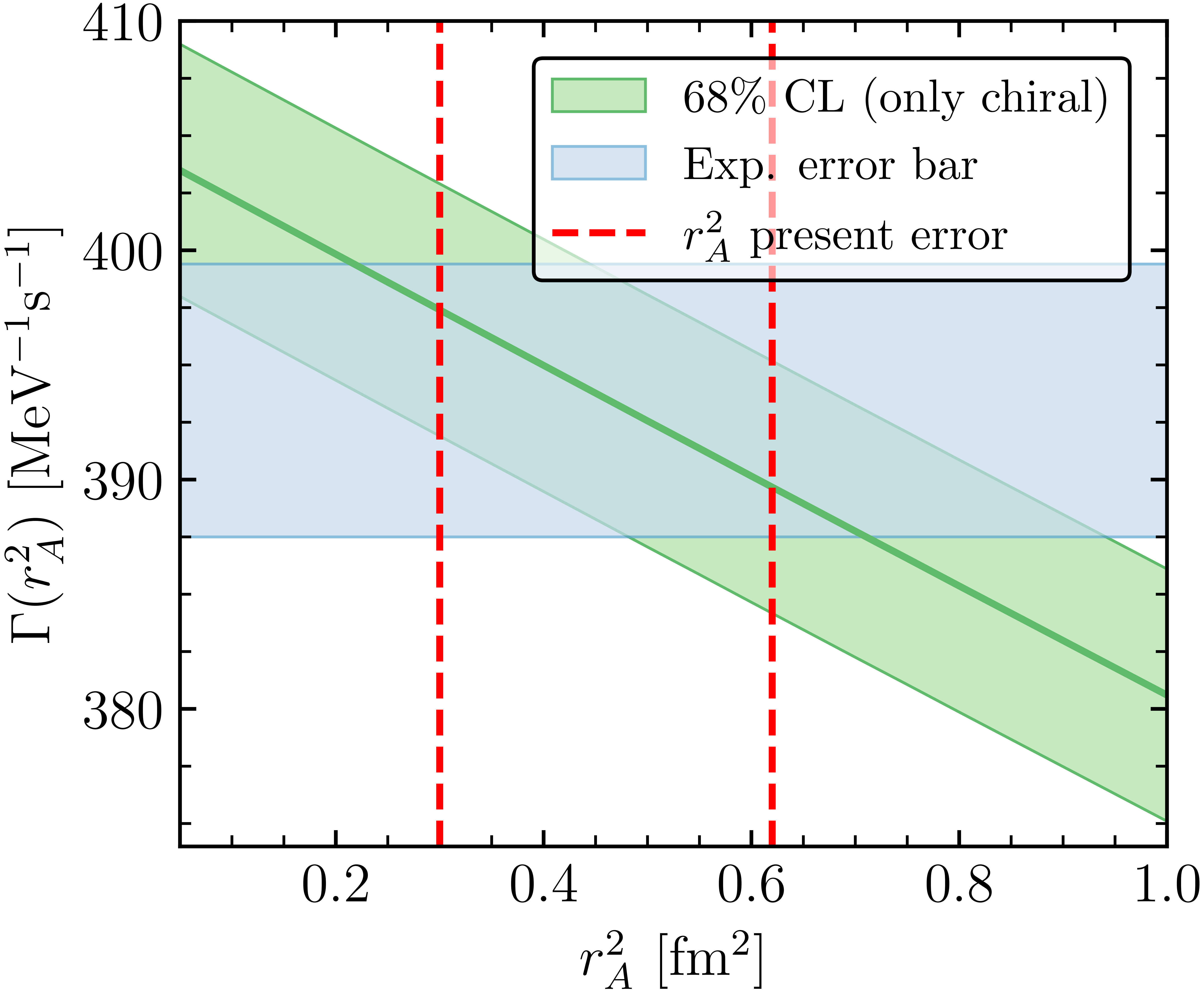
NVIa interaction only



# Extracting $r_A^2$

**Can we obtain a more precise value of $r_A^2$ with the present experimental and theoretical errors?**

NVIa interaction only



# Summary

## Many question marks about the analysis

- Which is the “correct” way to combine the chiral order of interaction and current?
- What is the power counting we should rely on?
- Which is the most reliable way to combine results from different models/cutoffs?

**Finding a reliable way to obtain the theoretical errors is crucial for extracting fundamental parameters from nuclear-electroweak processes**

# Acknowledgments


NTNP

DOE Topical Collaboration

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NERSC

National Energy Research Scientific Computing Center

Collaborators:

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- Michele Viviani (INFN Pisa)

# Sparse