Renormalizability Criteria in Nuclear Chiral EFT

A. M. Gasparyan, Ruhr-Universität Bochum

in collaboration with E. Epelbaum, N. Jacobi

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Outline

Motivation: explicit renormalization in chiral EFT Motivation: finite (of the order of the hard scale) cutoff scheme Renormalizability of the <u>NN amplitude at NLO</u>. Counterexamples Renormalizability in the nonperturbative regime. Counterexamples Summary

Renormalization in chiral EFT

 $\begin{array}{l} \text{Expansion parameter: (soft scale)/(hard scale)} \ Q = \frac{q}{\Lambda_b} \\ \\ q \in \left\{ \left| \vec{p} \right|, M_\pi \right\}, \qquad \Lambda_b \sim M_\rho \end{array} \end{array}$

"Perturbative" calculation of observables

Renormalization in chiral EFT

Expansion parameter: (soft scale)/(hard scale) $Q = \frac{q}{\Lambda_b}$ $q \in \{ |\vec{p}|, M_{\pi} \}, \qquad \Lambda_b \sim M_{\rho}$

"Perturbative" calculation of observables

$$\mathcal{L}_{eff} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \dots$$

Bare parameters of the Lagrangian

Renormalization: power counting for renormalized quantities

Implicit renormalization

 $T = T_0 fit bare C_i^{(0)} T = T_0 + T_2 (re) fit bare C_i^{(0)}, C_i^{(2)} T = T_0 + T_2 + T_4 (re) fit bare C_i^{(0)}, C_i^{(2)}, C_i^{(4)} C_i^{(4)}$

Balancing at the border of phenomenology

• • •

Implicit renormalization

 $T = T_0 fit bare C_i^{(0)}$ $T = T_0 + T_2 (re) fit bare C_i^{(0)}, C_i^{(2)}$ $T = T_0 + T_2 + T_4 (re) fit bare C_i^{(0)}, C_i^{(2)}, C_i^{(4)}$

Balancing at the border of phenomenology

• • •

Explicit renormalization

 $T = \mathbb{R}(T_0) + \mathbb{R}(T_2) + \mathbb{R}(T_4) + \dots$

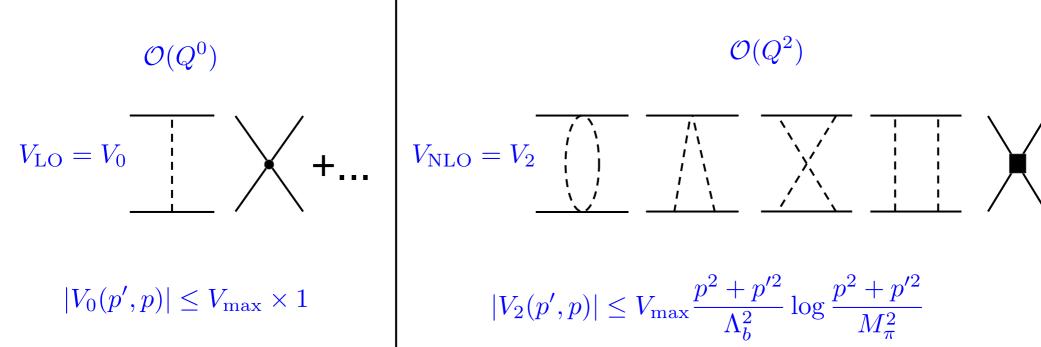
 $C_i = C_i^r + \delta C_i$ bare =renormalized + counter term (absorb divergent and power counting breaking contributions)

Identify each term individually, or at least prove this is possible

Justifies theoretical error estimation!

Power counting for NN chiral EFT: LO and NLO

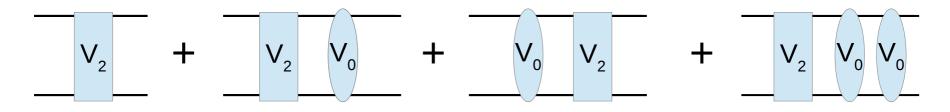
Weinberg, S., **NPB363**, 3 (1991)



Power counting for NN chiral EFT: LO and NLO Weinberg, S., NPB363, 3 (1991) $\mathcal{O}(Q^2)$ $\mathcal{O}(Q^0)$ +... $V_{\rm LO} = V_0$ $|V_2(p',p)| \le V_{\max} \frac{p^2 + p'^2}{\Lambda_t^2} \log \frac{p^2 + p'^2}{M_{\pi}^2}$ $|V_0(p',p)| \le V_{\max} \times 1$

LO potential has to be iterated (resummed):

 $T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \dots$



Motivation: Finite cutoff scheme

Rob Timmermans: If you cannot go with the cutoff above the breakdown scale, what, at all, are you doing?

Answer: The finite cutoff scheme is the most well grounded approach to date (with nonperturbative pions)

LO: $T_0^{[n]} = V_0 (GV_0)^n \sim \mathcal{O}(Q^0)$

We cannot absorb all positive powers of Λ . Absorb only those that are not compensated by the inverse powers of Λ_b

Expectation:
$$\Lambda \approx \Lambda_b : \int \frac{p^{n-1}dp}{(\Lambda_b)^n} \sim \left(\frac{\Lambda}{\Lambda_b}\right)^n \sim \left(\frac{\Lambda_b}{\Lambda_b}\right)^n \sim \mathcal{O}(Q^0)$$
 G. P. Lepage, nucl-th/9706029 J. Gegelia, JPG25, 1681 (1999)

NLO:
$$T_2^{[m,n]} = (V_0 G)^m V_2 (GV_0)^n \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting violating contributions from large loop momenta: $p \sim \Lambda, p' \sim \Lambda ext{ in } V_2(p',p)$

Expectation: power-counting breaking contributions can be absorbed by lower order contact (counter) terms

Variation of the cutoff

 $\delta T_{\Lambda} = (\mathbb{1} + T_0 G) \delta V_{\Lambda} (\mathbb{1} + G T_0)$

$$\Delta T_{\Lambda} = (\mathbb{1} + T_0 G) \Delta V_{\Lambda} (\mathbb{1} + G T_0) \approx |T_0|^2 m_N \Lambda \frac{\Delta \Lambda}{\Lambda}$$

Not small If T₀ is enhanced (S-waves): $T_0 \sim Q^{-1}$

After renormalization: adjusting counter terms

$$\triangle \mathbb{R}(T_{\Lambda}) = (\mathbb{1} + T_0 G)(\triangle V_{\rm CT} + \triangle V_{\Lambda})(\mathbb{1} + GT_0) \approx |T_0|^2 m_N \Lambda \frac{\triangle \Lambda}{\Lambda} \frac{p^2}{\Lambda^2}$$

Effective Lagrangian and the regulator

 $\mathcal{L} = \mathcal{L}_{LO} + \mathcal{L}_{>LO}$ -RG-invariance $\mathcal{L}_{LO} \Longrightarrow V_{LO} = V_{\Lambda}^{(0)} \longrightarrow \text{ regulated potential}$ $V_{>LO} = V - V_{LO} \quad \text{contains} \quad \delta_{\Lambda} V^{(0)} = V_{\Lambda}^{(0)} - V_{\Lambda=\infty}^{(0)}$

Effective Lagrangian and the regulator

 $\begin{aligned} \mathcal{L} &= \mathcal{L}_{\mathrm{LO}} + \mathcal{L}_{>\mathrm{LO}} \\ \mathcal{L}_{\mathrm{LO}} &\Longrightarrow V_{\mathrm{LO}} = V_{\Lambda}^{(0)} \longrightarrow \text{ regulated potential} \\ V_{>\mathrm{LO}} &= V - V_{\mathrm{LO}} \quad \text{contains} \quad \delta_{\Lambda} V^{(0)} = V_{\Lambda}^{(0)} - V_{\Lambda=\infty}^{(0)} \end{aligned}$

Lagrangian and amplitude are **formally** cutoff (regulator) independent

$$\frac{d\mathcal{L}}{d\Lambda} = 0 \qquad \longrightarrow \qquad \frac{dT}{d\Lambda} = 0$$

 Λ specifies the non-perturbative regime (the renormalization scheme)

Nonperturbative effects (bound states) cannot be generated this way

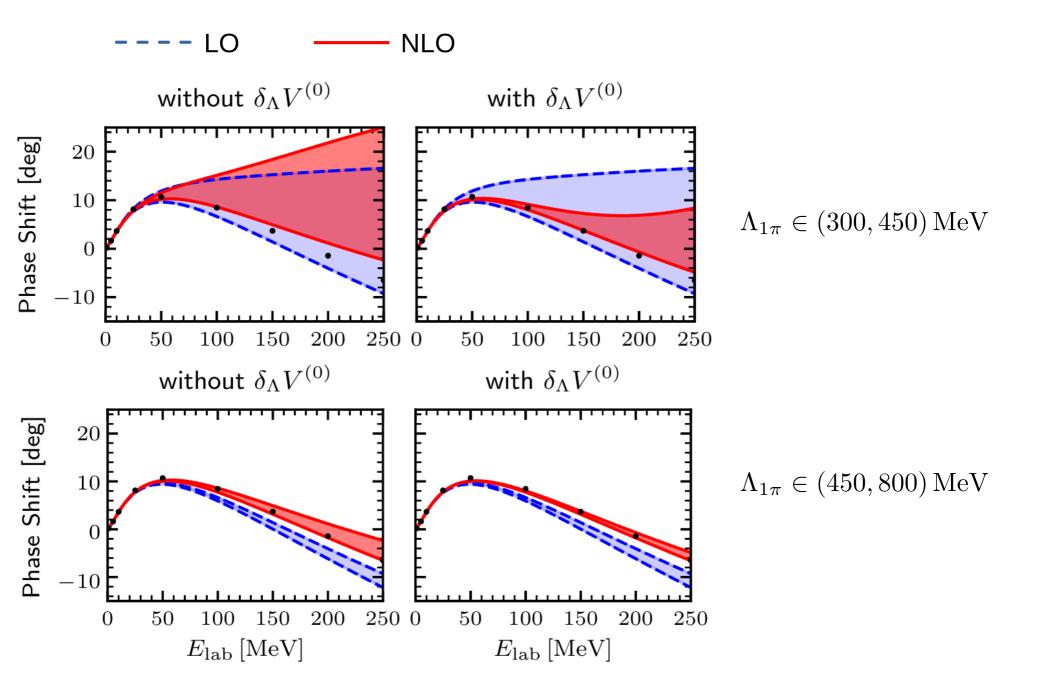
The remaining Λ -dependence is removed perturbatively by expansion δV_{Λ}

For locally regulated long-range potentials, $\delta_{\Lambda}V^{(0)}$ can be expanded in 1/ Λ and absorbed by contact interactions,

or can be kept explicit to access lower values of the cutoff

Explicit inclusion of the regulator corrections $\delta V_{\Lambda}^{(0)}$, ${}^{3}P_{0}$

AG, E.Epelbaum, **PRC107**, 044002 (2023)



Misconceptions: RG-invariance in Perturbative QFT

S-matrix is renormalization scale μ independent (only formally and up to higher order)

Perturbation theory converges equally well for all μ :

QED: Landau pole

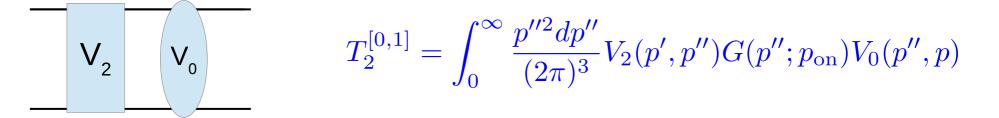
QCD: effective coupling constant. Given a scheme, our procedure automatically determines the couplingconstant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and

Brodsky, Lepage, Mackenzie, PRD28 (1), 228 (1983)

$$\begin{split} & \text{for } \Lambda_1, \Lambda_2 > \bar{\Lambda}_{\text{EFT}} : \\ & \frac{\mathcal{O}(k, p_{\text{typ}}; \Lambda_1) - \mathcal{O}(k, p_{\text{typ}}; \Lambda_2)}{\mathcal{O}(k, p_{\text{typ}}; \Lambda_1)} = \left(\frac{k, p_{\text{typ}}}{\bar{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \dots \end{split} \\ & \text{H. Grießhammer, EPJA 56 (4), 118 (2020)} \\ & \frac{1}{\mathcal{O}(k, p_{\text{typ}}; \Lambda_1)} \\ & \frac{1}{\mathcal{O}(k, p_{\text{typ}}; \Lambda_1)} = \mathcal{O}\left(\frac{Q^{\mathcal{V}+1}}{\bar{\Lambda}_{\text{hi}}^{\mathcal{V}} \bar{\Lambda}}\right) \\ & \text{H. W. Hammer, S. König, U. van Kolck, Rev. Mod. Phys. 92(2), 025004 (2020)} \end{split}$$

None of these criteria can be fulfilled (exceptional cutoffs), but this is not necessary

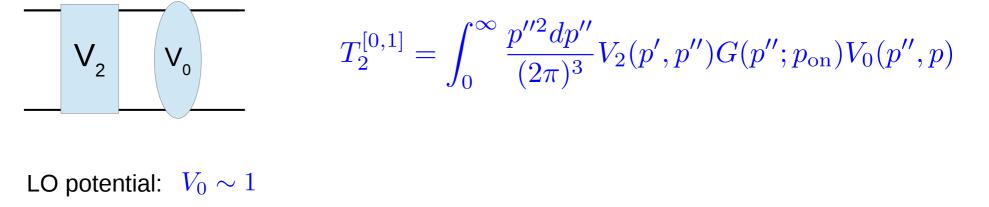
Technicalities of renormalization: estimating integrals using bounds on potentials



LO potential: $V_0 \sim 1$

NLO potential: $V_2 \sim \frac{p^2}{\Lambda_b^2} \log \frac{p^2}{M_\pi^2}$ 2-nucleon Green's function: $G \sim \frac{1}{p^2}$ Integral converges at $p \sim \Lambda$ (regulator) $\longrightarrow T_2^{[0,1]} \sim \frac{\Lambda^3}{\Lambda_b^3} \log \frac{\Lambda}{M_\pi} \neq \mathcal{O}(Q^2)$

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Renormalization \rightarrow Subtraction \rightarrow Counter term δC_0

Structure of the interaction in chiral EFT

Interaction obtained from chiral EFT: $V(\vec{p}', \vec{p}) = V_{\text{short}}(\vec{p}', \vec{p}) + V_{\text{long}}(\vec{p}', \vec{p})$

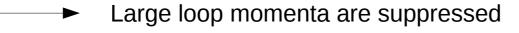
 $V_{\text{short}}(\vec{p}', \vec{p}) = \text{Polynomial}(\vec{p}', \vec{p}) F_{\Lambda}(\vec{p}', \vec{p})$

 $V_{\text{long}}(\vec{p}', \vec{p}) = V_L(\vec{q} = \vec{p}' - \vec{p}) \tilde{F}_{\Lambda}(\vec{p}', \vec{p}), \qquad V_L = V_{1\pi} + V_{2\pi} + \dots$

Subtractions:
$$|V(p',p) - V(p',0)| \leq \left|\frac{p}{p'}\right| \times (\dots) \text{ if } |p'| > |p|$$

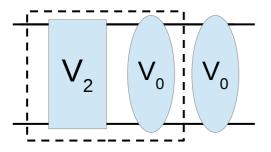
 $\left|V(p',p) - \sum_{i=0}^{n} \frac{\partial^{i} V(p',p)}{i!(\partial p)^{i}}\right|_{p=0} p^{i} \leq \left|\frac{p}{p'}\right|^{n+1} \times (\dots) \text{ if } |p'| > |p|$
AG E Epelbaum PR

AG, E.Epelbaum, **PRC 105**, 024001 (2022)



Renormalizability

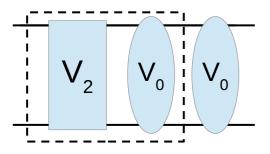
More iterations of V_0



LO potential: $V_0 \sim 1$

Renormalized NLO amplitude: $\mathbb{R}\left(T_2^{[0,1]}\right) \sim \frac{p^2}{\Lambda_b^2} \frac{\Lambda}{\Lambda_b} \log \frac{\Lambda}{M_{\pi}}$ 2-nucleon Green's function: $G \sim \frac{1}{p^2}$ Integral converges at $p \sim \Lambda$ \longrightarrow $T_2^{[0,2]} \sim \frac{\Lambda^4}{\Lambda_b^4} \log \frac{\Lambda}{M_{\pi}} \neq \mathcal{O}(Q^2)$

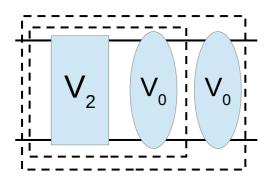
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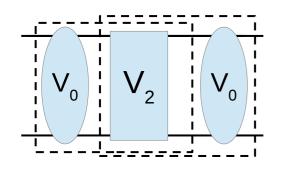
One more subtraction



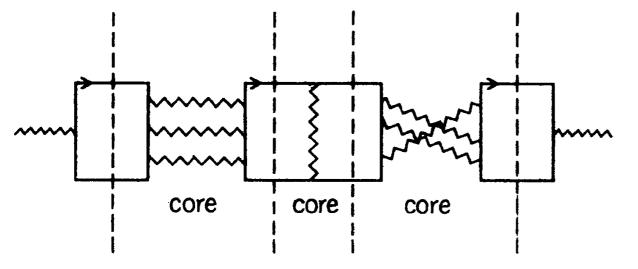
The same form of a counter term δC_0

$$\mathbb{R}\left(T_2^{[0,2]}\right) \sim \frac{p^2}{\Lambda_b^2} \frac{\Lambda^2}{\Lambda_b^2} \log \frac{\Lambda}{M_{\pi}}$$

Overlapping diagrams



Early history of QFT. Using tricks: Ward identities, derivatives w.r.t external momenta



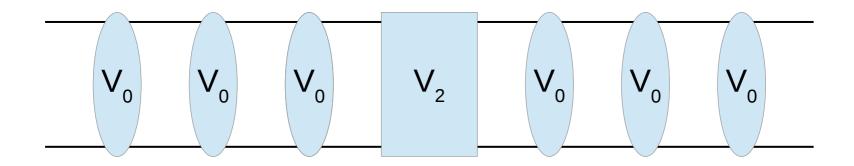
Mills, Yang, Prog.Theor.Phys.Suppl. 37 (1966)

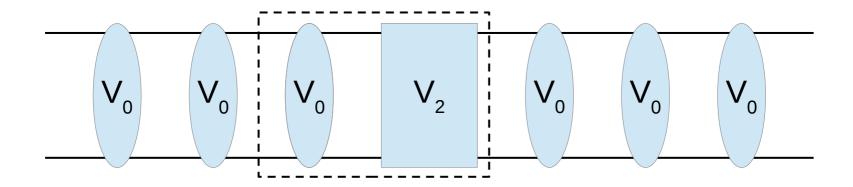
For this 16-th order diagram tricks do not work

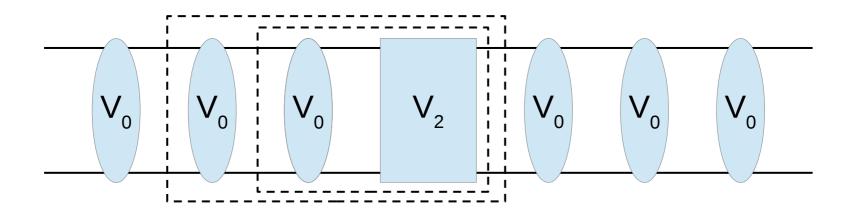
General method. BPHZ scheme:

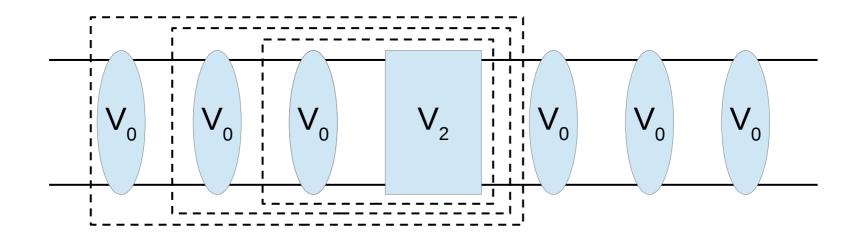
subtractions in all possible nested sets of diagrams (forests)

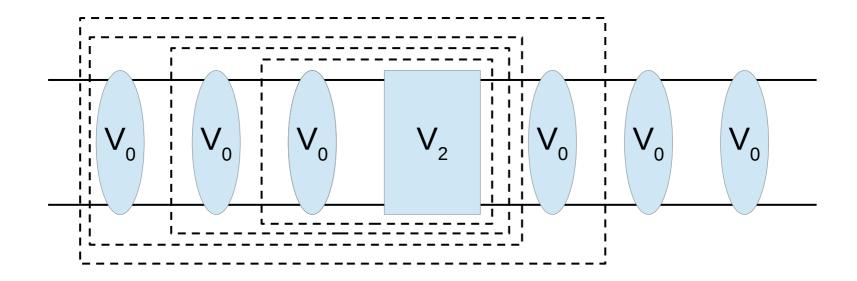
N. N. Bogoliubov, O. S. Parasiuk, AM97, 227 (1957); K. Hepp, CMP2, 301 (1966); W. Zimmermann, CMP15, 208 (1969)

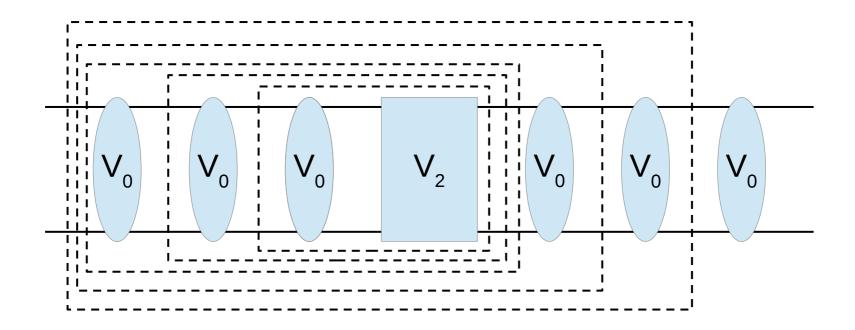


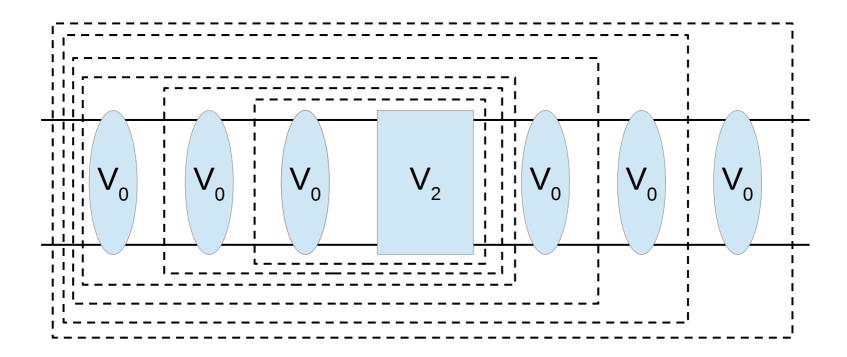


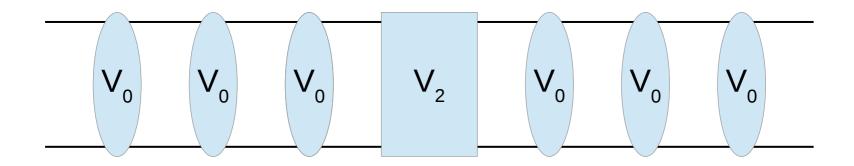


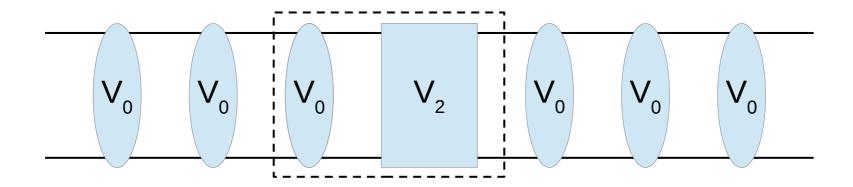


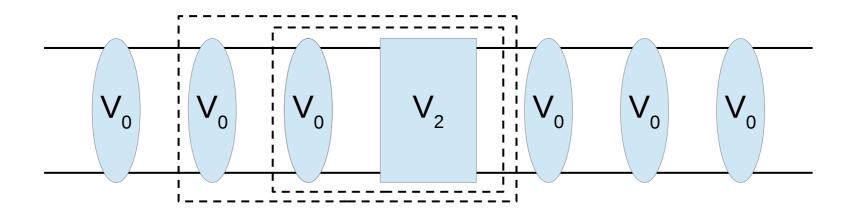


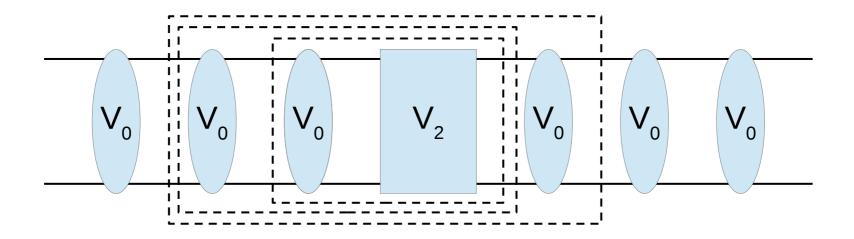


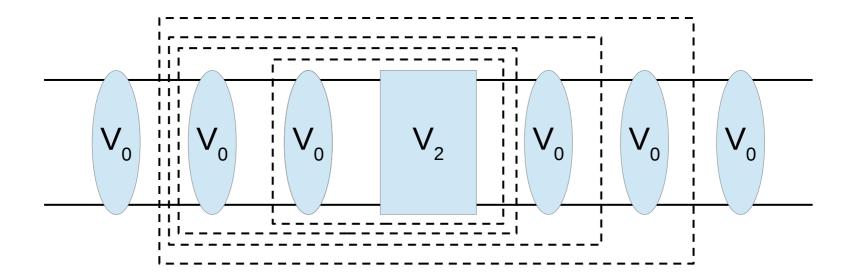


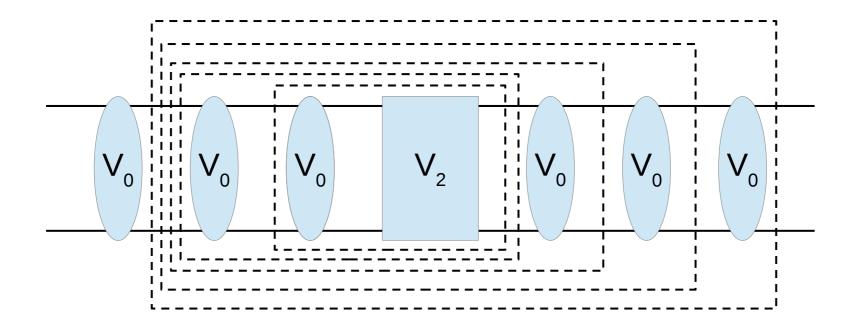


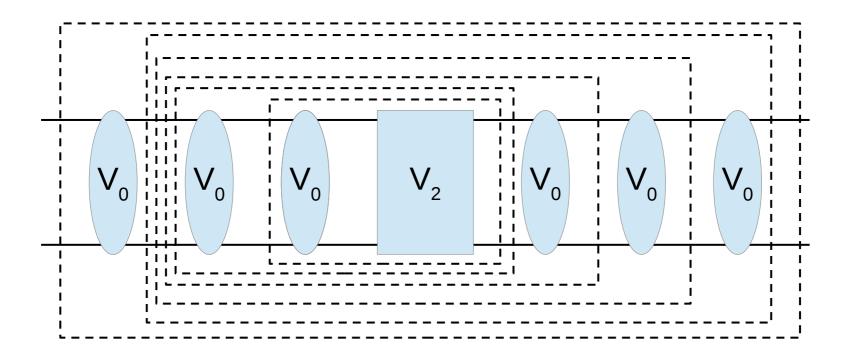


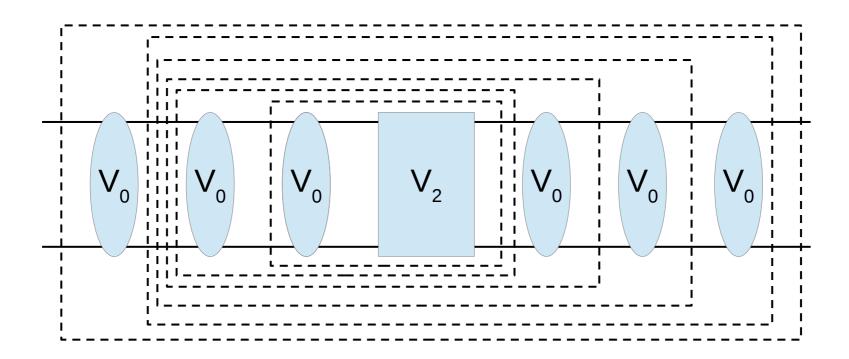












Partition integrations over p_i into sectors to avoid double counting All counter terms add up to a single counter term δC_0

Power counting in the finite-cutoff scheme, NLO

AG, E.Epelbaum, PRC 105, 024001 (2022)

Renormalized amplitude:

 ∞ $\mathbb{R}(T_2) = \sum_{n=1}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)$ -Perturbative (convergent) sum m,n=0

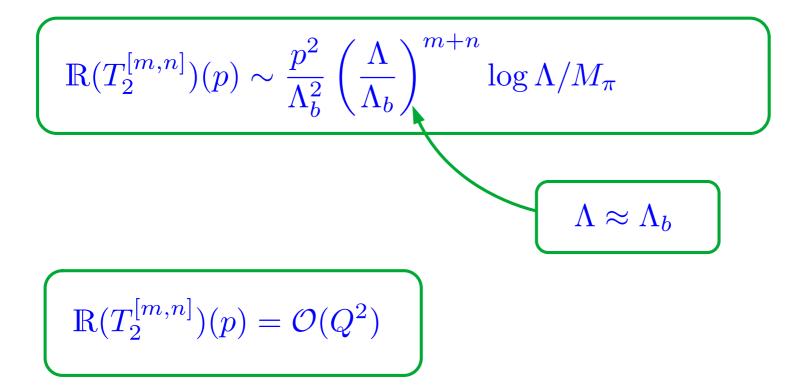
$$\mathbb{R}(T_2^{[m,n]})(p) \sim \frac{p^2}{\Lambda_b^2} \left(\frac{\Lambda}{\Lambda_b}\right)^{m+n} \log \Lambda / M_{\pi}$$

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AG, E.Epelbaum, **PRC 105**, 024001 (2022)

Renormalized amplitude:

 $\mathbb{R}\left(T_{2}
ight)=\sum \mathbb{R}\left(T_{2}^{[m,n]}
ight)$ -Perturbative (convergent) sum m,n=0



Non-local separable long-range interaction

AG, E.Epelbaum, N.Jacobi, in preparation

$$V_{0} = C_{0}F_{\Lambda}(p')F_{\Lambda}(p) + \dots,$$

$$V_{2} = C_{2}\frac{p'^{2} + p^{2}}{\Lambda_{b}^{2}}\frac{p'^{2}p^{2}}{(M_{\pi}^{2} + p'^{2})(M_{\pi}^{2} + p^{2})}F_{\Lambda}(p')F_{\Lambda}(p).$$
 two-pion exchange

$$F_{\Lambda}(p) = rac{\Lambda^2}{(\Lambda^2 + p^2)}$$

$$V_0 G V_2 \sim \frac{\Lambda^2}{\Lambda_b^2} \frac{p^2}{(M_\pi^2 + p^2)} \sim O(Q^0)$$

$$\int dp', \qquad p' \sim \Lambda$$

$$|V(p',p) - V(p',0)| \ge \left|\frac{p}{p'}\right| \times (\dots) \text{ if } |p'| > |p|$$

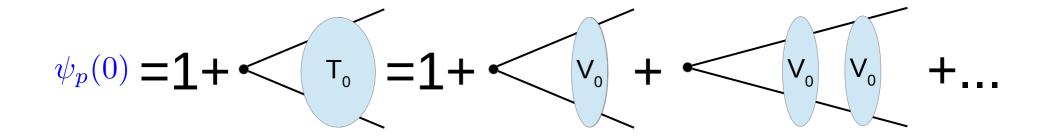
- Long-range power-counting-breaking terms
 - Nonrenormalizability (in terms of local counter terms)

Renormalization in the non-perturbative regime

AG, E.Epelbaum, PRC107, 044002 (2023)

The series for $R(T_2^{[m,n]})$ can be summed explicitly:

$$\mathbb{R}(T_2)(p) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)(p) = T_2(p) + \delta C_0 \psi_p(0)^2, \qquad \delta C_0 = -\frac{T_2(0)}{\psi_0(0)^2}$$



Using Fredholm formula to match to the perturbative regime

$$T_2(p) = (1 + T_0 G)V_2(1 + GT_0) = \frac{N_2(p)}{D(p)^2}$$

D(p)-Fredholm determinant

Convergent series in V₀:
$$N_2 = \sum_{i=0}^{\infty} N_2^{[i]}$$
, $D = \sum_{i=0}^{\infty} D^{[i]}$

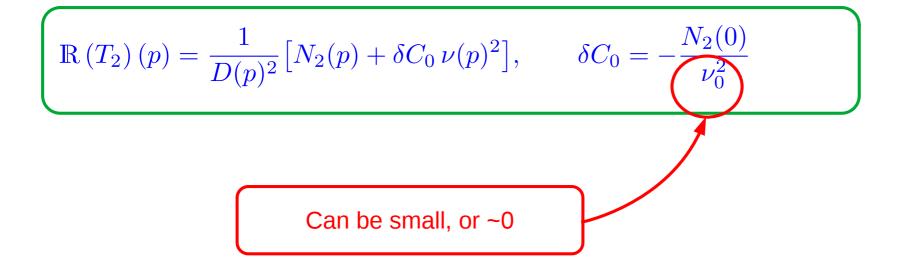
The same for the counter terms:

$$\delta T_2 = (1 + T_0 G) \delta V_0^{ct} (1 + G T_0) = \delta C_0 [\psi_p(0)]^2$$
$$\psi_p(0) = \frac{\nu(p)}{D(p)} \qquad \nu(p) = \sum_{i=0}^{\infty} \nu^{[i]}(p)$$

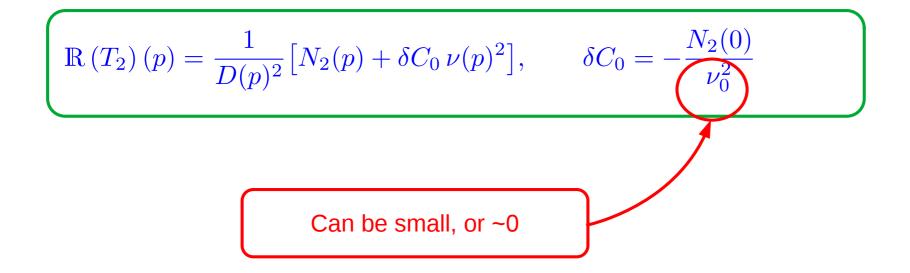
Renormalizability constraints

$$\mathbb{R}(T_2)(p) = \frac{1}{D(p)^2} \left[N_2(p) + \delta C_0 \nu(p)^2 \right], \qquad \delta C_0 = -\frac{N_2(0)}{\nu_0^2}$$

Renormalizability constraints



Renormalizability constraints



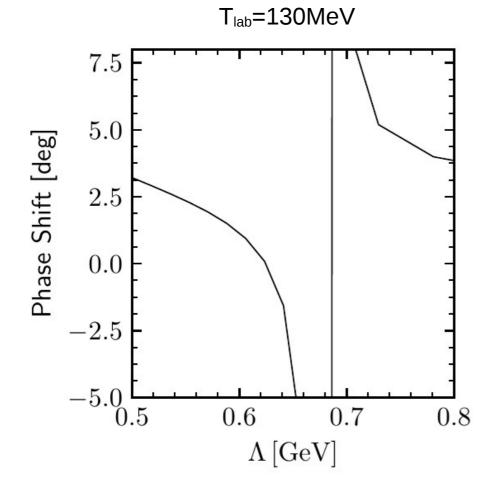
Renormalizability constraints on (the short-range part of) the LO potential. The simplest formulation: LECs must be of natural size (If $\Lambda \sim \Lambda_b$).

Constraints on the choice of the cutoff are not driven by data!

For realistic interactions works well for Λ <650-750 MeV

Failure of renormalizability for $\Lambda > \Lambda_b$,³ P_0

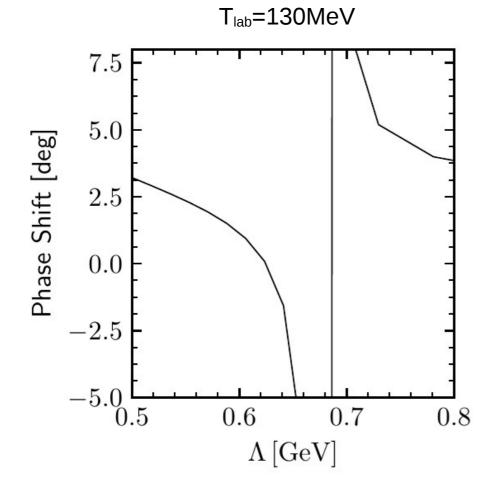
AG, E.Epelbaum, **PRC107**, 034001 (2023)



Sharp cutoff, harder than smooth regulators 0.7 GeV \rightarrow above 1 GeV

Failure of renormalizability for $\Lambda > \Lambda_b$,³ P_0

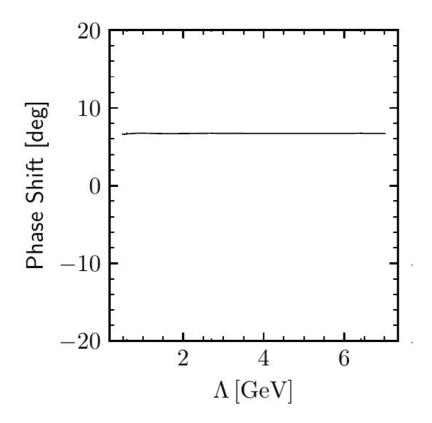
AG, E.Epelbaum, **PRC107**, 034001 (2023)



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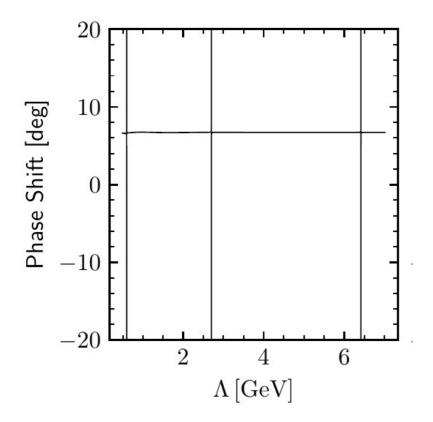
Adding one more counter term at NLO to reduce cutoff dependence: "RG-invariance"

B. Long, C. J. Yang, **PRC84**, 057001 (2011) AG, E.Epelbaum, **PRC107**, 034001 (2023)



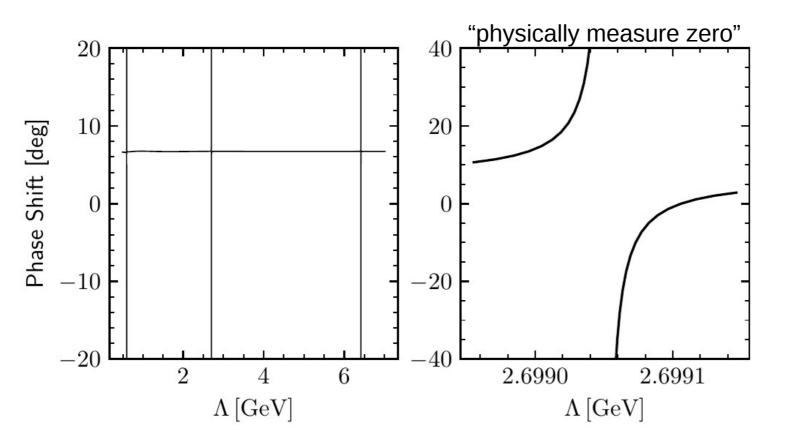
B. Long, C. J. Yang, **PRC84**, 057001 (2011)

AG, E.Epelbaum, **PRC107**, 034001 (2023)



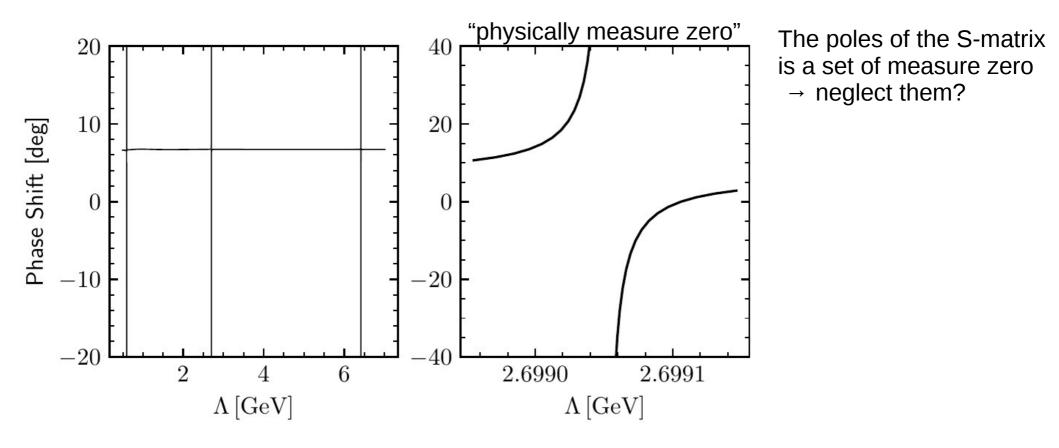
B. Long, C. J. Yang, **PRC84**, 057001 (2011)

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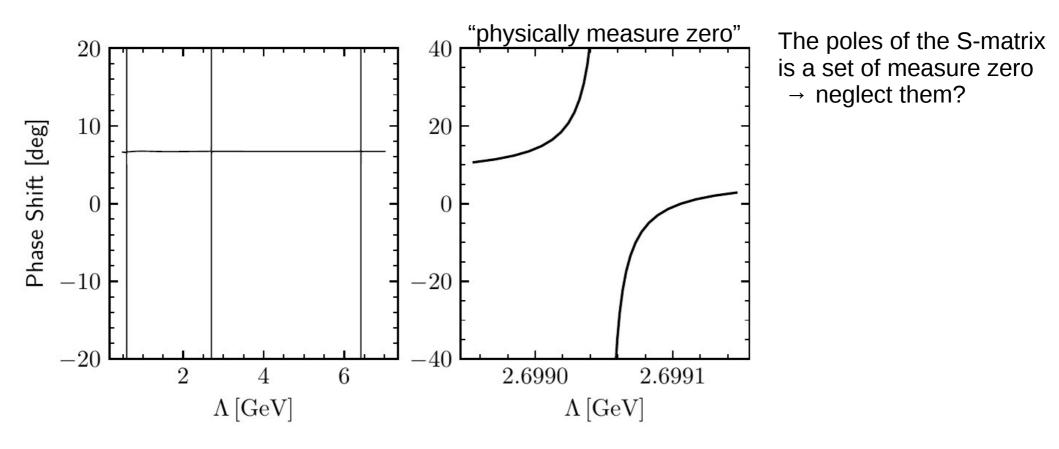
AG, E.Epelbaum, **PRC107**, 034001 (2023)



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AG, E.Epelbaum, **PRC107**, 034001 (2023)

R. Peng, B. Long, F. Xu, 2407.08342 (2024)



Relying on numerical simulations without a deeper understanding of the physics might be dangerous

(In)Consistency of Weinberg power counting and its modifications

Large cutoff arguments are irrelevant in the finite cutoff scheme: Divergencies \rightarrow positive (uncompensated) power powers of Λ

 $^{1}S_{0}: M_{\pi}^{2} \log M_{\pi} / \Lambda \sim O(Q^{2})$ for $\Lambda \sim \Lambda_{b}$

Mismatch of ultraviolet divergencies and infrared power counting is typicall: covariant ChPT in the 1-nucleon sector, especially Δ -full \rightarrow scheme dependence as a higher order effect

Consistent in the EFT sense: systematic expansion preserving symmetries

 $^{3}P_{0}$: cutoff variation is a higher order effect, it is phenomenologically enhanced (reduce g_A by a factor of 2)

The scheme is inefficient \rightarrow promote some contact terms: ${}^{1}S_{0}$, ${}^{3}P_{0}$ Analogy in perturbative EFT: promotion of the Q⁵ contact term in yN \rightarrow yN ChPT

H. Griesshammer, J. McGovern, D. Phillips, Eur.Phys.J. A 52 (2016) 5, 139

Going to larger cutoff values: "RG-invariance"

H. W. Hammer, S. König, U. van Kolck, **Rev. Mod. Phys. 92(2)**, 025004 (2020)

However, while in analytical calculations Eq. (...) can be verified explicitly, in numerical calculations varying the regulator parameter widely above the breakdown scale is usually the only tool available to check RG invariance.

After renormalization, when the contribution from momenta of the order of the large cutoff have been removed, the dominant terms in loop integrals come from momenta of O(Q).

What about momenta of order Λ_b ?

Large cutoffs do not solve any problems of finite cutoffs

Price: Promoting many (∞) counter terms to make the amplitude insensitive (independent) of the cutoff Relying on purely (potentially dangerous) numerical analysis

Summary

Explicit renormalization of an EFT provides a justified systematic expansion of observables and theoretical error estimate

Sufficient conditions for renormalizability:

- (1) Locality of the long-range forces
- (2) Cutoff of the order of the hard scale $\Lambda \approx \Lambda_b$
- (3) Naturalness of the counter terms

(1)+(2) in most cases imply (3): (1)+(2) \rightarrow (3)

Questions

Should we insist on cutoff insensitivity of a scheme for ∧>∧b by promoting many (∞) counter terms and relying on semi-phenomenological (potentially dangerous) numerical analysis or

> should we stick to the practically more affordable and better justified fundamentally finite cutoff scheme ?