Nucleon-nucleon potentials in comparison: physics or polemics?

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Motivations

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PHYSICS REPORTS

Nucleon–nucleon potentials in comparison: Physics or polemics?†

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Abstract

Guided by history, we review the major developments concerning realistic nucleon–nucleon (NN) potentials since the pioneering work by Kuo and Brown on the effective nuclear interaction. Our main emphasis is on the physics underlying various models for the NN interaction developed over the past quarter-century. We comment briefly on how to test the quantitative nature of nuclear potentials properly. A correct calculation (performed by independent researchers) of the χ^2/datum for the fit of the world NN data yields 5.1, 3.7, and 1.9 for the Nijmegen, Paris, and Bonn potential, respectively. Finally, we also discuss in detail the relevance of the on- and off-shell properties of NN potentials for microscopic nuclear structure calculations.

Motivations

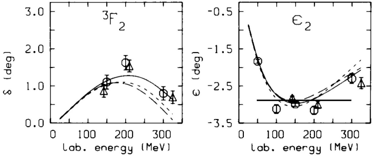

Fig. 11. (continued)

3. Polemics

Recently, there has been some debate about the 'quality' of different NN potentials. In particular, the χ^2 of the fit of the experimental NN data by a potential has sometimes become an issue. Unfortunately, this debate has not always been conducted in a strictly scientific manner. Therefore, we like to take this opportunity for a few comments, in the hope that this may help to lead the discussion back to more scientific grounds.

(1) *The χ^2 is not a magic number.*

Its relevance with regard to the 'quality' of a potential is limited. Consider, for example, a model based on little theory, but with many parameters; this model will easily fit the data well and produce a very low χ^2 (e.g., χ^2/datum $\approx$ 1). But we will not learn much basic physics from this. On

Motivations

The question now is: how do these large on-shell differences affect nuclear structure results?

To answer this question, it is best to consider a nuclear structure quantity for which an exact calculation can be performed. We choose the binding energy of the triton. Rigorous Faddeev calculations, which solve the three-body problem exactly, are feasible and have actually been performed for the two Nijmegen potentials under discussion. The results are summarized in Table 3: we see that the old Nijmegen potential [13] with a χ^2/datum of 6.5 predicts 7.63 MeV [34] for the triton binding, while the new potential with a χ^2/datum of 1.0 yields 7.62 MeV [35]. Thus, in spite of the seemingly very large differences on-shell in terms of the χ^2, the difference in the nuclear structure quantity under consideration is negligibly small.

On the other hand, consider two potentials that have an almost identical χ^2 for the world np data (implying that they are essentially identical on-shell). Accidentally, this is true for the Paris [14] and the Bonn *B* [6] potential, which both have a χ^2/datum of about 2 (cf. Table 3). The triton binding energy predictions derived from these two potentials are 7.46 MeV for the Paris potential and 8.13 MeV for Bonn *B*. This appears like a contradiction to the previous results. With the χ^2 so close, naively, one would have expected identical triton binding energy predictions. Obviously there is another factor, even more important than the χ^2: this factor is the off-shell behavior of a potential (particularly, the off-shell tensor force strength, that can vary substantially for different realistic potentials). A simple measure for this strength is the D-state probability of the deuteron, P_D, with a smaller P_D implying a weaker (off-shell) tensor force. For this reason, we are also giving in Table 3 the P_D for each of the potentials under consideration. The real reason for the differences in the predictions by Paris and Bonn *B* is the difference in the P_D with Paris predicting 5.8% and Bonn *B* 5.0% (cf. Table 3).

Summarizing: on-shell differences between potentials are seen in differences in the χ^2 for the fit of the NN data; off-shell differences are seen in differences in P_D. As Table 3 reveals, off-shell differences are much more important than on-shell differences for the triton binding energy.[6]

A similar consideration can be done for nuclear matter. This is summarized in Table 4. In this example, we are more specific as far as the χ^2 is concerned. We have choosen a particular set of NN

Table 3
Correlations between two-nucleon and three-nucleon properties as predicted by different NN potentials. The χ^2/datum is related to the on-shell properties of a potential, while P_D depends essentially on the off-shell behavior

	Nijmegen [13]	Nijm.–Reid [33]	Bonn B [6]	Paris [14]
Two-nucleon data:				
χ/datum[a]	3.8	1.0	2.1	2.0
P_D (%)[b]	5.4	5.6	5.0	5.8
Triton binding (MeV):	7.63	7.62	8.13	7.46

[a] for the fit of the world np data (without σ_{tot}) in the range 10–300 MeV (cf. Table 1).
[b] D-state probability of the deuteron.

Motivations

30 R. Machleidt, G.Q. Li / Physics Reports 242 (1994) 5–35

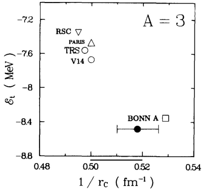

Fig. 13. Triton energy, $\mathscr{E}_t$, versus the invers charge radius of ^{3}He, $1/r_c$, as predicted by various NN potentials. The experimental value is given by the horizontal error bar.

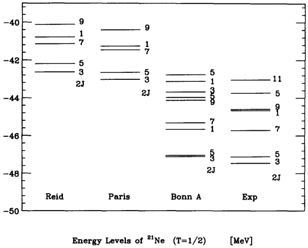

Fig. 14. The spectrum of ^{21}Ne. Predictions by NN potentials are compared with experiment. (From Ref. [37].)

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Motivations

R. Machleidt, G.Q. Li / Physics Reports 242 (1994) 5–35 31

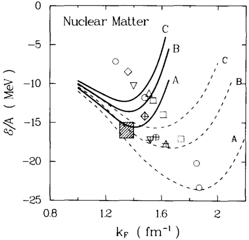

Fig. 15. Energy per nucleon in nuclear matter, $\mathcal{E}/A$, versus density expressed in terms of the Fermi momentum k_F. Dashed lines represent results from non-relativistic Brueckner calculations, while solid lines are Dirac-Brueckner results. The letters *A*, *B*, and *C* refer to the Bonn *A*, *B*, and *C* potential, respectively. The shaded square covers empirical information on nuclear saturation. Symbols in the background denote saturation points obtained for a variety of NN potentials applied in conventional many-body theory. (From Ref. [38].)

Table 5. D-state probability of the deuteron, P_D, as predicted by various potential applied in Figs. 13–15

Potential	P_D (%)
Bonn *A* [6]	4.4
Bonn *B* [6]	5.0
Bonn *C* [6]	5.6
Paris [14]	5.8
TRS [7]	5.9
Argonne V_{14} [8]	6.1
Reid (RSC) [5]	6.5

Food for thought

- Evaluating the relevance of a consistent treatment of meson theory: connection with the results for the two-nucleon system
- Discussing the need of getting a low χ^2/datum
- Addressing the role of the off-shell component of the *NN* potential with respect to the on-shell matrix elements

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Inspiration

The relative weight of on-shell and off-shell in recostructing the structure and observed properties of nuclear systems is quite intriguing
As Ruprecht pointed out:

On the other hand, consider two potentials that have an almost identical χ^2 for the world np data (implying that they are essentially identical on-shell). Accidentally, this is true for the Paris [14] and the Bonn B [6] potential, which both have a χ^2/datum of about 2 (cf. Table 3). The triton binding energy predictions derived from these two potentials are 7.46 MeV for the Paris potential and 8.13 MeV for Bonn B. This appears like a contradiction to the previous results. With the χ^2 so close, naively, one would have expected identical triton binding energy predictions. Obviously there is another factor, even more important than the χ^2: this factor is the off-shell behavior of a potential (particularly, the off-shell tensor force strength, that can vary substantially for different realistic potentials). A simple measure for this strength is the D-state probability of the deuteron, P_D, with a smaller P_D implying a weaker (off-shell) tensor force. For this reason, we are also giving in Table 3 the P_D for each of the potentials under consideration. The real reason for the differences in the predictions by Paris and Bonn B is the difference in the P_D with Paris predicting 5.8% and Bonn B 5.0% (cf. Table 3).

Summarizing: on-shell differences between potentials are seen in differences in the χ^2 for the fit of the NN data; off-shell differences are seen in differences in P_D. As Table 3 reveals, off-shell differences are much more important than on-shell differences for the triton binding energy.[6]

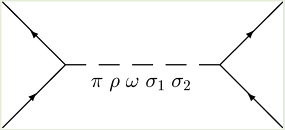
On-shell-equivalent *NN* potentials

- In order to investigate such a topic, we need a class of on-shell-equivalent *NN* potentials, characterize their off-shell behavior, and study their response in nuclear structure
- We have met such an issue by way of the $V_{\text{low-}k}$ approach: choose a realistic *NN* potential, then renormalize it introducing a set of different cutoffs Λ
- Then, the $V_{\text{low-}k}$s with different cutoffs are employed to calculate specific relevant properties of nuclear systems

The CD-Bonn *NN* potential

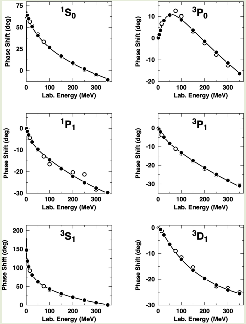
Our starting point is the CD-Bonn *NN* potential
The motivations to consider this potential are twofold:

The CD-Bonn potential is rooted in meson theory



R. Machleidt, Phys. Rev. C **63**, *024001 (2001)*

The CD-Bonn potential is a high-precision potential, with a χ^2/datum=1.01



The $V_{\mathrm{low}\text{-}k}$ approach

In 1997, Tom Kuo started to develop a method to perform a unitary transformation of the two-nucleon Hamiltonian, corresponding to a certain V_{NN}, into a new one defined up to a momentum cutoff Λ
In the full Hilbert space, the two-nucleon hamiltonian is written as:

$$\int_0^{\infty} [H_0(k,k') + V_{NN}(k,k')] \langle k|\Psi_\nu\rangle k^2 dk = E_\nu \langle k'|\Psi_\nu\rangle$$

In a reduced model space $P = \int_0^{\Lambda} |k\rangle\langle k| k^2 dk$, a new effective Hamiltonian is defined, which solves the equation

$$\int_0^{\Lambda} [H_0(k,k') + V_{\mathrm{low-k}}(k,k')] \langle k|\Phi_\mu\rangle k^2 dk = \tilde{E}_\mu \langle k'|\Phi_\mu\rangle \ ,$$

with the fundamental constraint: $\tilde{E}_\mu \in \{E_\nu\}$

The effective Hamiltonian H_{eff} can be easily constructed by way of the Lee-Suzuki transformation (plus a hermitization procedure)

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The $V_{\text{low-}k}$ approach

The $V_{\text{low-}k}$ transformation provides a set of on-shell-equivalent potentials, for any choice of the cutoff Λ

Deuteron binding energy

Λ (in fm^{-1})	V_{eff}	V_{NN}
2.1	-2.225	-2.225
2.2	-2.225	
2.3	-2.225	
2.4	-2.225	
2.5	-2.225	
2.6	-2.225	

$^1S_0$ phase shifts

E_{lab} (MeV)	CD-Bonn	$\Lambda = 2.1$ fm^{-1}	Expt.
1	62.1	62.1	62.1
10	60.0	60.0	60.0
25	50.9	50.9	50.9
50	40.5	40.5	40.5
100	26.4	26.4	26.8
150	16.3	16.3	16.9
200	8.3	8.3	8.9
250	1.6	1.6	2.0
300	-4.3	-4.3	-4.5

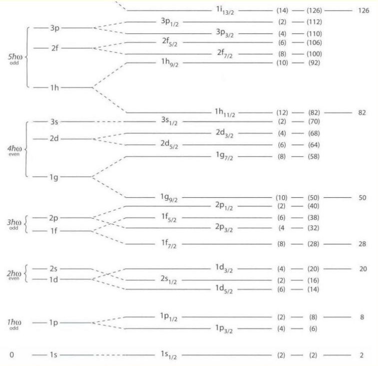
However, the $V_{\text{low-}k}$s are characterized by a different off-shell behavior

Deuteron D-state probability

Λ (in fm^{-1})	2.1	2.2	2.3	2.4	2.5	2.6	CD-Bonn
P_D	3.96	4.09	4.21	4.32	4.41	4.49	4.85

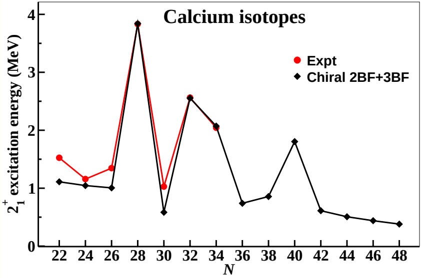
Nuclear structure with on-shell-equivalent $V_{\text{low-}k}$s

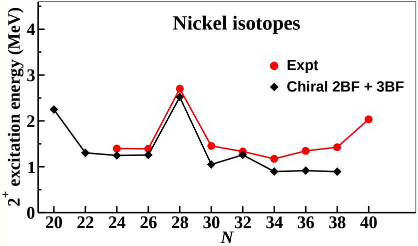
A relevant feature that *NN* potentials are expected to achieve is the reproduction of the observed shell structure in terms of single-body energy orbitals, that is the cornerstone of the nuclear shell model



Nuclear structure with on-shell-equivalent $V_{\text{low-}k}$s

In the nuclear shell model the energy spacings of the single-particle orbitals drive the shell evolution, being the most relevant contribution to the monopole component of the shell-model Hamiltonian

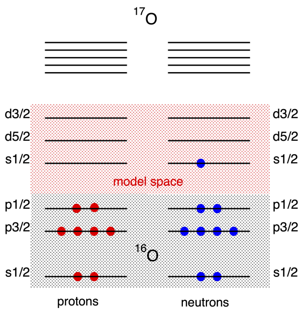




Then, we have investigated the ability of on-shell-equivalent $V_{\text{low-}k}$s to reproduce experimental single-particle energy spacings

Shell model with a core

Our framework is the “shell model with a core”, namely the nucleons occupying the filled shells are considered frozen, and we consider as degrees of freedom of the shell-model Hamiltonian only those of the nucleons acting in the model space placed above the doubly-closed core



Then, we need to construct an effective Hamiltonian which accounts for the degrees of freedom of the full Hamiltonian that have not been included explicitly within the model space

The effective shell-model Hamiltonian

We start from the many-body Hamiltonian H defined in the full Hilbert space:

$$H = H_0 + H_1 = \sum_{i=1}^{A} (T_i + U_i) + \sum_{i<j} (V_{ij}^{NN} - U_i)$$

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \quad \begin{array}{c} \mathcal{H} = \Omega^{-1} H \Omega \\ \Longrightarrow \\ Q\mathcal{H}P = 0 \end{array} \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$

$$H_{\mathrm{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow \Omega = e^{\omega}$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$

$$H_1^{\mathrm{eff}}(\omega) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P - $$

$$- PH_1Q \frac{1}{\epsilon - QHQ} \omega H_1^{\mathrm{eff}}(\omega)$$

The perturbative approach to the shell-model H_{eff}

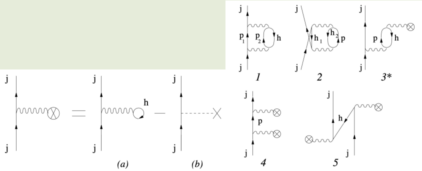
The $\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

Exact calculation of the $\hat{Q}$-box is computationally prohibitive for many-body system $\Rightarrow$ we perform a perturbative expansion

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

$\hat{Q}$-box: 1st- & 2nd-order 1-b diagrams



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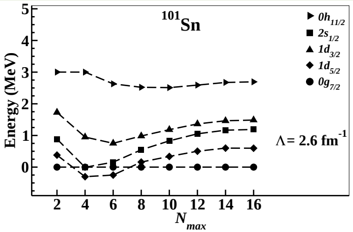
On-shell-equivalent *NN* potentials and s.p. energies

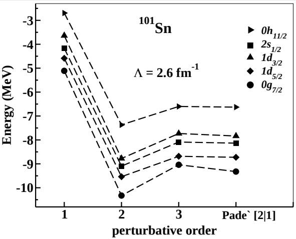
As case study, we have considered the calculation of the single-particle energy spacings of the $0g_{7/2}1d2s0h_{11/2}$ shell, namely those corresponding to ^{101}Sn and ^{133}Sb, outside doubly-closed ^{100,132}Sn cores

First, we have examined the perturbative behavior of the calculated single-particle energies, both with respect to the number of intermediate states and the perturbative order of the $\hat{Q}$ box expansion, considering the "hardest cutoff" $\Lambda = 2.6$ fm^{-1}

Intermediate-states convergence



Order-by-order convergence



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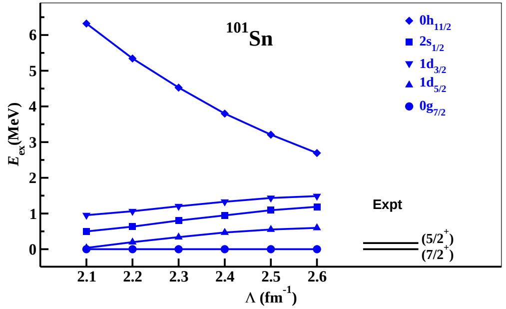
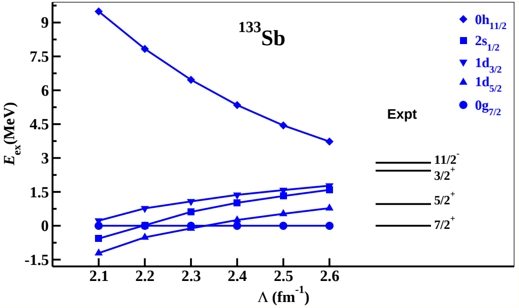
^{101}Sn single-particle energy spacings



- The spin-orbit splitting $1d_{5/2} - 1d_{3/2}$ is rather insensitive with respect to the cutoff ("softeness" of the *NN* potential)
- The relative position of the $0g_{7/2}, 0h_{11/2}$ orbitals (which lack of their spin-orbit counterpart) with respect to the other ones shows a cutoff dependence
- In general, the "harder" is the cutoff, the better are the results compared with the phenomenology

^{133}Sb single-particle energy spacings



- The spin-orbit splitting $1d_{5/2} - 1d_{3/2}$ is rather insensitive with respect to the cutoff ("softeness" of the *NN* potential)
- The relative position of the $0g_{7/2}, 0h_{11/2}$ orbitals (which lack of their spin-orbit counterpart) with respect to the other ones shows a cutoff dependence
- In general, the "harder" is the cutoff, the better are the results compared with the phenomenology

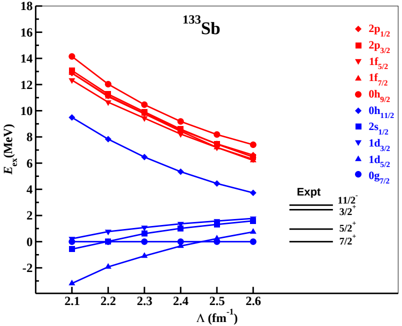
Questions

The calculated energy spectra of ^{101}Sn and ^{133}Sb as a function of the cutoff Λ, namely of the off-shell component of the *NN* potential, raise some considerations:

- The energy spacing of the spin-orbit-partner $1d$ orbitals is almost independent from the cutoff, and in ^{133}Sb in a good agreement with the experimental one
- The energy spacing of the $0g_{7/2}, 0h_{11/2}$ orbitals, whose spin-orbit partners are placed outside of the model space, is markedly dependent from the cutoff, and the comparison with experiment favors a "harder" off-shell component
- What is the component of the one-body potential that is mostly affected by the off-shell component, and that drives the different outcomes?

In order to look for an answer, we have derived the single-particle energy spacings in a larger model space, adding the orbitals belonging to the shell above

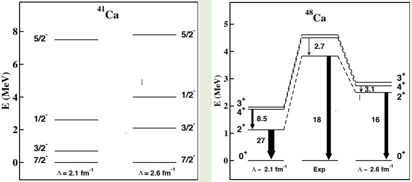
^{133}Sb single-particle energies in a larger model space



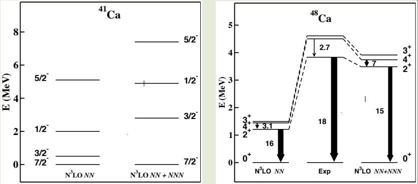
- The spin-orbit splitting is then rather independent with respect to the cutoff for all spin-orbit partners
- The $\mathbf{L^2}$ component of the one-body shell-model Hamiltonian is strongly affected by the off-shell behavior of the *NN* potential
- In general, the “harder” is the cutoff, the stronger is the lowering of the orbitals with larger orbital angular momenta l, then approaching the observed phenomenology

Shell evolution

$V_{\text{low-}k}$



ChPT



- The role of high-momentum components of meson-exchange *NN* potentials mirrors the one of the three-body force for nuclear potentials derived within ChPT
- Both of them are crucial to drive the observed shell evolution
- ChPT three-body force is contributes also to the spin-orbit splitting, the main role being played by the 2π-exchange term *(see T. Fukui et al., Phys. Lett. B* **855**, *138839 (2024))*

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Reflections and conclusions

- Ruprecht's observation was right: in many-nucleon systems the role of the off-shell component of the nucleon-nucleon force is quite relevant
- This component impacts largely on the monopole component of the shell-model Hamiltonian, namely on the single-particle energy spacings
- High-momentum components of the *NN* potential contribute to a correct reproduction of the observed shell evolution
- Starting from ChPT nuclear Hamiltonians, the role of the three-body force on the shell evolution reflects in the one of high-momentum components of meson-theoretic *NN* potentials

Questions

- If high-momentum components are important to reproduce shell evolution in many-nucleon systems, namely dealing with low-energy excited states, then are nuclear forces really framed in a low-energy regime?
- Consequently, in an EFT perspective, what is the break-down scale and what are the degrees of freedom to be considered to construct a ChPT Hamiltonian which may provide the observed nuclear structure but whose many-body components could be perturbatively manageable?

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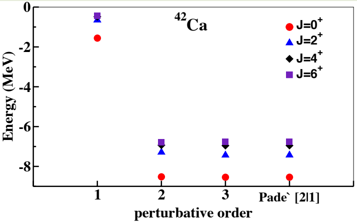
Wish you were here

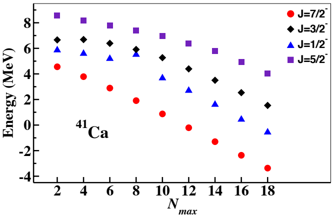
Backup slides

Perturbative properties

Order-by-order convergence



Intermediate-state convergence



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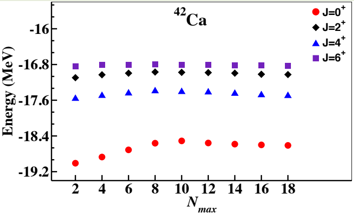
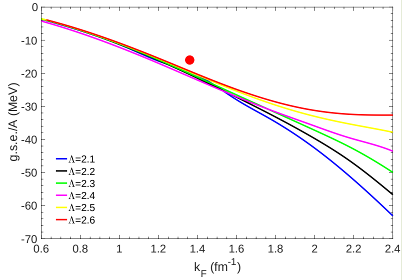


Y. Z. Ma, L. C., L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. R. Xu, Phys. Rev. C **100**, *034324 (2019)*

EOS of infinite nuclear matter

$V_{\text{low-}k}$



ChPT