Cluster EFT calculation of electromagnetic breakup reactions with the Lorentz Integral Transform method

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ECT* Workshop
"The nuclear interaction: post-modern developments"
August 22, 2024

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  Thesis:

  "Cluster EFT calculation of electromagnetic breakup reactions with the Lorentz Integral Transform method"

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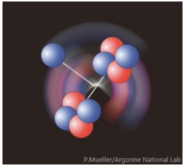
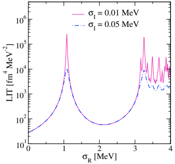
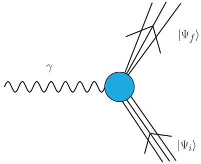
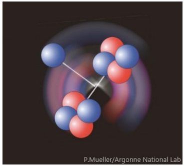






INTRODUCTION and OUTLINE

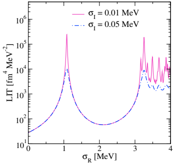
Cluster EFT calculation
of electromagnetic breakup reactions
with the Lorentz Integral Transform method



MODEL

- **Effective particles**
  - nucleons and α-particles
- **Interaction**
  - potential models from Effective Field Theory (EFT)

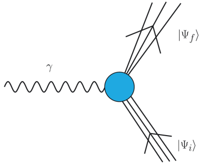
Cluster EFT calculation of electromagnetic breakup reactions with the **Lorentz Integral Transform method**



Choice of the **METHOD**

- **Bound-state problem**
  - variational method
  - Non-Symmetrized Hyperspherical Harmonics (NSHH) method
- **Continuum-states problem**
  - integral transform approach: Lorentz Integral Transform (LIT) method

Cluster Effective Field Theory calculation of **electromagnetic breakup reactions** with the Lorentz Integral Transform method



$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n$$

$$\gamma + {}^{12}\text{C} \rightarrow \alpha + \alpha + \alpha$$

- Study of the reactions of *astrophysical relevance* in a **three-body *ab initio*** approach and in the **low-energy regime**
  - three-body binding energies
  - cross sections
- Comparison of the results with the **experimental data**

Cluster Effective Field Theory (EFT) approach

[Hammer, *et al.* (2017), Hammer, *et al.* (2020)]

Borromean systems: ^{9}Be $\sim \alpha\alpha n$ $\quad S_3 \approx 1.573$ MeV $\ll S_p(^4\text{He}) \approx 19.81$ MeV

^{12}C $\sim \alpha\alpha\alpha$ $\quad S_3 \approx 7.275$ MeV $< S_p(^4\text{He}) \approx 19.81$ MeV

$$^6\text{He} \sim \alpha n n\,,\ ^{10}\text{Be} \sim \alpha\alpha n n\,,\ ^{16}\text{O} \sim \alpha\alpha\alpha\alpha\,,\dots$$

$\Rightarrow$ 3-body (or 4-body) *effective clustering* systems in the **low-energy regime**

Separation of energy scales $\rightarrow$ halo/cluster **EFT approach**

momentum scales: M_{low}, M_{high}

$\downarrow$

EFT expansion in $\left(\frac{M_{low}}{M_{high}}\right)^\nu$

$\downarrow$

error estimate

Two-body effective potentials

[Hammer, *et al.* (2017), Ji]

Effective potential defined in *momentum space* and in the partial wave ℓ:

$$\mathcal{V}_\ell(p,p') = \left[\lambda_0 + \lambda_1 (p^2 + p'^2)\right] p^\ell p'^\ell \, g(p;\Lambda) \, g(p';\Lambda)$$

- sum of **contact terms** parametrized by the LECs
- **momentum-regulator function** $g(p;\Lambda)$

$$g(p;\Lambda) = e^{-\left(\frac{p}{\Lambda}\right)^{2m}} \qquad m = 1,2$$

Two-body effective potentials

[Hammer, *et al.* (2017), Ji]

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$$g(p;\Lambda) = e^{-\left(\frac{p}{\Lambda}\right)^{2m}} \qquad m = 1, 2$$

✓ We calculate the $\mathcal{T}$-matrix by solving the Lippmann-Schwinger equation (on-shell: $\boldsymbol{p} = \boldsymbol{p}' = \boldsymbol{k}$)

$\Rightarrow$ α–n : $\mathcal{T}_\ell(k)$, α–α : $\mathcal{T}_\ell^{SC}(k)$ Coulomb-distorted strong term

✓ We compare the calculated low-energy $\mathcal{T}$-matrix with its ERE or Coulomb-modified ERE up to terms $O(k^2)$

$$\Rightarrow \lambda_i = \lambda_i(a_\ell, r_\ell, \Lambda)$$

✓ The LECs are fixed on the experimental values of the scattering length a_ℓ^{exp} and the effective range r_ℓ^{exp}

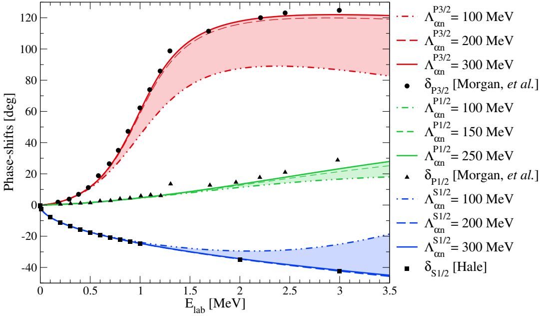
$$\lambda_i = \lambda_i(a_\ell^{\text{exp}}, r_\ell^{\text{exp}}, \Lambda)$$

The effective potentials $\mathcal{V}_\ell^{\alpha n}$ and $\mathcal{V}_\ell^{\alpha\alpha}$ reproduce the experimental low-energy α–n and α–α phase-shifts ➠

Two-body effective potentials

Calculated phase-shifts for different two-body cut-offs $\Lambda < \Lambda^{max}$ ($\Leftarrow$ Wigner bound)

$\alpha - n$

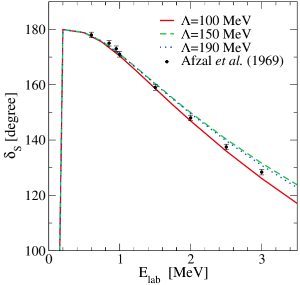


$P_{3/2}$ Resonance

$P_{1/2}$

$S_{1/2}$

$\alpha - \alpha$



S_0 Resonance

Power counting ➠

Two-body effective potentials

α–n

$$\frac{k^{2\ell}}{\mathcal{T}_\ell(k)} = \frac{\mu}{2\pi}\left[\frac{1}{a_\ell} - \frac{1}{2}r_\ell k^2 + O(k^4) + ik^{2\ell+1}\right]$$

$$M_{hi} = \sqrt{2\mu S_p(^4\mathrm{He})} \approx 170 \text{ MeV}$$

Power counting

α–α

$$\frac{1}{\mathcal{T}^{SC}_{\ell=0}(k)} \propto \frac{\mu}{2\pi}\left[\frac{1}{a_0} - \frac{r_0}{2}k^2 + \mathcal{O}(k^4) + 2k_C H(\eta)\right]$$

$$M_{hi} = \sqrt{2\mu S_p(^4\mathrm{He})} \approx 270 \text{ MeV}, \quad k_C = Z_\alpha^2 \alpha_{\mathrm{em}} \mu \approx 60 \text{ MeV}$$

Two-body effective potentials

Power counting

$\alpha - n$

$$\frac{k^{2\ell}}{\mathcal{T}_\ell(k)} = \frac{\mu}{2\pi}\left[\frac{1}{a_\ell} - \frac{1}{2} r_\ell k^2 + O(k^4) + i k^{2\ell+1}\right]$$

$$M_{hi} = \sqrt{2\mu S_p(^4\text{He})} \approx 170 \text{ MeV}$$

$P_{3/2}$: resonance $\Rightarrow$ **enhanced** partial wave

$$M_{lo} = \sqrt{2\mu E_R(^5\text{He})} \approx 30 \text{ MeV} \ll M_{hi}$$

$$\boxed{\frac{1}{a_1} \sim M_{lo}^2 M_{hi}\,,\quad r_1 \sim M_{hi} \quad \Rightarrow \quad a_1, r_1 \quad \textbf{LO}}$$

[Bedaque, *et al.* (2003)]

$a_1^{\text{exp}}, r_1^{\text{exp}} \Rightarrow M_{lo}/M_{hi} \approx 30 \text{ MeV}/170 \text{ MeV} \approx 0.2$

nonperturbative approach

$\alpha - \alpha$

$$\frac{1}{\mathcal{T}^{SC}_{\ell=0}(k)} \propto \frac{\mu}{2\pi}\left[\frac{1}{a_0} - \frac{r_0}{2} k^2 + \mathcal{O}(k^4) + 2k_C H(\eta)\right]$$

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S_0: resonance $\Rightarrow$ **enhanced** partial wave

$$M_{lo} = \sqrt{2\mu E_R(^8\text{Be})} \approx 20 \text{ MeV} \ll M_{hi}$$

$$\boxed{\frac{1}{a_0} \sim \frac{M_{\text{lo}}^3}{M_{hi}^2}\,,\quad r_0 \sim \frac{1}{M_{\text{hi}}} \quad \Rightarrow \quad a_0, r_0 \quad \textbf{LO}}$$

[Higa, *et al.* (2008)]

$a_0^{\text{exp}}, r_0^{\text{exp}} \Rightarrow M_{lo}/M_{hi} \approx 20 \text{ MeV}/180 \text{ MeV} \approx 0.1$

nonperturbative approach

Two-body effective potentials

Power counting

α–n

$$\frac{k^{2\ell}}{\mathcal{T}_\ell(k)} = \frac{\mu}{2\pi}\left[\frac{1}{a_\ell} - \frac{1}{2} r_\ell k^2 + O(k^4) + ik^{2\ell+1}\right]$$

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nonperturbative approach

$S_{1/2}$ and **$P_{1/2}$**: **non-enhanced** partial waves

a_0 LO, r_0, a_1, r_1 subleading [Bedaque, *et al.* (2003)]

we use the same nonperturbative approach

α–α

$$\frac{1}{\mathcal{T}^{SC}_{\ell=0}(k)} \propto \frac{\mu}{2\pi}\left[\frac{1}{a_0} - \frac{r_0}{2}k^2 + \mathcal{O}(k^4) + 2k_C H(\eta)\right]$$

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$$a_0^{\text{exp}}, r_0^{\text{exp}} \Rightarrow M_{lo}/M_{hi} \approx 20 \text{ MeV}/180 \text{ MeV} \approx 0.1$$

nonperturbative approach

Three-body potential

	^{9}Be	^{12}C
Leading Order (LO)	$\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}^{\alpha n}_{P_{3/2}}(\Lambda^{\alpha n}_{P_{3/2}}) + \mathcal{V}_{\mathbf{3}}(\Lambda_3, \lambda_3)$ $+ \mathcal{V}^{\alpha n}_{P_{1/2}}(\Lambda^{\alpha n}_{P_{1/2}})$ $+ \mathcal{V}^{\alpha n}_{S_{1/2}}(\Lambda^{\alpha n}_{S_{1/2}})$	$\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}_{\mathbf{3}}(\Lambda_3, \lambda_3)$

To avoid the dependence of the three-body results on the two-body cut-offs,
we add a three-body potential

$$\mathcal{V}_3(Q, Q') = \lambda_3 \; e^{-\left(\frac{Q}{\Lambda_3}\right)^2} \; e^{-\left(\frac{Q'}{\Lambda_3}\right)^2}$$

- we choose the two-body cut-offs $\Lambda^{\alpha n}$ and/or $\Lambda^{\alpha\alpha}$
- for each value of the three-body cut-off Λ_3, the LEC λ_3 is fixed on a three-body observable

Methods

Electromagnetic inclusive reactions

Cross section

$$\sigma_{\text{EM}} \propto \mathcal{R}(\omega)$$

Response function

$$\mathcal{R}(\omega) = \int df \, |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2 \, \delta(E_f - E_0 - \omega)$$

1. calculation of the initial bound state $|\Psi_0\rangle$ ➛ bound-state method
2. calculation of the final states $|\Psi_f\rangle$ in the continuum ➛ integral transform approach
3. determination of the operator $\hat{\mathcal{O}}$ ➛ photodisintegration processes

Non-Symmetrized Hyperspherical Harmonics (NSHH) method

[Gattobigio, *et al.* (2011), Deflorian, *et al.* (2013)]

$$\mathcal{R}(\omega) \sim |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2$$

Variational method + Non-Symmetrized HH basis

- potentials from EFT ← INPUT
- $\hat{\mathcal{H}}$ is represented on a Non-Symmetrized **basis in momentum space**

$$\Psi = \sum_{\nu} c_\nu \Psi_\nu \equiv \sum_{m\{K\}} c_{m\{K\}} \, f_m(Q) \, \mathcal{Y}_{\{K\}}(\Omega_Q)$$

$f_m(Q) \propto$ Laguerre polynomials basis

$\mathcal{Y}_{\{K\}}(\Omega_Q) =$ complete basis of the HH functions

- $\hat{\mathcal{H}}$ is diagonalized

$$\sum_{\nu'} \langle \Psi_\nu | \hat{\mathcal{H}} | \Psi_{\nu'} \rangle \, c_{\nu'} = E \, c_\nu \qquad E_0, \{ c_\nu^0 \} \Rightarrow \Psi_0$$

- Convergence is reached by enlarging the dimension of the basis ($m = 1, \ldots, N_{\text{Lag}}$, $K = 1, \ldots, K^{\text{max}}$)

Lorentz Integral Transform (LIT) method

[Efros, *et al.* (2007)]

$$\mathcal{R}(\omega) \sim |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2$$

$\mathcal{R}(\omega)$: states in the continuum spectrum are involved $\hat{\mathcal{H}} |\Psi_f\rangle = E_f |\Psi_f\rangle \Rightarrow$ direct calculation is DIFFICULT

Integral transform approach

- Definition of an **Integral Transform** $\mathcal{L}(\sigma)$ with a **Lorentzian kernel** $\mathcal{K}(\sigma, \omega)$ $(\sigma = \sigma_R + i\sigma_I)$

$$\mathcal{L}(\sigma) = \int d\omega \mathcal{K}(\sigma, \omega) \mathcal{R}(\omega) , \quad \mathcal{K}(\sigma, \omega) = \frac{1}{(\omega - E_0 + \sigma_R)^2 + \sigma_I^2} \quad \Rightarrow \quad \mathcal{L}(\sigma) \xrightarrow{\text{INVERSION}} \mathcal{R}(\omega)$$

- It can be demonstrated that $\mathcal{L}(\sigma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle$, where $|\tilde{\Psi}\rangle \equiv$ LIT states
  $|\tilde{\Psi}\rangle$ can be calculated using **bound-state methods**

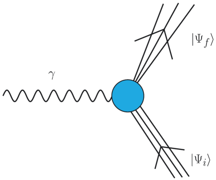
Continuum-states problem $\mathcal{R}(\omega)$ $\xrightarrow{\text{reformulation}}$ Bound-state-like problem $\mathcal{L}(\sigma)$

Photodisintegration reactions

[Bacca and Pastore (2014)]

$$\mathcal{R}(\omega) \sim |\langle\Psi_f|\,\hat{\mathcal{O}}\,|\Psi_0\rangle|^2$$

Photon γ:
$\boldsymbol{A}(\boldsymbol{x}), \hat{\epsilon}_{\boldsymbol{q},\lambda}$
choice: $\boldsymbol{q} \parallel \hat{\boldsymbol{z}}$



Nucleus:
$\boldsymbol{J}(\boldsymbol{x})$ nuclear current operator
$\rho(\boldsymbol{x})$ nuclear charge operator

$\omega = |\boldsymbol{q}|$

$|\Psi_0\rangle \rightarrow |\Psi_f\rangle$
transition

$$\mathcal{R}_\gamma(\omega) \sim \left|\langle\Psi_f|\,\hat{\epsilon}_{\boldsymbol{q},\lambda}\cdot\boldsymbol{J}(\boldsymbol{q})\,|\Psi_0\rangle\right|^2$$

Nuclear current matrix element

Siegert operator

[Siegert (1937), Golak, *et al.* (2011)]

$$\mathcal{R}(\omega) \sim \left| \langle \Psi_f | \, \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) \, | \Psi_0 \rangle \right|^2$$

- Multipole decomposition: $\hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) = -\sum_J \sqrt{2\pi(2J+1)} \left[\mathcal{T}^{el}_{J\lambda}(q;\boldsymbol{J}) + \lambda \mathcal{T}^{mag}_{J\lambda}(q;\boldsymbol{J}) \right]$

$$\mathcal{T}^{el}_{J\lambda}(q;\boldsymbol{J}) = \mathcal{T}^{el,I}_{J\lambda}(q;\boldsymbol{J}) + \mathcal{T}^{el,II}_{J\lambda}(q;\boldsymbol{J})$$

Dominant: $EJ = E1, E2$

- Siegert theorem (continuity equation)

$$\mathcal{T}^{el,I}_{J\lambda}(q;\boldsymbol{J}) \propto \int \mathrm{d}\hat{q}' \, \boldsymbol{q}' \cdot \boldsymbol{J}(\boldsymbol{q}') \, Y_{J\lambda}(\hat{q}') \quad \leftarrow \quad \omega\rho(\boldsymbol{q}) - \boldsymbol{q} \cdot \boldsymbol{J}(\boldsymbol{q}) = 0$$

$$\propto \int \mathrm{d}^3\boldsymbol{x} \, j_J(qx) \, \rho(\boldsymbol{x}) \, Y_{J\lambda}(\hat{x}) \propto \mathcal{T}^{el,S}_{J\lambda}(q;\rho)$$

- long-wavelength approximation ($qR \ll 1$)

$$\mathcal{T}^{el,S}_{J\lambda}(q;\rho) \propto \omega^J \int \mathrm{d}^3\boldsymbol{x} \, x^J \rho(\boldsymbol{x}) Y_{J\lambda}(\hat{x})$$

$\leftarrow$ dipole $\boldsymbol{d}_{\boldsymbol{\lambda}}$ or quadrupole $\boldsymbol{u}_{\boldsymbol{\lambda}}$ operator

$$\mathcal{T}^{el,II}_{J\lambda}(q;\boldsymbol{J}) \propto \omega^{J+1}$$

$\leftarrow$ correction

One-body convection current

[Bacca and Pastore (2014)]

$$\mathcal{R}(\omega) \sim \Big| \langle \Psi_f | \, \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) \, | \Psi_0 \rangle \Big|^2$$

- The nuclear current operator is a sum of terms

$$\boldsymbol{J} = \boldsymbol{J}^{[1]} + \boldsymbol{J}^{[2]} + \boldsymbol{J}^{[3]} + \dots$$

← the nuclear current matrix element can be calculated by using only the **one-body term** i.e. the nuclear convection current

- Specifically with our EFT, the continuity equation is **NOT** fully satisfied.

Motivations

[Bacca and Pastore (2014)]

Why this twofold calculation?

1. Siegert operator

$$\mathcal{T}^{el,S}_{J\lambda}(q;\rho)$$

2. One-body current

$$J^{[1]}_{\lambda}(q)$$

continuity equation: $\boldsymbol{q}\cdot\boldsymbol{J}(\boldsymbol{q}) = [\mathcal{H},\rho(\boldsymbol{q})]\,, \qquad \mathcal{H} = T + V_{2B} + \ldots$

$\boldsymbol{q}\cdot\boldsymbol{J}^{[1]}(\boldsymbol{q}) = [\mathcal{T},\rho(\boldsymbol{q})]$ always satisfied , $\quad [\mathcal{V}_{2B},\rho(\boldsymbol{q})] \neq 0 \Rightarrow \boldsymbol{J}^{[2]}(\boldsymbol{q})$ such that $\boldsymbol{q}\cdot\boldsymbol{J}^{[2]}(\boldsymbol{q}) = [\mathcal{V}_{2B},\rho(\boldsymbol{q})]$

$\ldots$

- With the **Siegert theorem**, since the continuity equation is used explicitly, the contribution due to the many–body current operators is implicitly included in the calculation

- Comparison between the two calculations

$\Rightarrow$ the contribution due to the many–body currents can be quantified

Practical calculation of the LIT

Eigenvalue method

[Efros, *et al.* (2007)]

$$\mathcal{L}(\sigma) = \sum_L \frac{|\langle \Psi_L | \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) | \Psi_0 \rangle|^2}{(E_L - E_0 - \sigma_R)^2 + \sigma_I^2}$$

$$\sum_{\nu'} \langle \Psi_\nu | \hat{\mathcal{H}} | \Psi_{\nu'} \rangle \, c_{\nu'} = E \, c_\nu$$

$$E_0, \{ c_\nu^0 \} \Rightarrow \Psi_0 \qquad E_L, \{ c_\nu^L \} \Rightarrow \Psi_L$$

1. Siegert operator [This work]

$$E1 : \langle \Psi_L | d_\lambda | \Psi_0 \rangle \qquad E2 : \langle \Psi_L | u_\lambda | \Psi_0 \rangle$$

coordinate space calculation

$$\Psi^{0,L}(\rho, \Omega_\rho) = \sum_{m\{K\}} c^{0,L}_{m\{K\}} g_{m,K}(\rho) \, \mathcal{Y}_{\{K\}}(\Omega_\rho)$$

2. One-body current [Filandri (2022)]

$$\langle \Psi_L | J_\lambda^{[1]} | \Psi_0 \rangle$$

momentum space calculation

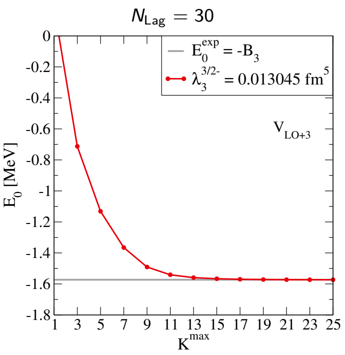
$$\Psi_{0,L}(Q, \Omega_Q) = \sum_{m\{K\}} c^{0,L}_{m\{K\}} f_m(Q) \, \mathcal{Y}_{\{K\}}(\Omega_Q)$$

a Fourier Transform of the basis $f_m(Q)$ originally defined in momentum space is needed [Viviani, *et al.* (2006)]

$$g_{m,K}(\rho) = (-i)^K \int_0^\infty dQ \, \frac{Q^{3N-1}}{(Q\rho)^{\frac{3N}{2}-1}} \, J_{K+\frac{3N-3}{2}+\frac{1}{2}}(Q\rho) \, f_m(Q) \qquad N = A - 1$$

$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n$ $|0\rangle : J^\pi = 3/2^- \rightarrow |f\rangle : J^\pi = 1/2^+$

^{9}Be ground state (LO)



- Basis $\sim f_m(Q)\,\mathcal{Y}_{\{K\}}(\Omega_Q)$ with $m = 1, \ldots, N_{\text{Lag}}$, $K = 1, \ldots, K^{\max}$
- EFT potential

$$\mathcal{V}_{LO+3} = \mathcal{V}_S^{\alpha\alpha} + \mathcal{V}_{P_{3/2}}^{\alpha n} + \mathcal{V}_3(300, \lambda_3^{3/2^-})$$

$\downarrow$

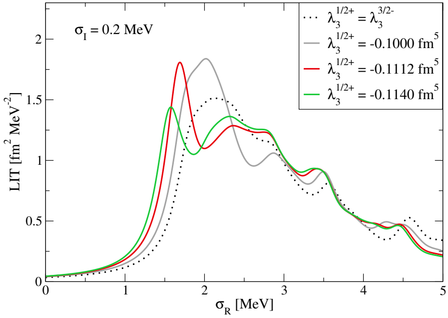
The LEC $\lambda_3^{3/2^-}$ is fixed to reproduce the experimental binding energy $B_3 = -1.573$ MeV

- $\mathcal{V}_3 = 0$ with $N_{\text{Lag}} = 30$, $K^{\max} = 25$

$\mathcal{V}_{LO}$	$E_0 = -1.965$ MeV	
$\mathcal{V}_{LO} + \mathcal{V}_{S_{1/2}}^{\alpha n}$	$E_0 = -1.982$ MeV	subleading!
$\mathcal{V}_{LO} + \mathcal{V}_{P_{1/2}}^{\alpha n}$	$E_0 = -2.041$ MeV	subleading!

$\gamma + {}^9\text{Be} \to \alpha + \alpha + n$ $|0\rangle : J^\pi = 3/2^- \to |f\rangle : J^\pi = 1/2^+$

$1/2^+$ LIT (LO)



- EFT potential

$$\mathcal{V}_{LO+3} = \mathcal{V}_S^{\alpha\alpha} + \mathcal{V}_{P_{3/2}}^{\alpha n} + \mathcal{V}_3(300, \lambda_3^{1/2^+})$$

$\downarrow$

The LEC $\lambda_3^{1/2^+}$ is fixed to locate the resonance peak at the experimental energy

- dipole operator

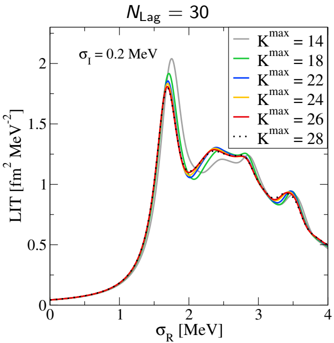
$$\mathcal{L}(\sigma_R) = \sum_L \frac{|\langle \Psi_L | \hat{d}_\lambda | \Psi_0 \rangle|^2}{(E_L - E_0 - \sigma_R)^2 + \sigma_I^2}$$

$\downarrow$

$\sigma_I \sim$ resolution

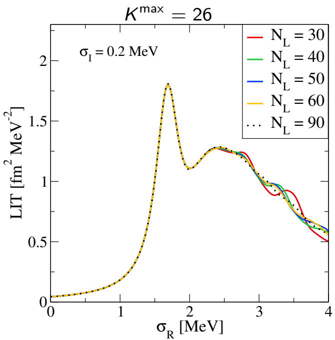
$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \rightarrow |f\rangle : J^{\pi} = 1/2^{+}$$

$1/2^{+}$ LIT (LO)



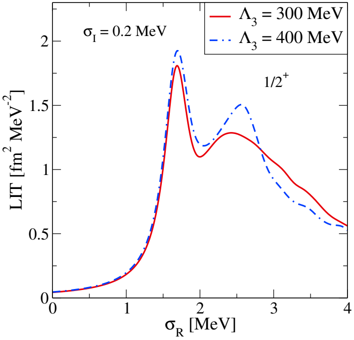
← convergence: $K^{max} = 26$

convergence: $N_{\text{Lag}} > 60$ →



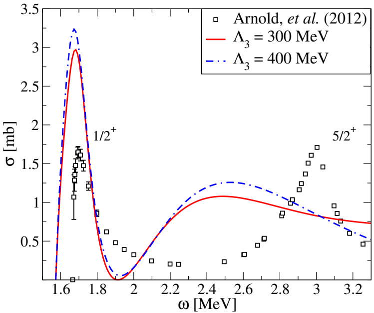
$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n$ $\qquad |0\rangle : J^\pi = 3/2^- \rightarrow |f\rangle : J^\pi = 1/2^+$

$1/2^+$ LIT and cross section (LO)



← *slight* dependence on the variation of the cut-off Λ_3

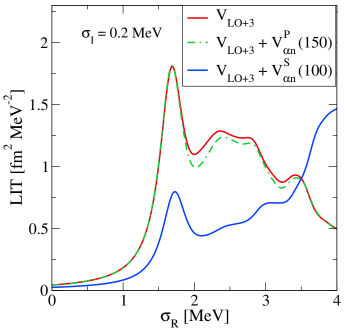
$\mathcal{L}(\sigma_R) \rightarrow \mathcal{R}(\omega) \rightarrow \sigma(\omega)$



Inclusion of other α–n partial waves ➠

$\gamma + {}^9\text{Be} \to \alpha + \alpha + n$ $|0\rangle : J^\pi = 3/2^- \to |f\rangle : J^\pi = 1/2^+$

$1/2^+$ LIT (LO + $P_{1/2}$, LO + $S_{1/2}$)



- $\mathcal{V}^{\alpha n}_{P_{1/2}} \to$ almost no effect
- $\mathcal{V}^{\alpha n}_{S_{1/2}} \to$ peak lower in height! ($\Lambda^{\alpha n}_{S_{1/2}} = 100$ MeV)

Choice: cut-offs $\Lambda^{\alpha n}_{S_{1/2}} > 100$ MeV

$\Rightarrow$ to "project out" the αn deep bound state we add a **projection potential**

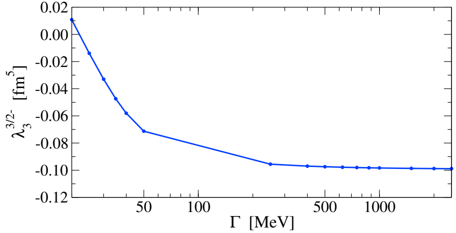
$$\mathcal{V}_{PR}(p, p') = \psi_{S_{1/2}}(p)\, \frac{\Gamma}{4\pi}\, \psi_{S_{1/2}}(p')$$

Theoretically: $\Gamma \to \infty$

In practice: Γ-independence of the three-body results ➠

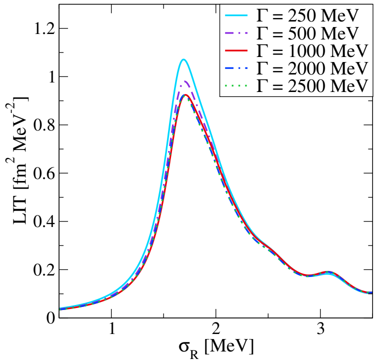
$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \rightarrow |f\rangle : J^{\pi} = 1/2^{+}$$

➠ Γ-independence of the three-body results



- calculation of the ^{9}Be ground state
  - calculation of the $1/2^{+}$ LIT

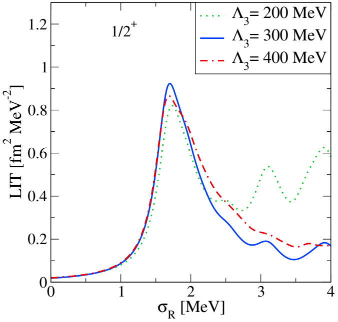
$\Gamma \gtrsim 1000$ MeV ⇒ **not** a free parameter



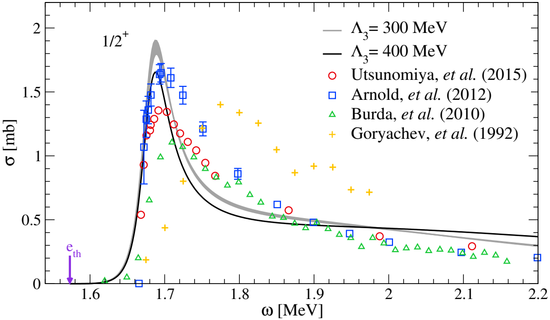
$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \rightarrow |f\rangle : J^{\pi} = 1/2^{+}$$

$1/2^+$ LIT and cross section

[YC, Filandri, Ji, Leidemann, Orlandini, In preparation]



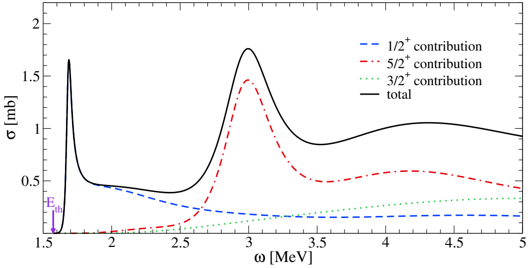
slight dependence on the variation of the cut-off Λ_3

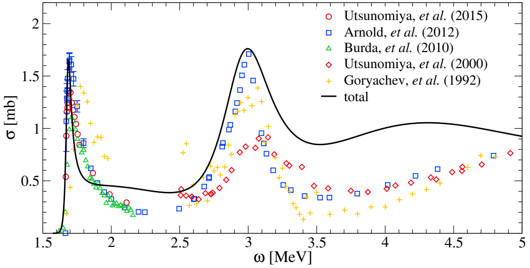


shade: *error* due to the inversion procedure $\mathcal{L}(\sigma_R) \rightarrow \mathcal{R}(\omega)$

$$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^\pi = 3/2^- \rightarrow |f\rangle : J^\pi = 1/2^+, 5/2^+, 3/2^+$$

Total photodisintegration cross section





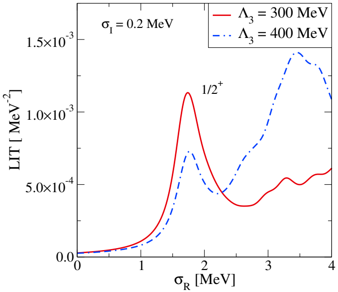
$1/2^+, 3/2^+$: $N_{\text{Lag}} = 60$, $K^{\text{max}} = 26$

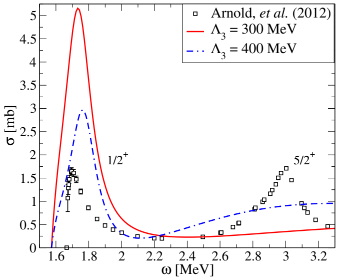
$5/2^+$: $N_{\text{Lag}} = 80$, $K^{\text{max}} = 30 \rightarrow$ **not** fully convergent results

this needs further check (ongoing!)

$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \rightarrow |f\rangle : J^{\pi} = 1/2^{+}$$

$1/2^{+}$ LIT and cross section (LO) - one-body current calculation

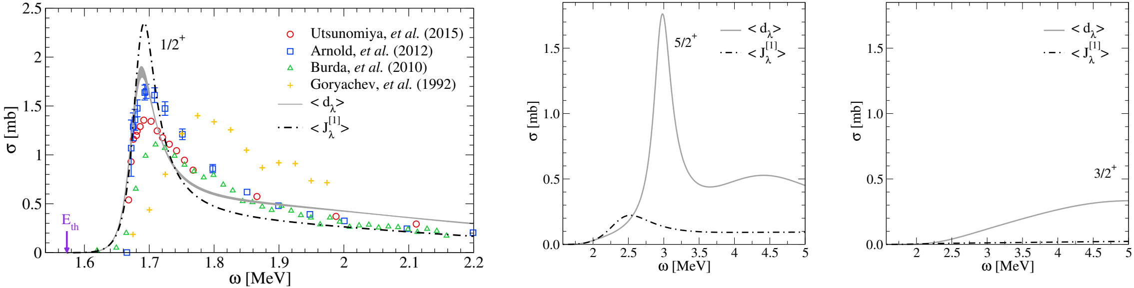




- $R(\omega) \sim |\langle \Psi_L | J_{\lambda}^{[1]} | \Psi_0 \rangle|^2$
  one-body convection current operator [Filandri, PhD Thesis (2022)]
- *strong dependence* on the variation of the cut-off Λ_3
  $\Rightarrow$ non-vanishing contribution due to the many-body currents

$$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^\pi = 3/2^- \rightarrow |f\rangle : J^\pi = 1/2^+, 5/2^+, 3/2^+$$

Photodisintegration cross section (NLO) - comparison

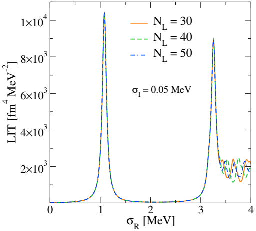


⇒ *non-negligible* contribution of the many–body currents

⇒ *dominant* effect of the many–body currents (although the results are not fully convergent)

$\gamma + {}^{12}\mathrm{C} \to \alpha + \alpha + \alpha$ $|0\rangle : J_n^\pi = 2_1^+$ (bound state) $\to |f\rangle : J_n^\pi = 0_2^+$ (Hoyle state)

0^+ LIT (LO)

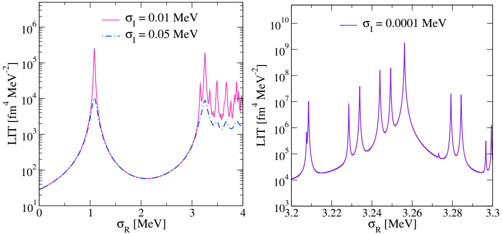


- quadrupole operator
- $\mathcal{V}_{LO+3} = \mathcal{V}_{S_0}^{\alpha\alpha} + \mathcal{V}_{\mathbf{3}}(\Lambda_3, \lambda_3)$

  λ_3^{2+} chosen to reproduce $E^{exp}(2_1^+) = -2.875$ MeV,

  λ_3^{0+} chosen to locate the resonance peak at the exp. en. ≈ 3.25 MeV
- 0_1^+ (ground state) and 0_2^+ are **NOT** simultaneously reproduced

  $E(0_1^+) = -1.792$ MeV $\neq E^{exp}(0_1^+) = -7.275$ MeV



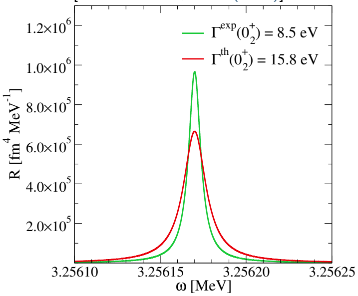
Construction of the response function:

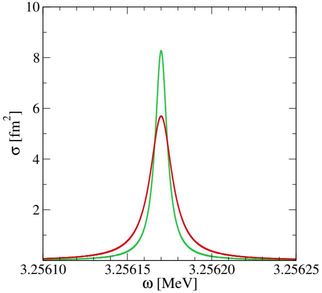
- we take the highest peak as a single "LIT state"
- we impose the experimental width of the resonance ($\Gamma^{exp}(0_2^+) \sim$ eV, very narrow!)

$\gamma + {}^{12}\mathrm{C} \rightarrow \alpha + \alpha + \alpha$ $|0\rangle : J_n^\pi = 2_1^+$ (bound state) $\rightarrow$ $|f\rangle : J_n^\pi = 0_2^+$ (Hoyle state)

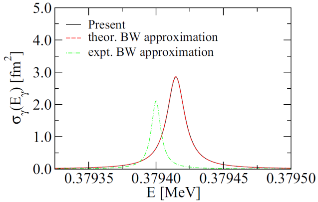
0^+ cross section (LO)

[Γ^{th} from Suno, *et al.* (2016)]





[Suno, *et al.* (2016)]



Comparison

$\Rightarrow$ factor $\sim$ 4 or $\sim$ 2

Conclusions and outlook

Photodisintegration reactions of three-body cluster nuclei *at low energy*

- potentials from Cluster EFT
- LIT method: cross sections
- study of the effect of the many-body currents

^{9}Be

- LO cross section: overestimation of the data
- LO+$S_{1/2}$ cross section: agreement with the data $\longrightarrow$ ❶ comparison with other calculations (phenomenological potentials)
- non-vanishing effect of the many-body currents, dominant in the $5/2^{+}$ channel

☞ Calculation of the magnetic transition $M1$

^{12}C

- LO calculation (early stage!): overestimation of the data $\longrightarrow$ ❷ How the calculation can be improved? (*D*-wave potential)

☞ Improvement in the convergence

- Effect of the many-body currents in ^{12}C nucleus
- Four-body calculations
- *NN* interaction from EFT (^{10}Be)
- Final goal: $\gamma + {}^{16}\mathrm{O} \rightarrow \alpha + \alpha + \alpha + \alpha$ $\longrightarrow$ ❸ Is the EFT approach still valid?