

# *Computing atomic nuclei based on Chiral EFT and HALQCD interactions*



The Nuclear Interaction: Post-Modern Developments August 19-23, 2024



Carlo Barbieri **•HAL QCD and nuclei •Results with ChEFT •Diagrammatic Monte Carlo** (for nuclei, eventually…)





## **Reach of ab initio methods across the nuclear chart**

- 
- 



# **Nuclei with HAL QCD forces**

C. McIlroy, CB et al. Phys. Rev. C**97**, 021303(R) (2018) D. Lonardoni et al. - in preparation



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*In collaboration with:*











\* Manifest gauge invariance



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$$
\int_{\mu}^{a} q q
$$

## Vacuum expectation value

path integral  
\n
$$
lq e^{-S(q,q,U)} O(q,q,U)
$$
  
\n $D(U)e^{-S_U(U)} O(D^{-1}(U))$   
\nquark propagator  
\n $O(D^{-1}(U_i))$ 

 $\{U_i\}$ : ensemble of gauge conf. U generated w/ probability det  $D(U) e^{-S_U(U)}$ 

# \* Highly predictive

*Slide, courtesy of T. Inoue (YITP talk, Oct. 8th 2015)*

O



Define a general potential U(*r*,*r*') which is and non-local but energy independent up to inelastic threshold,

such that:  $-\frac{\nabla^2}{2\mu}\varphi_{\vec{k}}(\vec{r})\ +\ \int d\vec{r}^{\,\prime}U(\vec{r},\vec{r}^{\,\prime})\varphi_{\vec{k}}(\vec{r}^{\,\prime})$ 

$$
) = E_{\vec{k}} \varphi_{\vec{k}}(\vec{r})
$$

$$
, \ \ \varphi_{\vec{k}}(\vec{r}) = \sum \langle 0|B_i(\vec{x} + \vec{r}, t)B_j(\vec{x}, t)|B = 2, \vec{k} \rangle
$$

$$
B_i(\vec{x} + \vec{r}, t)B_j(\vec{x}, t) J(t_0)|0\rangle = \sum_{\vec{k}} A_{\vec{k}} \varphi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} +
$$

$$
d\vec{r}^{\,\prime}U(\vec{r},\vec{r}^{\,\prime})\psi(\vec{r}^{\,\prime},t)\;=\;-\frac{\partial}{\partial t}\psi(\vec{r},t)
$$

for the Nambu-Bethe-Salpeter (NBS) wave function, Operationally, measure the 4-pt function on the QCD Lattice  $\psi(\vec{r},t)=\sum_{\vec{\sigma}}\langle 0|B\rangle$ 

and extract  $U(r,r')$  from:  $\left\{ 2M_B - \frac{\nabla^2}{2\mu} \right\} \psi(\vec{r},t) + \int di$ 

A *local potential* V(*r*) is then obtained through a derivative expansion of U(*r*,*r*'), which *must give the same observables* of the LQCD simulation:

$$
U(\vec{r},\vec{r}')=\delta(\vec{r}-\vec{r}')V(\vec{r},\nabla):
$$

$$
V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\frac{\partial}{\partial t} \psi(\vec{r}, t)}{\psi(\vec{r}, t)}
$$

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## **The HAL-QCD Method**



*Prog. Theor. Phys. 123 89 (2010); Phys. Lett. B712 , 437 (2012); Prog. Theor. Exp. Phys. 01A105 (2012)*



## **Two-Nucleon HAL potentials in flavour SU(3) symm.**

## **Quark mass dependence of** *V(r)* **for NN partial wave (<sup>1</sup>S<sub>0</sub>, <sup>3</sup>S<sub>1</sub>, <sup>3</sup>S<sub>1</sub>-<sup>3</sup>D<sub>1</sub>)**

 $\rightarrow$  Potentials become stronger  $m_{\pi}$  as decreases.

e.<br>Avis de la propincia de la pr



*Prog. Theor. Exp. Phys. 01A105 (2012)*  $109.$  Theor. LAP. Thys. OIA109 (2012)  $\frac{1}{2}$ 

# Quark mass dependence

● Quark mass dependence of potentials in NN 3 S1





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## **Infrared convergence**

*Short-range repulsion in the HALQCD-type potentials can be tamed correctly even for large nuclei.*  C. McIlroy, CB, et al., Phys. Rev. C**97**, 021303(R) (2018)









## **Binding of 16O and 40Ca:**

➔ 16O at mπ≈ 470 MeV is unstable toward 4-α breakup*!*

C. McIlroy, CB, et al., Phys. Rev. C**97**, 021303(R) (2018)



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Binding energies are ~17 MeV 16O and 70-75MeV for <sup>40</sup>Ca. Possibly being underestimated by 10%

certainties in the model space extrapolation [52]. The second

error corresponds to many-body truncations and we estimate

it to be 10% based on the finding for 4He. The SCGF results

![](_page_7_Picture_260.jpeg)

tracted the kinetic energy of the kinetic energy of the center of  $\alpha$  mass according to  $\alpha$  mass according to  $\alpha$ 

![](_page_7_Picture_10.jpeg)

*NB*: All calculations assuming spherical wave functions…

![](_page_8_Picture_5.jpeg)

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### *g*(!) using the Koltun sum rule. The same lattice simulation setup used to generate the HAL469 interaction gives a nucleon gives a nucleon gives a nucleon gives a nucleon  $\alpha$

tracted the kinetic energy of the center of mass according to

![](_page_8_Figure_1.jpeg)

### **Results for binding** error corresponds to many-body truncations and we estimate it to be 10 method on the finding for 4He. The final state of 4He. The SCGF results is a set of 4He. The SCGF are sensible less bound than our previous BHF results [22]. The sensible previous BHF results [22]. Th

certainties in the model space extrapolation [52]. The second

Slides from *S. Aoki* at Kavli institute, Oct. 2016

![](_page_9_Picture_6.jpeg)

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Physical mass now under reach  $(m_\pi \approx 145 \text{ MeV})$  for hyperons

## **HAL QCD interactions with hyperons and (near)physical pain mass**

- 
- Need to improve on statistic for the NN sector
- $\Omega$  potential

![](_page_9_Figure_4.jpeg)

S. Aoki, T. Doi, Front. Phys. 8:307 (2020).

![](_page_9_Picture_12.jpeg)

![](_page_9_Picture_13.jpeg)

![](_page_9_Picture_14.jpeg)

## **Quantum MC calculations for** *Y***s**

- AV4' + UIX with phenomenological hypernuclear forces requires large ΛNN 3-baryon force
- Physical mass now under reach  $(m_\pi \approx 145 \text{ MeV})$  for hyperons
- HALQCD ΛN 3-baryon force is already very close to experiment

![](_page_10_Figure_4.jpeg)

![](_page_10_Picture_5.jpeg)

 $H=-\frac{\hbar^2}{2m_N}\sum_i \nabla_i^2 + \sum_{i < i} v_{ij} + \sum_{i < i < k} V_{ijk} - \frac{\hbar^2}{2m_\Lambda}\nabla_\Lambda^2 + \sum_i v_{i\Lambda}$  $v_{ij} = \sum_{p=1,4} v^p(r_{ij}) O_{ij}^p$ Argonne  $v'_{4}$  (AV4') nucleon-nucleon (NN) interaction

central component of the Urbana IX (UIX<sub>c</sub>)  $V_{ijk} = A_R \sum_{m} T^2 (m_{\pi} r_{ij}) T^2 (m_{\pi} r_{ik})$ 

The hyperon-nucleon  $(YN)$  potential

$$
v_{i\Lambda} = \sum_{p=c,\sigma,t} v^p(r_{i\Lambda}) O_{i\Lambda}^p
$$

### **Diffusion Monte Carlo:**

$$
\langle X|\Psi_T\rangle = \langle X|\left(\prod_{i
$$

### **AFDMC:**

D. Lonardoni, A. Lovato, et al, Phys. Rev. Lett. 114, 092301 (2015) & arXiv:1506.04042

![](_page_10_Picture_16.jpeg)

D. Lonardoni, A. Lovato, CB, T. Inoue, HALQCD coll — unpublished ardoni A Lovat

![](_page_11_Picture_14.jpeg)

bath sampling among the four normalized weights *wi*/*W*, with

timated according to  $E$  and  $\mathcal{L}$  the asymptotic value is found to  $E$ 

- *K* requires very large with phenomenologic matter is a contract of the co<br>The contract of the contract of
- Physical mass now under reach  $(m_\pi \approx 145 \text{ MeV})$  for hyperons Physical mass now under reach  $(m \approx 145 \text{ MeV})$  for b puting observables. This is not plus and minutes and minus and minutes and minutes and minus  $\mathcal{L}$
- HALQCD AN 3-baryon force is already very close to experiment  $HAIOC$

![](_page_11_Figure_9.jpeg)

 $10<sub>1</sub>$ 

Table 1:  $\Lambda$  separation energies (in MeV) for different hypernuclei with the hyperon in different single-particle states. Second column reports the AFDMC results using the original HALQCD96 AN potential. Third column shows the results for the modified HALQCD96 AN potential (see text for details). In the last column, the available experimental data [] are reported.

$^{A}_{\Lambda}Z$	$J^{\pi}$ (state)	HALQCD96	HALQCD96*	Exp
$^{5}_{\Lambda}$ He	$1/2^+(s)$	0.21(5)	1.02(3)	3.12(2)
$^{16}_{\Lambda}$ O	$1^{-}(s)$	9.5(5)	13.5(2)	13.4(4)
	$2^{+} (p)$	$-1.3(2)$	0.5(1)	2.5(2)
$^{40}$ Ca	$2^{+}$ (s)	21.0(5)	26.8(5)	19.3(1.1)
	$3^{-}(p)$	9.3(6)	13.7(6)	11.0(5)

### **Future application for** *Y***s in nuclei now possible** *W* = P<sup>4</sup> *<sup>i</sup>*=<sup>1</sup> *wi*/4 being the cumulative weight. The latter is then urure by fitting the imaginary-time behavior of the unconstrained enuture application for Ys in n Unconstrained propagations have been performed in the latest

![](_page_11_Figure_4.jpeg)

![](_page_11_Picture_5.jpeg)

and for cases in the case in which a real wave function can be used, the use of the use of  $\mathsf{D}$ 

# **Self-Consistent Green's function computations based on Chiral EFT interactions (NN+3N forces)**

![](_page_12_Picture_1.jpeg)

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![](_page_12_Picture_4.jpeg)

![](_page_12_Picture_5.jpeg)

*F-RPA: Phys. Rev. C63, 034313 (2001)*  Compute the nuclear self energy to extract both scattering (optical potential) and spectroscopy. Both ladders and rings are needed for atomi nuclei: Both ladders and rings are needed for atomi nuclei: ielf energy to extract both scatteri ducible self-energy up to the third order is given by the exact order is given by  $\int$  cushuto the m i.e., correlate particle propagators are correlated, the interest single-particle propegators are correlated, ducible self-energy up to third distribution by the exact is given by the exact is given by the exact is given by the exact in the exact is given by the exact in the exac

skeleton and interaction-irreducible diagrams up to third order

### The Faddev-RPA and ADC(3) methods in a few words *<sup>W</sup>*αβϵ*,*γ δη ρηϵ\$ *a†* α*a†* <sup>β</sup> *a*δ*a*<sup>γ</sup> *,* (15) For the irreducible self-energy, all one-particle irreducible, *<sup>W</sup>*αβϵ*,*γ δη ρηϵ\$ *a†* α*a†* <sup>β</sup> *a*δ*a*<sup>γ</sup> *,* (15) 3NF *W*ˆ . For the irreducible self-energy, all one-particle irreducible,

![](_page_13_Picture_8.jpeg)

**Since I** UNIVERSITÀ DEGLI ST

![](_page_13_Figure_11.jpeg)

![](_page_13_Picture_12.jpeg)

(a) (b)

 $\mathcal{L}(\mathcal{$ 

skeleton and interaction-irreducible diagrams up to third order

![](_page_13_Figure_2.jpeg)

All Ladders (GT) and ring modes (GW) are coupled to all orders. Two approaches:  $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$  and ( entering  $\overline{\phantom{a}}$ All Ladders (GI) a true correlated propagators; i.e., they are not computed from  $\mathbf{r}$  $\overline{\phantom{a}}$ the contraction of the contraction of the interaction of the interaction vertex are obtained from the interact true correlated propagators; i.e., they are not computed from  $\sim$ 

- . Faddev-RPA allows for RPA modes perturbation theory (or on allows): it allows  $6.$  Eaddey DDA perturbation theory (or orders resummations): it also results in the set of the
- ADC(3) Tamn-Dancoff version using 3rd order diagrams as 'seeds':  $u_{\rm eff}$ us tode $u_{\rm eff}$  a reference state upon which a perturbative series  $v_{\rm eff}$ function in Feynman diagrams. Nevertheless, the auxiliary diagrams as *see*ds Considering the decomposition of Eq. (9), the irreducible function in Feynman diagrams. Nevertheless, the auxiliary potential *diagrams as 's* considering the decomposition of Eq. (9), the interaction of Eq. (9), the interact

![](_page_13_Figure_6.jpeg)

![](_page_13_Picture_7.jpeg)

### The Self-Consistent Green's Function with Faddev-RPA the best precision for quasiparticle energies [49], it still yields proper predictions for the trend of binding energies [22]. We plot the Gorkov-predicted binding energies for all through removal and addition of one proton. This confirms that  $\mathbf{r}$ all considerations made regarding the effects of leading-order 3NFs on the oxygens also apply to their neighboring chains. In

![](_page_14_Figure_1.jpeg)

The present implementation of the Gorkov-GF approach

oxygen is  $\mathcal{O}_{\mathcal{A}}$  and compare them to the Dyson-Dyso

![](_page_14_Picture_4.jpeg)

Figure 7 shows the analogous information for the binding

in determining the neutron driplines at 23N and 24O. Fluorine

![](_page_14_Picture_3515.jpeg)

oxygen isotopes in Fig. 6 and compare them to the Dyson-A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013) and Phys. Rev. C 92, 014306 (2015) isotopes have been observed experimentally up to 31F but with the 31F but with the 31F but with the 31F but wi<br>The 31F but with with the 31F but with the

energies of the nitrogen and fluorine isotopic chains, obtained

 $\sum_{i=1}^{\infty} a_i$ 7. CI. MICPOSCOPIC SHEIL MOUEL LOTSUKU ET UI, PREI 00, OSZSOT (ZOIO).] 014306-8 cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

![](_page_15_Picture_5.jpeg)

. DEGLI STUDI DI MILA<mark>N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm<sup>-1</sup>)</mark> DIPARTIMENTO DI FISICA N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm-1)

![](_page_15_Picture_7.jpeg)

![](_page_15_Picture_8.jpeg)

![](_page_15_Figure_2.jpeg)

 $\rightarrow$  3NF crucial for reproducing bindir Hamiltonians. The shaded area highlights the changes owing to the  $\rightarrow$  3NF crucial for reproducing binding energies and dr remergies and driplines around oxygen Experiment are from Refs. [56–58,63,72].  $\rightarrow$  3NF crucial for reproducing binding energies and driplines around oxygen

### [i.e., at the ADC(2) level]. Although this does not guarantee  $\bullet$  fan the NLMLF che proper predictions for the trend of binding energies [22]. We plot the Gordon plot the Gordon predicted binding energies for all  $\Delta$  Cinolli all considerations made regarding the effects of leading-orderations  $\mathcal{C}$ 3NFs on the oxygens also apply to their neighboring chains. In *NN* + 3*N*-induced Hamiltonian systematically underbinds the full isotopic chain and predicts and predicts and predicts and predicts are a continuously about the continuously continuously and predicts  $\mathbf{r}$ -60 a 29F that is very weakly bound. Figure 7 clearly demonstrates  $\mathcal{L}^{\text{max}}$ that this is attributable to an very subtle cancellation between the republic form 3NFs and the attraction form 3NFs and the attraction generated by one of the attraction gener<br>The attraction generated by one of the attraction generated by one of the attraction generated by one of the a **Results for the N-O-F chains**

isotopes have been observed experimentally up to 31F but with

![](_page_16_Figure_1.jpeg)

![](_page_16_Picture_2.jpeg)

![](_page_16_Figure_4.jpeg)

A. Cipollone, CB, P. Navrátil, *Phys. Rev. C 92, 014306 (2015)*

## Neutron spectral function of Oxygens

![](_page_16_Figure_7.jpeg)

![](_page_16_Picture_8.jpeg)

![](_page_16_Picture_9.jpeg)

![](_page_16_Picture_10.jpeg)

![](_page_17_Figure_2.jpeg)

![](_page_17_Picture_3.jpeg)

## **N3LO(500) + nln 3NF**

 $\mathbb{R}$  interpreted decli etiidi due to model space truncations are to  $\mathbb{R}$ ≈1% of the total binding energy for NNLOsat and 0.5% for *NN*+

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### SCGF – Gorkov-ADC(2) NOVEL CHIRAL HAMILTONIAN AND OBSERVABLES IN … PHYSICAL REVIEW C **101**, 014318 (2020)

recently measured masses of <sup>55</sup>−57Ca were taken from Ref. [91]. The computational errors due to model space truncations are below 2% of V. Somà, P. Navrátil, F. Raimondi, CB, T. Duguet – Phys. Rev. C**101**, 014318 (2020) the total binding energy for NNLOsat and below 1% for *NN*+ 3*N*(lnl) and *NN*+ 3*N*(400). Note that the ADC(3) truncation accounts for Eur. Phys. J. A**57** 135 (2021)

![](_page_17_Picture_7.jpeg)

### Validated by charge distributions and neutron quasiparticle spectra:

![](_page_18_Figure_3.jpeg)

![](_page_18_Picture_4.jpeg)

- 34Si is unstable, charge distribution is still unknown
- Suggested central depletion from mean-field simulations
- Ab-initio theory confirms predictions
- Other theoretical and experimental evidence: *Phys. Rev. C 79, 034318 (2009), Nature Physics 13, 152–156 (2017).*

![](_page_18_Figure_12.jpeg)

![](_page_18_Picture_13.jpeg)

![](_page_18_Picture_14.jpeg)

Duguet, Somà, Lecuse, CB, Navrátil, Phys.Rev. C**95**, 034319 (2017)

## **Bubble nuclei... 34Si prediction**

![](_page_18_Figure_1.jpeg)

## **46Ar(3He,d)47K at GANIL**

## d3/2 - s1/2 inversion revisited from adding protons to 46Ar

D. Brugnara, A. Gottardo, CB et al…

![](_page_19_Picture_8.jpeg)

![](_page_19_Picture_9.jpeg)

![](_page_19_Figure_1.jpeg)

![](_page_19_Picture_2.jpeg)

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![](_page_19_Figure_6.jpeg)

### d3/2 - s1/2 inversion revisited from adding protons to 46Ar

### **46Ar(3He,d)47K at GANIL : New charge bobble in 46Ar** ferred nucleon. In particular, the addition to an s-wave chargetribution for the ejected deuteron for the ejected deuteron that is peaked that is peaked that is peaked that 0*.*0 0*.*2 0*.*4 0*.*6 0*.*8 1*.*0  $C^2 S(\ell = 2) / C^2 S(\ell = 0)$ Fig. 5: Comparison of experimental results and theoretical models. The horizontal models. The horizontal  $\mathcal{C}$

![](_page_20_Figure_1.jpeg)

**Theory & experiment for relative SFs agree within 1 sigma and confirms charge depletion in 46Ar**

![](_page_20_Picture_5.jpeg)

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![](_page_20_Figure_8.jpeg)

![](_page_20_Picture_9.jpeg)

 $\sim$ 

 $8 - 6$ 

![](_page_21_Figure_1.jpeg)

FIG. 1. Overview of the SCRIT electron scattering facility.

## **Electron-Ion Trap colliders…**

![](_page_21_Figure_7.jpeg)

FIG. 3. Reconstructed momentum spectra of  $^{132}Xe$  target after background subtraction. Red shaded lines are the simulated radiation tails following the elastic peaks.

![](_page_21_Figure_9.jpeg)

First ever measurement of charge radii through electron scattering with and ion trap setting that can be used on radioactive isotopes !!

K. Tsukada *et al*., Phy rev Lett **118**, 262501 (2017)

![](_page_21_Picture_5.jpeg)

*P. Arthuis, CB, M. Vorabbi, P. Finelli, Phys. Rev. Lett. 125, 182501 (2020)*

![](_page_21_Picture_11.jpeg)

![](_page_21_Picture_12.jpeg)

### sections multiplied by the luminosity as a function of the luminosity as a function of the luminosity as a function<br>The luminosity as a function of the luminosity as a function of the luminosity as a function of the lumino the effective momentum transfer for the three experimental electron beam energies of Eestima energies of Eestima energies of Eestima energies of Eestima energies of Ees **Charge density for Sn and Xe isotopes**

![](_page_22_Figure_1.jpeg)

**Having proved the capacity of SCGF and NNLOS** now @ TU Darmstadt,

important precedent in the use of SCGF with the NNLOsat

![](_page_22_Picture_3.jpeg)

Gorkov ADC(2) and Dyson ADC(3) with N3LO-Inl and NNLOsat Hamiltonians

![](_page_22_Picture_411.jpeg)

![](_page_22_Picture_7.jpeg)

# **Ab initio optical potentials from propagator theory**

![](_page_23_Figure_7.jpeg)

Relation to Fesbach theory: Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991) Escher & Jennings Phys. Rev. C**66**, 034313 (2002)

Previous SCGF work:

CB, B. Jennings, Phys. Rev. C**72**, 014613 (2005) S. Waldecker, CB, W. Dickhoff, Phys. Rev. C**84**, 034616 (2011) A. Idini, CB, P. Navrátil, Phys. Rv. Lett. **123**, 092501 (2019) M. Vorabbi, CB, et al., Phys. Rev. C **109**, 034613 (2024)

![](_page_23_Figure_4.jpeg)

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Solve scattering and overlap functions directly in momentum space:  $\alpha$ ov<del>c</del>hap functions allectly in moniclitum space. olve scattering and overlap functions directly in n  $\alpha$  is the center of mass motion  $\alpha$ ichtum space.

effective two-body interaction, averaged on the correlated

performed calculations with the third order algebraic

up to 400 MeVof excitation energy and partial waves of the

which is nonlocal and energy dependent, where Rn;lðkÞ are

$$
\Sigma^{\star\ l,j}(k,k';E) = \sum_{n,\,n'} R_{n\,l}(k) \Sigma^{\star\ l,j}_{n,\,n'} R_{n\,l}(k')
$$

$$
\frac{k^2}{2\mu} \psi_{l,j}(k) + \int \mathrm{d}k' \, k'^2 \, \Sigma^{\star l,j}(k,k';E_{c.m.}) \psi_{l,j}(k') = E_{c.m.} \psi_{l,j}(k)
$$

 $\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}})$  is incorporated by the configurations of  $\mathcal{L}^{\mathcal{L}}$ 

![](_page_24_Picture_7.jpeg)

 $\mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L}$ 

effective two-body interaction, averaged on the correlated

• it is proven to be a Feshbach opt. pot  $\rightarrow$  in general it is *non-local!* Particle-vibration <u>and B label the single particle</u> particle particle particle particle particle particle particle couplings: has the following general spectral representation: the following general representation: the following general<br>The following general representation: the following general representation: the following general representatio **N**  $\int_{\alpha}^{\dagger} \left( \frac{1}{\Gamma(\alpha)} \right)$ ) –  $\equiv$ 11 12 13 14 15 16 17 17 18  $\sum_{j,\beta}$  $\overline{\phantom{a}}$  $\sqrt{}$  $r_{s,s}$  $\overline{\mathbf{N}}_{\alpha,r}$  $\begin{pmatrix} 1 \end{pmatrix}$  $E - (\mathbf{K}^{\lt} + \mathbf{D}) - i\Gamma$ "  $r_{s,s}$  $\mathbf{N}^{\dagger}_{s,\beta}$  $\frac{1}{\sqrt{2}}$  and  $\frac{1}{\sqrt{2}}$  $i,j$  $\textbf{M}^{\dagger}_{\alpha}$  $\alpha, i$  $\begin{pmatrix} 1 \end{pmatrix}$  $E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma$ "  $i,j$  $\overline{\mathbf{M}}_{j,\beta}$  $r_{\rm s}$ r;s  $\overline{E-(\mathbf{K}^{\lt}\,+)}$  $Parricle-vibration$  is a and  $Parricle-vibration$ the Couplings: 2000 independent and energy in the couplings: mean field, and  $\mathcal{L}$  sets the correct boundary conditions. We conclude the correct boundary conditions. We consider  $\mathcal{L}$ al it is  $n$ many-body problem that comes from the diagonalization  $\begin{pmatrix} -1 & 1 & 1 \end{pmatrix}$  $\int_{r,s}$  introduce a small finite width for the  $i\Gamma/\int_{r,s}$  and  $s$ 2h1p configurations, which would otherwise be discretized in the present approach. We checke that the present approach. We checke that the present approach. We checke  $\sqrt{2}$  $\frac{d}{dx}$ enough HO spaces so that the intrinsic ground state interior so that the intrinsic ground state intrinsic ground state interior state in the interior state interior state in the interior state interior state in the interio

![](_page_24_Picture_8.jpeg)

<sup>Σ</sup>⋆l;jðk; k<sup>0</sup>

n;n0

which is non-local and energy dependent, where  $R$ 

### Microscopic optical potential and as discussed in Refs. 20,35 and 20,35 and 20,35 and 20,35 and 20,35 and 20,35 calculation proceeds by solving the Dyson equation, propagator antical natantial calculation process by solving the Dyson equation,

![](_page_24_Figure_1.jpeg)

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Nuclear self-energy  $\Sigma^{\star}(\mathbf{r},\mathbf{r}';\varepsilon)$ : • contains *both particle* and *hole* props.  $\mathbf{g}$  $\frac{1}{2}$ gður salf angrav $\nabla^{\star}(n, n')$ (Holder seit-energy  $\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon)$ ). contains both particle and hole props.

### NCSM/RGM [**without** core excitations]

 $EM500: NN-SRG$   $\lambda_{SRG}$ = 2.66 fm<sup>-1</sup>, Nmax=18 (IT) [PRC**82**, 034609 (2010)]

NNLOsat: Nmax=8 (IT-NCSM)

 $SCGF$   $[\Sigma^{(\infty)}$  only], always Nmax=13

### Benchmark with NCSM-based scattering.

### Scattering from mean-field only:

![](_page_25_Figure_3.jpeg)

![](_page_25_Picture_4.jpeg)

![](_page_25_Picture_10.jpeg)

![](_page_25_Picture_11.jpeg)

![](_page_25_Picture_12.jpeg)

![](_page_25_Picture_13.jpeg)

![](_page_25_Picture_14.jpeg)

## **Low energy scattering – from SCGF**

[A. Idini, CB, Navratil, Phys. Rev. Lett. **123**, 092501 (2019) ]

### Benchmark with NCSM-based scattering.

### Scattering from mean-field only:

![](_page_26_Figure_3.jpeg)

![](_page_26_Picture_4.jpeg)

## **Low energy scattering – from SCGF**

### Full self-energy from SCGF:

![](_page_26_Picture_7.jpeg)

### [A. Idini, CB, Navratil, Phys. Rev. Lett. **123**, 092501 (2019) ]

state-of-the-art SCGF calculations to test current ab initio

## actions. We then use a saturations. We then use a saturations a saturation of the saturation of the saturation<br>The saturation of the saturation of th

![](_page_27_Figure_2.jpeg)

[A. Idini, CB, Navrátil, n-<sup>16</sup>O, total elastic cross section electricity of neutrons from the phys. Rev. Lett. **123**, 092501 (2019)] Phys. Rev. Lett. 123, 092501 (2019)]

NCSMC computations with NN participations with NN participations with NN participations with NN participations

![](_page_27_Figure_6.jpeg)

### **Role of intermediate state configurations (ISCs)** methods and compare our results to NCSM-RGM and NCSMC computations with NN and NN þ 3N inter<sup>j</sup>Ψ<sup>A</sup>þ<sup>1</sup> n i in the corresponding overlaps of the corresponding over figurations (ISCs) scattering wave function through Feshbach theory [1,38]. actions. We then use a saturations. We then use a saturations. We then use a saturation of the chiral Hamilton<br>The chiral Hamiltonian to the chiral Hamiltonian to the chiral Hamiltonian to the chiral Hamiltonian Control H ate contigurations (LSCs) Formalism.—The Hamiltonian used to compute the

scattering wave function through Feshbach theory [1,38].

energy Σ<sup>⋆</sup>ðωÞ associated with the optical potential is

localized, and it can be efficiently expanded on square

![](_page_27_Picture_7.jpeg)

![](_page_27_Picture_8.jpeg)

![](_page_28_Figure_0.jpeg)

DIPARTIMENTO DI FISICA

![](_page_28_Picture_2.jpeg)

S. Brolli (Masters thesis)

![](_page_28_Picture_4.jpeg)

![](_page_28_Picture_5.jpeg)

![](_page_28_Picture_6.jpeg)

## **Green's function theory beyond ADC(3)?**

The Green's function is found as the exact solution of the Dyson equation:

$$
G_{\alpha\beta}\left(\omega\right)=G_{\alpha\beta}^{\left(0\right)}\left(\omega\right)+\sum_{\gamma\delta}G_{\alpha\gamma}^{\left(0\right)}\left(\omega\right)\Sigma_{\gamma\delta}^{\star}\left(\omega\right)G_{\delta\beta}\left(\omega\right)
$$

It requires knowing the self-energy which is the sum of an *infinite series* of Feynman diagrams:

![](_page_29_Figure_4.jpeg)

![](_page_29_Picture_5.jpeg)

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![](_page_29_Picture_8.jpeg)

- 
- $G_{\gamma\delta}^{\star}\left(\omega\right)G_{\delta\beta}\left(\omega\right),$

![](_page_29_Picture_11.jpeg)

![](_page_29_Picture_12.jpeg)

![](_page_30_Figure_1.jpeg)

## Diagrams grow factorially (more than exponentially) with the order A direct calculation of all diagrams beyond order three is unfeasible.

Diagrammatic Monte Carlo (DiagMC) samples diagrams in their topological space using a Markov chain.

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![](_page_30_Picture_7.jpeg)

![](_page_30_Picture_8.jpeg)

*<sup>W</sup>o*(*N*) <sup>1</sup>*<sup>T</sup>* <sup>2</sup>*S*⌃? *. W*<sub>o</sub>(*N*) is an order dependent reweighting factor.  $W_o(N)$  is an order dependent reweighting factor.

is a probability distribution function function. The probability distribution function function. The probabili<br>In the probability distribution function. The probability distribution function function function function. Th

 $\beta$  (C)| $W_o(N)$  is a normalization factor.  $\frac{\omega}{\alpha\beta}\left(\mathcal{C}\right)\vert W_{o}(N)$  is a normalization factor.  $(C)|W_o(N)|$  is a normalization factor  $i\mathcal{Z}^{\omega}_{\alpha\beta}=\int d\mathcal{C}\;|\mathcal{D}^{\omega}_{\alpha\beta}\left(\mathcal{C}\right)|W_o(N)\;$  is a normalization factor

is a probability distribution function is a probability distribution function. is a probability distribution function. ↵(*C*)*|Wo*(*N*)  $\Theta$   $w_{\alpha\beta}^{\omega}(\mathcal{C}):=\frac{\Gamma_{\alpha\beta}(\mathcal{C})\Gamma\Gamma_{\beta}(\mathcal{C})}{\mathcal{Z}^{\omega}}$  is a probability distribution function

![](_page_31_Picture_17.jpeg)

![](_page_31_Picture_18.jpeg)

![](_page_31_Picture_19.jpeg)

### Diagrammatic Monte Carlo: overview Overview of the mathematics of t Oregiew of the mathematic control Monte Carlo: Overview of the math

$$
\Sigma_{\alpha\beta}^{\star}\left(\omega\right)=\sum_{\mathcal{T}}\sum_{\gamma_{1}...\gamma_{n}}\int d\omega_{1}...d\omega_{m}\;\mathcal{D}_{\alpha\beta}^{\omega}\left(\mathcal{T};\gamma_{1}...\gamma_{n};\omega_{1}\right)
$$

 $\bm{W}$ e define  $\bm{\mathcal{C}} := (\bm{\mathcal{T}}; \gamma_1...\gamma_n; \omega_1...\omega_m).$ We define  $C := (\mathcal{T}; \gamma_1...\gamma_n; \omega_1...$  $\mathsf{We define} \quad \mathcal{C} := (\mathcal{T}; \gamma_1...\gamma_n; \omega_1...\omega_m).$ We define *C* := (*T* ; 1*...n*; !1*...*!*m*).  $\mathcal{C} := (\mathcal{T}; \gamma_1...\gamma_n; \omega_1...\omega_m).$ 

$$
\textcolor{blue}{\text{O}} \; w_{\alpha\beta}^{\omega} \left( \mathcal{C} \right) := \frac{|\mathcal{D}^{\omega}_{\alpha\beta}(\mathcal{C})|W_o(N)}{\mathcal{Z}^{\omega}_{\alpha\beta}} \; \; \text{ is a probability}
$$

**WERSITÀ DEGLI STUE**<br>ARTIMENTO DI FISIC *|D*! ↵(*C*)*|Wo*(*N*) *NO*  $S_{\rm E}$ 

DIPARTIMENTO DI FISICA

$$
\Sigma^{\star}_{\alpha\beta}(\omega) = \int d\mathcal{C} \left[ \mathcal{D}^{\omega}_{\alpha\beta}(\mathcal{C}) \right] e^{i \arg[\mathcal{D}^{\omega}_{\alpha\beta}(\mathcal{C})]} \mathbb{1}_{\mathcal{T} \in \mathcal{S}_{\Sigma^{\star}}}
$$

orde *Wo*(*N*) is an order dependent reweighting factor.  $W_o(N)$  is an order  $\overline{M}$   $\overline{M}$   $\overline{M}$   $\overline{M}$   $\overline{M}$ ↵ (*C*)*|Wo*(*N*) is a normalization factor.

![](_page_31_Figure_13.jpeg)

$$
\Sigma^{\star}_{\alpha\beta}(\omega) = \mathcal{Z}^{\omega}_{\alpha\beta} \int d\mathcal{C} \, \frac{|\mathcal{D}^{\omega}_{\alpha\beta}(\mathcal{C})|W_o(N)}{\mathcal{Z}^{\omega}_{\alpha\beta}} \frac{e^{i \arg[\mathcal{D}^{\omega}_{\alpha\beta}(\mathcal{C})]}}{W_o(N)} \mathbf{1}_{\mathcal{T} \in \mathcal{S}_{\Sigma^{\star}}}
$$

$$
\mathcal{Z}^{\omega}_{\alpha\beta} = \int d\mathcal{C} \, | \mathcal{D}^{\omega}_{\alpha\beta}(\mathcal{C}) | W_o(N) \text{ is a non-}
$$

### S. Brolli (Masters thesis)

 $\frac{\omega}{\alpha\beta}\left(\mathcal{T}; \gamma_1...\gamma_n; \omega_1...\omega_m\right) 1_{\mathcal{T} \in \mathcal{S}_{\Sigma^{\star}}}$  $\left(\mathcal{I} ; \gamma_1...\gamma_n; \omega_1...\omega_m\right) 1_{\mathcal{T} \in \mathcal{S}_{\Sigma^{\star}}}$ *<sup>d</sup>*!1*...d*!*<sup>m</sup> <sup>D</sup>*!  $\tau$ <sup>*n*</sup>  $\gamma$ <sup>1</sup>,  $\gamma$ <sup>2</sup>, 2<sup>*u*</sup> (*1*) *n*<sub>*d*</sub> (*1*) *n*<sub>*d*</sub> (*1*) *n*<sub>*d*</sub> (*1*) *n*<sub>*d*</sub> (*1*) *n*<sub>d</sub> (*1*) *n* 

## **The updates** The updates

<sup>1</sup> *Change Frequency*

### <sup>2</sup> *Change Single-Particle Quantum Numbers*

![](_page_32_Picture_3.jpeg)

*Change Frequency***:** 

### Standard Monte Carlo

![](_page_32_Picture_14.jpeg)

![](_page_32_Picture_15.jpeg)

![](_page_32_Figure_5.jpeg)

correct equilibrium distribution (*w*!  $i$ *Change Single-Particle Quantum Numbers*:

![](_page_32_Figure_7.jpeg)

![](_page_32_Picture_8.jpeg)

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![](_page_33_Picture_8.jpeg)

![](_page_33_Figure_1.jpeg)

![](_page_33_Figure_2.jpeg)

 $\omega^\prime_1$  is drawn from the probability distribution  $W_f\left(\omega^\prime_1\right)$ .

The unphysical propagators are turned into The unphysical propagators are turned into physical ones when reconnected.

![](_page_33_Picture_12.jpeg)

$$
q_{AL} = \frac{\left|g\right|}{4\pi} \frac{1}{W_f\left(\omega_1'\right)} e^{-k\omega_1'^2} |G_\alpha\left(\omega\right)| \frac{W_o(3)}{W_o(2)}
$$

![](_page_33_Picture_5.jpeg)

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DIPARTIMENTO DI FISICA

*Reconnect*:

![](_page_33_Figure_10.jpeg)

![](_page_33_Picture_13.jpeg)

![](_page_33_Picture_14.jpeg)

### S. Brolli (Masters thesis)

self-energy is

WHERE A AND ARRIVE A AND A AND A AND A AND A LABEL TO MILLAND

**E**<br>
FORBIS, ENERGY INDENTIALED BY THE CORRECT AND ENERGY INDEPENDENTIALED AND ENERGY INDEPENDENTIALED AND ENERGY IN  $t$  fisical energy independent and  $t$ 

### It has never been tried for systems with discrete energy levels (nuclear This method has been implemented for infinite systems. It has never been tried for systems with discrete energy levels (nuclear study elastic scattering of neutrons from 16O and 40Ca. The study elastic scattering of neutrons from 16O and<br>And 40Ca. The study elastic scattering of neutrons from 16O and 40Ca. The study elastic scattering of neutrons Formalism.—The Hamiltonian used to compute the study elastic scattering of neutrons from <sup>16</sup>O and <sup>40</sup>Ca. Formalism.—The Hamiltonian used to compute the

actions. We then use a saturations are considered as a saturation of the saturation of the saturation of the s<br>And the saturation of the saturation of

![](_page_34_Figure_2.jpeg)

![](_page_34_Figure_3.jpeg)

Although the scattering waves are unbound, the self-

### **Results of the simulation for D=4** *4.3. RESULTS OF THE SIMULATION FOR D* = 4 57  $emol$ linulation for D=4 integrable functions. Hence, we proceed by calculating functions. Hence, we proceed by calculating functions.<br>Hence, we proceed by calculating functions. Hence, we proceed by calculating functions. Hence, we proceed the Although the scattering waves are unbound, the selfenergy  $b = 4$  associated with the optical potential potential  $b = 4$ localized, and it can be efficiently expanded on square

### Richardson pairing model with D states, nall It has never been tried for systems with discrete energy levels (nuclear dson pairing model with D states, half filled WILLI D'SLALL. **c**:  $\frac{1}{2}$ Richardson pairing model with D states, half filled:

This method has been implemented for infinite systems  $\mathcal{L}_\text{max}$  infinite systems. In finite systems.

$$
\Sigma^{\star}_{\alpha\beta}(\omega) = \Sigma^{(\infty)}_{\alpha\beta} + \sum_{i,j} \mathbf{M}^{\dagger}_{\alpha,i} \left( \frac{1}{E - (\mathbf{K}^{\gt} + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left( \frac{1}{E - (\mathbf{K}^{\lt} + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}^{\dagger}_{s,\beta} \qquad \frac{F_{\alpha,r}}{\alpha}
$$

![](_page_34_Picture_5.jpeg)

![](_page_34_Figure_9.jpeg)

![](_page_34_Figure_10.jpeg)

affect our conclusions below. The conclusions below. The conclusions below. The conclusions below. The conclusions of the concl

Figure 4.1: Components  $\alpha = 0$  and  $\alpha = 2$  of the imaginary part of the self-energy for different values of the coupling *g*. The blue line is the results obtained with the BDMC simulation, while the red line is the best fit as a sum of two Lorentzians. The results for the two values of  $\alpha = 0, 2$  $\ell$ ,  $\ell$ ,  $\delta$  are displayed respectively on the left and on the right of the graph. The error bars are calculated as explained in the main text. as explained in the main text.  $\sum_{s=1}^{\infty}$   $\sum_{s=1}^{\infty}$  salues of the coupling g. The blue line is the results obtained with the BDMC simulation, while<br>the red line is the best fit as a sum of two Lorentzians. The results for the two values of  $\alpha$ 

affect our conclusions below. The conclusions below. The conclusions below. The conclusions below. The conclusions of the concl

![](_page_34_Picture_13.jpeg)

## Reorganization in terms of ladders (T)

![](_page_35_Figure_2.jpeg)

## **Ongoing extensions to nuclei in no-core model spaces**

DiagMC is being extended to treat realistic microscopic nuclear Hamiltonians.

Example of DiagMC neutron  $p_{1/2}$  self-energy partial wave at 2nd order in harmonic oscillator space with dimension  $N_{\text{max}}=$  max  $\{2n+1\}=2$  in  $160$ .

![](_page_36_Figure_3.jpeg)

![](_page_36_Picture_4.jpeg)

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![](_page_36_Picture_9.jpeg)

![](_page_36_Picture_10.jpeg)

![](_page_36_Picture_11.jpeg)

## **Summary**

→ Occurrence of a charge bubble in 46Ar (second case "known")

*M. Vorabbi, P. Arthuis*

*P. Navrártil*

*V. Somà, T. Duguet, A. Scalesi*

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- $\rightarrow$  HAL QCD share characteristic with low energy EFT interactions though they are different. → HAL QCD forces allow studying (un)physical quark masses and improve in Y-N description.
- → SCGF Gorkov/ADC(3) computations with ChEFT reliable and evolving to large masses
	-
- $\rightarrow$  Diagrammatic Monte Carlo is a promising method to go forward on high precision simulations.

![](_page_37_Picture_17.jpeg)

![](_page_37_Picture_19.jpeg)

![](_page_37_Picture_20.jpeg)

![](_page_37_Picture_21.jpeg)

*G. Colò, E. Vigezzi, S. Brolli*

![](_page_37_Picture_8.jpeg)

![](_page_37_Picture_22.jpeg)

![](_page_37_Picture_23.jpeg)

![](_page_37_Picture_25.jpeg)

![](_page_37_Picture_26.jpeg)

## **And thanks to my collaborators (over the years…):**

![](_page_37_Picture_5.jpeg)

![](_page_37_Picture_6.jpeg)

![](_page_38_Picture_0.jpeg)

![](_page_38_Picture_1.jpeg)

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![](_page_38_Picture_4.jpeg)

![](_page_38_Picture_5.jpeg)

## **SCGF computations of infinite matter**

![](_page_39_Picture_1.jpeg)

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DIPARTIMENTO DI FISICA

![](_page_39_Picture_4.jpeg)

*F. Marino (PhD Thesis)*

![](_page_39_Picture_6.jpeg)

![](_page_39_Picture_7.jpeg)

![](_page_39_Picture_8.jpeg)

![](_page_39_Picture_9.jpeg)

*Machine-learn DFT functional on the nuclear equation of state Benchmark in finite systems both the nuclear equation of state* 

![](_page_40_Picture_9.jpeg)

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demanding: need to link the EDF to

### **Nuclear energy density functionals grounded in** *ab initio* **calculations** loor energy density functionals grounded in *ah initio c*alc

F. Marino  $\bullet$ , <sup>1,2,\*</sup> C. Barbieri  $\bullet$ , <sup>1,2</sup> A. Carbone, <sup>3</sup> G. Colò  $\bullet$ , <sup>1,2</sup> A. Lovato  $\bullet$ , <sup>4,5</sup> F. Pederiva, <sup>6,5</sup> X. Roca-Maza  $\bullet$ , <sup>1,2</sup> and E. Vigezzi  $\bullet$ <sup>2</sup> and E. Vigezzi  $\mathbf{P}^2$ <sup>1</sup>Dipartimento di Fisica "Aldo Pontremoli," Università degli Studi di Milano, 20133 Milano, Italy <sup>2</sup>Istituto Nazionale di Fisica Nucleare, Sezione di Milano, 20133 Milano, Italy <sup>3</sup>*Istituto Nazionale di Fisica Nucleare–CNAF, Viale Carlo Berti Pichat 6/2, 40127 Bologna, Italy* <sup>1</sup>*Dipartimento di Fisica "Aldo Pontremoli," Università degli Studi di Milano, 20133 Milano, Italy* Istituto Nazionale al Estea Nueleare, sezione al mitano, 20155 mitano, ne<br>Into Nazionale di Fisica Nucleare\_CNAF\_Viale Carlo Rerti Pichat 6/9 40197 Re

DFT is in principle exact – but the energy density functional (EDF) is not known  $\frac{2^{20}}{\epsilon}$ 

For nuclear physics this is even more and a state of the nuclear state of t

theories rooted in QCD!

### **Nuclear Density Functional from Ab Initio Theory** such as isospin-dependent spin-orbit contributions, cannot be expired spin-orbit contributions, cannot be expe<br>In the contributions, cannot be experimented by the contributions, cannot be experimented by the contributions somewhat simplified interactions, but it can be applied to

### PHYSICAL REVIEW C 104, 024315 (2021)

compute larger nuclei and nuclei<br>The computer matter is a matter of the computer matter in the computer state of the computer state of the comp

![](_page_40_Picture_11.jpeg)

![](_page_40_Figure_12.jpeg)

 $\overline{\phantom{0}}$ 

### Gradient terms are important (but SCGF/NNLO-sat : they seem to work!):

F. Marino, G. Colò, CB et al., Phys Rev. C**104**, 024315 (2021)  $C_{\alpha}$  is  $\alpha$  is monochromatic too and  $\alpha$   $\alpha$   $\alpha$   $\beta$   $\circ$ o, G. Colò, CB et al., Phys Rev. C**104**, 0

![](_page_41_Figure_4.jpeg)

![](_page_41_Picture_13.jpeg)

### Need to extract gradient information from non-uniform matter

![](_page_41_Picture_7.jpeg)

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sity and energy of the g.s. of the perturbed system are

called  $\mathcal{P}^{(x)}$ , respectively. If the external positively. If the external positively. If the external positive<br>If the external positively. If the external positively. If the external positively. If the external posit

tential is weak enough the tential is weak enough the treated perturbation of the treated perturbation of the<br>Interaction of the treated perturbation of the treated perturbation of the treated perturbation of the treated

batively (see e.g. Refs. [43, 47]). The density fluctuation is e.g. Refs. [43, 47]. The density fluctuation fluctuation is e.g. Refs. [43, 47]. The density fluctuation is e.g. Refs. [43, 47]. The density fluctuation is e.g

### **Benchmark on finite systems** batively (see e.g. Refs.  $\mathbb{R}^d$  ). The density fluctuation  $\mathbb{R}^d$  ,  $\$ induced by *v*(x), in particular, is linear in the external

*Machine-learn DFT functional on the nuclear equation of state* 

—>

![](_page_41_Figure_10.jpeg)

# and in the systems of the interest of  $\boldsymbol{p}$

![](_page_41_Figure_5.jpeg)

mine the occupied states of the A-particle FG g.s. once

![](_page_41_Picture_2.jpeg)

Finite size box (of length L) with periodic boundary conditions: **Construction of the Model Space** For simplicity, we assume a total number *A* of nucleons in the density periodic box. For boxes of length **L**, the density and periodic boundary condi

> ; (11.54) C)  $\overline{1}$  $\frac{1}{2}$

*H*  $\alpha$ 

![](_page_42_Figure_6.jpeg)

of the kinetic term and an auxiliary one-body operator *U*

b

. Its choice defines the

![](_page_42_Figure_7.jpeg)

"h˚*<sup>A</sup>*

<sup>0</sup> j*T* Œ*H*

b*I*

1.*t*1/::: *H*

b*I*

1.*tn*/*a<sup>I</sup>*

!

### **ADC(3) computations for infinite matter** retain the full momentum space and write the SCGF equations already in the full temperatures and in a full scale  $\sim$  for a function and will be discussed function and will be discussed function reference state, <sup>j</sup>˚*<sup>A</sup>* <sup>0</sup> i, and the corresponding unperturbed propagator *g*.0/.!/ that mputations tor intinite mattle *H* **1** <u>P</u>  $\frac{1}{2}$ , where ba<br>Barat<br>Barat *V* denotes the two-body interaction operator and *W* ba<br>Barat<br>Barat is the three-body interaction. In a second-quantized framework, the full Hamiltonian interaction. In a second-qua<br>The full Hamiltonian interaction in a second-quantized framework, the full Hamiltonian interaction in a second reads: *H* <sup>D</sup> <sup>X</sup> ˛*a*˛ !<sup>X</sup> "0 ˛ *a*! conditions coded by the ˙*i*% terms in the denominators. This implies a dispersion relations for infinite mothem to Correspondingly, the direct coupling of single particle orbits to ISCs (of 2p1h and  $2$ h1p character or more complex) imposes the separable structure of the separable structure of the residues. In conditions conditions coded by the denominators. This implies a dispersion and denominators. This implies a di relation that can link the real and imaginary parts of the self-energy [22, 26]. Corresponding the direct coupling of single particle orbits to ISCs (orbits to ISCs (orbits to ISCs (orbits to 2h1p character or more complex) imposes the separable structure of the residues. In

b

### ADC(3) self energy:

<sup>1</sup> D !*U*

b

C

b

*V* C *W*

b

b

*U*˛ˇ *a*!

˛*a*<sup>ˇ</sup> C

1

X

*V*˛";ˇı *a*!

 $\overline{F}$  Marino CB et al in preparation WFN- $\blacksquare$  ividinity, OD ot di., in proparation  $\blacksquare$  in  $\blacksquare$ F. Marino, CB et al., **in preparation**

b*I*

<sup>1</sup>.*t*/, *a<sup>I</sup>*

![](_page_42_Picture_8.jpeg)

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DIPARTIMENTO DI FISICA

˛.*t*/ and *a<sup>I</sup>*

ˇ

$$
\rho = \frac{A}{L^3} \qquad p_F = \sqrt[3]{\frac{6\pi^2 \rho}{\nu_d}}
$$

$$
\phi(x+L,y,z)=\phi(x,y,z)
$$

$$
\widehat{H} = \sum_{\alpha} \varepsilon_{\alpha}^{0} a_{\alpha}^{\dagger} a_{\alpha} - \sum_{\alpha \beta} U_{\alpha \beta} a_{\alpha}^{\dagger} a_{\beta} + \frac{1}{4} \sum_{\substack{\alpha \gamma \\ \beta \delta}} V_{\alpha \gamma, \beta \delta} a_{\alpha}^{\dagger} a_{\gamma}^{\dagger} a_{\delta} a_{\beta} + \frac{1}{36} \sum_{\substack{\alpha \gamma \epsilon \\ \beta \delta \eta}} W_{\alpha \gamma \epsilon, \beta \delta \eta} a_{\alpha}^{\dagger} a_{\gamma}^{\dagger} a_{\epsilon}^{\dagger} a_{\eta} a_{\delta} a_{\beta}.
$$

$$
\Delta \mathsf{DC}(3) \text{ self energy:}
$$
\n
$$
\Sigma_{\alpha\beta}^{(\star)}(\omega) = -U_{\alpha\beta} + \Sigma_{\alpha\beta}^{(\infty)} + M_{\alpha,r}^{\dagger} \left[ \frac{1}{\omega - [E^{\rangle} + C]_{r,r'} + i\eta} \right]_{r,r'} M_{r',\beta} + N_{\alpha,s} \left[ \frac{1}{\omega - (E^{\langle} + D) - i\eta} \right]_{s,s'} N_{s',\beta}^{\dagger}
$$

![](_page_42_Picture_15.jpeg)

![](_page_42_Picture_16.jpeg)

*...*