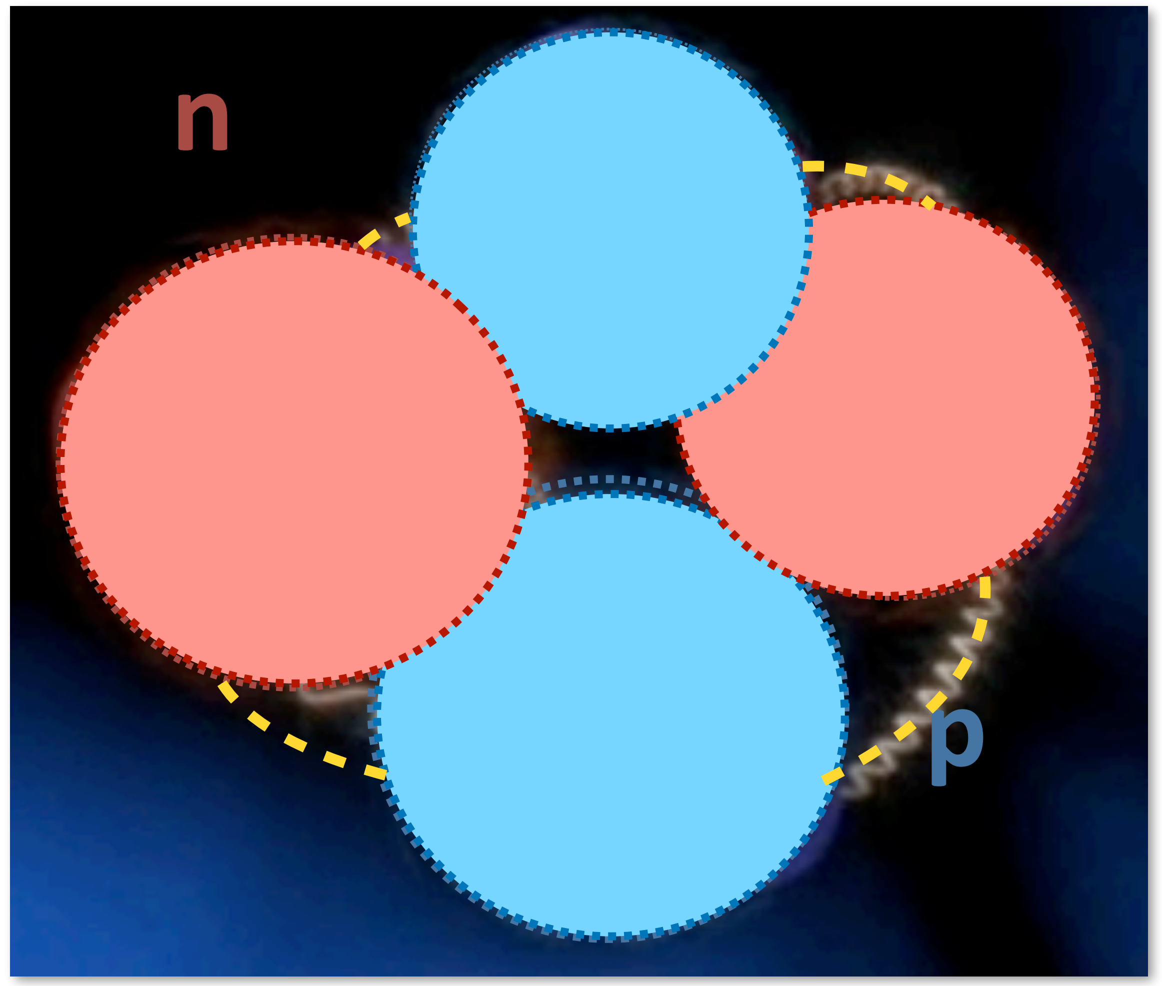
The 0^+ excited state of the α-particle

Sonia Bacca

ECT* Workshop, The nuclear interaction: post-modern developments, Aug 23rd, 2024

The α-particle



Few-body system
Well bound and stable system

The α-particle ground state

Benchmark with different few-body methods

Same ingredients

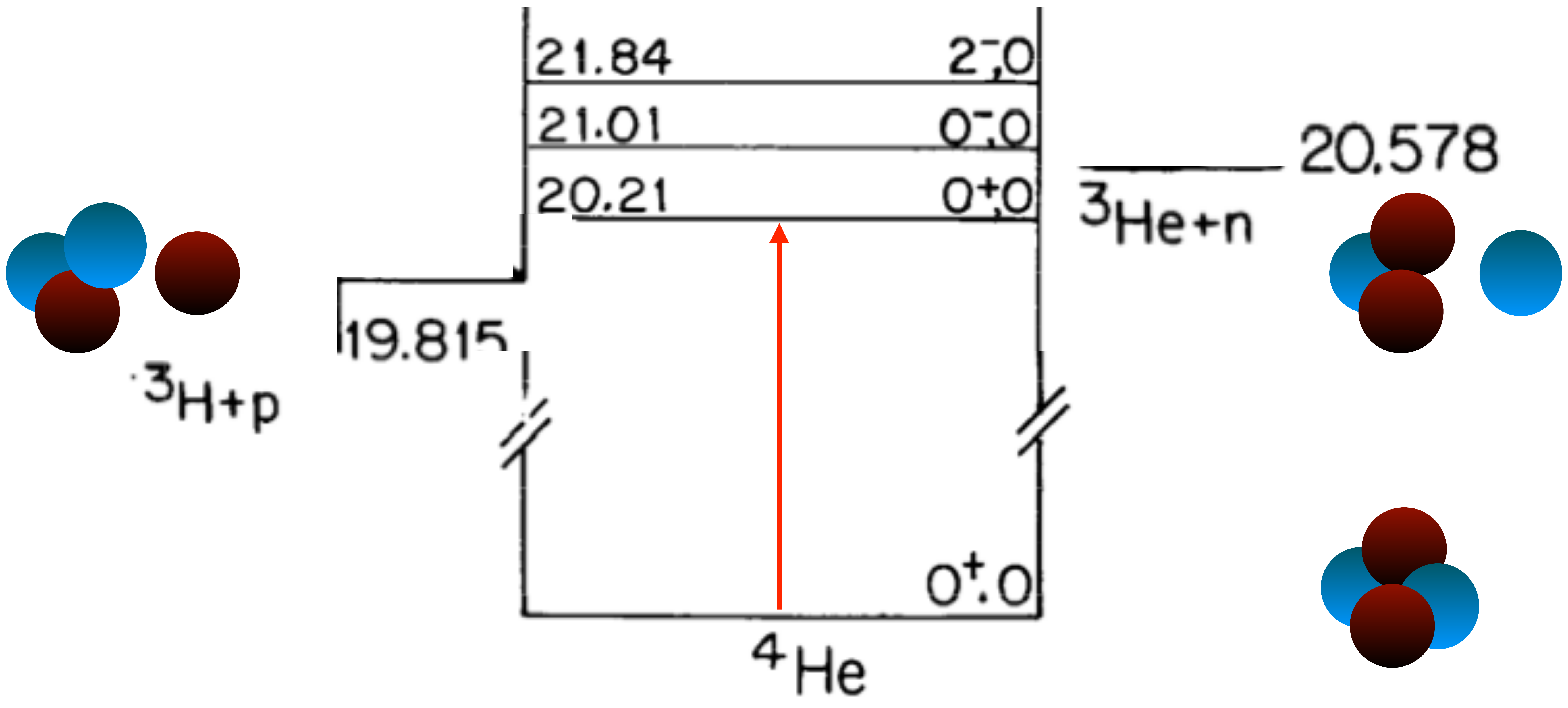
Different numerical methods

Same output

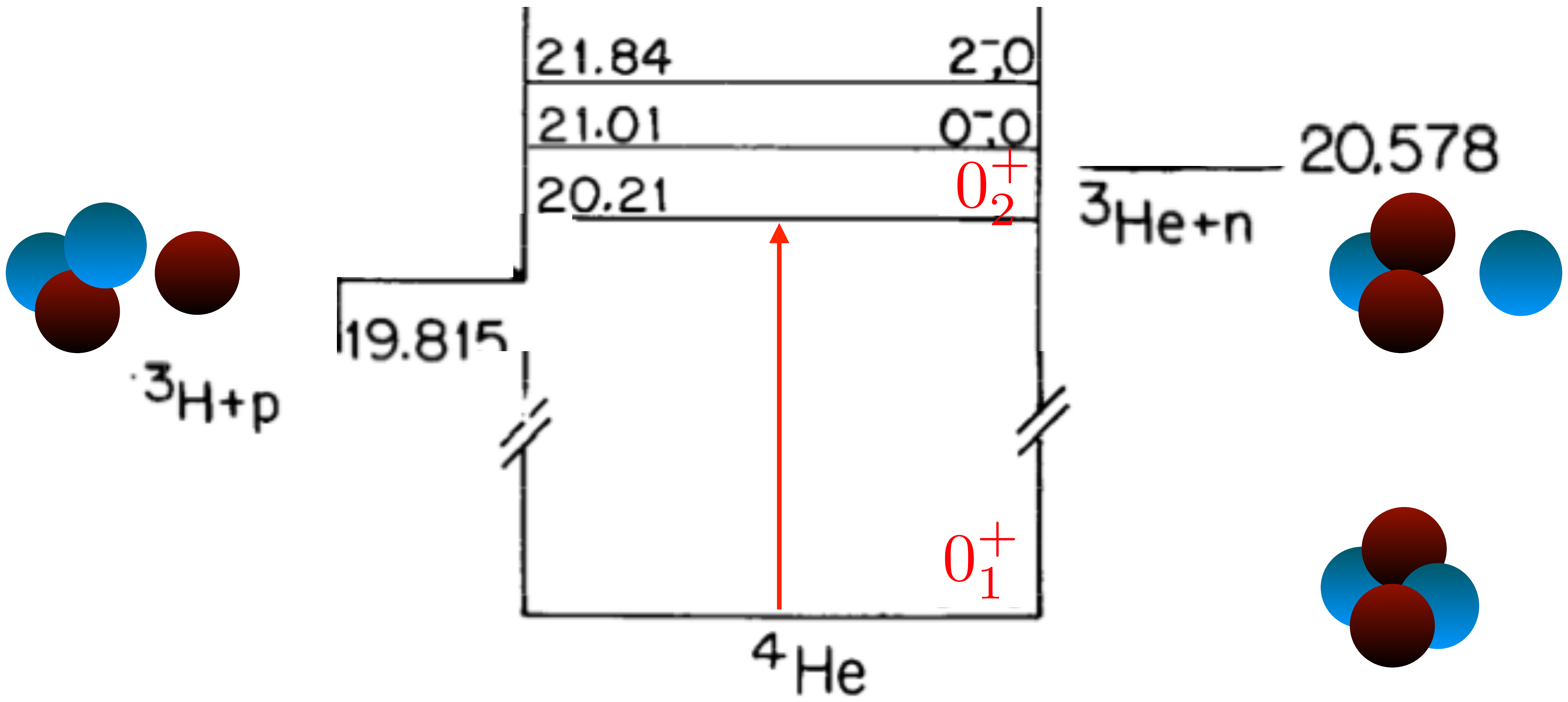
Kamada et al, PRC 64 (2001) 044001

Method	$\langle T \rangle$	$\langle V \rangle$	E_0	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

The α-particle



The α-particle



Gattobigio and Kievsky, FBS **64**, 86 (2023)

In the absence of Coulomb, the 0_2^+ state is a bound Efimofv state realization in nuclear physics. The Coulomb force transforms it to a resonance pushed above the particle-emission threshold.

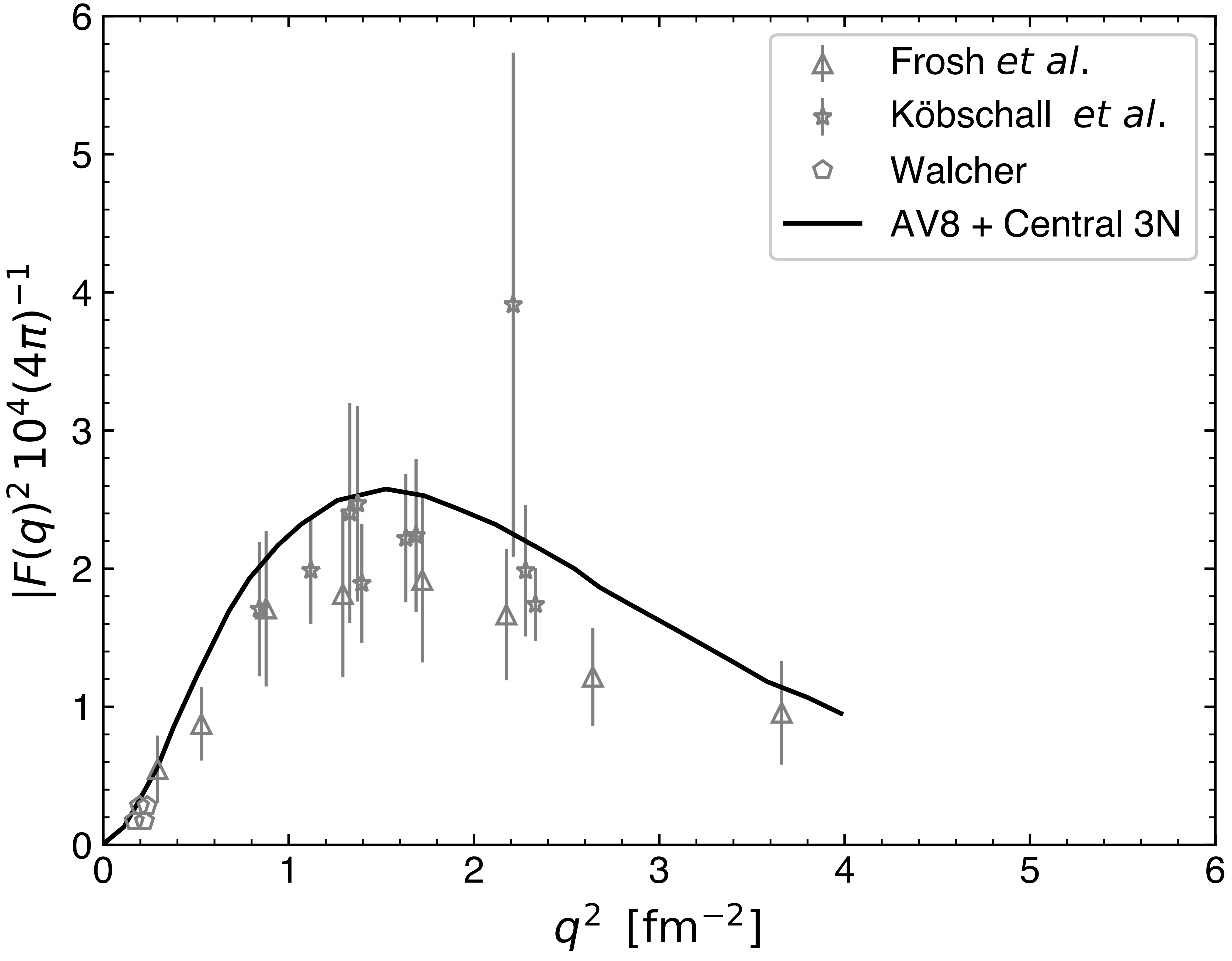
The α-particle transition form factor $0_1^+ \rightarrow 0_2^+$

Status quo in 2004

Hiyama et al. PRC **70**, 031001(R) (2004)

Method: Gaussian expansion basis

Potential: simplified force



The α-particle transition form factor $0_1^+ \to 0_2^+$

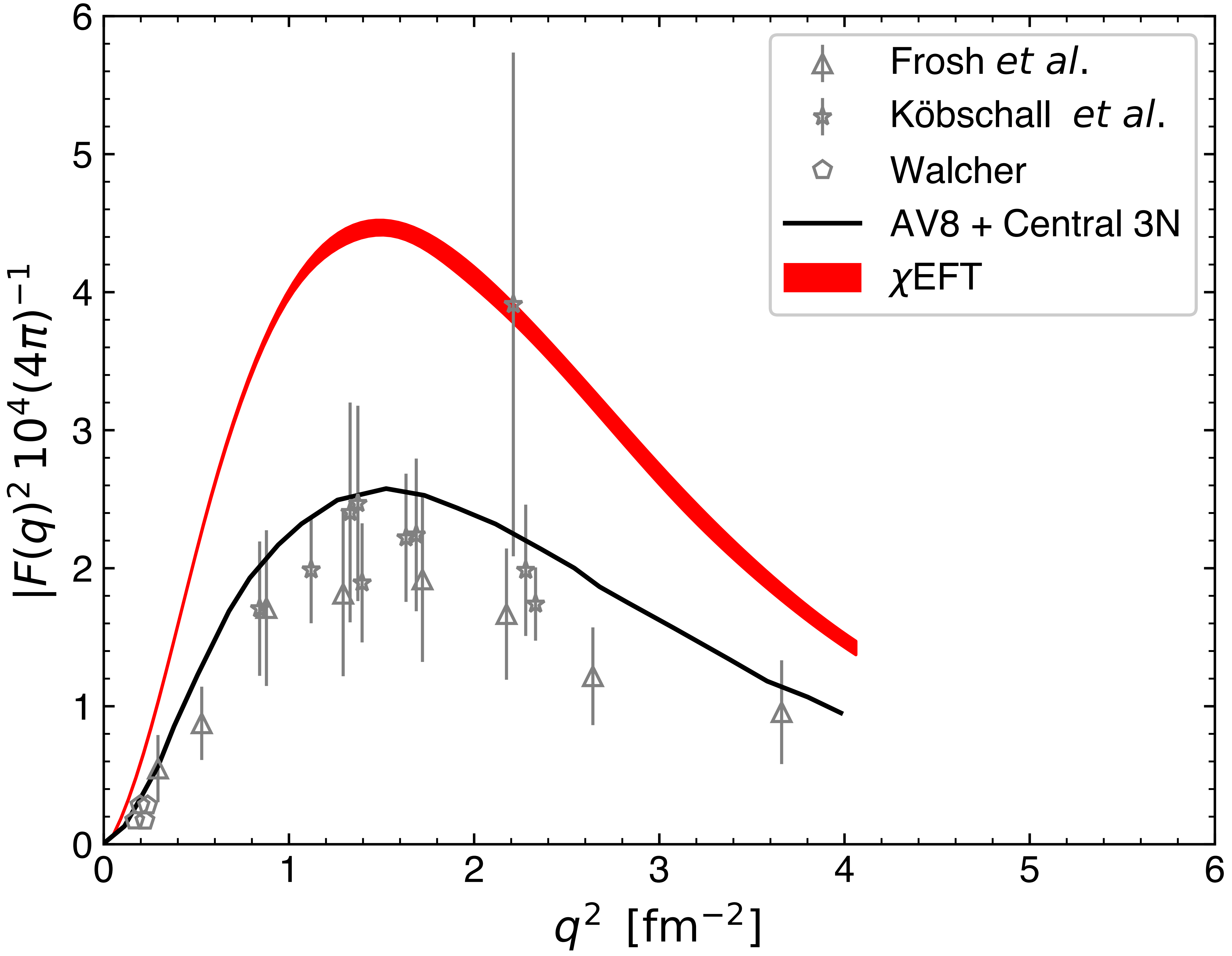
Status quo in 2013

Bacca et al. PRL **110**, 042503 (2013)

Method: Lorentz integral transform + hyperspherical harmonics (LIT + HH)

Potential: realistic NN+3N from χEFT

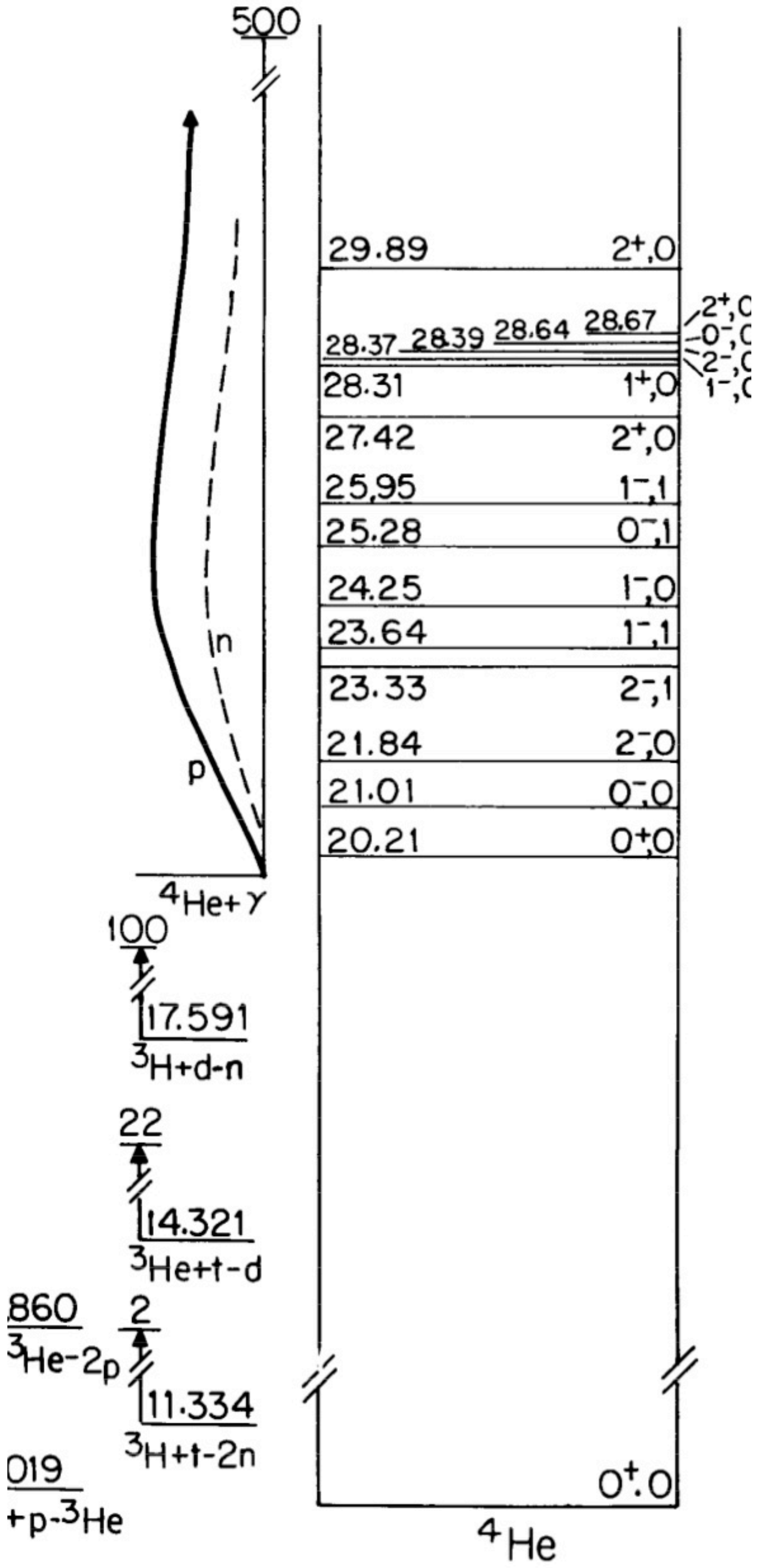
What is going on here?



The α-particle

Excitation of the 1$^-$ states

Success story for LIT+HH

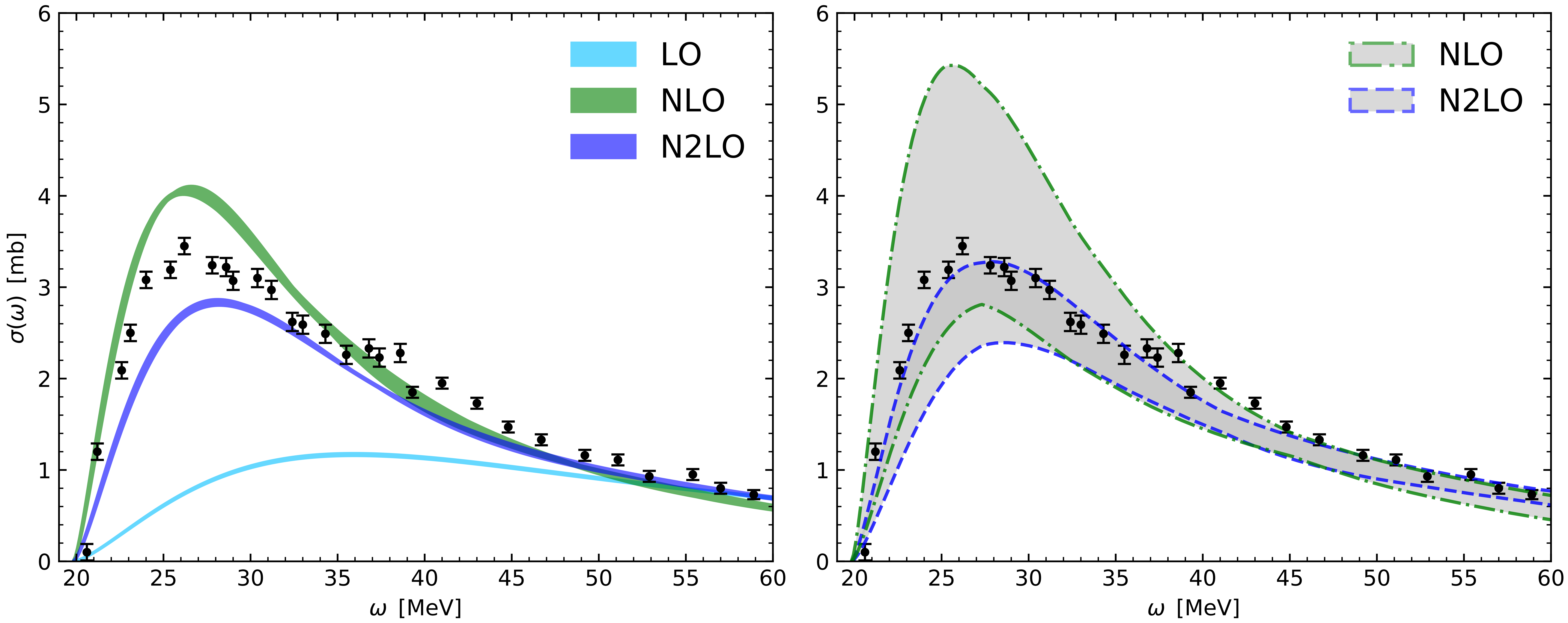


^{4}He photoabsorption cross-section

$$\sigma_\gamma = \frac{4\pi^2\alpha}{3}\omega R^{E1}(\omega)$$

Acharya, SB, Bonaiti, Li Muli, Sobczyk, Front. Phys.10:1066035 (2023)

With local chiral potentials from Phys. Rev. C **90**, 054323 up to N2LO, Method: LIT + HH



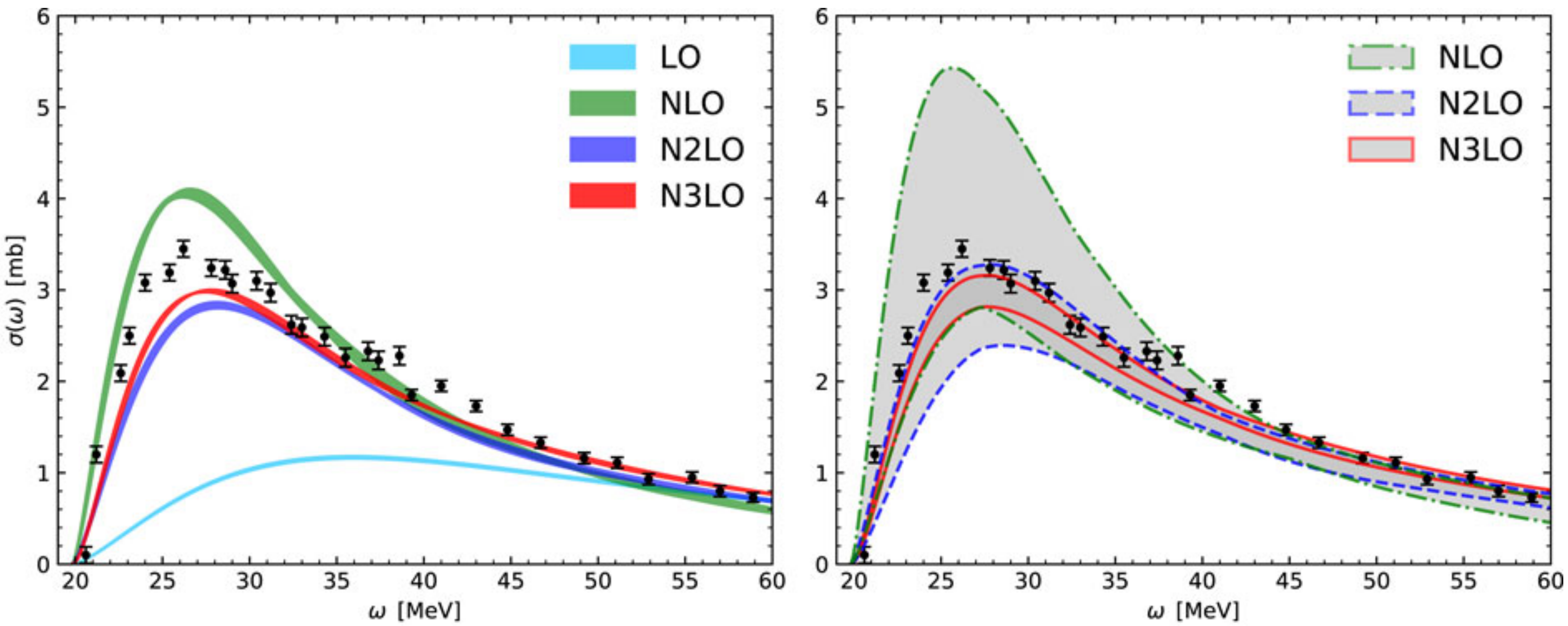
$$\delta_{\mathcal{O}}^{\chi\text{EFT}} = \max\left\{\left(\frac{Q}{\Lambda}\right)^{k+1}|\mathcal{O}_{\nu_0}|, \left(\frac{Q}{\Lambda}\right)^{k}|\mathcal{O}_{\nu_0+1}-\mathcal{O}_{\nu_0}|, \ldots, \left(\frac{Q}{\Lambda}\right)|\mathcal{O}_{\nu_0+k}-\mathcal{O}_{\nu_0+k-1}|\right\}$$

^{4}He photoabsorption cross-section

$$\sigma_\gamma = \frac{4\pi^2\alpha}{3}\omega R^{E1}(\omega)$$

Acharya, SB, Bonaiti, Li Muli, Sobczyk, Front. Phys.10:1066035 (2023)

Adding N3LO from Enter and Machleid + 3NF at N2LO from Navratil, Method: LIT + HH



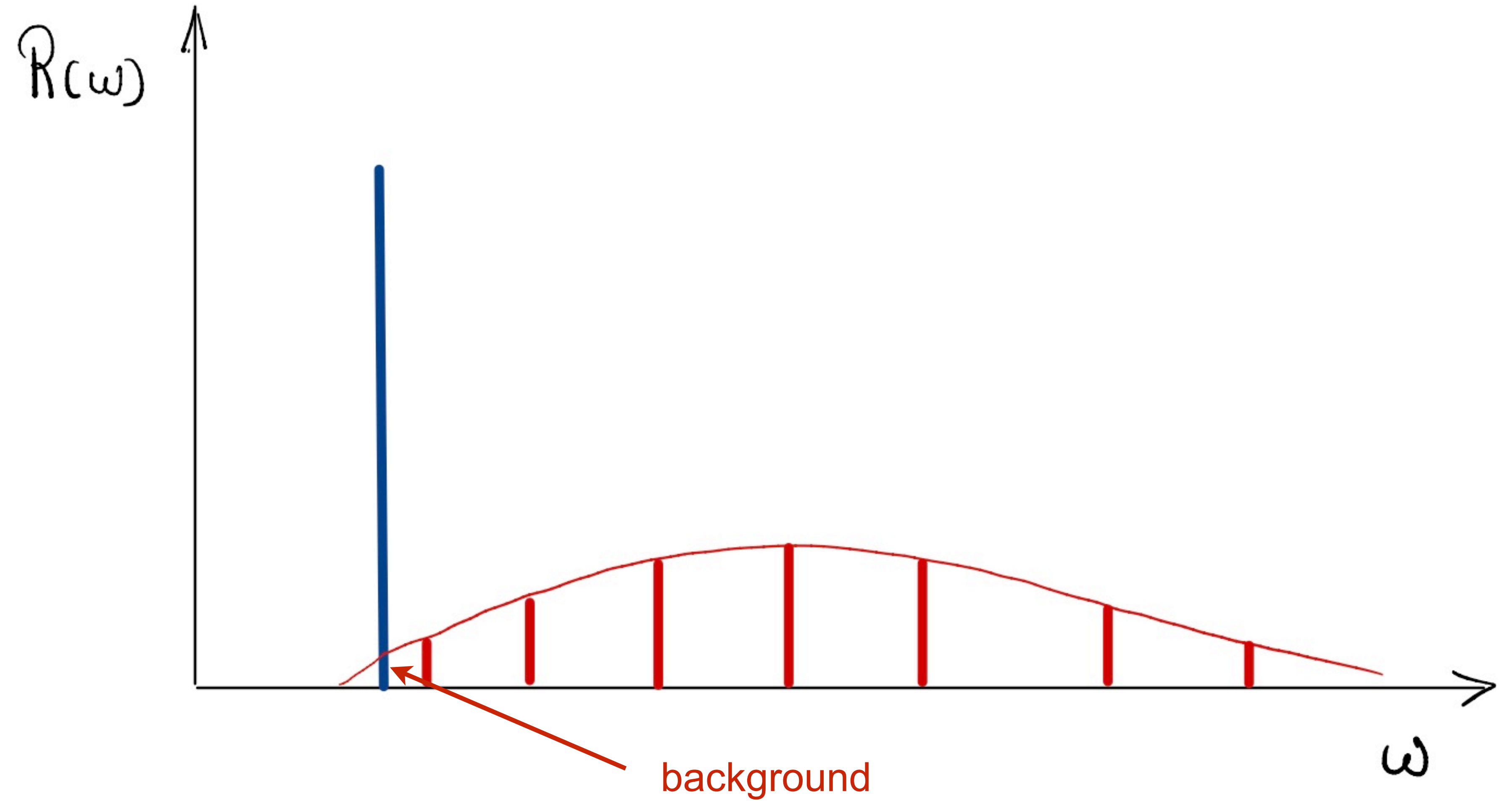
$$\delta_{\mathcal{O}}^{\chi\mathrm{EFT}} = \max\left\{\left(\frac{Q}{\Lambda}\right)^{k+1}|\mathcal{O}_{\nu_0}|, \left(\frac{Q}{\Lambda}\right)^{k}|\mathcal{O}_{\nu_0+1}-\mathcal{O}_{\nu_0}|, \ldots, \left(\frac{Q}{\Lambda}\right)|\mathcal{O}_{\nu_0+k}-\mathcal{O}_{\nu_0+k-1}|\right\}$$

LIT + HH for the monopole transition $0_1^+ \to 0_2^+$

$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2} \int d\omega R^{\rm res}_{\mathcal{M}}(q,\omega) \qquad R(q,\omega) = R^{\rm res}_{\mathcal{M}}(q,\omega) + R^{\rm bg}_{\mathcal{M}}(q,\omega)$$

For fixed *q*, apply integral transform

$$L(\sigma,\Gamma) = \int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \langle\tilde\psi|\tilde\psi\rangle$$



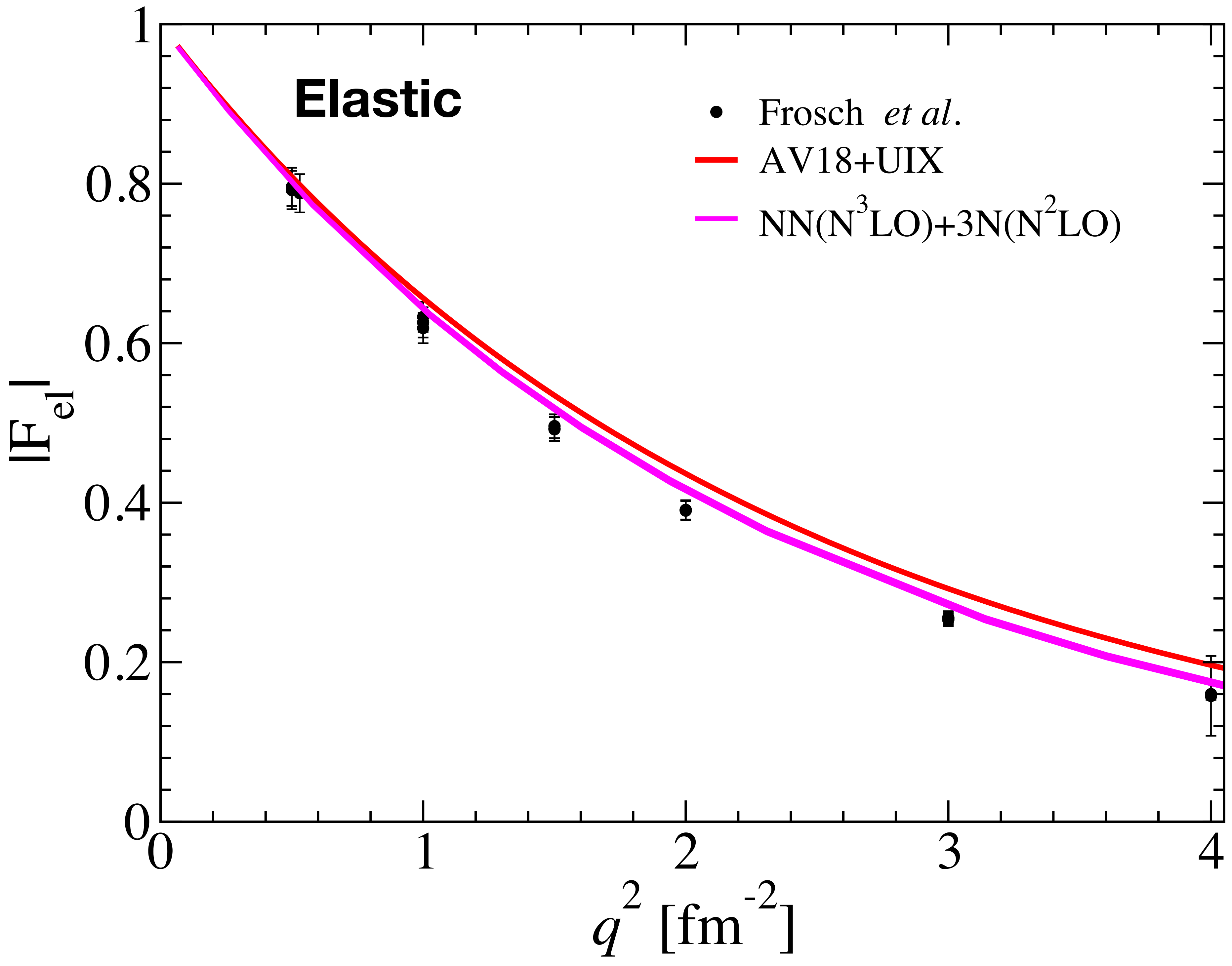
and solve bound-state-like equation
with hyperspherical harmonics (HH)

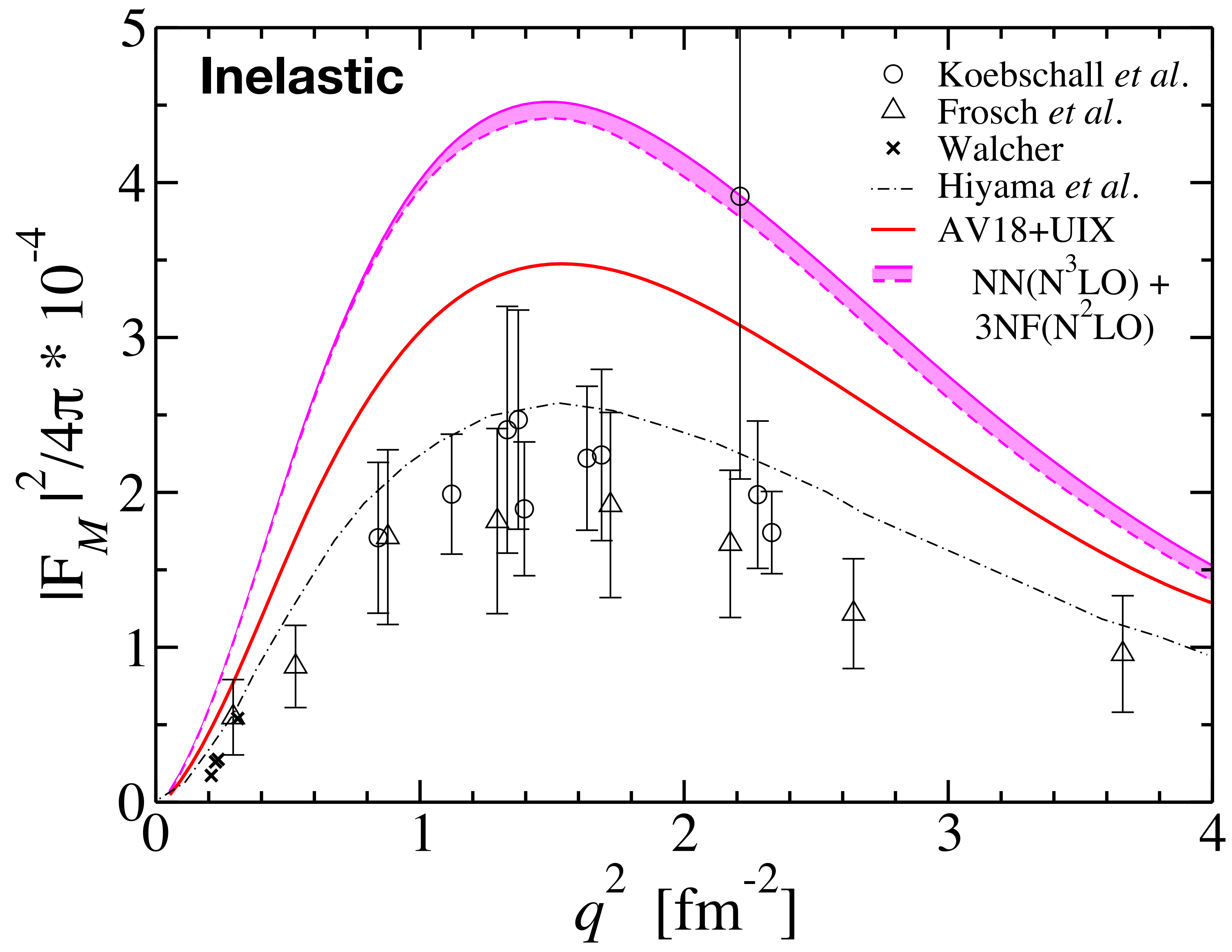
$$(H - E_0 - \sigma + i\Gamma)\,|\,\tilde\psi\rangle = \mathcal{M}|\,\psi_0\rangle$$

We see one pronounced peak at the resonant energy, and many others at higher energy
Exploit the power of the LIT method to subtract the background

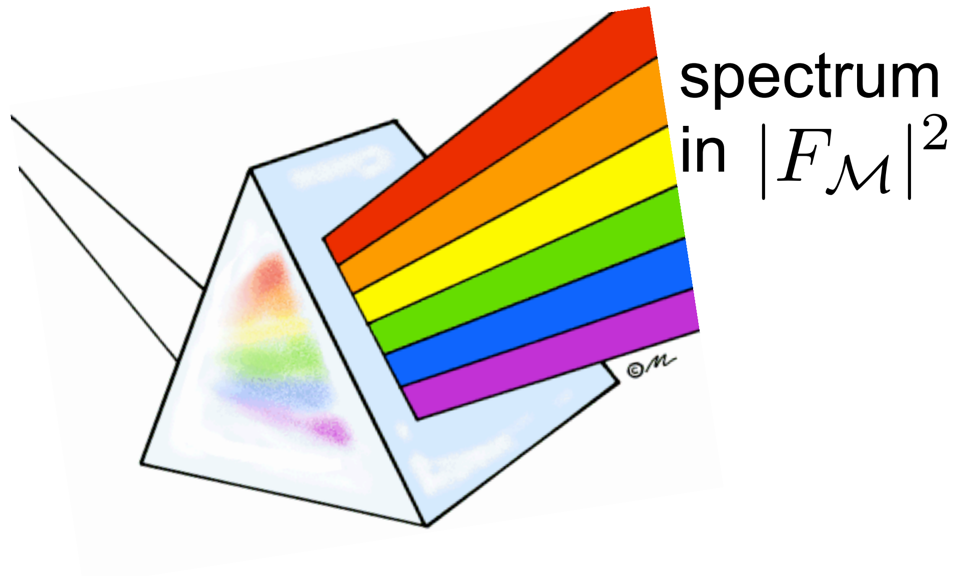
The α-particle transition form factor $0_1^+ \to 0_2^+$

Status quo in 2013





AV8' + central 3NF E_0 = -28.44 MeV
AV18+UIX E_0 = -28.40 MeV
NN(N³LO)+3NF(N²LO) E_0 = -28.36 MeV
E_0^{exp}= -28.30 MeV

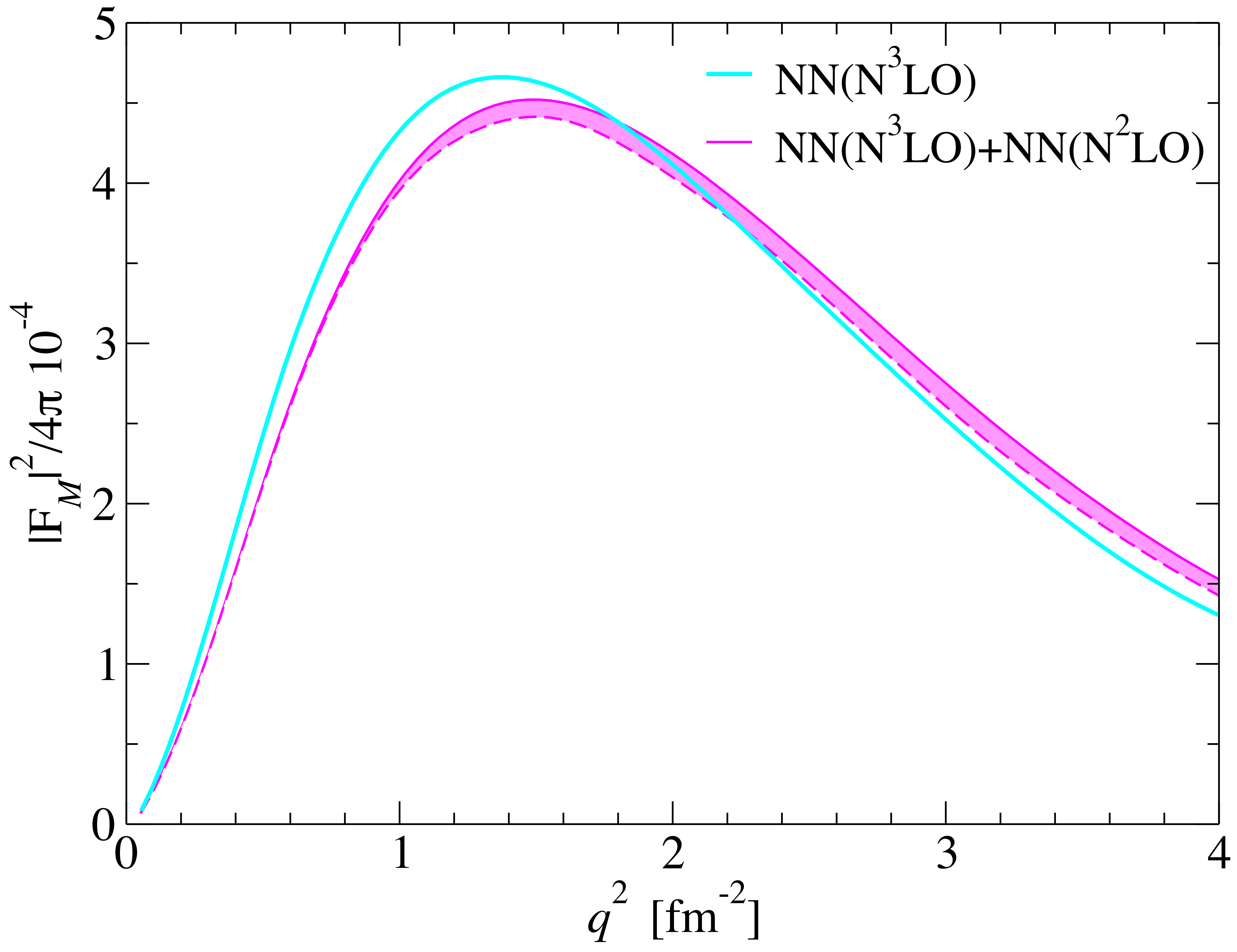


The α-particle transition form factor $0_1^+ \to 0_2^+$

Checks

- Numerics? Our calculations are well converged (few % level) in the HH basis

$K_{\max}$	12	14	16	18
$10^4 \lvert F_{\mathcal{M}} \rvert^2$	4.59	4.75	4.85	4.87



- Many-body charge operators?

Impulse approximation valid for elastic form factor below 2 fm^{-1} Viviani *et al.*, PRL **99** (2007) 112002

Many-body operators appear at high oder in EFT

- Higher order 3NF (N^3LO)? Unlikely...

- Can the experiment be wrong?

New experiment

Mainz

Status quo in 2023

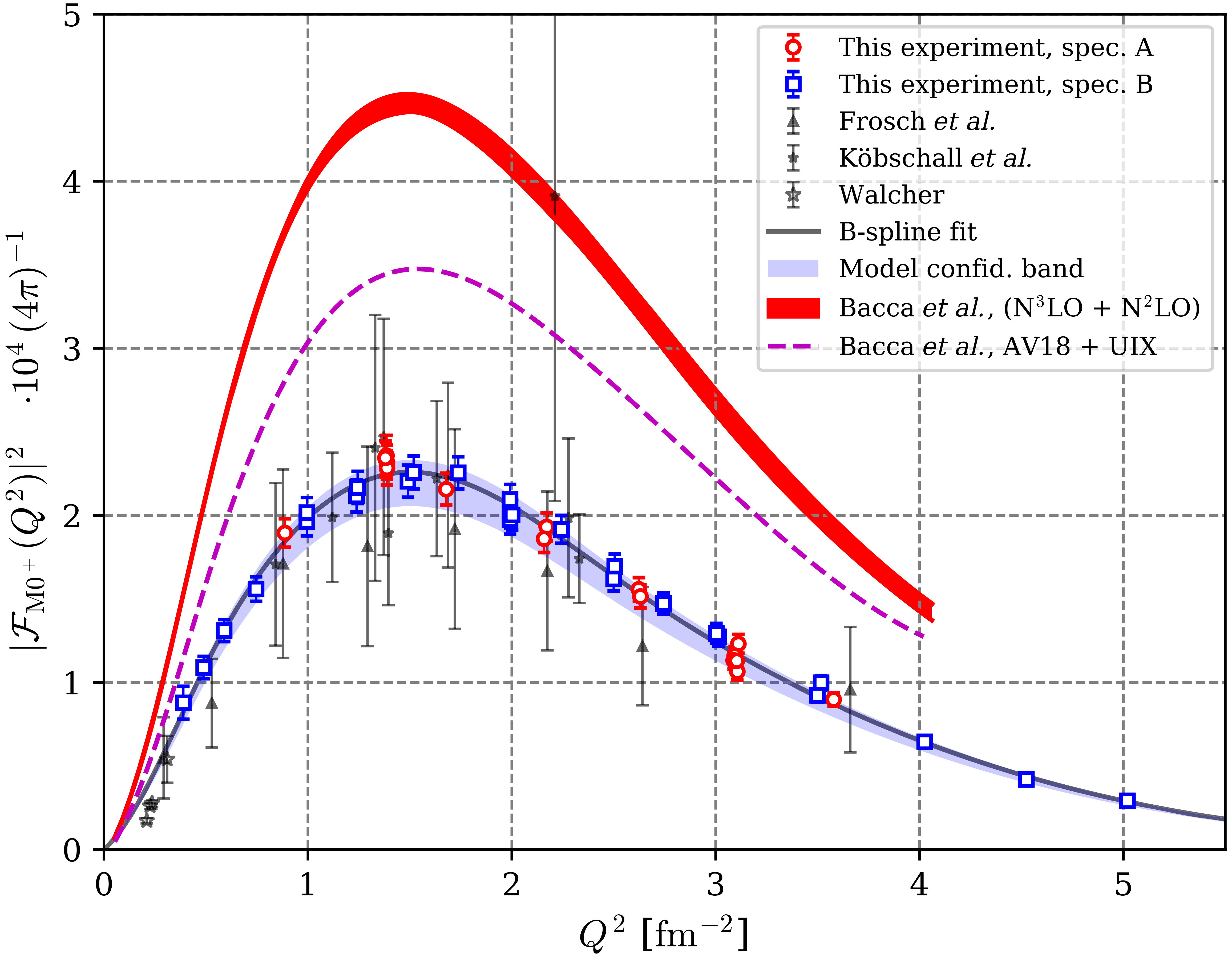
PHYSICAL REVIEW LETTERS **130,** 152502 (2023)

Editors' Suggestion Featured in Physics

Measurement of the α-Particle Monopole Transition Form Factor Challenges Theory: A Low-Energy Puzzle for Nuclear Forces?

S. Kegel,[1] P. Achenbach,[1] S. Bacca,[1,2] N. Barnea,[3] J. Beričič,[4] D. Bosnar,[5] L. Correa,[6,1] M. O. Distler,[1] A. Esser,[1] H. Fonvieille,[6] I. Friščić,[5] M. Heilig,[1] P. Herrmann,[1] M. Hoek,[1] P. Klag,[1] T. Kolar,[7,4] W. Leidemann,[8,9] H. Merkel,[1] M. Mihovilovič,[1,4] J. Müller,[1] U. Müller,[1] G. Orlandini,[8,9] J. Pochodzalla,[1] B. S. Schlimme,[1] M. Schoth,[1] F. Schulz,[1] C. Sfienti,[1,*] S. Širca,[7,4] R. Spreckels,[1] Y. Stöttinger,[1] M. Thiel,[1] A. Tyukin,[1] T. Walcher,[1] and A. Weber[1]

Problem with chiral EFT becomes stronger



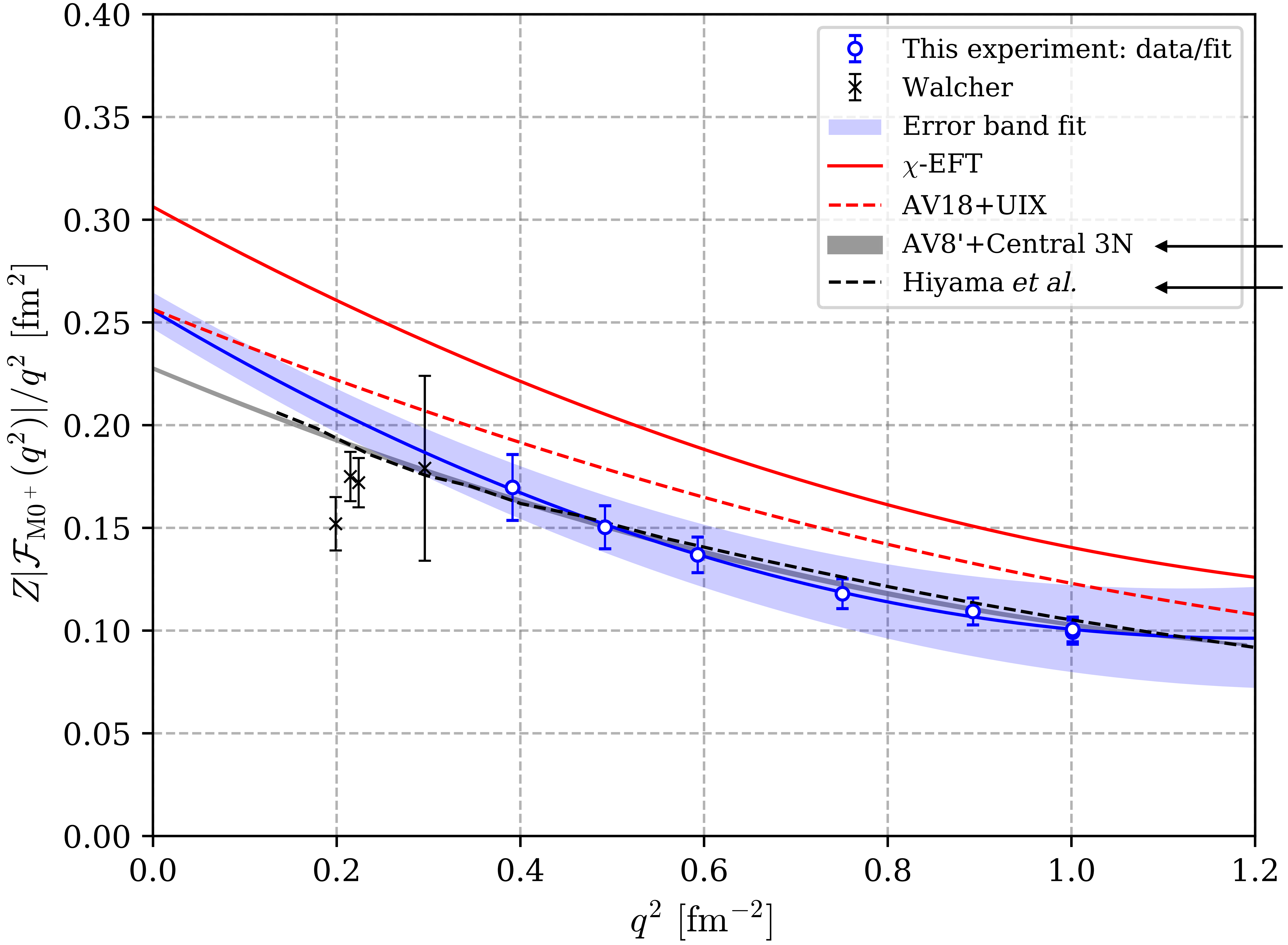
Benchmark

PHYSICAL REVIEW LETTERS **130,** 152502 (2023)

Editors' Suggestion   Featured in Physics

Measurement of the α-Particle Monopole Transition Form Factor Challenges Theory: A Low-Energy Puzzle for Nuclear Forces?

S. Kegel,[1] P. Achenbach,[1] S. Bacca,[1,2] N. Barnea,[3] J. Beričič,[4] D. Bosnar,[5] L. Correa,[6,1] M. O. Distler,[1] A. Esser,[1] H. Fonvieille,[6] I. Friščić,[5] M. Heilig,[1] P. Herrmann,[1] M. Hoek,[1] P. Klag,[1] T. Kolar,[7,4] W. Leidemann,[8,9] H. Merkel,[1] M. Mihovilovič,[1,4] J. Müller,[1] U. Müller,[1] G. Orlandini,[8,9] J. Pochodzalla,[1] B. S. Schlimme,[1] M. Schoth,[1] F. Schulz,[1] C. Sfienti,[1,*] S. Širca,[7,4] R. Spreckels,[1] Y. Stöttinger,[1] M. Thiel,[1] A. Tyukin,[1] T. Walcher,[1] and A. Weber[1]



LIT+HH (our work)

GEM

Using different few-body methods, we get same result

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Probing the Helium Nucleus be
Ground State
Evgeny Epelbaum
Physics and Astronomy, Ruhr University Bochum, Bochum, Germany
Das Physikportal
pro-physik.de
Rätsel um Anregung von α-Teilchen
26.04.2023 - Theoretisch bestimmte und gemessene
überein.
Am Mainzer Teilchenbeschleuniger „Mami" hat die A1-Kollaboration im Rah
Anregung eines α-Teilchens von seinem Grundzustand zum ersten angereg
Genauigkeit systematisch vermessen. Die Gegenüberstellung von Experime
zugehörigen Niederenergie-Theorie zeigt, dass die Anregung von α-Teilchen
Kernkräften nicht korrekt beschrieben wird – und wirft damit viele Fragen au
LIVESCIENCE
Scientists tried to solve the mystery of the helium nucleus — and ended up more confused than ever
News
By Anna Demming published June 27, 2023
Helium is the simplest element in the periodic table with more
ACCUEIL › PHYSIQUE
Le mystère du noyau d'hélium : une énigme persistante pour la physique nucléaire
PUBLIÉ LE 28 JUIN 2023 À 18H45 | MODIFIÉ LE 28 JUIN 2023
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By measuring inflated helium nuclei, physicists have challenged o
understanding of the force that binds protons and neutrons.
PHYSICS TODAY
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Theory and experiment disagree on alpha particles
12 May 2023
Electron-scattering experiments on excited helium nuclei open questions about the ac
art nuclear models.

Two important comments

E. Epelbaum, Physics **16** (2023) 58

Threshold energy:

"*The form factor may depend on the energy difference between the position of the resonance and the two-body breakup threshold, so any uncertainties in calculating the excitation energy would translate into relatively large uncertainties in the form factor predictions.*"

⇒ investigate role of threshold energy

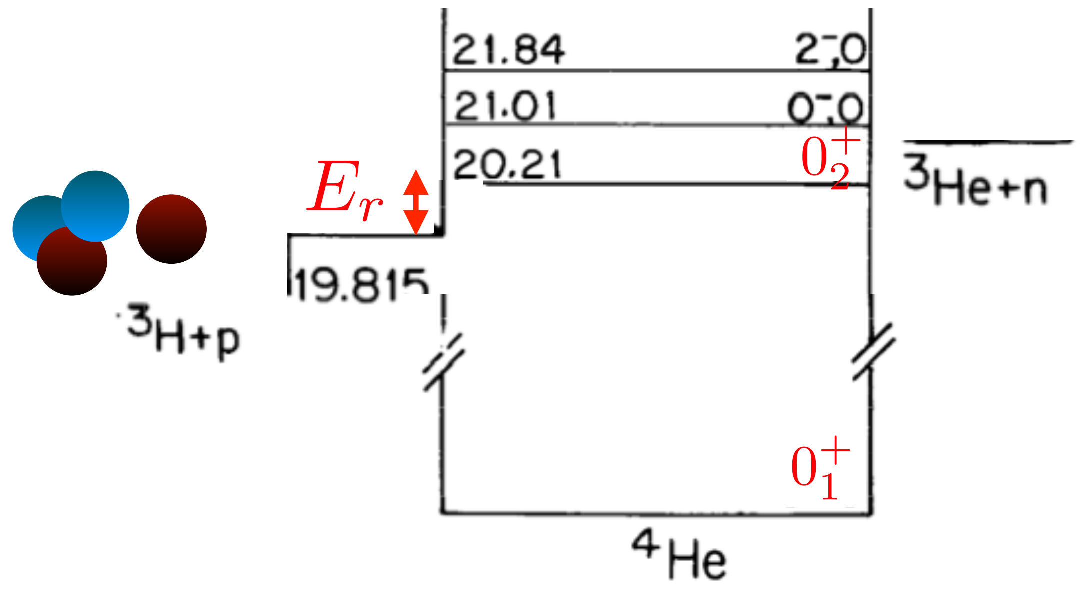
Kamimura, Prog. Theor. Exp. Phys. (Letter) Vol. 2023, 071D01 (2023)

Bound vs Continuum:

Claim that "*the large difference between the calculated form factors does not stem from numerical methods but from the Hamiltonian. However, the comparison between the methods was performed only for the calculation based on the bound-state approximation.*"

⇒ investigate role of continuum

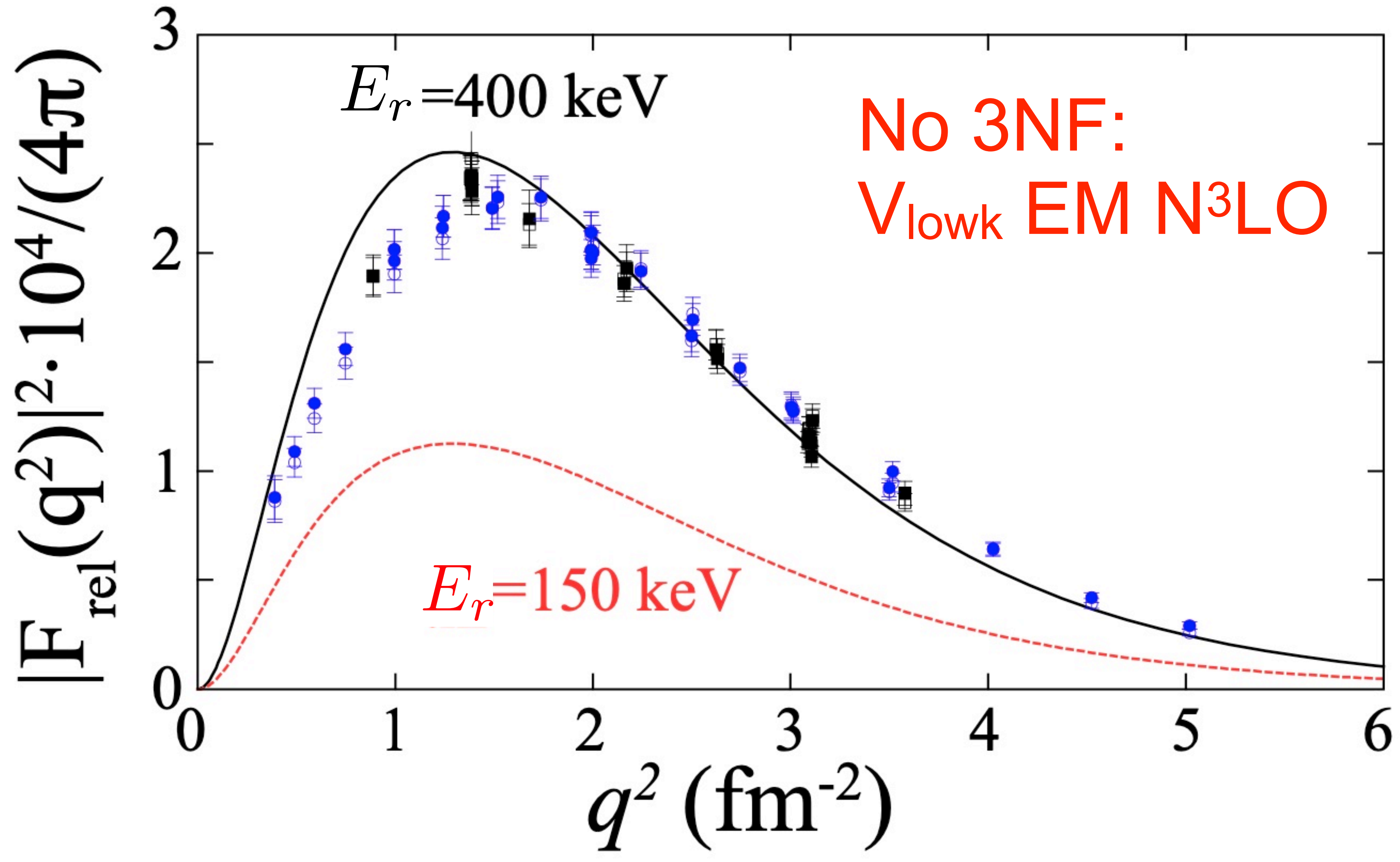
New Calculations after our PRL

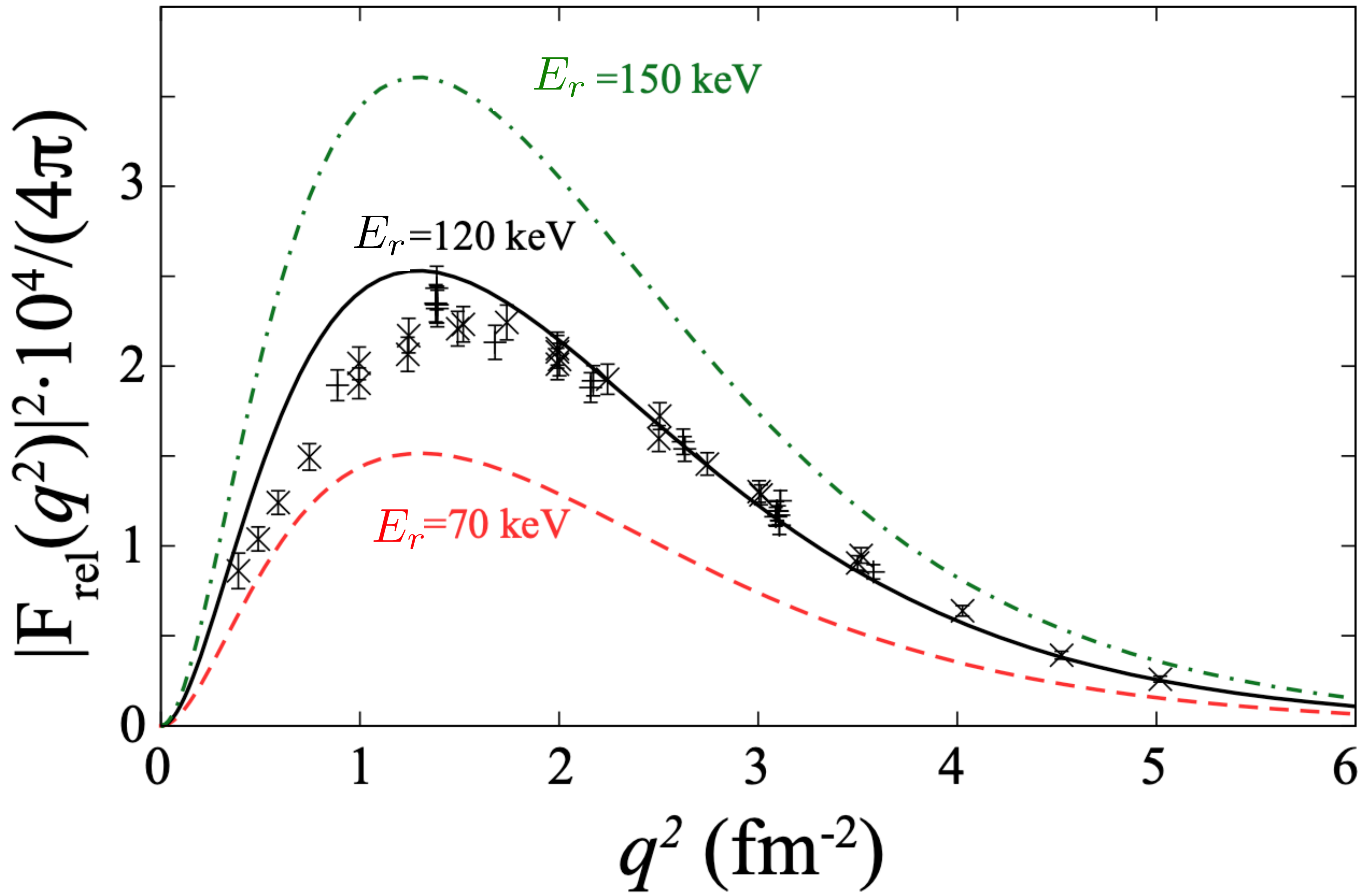


With simplified Hamiltonians

Michel, Nazarewicz, Ploszajczak et al., PRL 131, 242502 (2023)
Method: NCGSM-CC

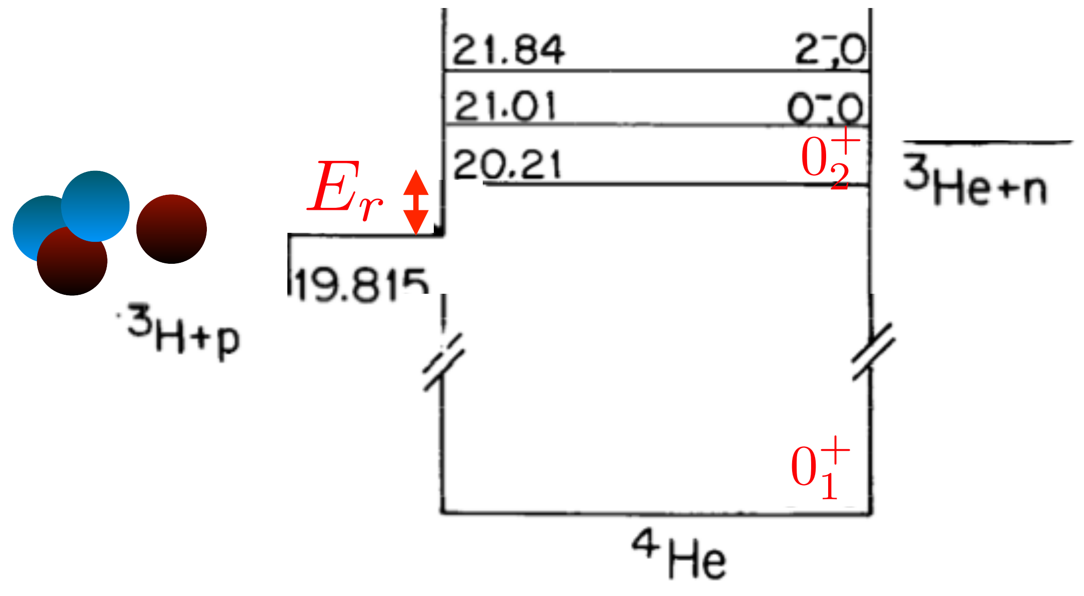
Peng et al. found out a 1/Z wrong factor in their derivation.
Update: courtesy of M. Ploszajczak, private communication (unpublished, 2024)





E_r fit to exp ⇒ monopole form factor agrees with data

~~E_r fit to exp ⇒ monopole form factor agrees with data~~

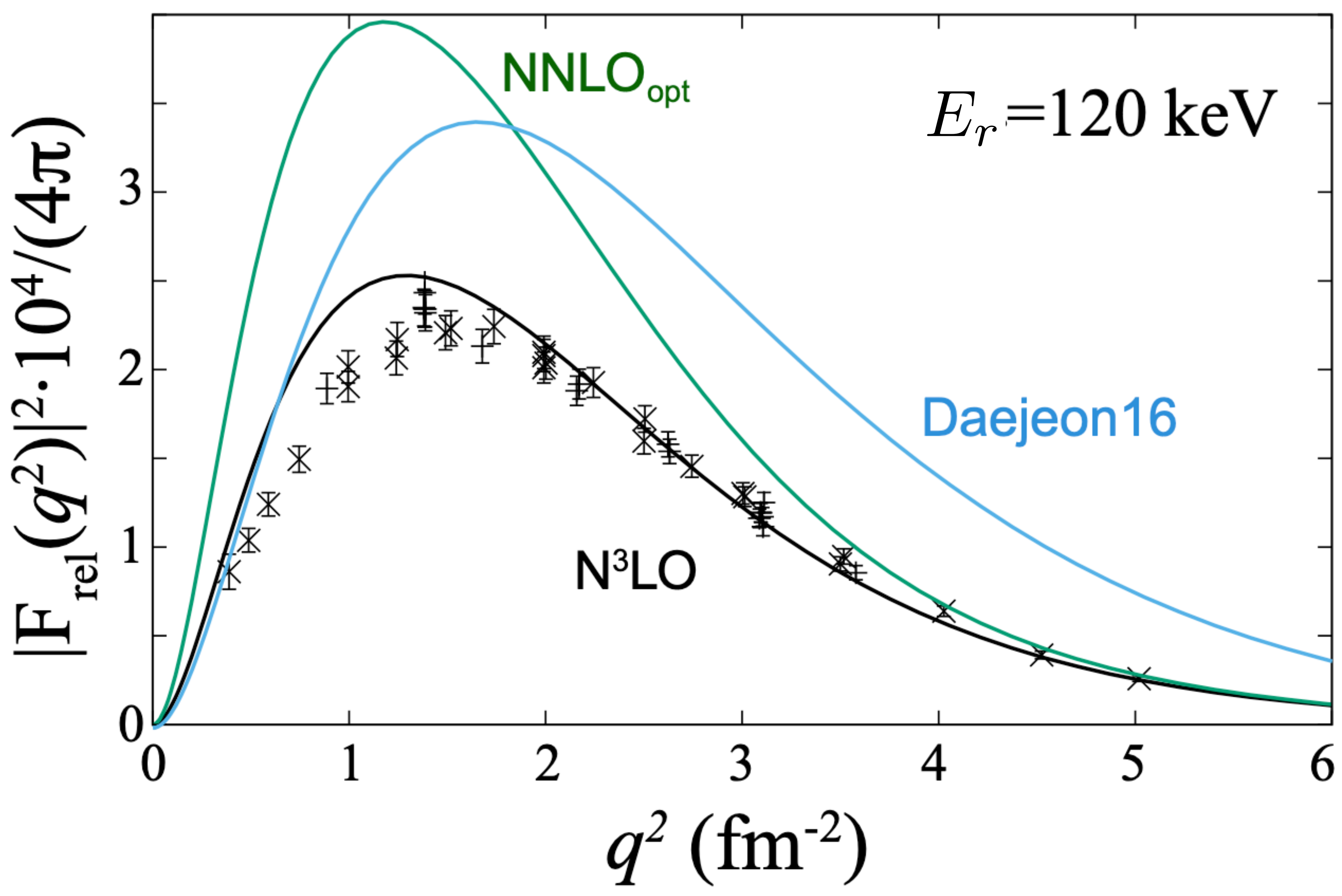


New Calculations after our PRL

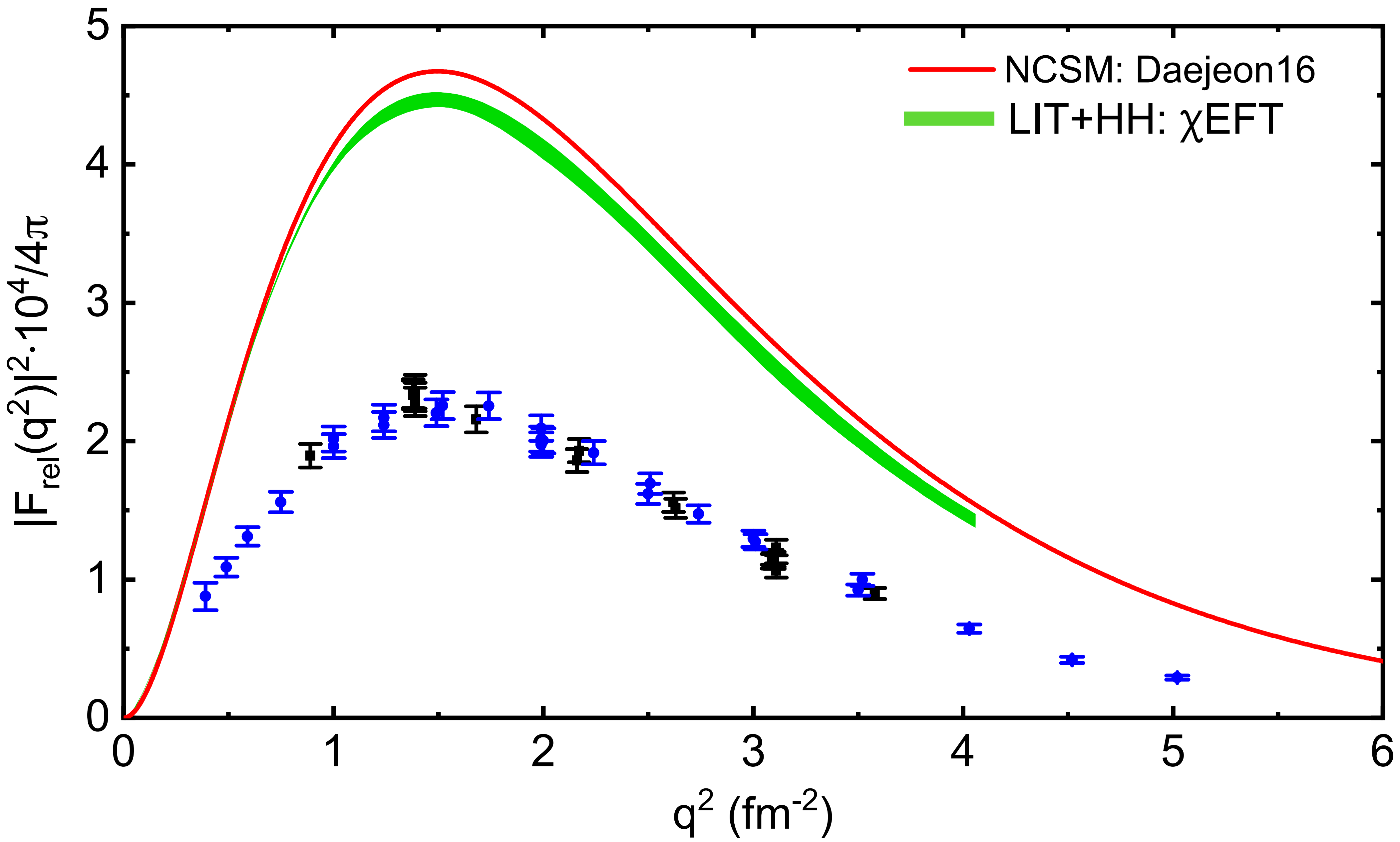
With simplified Hamiltonians

Michel et al., Errata (unpublished, 2024)
Method: NCGSM-CC

Peng et al., private communication (unpublished, 2024)



Large potential dependence is seen, consistent with our findings

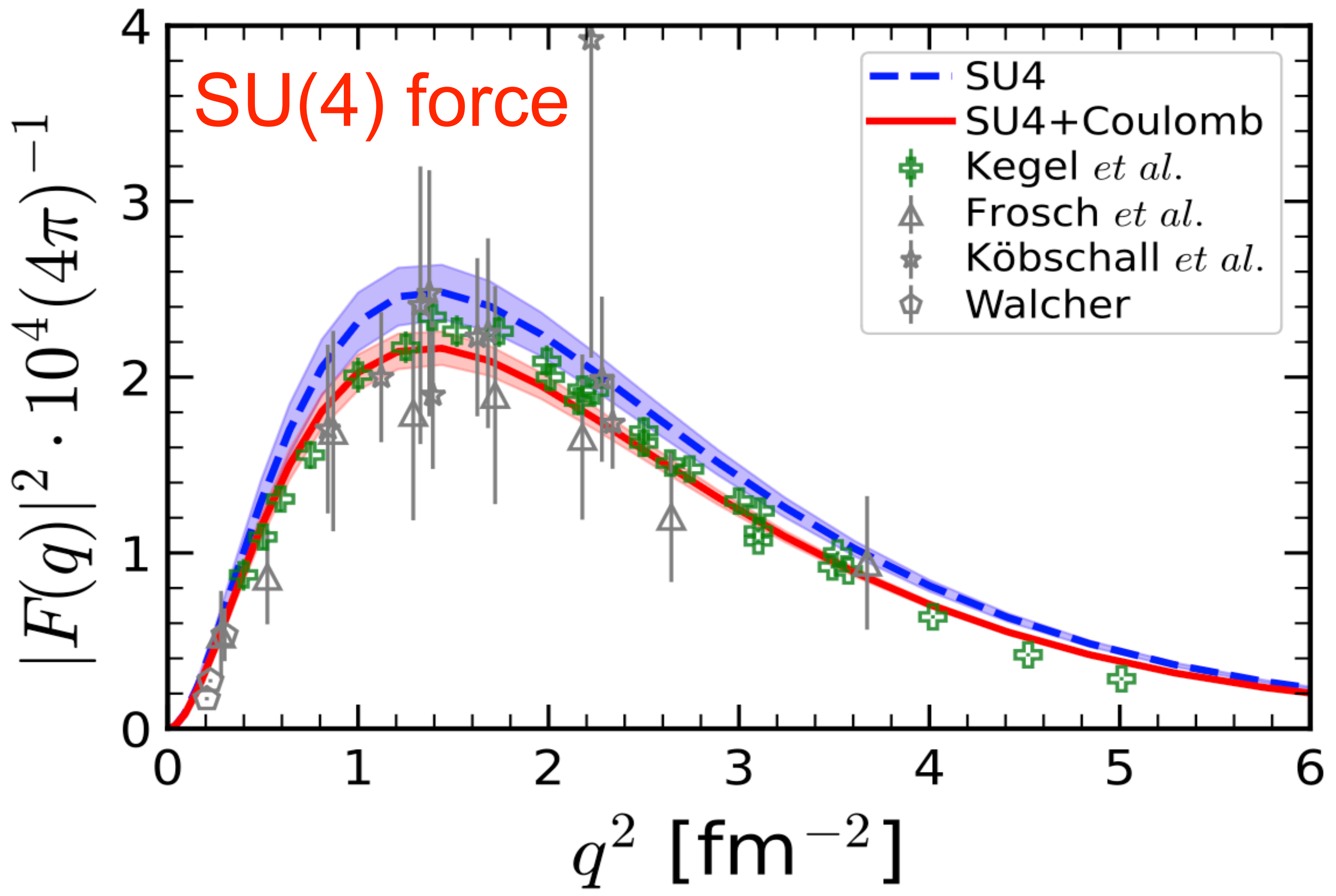


High monopole form factor is seen, like in our case, but E_r =60 keV

New Calculations after our PRL

With simplified Hamiltonians

Meißner, Shen, Elhatisari, Lee, PRL 132, 062501 (2024)
Method: LatticeEFT

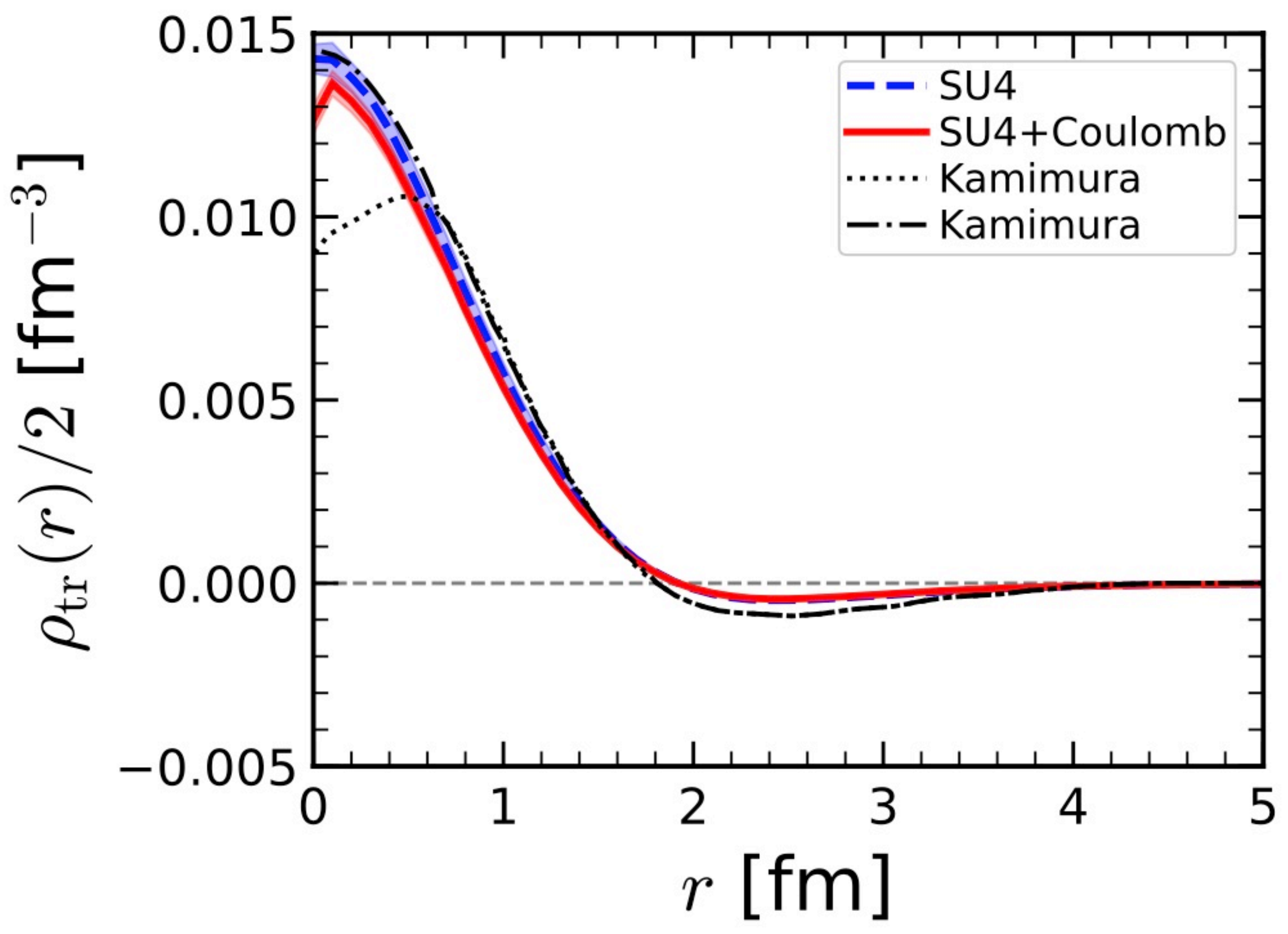


E_r not fit to experimental value, but comes out correct to be about 0.40(9) MeV and describes well the monopole form factor

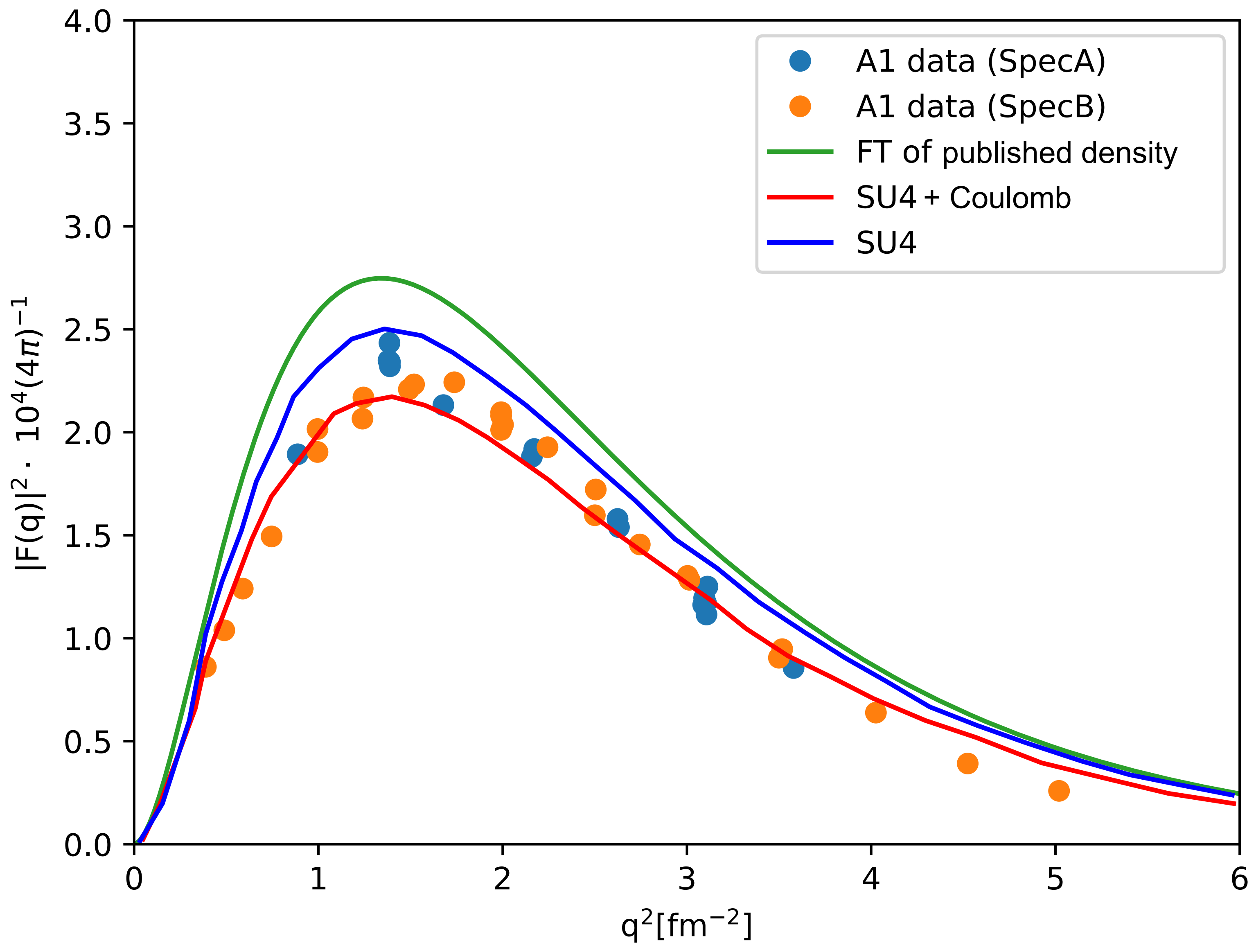
New Calculations after our PRL

With simplified Hamiltonians

Meißner, Shen, Elhatisari, Lee, PRL 132, 062501 (2024)
Method: LatticeEFT



Meißner and Shen, private communication (2024)

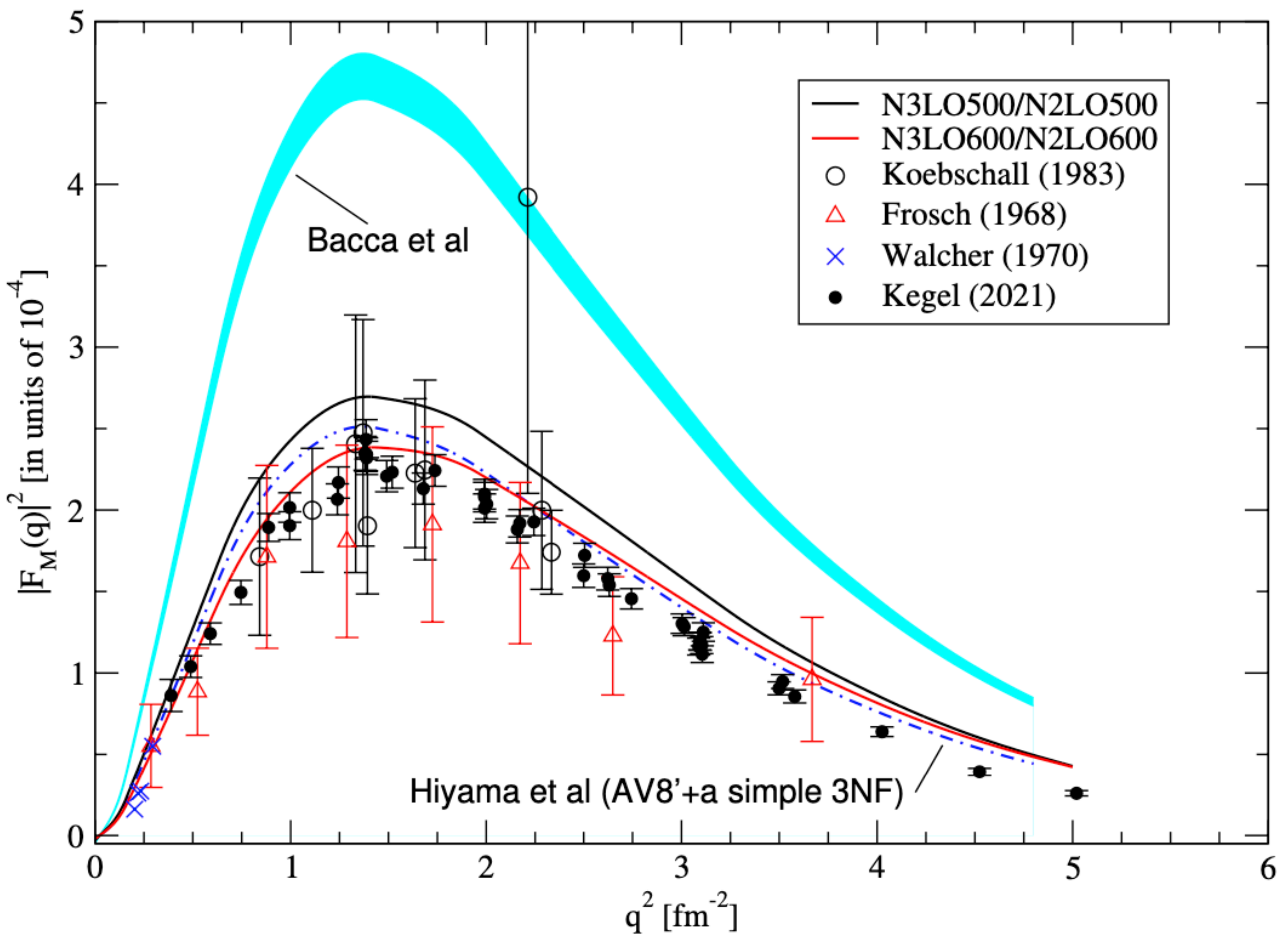


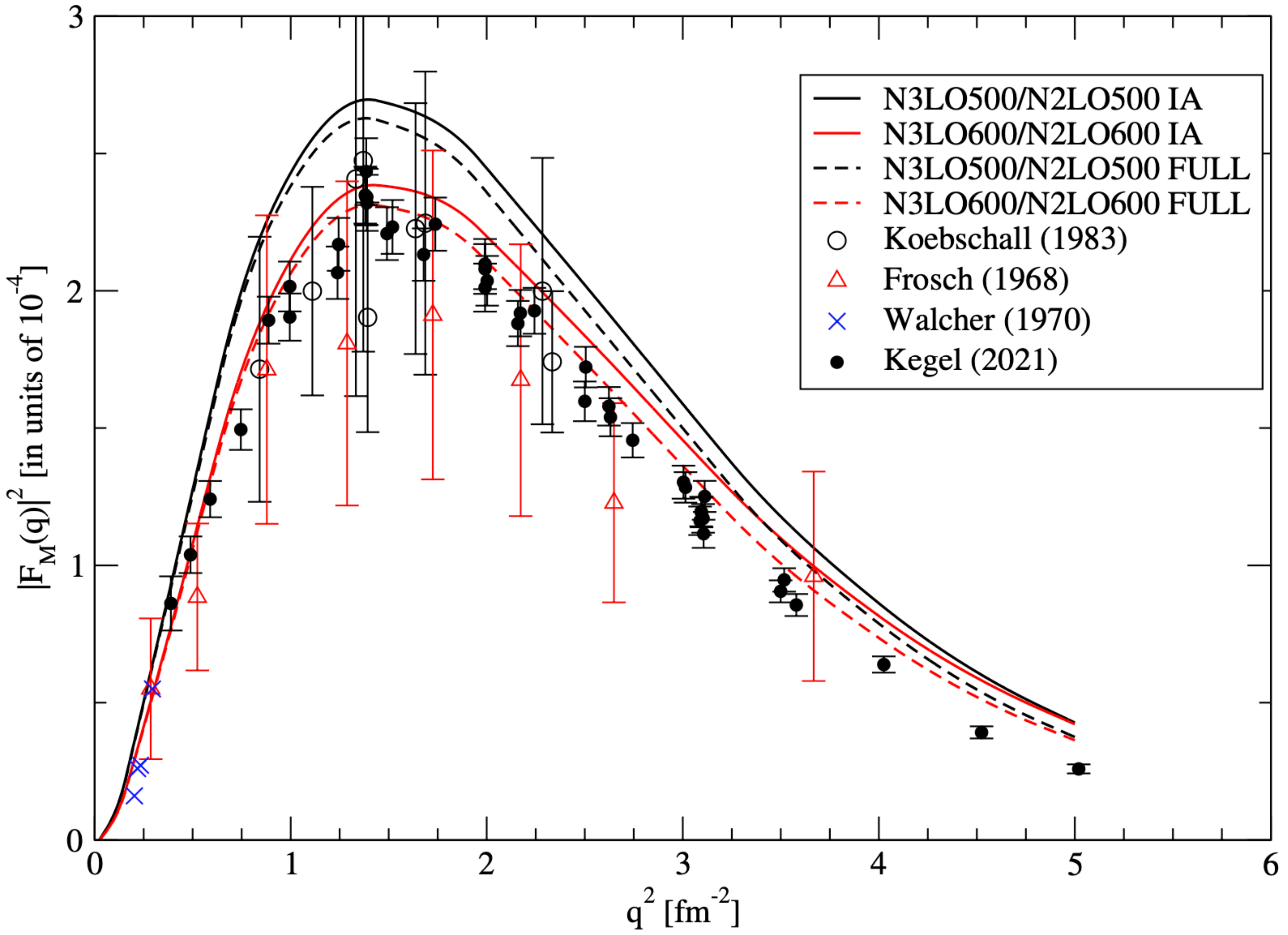
Fourier transform of transition density is higher than data.
Tail was modified to exp(-r$^{1.5}$) because resonance should fall off slower than a bound state

New Calculations after our PRL

With chiral Hamiltonians

Viviani et al, FBS (2024), Method HH





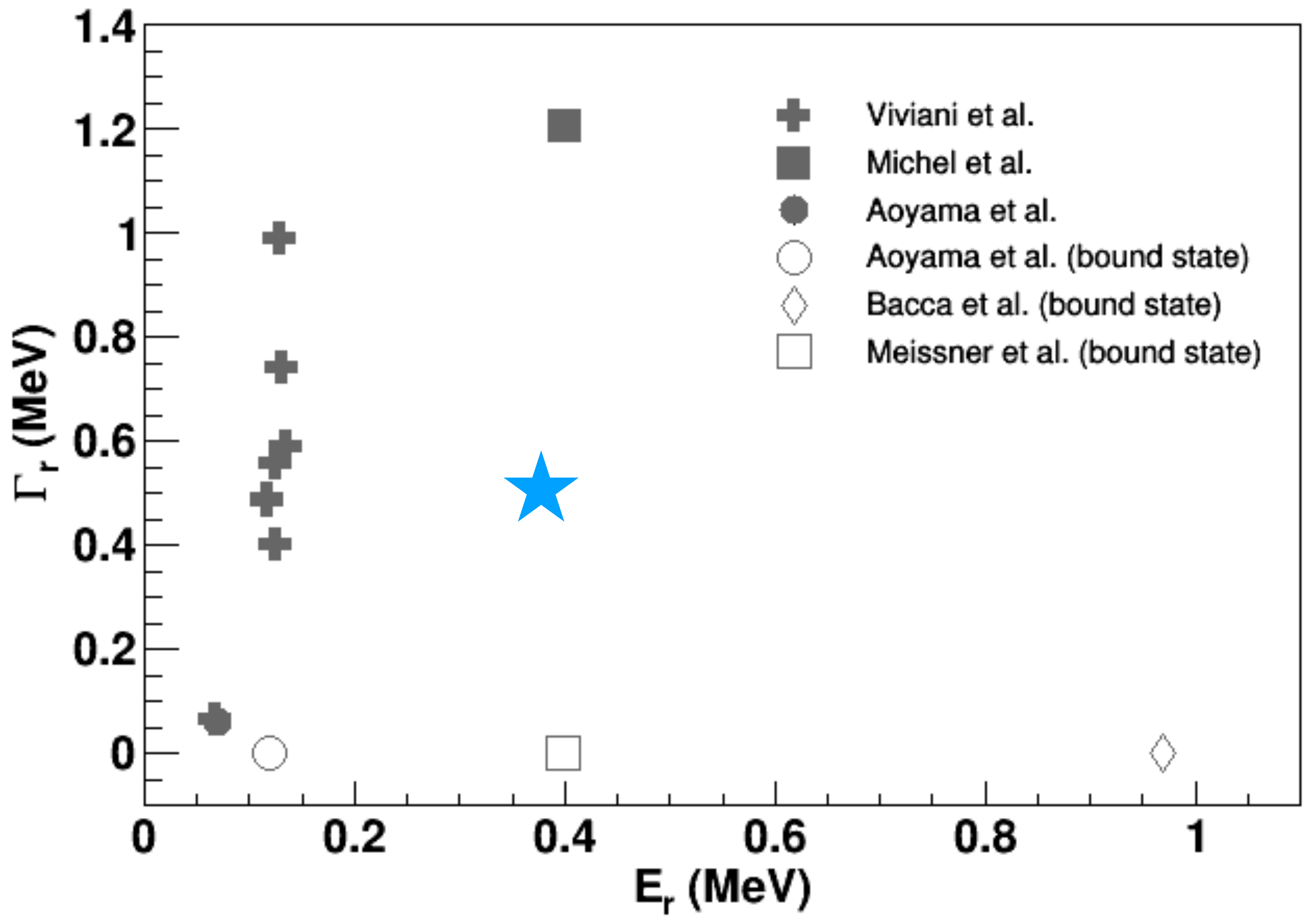
Chiral EFT force is different from the one used Bacca et al. (2013)

Interestingly, monopole data are reproduced but E_r is about 120 keV.

Effect of two-body currents is a few %

None of the theories gets E_r and Γ_r correct

Evaluation by Tilley et al, NPA 541 (1992) 1-104 ★ $E_r = 0.39$ MeV $\Gamma_r = 0.50$ MeV



Pionless EFT with RGM: Kirscher and Griesshammer, EPJA **54**, 137 (2018) $E_r = 0.38 \pm 0.25\text{MeV}$

Investigating the 0^+ with α scattering

Courtesy from Soukeras and Cappuzzello

^{4}He(e,e´)^{4}He*

- Pure electro-magnetic 😀
- Mixed isoscalar and isovector 🙁
- Contribution of several multipoles 🙁
- Low cross section 🙁

^{4}He(^{4}He,^{4}He)^{4}He*

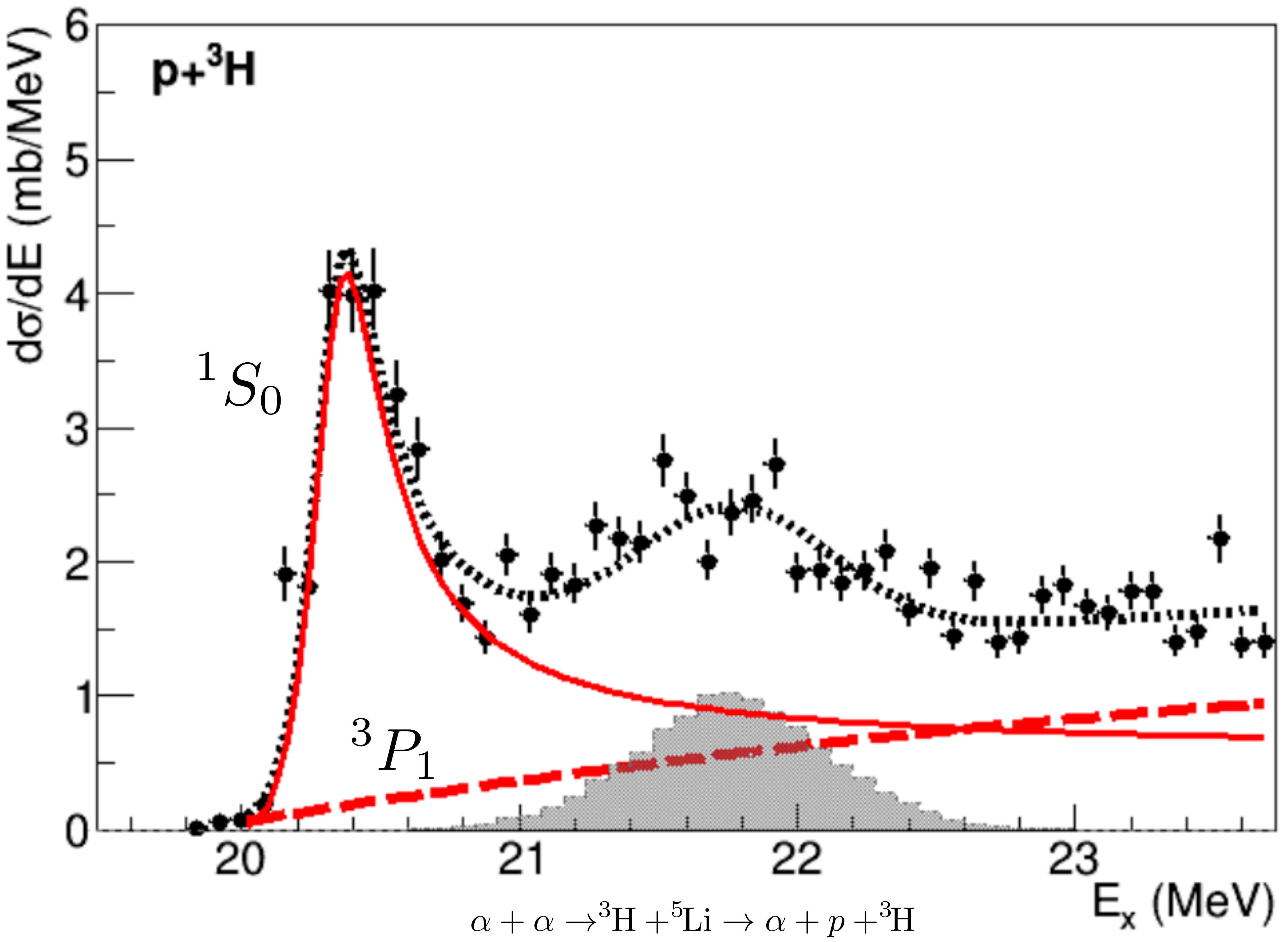
- Involves strong force ⇒ need modelling 🙁
- Purely isoscalar 😀
- Contribution of few other multipoles 😀
- Large cross section 😀

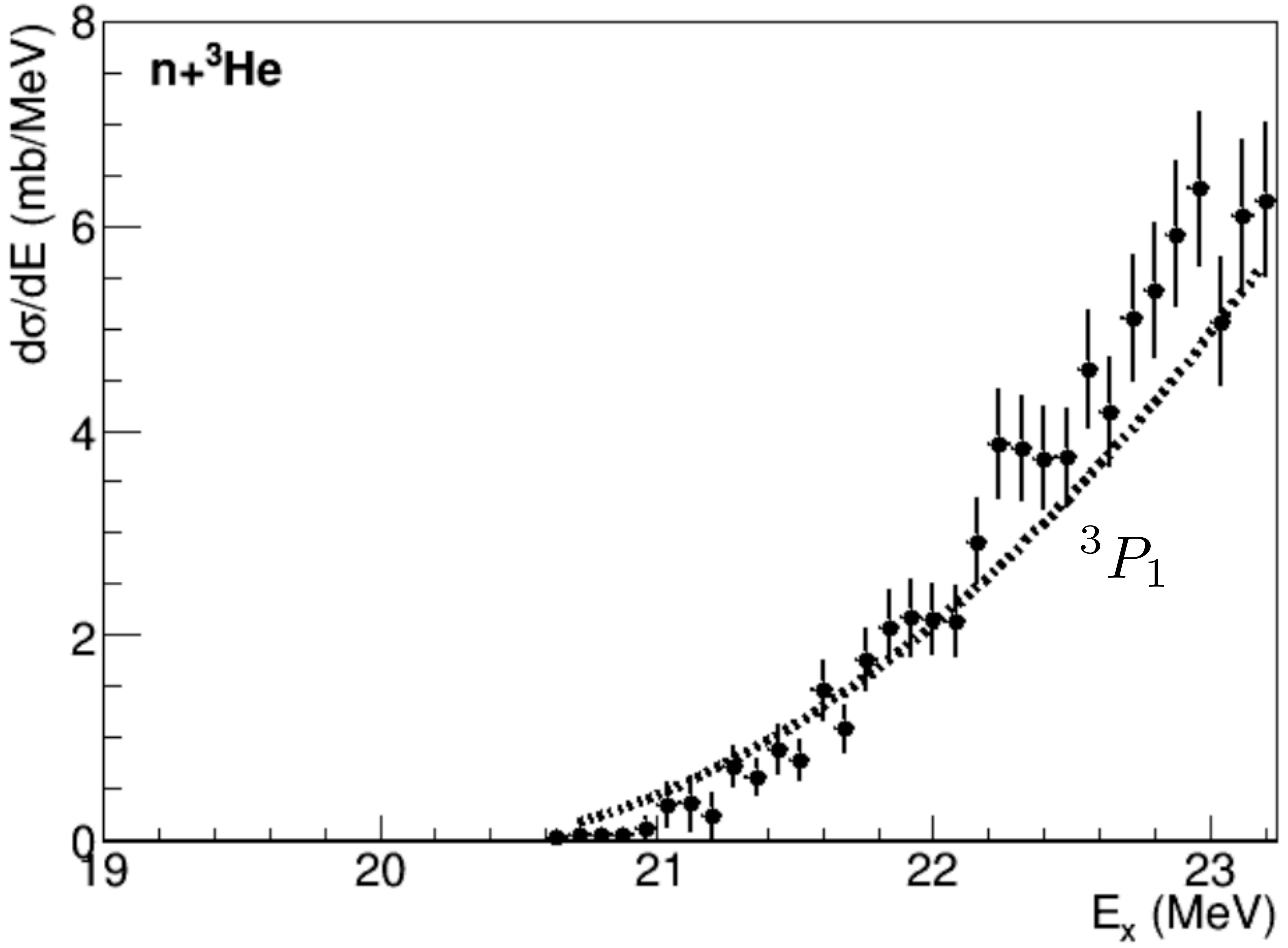
New experiment: α-α scattering

Catania

Experiment by Cappuzzello, Soukeras et al.

Fano analysis (accounts for asymmetries in the resonance)

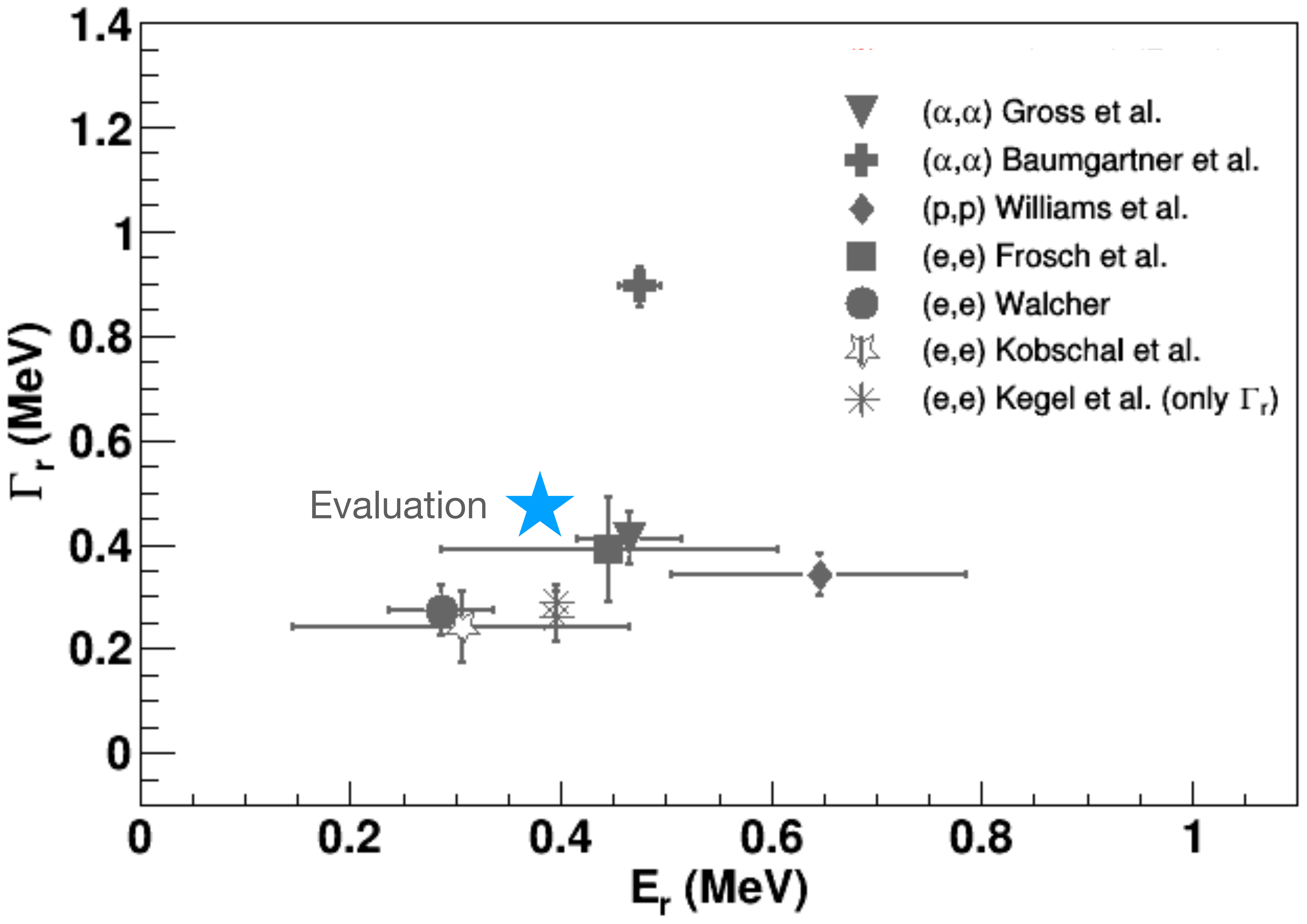




New experiment: α-α scattering

Catania

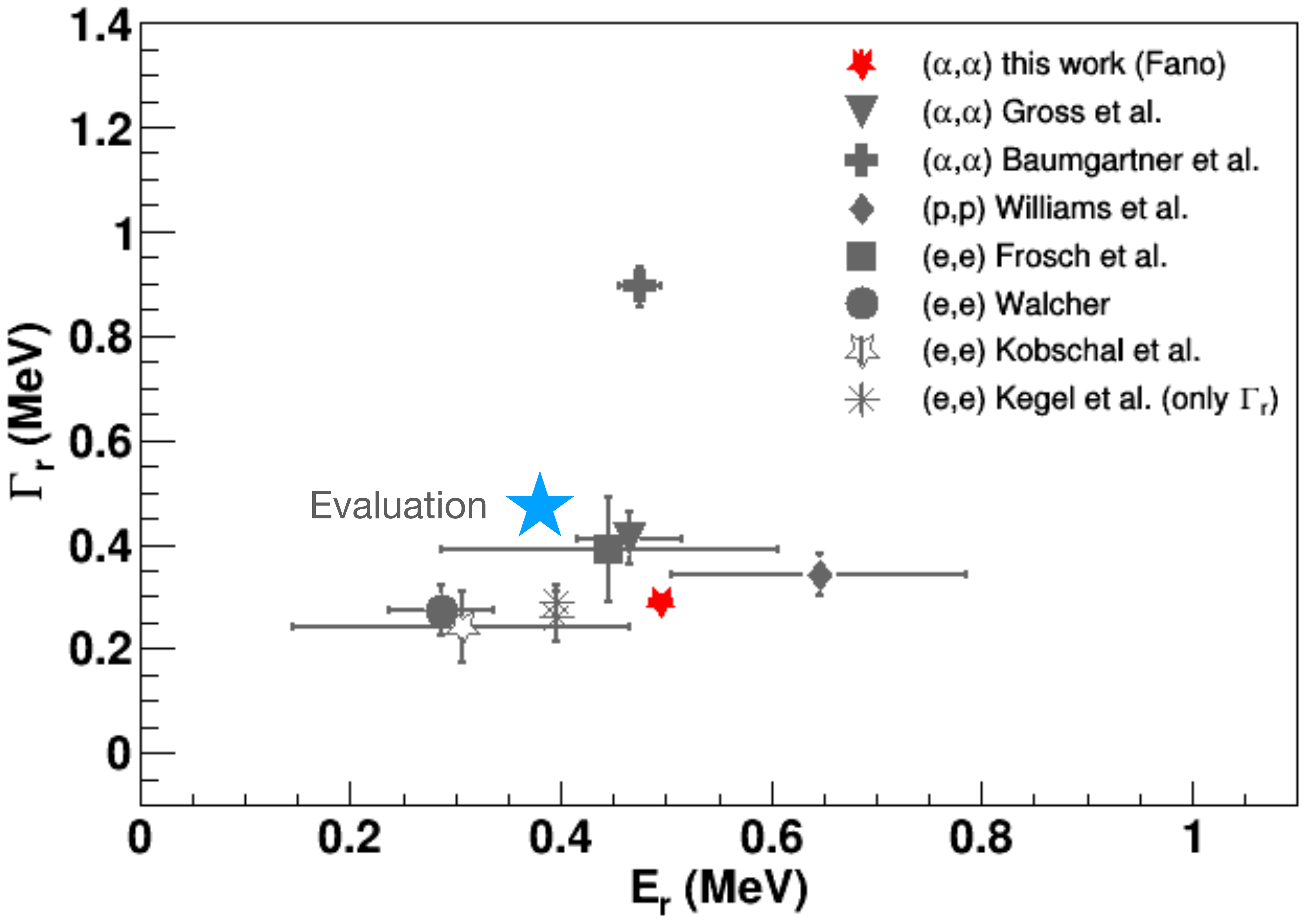
New characterization of the resonance



New experiment: α-α scattering

Catania

New characterization of the resonance



$$E_r = 20.31 \pm 0.01 \text{ MeV}, \quad \Gamma_r = 0.29 \pm 0.01 \text{ MeV}$$

α-α scattering: reaction theory

Luiz Chamon et al.

$$\begin{cases} \left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\frac{l(l+1)}{2\mu R^2}+U_1(R)\right]\psi_l^{(1)}+U_{\rm coup}(R)\psi_l^{(2)} = E_{\rm cm}\psi_l^{(1)} \\ \left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\frac{l(l+1)}{2\mu R^2}+U_2(R)\right]\psi_l^{(2)}+U^*_{\rm coup}(R)\psi_l^{(1)} = (E_{\rm cm}-E_X)\psi_l^{(2)} \end{cases}$$

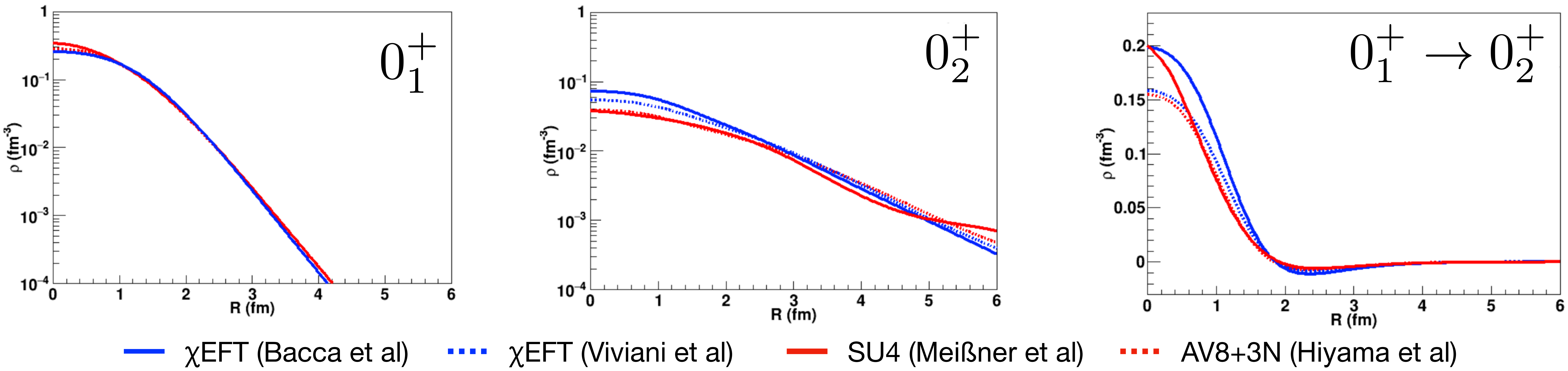
Set of coupled equations

$$U_j(R) = V_j^{(C)}(R) + N_{Rj}V_j^{(N)}(R) + iN_{Ij}V_j^{(N)}(R), j=1,2$$

Optical potentials

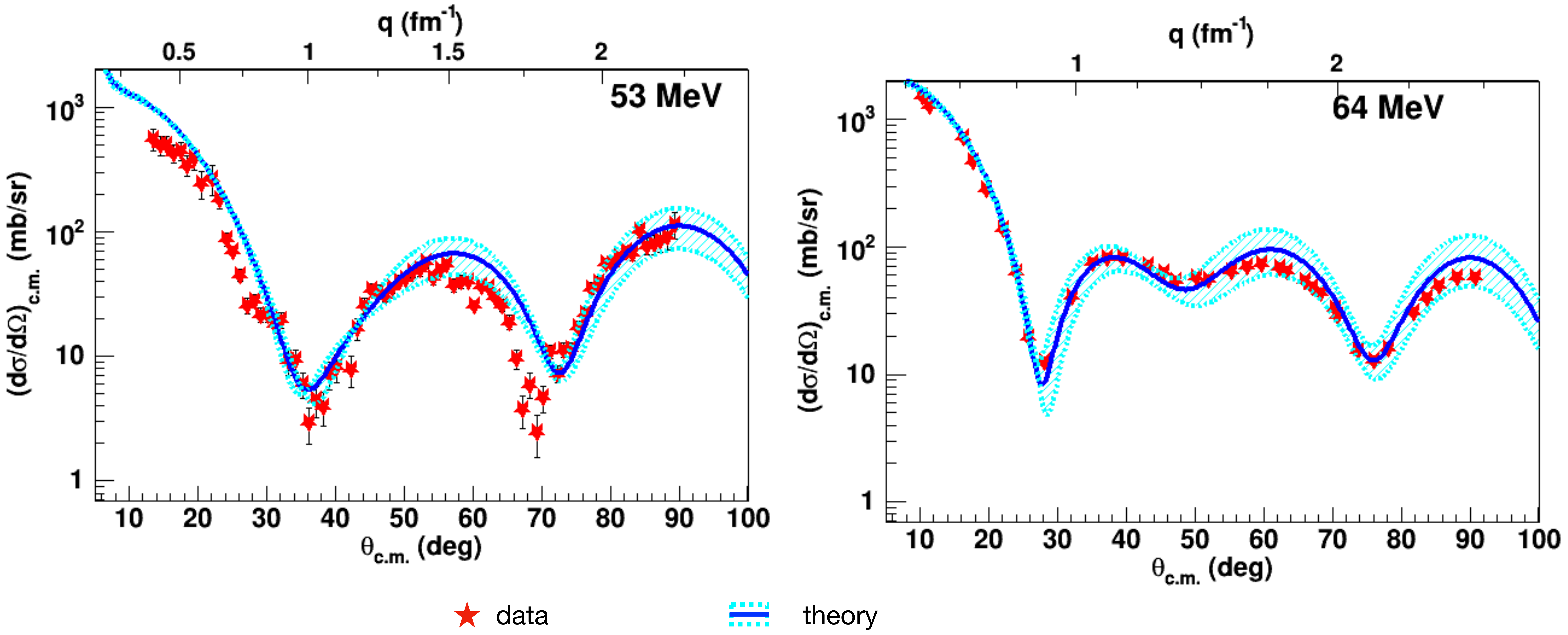
$$V^{(N)}(R) = \int \rho_{0_i^+}(\vec{r}_1)\,\rho_{0_j^+}(\vec{r}_2)\,v_{NN}(\vec{R}-\vec{r}_1+\vec{r}_2)\,d\vec{r}_1\,d\vec{r}_2, i,j=1,2$$

Double folding potentials



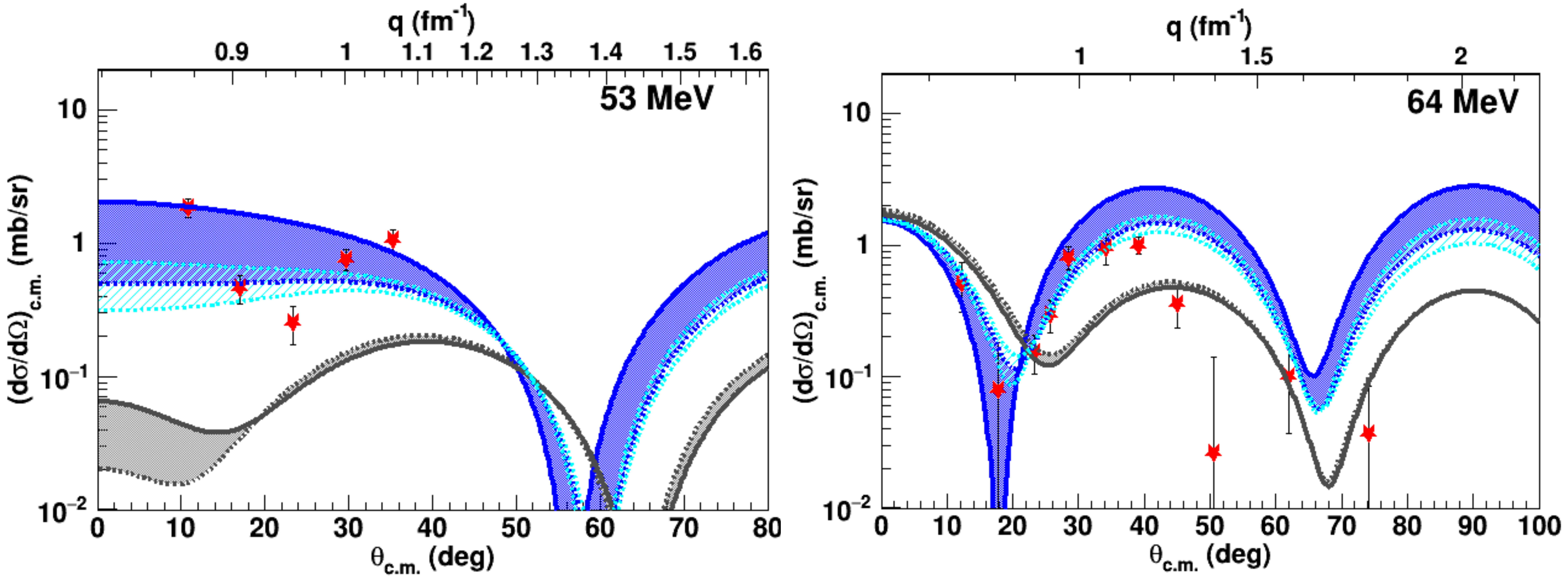
Elastic α-α scattering

Experiment by Cappuzzello, Soukeras et al., coupled-channel calculations by Luiz Chamon with double folding potential using the experimental ground-state density



Inelastic α-α scattering

Experiment by Cappuzzello, Soukeras et al., coupled-channel calculations by Luiz Chamon with double folding potential using ab-initio ground-state densities



★ data — χEFT (Bacca et al) ··· χEFT (Viviani et al) — SU4 (Meißner et al) ··· AV8+3N (Hiyama et al)

Conclusions and outlook

- The solution is not "just" a simple tuning of E_r
- More investigations are needed before we can call it a solved problem
- We need more benchmarks on the few-body methods on this resonance
- We need order-by-order EFT calculations to assess full uncertainties

Thanks to my collaborators:
N.Barnea, L.Chamon, W.Leidemann, G.Orlandini, F.Cappuzzello, V.Soukeras, C.Sfienti, S.Schlimme

and to the colleagues who provided us with their results:
U-G.Meißner, Y.Peng, M.Ploszajczak, A.Kievsky, M.Viviani

Thank you for your attention!

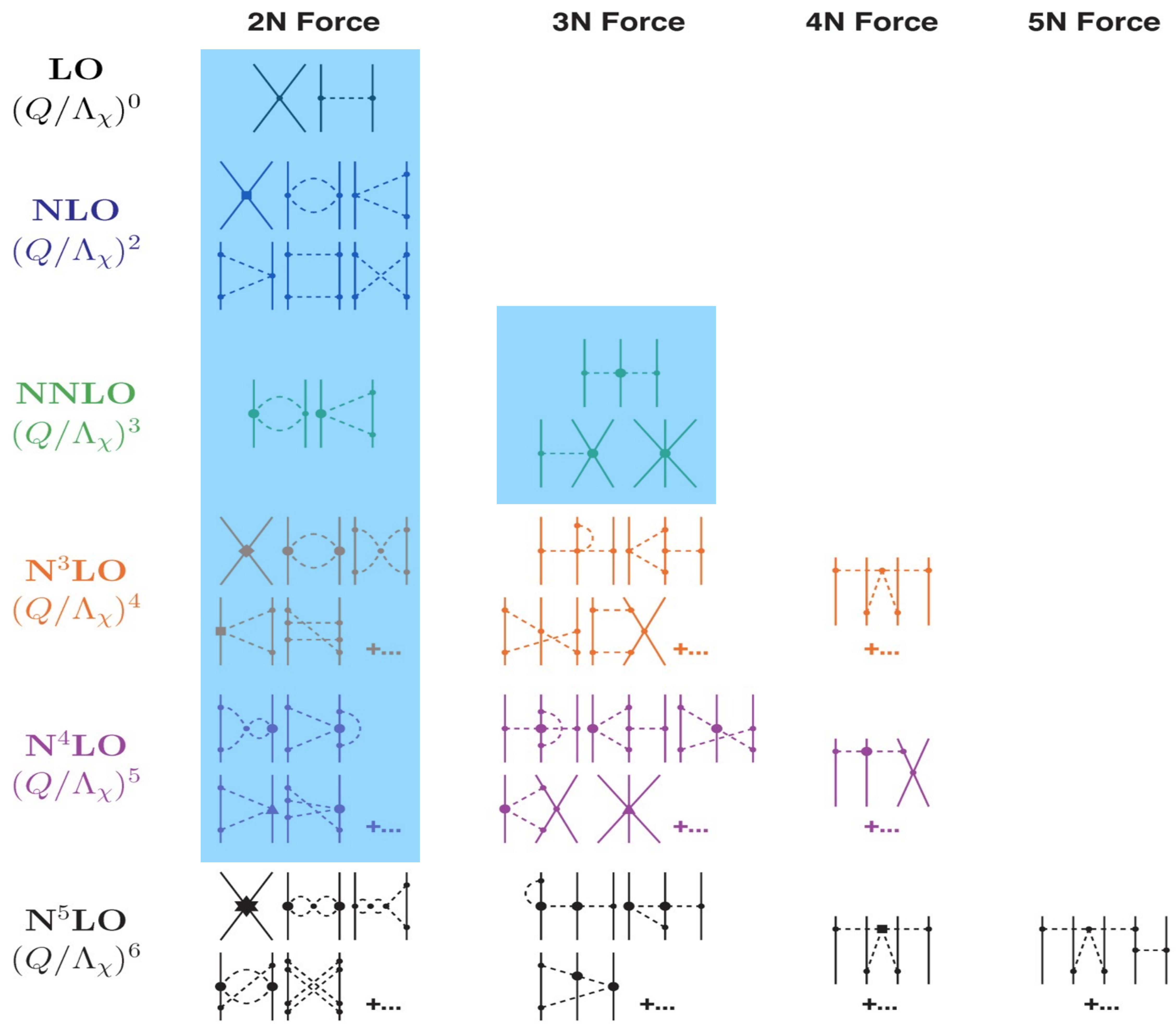
Open questions

- If this resonance state is fine-tuned, does it mean it is not interesting for calibrating forces?
- Could the problem be the “wrong” power counting?
- Could the problem be the relatively low-order used in chiral EFT?
- Is the problem in the force or in the few-body method?

Backup slides

Chiral effective field theory

Picture from R. Machleidt (2017)

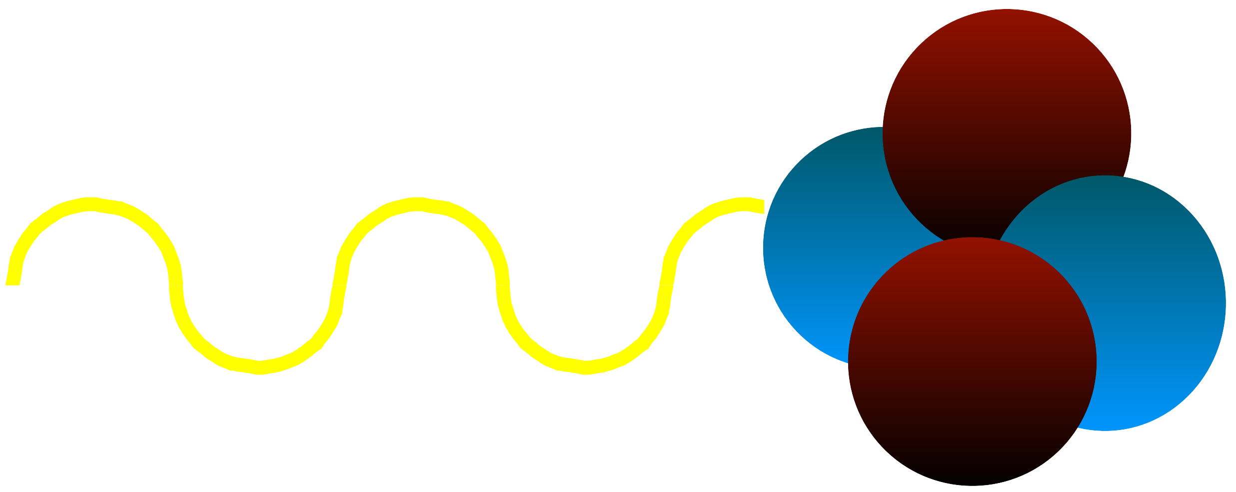


First excited state of ^{4}He explored within chiral EFT so far with

- 2N forces up to N4LO
- 3N forces up to N2LO

Chiral effective field theory

Coupling to the electromagnetic field



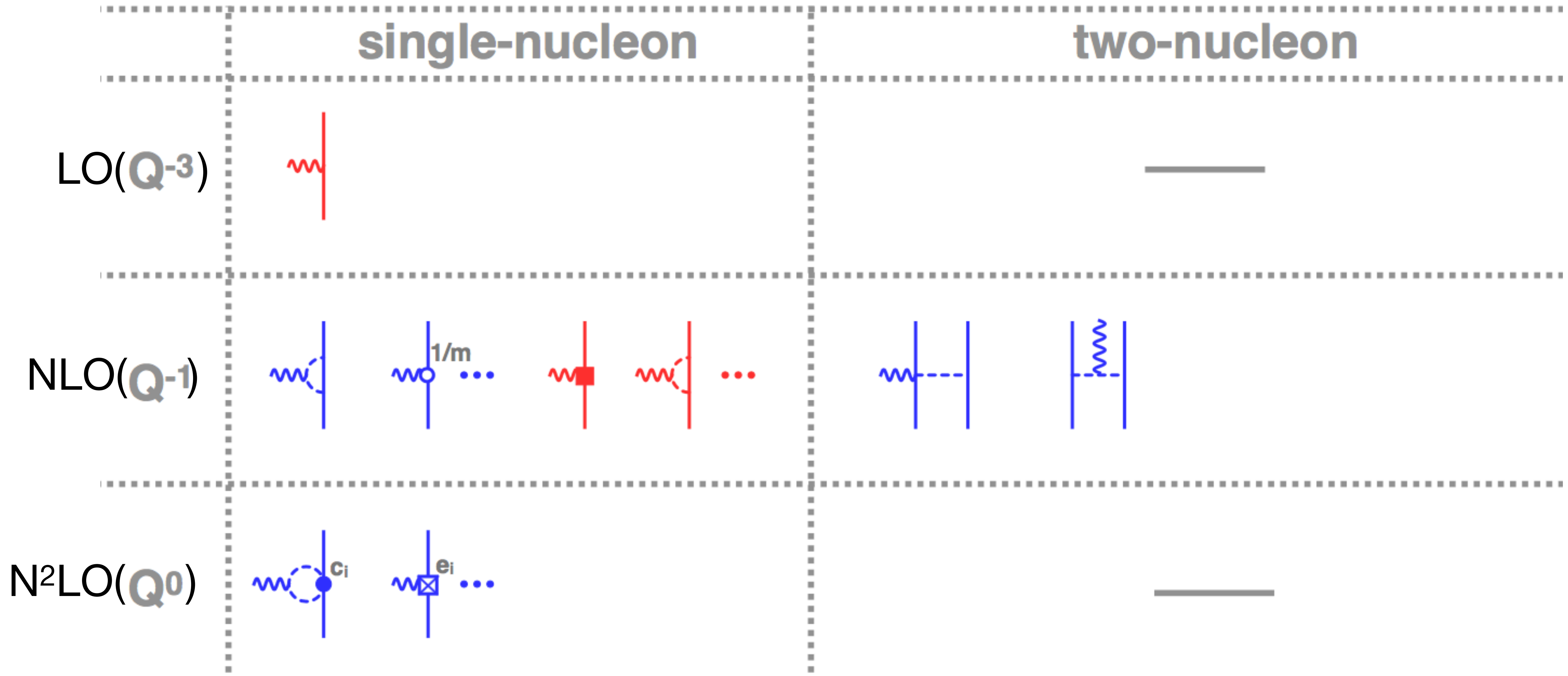
Cross Section $\sigma_{ew} \sim R(\omega) = \sum_f \left| \left\langle \psi_f \left| J^\mu \right| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$

Electroweak operator

Chiral effective field theory

Electromagnetic operator

$$J^{\mu} = J^{\mu}_{\mathrm{1BC}} + J^{\mu}_{\mathrm{2BC}}$$

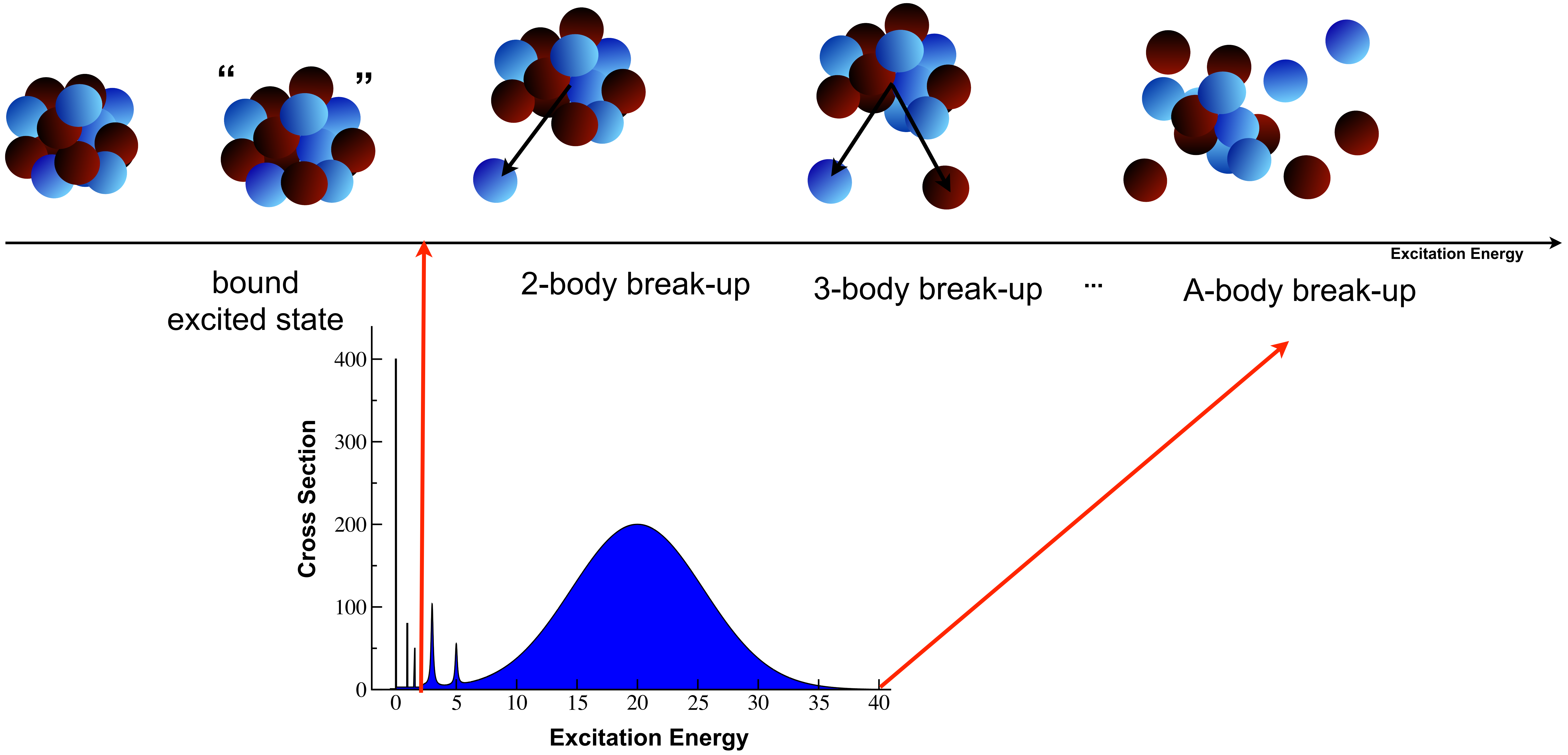


Credits: B.Acharya

The continuum problem

$$R(\omega) = \sum\!\!\!\!\!\!\!\int_f \left| \left\langle \psi_f \right| J^\mu \left| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

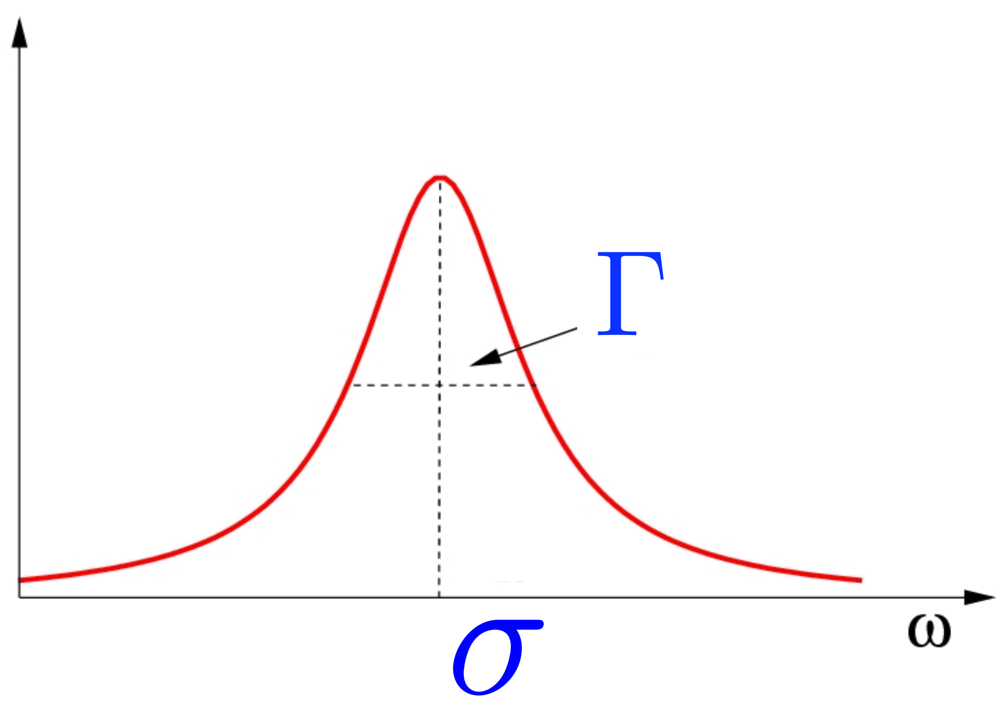
Depending on E_f, many channels may be involved



The Lorentz integral transform (LIT)

inversion

$$L(\sigma,\Gamma) = \frac{\Gamma}{\pi}\int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \langle\tilde{\psi}|\tilde{\psi}\rangle$$



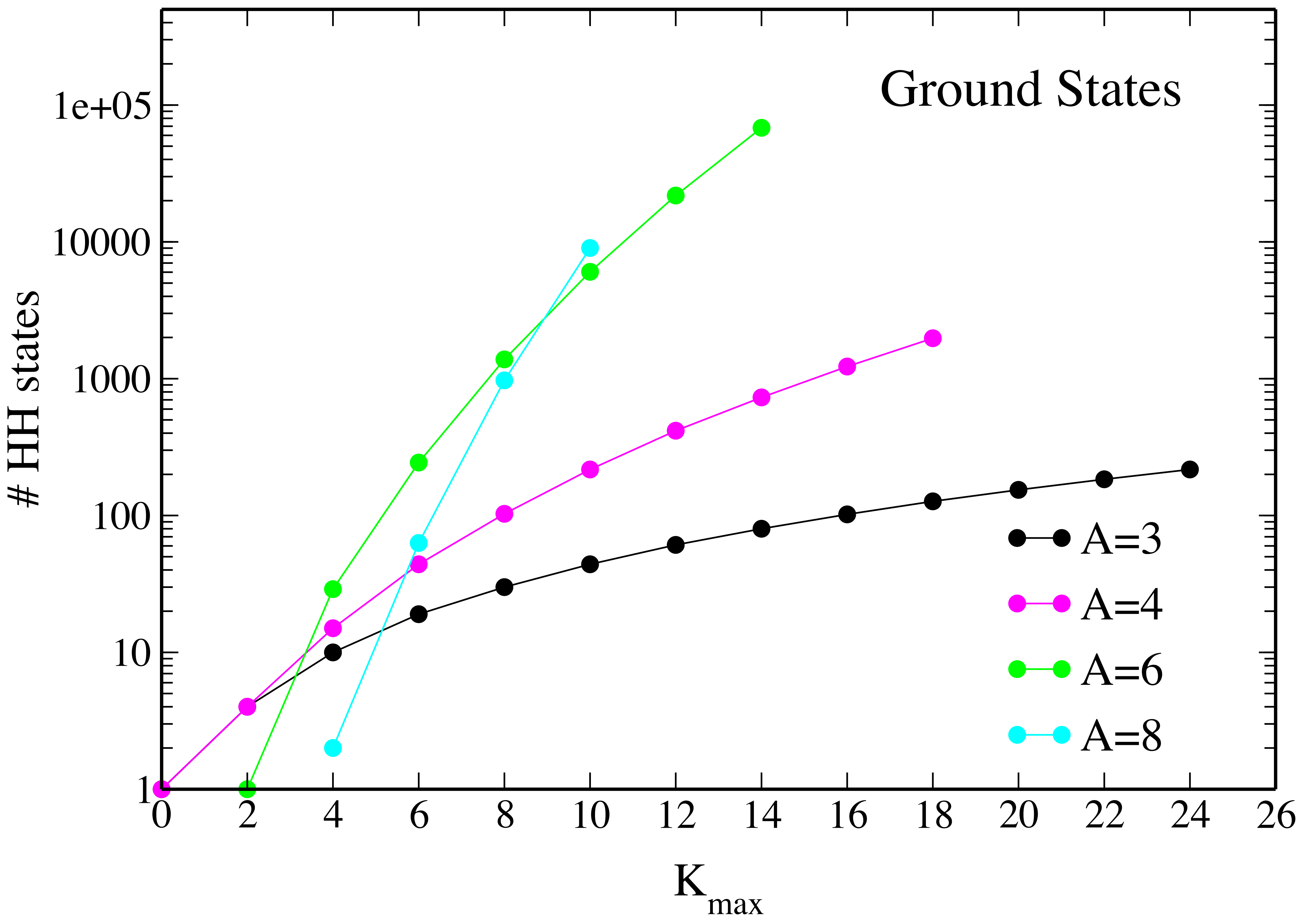
Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

$$(H - E_0 - \sigma + i\Gamma)\,|\,\tilde{\psi}\rangle = J^\mu\,|\,\psi_0\rangle$$

Reduce the continuum problem to a bound-state-like equation

Hypersperical Harmonics (HH)

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2} \rho^{n/2} L_\nu^n(\rho) [\mathcal{Y}_{[K]}^{\mu}(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^a$$



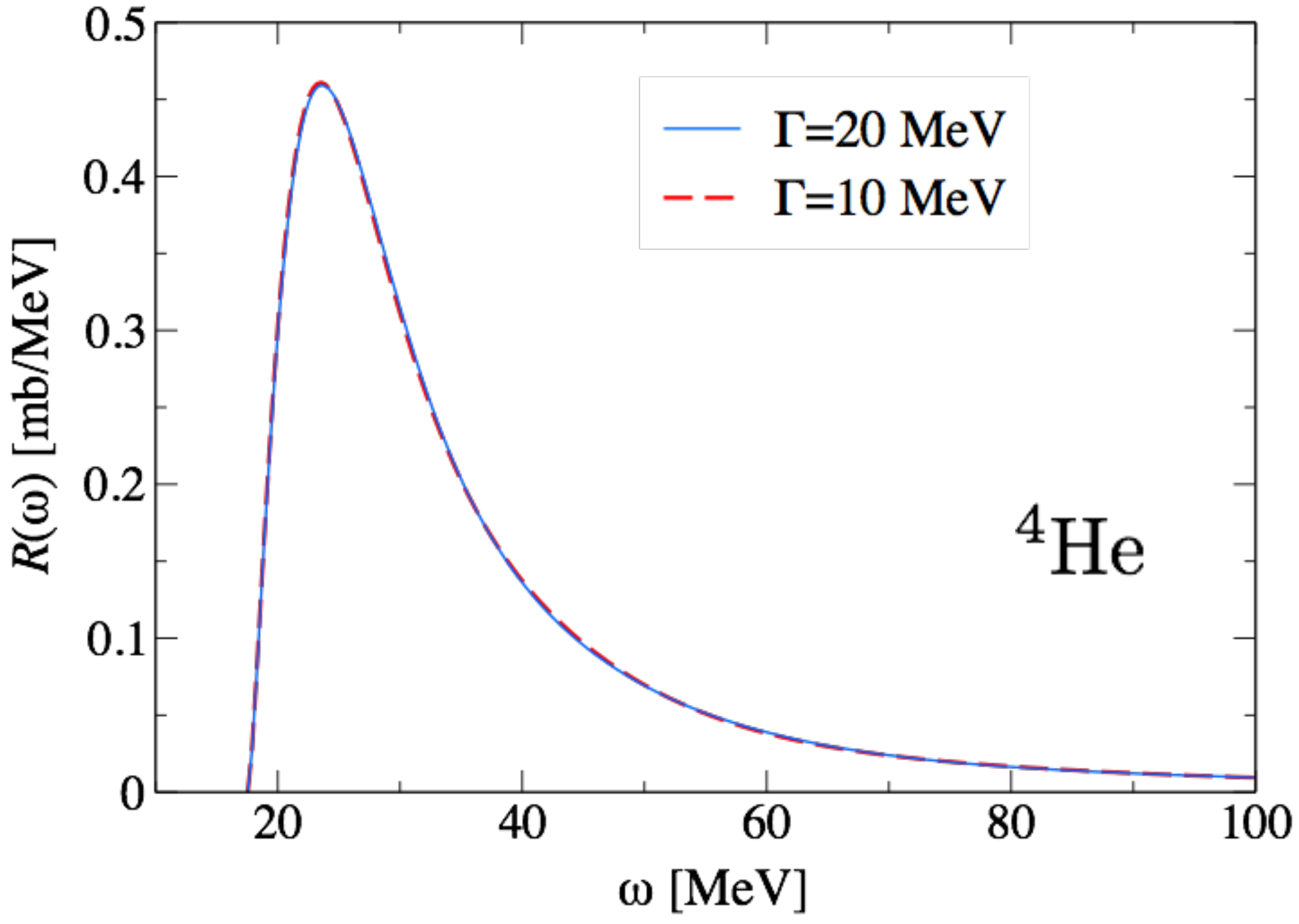
Inversion of the LIT

The inversion is performed numerically with a regularization procedure (ill-posed problem)

Ansatz

$$R(\omega) = \sum_{i}^{I_{\max}} c_i \chi_i(\omega, \alpha) \quad \Longrightarrow \quad L(\sigma, \Gamma) = \sum_{i}^{I_{\max}} c_i \mathcal{L}[\chi_i(\omega, \alpha)]$$

fit



Message: Inversions are stable if the LIT is calculated precisely enough

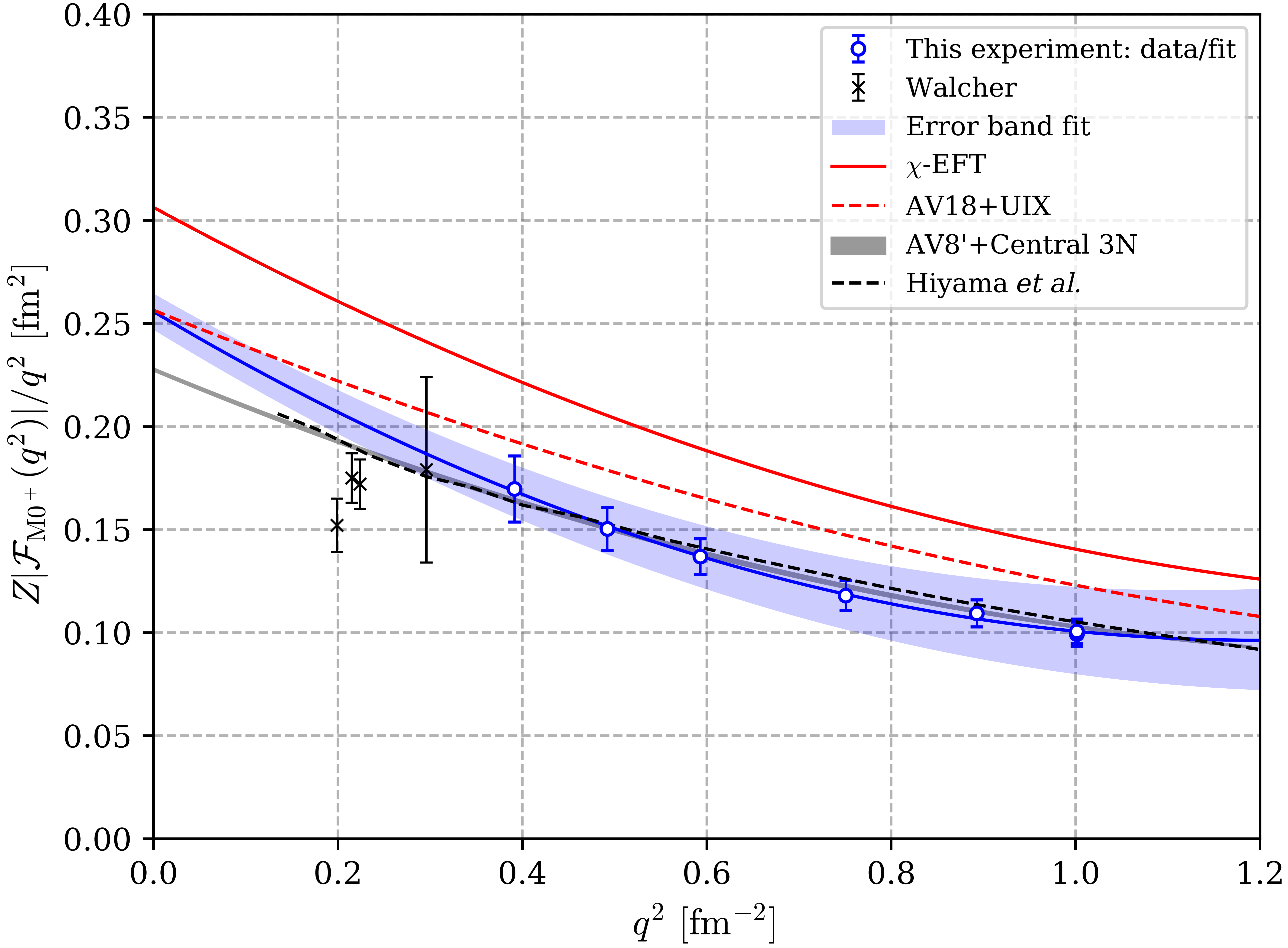
Benchmark

PHYSICAL REVIEW LETTERS **130**, 152502 (2023)

Editors' Suggestion | Featured in Physics

Measurement of the α-Particle Monopole Transition Form Factor Challenges Theory: A Low-Energy Puzzle for Nuclear Forces?

S. Kegel,[1] P. Achenbach,[1] S. Bacca,[1,2] N. Barnea,[3] J. Beričič,[4] D. Bosnar,[5] L. Correa,[6,1] M. O. Distler,[1] A. Esser,[1] H. Fonvieille,[6] I. Friščić,[5] M. Heilig,[1] P. Herrmann,[1] M. Hoek,[1] P. Klag,[1] T. Kolar,[7,4] W. Leidemann,[8,9] H. Merkel,[1] M. Mihovilovič,[1,4] J. Müller,[1] U. Müller,[1] G. Orlandini,[8,9] J. Pochodzalla,[1] B. S. Schlimme,[1] M. Schoth,[1] F. Schulz,[1] C. Sfienti,[1,*] S. Širca,[7,4] R. Spreckels,[1] Y. Stöttinger,[1] M. Thiel,[1] A. Tyukin,[1] T. Walcher,[1] and A. Weber[1]



Low Q^2-data → Information about the **spatial structure** of the resonance

$$\frac{Z|F(q^2)|}{q^2} = \frac{1}{6}\langle r^2\rangle_{\mathrm{tr}}\left[1 - \frac{q^2}{20}\mathcal{R}^2_{\mathrm{tr}} + \mathcal{O}(q^4)\right]$$

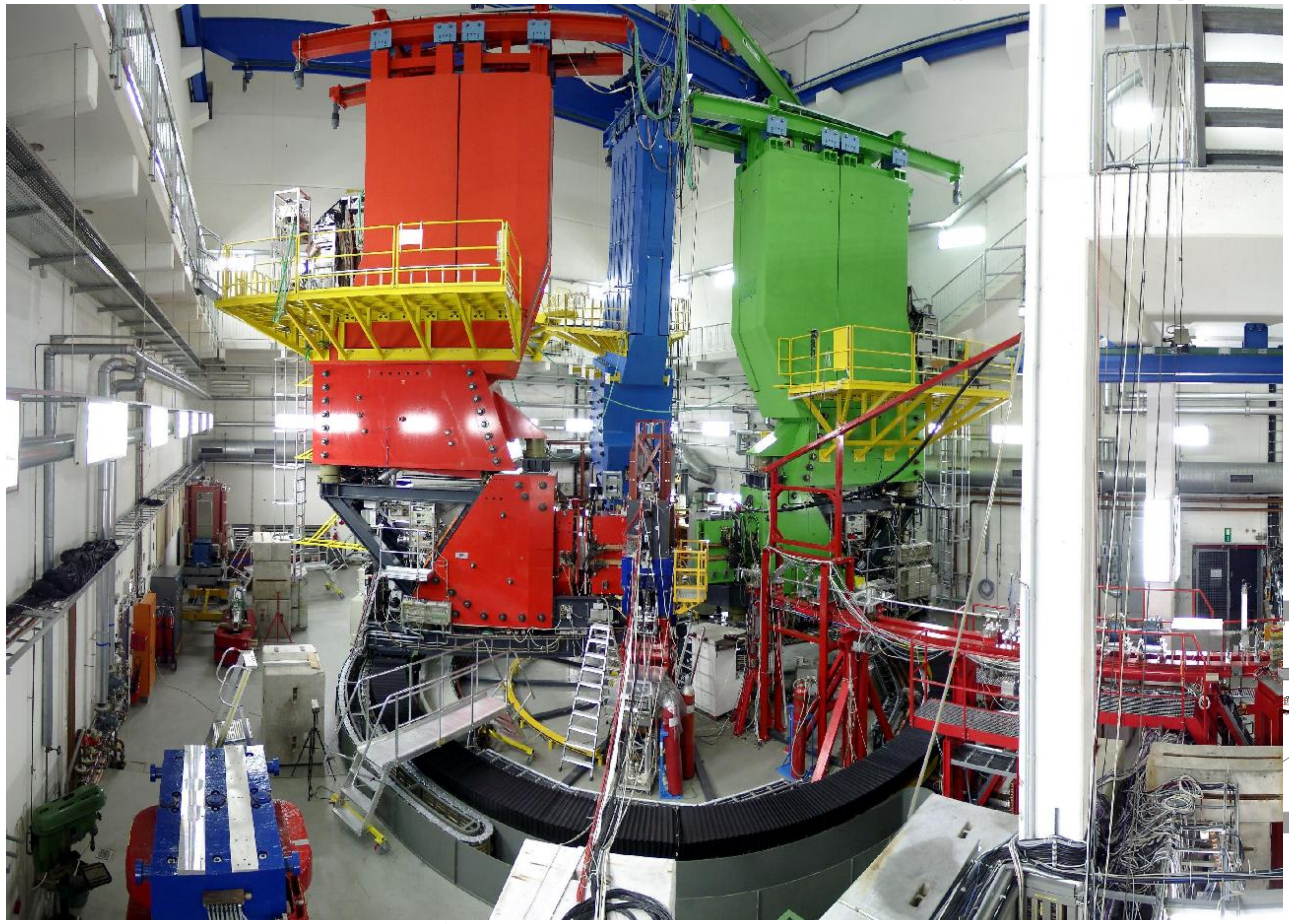
$\langle r^2\rangle_{\mathrm{tr}}$ ↑ Monopole transition matrix element

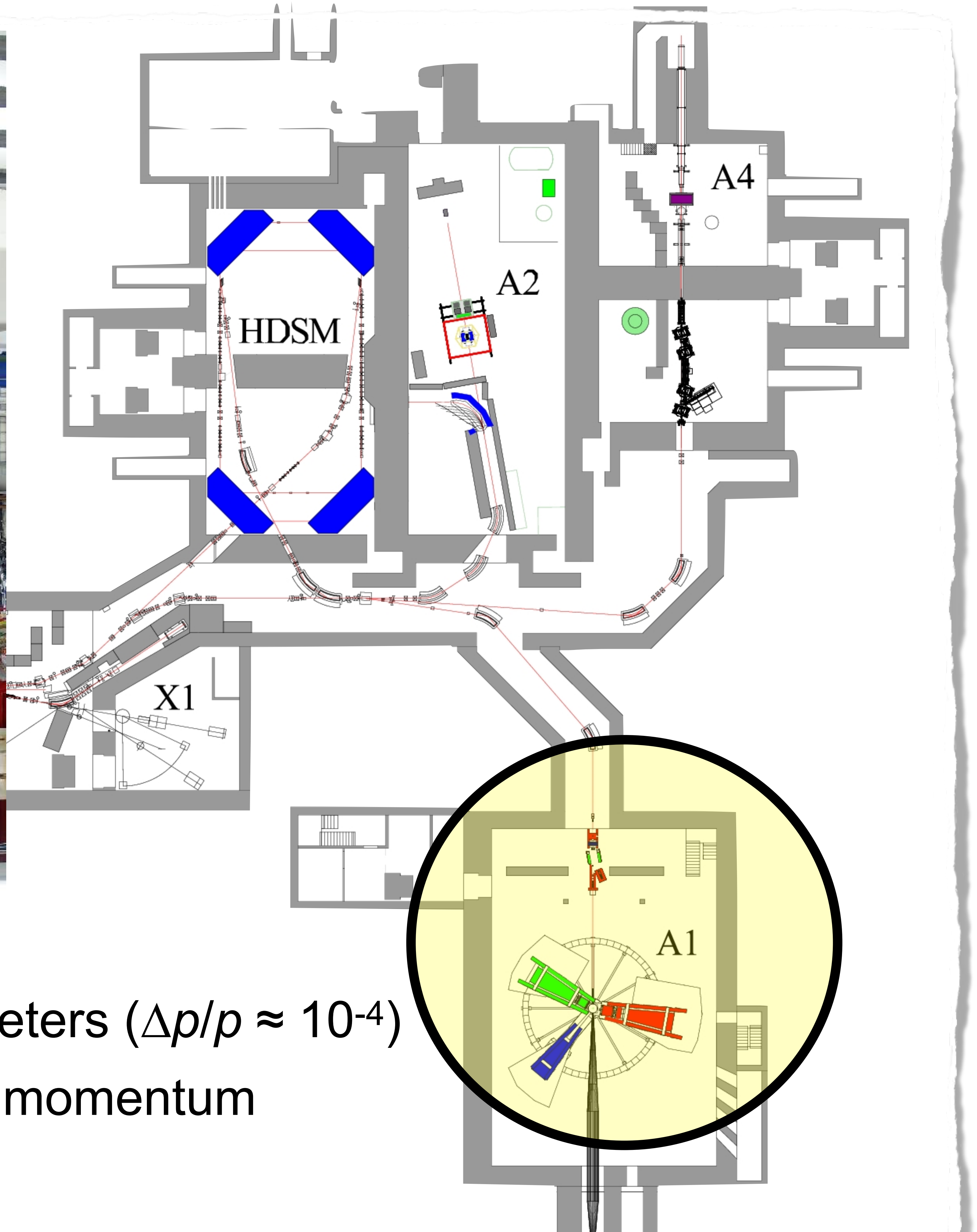
$\mathcal{R}^2_{\mathrm{tr}}$ ↑ $\langle r^4\rangle_{\mathrm{tr}}/\langle r^2\rangle_{\mathrm{tr}}$ Monopole transition radius

	$\langle r^2\rangle_{\mathrm{tr}}$ (fm^2)	$\mathcal{R}_{\mathrm{tr}}$ (fm)
Experiment	1.53 ± 0.05	4.56 ± 0.15
Theory (AV8'+ central 3N)	1.36 ± 0.01	4.01 ± 0.05
Theory (AV18 + UIX)	1.54 ± 0.01	3.77 ± 0.08
Theory (χEFT)	1.83 ± 0.01	3.97 ± 0.05

New experiment

Mainz





- Three high-momentum-resolution magnetic spectrometers ($\Delta p/p \approx 10^{-4}$)
- Reconstruction of scattering angle θ and scattered e^- momentum

New experiment

Mainz

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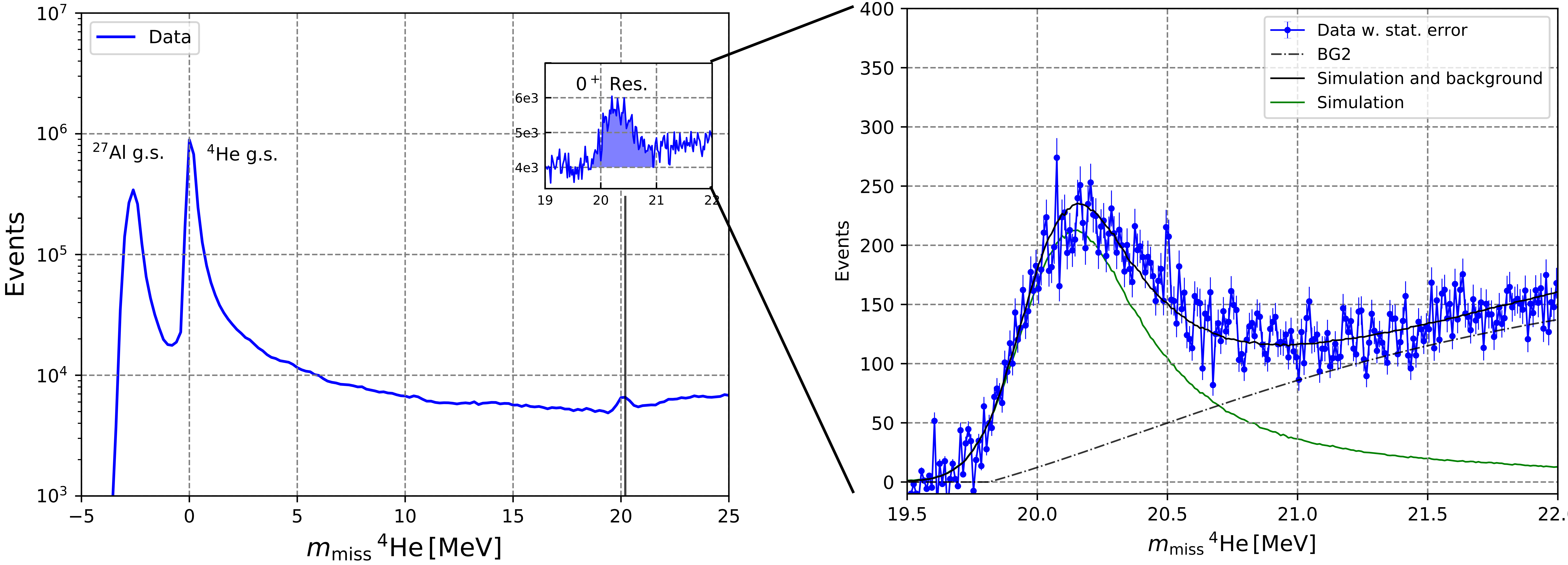
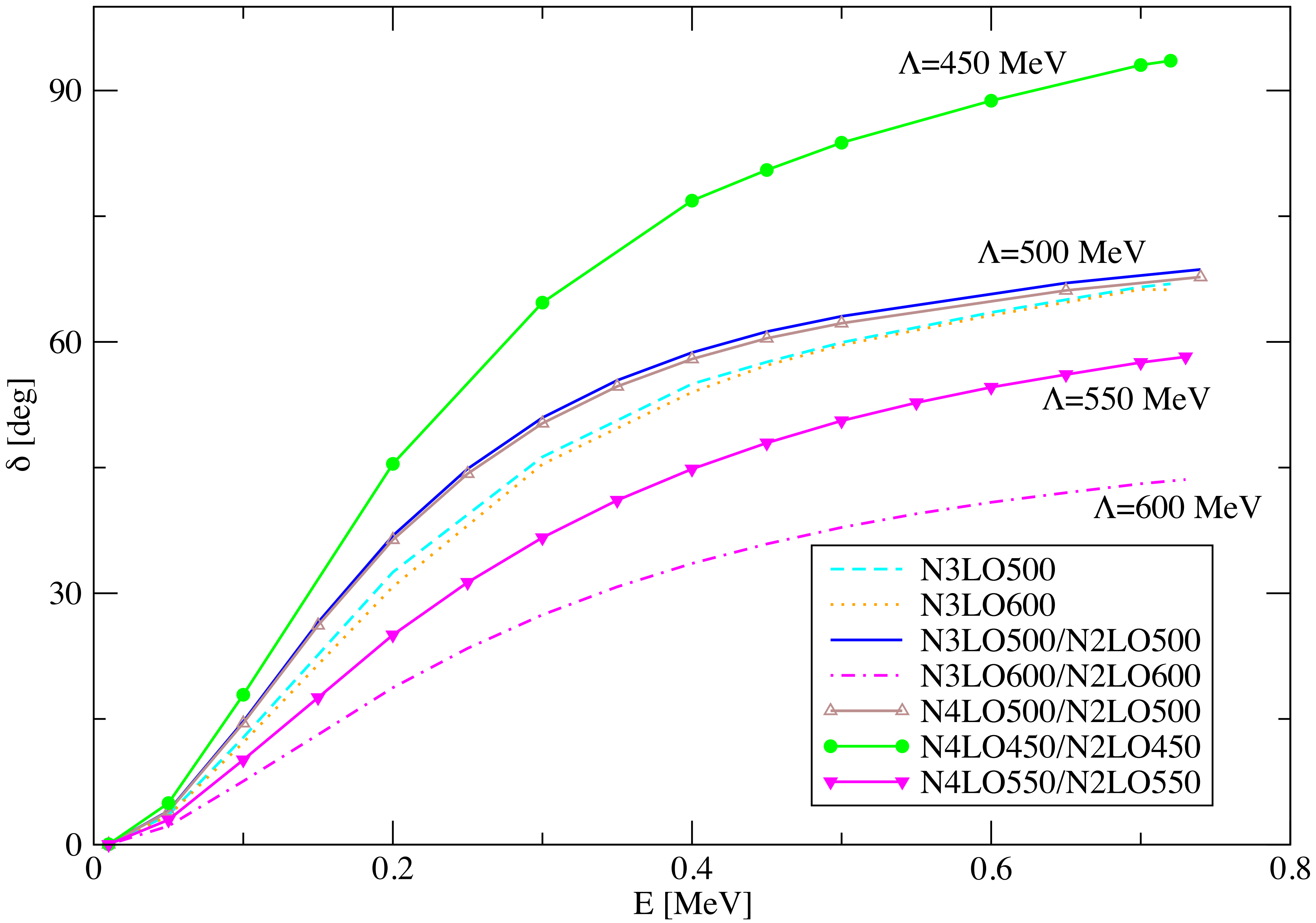


Resonance features

Viviani et al, Phys. Rev. C **102**, 034007 (2020)

Interaction	E_r (MeV)	Γ_r (MeV)
N3LO500	0.126	0.556
N3LO600	0.134	0.588
N3LO500/N2LO500	0.118	0.484
N3LO600/N2LO600	0.130	0.989
N4LO450/N2LO450	0.126	0.400
N4LO500/N2LO500	0.118	0.490
N4LO550/N2LO550	0.130	0.740
Expt.	0.39	0.50



Large dependence on the nuclear Hamiltonian,
In particular on 3N forces