SYMBOLIC REGRESSION AS A TOOL FOR STUDYING GPDS

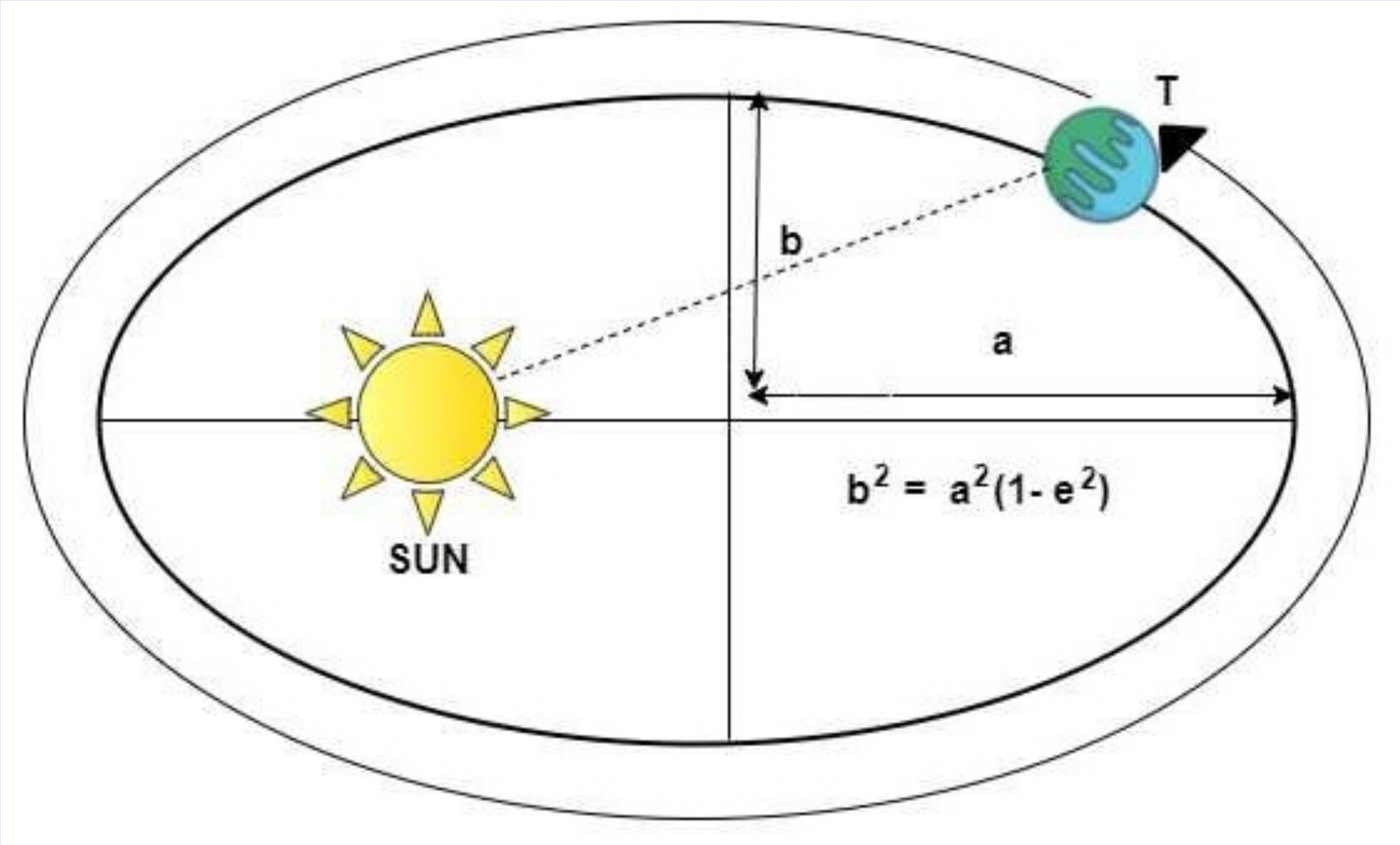
Marija Čuić (University of Virginia)
for the EXCLAIM collaboration

Towards improved hadron tomography with hard exclusive reactions
August 5-9, ECT*, Trento

CAN WE EXTRACT PHYSICAL LAWS OUT OF DATA?

Johannes Kepler's third law
of planetary motion: $(\text{period})^2 \propto (\text{radius})^3$

Planet	Semimajor axis (10^{10}m)	Period T (y)	T^2/a^3 ($10^{-34}y^2/m^3$)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

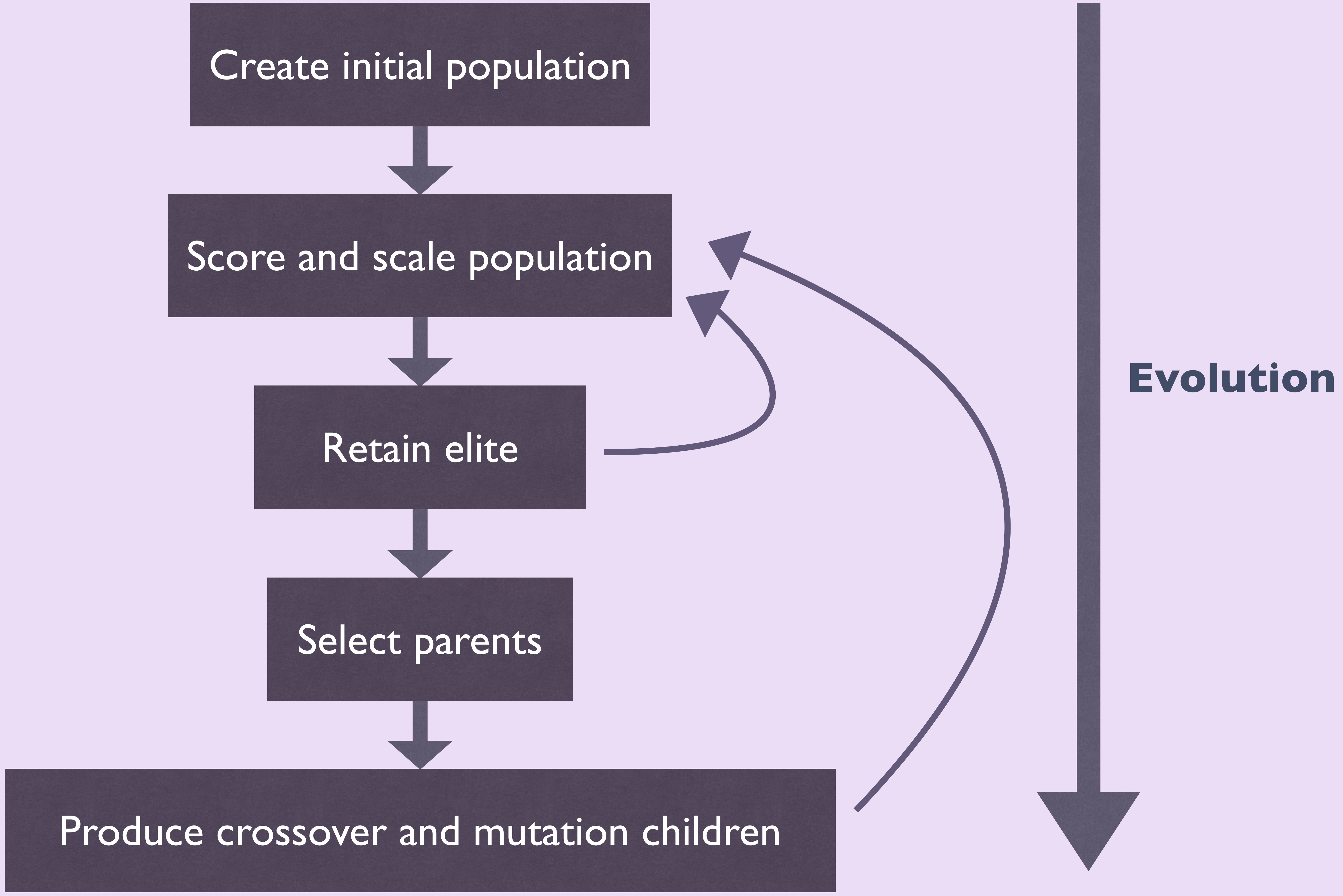


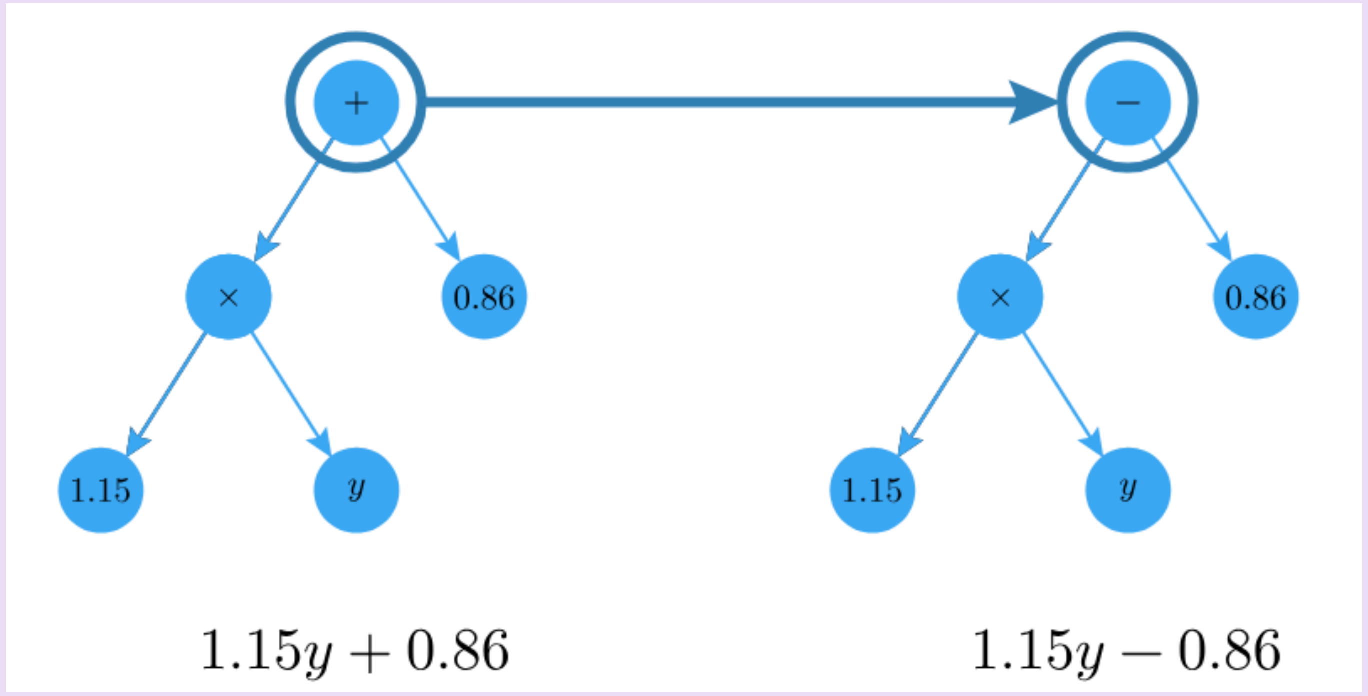
A. Anjum, S. Mishra, 2020

On the 8th of March of this year 1618, if exact information about the time is desired, it appeared in my head. But I was unlucky when I inserted it into the calculation, and rejected it as false. Finally, on May 15, it came again and with a new onset conquered the darkness of my mind, whereat there followed such an excellent agreement between my seventeen years of work at the Tychonic observations and my present deliberation that I at first believed that I had dreamed and assume the sought for in the supporting proofs. But it is entirely certain and exact that the proportion between the periodic times of any two planets is precisely one and a half times the proportion of the mean distances.

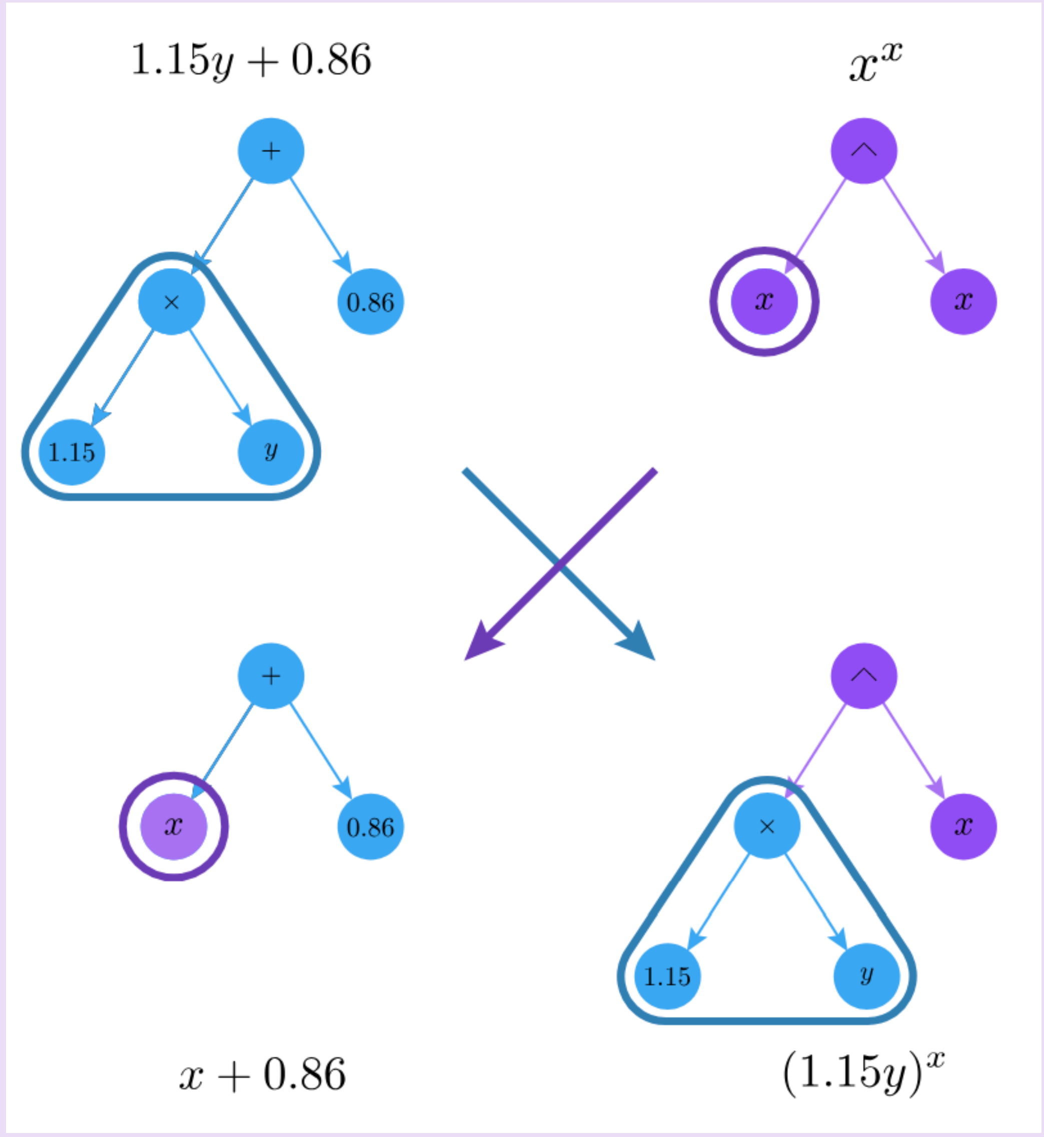
Johannes Kepler

GENETIC ALGORITHMS





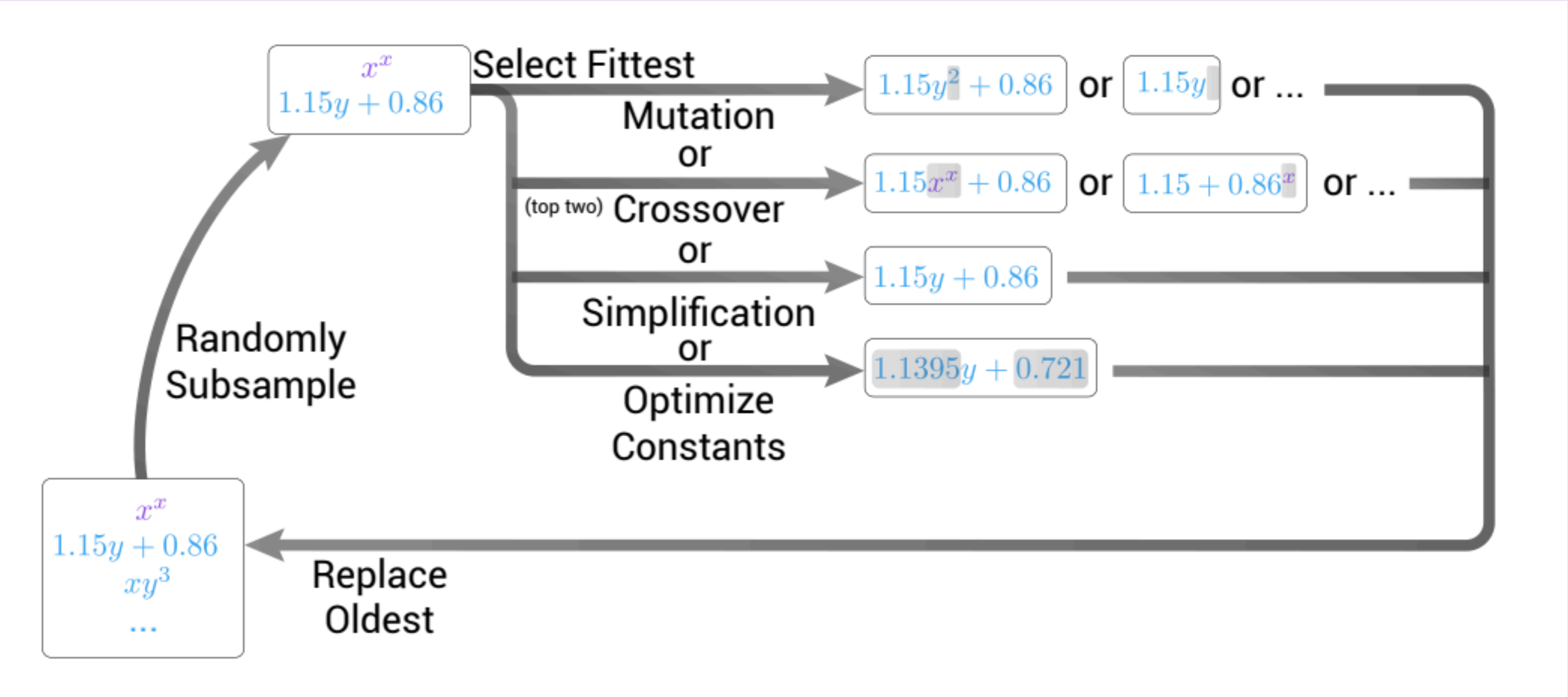
Mutation

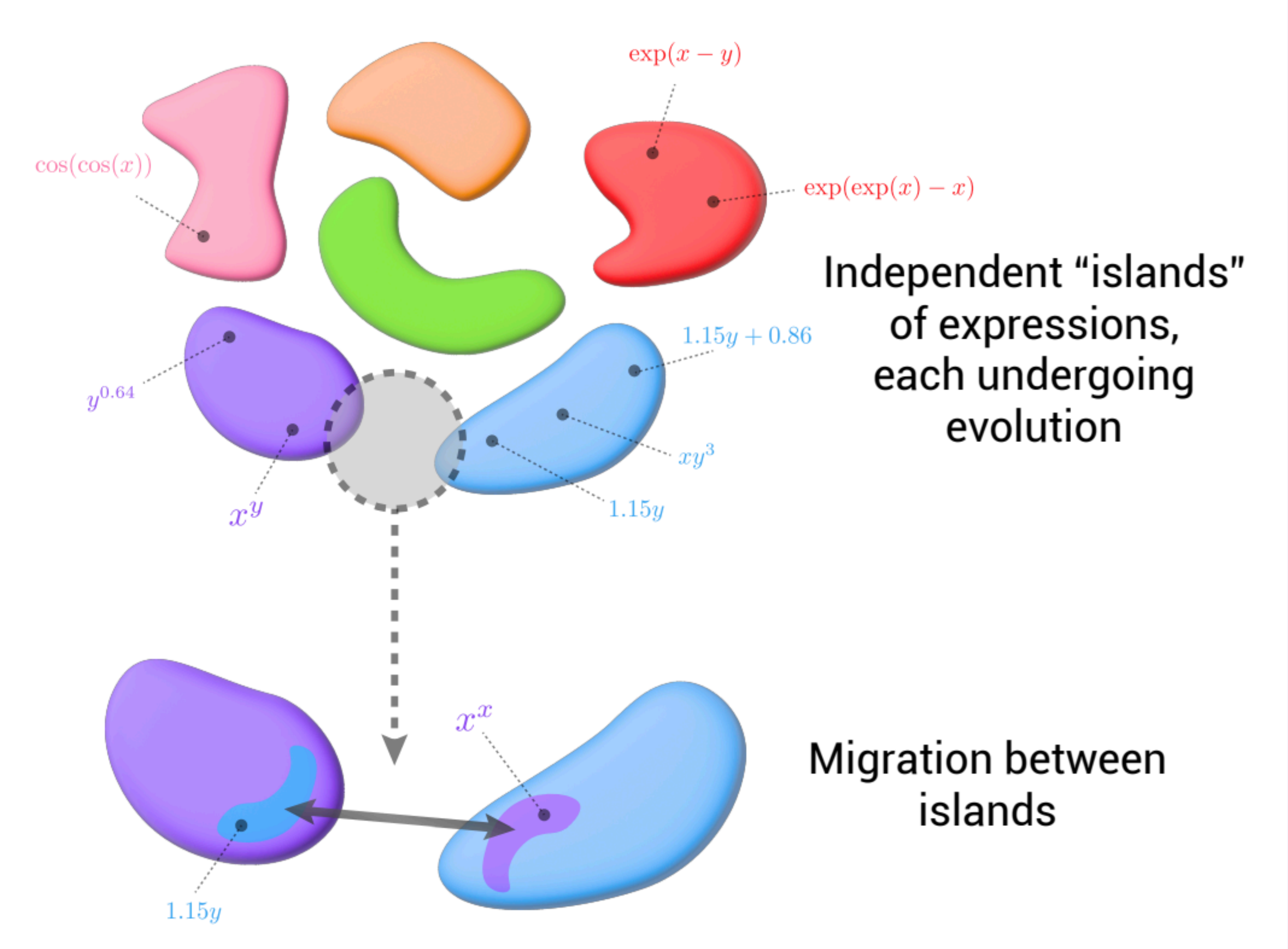


Crossover

PYSR

Computationally inexpensive and highly customizable





[ARXIV:2305.01582V3]

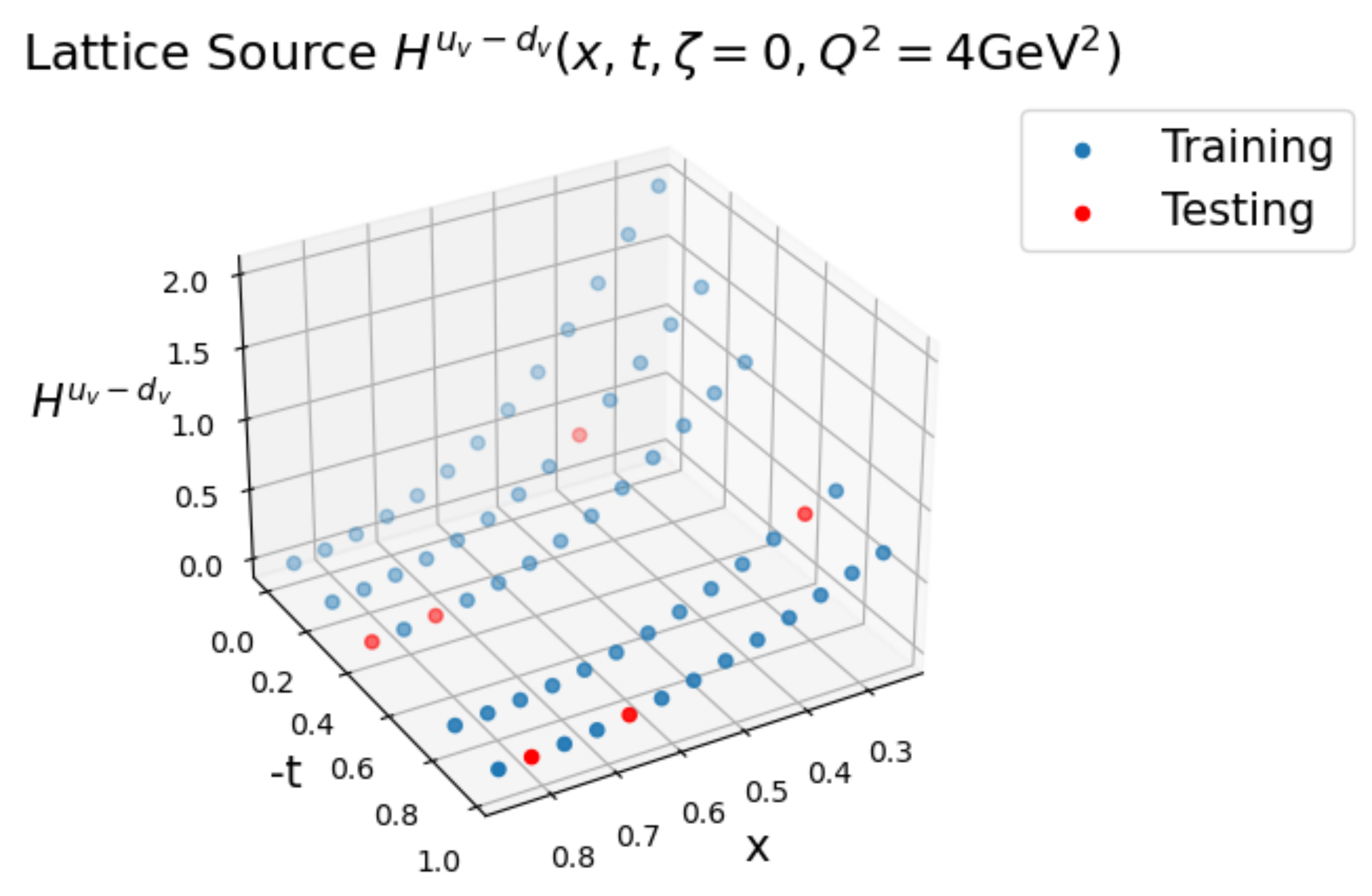
Input:
Data (with uncertainties)

Software :

- Goodness of form (max complexity, possible operators, etc…)
- Goodness of fit (MSE+custom loss)
- Hyperparameters (number of iterations, measure for choosing best model, etc…)

Output:
Pareto front/Hall of fame with best model

LATTICE GPDS AND PHENOMENOLOGICAL MODELS



Huey-Wen Lin, 2021

We encourage parametrizations:

1) $H^{u-d}(x,t) = P(x)F(t)$
2) $H^{u-d}(x,t) = x^{\alpha}(1-x)^{\beta}P(x,t)$
3) $H^{u-d}(x,t) = x^{\alpha}(1-x)^{\beta}P(x)F(t)$

with custom loss:

- MSE + Non-Factorization Penalty
- MSE + Regge behavior,

complexity:

- Maximum complexity allowed for power operator,

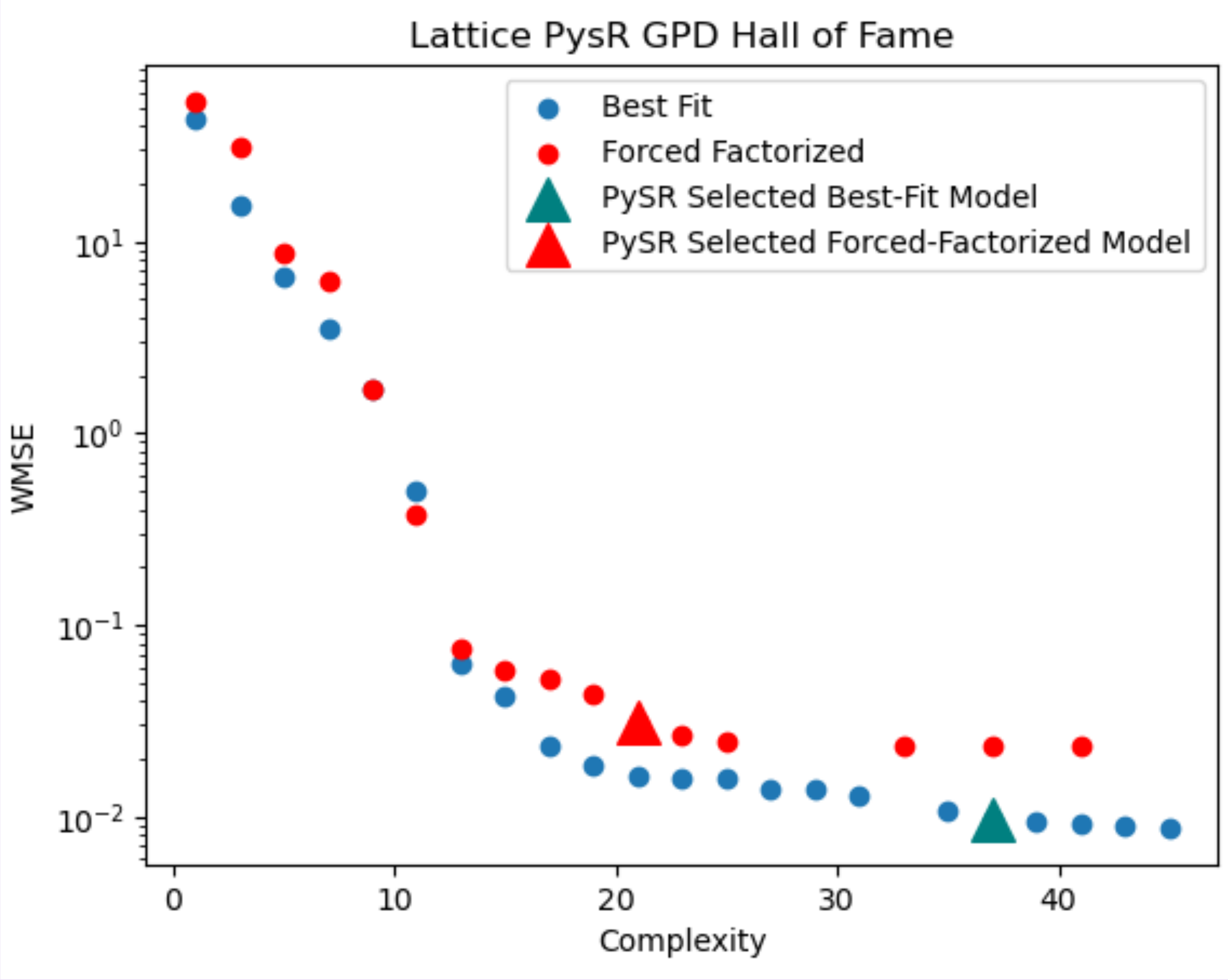
and forcing the PDF form:

- replace eq. 2) with $H^{u-d}(x,y,t) = x^{\alpha}y^{\beta}P(x,t)$, where $y = 1 - x$, and do a 3D fit.

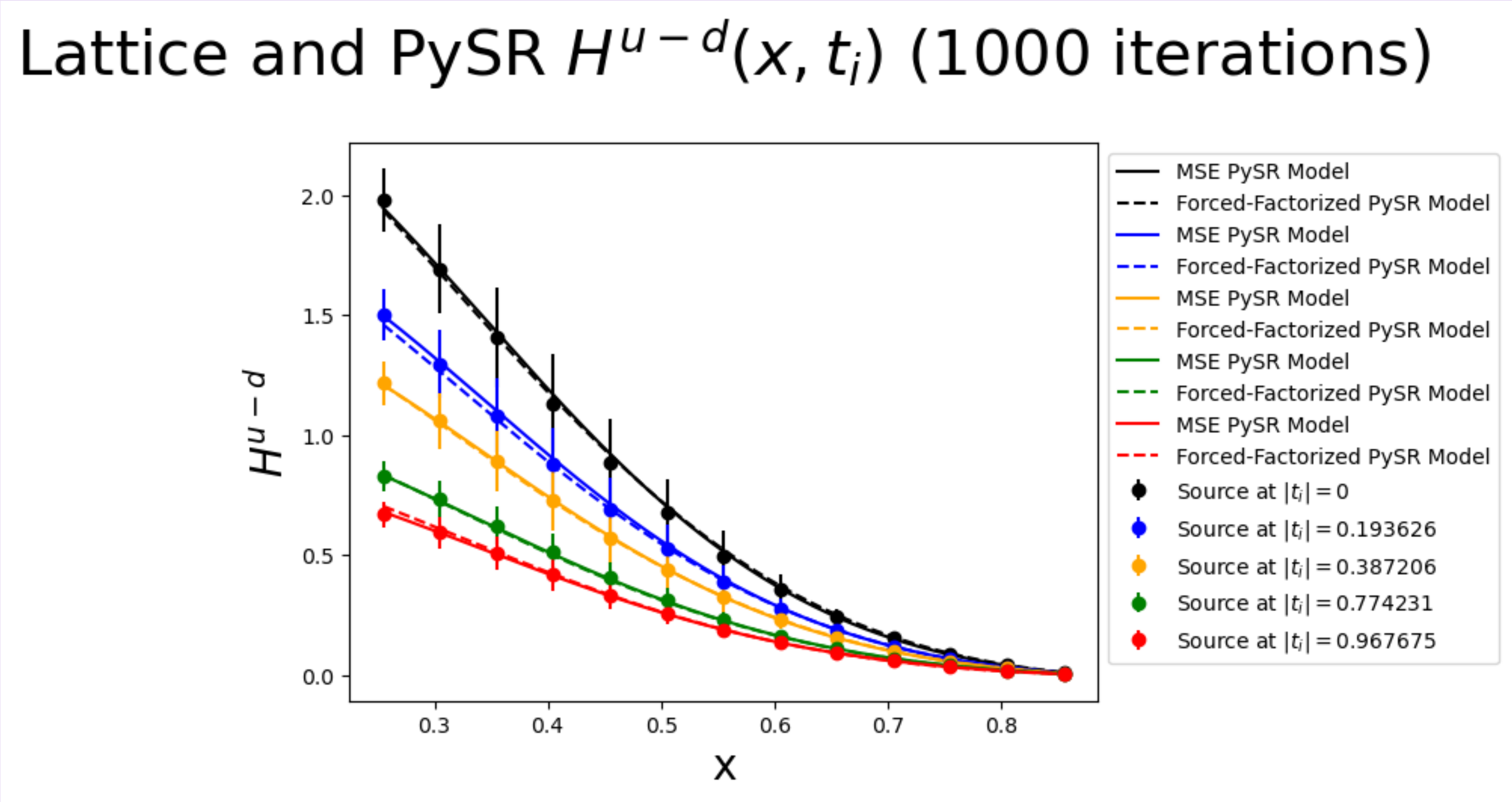
PARETO FRONT

Lattice unfactorized

COMPLEXITY	LOSS	EQUATION
1	43.2423	0.0616
3	15.5882	0.8262 - x
5	6.5251	(0.35775 / x) - 0.4188
7	3.4833	(0.8373 - x) / (Itl + x)
9	1.7014	(2.1328 - Itl) * ((0.0027^ x) ^ x)
11	0.4980	(((0.0006 ^ x) ^ x) ^ x) / (0.4583+ Itl)
13	0.0627	((-0.0184 + ((0.0053 ^ x) ^ x)) / (2.7513 ^ Itl)) * 2.6767
15	0.0425	((((0.0054 ^ x) ^ x) -0.0185) / (Itl +1.1378)) * (3.1019 - Itl)
17	0.0232	((((0.0034 ^ x) ^ x) -0.0126) / (Itl + 1.0742)) * ((2.7933 + x) - Itl)
19	0.0185	((((0.0034^ x) ^ x) -0.0126) / ((Itl ^ 0.9381) + 1.0721)) * (x + (2.8084 - Itl))
21	0.0162	((((0.0034^ x) ^ x) -0.0126) / ((Itl ^ 0.9381) + 1.0742)) * ((2.7654 + (x / 0.9003)) - Itl)
23	0.0160	(((2.7654 + (x / 0.9033)) - Itl) +0.0126) * ((((0.0034 ^ x) ^ x) -0.0126) / ((Itl ^ 0.9223) + 1.0742))
25	0.0159	((((0.0034 ^ x) ^ x) -0.0126) / ((Itl ^ 0.9223) + 1.0742)) * (((2.7654 + ((x / 1.0742) / 0.8533)) - Itl) +0.0126)
27	0.0138	((x + ((2.8084 - Itl) + x)) * (((((0.0030 / 1.0486) ^ x) ^ x) -0.0110) / (((x * 1.6675) ^ 0.0770) + Itl))) * 0.8197
29	0.0138	((x + (x + (2.8084 - Itl))) * (((((0.0030 / 1.0486) ^ x) ^ x) -0.0110) / (((x * (1.7154 - 0.0770)) ^ 0.0770) + Itl))) * 0.8197
31	0.0130	((-0.0107+ ((0.0028 ^ x) ^ x)) * (((((1.5143 / 1.2897) + 2.5251) + x) - Itl) - (((Itl / 0.4269) + Itl) ^ 0.4417)) * ((1.0796^ x) + -0.2719)
35	0.0106	(((((((((14.4235 / x) / 1.4622) + Itl) + (Itl / 0.7363)) ^ x) - ((Itl * (Itl +0.1756)) * Itl)) * (0.0004 ^ x)) ^ x) -0.0148) / (0.6474 + Itl)) * 1.6940
37	0.0096	(((((0.0004 ^ x) * ((((((14.4235 / x) / 1.4601) + Itl) + ((Itl / 0.5526) - 0.3370)) ^ x) - ((Itl * (Itl +0.1937)) * Itl))) ^ x) -0.0148) / (0.6474 + Itl)) * 1.6940
39	0.0094	(((((0.0004 ^ x) * ((((((14.4235 / x) / 1.4601) + (Itl + Itl)) + ((Itl * 0.7243) -0.2396)) ^ x) - ((Itl * (Itl +0.1937)) * Itl))) ^ x) -0.0148) / (0.6474 + Itl)) * 1.6940
41	0.0091	(((((((((14.4235 / x) / 1.4601) + 2Itl) + (Itl - 0.3370)) ^ x) - ((Itl * (Itl - (-0.1937 * (Itl +0.1937)))) * Itl)) * (0.0004 ^ x)) ^ x) -0.0148) / (0.6474 + Itl)) * 1.6940
43	0.0089	(((((((((14.4235 / x) / 1.4601) + 2Itl) + (Itl - 0.3885)) ^ x) - ((Itl * (Itl - (-0.1937 * ((Itl +0.1937) * Itl)))) * Itl)) * (0.0004 ^ x)) ^ x) -0.0148) / (0.6474 + Itl)) * 1.6940
45	0.0087	(((((((((14.4235 / x) / 1.4601) + 2Itl) + ((Itl ^ 1.4625) - 0.3370)) ^ x) - ((Itl * (Itl - ((-0.1937 * Itl) * (Itl +0.1937)))) * Itl)) * (0.0004 ^ x)) ^ x) -0.0148) / (0.6474 + Itl)) * 1.6940



LATTICE RESULTS



Non-factorized:

$$H^{u_v-d_v}(x,0,t) = \frac{1.69\left(\left(0.0004^x\left(-|t|^3-0.194|t|^2+\left(2.81|t|-0.337+\frac{9.88}{x}\right)^x\right)\right)^x-0.0148\right)}{|t|+0.647}$$

wMSE = 0.0096

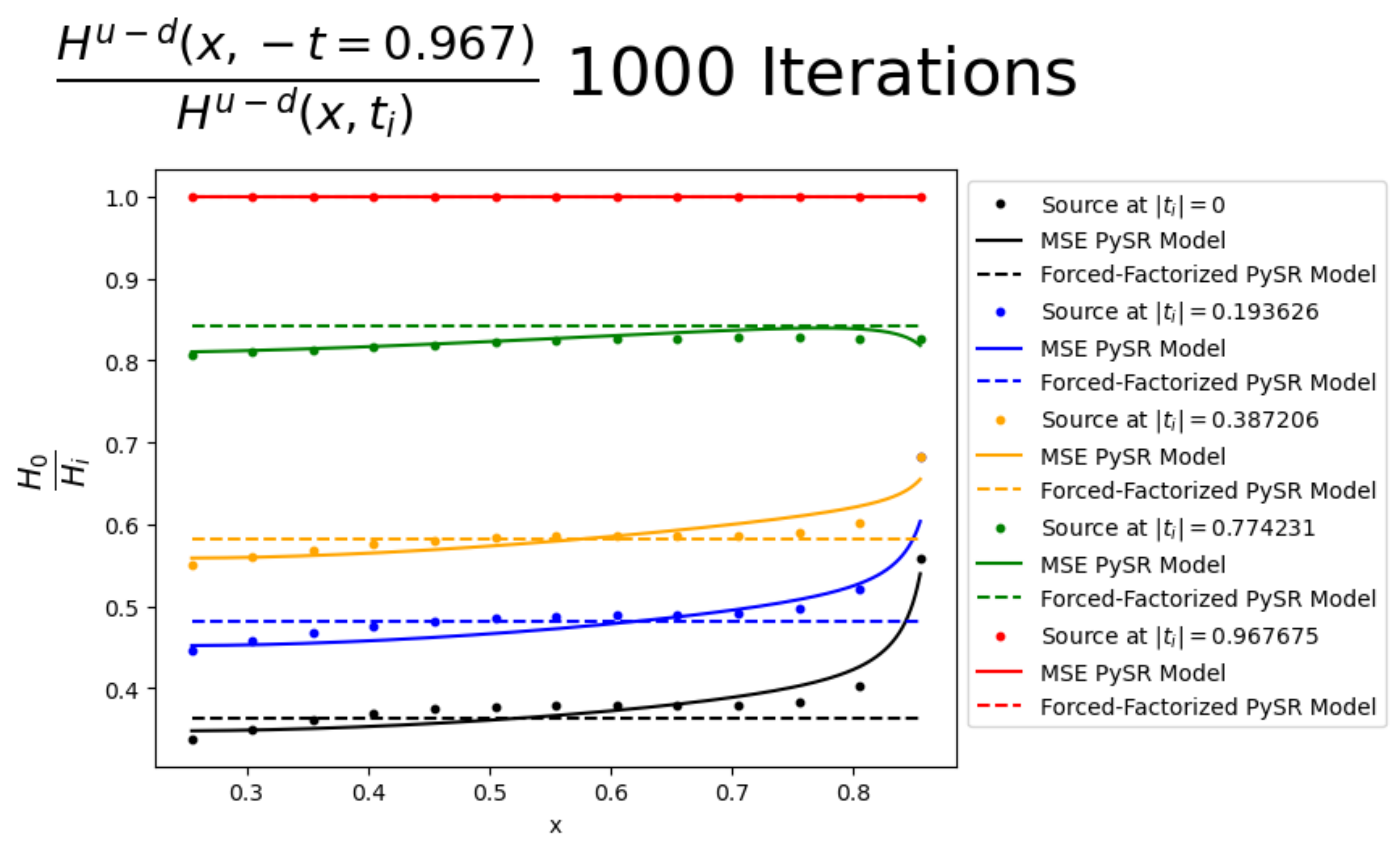
Forced-factorized:

$$H^{u_v-d_v}(x,0,t) = \frac{\left(0.0074^x-0.0135\right)^x}{\frac{0.131^{|t|}}{1.58|t|^{|t|}+1.11}+|t|}$$

wMSE = 0.031

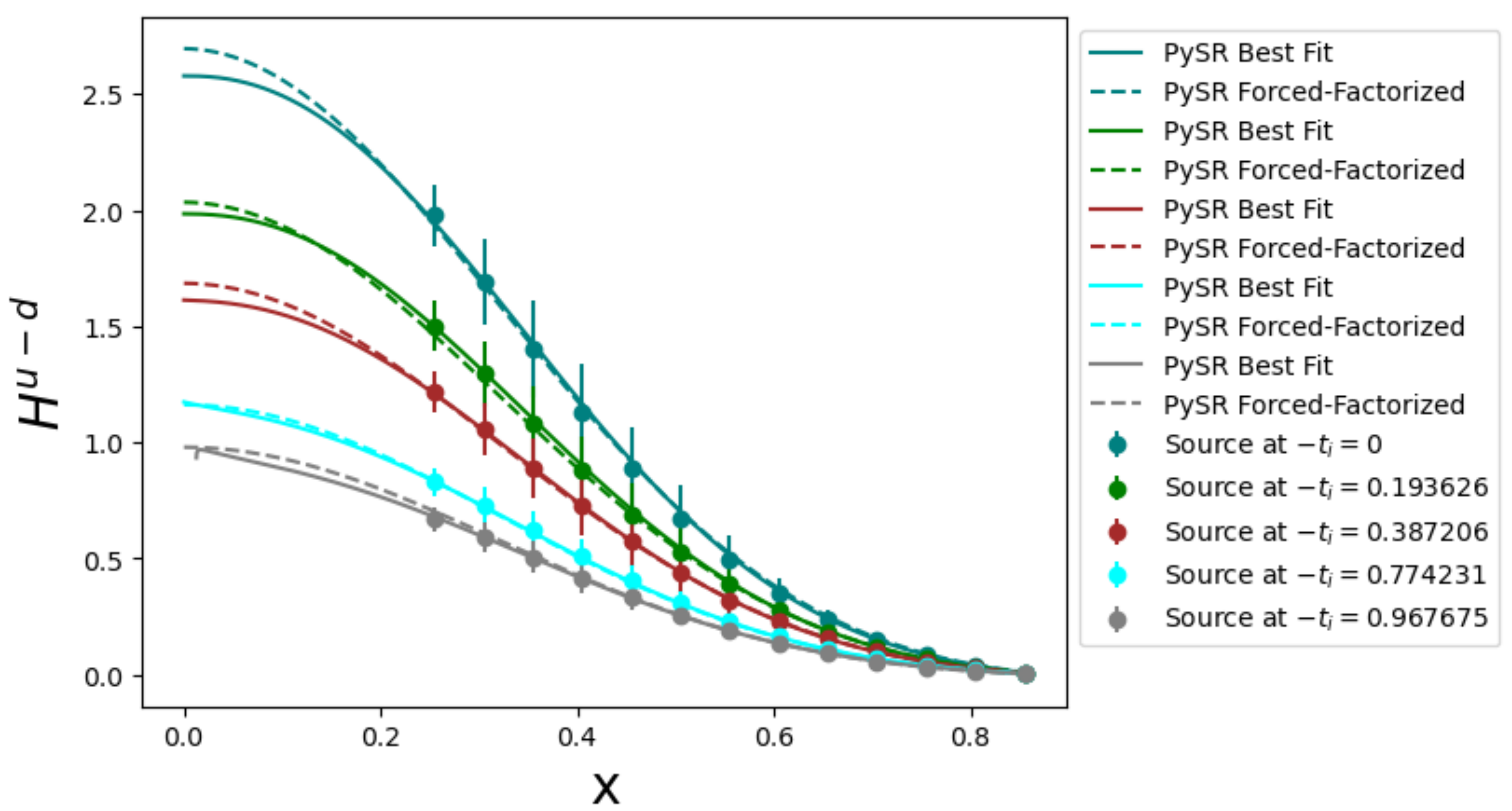
Figure by A. Dotson

X AND T FACTORIZATION ON LATTICE

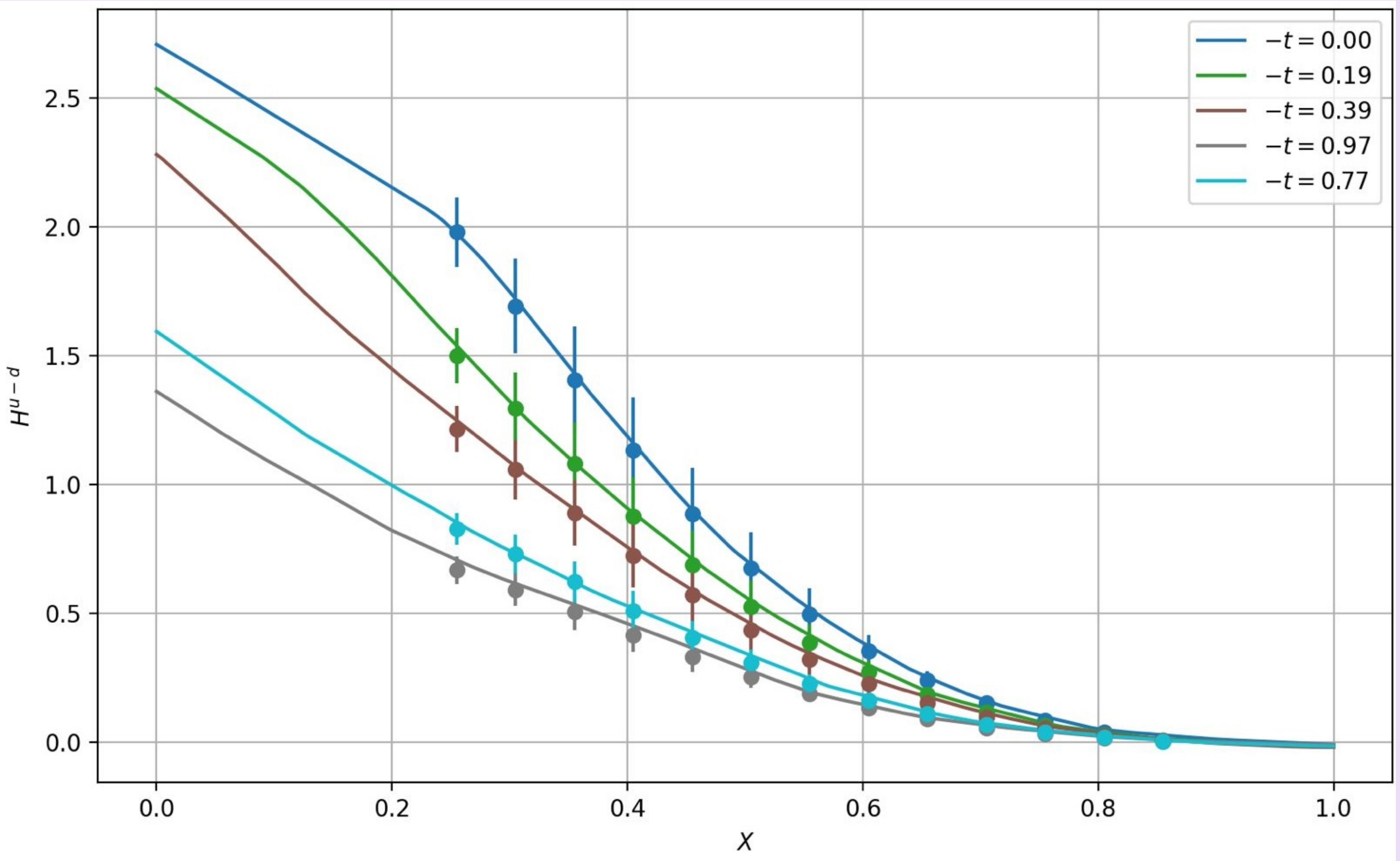


Lattice does not fully factorize, mostly deviates at smaller and larger x, and larger t

COMPARISON TO NEURAL NETWORKS



PySR extrapolation of lattice GPDs

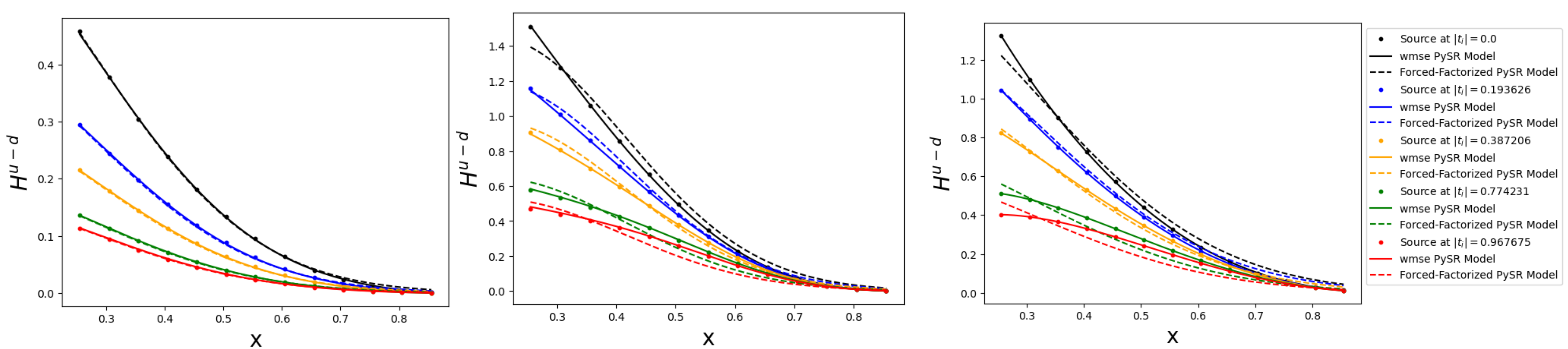


Neural network extrapolation of lattice GPDs

Neural networks known for poor extrapolation, SR seems to work better!

Figures by A. Dotson and Z. Panjsheeri

PHENOMENOLOGICAL RESULTS

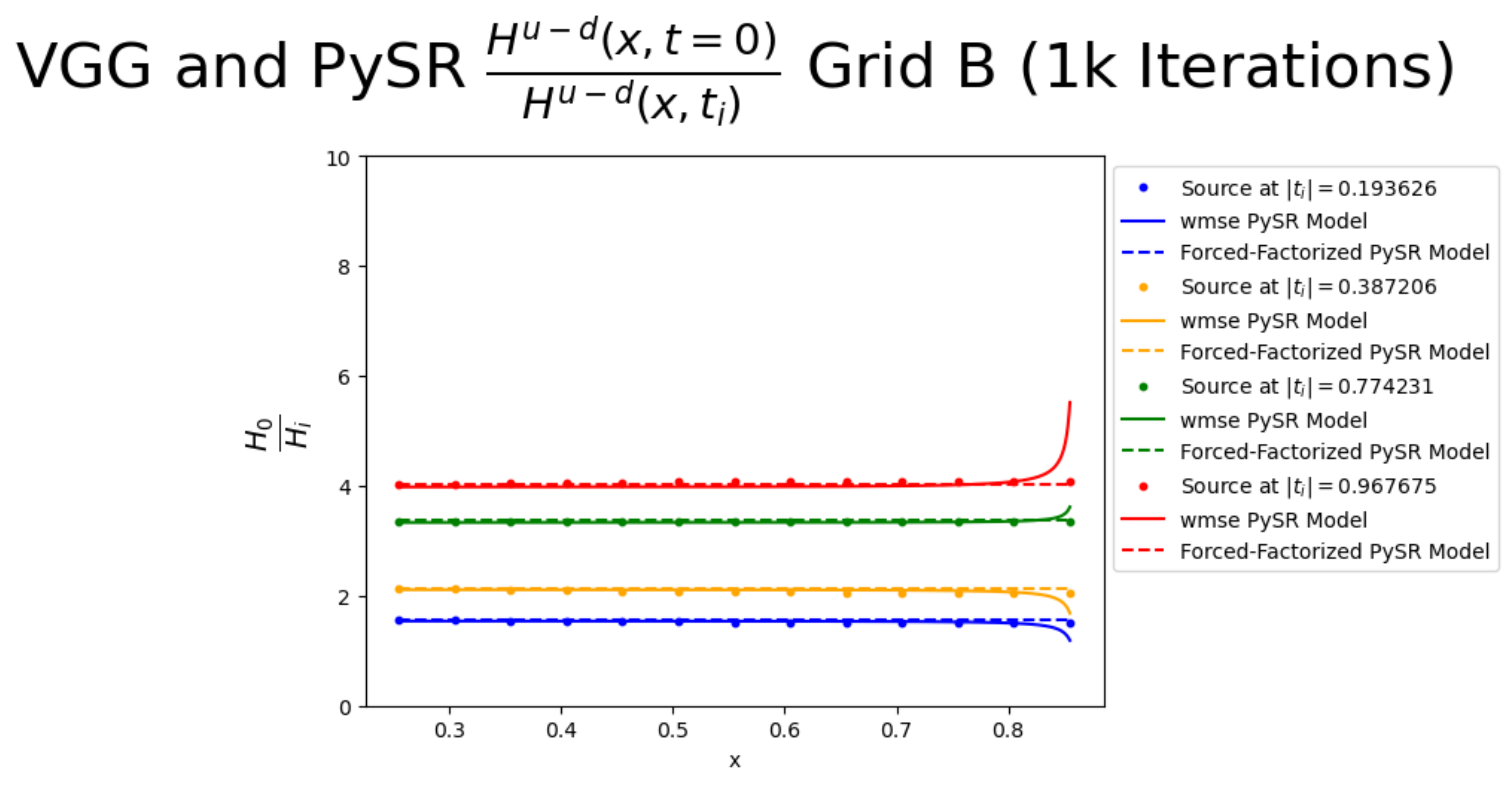


VGG | GGL | GK

MSE = 2×10^{-6} | MSE = 1×10^{-5} | MSE = 2×10^{-7}

Factorized MSE = 5×10^{-6} | Factorized MSE = 1×10^{-3} | Factorized MSE = 1×10^{-3}

MORE ON VGG…



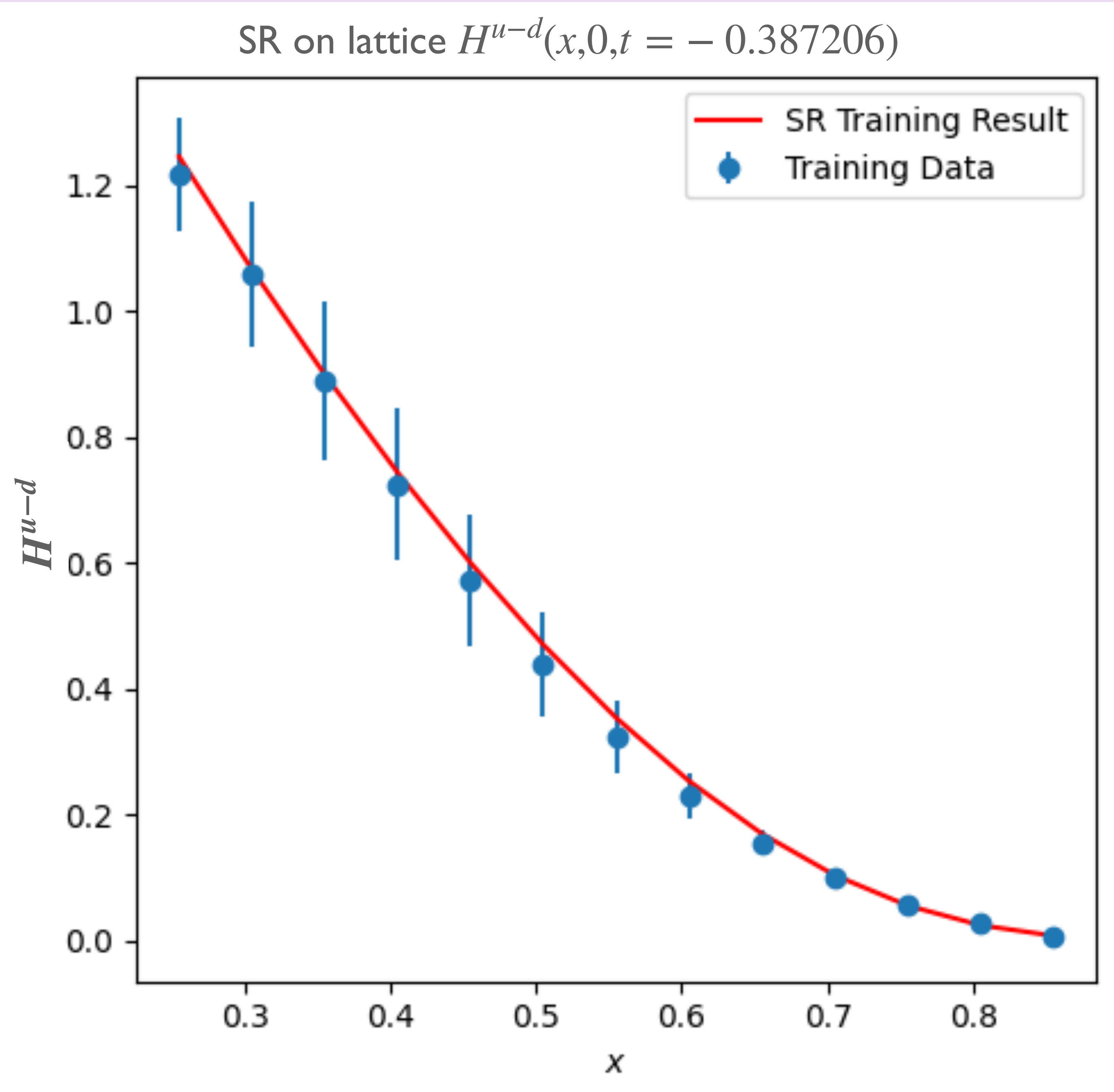
Non-factorized:

$$H^{u-d}(x,0,t) = -\frac{0.00239x}{|t| + 0.335} + \frac{\left(0.00174^{x}\right)^{x}}{|t| + 2.52\left(|t| + 0.663\right)^{1.33}}$$

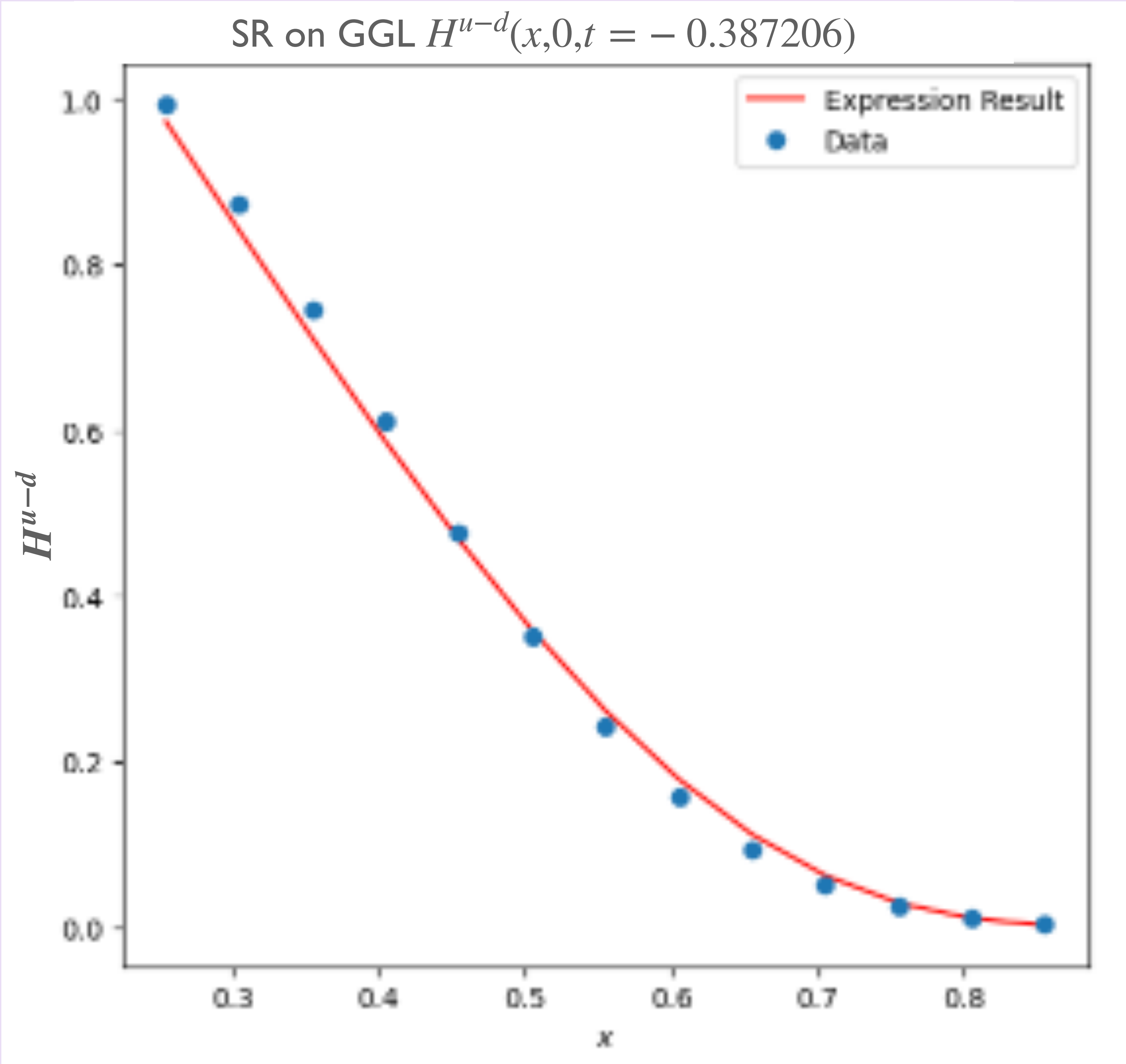
Factorized:

$$H^{u-v}(x,0,t) = \frac{0.582\left(\left(0.00269^{x}\right)^{1.11}\right)^{x}}{|t| + 0.836 + 1.35 \cdot 0.836^{-|t|}|t|}$$

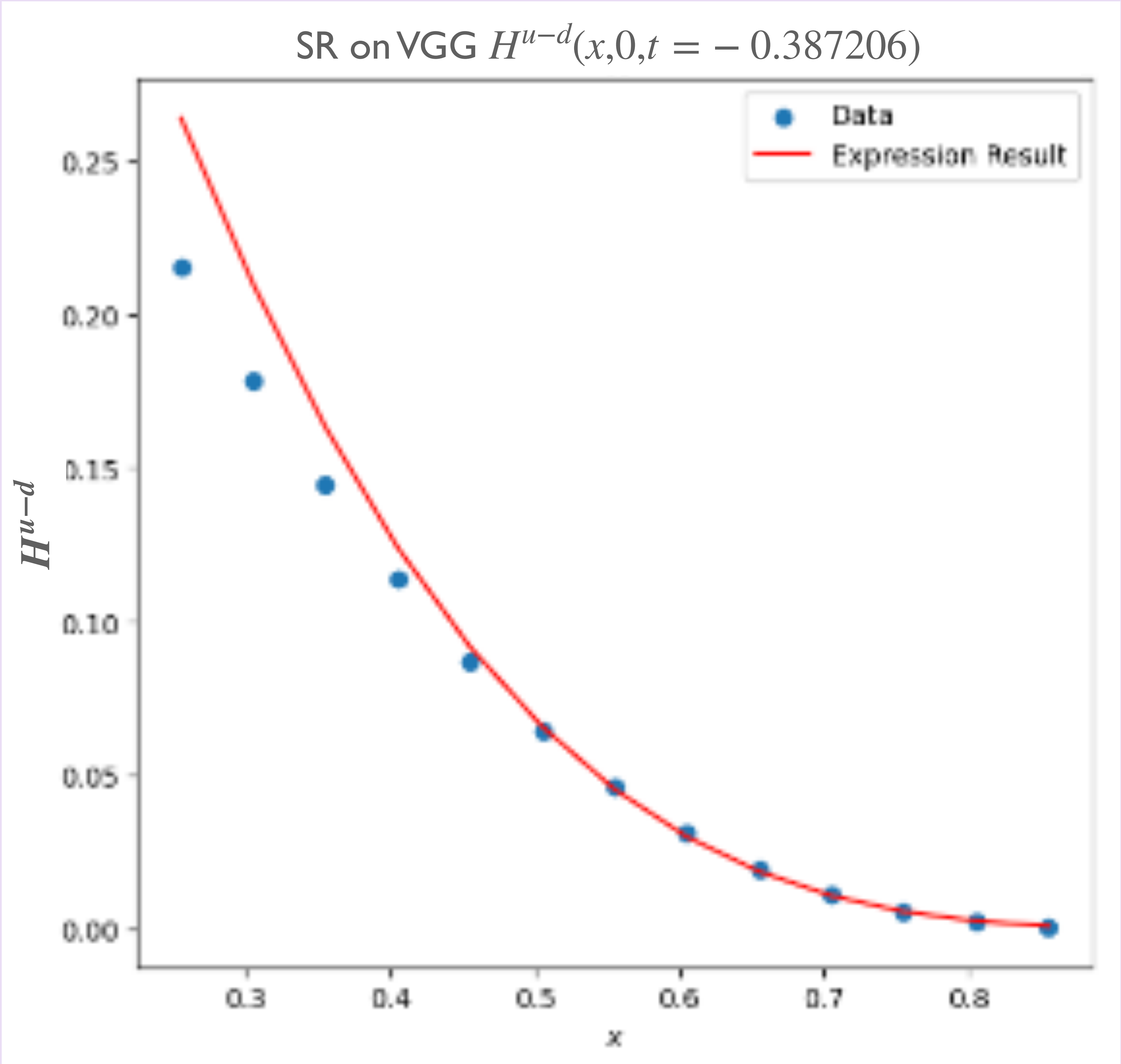
REGGE ANSATZ



Lattice



GGL model



VGG

$H^{u_v-d_v}(x,0,t) = x^{-0.09}(1-x)^{1.57}e^{-0.68(\ln(1-x))^2}e^{-t+1.01}$

MSE = 0.001

$H^{u_v-d_v}(x,0,t) = x^{t-0.38}(1-x)^{1.46}e^{(\ln(1-x))^2}e^{0.32t+0.38}$

MSE = 7×10^{-7}

$H^{u_v-d_v}(x,0,t) = x^{0.11}(1-x)^{3.58}e^{1.39t+0.41}$

MSE = 0.0003

Figures by A. Reddy Singireddy

OUTLOOK

- Detailed study of uncertainties
- Study of convergence of the Pareto front
- Detailed study of lattice GPDs and GPD moments
- Building a brute-force custom symbolic regression/approximator