# DVMP at higher-order and higher-twist revisited

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STRONG 2020

# Outline

- DVMP at NLO:
  - NLO global DIS+DVCS+DV$\rho^0$P fits

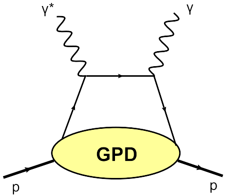
    [Čuić, Duplančić, Kumerički, P-K. '23]
  - improving meson description (DAs)

    $\rightarrow$ work in progress with Raj Kishore, K. Kumerički

- DVMP at twist-3:
  - lessons from wide-angle meson production [Kroll, P-K. '18, '21]
  - DV$\pi^0$P [Duplančić, Kroll, P-K., Szymanowski '24]

# GPDs from deeply virtual exclusive processes

## DVCS



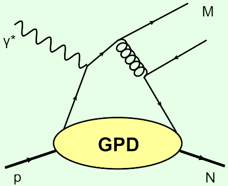

$$\gamma_T^* N \to \gamma N$$

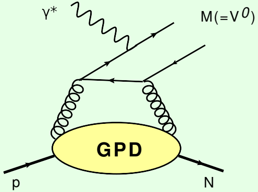
$H^q$, $E^q$, $\widetilde{H}^q$, $\widetilde{E}^q$

NLO:

$H^G$, $E^G$, $\widetilde{H}^G$, $\widetilde{E}^G$, $F_T^G$

## DVMP




$$\gamma_L^* N \to M N'$$

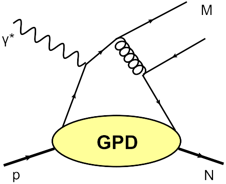
$M = V_L$: $H^{q_i}$, $E^{q_i}$; $H^G$, $E^G$

$M = PS$: $\widetilde{H}^{q_i}$, $\widetilde{E}^{q_i}$

$$\gamma_T^* N \to M N'$$

$M_{\text{twist-3}} \Rightarrow F_T^q$

# Meson production: handbag factorization




$\gamma^{(*)} N \to M N'$

DEEPLY VIRTUAL
$Q^2 \gg$, $-t \ll$

DVMP

[Collins, Frankfurt, Strikman '97]

- factorization
  $\mathcal{H}^a \otimes GPD$
- GPDs at small $(-t)$
- tw2: $\gamma_L^*$, tw3: $\gamma_T^*$

WIDE ANGLE
$-t$, $-u$, $s \gg$, $Q^2 \ll$ or 0

WAMP

[Huang, Kroll '00]

- arguments for factorization
  $\mathcal{H}^a (1/x \otimes GPD(\xi = 0))$
- GPDs at large $(-t)$

large scale $\mathcal{Q}^2$ $(Q^2, -t, s, \ldots)$

- twist expansion: $\langle \mathcal{H} \rangle^{tw2} + \dfrac{\langle \mathcal{H} \rangle^{tw3}}{\mathcal{Q}} + \ldots$
- $\alpha_S$ expansion for each twist: $\alpha_S(\mathcal{Q}) \langle \mathcal{H} \rangle^{LO} + \alpha_S^2(\mathcal{Q}) \langle \mathcal{H} \rangle^{NLO}$

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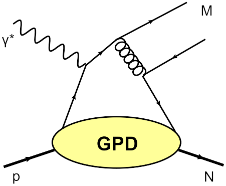


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$\mathcal{H}^a$ ... parton subprocess helicity amplitudes

$\Rightarrow \mathcal{M}$ ... hadron helicity amplitudes

$\Rightarrow$ observables (cross sections, asymmetries)

# Meson production status

- DV ($V_L$) P:
  - data show importance of $\gamma_L^*$ contributions ($Q^2 < 100$ GeV$^2$)
    $\Rightarrow$ twist-2 predictions describe $\sigma_L$ (small-$x_B$)
  - tw2 NLO corrections large
    $\Rightarrow$ global DIS+DVCS+DV$V_L$P fits at NLO [Čuić, Duplančić, Kumerički, P-K. '23]

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    $\Rightarrow$ $\gamma_T^*$ contributions with quark transversity GPDs and 2-body ($\pi = q\bar{q}$) twist-3 approximation (WW) [Goloskokov, Kroll '10, Goldstein, Hernandez, Liuti '13]

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- WA$\pi$P:
  - twist-2 results bellow the data for photoproduction

    & 2-body twist-3 contributions vanish

    $\Rightarrow$ 3-body ($\pi = q\bar{q}g$) tw3 contributions determined

    $\Rightarrow$ tw3 pion DA from photoproduction fits [Kroll, P-K. 18', '21]

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  - full (2- and 3-body) twist-3 contributions confronted with data

    [Duplančić, Kroll, P-K., Szymanowski '24]
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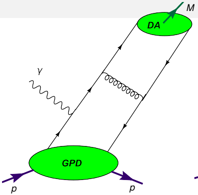
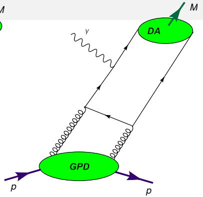
# DVMP at twist-2 NLO

# DVMP to NLO



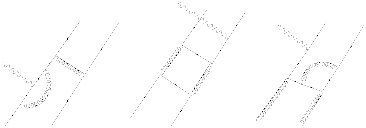




NLO DV PS$^{+}$ prod.: [Belitsky and Müller '01]

NLO DV V$_L$ prod.: [Ivanov et al '04,]

NLO DV V$_L$ (corr.), PS, (S, PV$_L$) prod.:

[Duplančić, Müller, P-K. '17]



- only few DVMP phenomenological analysis to NLO
- NLO corrections important: reduction of dependence on the scales and schemes, large (model dependent) NLO corrections
- NLO global DIS+DVCS+DVMP fits needed

# From momentum fraction to CPaW formalism

[Müller '06, Müller, Schäfer '06]

DVMP: Transition form factors $a = q, G\,,\ \int dz \equiv \overset{z}{\otimes}$

$$\mathcal{F}^a_M(\xi, t, Q^2) = F^a(x, \xi, t; \mu_F) \overset{x}{\otimes} T^{M,a}(x, \xi, u, Q^2; \ldots) \overset{u}{\otimes} \phi^M(u; \mu_\varphi)$$

$F^a$ ... GPD, $\phi_M$ ... DA, $T^a$ ... hard-scattering amplitude

- conformal partial wave expansion:

  as Mellin moments in DIS; $x^n \to$ Gegenbauer polynomials $C_n^{3/2}$ (quarks), $C_n^{5/2}$ (gluons)

  $F^q_j(\xi, \ldots) \sim \int \mathrm{d}x\, F^q(x, \xi, \ldots)\, C_j^{3/2}(x/\xi)$ and analogously $T^{M,a}_{j,k}$, $\phi^M_k$

- series summed using Mellin-Barnes integral over complex $j$

$$\mathcal{F}^a_M(\xi, t, Q^2) = F^a_j(\xi, t; \mu_F) \overset{j}{\otimes} T^{M,a}_{jk}(\xi, Q^2; \mu_R, \mu_F, \mu_\varphi) \overset{k}{\boxtimes} \phi^M_k(\mu_\varphi)$$

Advantages: easy evolution, interesting GPD modeling, moments accessible on lattice, stable numerics and efficient fitting

$x$-space advantages: more intuitive, widely used

# CPaW formalism for DVCS and DVMP to NLO

"Manuals":

- DVCS [Kumerički, Müller, P-K., Schäfer '07]
  - $T_j^a$ and GPD evolution ($\mathbb{E}$) to NLO
  - application to NNLO ready
  - modeling GPD moments: t-chanel SO(3) partial waves
- DVMP [Müller, Lautenschläger, P-K., Schäfer '14], [Duplančić, Müller, P-K. '17]
  - $T_{j,k}^{M,a}$ to NLO for $M = V_L, P, (S, PV_L)$ (in $x$ and $j$-space)
- compendium [Čuić, Duplančić, Kumerički, P-K. '23]
- Gepard software [Kumerički '22 on github]

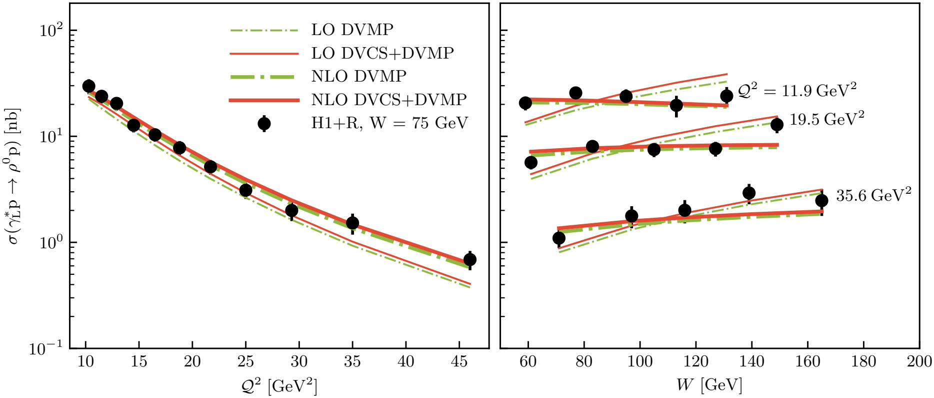
Applications:

- several applications to small-$x$ phenomenology at NLO [..., Lautenschlager, Müller, Schäfer '13 unpublished], some hybrid applications to JLab kinematics (LO) [Kumerički, Müller '09]
- attempts to describe different kinematical regions (LO) [Guo et al. '22, '23 - GUMP]

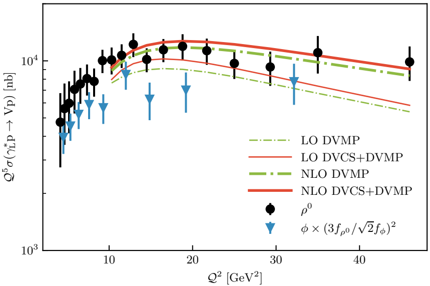
# Global NLO fits (DIS+DVCS+DVV$_L$P)

- small-x global fits to HERA collider data (DIS, DVCS, DV$\rho^0_L$P)

[Čuić, Duplančić, Kumerički, P-K. '23]



# Global NLO fits (DIS+DVCS+DVV$_L$P)



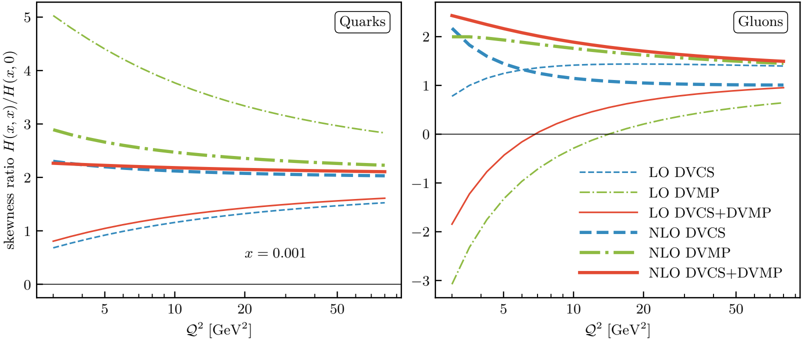

$\sigma_L$ asymptoticaly: $\dfrac{1}{Q^6}$

experimental data for fixed $x_B$: $\approx \dfrac{1}{Q^4}$, for fixed $W$: $\approx \dfrac{1}{Q^5}$

- successful description of $Q^2$ dependence

# Global NLO fits (DIS+DVCS+DVV$_L$P)

Skewness ratio $r = \dfrac{H(x,x)}{H(x,0)}$



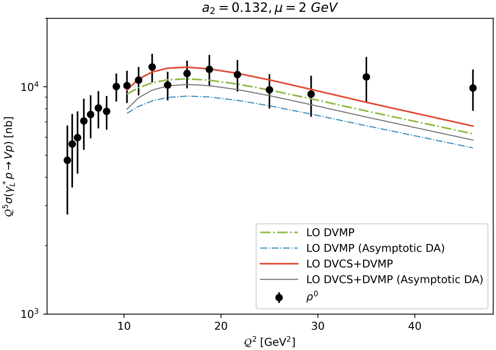

- conformal (Shuvaev) values (GPDs completely specified by PDFs): $r^q \approx 1.65$, $r^G \approx 1$,
- $r$ measures goodness of GPD extraction $\Rightarrow$ NLO fit successful

# Improving DA description

→ work in progress with Raj Kishore, K. Kumerički

$$\phi^\rho(u, \mu_\varphi) = 6u(1-u)\left[1 + a_2(\mu_\varphi)\, C_2^{3/2}(2u-1) + \ldots\right]$$

$a_2(\mu_0) = 0.132,\ \mu_0 = 2$ GeV [Braun et al. '16]




⇒ significant impact of the DA form

# Concluding remarks: DV$\rho_L^0$P at twist-2 NLO

- Global DIS+DVCS+DVMP fits show importance of NLO.

  $\Rightarrow$ universal GPDs

- DV$\rho_L^0$P can only be described at NLO.

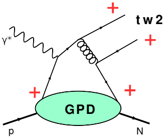
- Meson DA additional nontrivial nonperturbative input.

# DV$\pi$P at twist-3

# $\pi$ production to twist-3

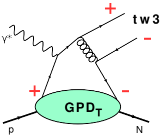
$\mu$ photon helicity, $\lambda \ldots$ quark helicities

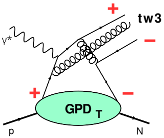
$\mathcal{H}^{\pi}_{0\lambda,\mu\lambda}$ ... non-flip subprocess amplitudes (twist-2)




$\widetilde{H},\ \widetilde{E}$

$\mathcal{H}^{\pi}_{0-\lambda,\mu\lambda}$ ... flip subprocess amplitudes (twist-3) $\boxed{\sim \mu_\pi/Q}$




$H_T,\ \bar{E}_T = 2\widetilde{H}_T + E_T,\ \ldots$

$\rightarrow$ just pion DA tw-3 contributions $\Leftarrow \mu_\pi = m_\pi^2/(m_u + m_d) \cong 2$ GeV

(see S. Bhattacharya talk)

distribution amplitudes (DAs):

twist-2 $(q\bar{q})$ : $\phi_\pi$

2-body $(q\bar{q})$ twist-3 $\phi_{\pi p}$, $\phi_{\pi\sigma}$    3-body $(q\bar{q}g)$ twist-3 $\phi_{3\pi}$

$\rightarrow$ connected by equations of motion (EOMs)

# Subprocess amplitudes: twist-3

$$\begin{aligned}\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\ &= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}}\, + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right) \\ &= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \quad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \quad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}\end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance
- WAMP:
  - photoproduction ($Q \to 0$): $\mathcal{H}^{\pi,\phi_{\pi p}} = 0$
  - no end-point singularities
- DVMP ($t \to 0$):
  - end-point singularities in $\mathcal{H}^{\pi,\phi_{\pi p}}$:

$$\int_0^1 \frac{d\tau}{\tau}\phi_{\pi p}(\tau)\ \frac{1}{(x+\xi+i\epsilon)^2} \overset{x}{\otimes} H_T(\bar{E}_T)$$

$$\phi_{\pi p}(\tau) = 1 + \omega_{\pi p}\, C_2^{1/2}(2\tau - 1) + \ldots$$

$\tau$ ... quark long. momentum fraction in $\pi$

# Treatment of end-point singularities: MPA

$\Rightarrow$ Modified perturbative approach (MPA) [Goloskov, Kroll, '10]

- $k_\perp$ quark transverse momenta in pion

$$\frac{1}{((x+\xi)\tau - k_T^2/Q^2(2\xi) + i\epsilon)} \frac{1}{(x+\xi+i\epsilon)}$$

- $\phi_\pi \to$ light-cone wave function $\Psi_\pi \sim \phi_\pi \exp\left[-a_\pi^2 k_\perp^2\right]$
- $\int_0^1 d\tau \quad \to \int d^2\mathbf{k}_T \int_0^1 d\tau \quad \overset{\text{FT}}{\to} \int d^2\mathbf{b} \int_0^1 d\tau$
- Sudakov form factor $\exp\left[-S(\tau, \mathbf{b}, Q^2)\right]$

- consistently treated 2- and 3-body tw3 contributions, as well as tw2
- involved multidimensional integrations
- calculation of NLO corrections would be complicated

# Treatment of end-point singularities: $m_g^2$

$\Rightarrow$ pure collinear picture with effective gluon mass $m_g^2$

[Schwinger '62, Cornwall '82, . . . , Shuryak, Zahed '21]

$$\int_0^1 d\tau \phi_{\pi p}(\tau) \frac{1}{((x+\xi)\tau - m_g^2/Q^2(2\xi) + i\epsilon)} \frac{1}{(x-\xi+i\epsilon)} \overset{x}{\otimes} H_T(\bar{E}_T)$$

$$m_g^2(Q^2) = \frac{m_0^2}{1 + (Q^2/M^2)^{1+p}}$$ [Aguilar, Binosi, Papavassiliou '14]

$m_g^2(0) = 0.01$ GeV$^2$

- proof of concept
- suitable for faster fitting
- easier determination of NLO corrections (already available for tw2)

# Soft physics input

GPDs

- double distribution representation [Müller '94, Radyushkin '99], double-distribution integral analytically evaluated [Goloskokov, Kroll '08]

DAs

$$\begin{aligned}\phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F) \quad &= \quad 360\tau_a\tau_b\tau_g^2\Big[1+\omega_{1,0}(\mu_F)\,\frac{1}{2}(7\tau_g-3)\\ &+\omega_{2,0}(\mu_F)\,(2-4\tau_a\tau_b-8\tau_g+8\tau_g^2)\\ &+\omega_{1,1}(\mu_F)\,(3\tau_a\tau_b-2\tau_g+3\tau_g^2)\Big]\end{aligned}$$ [Braun, Filyanov '90]

$\rightarrow \phi_{\pi p}$ using EOMs [Kroll, P-K '18]

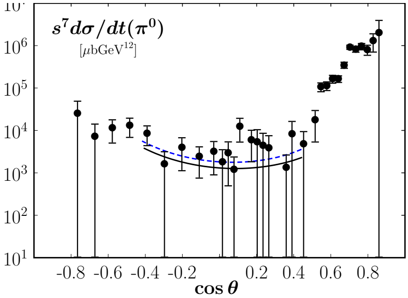
evolution taken into account

# Results from photoproduction ($\pi$)

- complete tw-3 prediction for $\pi_0$ photoproduction fitted to CLAS data

$\Rightarrow \phi_{3\pi}$ coefficients $\omega_{1,0}$, $\omega_{2,0}$, $\omega_{1,1}$ (set2)

[Kroll, P-K '21]



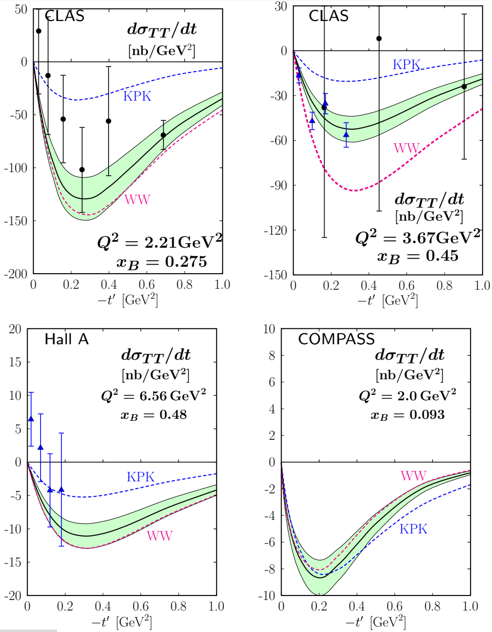

solid curve: set1 (DV$\pi^0$P)

dashed curve: set2

exp data:

full circles [CLAS '18]

# Modified perturbative approach (MPA): $d\sigma_{TT}$




solid curves: set1

dashed curves: set2, WW

exp data:

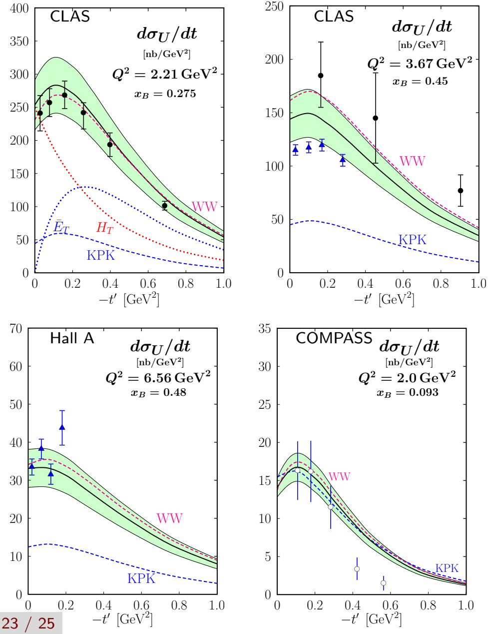
full circles [CLAS '14]

triangles [Hall A '20]

$$\frac{d\sigma_{TT}}{dt} : \bar{E}_T, \qquad \left|\frac{d\sigma_{TT}}{dt}\right| \leq \frac{d\sigma_T}{dt}$$

- $d\sigma_{TT}$ large
- good description with set1
- strong dependence on DA

# Modified perturbative approach (MPA): $d\sigma_U$




solid curves: set1

dashed curves: set2, WW

exp data:

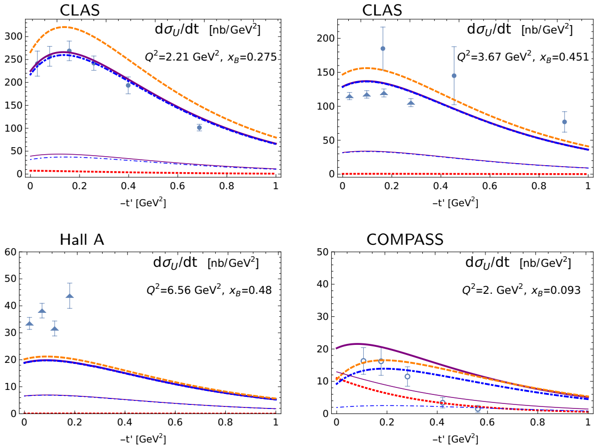
- full circles [CLAS '14]
- triangles [Hall A '20]
- open circles [COMPASS '19]

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt}$$

$$\frac{d\sigma_T}{dt} : H_T, \bar{E}_T \qquad \frac{d\sigma_L}{dt} : \widetilde{H}, \widetilde{E}$$

- $\sigma_L$ negligible except for COMPASS kin. ($40\%$)

# Collinear approach with $m_g^2$: $d\sigma_U$




set1:purple solid
set2: thin solid
WW: orange dashed

red curves: tw2
blue curves: tw3

exp data:
full circles [CLAS '14]
triangles [Hall A '20]
open circles [COMPASS '19]

- tw2 ($\sigma_L$) significant for COMPASS kinematics (small $x_B$)
- $Q^2$ dependence challenging

# Concluding remarks: DV$\pi$P at twist-3

- 3-body twist-3 contributions:
  - important for gauge invariance
  - smaller than 2-body contributions
  - change 2-body twist-3 DA, and thus 2-body tw3 contributions, through EOM
- Improved twist-3 analysis shows twist-3 dominates in DV$\pi^0$P at accessible energies, except for COMPASS kinematics (small xB).
- NLO corrections to twist-2 may be important for COMPASS kinematics.
- Wide-angle meson production also dominated by twist-3 and provides complementary information on pion DA and GPDs at large-t.

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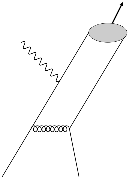
Thank you.

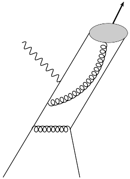
# Subprocess amplitudes $\mathcal{H}$: projectors



$q\bar{q} \to \pi$ projector [Beneke, Feldmann '00]

$$(\tau q' + k_\perp) + (\bar{\tau} q' - k_\perp) = q'$$

$$\mathcal{P}_2^\pi \sim f_\pi \Big\{ \gamma_5 \, \not{q}' \phi_\pi(\tau, \mu_F) + \mu_\pi(\mu_F) \Big[ \gamma_5 \, \phi_{\pi p}(\tau, \mu_F) - \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \frac{q'^\mu n^\nu}{q' \cdot n} \phi'_{\pi\sigma}(\tau, \mu_F) + \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, q'^\mu \phi_{\pi\sigma}(\tau, \mu_F) \frac{\partial}{\partial k_{\perp\nu}} \Big] \Big\}_{k_\perp \to 0}$$



$q\bar{q}g \to \pi$ projector [Kroll, P-K '18]

$$\tau_a q' + \tau_b q' + \tau_g q' = q'$$

$$\mathcal{P}_3^\pi \sim f_{3\pi}(\mu_F) \, \frac{i}{g} \gamma_5 \, \sigma_{\mu\nu} q'^\mu g_\perp^{\nu\rho} \, \frac{\phi_{3\pi}(\tau_a, \tau_b, \tau_g, \mu_F)}{\tau_g}, \quad f_{3\pi} \sim \mu_\pi$$

$\mu_\pi = m_\pi^2/(m_u + m_d) \cong 2$ GeV

# DAs and EOMs

$$\tau\phi_{\pi p}(\tau) + \frac{\tau}{6}\,\phi'_{\pi\sigma}(\tau) - \frac{1}{3}\,\phi_{\pi\sigma}(\tau) \quad = \quad \phi_\pi^{EOM}(\bar\tau)$$

$$\bar\tau\phi_{\pi p}(\tau) - \frac{\bar\tau}{6}\phi'_{\pi\sigma}(\tau) - \frac{1}{3}\phi_{\pi\sigma}(\tau) \quad = \quad \phi_\pi^{EOM}(\tau)$$

$$\phi_\pi^{EOM}(\tau) = 2\frac{f_{3\pi}}{f_\pi\mu_\pi}\int_0^{\bar\tau}\frac{d\tau_g}{\tau_g}\,\phi_{3\pi}(\tau,\bar\tau-\tau_g,\tau_g)$$

- EOMs and symmetry properties
  $\Rightarrow$ the subprocess amplitudes in terms of two twist-3 DAs
  and 2- and 3-body contributions combined
- combined EOMs $\rightarrow$ first order differential equation $\Rightarrow$ from known form of $\phi_{3\pi}$ [Braun, Filyanov '90] one determines $\phi_{\pi p}$ (and $\phi_{\pi\sigma}$)

Note: $q\bar{q}g$ projector and EOMs were derived using light-cone gauge for constituent gluon

# Subprocess amplitudes: twist-3

$$
\begin{aligned}
\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}}\, + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \qquad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \qquad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}
\end{aligned}
$$

- DVMP ($\hat{t} \to 0$): $\hat{s} = -\frac{\xi - x}{2\xi} Q^2, \hat{u} = -\frac{\xi + x}{2\xi} Q^2$

$$
\mathcal{H}^{\pi,\phi_{\pi p}}_{0-\lambda,\mu\lambda} \sim (2\lambda + \mu)\ f_\pi \mu_\pi C_F \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2}\right) \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}} \phi_{\pi p}(\tau)}
$$

$$
\begin{aligned}
\mathcal{H}^{\pi,\phi_{3\pi},C_F}_{0-\lambda,\mu\lambda} &\sim -(2\lambda + \mu)\ f_{3\pi}\, C_F \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2}\right) \\
&\quad \times \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}^2} \int_0^{\bar{\tau}} \frac{d\tau_g}{\tau_g(\bar{\tau} - \tau_g)}\, \phi_{3\pi}(\tau, \bar{\tau} - \tau_g, \tau_g)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}^{P,\phi_{3\pi},C_G}_{0-\lambda,\mu\lambda} &\sim -(2\lambda + \mu)\ f_{3\pi}\, C_G \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2} + \frac{e_a + e_b}{\hat{s}\hat{u}}\right) \\
&\quad \times \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}} \int_0^{\bar{\tau}} \frac{d\tau_g}{\tau_g(\bar{\tau} - \tau_g)}\, \phi_{3\pi}(\tau, \bar{\tau} - \tau_g, \tau_g)}
\end{aligned}
$$

# Pion distribution amplitudes

Twist-2 DA: $\phi_\pi(\tau,\mu_F) = 6\tau\bar\tau\left[1 + a_2(\mu_F)\, C_2^{3/2}(2\tau-1)\right]$

Twist-3 DAs:

$$\begin{aligned}\phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F) \quad = \quad & 360\tau_a\tau_b\tau_g^2\Big[1+\omega_{1,0}(\mu_F)\,\frac{1}{2}(7\tau_g-3)\\ & +\omega_{2,0}(\mu_F)\,(2-4\tau_a\tau_b-8\tau_g+8\tau_g^2)\\ & +\omega_{1,1}(\mu_F)\,(3\tau_a\tau_b-2\tau_g+3\tau_g^2)\Big]\end{aligned}$$ [Braun, Filyanov '90]

using EOMs [Kroll, P-K '18]:

$$\begin{aligned}\phi_{\pi p}(\tau,\mu_F) \quad = \quad & 1+\frac{1}{7}\,\frac{f_{3\pi}(\mu_F)}{f_\pi\mu_\pi(\mu_F)}\Big(7\,\omega_{1,0}(\mu_F)-2\,\omega_{2,0}(\mu_F)-\omega_{1,1}(\mu_F)\Big)\\ & \times\Big(10\,C_2^{1/2}(2\tau-1)-3\,C_4^{1/2}(2\tau-1)\Big)\,, \quad \phi_{\pi\sigma}(\tau)=\ldots\end{aligned}$$

---

Parameters:

- $a_2(\mu_0) = 0.1364 \pm 0.0213$ at $\mu_0 = 2$ GeV [Braun et al '15] (lattice)
- $\omega_{10}(\mu_0) = -2.55\,, \omega_{10}(\mu_0) = 0.0$ and $f_{3\pi}(\mu_0) = 0.004\ \mathrm{GeV}^2$ . [Ball '99]
- $\omega_{20}(\mu_0) = 8.0$ [Kroll, P-K '18] fit to $\pi^0$ photoproduction data [CLAS '17]

Evolution of the decay constants and DA parameters taken into account.

# Form factors and GPDs at large $t$

$R_i \ldots 1/x$ moment of $\xi = 0$ GPD ($K_i$)

- $R_V(\leftarrow H)$, $R_T(\leftarrow E)$ from nucleon form factor analysis [Diehl, Kroll '13]
- $R_A(\leftarrow \tilde{H})$ form factor analysis and WACS KLL asymmetry [Kroll '17]
- $S_T(\leftarrow H_T)$, $\bar{S}_T(\leftarrow \bar{E}_T)$ low $-t$ from DVMP analysis [Goloskokov, Kroll '11]
- $S_S(\leftarrow \tilde{H}_T) \cong \bar{S}_T/2$ ($\bar{E}_T = 2\tilde{H}_T + E_T$)

GPD parameterization [Diehl, Feldmann, Jakob, Kroll '04, Diehl, Kroll '13]

$$K_j^a(x, \xi = 0, t) = k_j^a(x) \exp\left[t\, f_j^a(x)\right]$$

$$f_j^a(x) = \left(B_j^a - \alpha_i'^a \ln x\right)(1-x)^3 + A_j^a x (1-x)^2$$

- strong $x - t$ correlation
- power behaviour for large $(-t)$
- choice for transversity GPDs $A = 0.5$ GeV$^{-2}$

# Parameterization of GPDs at small $t$

double distribution representation [Müller '94, Radyushkin '99]

$$K_j^a(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\ \delta(\rho+\xi\eta-x)\, K_j^a(\rho,\xi=0,t)\, w_j^a(\rho,\eta)$$

- weight function $w_j^a \to$ generates $\xi$ dependence
- zero-skewness GPD:

$$K_j^a(x,\xi=0,t) = k_j^a(x)\ \exp\left[(b_j^a - {\alpha'}_j^a \ln x)\, t\right]$$

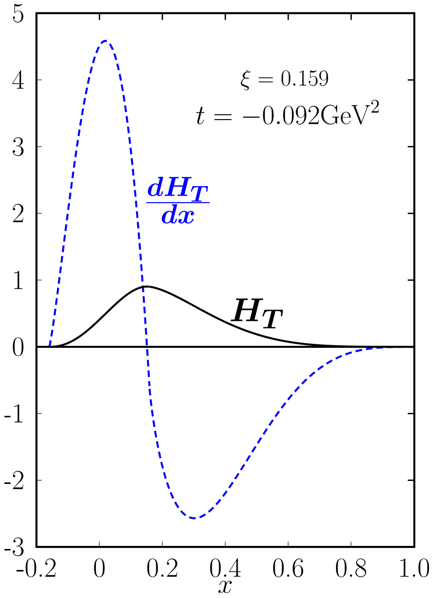
  - $H$ - GPDs: $k_j^a(x)$ from PDFs ($q$, $\Delta q$, $\delta q$)
  - $E$ - GPDs: $k_j^a(x) = N_j^a x^{-\alpha_j^a(0)}(1-x)^{\beta_j^a}$
- double-distribution integral analytically evaluated [Goloskokov, Kroll '08]

---

Parameters:

- $\{\ N_j^a,\ b_j^a,\ {\alpha'}_j^a,\ \alpha_j^a(0),\ \beta_j^a\ \}$ [Goloskokov, Kroll '11, '14] [Duplančić, Kroll, P-K., Szymanowski '24]
- moments of $H_T$ and $\bar{E}_T$ compared to lattice results

# Parameterization of GPDs at small $t$

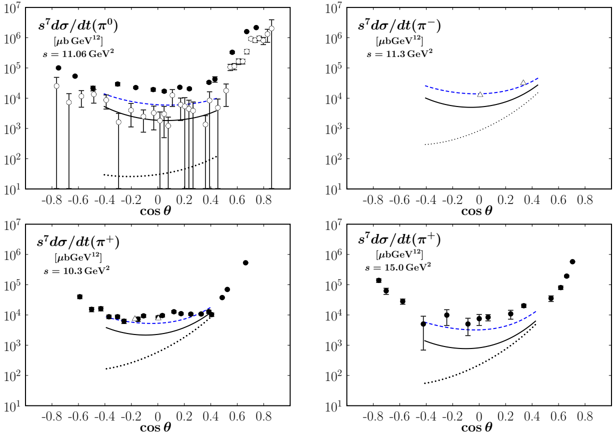



$\mu_0 = 2$ GeV

# Photoproduction ($\pi$)

- complete tw-3 prediction for $\pi_0$ photoproduction fitted to CLAS data and obtained predictions for $\pi^{\pm}$

[Kroll, P-K '21]



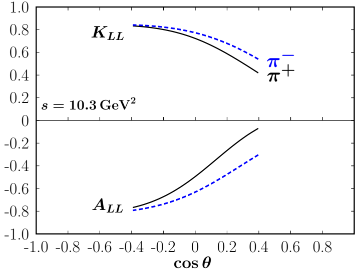

solid curves:
complete twist-3
dotted curves: twist-2

exp data:
full circles [SLAC '76]
open circles [CLAS '17]
triangles [JLab, Hall A '05]

- twist-2 prediction well below the data

# Spin effects - photoproduction

[Kroll, P-K '21]: $\pi^{\pm}$

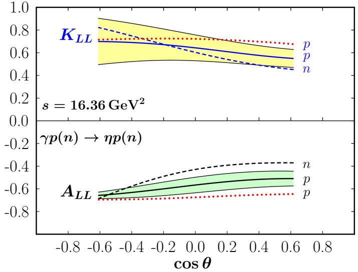


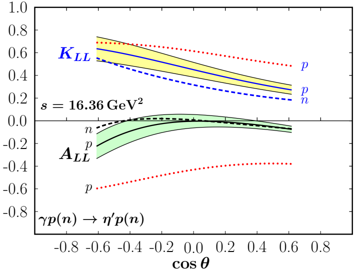

$A_{LL}(K_{LL})\ldots$ correlation of the helicities of the photon and incoming (outgoing) nucleon

$$A_{LL}^{P,tw2} = K_{LL}^{P,tw2}$$
$$A_{LL}^{P,tw3} = -K_{LL}^{P,tw3}$$

→ characteristic signature for dominance of twist-3 (like $\sigma_T \gg \sigma_L$ in DVMP)

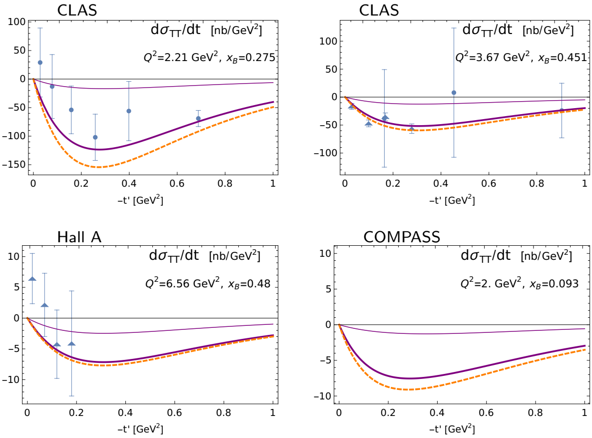
[Kroll, P-K '22]: $\eta$, $\eta'$




→ in contrast to $\pi$ and $\eta$, for $\eta'$ dominance of twist-2 and sensitivity to gluons

# Collinear approach with $m_g^2$: $d\sigma_{TT}$




set1:purple solid

set2: thin solid

WW: orange dashed

exp data:

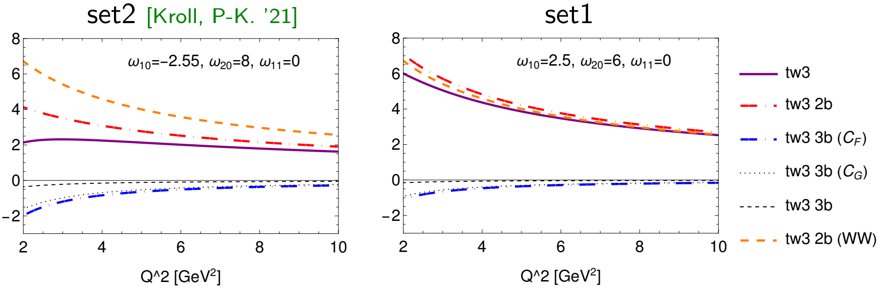
full circles [CLAS '14]

triangles [Hall A '20]

# Collinear approach with $m_g^2$

illustration: approximate factorization of $x$ and $\tau$ integration

$\Rightarrow$ $\tau$ integrals:




- 3-body contributions smaller but influence the $Q^2$ behaviour

# NLO for DV $V_L$ production

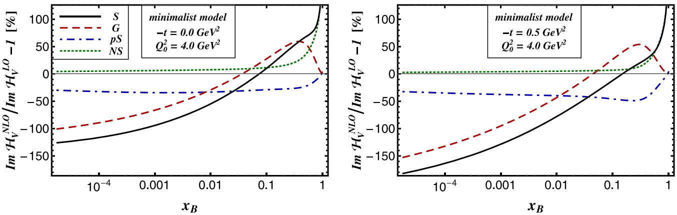



Fig. 6. Relative NLO corrections to the imaginary part of the flavor singlet TFF $\mathcal{F}_V^S$ (solid) broken down to the gluon (dashed), pure singlet quark (dash-dotted) and 'non-singlet' quark (dotted) at $t = 0\ \mathrm{GeV}^2$ (left panel) and $t = -0.5\ \mathrm{GeV}^2$ (right panel) at the initial scale $\mathcal{Q}_0^2 = 4\ \mathrm{GeV}^2$.

[Müller, Lautenschlager, P-K., Schäfer '14]

- big $\ln(1/\xi)$ terms for $\xi \ll$ ($j = 0$ pole) in gluon evolution kernel and gluon coefficient function

$$\Downarrow$$

large NLO corrections for small $\xi$ ($x_B$)

# From momentum fraction to CPaW formalism

DVCS: Compton form factors

$$\mathcal{F}^a(\xi, t, Q^2) = \int \mathrm{d}x \, T^a(x, \xi, Q, \mu_F; \mu_R) \, F^a(x, \xi, t, \mu_F)$$

$a = q, G$ or NS,S

DVMP: Transition form factors

$$\mathcal{F}^a_M(\xi, t, Q^2) = \int \mathrm{d}x \int \mathrm{d}u \, T^{M,a}(x, \xi, u, \dots) \, F^a(x, \xi, t, \mu_F) \, \phi_M(u, \mu_\varphi)$$

$F^a$ ... GPD, $\phi_M$ ... DA, $T^a$ ... hard-scattering amplitude

- conformal partial wave expansion: $C_n^{3/2}(x)$ (quarks), $C_n^{5/2}(x)$ (gluons)

$$F_j^q(\xi, \dots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \xi^{j-1} C_j^{3/2}(x/\xi) F^q(x, \xi, \dots), \; \dots, \; T_j^a, \; T_{j,k}^{M,a}$$

- series summed using Mellin-Barnes integral over complex $j$

$$\int_{-1}^{1} \frac{dx}{2\xi} \to 2 \sum_{j=0}^{\infty} \xi^{-j-1} \to \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \, \xi^{-j-1} \left[ i \pm \left\{ \begin{matrix} \tan \\ \cot \end{matrix} \right\} \left( \frac{\pi j}{2} \right) \right] \equiv \overset{j}{\otimes}$$

[Müller '06, Müller, Schäfer '06]

# Global NLO fits (DIS+DVCS+DVV$_L$P)

small-x global fits to HERA collider data ($\rho_0$)

- only NLO predecessor: [Lautenschlager, Müller, Schäfer '13 unpublished]
- hard scattering amplitude corrected [Duplančić, Müller, P-K. '17]
- new NLO fit [Čuić, Duplančić, Kumerički, P-K. '23]: improved treatment of experimental data

GPD model: [Kumerički, Müller, P-K., Schäfer '07, Kumerički, Müller '10]

- $H^a_j(\xi, t) = q^a_j \dfrac{1 + j - \alpha^a_0}{1 + j - \alpha^a_0 - \alpha'^a t} \left(1 - \dfrac{t}{m^2_a}\right)^{-2} \left(1 + s^a_2\, \xi^2 + s^a_4\, \xi^4\right)$

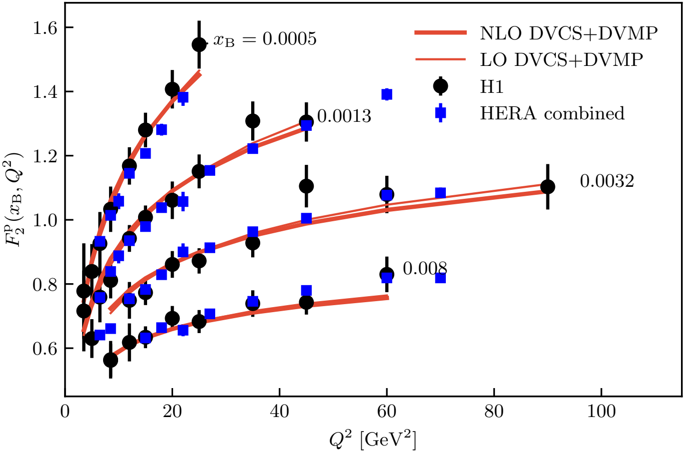
$$q^a_j = N_a \frac{B(1 - \alpha^a_0 + j, \beta^a + 1)}{B(2 - \alpha^a_0, \beta^a + 1)}$$

- small-$x$ kinematics $\Rightarrow a \in \{\mathsf{sea}, \mathsf{G}\}$, only dominant $H$ GPD

Fit parameters:

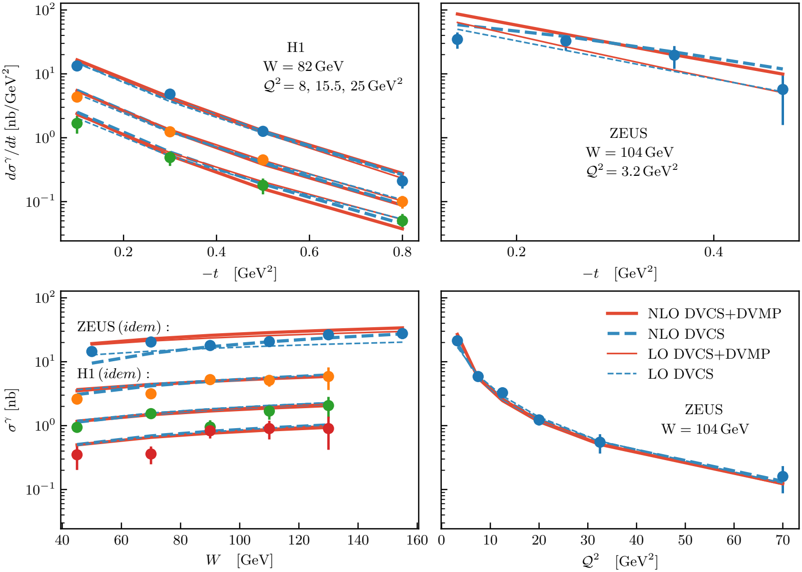
- DIS: $\{N_{\mathsf{sea}},\, \alpha^{\mathsf{sea}}_0,\, \alpha^{\mathsf{G}}_0\}$
- DVCS+DVMP: $\{\alpha'^{\mathsf{sea}},\, \alpha'^{\mathsf{G}},\, m^2_{\mathsf{sea}},\, m^2_{\mathsf{G}},\, s^{\mathsf{sea}}_2,\, s^{\mathsf{G}}_2,\, s^{\mathsf{sea}}_4,\, s^{\mathsf{G}}_4\}$

# Global NLO fits (DIS+DVCS+DV$V_L$P)




- may seem trivial, but not all popular models describe DIS

# Global NLO fits (DIS+DVCS+DVV$_L$P)



# Global NLO fits (DIS+DVCS+DV$V_L$P)

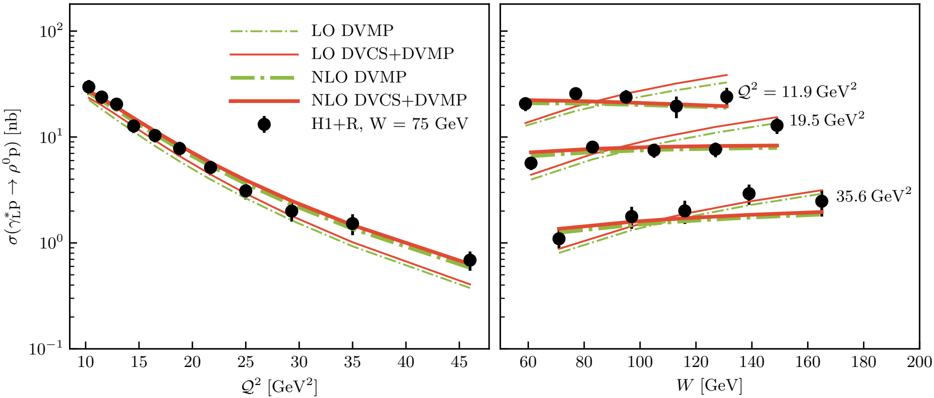
| Dataset | Refs. | $n_{\text{pts}}$ | LO- | | | NLO- | | |
|---|---|---|---|---|---|---|---|---|
| | | | DVCS | DVMP | DVCS-DVMP | DVCS | DVMP | DVCS-DVMP |
| DIS | [90] | 85 | 0.6 | 0.6 | 0.6 | 0.8 | 0.8 | 0.8 |
| DVCS | [92–95] | 27 | 0.4 | $\gg 1$ | 0.6 | 0.6 | $\gg 1$ | 0.8 |
| DVMP | [88, 89] | 45 | $\gg 1$ | 3.1 | 3.3 | $\gg 1$ | 1.5 | 1.8 |
| Total | | 157 | $\gg 1$ | $\gg 1$ | 1.4 | 3.7 | $\gg 1$ | 1.1 |

**Table 3**. Values of $\chi^2/n_{\text{pts}}$ for each LO or NLO model (columns) for the total DIS + DVCS + DVMP dataset and for subsets corresponding to different processes (rows). (The values denoted by $\gg 1$ are greater than 10.).

- NLO DVCS-DVMP fit describes the data well

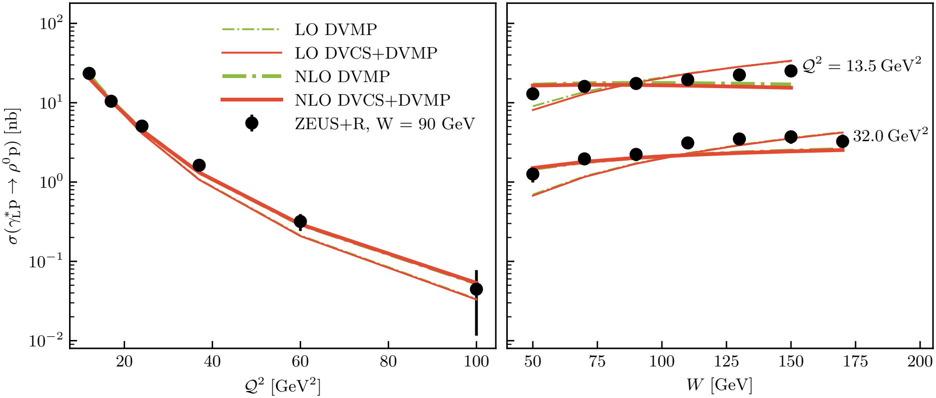
# Global NLO fits (DIS+DVCS+DVV$_L$P)

[Čuić, Duplančić, Kumerički, P-K. '23]

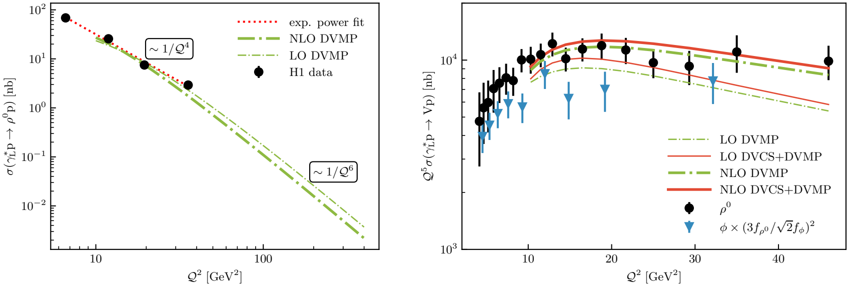



$$R \equiv \frac{\sigma_L^{\rho^0}}{\sigma_T^{\rho^0}} \to R(W, \mathcal{Q}^2) = \frac{\mathcal{Q}^2}{m_{\rho^0}^2}\left(1 + a\frac{\mathcal{Q}^2}{m_{\rho^0}^2}\right)^{-p}\left(1 + b\frac{\mathcal{Q}^2}{W}\right) \text{ fit}$$

# Global NLO fits (DIS+DVCS+DVV$_L$P)



# Global NLO fits (DIS+DVCS+DVV$_L$P)



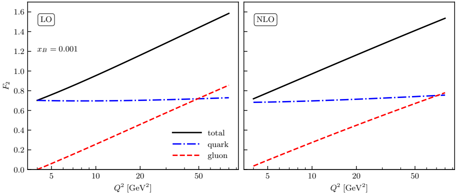

$\sigma_L$ asymptoticaly: $\dfrac{1}{Q^6}$

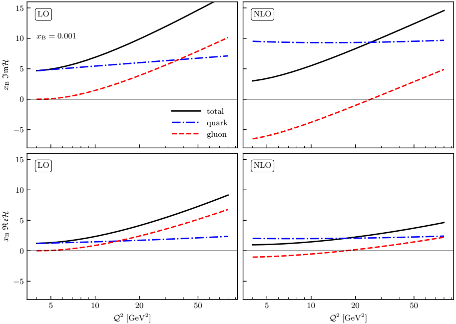
experimental data for fixed $x_B$: $\approx \dfrac{1}{Q^4}$, for fixed $W$: $\approx \dfrac{1}{Q^5}$

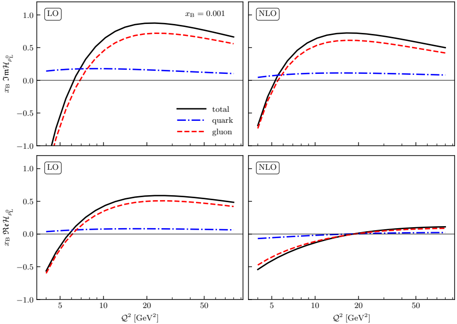
- successful description of $Q^2$ dependence

# Global NLO fits (DIS+DVCS+DVV$_L$P)

## quark-gluon structure?



# DVMP differential cross-sections

$$\frac{d^4\sigma}{dW^2 dQ^2 dt d\varphi} = \frac{\alpha_{em}(W^2 - m_N^2)}{16\pi^2 E_L^2 m_N^2 Q^2 (1-\varepsilon)} \left( \frac{d\sigma_T}{dt} + \varepsilon \frac{d\sigma_L}{dt} + \varepsilon \cos(2\varphi) \frac{d\sigma_{TT}}{dt} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\varphi \frac{d\sigma_{LT}}{dt} \right)$$

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}$$

$$\frac{d\sigma_L}{dt} : \widetilde{H}, \widetilde{E} \qquad \frac{d\sigma_T}{dt} : H_T, \bar{E}_T \qquad \frac{d\sigma_{TT}}{dt} : \bar{E}_T \qquad \frac{d\sigma_{LT}}{dt} : \widetilde{E}, H_T$$