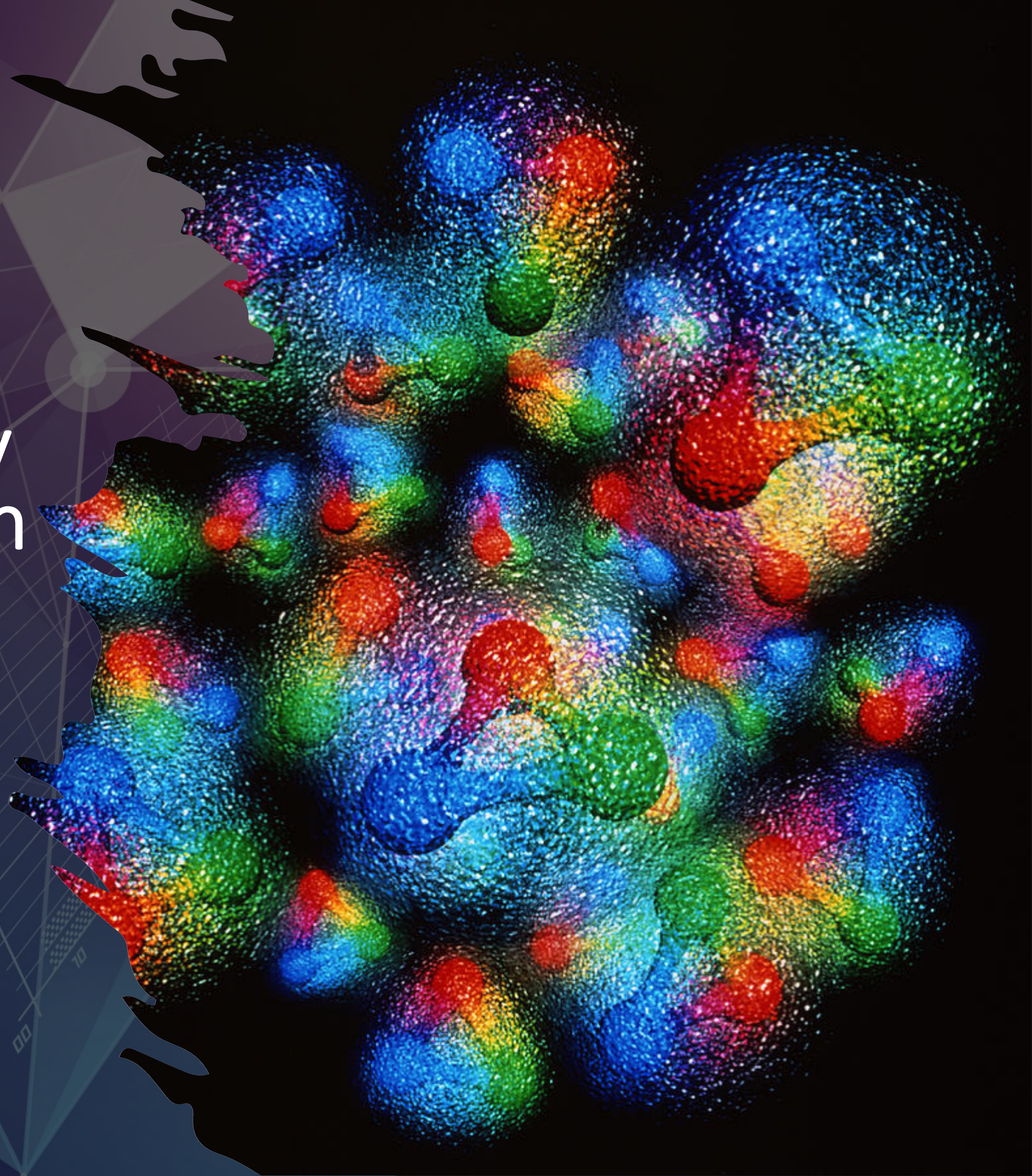
The correlated structure of the nucleon: two body GPDs and their extraction from UPCs

Simonetta Liuti.



With graduate students: Joshua Bautista, Zaki Panjsheeri

The Correlated Spatial Structure of the Proton as a framework for dynamical imaging

Joshua Bautista,* Zaki Panjsheeri,† and Simonetta Liuti‡

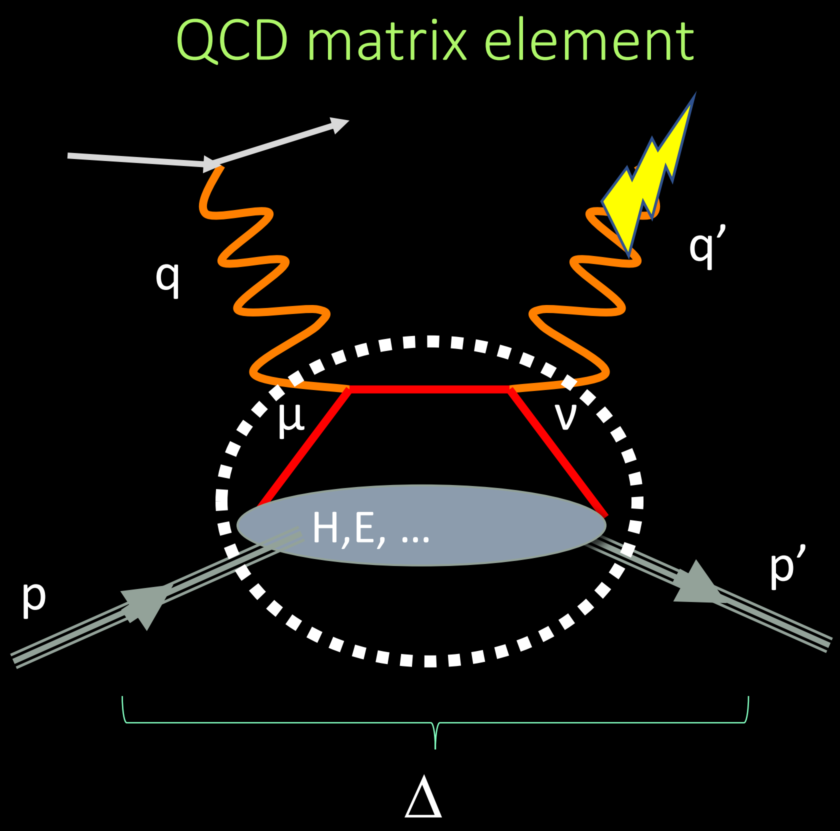
Soon on arXiv!

- Main Motivation

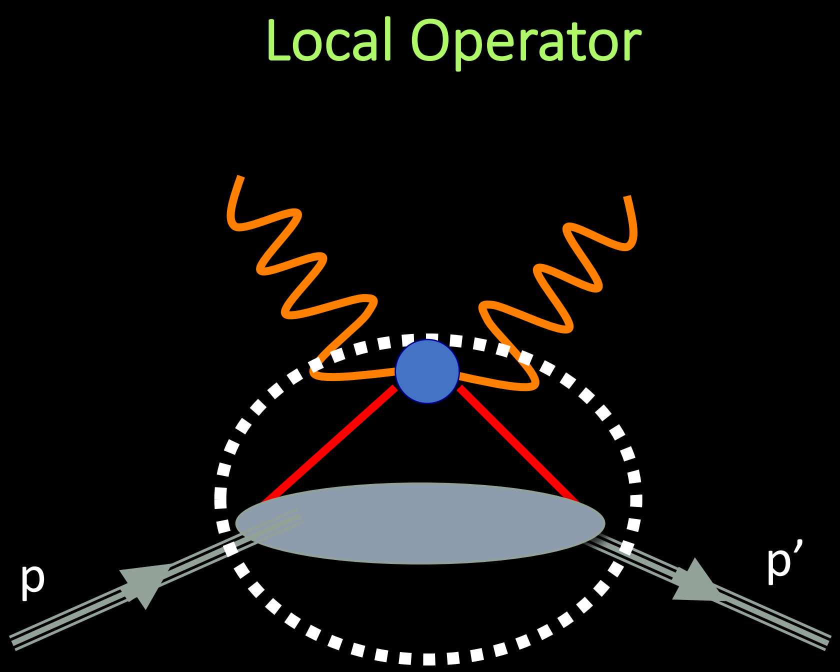
What does it really take to have an image of the proton and can we really ***observe*** what goes on inside it?

Can we establish where the quarks and gluons are located inside the proton?

Where are the quarks and gluons located with respect to one another?



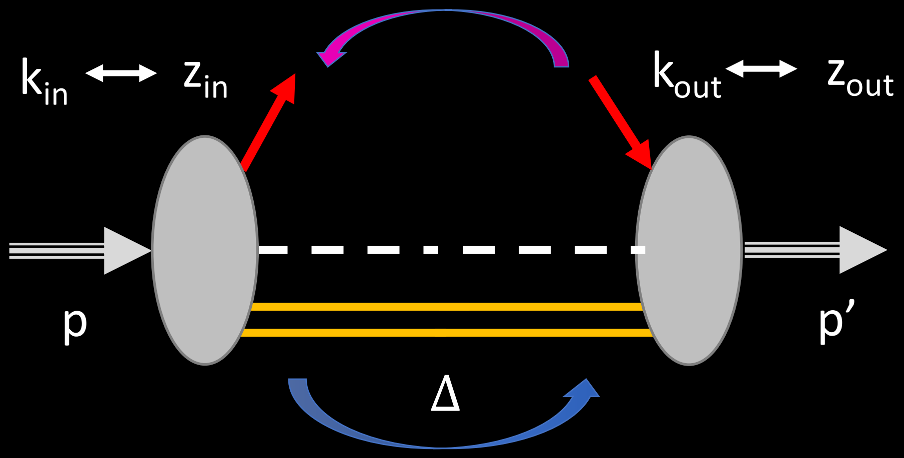
Generalized Parton Distribution



2nd GPD Moment → (EMT) Form Factor

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz_{in}}{(2\pi)} \int \frac{dz_{out}}{(2\pi)^2}\, e^{i(k_{in} z_{in})}\, e^{-i(k_{out} z_{out})}\, \langle p', \Lambda' | \overline{\psi}(z_{out}) \Gamma \mathcal{U}(z_{in}, z_{out}) \psi(z_{in}) | p, \Lambda \rangle \Big|_{z^+_{in(out)}=0},$$

$z = z_{in} - z_{out}$ = Ioffe time



Quark-quark correlation function

*Analogous treatment for gluons

Two sets of variables/Fourier conjugates

$$\begin{cases} b = \dfrac{z_{in} + z_{out}}{2} \\ \Delta = k_{in} - k_{out} = p - p' \end{cases}$$

External d.o.f., directly measurable

$$\begin{cases} z = z_{in} - z_{out} \\ k = \dfrac{k_{in} + k_{out}}{2} \rightarrow X p^{+} \end{cases}$$

Ioffe time

Loop variables

Changing variables...

$$W^{\Gamma}_{\Lambda,\Lambda'} = \delta^3(\Delta - p + p') \int \frac{d^2\mathbf{z}_T}{(2\pi)^2} \int \frac{dz^-}{(2\pi)} e^{i(k^+ + \Delta^+/2)z^-} \, e^{-i(\mathbf{k}_T + \mathbf{\Delta}_T/2)\cdot\mathbf{z}_T} \langle p', \Lambda' | \overline{\psi}\,(0,0,0)\; \Gamma\, \psi\,(0, z^-, \mathbf{z}_T)\, |p, \Lambda\rangle$$

Underlying partonic structure (leading twist): insert a set of complete states and translate the quark operators

$$H_q(X,0,t) = \int d^2\mathbf{k}_T^{\mathcal{X}}\, dk_{\mathcal{X}}^+\, \delta(k_{\mathcal{X}}^+ - (1-X)p^+)\, \langle p-\Delta \mid \bar{\psi}_+(0) \mid \mathcal{X}\rangle \langle \mathcal{X} \mid \psi_+(0) \mid p\rangle$$

$$\phi(k_{\mathcal{X}}^+, \mathbf{k}_{T,\mathcal{X}}) \to \phi(X, \mathbf{k}_{T,in}) = \langle \mathcal{X} \mid \psi_+(0) \mid p\rangle,$$

$$H_q(X,0,t) = \int d^2\mathbf{k}_{T,in}\, \phi^*(X, \mathbf{k}_{T,in} - \mathbf{\Delta})\phi(X, \mathbf{k}_{T,in}).$$

***one-body** non-diagonal density in transverse momentum*

$$H_q(X,0,t) = \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{\Delta}}\, \tilde{\phi}^*(X,\mathbf{b})\, \tilde{\phi}(X,\mathbf{b})$$

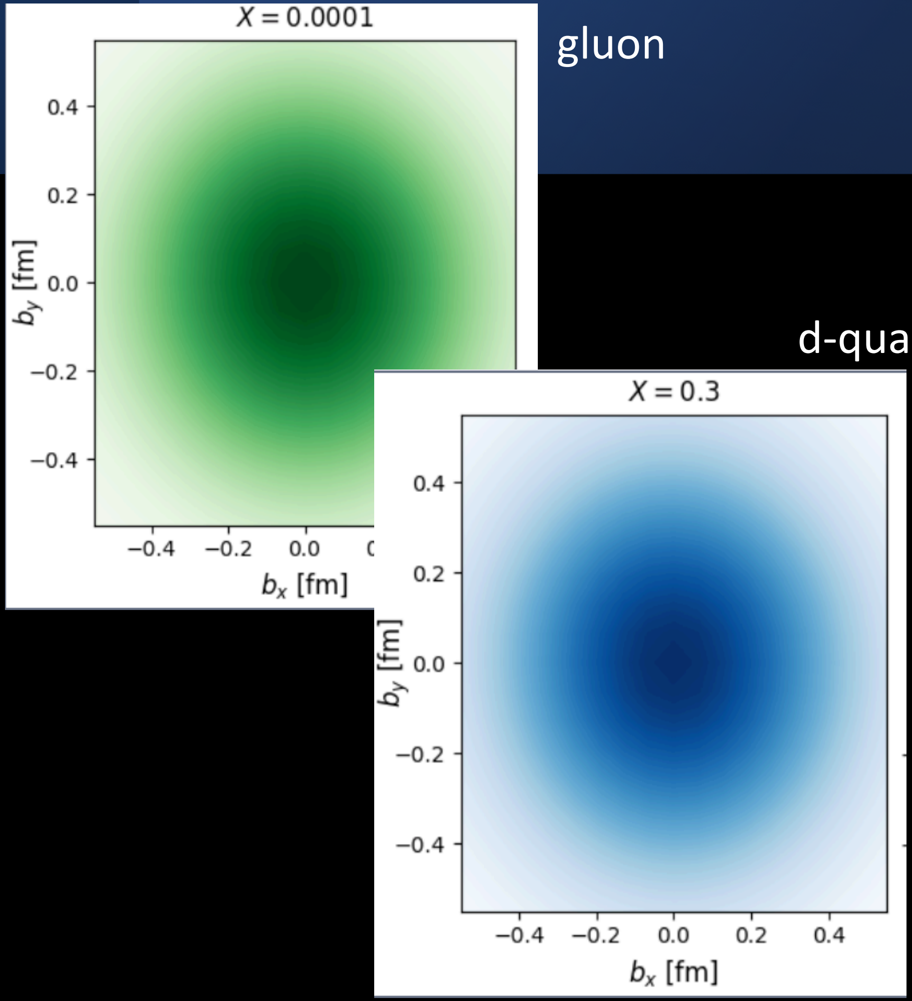
***one-body** diagonal density in transverse space*

3D Coordinate Space Representation

GPDs can be Fourier transformed from momentum space into coordinate space, providing insight into the spatial distributions of quarks and gluons inside the proton, besides matter and charge distributions.

gluon

d-quark



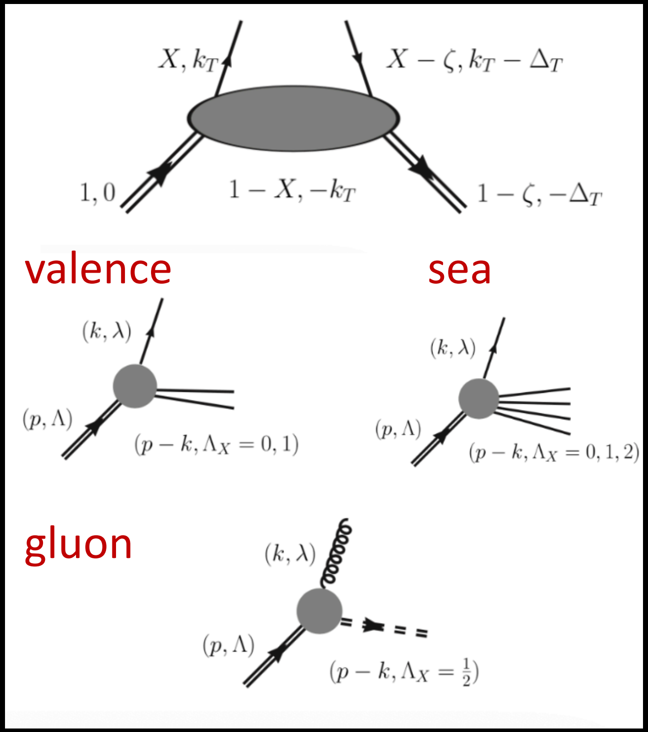
Slice of Wigner phase space distribution

$$\mathcal{H}^q(X, 0, b_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} H^q(X, 0, \Delta_T) e^{-i\Delta_T \cdot b_T}$$

GPD

With Z. Panjsheeri and J. Bautista

The “GGL” flexible parametrization



Reggeized spectator model-based, with all flavor components + gluons

DGLAP

$$F_{DGLAP}(X,\zeta,t) = F^{M_\Lambda}_{M_X,m}(X,\zeta,t)\, R_p^{\alpha,\alpha'}(X,\zeta,t)$$

Regge term

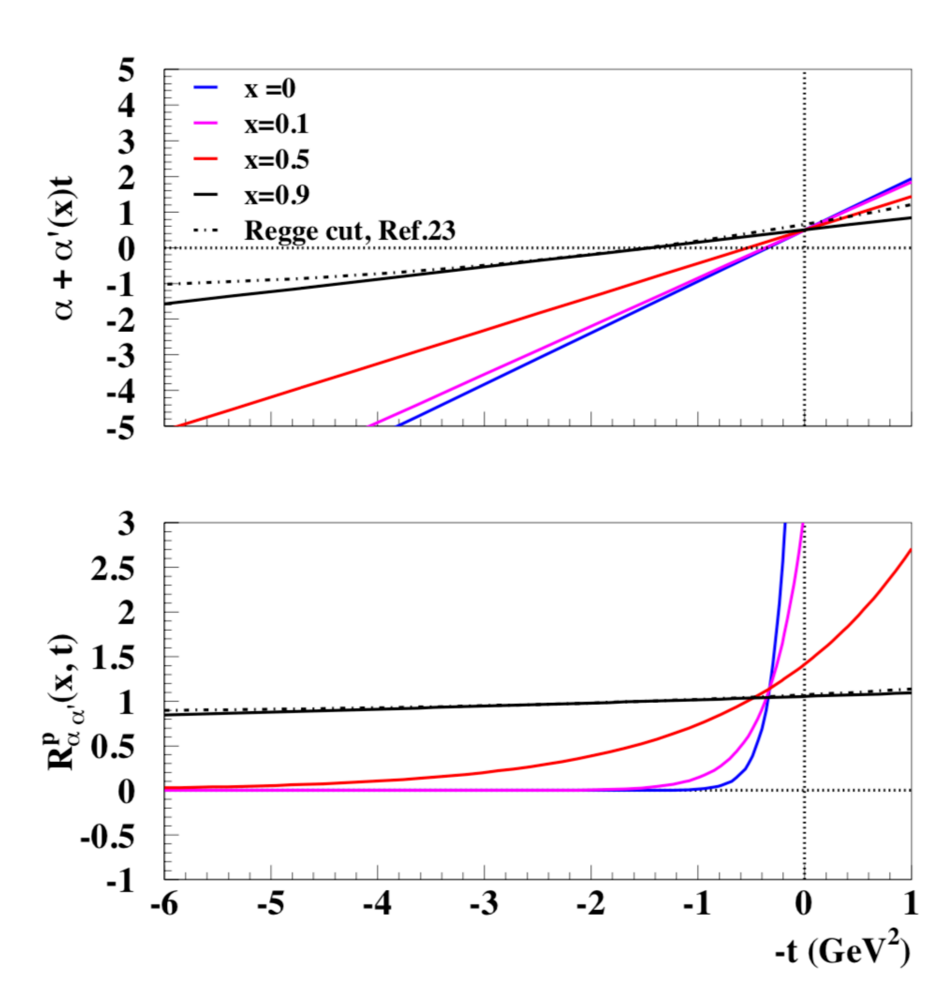
B.Kriesten et al, Phys. Rev. D1055, 056022 (2022)
J.O. Gonzalez Hernandez, S. Liuti, G. Goldstein, K. Kathuria, Phys. Rev. C88, 065206 (2013)
G.Goldstein, J.O.Gonzalez Hernandez and S. Liuti, Phys. Rev. D84, 034007 (2011)
S. Ahmad, H. Honkanen, S. Liuti and S.K. Taneja, EPJC 63 , 407 (2009)

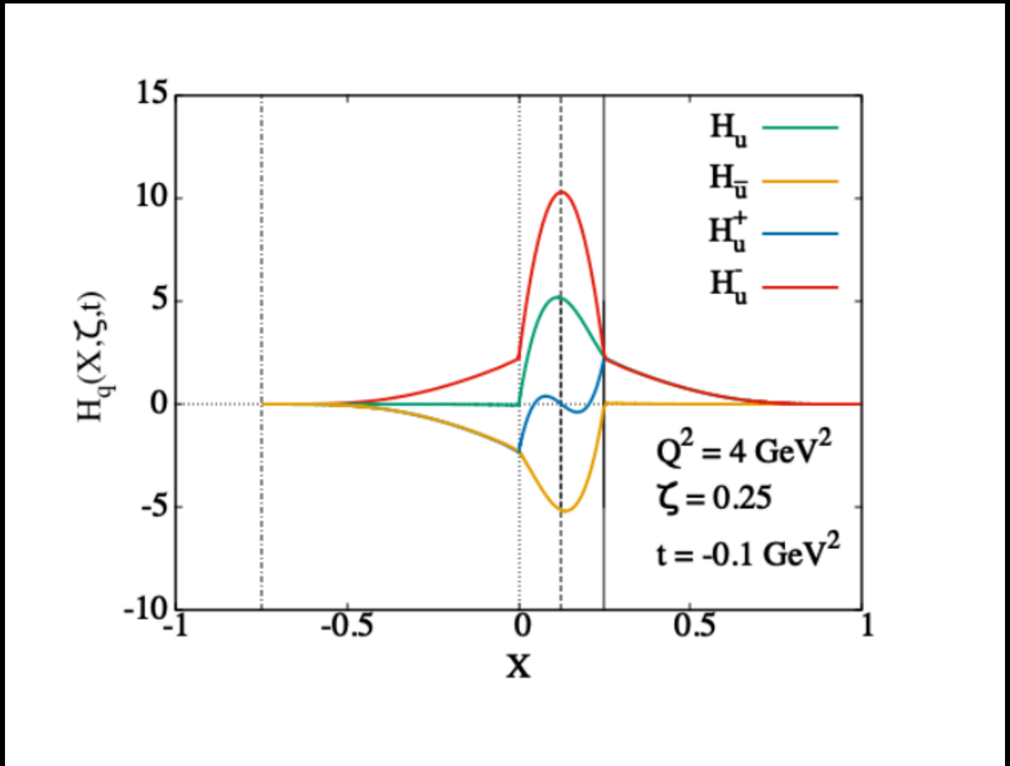
$$R_p^{\alpha,\alpha'} = X^{-[\alpha+\alpha'(1-X)^p t]}$$

Interpretation of the flavor dependence of nucleon form factors in a generalized parton distribution model

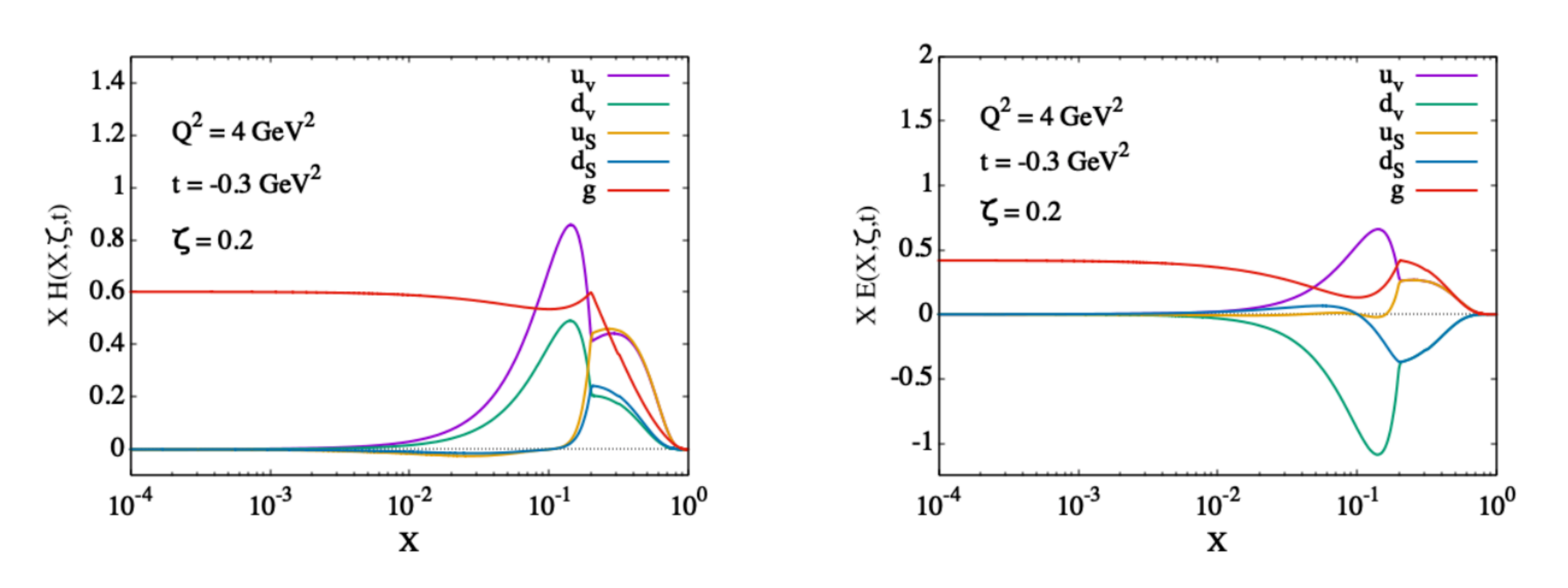
J. Osvaldo Gonzalez-Hernandez, Simonetta Liuti, Gary R. Goldstein, and Kunal Kathuria

Phys. Rev. C **88**, 065206 – Published 27 December 2013





ERBL region from symmetries



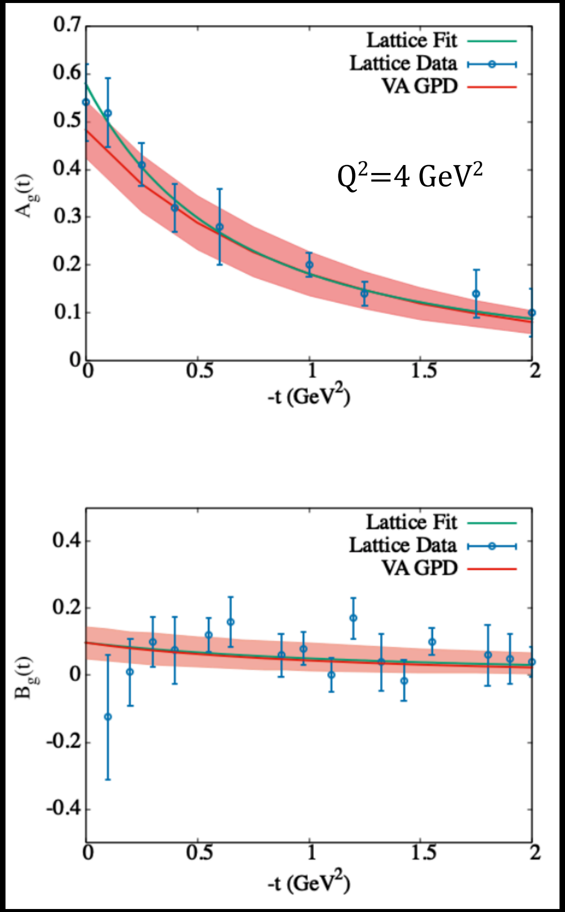
B. Kriesten. P. Velie, E. Yeats, F. Y. Lopez, & S. Liuti,
Phys.Rev.D 105 (2022) 5, 056022 and references therein

More work in progress with A. Khawaja

Gluons

Constraint from lattice moments

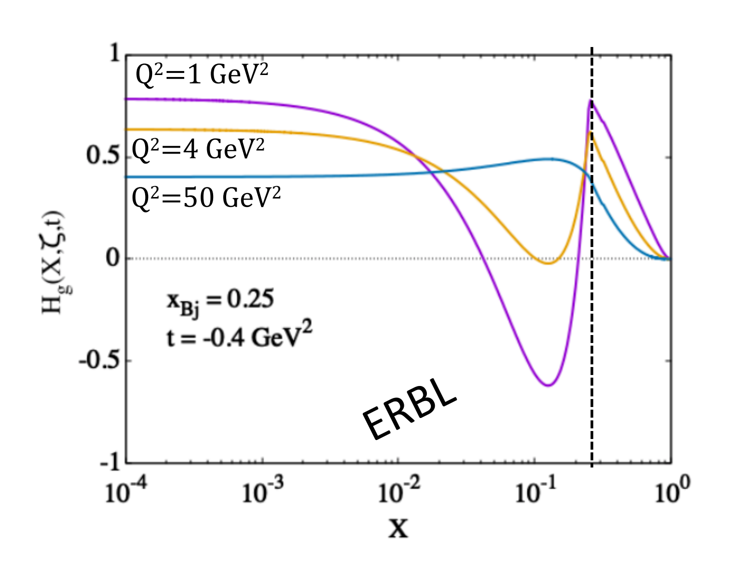
$$\int_0^1 dx H^g(x,\xi,t;Q^2) = A_g(t) + (2\xi)^2 C_g(t)$$
$$\int_0^1 dx E^g(x,\xi,t;Q^2) = B_g(t) - (2\xi)^2 C_g(t)$$



Lattice points from P. Shanahan and W. Detmold, Phys. Rev. D 99, 014511(2019), 1810.04626

NLO Evolution

Gluon

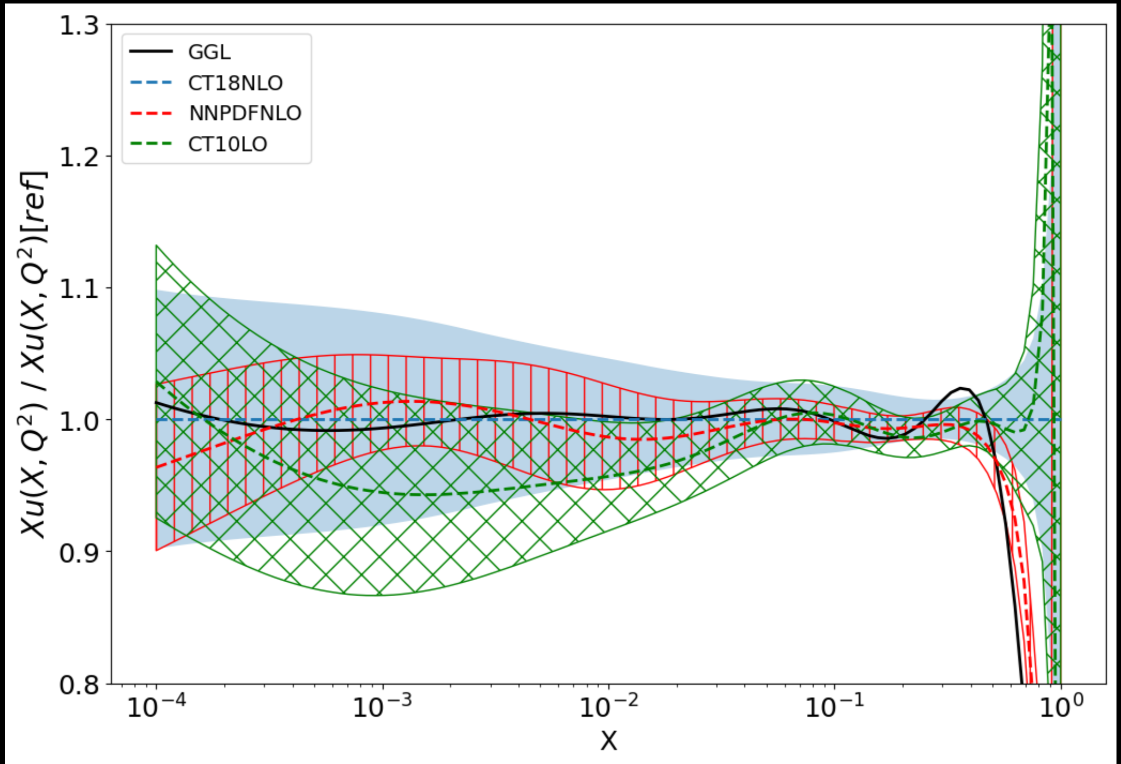


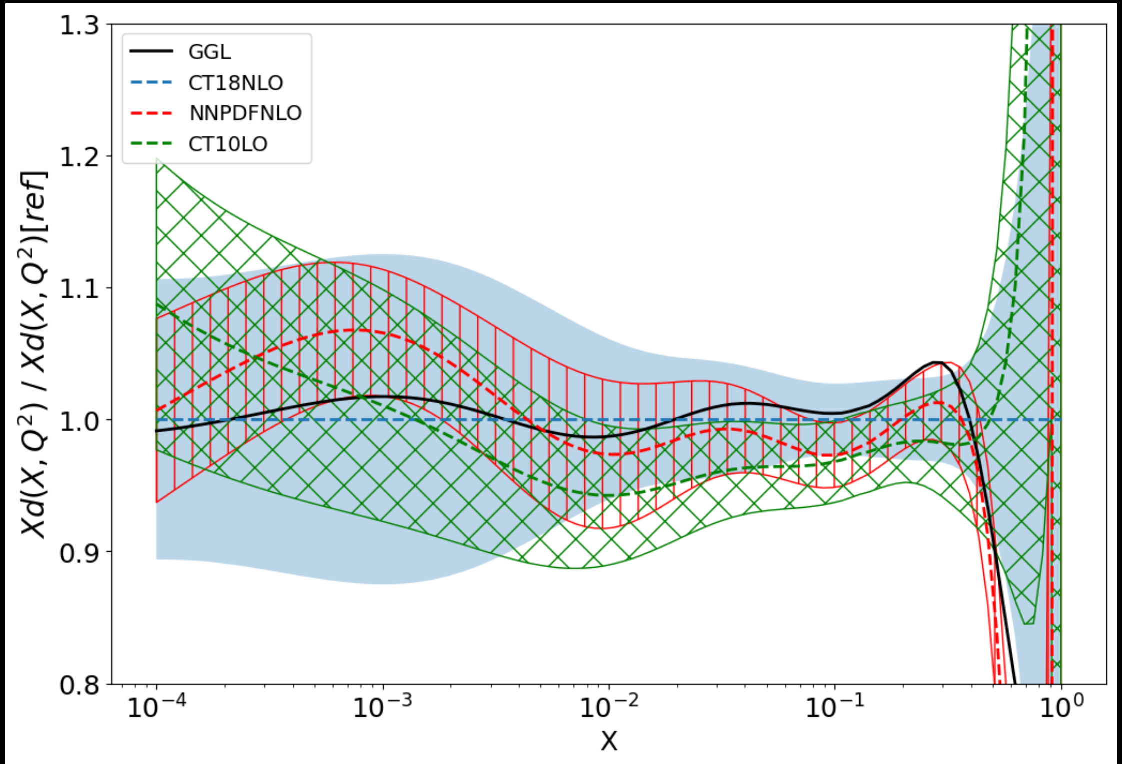
$$\frac{\partial}{\partial \ln Q^2} F_{q_v}(X,\zeta,Q^2) = \frac{\alpha_S}{2\pi} P_{qq}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F_{q_v}(Z,\zeta,Q^2) \quad (96)$$

$$\frac{\partial}{\partial \ln Q^2} F^{\Sigma}(X,\zeta,Q^2) = \frac{\alpha_S}{2\pi}\left[P_{qq}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F^{\Sigma}(Z,\zeta,Q^2) + 2N_f P_{qg}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F_g(Z,\zeta,Q^2)\right] \quad (97)$$

$$\frac{\partial}{\partial \ln Q^2} F_g(X,\zeta,Q^2) = \frac{\alpha_S}{2\pi}\left[P_{gq}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F^{\Sigma}(Z,\zeta,Q^2) + P_{gg}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F_g(Z,\zeta,Q^2)\right] \quad (98)$$

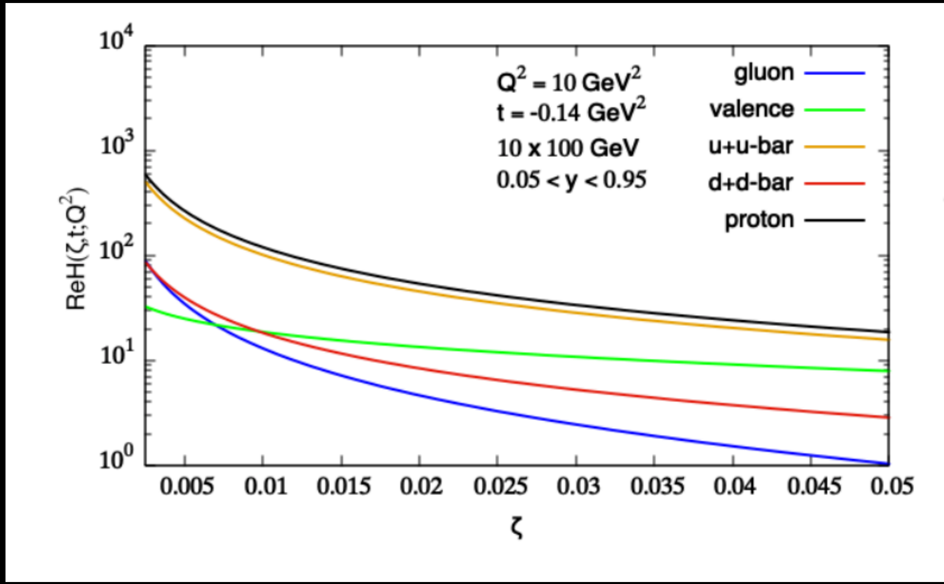
New version, A. Khawaja, J. Bautista, M. Čuič, Z. Panjsheeri, D. Adams, SL (soon on arXiv)



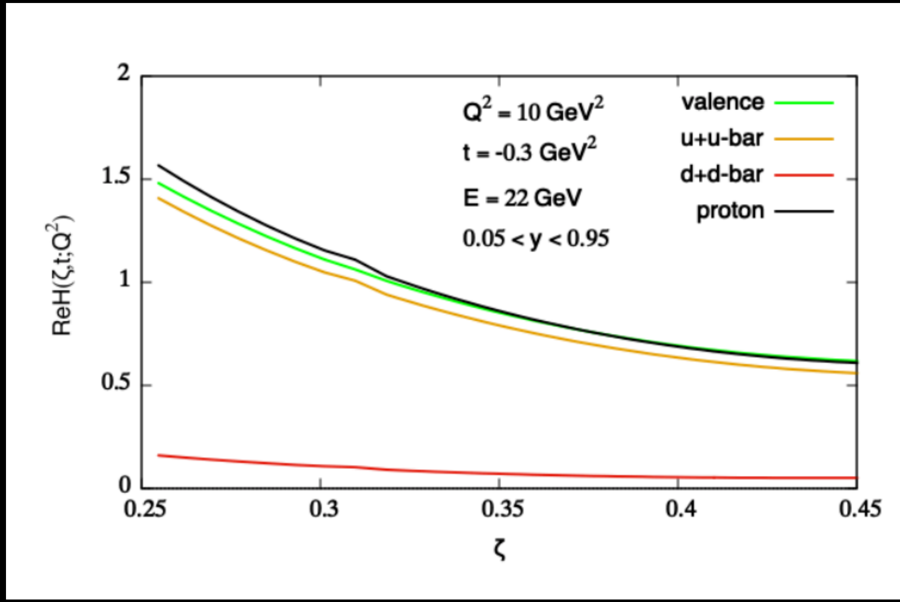


Quark and gluon content of Compton Form Factors

EIC



Jlab 12 GeV+



At Jlab kinematics

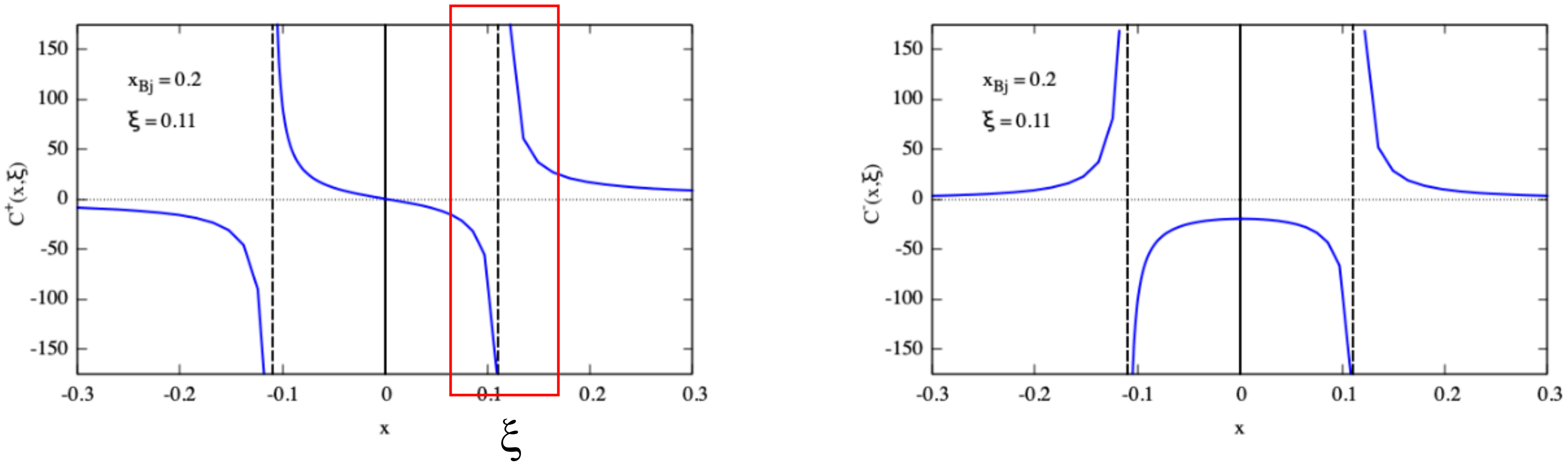


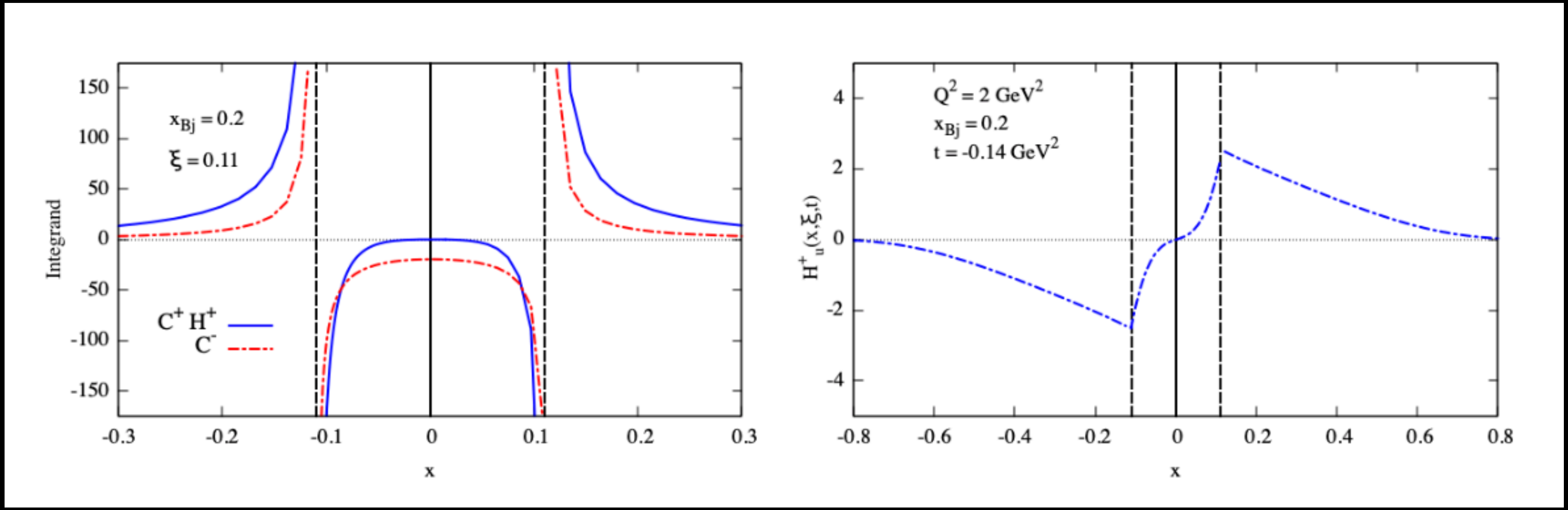
FIG. 7. Coefficients of the Compton form factors, C^+ (left) and C^- (right) (Eqs.(44)). The figure visualizes the symmetries around the $x = 0$, displayed in Eqs.(47).

$$\Re e\mathcal{H}_q = P.V.\int_{-1}^{1} dx\, H_q^+(x,\xi,t)\left(\frac{1}{x-\xi}+\frac{1}{x+\xi}\right) = P.V.\int_0^1 dx\,\frac{H_q^+(x,\xi,t)}{x-\xi} + \int_0^1 dx\,\frac{H_q^+(x,\xi,t)}{x+\xi} \quad (57)$$

$$\Re e\widetilde{\mathcal{H}}_q = P.V.\int_{-1}^{1} dx\, \widetilde{H}_q(x,\xi,t)\left(\frac{1}{x-\xi}-\frac{1}{x+\xi}\right) = P.V.\int_0^1 dx\,\frac{\widetilde{H}_q^+(x,\xi,t)}{x-\xi} - \int_0^1 dx\,\frac{\widetilde{H}_q^+(x,\xi,t)}{x+\xi} \quad (58)$$

With B. Kriesten et al., in preparation

DVCS experiments are sensitive to the anti-symmetric (flavor singlet) part of GPDs only

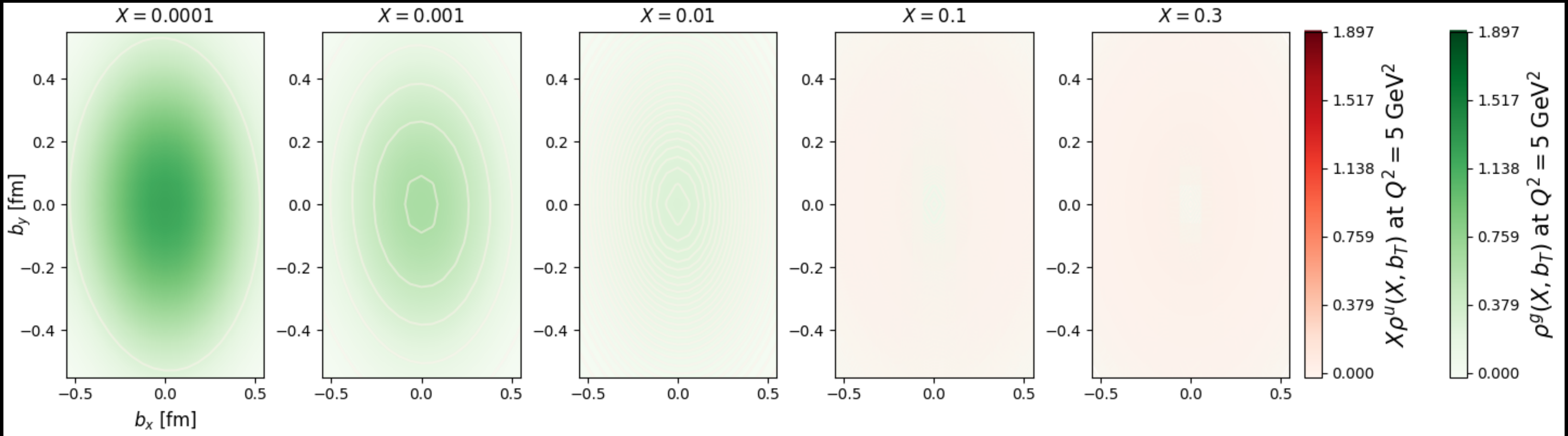


integrand

Anti-symmetric GPD component

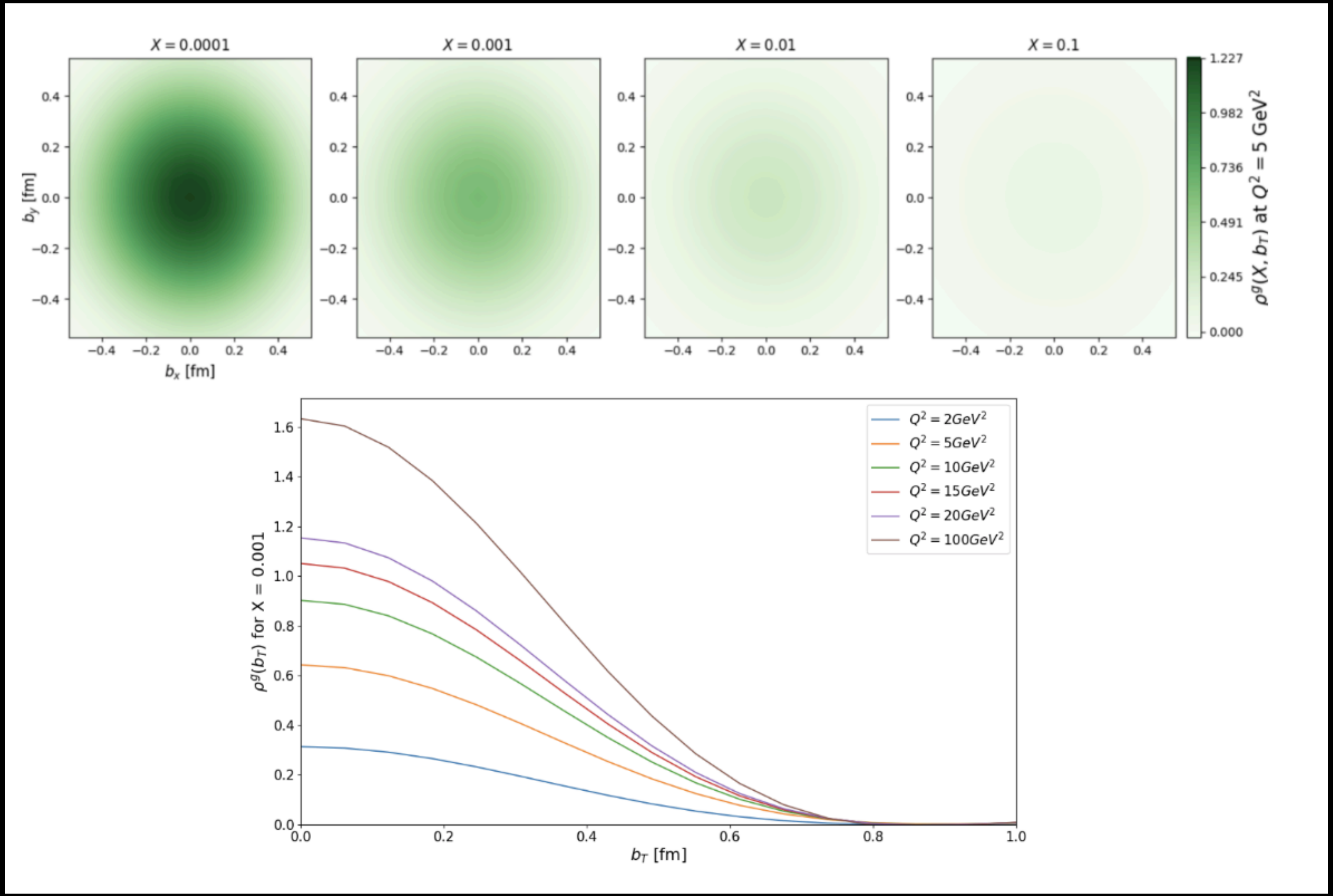
Fourier transforms

Relative weight of u and g distributions



Z. Panjsheeri

Gluon



$X = 0.0001$
$X = 0.001$
$X = 0.01$
$X = 0.1$
b_y [fm]
b_x [fm]
1.227
0.982
0.736
0.491
0.245
0.000
$\rho^g(X, b_T)$ at $Q^2 = 5$ GeV^2
$Q^2 = 2GeV^2$
$Q^2 = 5GeV^2$
$Q^2 = 10GeV^2$
$Q^2 = 15GeV^2$
$Q^2 = 20GeV^2$
$Q^2 = 100GeV^2$
$\rho^g(b_T)$ for X = 0.001
b_T [fm]

Gluon and quark matter density radius

$$< b_T^2 >^q (X) = \frac{\int_0^\infty d^2 b_T b_T^2 \mathcal{H}^q(X,0,b_T)}{\int_0^\infty d^2 b_T \mathcal{H}^q(X,0,b_T)}$$

Bautista, Panjsheeri, SL (2024)

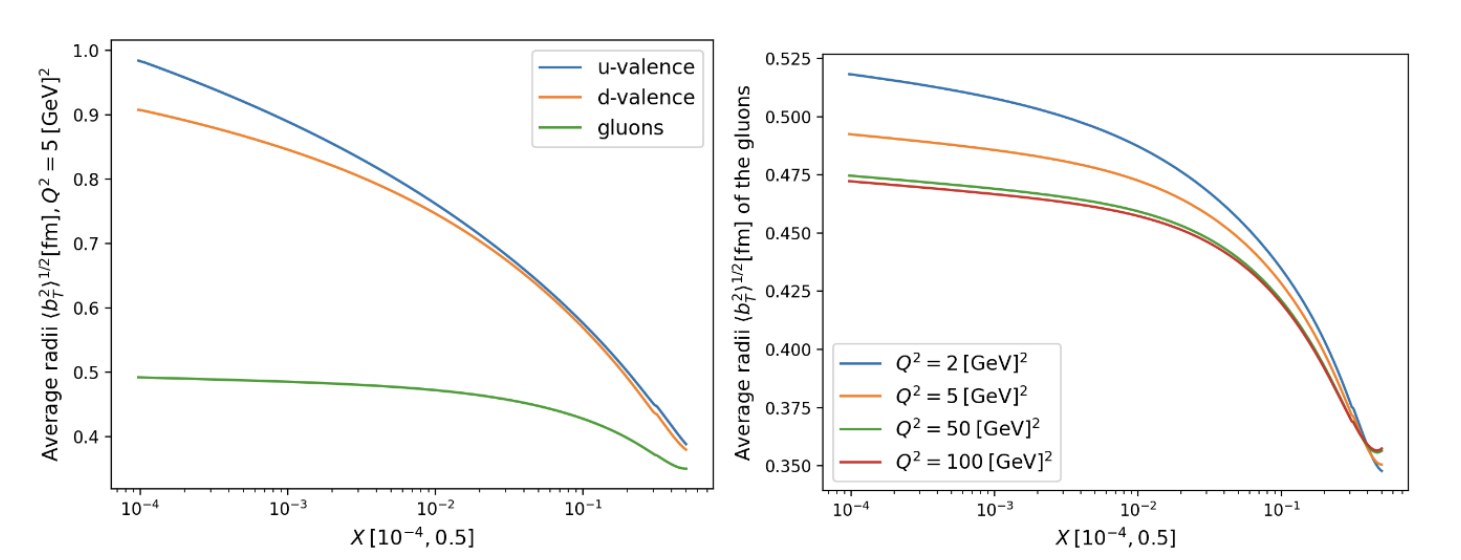
$\sqrt{\langle b_T^2 \rangle}$

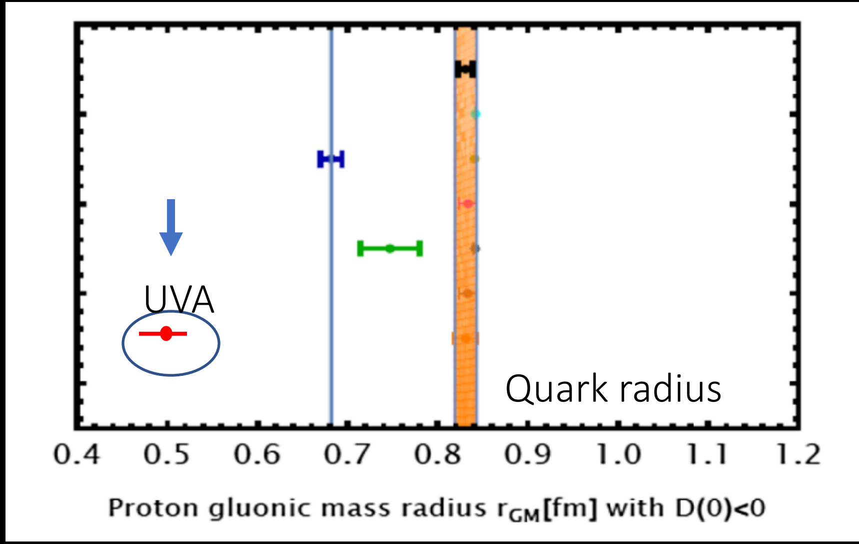
Compare to lattice and AdS/CFT integrated value

K. Mamo and I. Zaeed

PRD106, 086004 (2022)

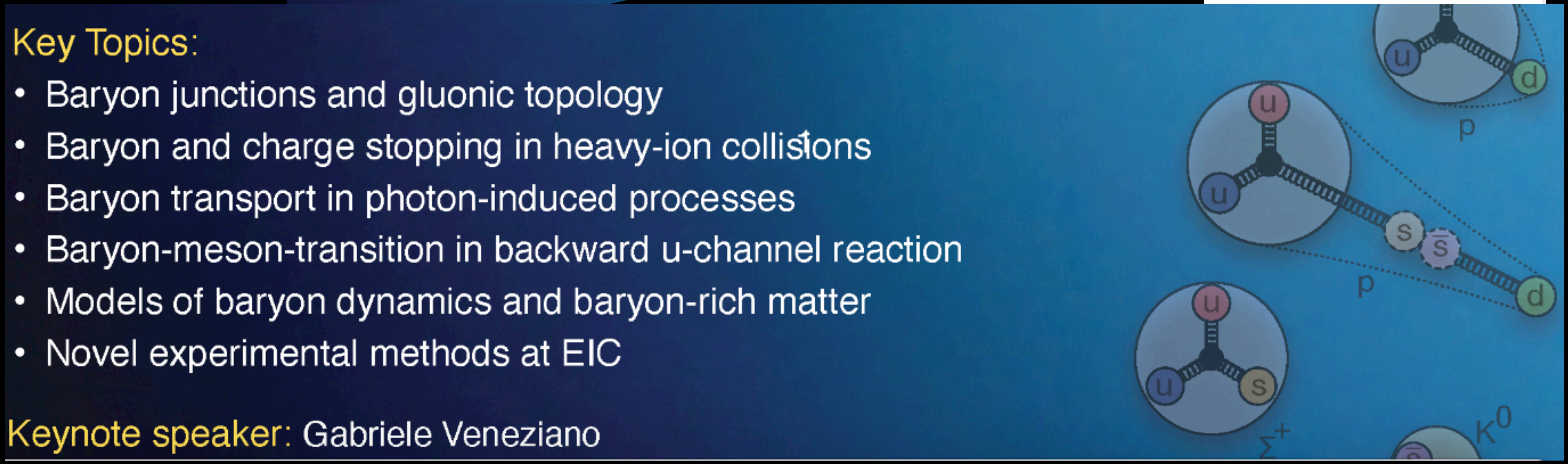
LQCD: Detmold and Shanahan



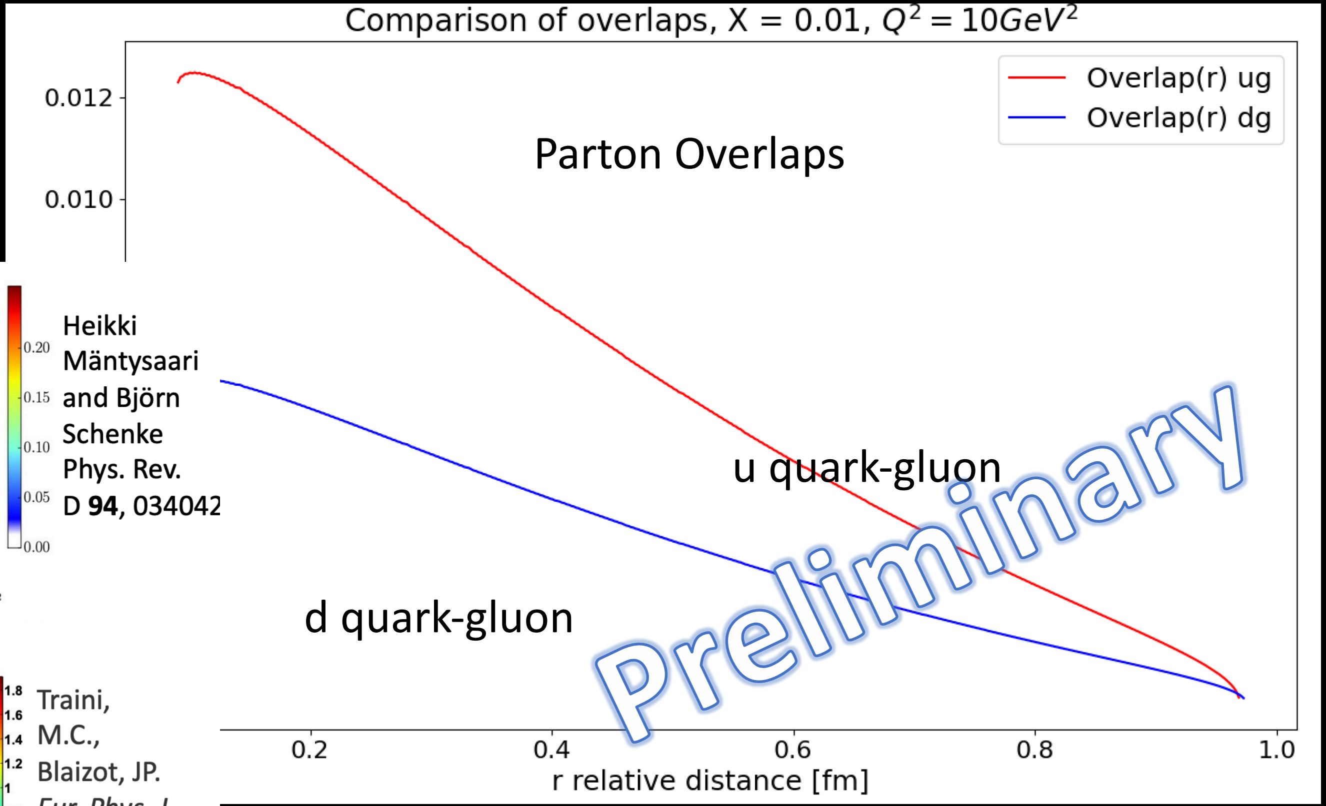


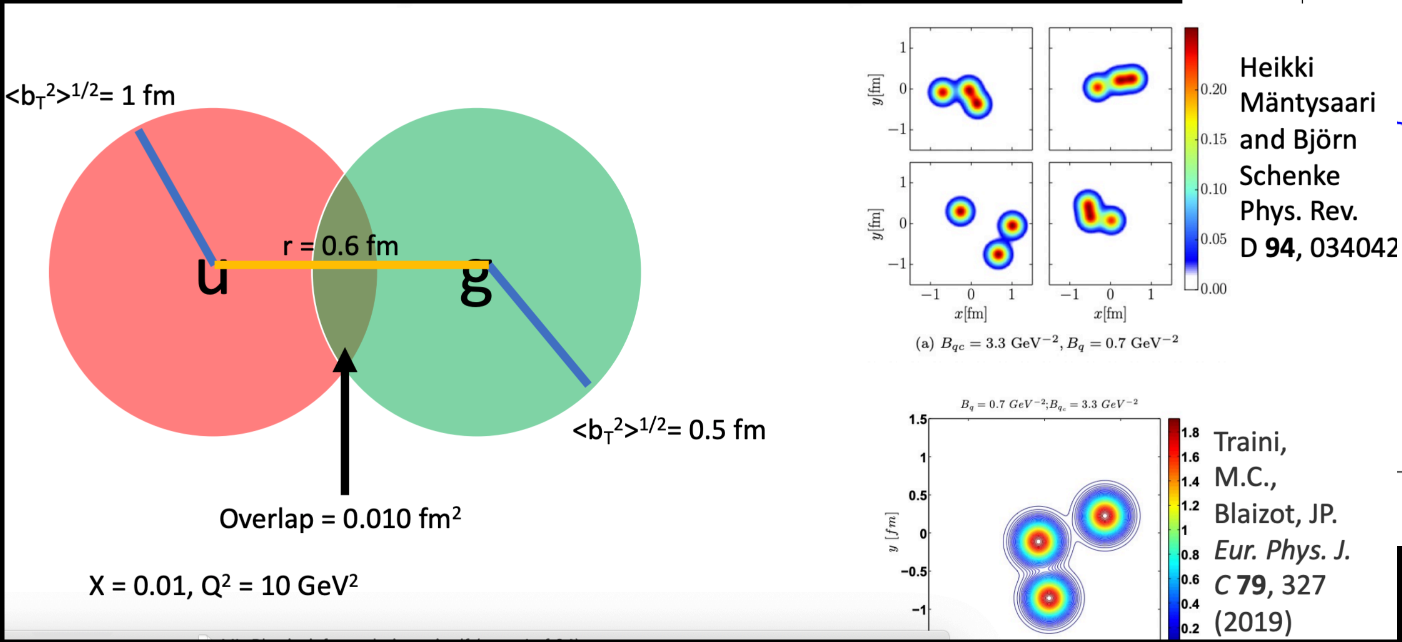
arXiv:2405.05842

- This emerging picture supports the idea of the gluons being at the core of the nucleon and carrying baryon number



Beyond one-body densities: two body densities and parton overlaps





Z. Panjsheeri, J. Bautista, SL, soon on arxiv

Z. Panjsheeri, SPIN 2023

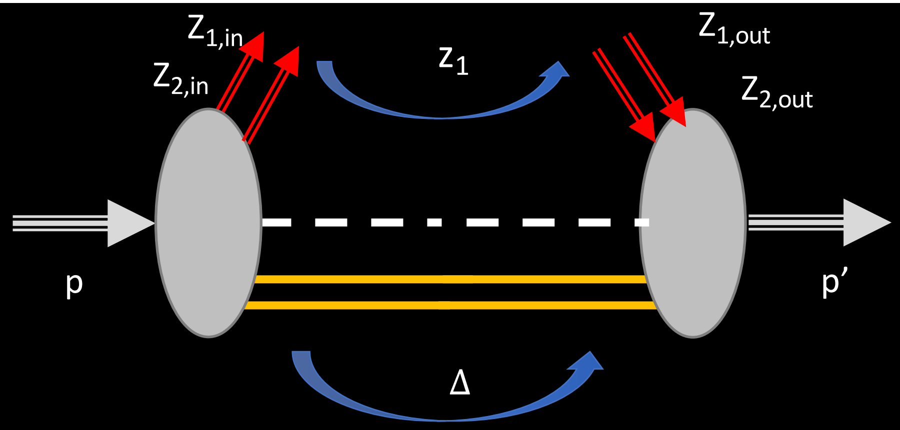
and UPC 2023, Playa de Carmen (Mexico) 12/2023

Beyond one-body densities: two body densities and parton overlaps

B. Two-body correlation function

The two-body correlation function is defined by a bilinear expression [17],

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{1,in} d\mathbf{z}_{1,T,in}}{(2\pi)^3} \frac{dz^-_{2,in} d\mathbf{z}_{2,T,in}}{(2\pi)^3} \int \frac{dz^-_{1,out} d\mathbf{z}_{1,T,out}}{(2\pi)^3} \frac{dz^-_{2out} d\mathbf{z}_{2,T,out}}{(2\pi)^3}$$
$$\times\, e^{i(k_{1,in} z_{1,in} + k_{2,in} z_{2,in})}\, e^{-i(k_{1,out} z_{1,out} + k_{2,out} z_{2,out})} \langle p', \Lambda' | \overline{\psi}(z_{1,out})\, \Gamma \psi(z_{1,in})\, \overline{\psi}(z_{2,out})\, \Gamma \psi(z_{2,in}) | p, \Lambda\rangle \Big|_{z_1^+ = z_2^+ = 0}$$



...manipulate the hadronic tensor similarly to the one-body case

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz_1^- d\mathbf{z}_{1,T}}{(2\pi)^3} \frac{dz^- d\mathbf{z}_T}{(2\pi)^3} \int \frac{dy^- d\mathbf{y}_T}{(2\pi)^3}$$
$$\times\, e^{i\Delta_2 y}\, e^{i(k_1+\Delta/2+k_2)z_1}\, e^{i(k_2 z)} \sum_{\mathcal{X}} \langle p',\Lambda' | \overline{\psi}_+(0)\, \overline{\psi}_+(y-z/2) | \mathcal{X}\rangle\langle\mathcal{X}| \psi_+(z_1)\, \psi_+(y+z/2+z_1) | p,\Lambda\rangle$$

qq scattering

Double GPD

$$H_{qq}(X_1,X_2,0,t_1,t_2) = \int d^2\mathbf{k}_T \int \frac{dy^- d\mathbf{y}_T}{(2\pi)^3} e^{i(\Delta_2-k_2)y} \langle p',\Lambda' | \overline{\psi}_+(0)\, \overline{\psi}_+(y) | \mathcal{X}\rangle\langle\mathcal{X}| \psi_+(0)\, \psi_+(y) | p,\Lambda\rangle$$

Relevant distances

$y = b_1 - b_2$ ➔ relative position of parton 1 and 2

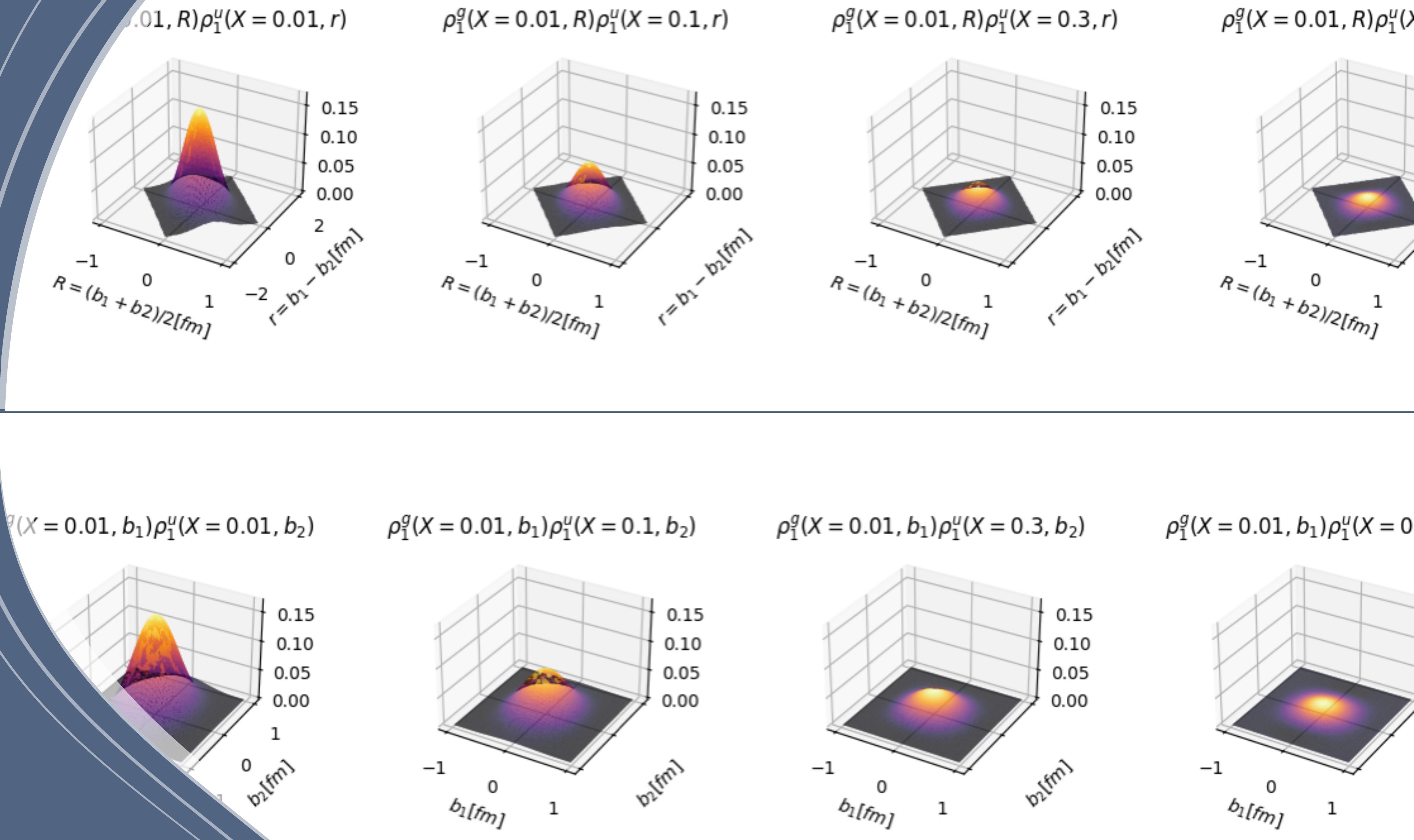
z_1, z_2 ➔ LC distance between “in” and “out” partons

Fourier transform

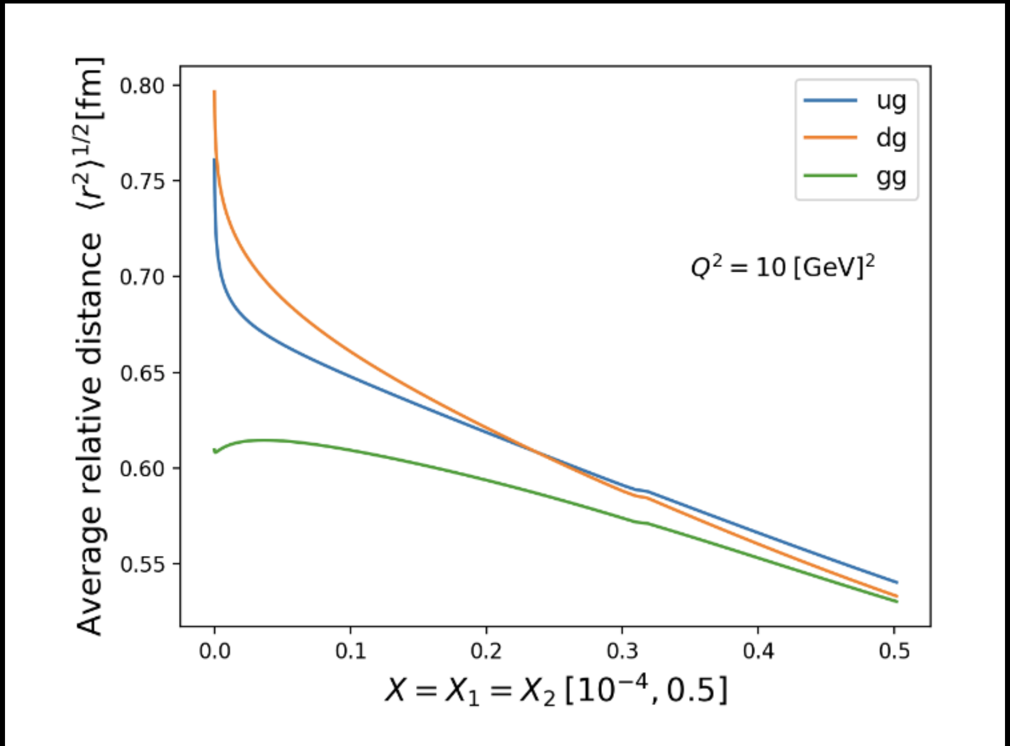
y ⟷ r

$$
\begin{aligned}
\mathbf{r} &= \mathbf{b}_1 - \mathbf{b}_2 \\
\mathbf{R}_{12} &= \frac{\mathbf{b}_1 + \mathbf{b}_2}{2}
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathbf{r}^2 \rangle (X_1, X_2) &= \frac{1}{\mathcal{N}} \int\int d^2\mathbf{r}\, d^2\mathbf{R}_{12}\, \left|\mathbf{r}^2\right|\ \rho_2\left(X_1, \mathbf{R}_{12} + \frac{\mathbf{r}}{2}; X_2, \mathbf{R}_{12} - \frac{\mathbf{r}}{2}\right) \\
\langle \mathbf{R}_{12}^2 \rangle (X_1, X_2) &= \frac{1}{\mathcal{N}} \int\int d^2\mathbf{r}\, d^2\mathbf{R}_{12}\, \left|\mathbf{R}_{12}^2\right|\ \rho_2\left(X_1, \mathbf{R}_{12} + \frac{\mathbf{r}}{2}; X_2, \mathbf{R}_{12} - \frac{\mathbf{r}}{2}\right) \\
\mathcal{N} &= \int\int d^2\mathbf{r}\, d^2\mathbf{R}_{12}\ \rho_2\left(X_1, \mathbf{R}_{12} + \frac{\mathbf{r}}{2}; X_2, \mathbf{R}_{12} - \frac{\mathbf{r}}{2}\right)
\end{aligned}
$$

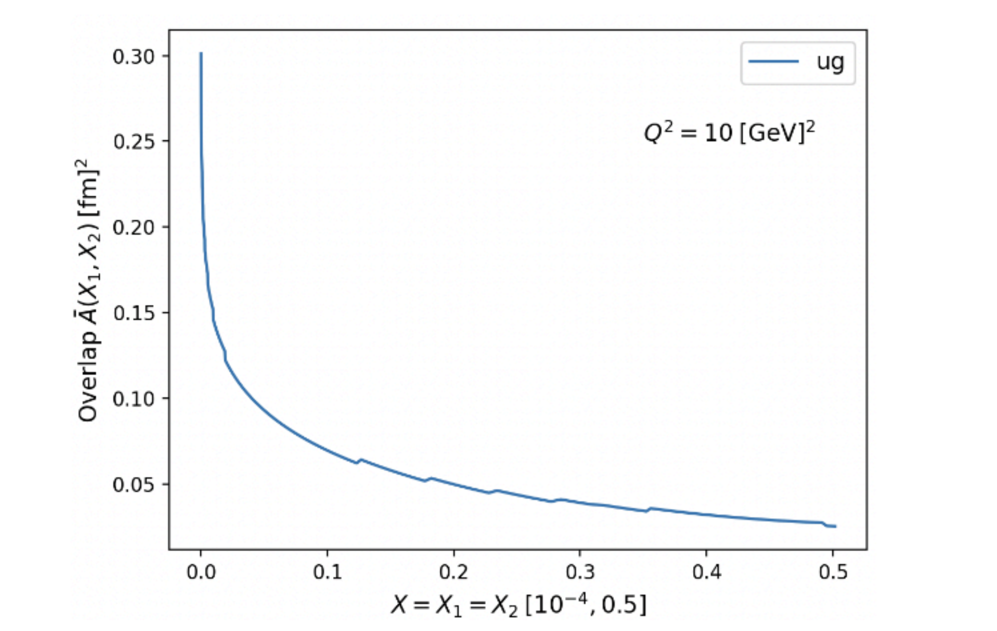


$.01, R)\rho_1^u(X=0.01, r)$
$\rho_1^g(X=0.01, R)\rho_1^u(X=0.1, r)$
$\rho_1^g(X=0.01, R)\rho_1^u(X=0.3, r)$
$\rho_1^g(X=0.01, R)\rho_1^u($
0.15
0.10
0.05
0.00
−1
0
1
2
−2
$R=(b_1+b2)/2[fm]$
$r=b_1-b_2[fm]$
$(X=0.01, b_1)\rho_1^u(X=0.01, b_2)$
$\rho_1^g(X=0.01, b_1)\rho_1^u(X=0.1, b_2)$
$\rho_1^g(X=0.01, b_1)\rho_1^u(X=0.3, b_2)$
$\rho_1^g(X=0.01, b_1)\rho_1^u(X=0$
$b_1[fm]$
$b_2[fm]$



arXiv:2405.05842

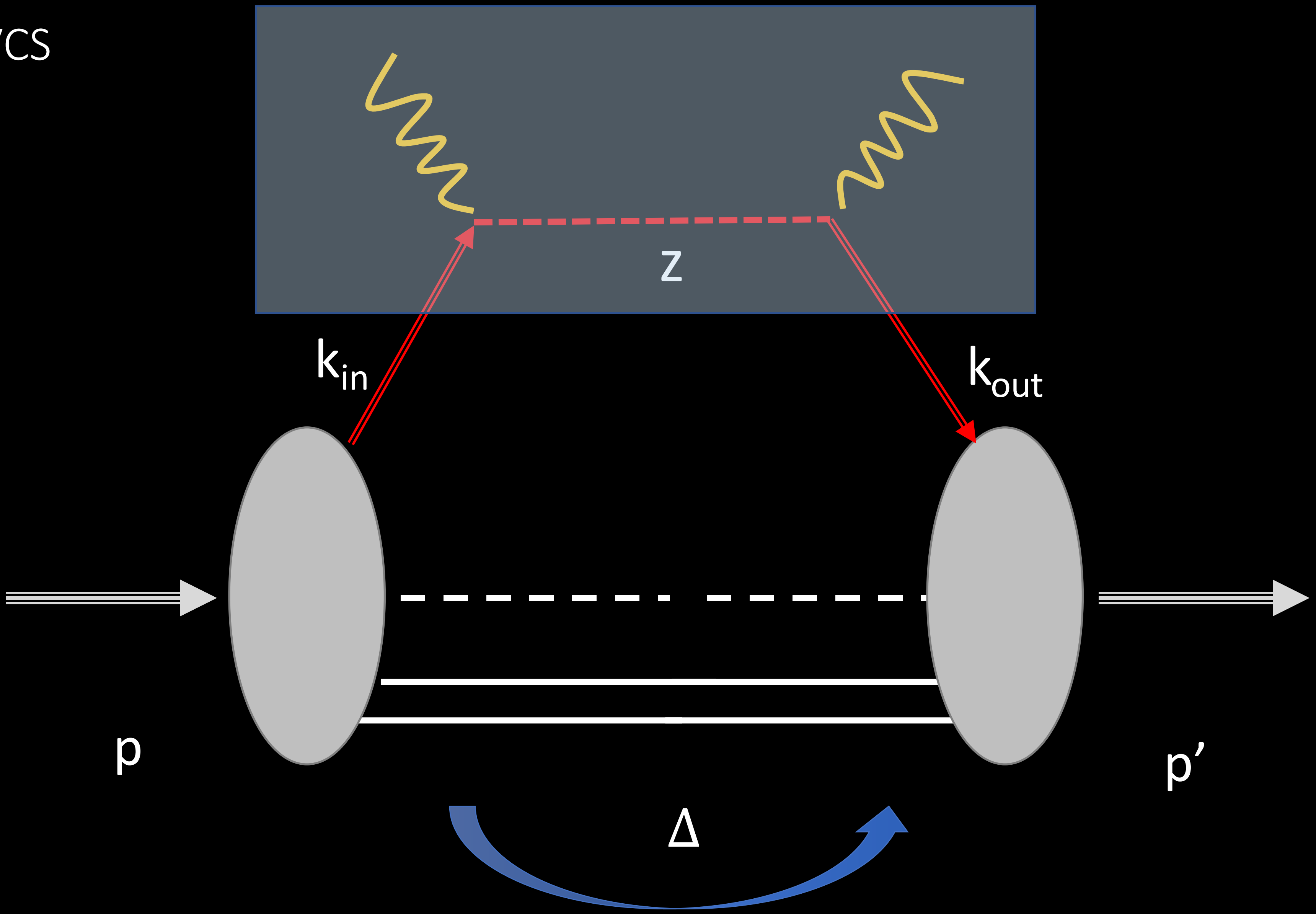
Overlap probability

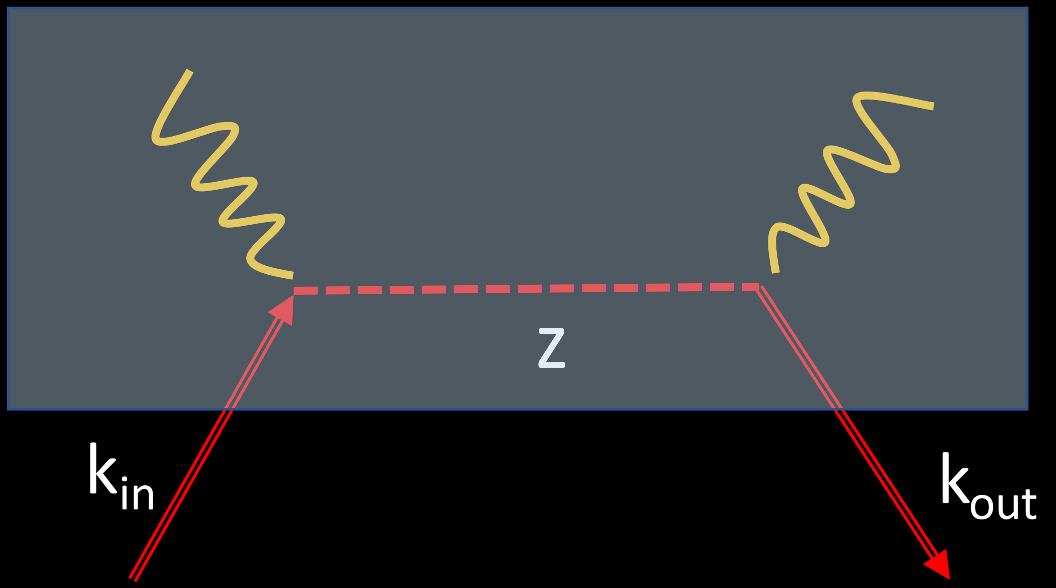


$$\langle A(X_1, X_2)\rangle \;=\; \frac{1}{\mathcal{N}}\int\int d^2\mathbf{r}\, d^2\mathbf{R}_{12}\, A(r)\, \rho_2^{qg}\left(X_1, \mathbf{R}_{12} + \frac{\mathbf{r}}{2}; X_2, \mathbf{R}_{12} - \frac{\mathbf{r}}{2}\right)$$

How to probe all this

Back to DVCS





At leading order in pQCD

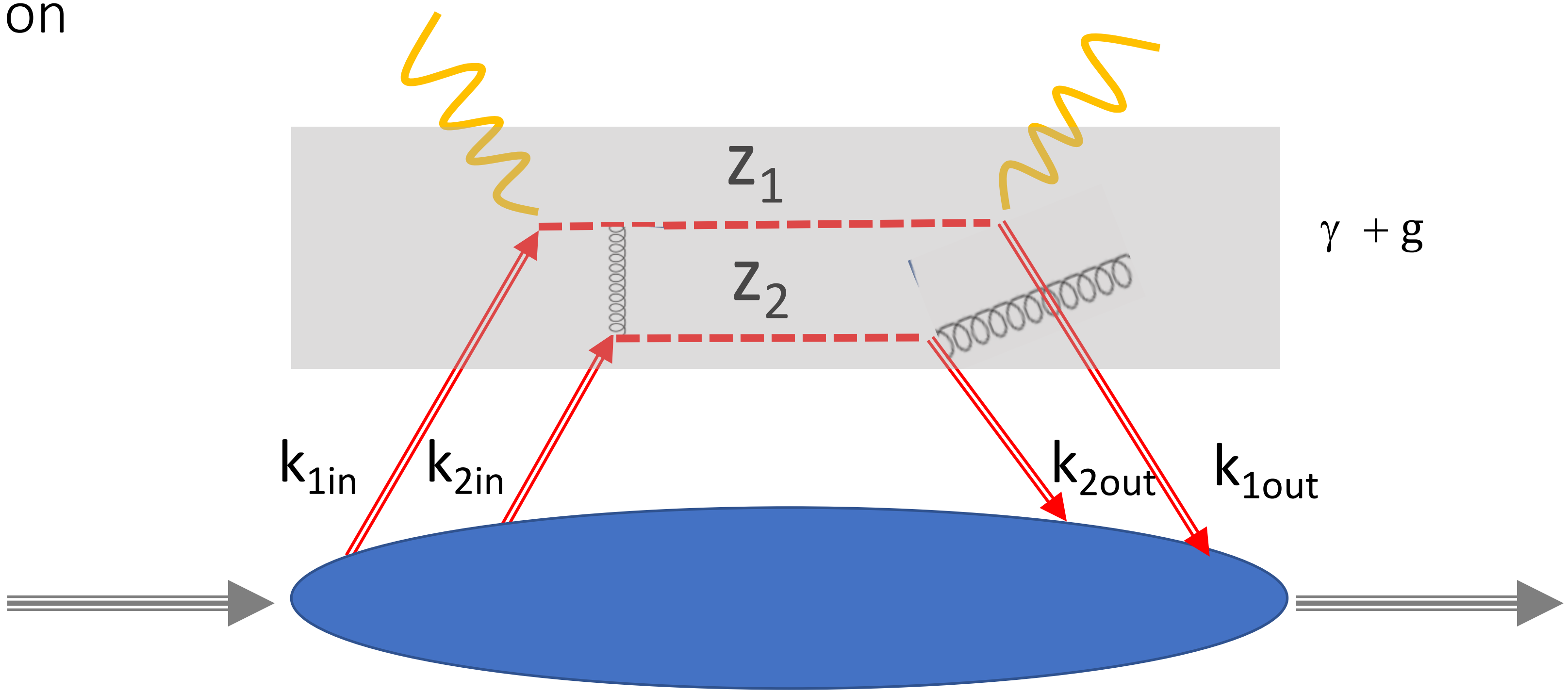
$$\int_{-1}^{1} dX\, \frac{1}{X-\zeta+i\epsilon} = P.V. \int_{-1}^{1} dX\, \frac{1}{X-\zeta} - i\pi\delta(X-\zeta)$$

- ✓ Because of many intricate reasons, despite the efforts at HERMES, HERA, Jlab, COMPASS, no GPD let alone image of the quark gluon structure is yet available

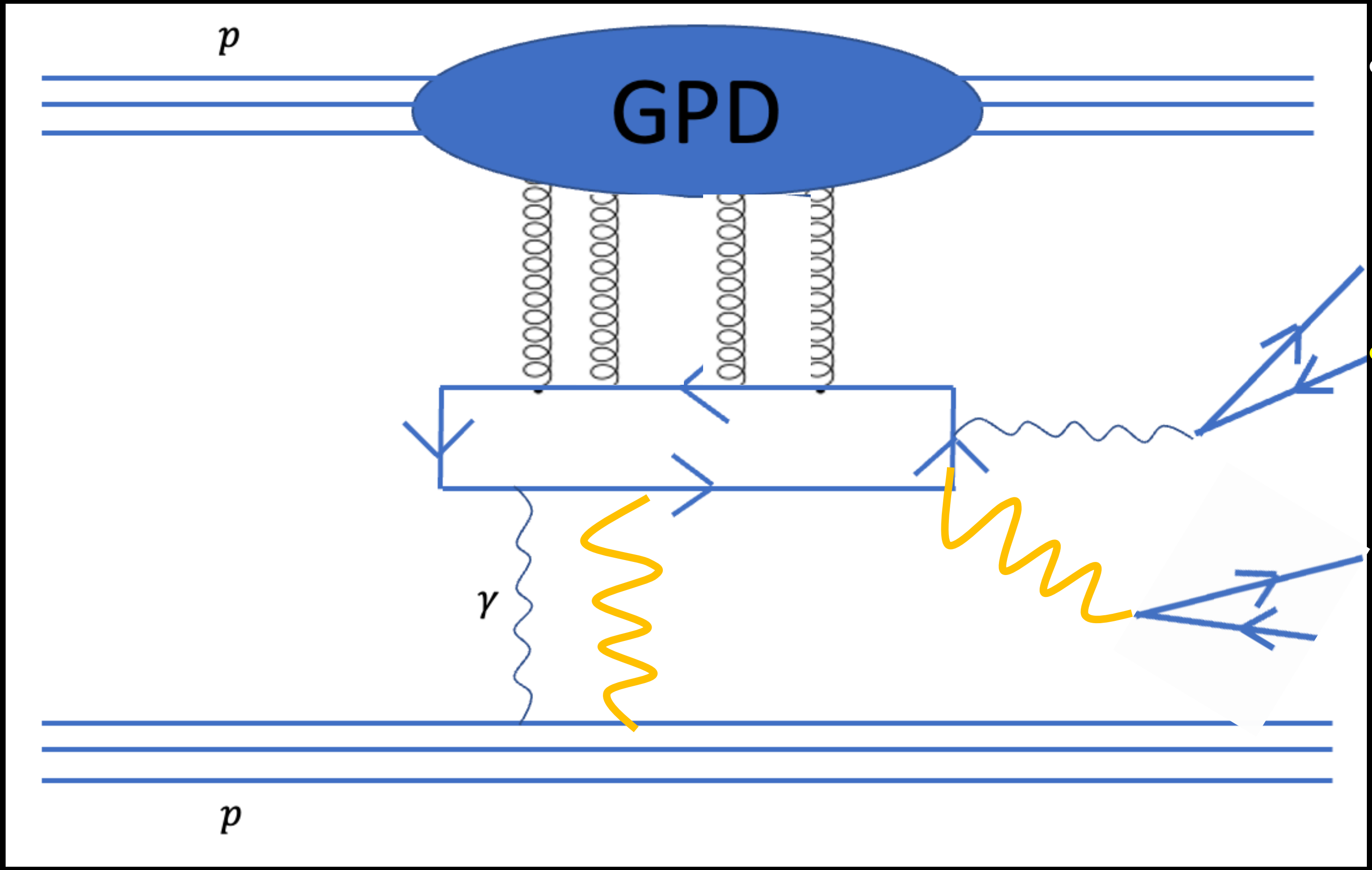
- ✓ New, alternative and/or more refined numerical methods are a must

Beyond one-body exclusive detection

Two jets detection



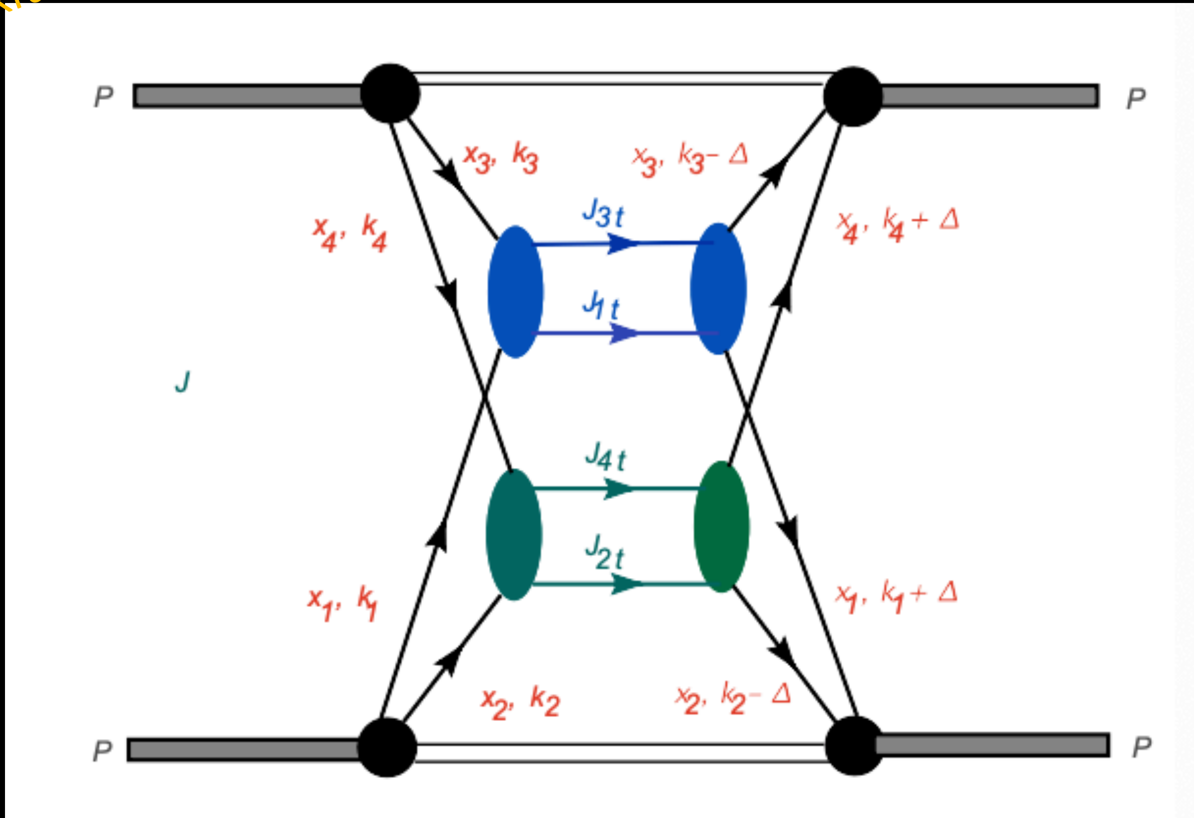
Connection to UPCs



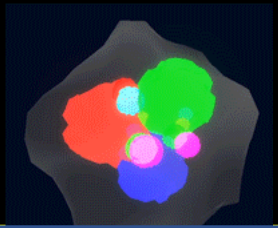
- GPDs can be observed in UPCs as in this diagram
- This figure is equivalent to time-like Compton scattering (TCS) because a lepton pair is created in the end
- We can obtain DPDs through UPCs by observing two such scatterings

The four jet production at LHC and Tevatron in QCD.
B. Blok, Yu. Dokshitzer, L. Frankfurt, M.Strikman (PRD 2011)

Alternative?



The EXCLAIM project (EXCLusives with Artificial Intelligence and Machine learning)



OUR PEOPLE

Computer Science/Machine Learning: Douglas Adams, Tareq Alghamdi, Fayaz, GiaWei Chern,, Yaohang Li, Anusha Singireddi Reddy

Lattice QCD: Michael Engelhardt, Huey Wen Lin, Emmanuel Ortiz

Phenomenology/Theory: Joshua Bautista, Marija Cuic, Andrew Dotson, Gary Goldstein, Carter Gustin, Adil Khawaja, SL, Zaki Panjsheeri, Matt Sievert, Dennis Sivers, Saraswati Pandey

Experiment: Marie Boer, Kemal Tegzin

Conclusions

- Extracting spatial information from data is an unprecedented challenging problem which is uniquely highly-dimensional with respect to what done so far
- Two-body densities are a must to investigate the relative distance among particles: this information is needed to locate the gluons
- How to extract it from data:
  - design experiments testing beyond "one-body" DVCS-type scenarios
  - keep developing refined numerical/ML-based approaches, as the complexity of the problems increase
  - build a platform with benchmarks for the community to compare results using consistent treatments of the uncertainties
  - UPC data at LHC are readily available!!