Exclusive photoproduction of a photon-meson pair

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Towards improved hadron tomography
with hard exclusive reactions
ECT Trento, Aug 8th,2024*

HRZZ
Croatian Science Foundation

Outline

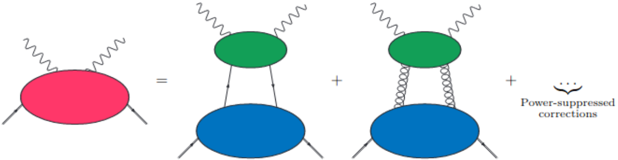
- Introduction and motivation
- Analytical results and challenges
- Preliminary numerical results
- Summary and outlook

work in progress in collaboration with K. Passek-K., G. Duplančić, S. Nabeebaccus, B. Pire, L. Szymanowski, S. Wallon

Exclusive processes and factorization

exclusive hard-scattering: $\exists$ large scale(s) $\rightarrow$ factorization:

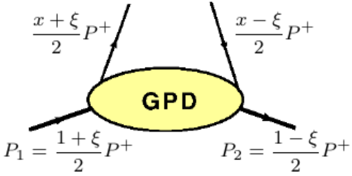
hard scattering amplitude	=	elementary hard-scattering amplitude	$\otimes$	hadron wave functions (GPDs, DAs)
		pQCD		evolution, input



[Mezrag 22']

DA . . . distribution amplitude (meson, nucleon)
GPD . . . generalized parton distribution

Generalized parton distributions (GPDs)



[Müller '92, et al. '94, Ji, Radyushkin '96]

$$P = P_1 + P_2$$
$$\Delta = P_2 - P_1$$

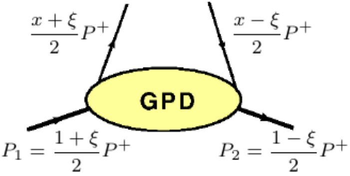
$a^\mu = (a^0, a^1, a^2, a^3) \to a^\pm = a^0 \pm a^3$

$$F^a(x, \xi, t = \Delta^2; \mu) = \int \frac{d\tau^-}{2\pi} e^{ixP^+\tau^-} \langle N(P_2)|\mathcal{O}^a(\tau)|N(P_1)\rangle\Big|_{\tau^+=0,\,\tau_\perp=0}$$

$a \in q, g$, $\mu \ldots$ factorization scale

- x : parton's average longitudinal momentum fraction, $-1 \leq x \leq 1$
- $\xi = -\frac{\Delta^+}{P^+}$: longitudinal momentum transfer (skewness), $\xi = \frac{x_B}{2-x_B}$
- $t = \Delta^2$: momentum transfer squared (Mandelstam variable)

Generalized parton distributions (GPDs)



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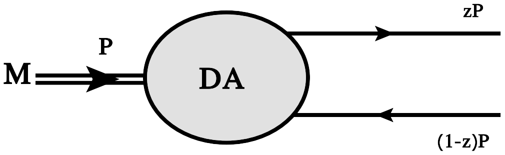
$a \in q, g$, $\mu \ldots$ factorization scale

- vector (H^a,E^a) and axial vector GPDs ($\tilde{H}^a$, $\tilde{E}^a$)
  $\to$ chiral-even $\quad \mathcal{O}^q(\tau) = \bar{q}(\tau)\gamma^+(\gamma^+\gamma_5)q(-\tau)$
- transversity GPDs (H^a_T,E^a_T, $\tilde{H}^a_T$,$\tilde{E}^a_T$)
  $\to$ chiral-odd $\quad \mathcal{O}^q(\tau) = \bar{q}(\tau)i\sigma^{+i}q(-\tau)$

Meson distribution amplitudes (DAs)



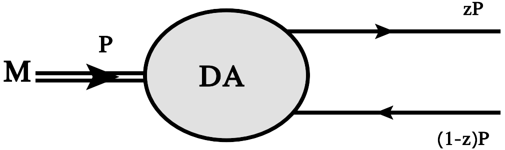
[Efremov, Radyushkin '80, Lapage, Brodsky '79,'80]

$$\phi^a_M(z;\mu) = \int \frac{d\tau^-}{2\pi}\, e^{i(2z-1)P^+\tau^-} \langle 0|\mathcal{O}^a(\tau)|M(P)\rangle\Big|_{\tau^+=0,\,\tau_\perp=0}$$

$a \in q, g$, $\mu \ldots$ factorization scale

- z : parton's longitudinal momentum fraction, $0 \leq z \leq 1$

Meson distribution amplitudes (DAs)



[Efremov, Radyushkin '80, Lapage, Brodsky '79,'80]

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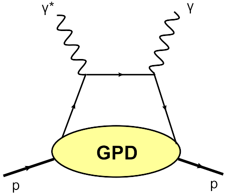
$a \in q, g$, $\mu \ldots$ factorization scale

- pseudoscalar mesons $(\pi, \eta, ...)$ $\quad \mathcal{O}^q(\tau) = \bar{q}(\tau)\gamma^+\gamma_5 q(-\tau)$
- longitudinally polarized vector mesons (ρ_L) $\quad \mathcal{O}^q(\tau) = \bar{q}(\tau)\gamma_5 q(-\tau)$
- transversely polarized vector mesons (ρ_T) $\quad \mathcal{O}^q(\tau) = \bar{q}(\tau) i\sigma^{+i} q(-\tau)$

Selected exclusive processes of interest

Deeply virtual Compton scattering (DVCS)



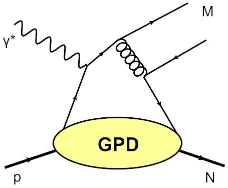
$$\boxed{\gamma^* p \to \gamma p}$$

factorization: [Collins, Freund '99]

$H^q, E^q, \tilde{H}^q, \tilde{E}^q$

$H^G, E^G, \tilde{H}^G, \tilde{E}^G$ (NLO)

Deeply virtual meson production (DVMP)



$$\boxed{\gamma^* p \to MN}$$

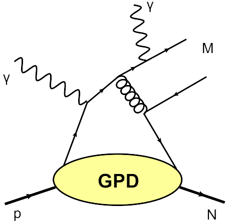
factorization:

[Collins, Frankfurt, Strikman '97]

$H^{q_i}, E^{q_i}; H^G, E^G$ (V_L)

$\tilde{H}^{q_i}, \tilde{E}^{q_i}$ (PS)

Photon-meson photoproduction



$$\boxed{\gamma p \to \gamma MN}$$

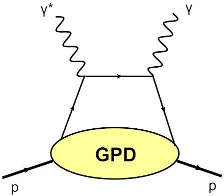
factorization: [Qiu, Yu '22,'23]

$H^a, E^a, \tilde{H}^a, \tilde{E}^a$

$H^a_T, E^a_T, \tilde{H}^a_T, \tilde{E}^a_T$

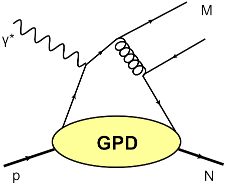
Selected exclusive processes of interest

Deeply vitual Comptom scattering (DVCS)



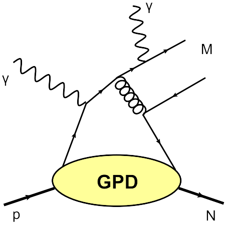
$\gamma^* p \to \gamma p$

Deeply virtual meson production (DVMP)



$\gamma^* p \to MN$

Photon-meson photoproduction



$\gamma p \to \gamma MN$

2→ 2

2→ 3

$$\frac{1}{x-\xi} \otimes GPD(x, \xi, t)$$

(LO)

$$f(x, \xi, \alpha) \otimes GPD(x, \xi, t)$$

(LO)

Motivation

- The richer kinematics of the $2 \to 3$ processes allow for an efficient probing of the sensitivity of the GPDs x dependence [Qiu,Yu,'23]

  $\to$ two large scales similarly as in DDVCS

- Transversity GPDs accessible at LO

$2 \to 3$ processes:

- $\gamma N \to (MM)N'$ [El Beiyad, Enberg, Ivanov, Pire, Segond, Szymanowski, Teryaev, Wallon: hep-ph/0209300, hep-ph/0601138, 1001.4491]
- $\gamma N \to (\gamma M)N'$ [Boussarie, Duplančić, Nabeebaccus, Passek-K., Pire, Szymanowski, Wallon: 1609.03830, 1809.08104, 2212.00655, 2302.12026]
- $\gamma N \to (\gamma\gamma)N'$ [Grocholski, Pedrak, Pire, Sznajder, Szymanowski, Wagner: 1708.01043, 2003.03263, 2110.00048, 2204.00396]
- $\pi N \to \gamma\gamma N'$ [Qiu, Yu: 2205.07846]

Motivation

- The richer kinematics of the $2 \to 3$ processes allow for an efficient probing of the sensitivity of the GPDs x dependence [Qiu,Yu,'23]
  $\rightarrow$ two large scales similarly as in DDVCS
- Transversity GPDs accessible at LO

$\gamma N \to (\gamma M) N'$

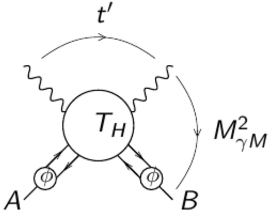
▷ completing and systematizing the results for all channels

▷ fast numerical code for different DA models with evolution included

▷ investigate the photoproduction of neutral pseudoscalar mesons

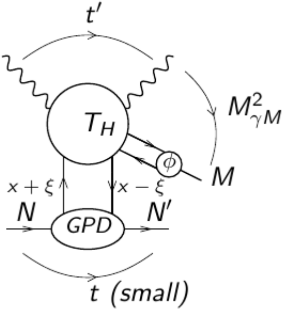
Photon-meson photoproduction: Factorization

$$\gamma + N \to \gamma + M(\pi, \eta, \rho, \dots) + N'$$



large angle factorisation
à la Brodsky Lepage

$\longrightarrow$

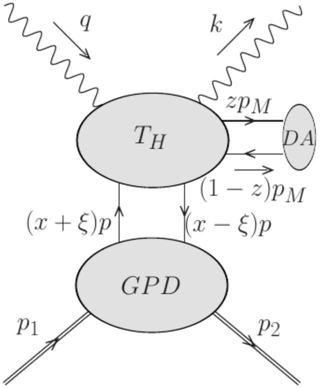


(à la "time-like" DVCS)

- factorization proof for $\pi^{\pm} N \to \gamma\gamma N'$ [Qiu, Yu '22] and other selected $2 \to 3$ processes [Qiu, Yu '23]

Photon-meson photoproduction: Kinematics

$$\gamma^*(q) + N(p1) \to \gamma(k) + M(p_M) + N'(p_2)$$



$$u' = (p_M - q)^2 \gg$$
$$t' = (k - q)^2 \gg$$
$$s' = M^2_{\gamma M} = (k + p_M)^2 \gg$$

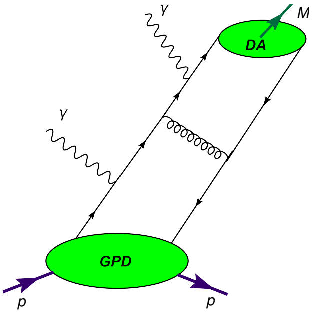
$$t = (p_2 - p_1)^2 \ll$$
$$s = S^2_{\gamma N} = (q + p_1)^2$$
$$\xi = \frac{\tau}{2-\tau}, \ \tau = \frac{M^2_{\gamma M}}{S^2_{\gamma N} - m^2_N}$$

$$-u', -t' > 1 \text{ GeV}^2, \ (-t)_{min} \leq -t \leq .5 \text{ GeV}^2$$

- amplitudes depend on ξ, $-t$, s, $\alpha = -u'/s'$
- convenient to introduce $y = \frac{\xi+x}{2\xi}$, $\bar{y} = 1 - y = \frac{\xi-x}{2\xi}$, $\boxed{\frac{\xi-1}{2\xi} \leq y \leq \frac{\xi+1}{2\xi}}$

Photon-meson photoproduction

$$\gamma q \to \gamma q(q\bar{q})$$



LO ρ mesons: [Boussarie, Pire, Szymanowsky, Wallon '16]

LO $\pi^\pm$ mesons: [Duplančić, Passek-K, Pire, Szymanowski, Wallon '18]

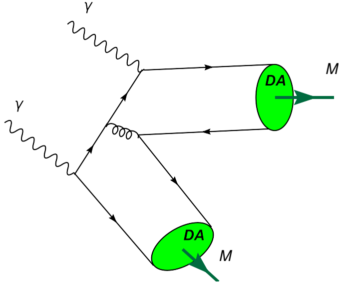
$$\pi^\pm : \widetilde{H}^q, \widetilde{E}^q, H^q, E^q$$

$$\rho^0_L : H^q, E^q, \widetilde{E}^q, \tilde{H}^q$$

$$\rho^0_T : H^q_T, E^q_T, \widetilde{E}^q_T, \tilde{H}^q_T$$

Meson pair production

$$\gamma\gamma \to (q\bar{q})(q\bar{q})$$

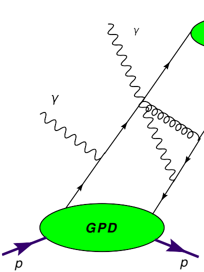


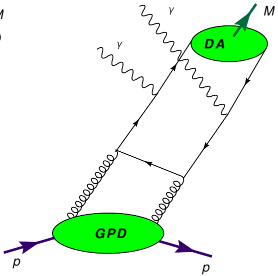
NLO: [Nižić '87, Duplančić, Nižić '06]

t-channel: $\gamma\gamma \to (q\bar{q})_1(q\bar{q})_2$
s-channel: $\gamma(q\bar{q})_1 \to \gamma(q\bar{q})_2$

Photon-π^0 photoproduction

$$\gamma q \to \gamma(q\bar{q})q,\ \gamma g \to \gamma(q\bar{q})g$$





$\widetilde{H}^q, \widetilde{E}^q$
H^q, E^q

$\widetilde{H}^G, \widetilde{E}^G$
H^G, E^G
$F_T^G, \widetilde{F}_T^G$

$(M)\pi^0$ photoproduction

$$\gamma\gamma \to (q\bar{q})(q\bar{q})$$
$$\gamma\gamma \to (gg)(q\bar{q})$$

$\gamma\gamma \to (PS)\pi^0 \quad \to \widetilde{H}^q, \widetilde{E}^q$

$\gamma\gamma \to (S)\pi^0 \quad \to H^q, E^q$

$\gamma\gamma \to (PS)_g\pi^0 \quad \to \widetilde{H}^G, \widetilde{E}^G$

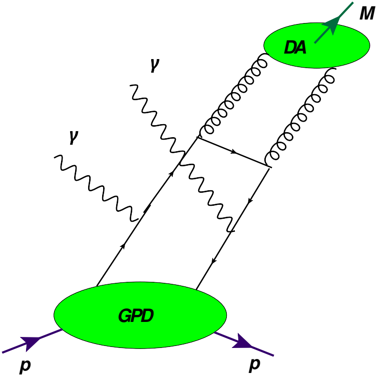
$\gamma\gamma \to (S)_g\pi^0 \quad \to H^G, E^G$

$\gamma\gamma \to (T)_g\pi^0 \quad \to F_T^G, \widetilde{F}_T^G$

LO: [Bayer, Grozin '85]

Photon-η, η' photoproduction

additionally $\gamma q \to \gamma(gg)q$



Valence Fock components of $M = \eta, \eta'$:

$$|q\bar{q}_8\rangle = |(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}\rangle \quad \text{(flavour-octet)}$$

$$\left\{ \begin{array}{l} |q\bar{q}_1\rangle = |(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}\rangle \quad \text{(flavour-singlet)} \\ |gg\rangle \end{array} \right.$$

Novel features:

1. flavour-mixing (singlet-octet mixing)
2. $|gg\rangle$ states contribute

   $\Rightarrow$ mixing of $q\bar{q}_1$ and gg DAs under evolution

Factorization

- Scattering amplitude $\mathcal{M}$ is expressed in terms of form factors $\mathcal{F}$ analogous to CFFs in DVCS

$$^a\mathcal{F} = \int_{-1}^{1} \mathrm{d}x \ \int_0^1 \mathrm{d}z \ T^a(x, \xi, z, \mu_R, \mu_F, \mu_\varphi; s, \alpha) \ F^a(x, \xi, t, \mu_F) \ \phi_M(z, \mu_\varphi)$$

$a = q, g$ $\quad \mu_F, \mu_\varphi \ldots$ factorization scales

$\mu_R \ldots$ renormalization scale

- T^a: subprocess hard-scattering amplitudes $\rightarrow$ pQCD
  - depend on two scales: $s, \alpha = -u'/s'$
  - more complicated expressions than in the case of DVCS and DVMP
    $\Rightarrow$ more demanding integrations with F^a and ϕ_M

- F^a: GPD

- ϕ_M: meson DA

Computation: T^a

- generic building blocks

$$y = \frac{\xi + x}{2\xi}, z \ldots y \in \mathbb{R} \ , \ z \in [0,1]$$

$$\alpha = -u'/s' = \frac{1 + \cos\theta'}{2}$$

$$\bar{a} \equiv 1 - a$$

  - $\frac{1}{z(y \pm i\epsilon)}$, $\frac{1}{z(\bar{y} \pm i\epsilon)}$ $\rightarrow$ usual "moment" type terms
  - $\frac{1}{z(y\bar{z} - \alpha z\bar{y} + i\epsilon)}, \ldots$ $\rightarrow$ terms that provide better sensitivity to GPD and DA form

- $\boxed{\pm i\epsilon} \Leftarrow$ time-like (s') and space-like (u', t') scales

$\gamma q \to (q\bar{q})q$ contribution

- no problem with integrations (factorization proven [Qiu, Yu '22, '23])

▷ obtained simpler closed expression suited for improved numerical integration (inclusion of different DA models, evolution taken into account, faster code)

▷ PV formalism enables efficient treatment of poles:
example, mixed term

$$\frac{1}{z\bar{z}\,(y\bar{z}\,-\alpha z\bar{y}\,+i\epsilon)} \to \frac{1}{\alpha\, z\bar{z}\,(\alpha+\bar{\alpha}\,y)}\left(\mathrm{PV}\frac{1}{z-\frac{y}{\alpha+\bar{\alpha}\,y}}+i\pi\;\delta\left(z-\frac{y}{\alpha+\bar{\alpha}\,y}\right)\right)$$

$\gamma g \to (q\bar{q})g$ contributions (π^0 production)

- gluon GPDs: $\mathcal{O}^g(\tau) = G^{+\mu}(-\tau)\widetilde{G}_\mu{}^+(\tau)$

$$\langle N(P_2)|A^\mu A^\nu|N(P_1)\rangle \to \frac{1}{(y \pm i0)(\bar{y} \pm i0)}(g^{\mu\nu}F^g - i\varepsilon_\perp^{\mu\nu}\tilde{F}^g + \ldots)$$

- due to gg projector with factor $\frac{1}{(y+i\epsilon)(\bar{y}+i\epsilon)}$, contributions

$$\frac{1}{(y+i\epsilon)(\bar{y}+i\epsilon)} \frac{N(y,z,\alpha)}{z\bar{z}\,(y-i\epsilon)(\bar{y}-i\epsilon)(y\bar{z}-\alpha z\bar{y}+i\epsilon)} \overset{y}{\otimes} F^g \overset{z}{\otimes} \phi_\pi$$

$$\to \frac{1}{(y+i\epsilon)(\bar{y}+i\epsilon)} \frac{N(y,z,\alpha)}{z\bar{z}\,(y-i\epsilon)(\bar{y}-i\epsilon)((y+i\epsilon)\bar{z}-\alpha z(\bar{y}-i\epsilon))}$$

$N(0,0,\alpha) = N(1,1,\alpha) = 0;\ \phi_\pi(z) \sim z\,\bar{z}$

demand additional attention $(F^g, \widetilde{F}^g(x = \pm\xi) \neq 0\ )$

$\Rightarrow$ additional regularization?($k_\perp$), breakdown of factorization?

[Nabeebaccus, Schoenleber, Szymanowski, Wallon '23]

$\gamma q \to (gg) q$ contributions $(\eta, \eta'$ production)

- gluon DAs: $\mathcal{O}^g(\tau) = G^{+\mu}(-\tau)\widetilde{G}_{\mu}{}^{+}(\tau)$

$$\langle 0|A^\mu A^\nu|M(P)\rangle \to \frac{1}{z\,\bar{z}}\; i\varepsilon^{\mu\nu}\;\phi_g(z)$$

- gg projector $\sim \dfrac{1}{z\,\bar{z}}$, $\phi_g \sim z^2\,\bar{z}^{\,2}$

$$\frac{1}{z\,\bar{z}}\,\frac{N(z,y,\alpha)}{z\,\bar{z}\,(y-i\epsilon)(\bar{y}\;-i\epsilon)\,(\bar{y}\,z-\alpha\bar{z}\,y+i\epsilon)} \overset{y}{\otimes} F^g \overset{z}{\otimes} \phi_g$$

▷ shown that there is no problem with factorization

▷ closed expressions obtained

▷ numerical analysis underway

GPD, DA models

- F^a: GPD
  $\rightarrow$ GK model [Radyushkin '98],[Goloskokov, Kroll '10] for valence quarks

- ϕ_M: meson DA

  $\rightarrow$ models:

$$\phi(z) = 6z(1-z) \text{ ... asymptotic DA}$$

$$\phi(z) = \frac{8}{\pi}\sqrt{z(1-z)} \text{ ... holographic DA}$$ [Brodsky, de Teramond '06]

$$\phi_q^M(z, \mu_\varphi) = 6z(1-z)\left[1 + \sum_{n=2}^{\infty}{}' a_n^M(\mu_\varphi) C_n^{3/2}(2z-1)\right]$$

$$\phi_g(z, \mu_\varphi) = 30z^2(1-z)^2\left[\sum_{n=2}^{\infty}{}' a_n^g(\mu_\varphi) C_{n-1}^{5/2}(2z-1)\right]$$

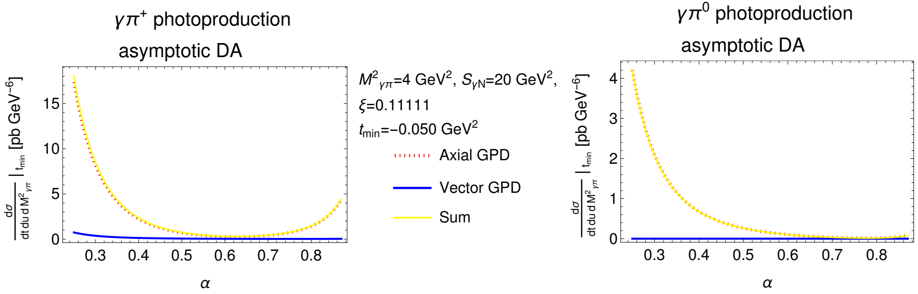
... expansion in Gegenbauer pol. $\Rightarrow$ evolution

$a_2^\pi = 0.1364,\ \mu_0 = 2GeV$ [Bali ,'19],

$a_2^1 = -0.12,\ a_2^g = 0.63,\ a_2^8 = -0.05,\ \mu_0 = 1GeV$ [Kroll, Passek-K. ,'12]

Numerics (preliminary!)

$$\frac{d\sigma}{dt\,du\,dM^2_{\gamma\pi}} = \frac{|\overline{\mathcal{M}}|^2_{t_{min}}}{32 S^2_{\gamma N} M^2_{\gamma\pi} (2\pi)^3} \qquad \alpha = -u'/s' = \frac{1+\cos\theta'}{2}$$



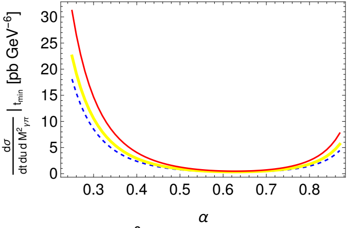
- ▷ contributions of axial GPDs $(\widetilde{H}^q, \widetilde{E}^q)$ dominate $\rightarrow$ probe for axial GPDs

Numerics (preliminary!)

$$\frac{d\sigma}{dt\,du\,dM^2_{\gamma\pi}} = \frac{|\overline{\mathcal{M}}|^2_{t_{min}}}{32 S^2_{\gamma N} M^2_{\gamma\pi} (2\pi)^3}$$

$$\alpha = -u'/s' = \frac{1+\cos\theta'}{2}$$

$\gamma\pi^+$ photoproduction



$M^2_{\gamma\pi}$=4 GeV^2, $S_{\gamma N}$=20 GeV^2,

ξ=0.11111

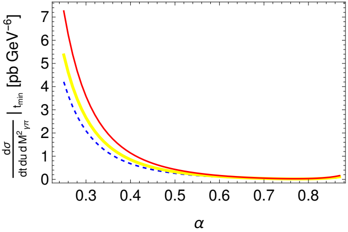
t_{min}=−0.050 GeV^2

asymptotic DA

Gegenbauer DA; a_2=0.1364

holographic DA

$\gamma\pi^0$ photoproduction



Numerics (preliminary!)

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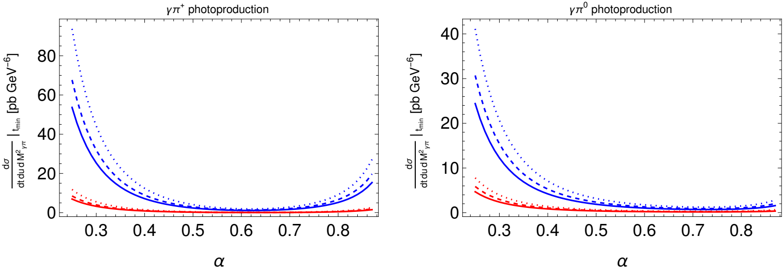
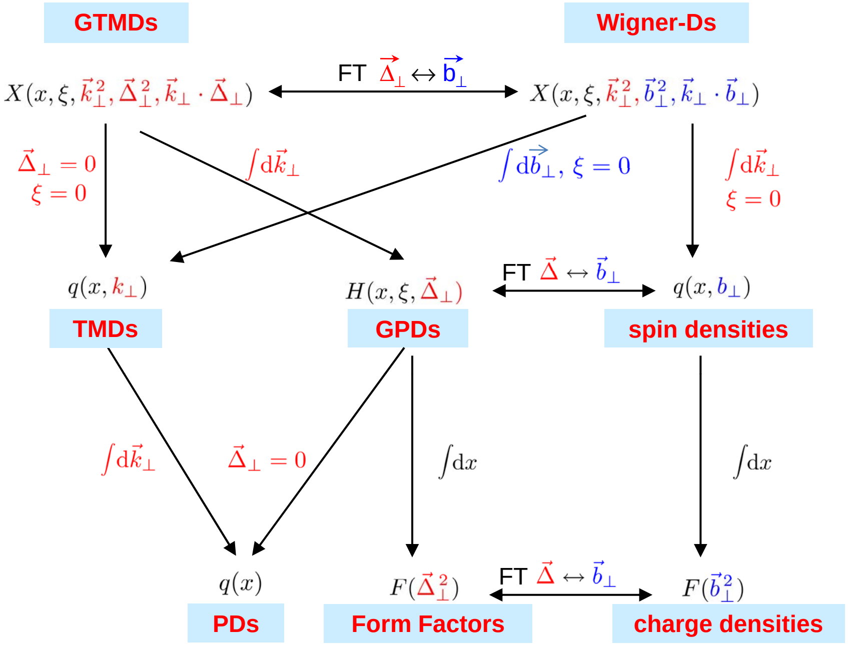


- $S_{\gamma N} = 20\ GeV^2$
- $s' = M^2_{\gamma\pi} = 3\ GeV^2$, $\xi = 0.081081$
- $s' = M^2_{\gamma\pi} = 5\ GeV^2$, $\xi = 0.14286$
- DAs: Asymptotic (straight line), Gegenbauer $a_2 = 0.1364$ (dashed), Holographic (dotted)

Summary: Photon-meson photoproduction

- Additional channel for extracting GPDs which provides greater sensitivity to x dependence and access to transversity GPDs at LO.
- Recent proof of factorization for quark GPDs.
- $(\gamma\pi^0)$: access to gluon GPDs but regularization of singularities needed.
- $(\gamma\eta, \eta')$: contributions with 2-gluon DA evaluated and proven to be finite.
- (γM): efficient closed analytical forms found, development of faster numerical code with evolution included.

Thank you!


GTMDs
Wigner-Ds
FT $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$
$X(x,\xi,\vec{k}_\perp^2,\vec{\Delta}_\perp^2,\vec{k}_\perp\cdot\vec{\Delta}_\perp)$
$X(x,\xi,\vec{k}_\perp^2,\vec{b}_\perp^2,\vec{k}_\perp\cdot\vec{b}_\perp)$
$\vec{\Delta}_\perp = 0$
$\xi = 0$
$\int d\vec{k}_\perp$
$\int d\vec{b}_\perp,\ \xi = 0$
$\int d\vec{k}_\perp$
$\xi = 0$
$q(x,k_\perp)$
$H(x,\xi,\vec{\Delta}_\perp)$
FT $\vec{\Delta} \leftrightarrow \vec{b}_\perp$
$q(x,b_\perp)$
TMDs
GPDs
spin densities
$\int d\vec{k}_\perp$
$\vec{\Delta}_\perp = 0$
$\int dx$
$\int dx$
$q(x)$
$F(\vec{\Delta}_\perp^2)$
FT $\vec{\Delta} \leftrightarrow \vec{b}_\perp$
$F(\vec{b}_\perp^2)$
PDs
Form Factors
charge densities

Definition of GPDs

- without helicity flip (chiral-even Γ matrices): 8 chiral-even GPDs:

$$F^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\, z_\perp=0}$$

$$F^g(x,\eta,t=\Delta^2) = \frac{2}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)G_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$

$a^\mu = a^0, a^1, a^2, a^3) \to a^\pm = a^0 \pm a^3$

- Lorentz decompozition:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

Definition of GPDs

- without helicity flip (chiral-even Γ matrices): 8 chiral-even GPDs:

$$\widetilde{F}^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2 | \bar{q}(-z)\gamma^+\gamma_5 q(z) | P_1 \rangle \Big|_{z^+=0,\, z_\perp=0}$$

$$\widetilde{F}^g(x,\eta,t=\Delta^2) = \frac{4}{P^+} \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2 | G^{+\mu}(-z)\widetilde{G}_\mu{}^+(z) | P_1 \rangle \Big|_{\dots}$$

- Lorentz decompozition:

$$\widetilde{F}^a = \frac{\bar{u}(P_2)\gamma^+\gamma_5 u(P_1)}{P^+}\widetilde{H}^a + \frac{\bar{u}(P_2) i\sigma^{+\nu}\gamma_5 u(P_1)\Delta_\nu}{2MP^+}\widetilde{E}^a \qquad a = q, g$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

Definition of GPDs

- With helicity flip (chiral-odd Γ matrices): 8 chiral-odd GPDs (transversity GPD):

$$F_T^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2 | \bar{q}(-z) i\sigma^{+i} q(z) | P_1 \rangle \Big|_{z^+=0,\, z_\perp=0}$$

- Lorentz decompozition:

$$F_T^q = \frac{\bar{u}(P_2) i\sigma^{+i} u(P_1)}{P^+} H_T^q + \frac{\bar{u}(P_2)(P^+\Delta^i - \Delta^+ P^i) u(P_1)}{M^2 P^+} \widetilde{H}_T^q + \\ + \frac{\bar{u}(P_2)(\gamma^+\Delta^i - \Delta^+\gamma^i) u(P_1)}{2MP^+} E_T^q + \frac{\bar{u}(P_2)(\gamma^+ P^i - P^+\gamma^i) u(P_1)}{MP^+} \widetilde{E}_T^q$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

Definition of GPDs

- With helicity flip (chiral-odd Γ matrices): 8 chiral-odd GPDs (transversity GPD):

$$F_T^g(x,\eta,t=\Delta^2) = \frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle P_2|\mathbf{S}\, G^{+i}(-z)G^{j\,+}(z)|P_1\rangle\Big|_{z^+=0,}$$

- Lorentz decompozition:

$$F_T^g = \mathbf{S}\frac{(P^+\Delta^j - \Delta^+P^j)}{2MP^+}\frac{\bar{u}(P_2)}{P^+}\Big[i\sigma^{+i}H_T^g + \frac{(P^+\Delta^i - \Delta^+P^i)}{M^2}\widetilde{H}_T^g + \\ + \frac{(\gamma^+\Delta^i - \Delta^+\gamma^i)}{2M}E_T^g + \frac{(\gamma^+P^i - P^+\gamma^i))}{M}\widetilde{E}_T^g\Big]u(P1)$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

Polynomiality and positivity constraints of GDP

- Polynomiality

  Lorentz covariance $\Rightarrow \int_{-1}^{1} dx x^m H^q(x,\xi,t) = \sum_{k=0}^{m} \xi^k A^q_{m,k}(t)$

- Positivity

  Positivity of Hilbert space norm $\Rightarrow H^q(x,\xi,t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$

DAs

$$\phi_\pi(x,\mu_F) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle 0|\bar{q}(-z)\gamma^+\gamma_5 q(z)|\pi(P)\rangle\Big|_{z^+=0,\, z_\perp=0}$$

$$\phi_\pi(x,\mu_F) = 6x\,(1-x)\left[1 + \sum_{n=2,4,\ldots} a_{\pi,n}(\mu_F)\; C_n^{3/2}(2x-1)\right]$$

$$a_{M,n}(\mu_F) = a_{M,n}^{LO}(\mu_F) + \frac{\alpha_S(\mu_F)}{4\pi} a_{M,n}^{NLO}(\mu_F)$$

$$a_{M,n}^{LO}(\mu_F) \;=\; \left(\frac{\alpha_S(\mu_O)}{\alpha_S(\mu_F)}\right)^{\gamma_n/\beta_0} a_{M,n}^{LO}(\mu_O) \qquad (\leq a_{M,n}^{LO}(\mu_O))$$

$\eta,\ \eta'$ DAs

1. flavour-mixing [review Feldman '00]

$\downarrow$

simplest possibility: to take particle dependence and the flavour-mixing to be solely embedded in the decay constants f_M^i

- $\boxed{\Phi_{Mi} = f_M^i\,\phi_i}$ $i \in \{1, 8\}$
- the decay constants parametrized as

$$f_\eta^8 = f_8 \cos\theta_8 \qquad f_\eta^1 = -f_1 \sin\theta_1$$

$$f_{\eta'}^8 = f_8 \sin\theta_8 \qquad f_{\eta'}^1 = f_1 \cos\theta_1$$

[Leutwyler '98, Felmann, Kroll, Stech, '98,'99]

2. $|gg\rangle$ states contribute

$\Rightarrow$ mixing of $q\bar{q}_1$ and gg DAs under evolution

$$(\Phi_{M1} \equiv \Phi_{Mq}) \quad (\Phi_{Mg})$$

$$\downarrow$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix} = \begin{pmatrix} V_{qq} & V_{qg} \\ V_{gq} & V_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix}$$

Factorization

$$ {}^a\mathcal{F} = \int_{-1}^{1} \mathrm{d}x \int_0^1 \mathrm{d}z \; T^a(x, \xi, z, \mu_F, \mu_\varphi; s, \alpha) \; F^a(x, \xi, t, \mu_F) \; \phi_M(z, \mu_\varphi) $$

$a = q, g$ μ_F, μ_φ ... factorization scales

- T^a: subprocess hard-scattering amplitudes $\rightarrow$ pQCD

  more complicated expressions than in the case of DVCS and DVMP
  $\Rightarrow$ more demanding integrations with F^a and ϕ_M

- F^a: GPD
  $\rightarrow$ different models based on [Radyushkin '98],[Goloskokov, Kroll '10]

- ϕ_M: meson distribution amplitude (DA)

  $\rightarrow$ models:

$\phi(z) = 6z(1-z)$ ... asymptotic DA

$\phi(z) = \frac{8}{\pi}\sqrt{z(1-z)}$ ... holographic DA [Brodsky, de Teramond '06]

$\phi(z, \mu_\varphi) = 6z(1-z)\left[1 + \sum_n a_n(\mu_\varphi) C_n^{3/2}(2z-1)\right]$

... expansion in Gegenbauer pol. $\Rightarrow$ evolution

Factorization II

- Scattering amplitude $\mathcal{M}$ is expressed in terms of form factors $\mathcal{H}$, $\mathcal{E}$, $\mathcal{H}$, $\mathcal{E}$ analogous to CFFs in DVCS

$$\mathcal{M}_\pi = \frac{1}{n\cdot p}\bar{u}(p_2,\lambda')[\hat{n}\mathcal{H}_\pi + \frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\mathcal{E}_\pi + \hat{n}\gamma_5\widetilde{\mathcal{H}}_\pi + \frac{n\cdot\Delta}{2\pi}\gamma_5\widetilde{\mathcal{E}}_\pi]u(p_1,\lambda)$$

$$TA = \varepsilon_{q_\perp}\cdot\varepsilon^*_{q_\perp},$$

$$TB = (\varepsilon_{q_\perp}\cdot p_\perp)(\varepsilon^*_{q_\perp}\cdot p_\perp)$$

$$TA5 = (\varepsilon^*_{k_\perp}\cdot p_\perp)\epsilon^{np\varepsilon_{q_\perp}p_\perp}$$

$$TB5 = -(\varepsilon_{q_\perp}\cdot p_\perp)\epsilon^{np\varepsilon^*_{k_\perp}p_\perp}$$

Recent work: $\gamma\pi^0$ photoproduction

quark contributions

- obtained simpler closed expression for sum of diagrams suited for improved numerical integrations and inclusion of different DA models
- PV formalism enables efficient treatment of poles;
  example, mixed term:

$$\frac{1}{z\bar{z}\;(y\bar{z}\;-\alpha z\bar{y}\;+i\epsilon)} \to \frac{1}{\alpha\, z\bar{z}\,(\alpha+\bar{\alpha}\,y)}\left(\mathrm{PV}\frac{1}{z-\frac{y}{\alpha+\bar{\alpha}\,y}} + i\pi\;\;\delta\left(z-\frac{y}{\alpha+\bar{\alpha}\,y}\right)\right)$$

$y = \frac{\xi + x}{2\xi}, z \ldots$ partons momentum fractions , $\alpha = -u'/s'$, and $\bar{a} \equiv 1 - a$

gluon contributions

- determined and closed expressions for the sum of diagrams obtained
- due gg projector with factor $\frac{1}{(y+i\epsilon)(\bar{y}\;+i\epsilon)}$, contributions

$$\frac{1}{(y+i\epsilon)(\bar{y}\;+i\epsilon)}\frac{N(y,z,\alpha)}{z\bar{z}\;(y-i\epsilon)(\bar{y}\;-i\epsilon)(y\bar{z}\;-\alpha z\bar{y}\;+i\epsilon)}$$

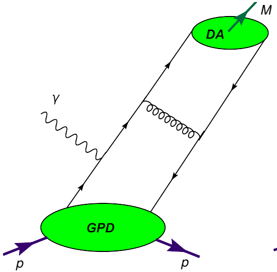
demand additional attention

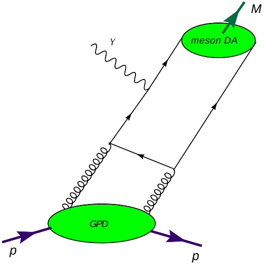
$\Rightarrow$ additional regularization?($k_\perp$), breakdown of factorization?

$\rightarrow$ work in progress

DVMP

$$\gamma^* q \to (q\bar{q})q,\ \gamma^* g \to (q\bar{q})g$$





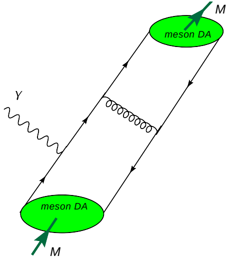
NLO DV PS^+ prod.: [Belitsky and Müller '01]

NLO DV V_L prod.: [Ivanov et al '04,]

NLO DV V_L (corr.), PS, (S, PV_L) prod.: [Duplančić, Müller, Passek-K. '17]

Meson em form factor

$$\gamma^*(q\bar{q}) \to (q\bar{q})$$



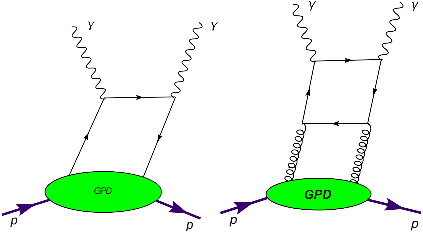
NLO: [..., Melić et al '99]

$$\gamma_L^*(M^\pm) \to M^\pm,$$
$$\gamma_L^*(S) \to V_L,\ \gamma_L^*(V_L) \to S$$
$$\gamma_L^*(PV_L) \to PS,\ \gamma_L^*(PS) \to PV_L$$
$$\Rightarrow \text{DVMP}$$

(D)DVCS

$$\gamma^* q \to \gamma^{(*)} q,\ \gamma^* g \to \gamma^{(*)} g$$

24



NLO: [Ji, Belitsky et al, Mankiewicz et al, '97]
[Pire, Szymanowski, Wagner '11]

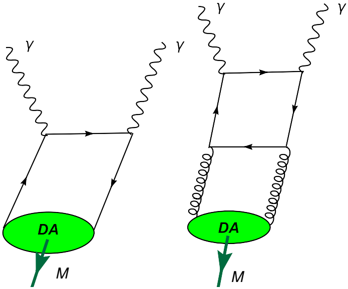
β_0 proportional NNLO: [Belitsky, Schäfer '98]

NNLO from conf. sym: [Müller '05, Kumerički, Müller, Passek-K '07,]

NNLO: [Braun, Manashov,Moch, Schoehleber '20,'21, Braun, Ji,Schoehleber '22]

Meson transition form factor

$$\gamma^* \gamma^{(*)} \to (q\bar{q}),\ \gamma^* \gamma^{(*)} \to (gg)$$



NLO: [..., Kroll, Passek-K '02] [Kroll, Passek-K '19]

β_0 proportional NNLO: [Melić, Nižić, Passek '01]

NNLO from conf. sym: [Melić, Müller, Passek '02]

NNLO [Braun, Manashov,Moch, Schoehleber '21, Gau, Huber,Ji, Wang '22]

Status

- experiment
  - DVCS, vector (ρ, J/Ψ, ϕ) and pseudoscalar (π, η) meson production measured by H1, ZEUS, HERMES (HERA, DESY), COMPASS (SPS, CERN), CLAS, Hall-A,C (JLab) ...
  - LHeC, EicC proposed
  - EIC (Electron Ion Collider at Brokhaven, 2030) under construction (luminosity 100-1000 times HERA)