Exclusive processes amplitudes beyond kinematic leading twist

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Outline

- Generalized parton distributions.
- DVCS, TCS & DDVCS.
- Spin-1/2 target: LO + LT.
- Spin-0 target: LO + LT + kinematic twist-3 + 4.
- Phenomenology.
- Summary and conclusions.

Partonic distribution

GPD

Generalized Parton Distribution ≈ "3D version of a PDF (Parton Distribution Function)." With x the fraction of the hadron's longitudinal momentum carried by a quark:

$$\mathrm{GPD}_f(x,\xi,t)=\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ix\bar{p}^+z^-}\langle p'|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2,z/2]\mathfrak{q}_f(z/2)|p\rangle\Big|_{z_\perp=z^+=0}$$

$$t=\Delta^2=(p'-p)^2,\quad \xi=-\frac{\Delta n}{2\bar{p}n},\quad \rho=\xi\frac{qq'}{\Delta q'},\quad \bar{p}=\frac{p+p'}{2}$$

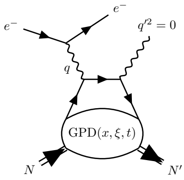
Importance

- Connected to **QCD energy-momentum tensor**. GPDs are a way to study "mechanical" properties and to address the hadron's spin puzzle.
- **Tomography:** distribution of quarks in terms of the longitudinal momentum and in the transverse plane.

$$q(x,\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{4\pi^2}\;e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}H^q(x,0,t=-\vec{\Delta}_\perp^2)$$

Deeply virtual Compton scattering (DVCS)

- In the 1990s, Müller, Ji and Radyushkin introduced the Generalized Parton Distributions (GPDs) to study the DVCS process:



Feynman diagram for DVCS

- At LO$\sim O(\alpha_s^0)$ and LT$\sim O(1/Q^0)$:

$$\mathrm{CFF}_{\mathrm{DVCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\xi} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx\ \ i\pi\delta(x-\xi)\mathrm{GPD}(x,\xi,t) + \cdots$$

- $\xi = \frac{-n\Delta}{2\bar{p}n}\,, \quad \bar{p} = \frac{p+p'}{2}\,, \quad \Delta = p' - p\,, \quad t = \Delta^2$

Inclusion of power corrections (kinematic higher twists) $\rightarrow$ **V. Braun's talk.**

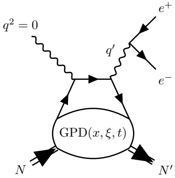
DVCS has been measured in DESY, CERN and JLab.

Timelike Compton scattering (TCS)

- Complementary to DVCS.

  E. R. Berger, M. Diehl and B. Pire, EPJC 23, 675–689 (2002).

  1st measurement of TCS by the CLAS collaboration at JLab: P. Chatagnon et al., PRL **127**, 262501 (2021).



Feynman diagram for TCS

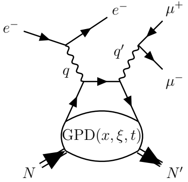
- At LO LT:

$$\mathrm{CFF_{TCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x+\xi} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\, i\pi\delta(x+\xi)\mathrm{GPD}(x,\xi,t) + \cdots$$

- Like DVCS but $\xi \to -\xi$.

Double deeply virtual Compton scattering (DDVCS)

- **DDVCS vs DVCS/TCS:** extra virtuality $\Rightarrow$ *generalized* Björken variable $\rho \Rightarrow$ GPDs for $x = \rho \neq \xi$.



Double DVCS (DDVCS)

- At LO LT:

$$\text{CFF}_{\text{DDVCS}} \sim \text{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\rho} \text{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\; i\pi\delta(x-\rho)\text{GPD}(x,\xi,t) + \cdots$$

$$\xi = -\frac{\Delta n}{2\bar{p}n}, \quad \rho = \xi \frac{qq'}{\Delta q'}, \qquad \rho = \xi \text{ (DVCS)}, \quad \rho = -\xi \text{ (TCS, LT approach)}$$

LO + LT: Belitsky & Müller, PRL 90, 022001 (2003); Guidal & Vanderhaeghen, PRL 90, 012001 (2003); Belitsky & Müller, PRD 68, 116005 (2003); K. Deja, VMF, B. Pire, P. Sznajder & J. Wagner, PRD 107, 094035 (2023).

Experimental projections for DDVCS at EIC and JLab $\rightarrow$ **today's talk by J. S. Alvarado.**

Spin-1/2 target, DDVCS subprocess: LO + LT

- DDVCS subprocess amplitude:

$$i\mathcal{M}_{\mathrm{DDVCS}}=\frac{ie^4\bar{u}(\ell_-,s_\ell)\gamma_\mu v(\ell_+,s_\ell)\bar{u}(k',s)\gamma_\nu u(k,s)}{(q^2+i0)(q'^2+i0)}T^{\mu\nu}_{s_2s_1}$$

- Compton tensor decomposition at LT:

$$T^{\mu\nu}_{s_2s_1}=-\frac{1}{2}g^{\mu\nu}_\perp\bar{u}(p',s_2)\Big[(\mathcal{H}+\mathcal{E})\not{n}-\frac{\mathcal{E}}{M}\bar{p}^+\Big]u(p,s_1)-\frac{i}{2}\epsilon^{\mu\nu}_\perp\bar{u}(p',s_2)\left[\widetilde{\mathcal{H}}\not{n}+\frac{\widetilde{\mathcal{E}}}{2M}\Delta^+\right]\gamma^5u(p,s_1)$$

- **Longitudinal plane is built with $\{\bar{q},\bar{p}\}$.**
- $q^\nu_\perp\sim\Delta^\nu_\perp\Rightarrow g^{\mu\nu}_\perp q_\nu\neq 0\Rightarrow$ EM gauge-violation. Solution:

$$q^\nu_\perp|_{t=t_0}\sim\Delta^\nu_\perp|_{t=t_0}=0$$

- This procedure is consistent with the longitudinal factorization which is at the core of the GPD description.

DDVCS subprocess *à la* Kleiss-Stirling

- DDVCS subprocess amplitude:

$$i\mathcal{M}_{\mathrm{DDVCS}}=\frac{-ie^4}{(Q^2-i0)(Q'^2+i0)}\left(i\mathcal{M}^{(V)}_{\mathrm{DDVCS}}+i\mathcal{M}^{(A)}_{\mathrm{DDVCS}}\right)$$

- Vector contribution:

$$i\mathcal{M}^{(V)}_{\mathrm{DDVCS}}=-\frac{1}{2}\left[f(s_\ell,\ell_-,\ell_+;s,k',k)-g(s_\ell,\ell_-,n^\star,\ell_+)g(s,k',n,k)-g(s_\ell,\ell_-,n,\ell_+)g(s,k',n^\star,k)\right]$$
$$\times\left[(\mathcal{H}+\mathcal{E})[Y_{s_2s_1}g(+,r'_{s_2},n,r_{s_1})+Z_{s_2s_1}g(-,r'_{-s_2},n,r_{-s_1})]-\frac{\mathcal{E}}{M}\mathcal{J}^{(2)}_{s_2s_1}\right]$$

- Axial contribution:

$$i\mathcal{M}^{(A)}_{\mathrm{DDVCS}}=\frac{-i}{2}\epsilon^{\mu\nu}_{\perp}j_\mu(s_\ell,\ell_-,\ell_+)j_\nu(s,k',k)\left[\widetilde{\mathcal{H}}\mathcal{J}^{(1,5)+}_{s_2s_1}+\widetilde{\mathcal{E}}\frac{\Delta^+}{2M}\mathcal{J}^{(2,5)+}_{s_2s_1}\right]$$

Single beam-spin asymmetry

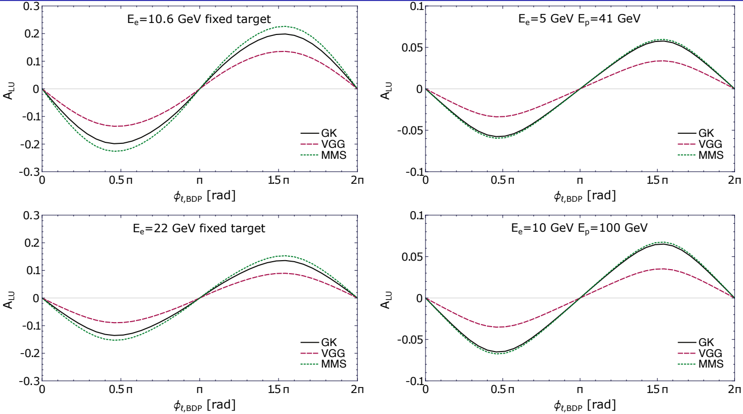
- Single beam-spin asymmetry for longitudinally polarized electrons:

$$A_{LU}(\phi_{\ell,\mathrm{BDP}}) = \frac{\Delta\sigma_{LU}(\phi_{\ell,\mathrm{BDP}})}{\sigma_{UU}(\phi_{\ell,\mathrm{BDP}})}$$

$$\Delta\sigma_{LU}(\phi_{\ell,\mathrm{BDP}}) = \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} d\theta_{\ell,\mathrm{BDP}} \;\; \sin\theta_{\ell,\mathrm{BDP}}$$

$$\times \left(\frac{d^7\sigma^{\rightarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\mathrm{BDP}}} - \frac{d^7\sigma^{\leftarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\mathrm{BDP}}} \right)$$

- We consider $Q'^2 > Q^2$: timelike dominated DDVCS.



JLab12, JLab20+: up to **15-20%** EIC 5x41, EIC 10x100: **3-7%**

Experiment	Beam energies [GeV]	y	$\lvert t\rvert$ [GeV^2]	Q^2 [GeV^2]	Q'^2 [GeV^2]
JLab12	$E_e = 10.6,\ E_p = M$	0.5	0.2	0.6	2.5
JLab20+	$E_e = 22,\ E_p = M$	0.3	0.2	0.6	2.5
EIC	$E_e = 5,\ E_p = 41$	0.15	0.1	0.6	2.5
EIC	$E_e = 10,\ E_p = 100$	0.15	0.1	0.6	2.5

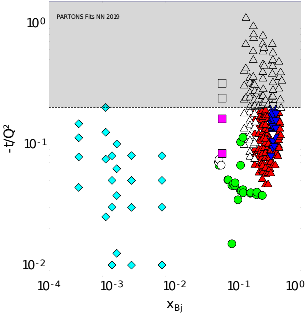
Ongoing studies for future experiments $\rightarrow$ **Stepanyan's talk.**

Beyond LT: why higher twists?

1. Nucleon tomography is a Fourier transform in $\Delta_\perp$ that requires data on a sizable range of t:

$$q(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{4\pi^2}\ e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H^q(x,0,t=-\vec{\Delta}_\perp^2)$$

2. Increase the range of useful experimental data:



Data come from the Hall A (▼, ▽), CLAS (▲, △), HERMES (●, ○), COMPASS (■, □) and HERA H1 and ZEUS (♦, ◇) experiments. The gray bands (open markers) indicate phase-space areas (experimental points) being excluded in the analysis of **H. Moutarde, P. Sznajder, J. Wagner, EPJC 79, 614 (2019).**

Scale of DDVCS and experiments

- **Scale of DDVCS, not imposed but inferred from the twist expansion:**

$$\mathbb{Q}^2 = Q^2 + Q'^2 + t$$

$\mathbb{Q}^2 \xrightarrow{\text{DVCS}} Q^2 + t$, see **V. Braun's talk.**

- **Conclusion:** you may choose

$$\begin{cases} \sigma_{\text{DDVCS}} \text{ grows rapidly with small } Q^2 \Rightarrow Q^2 < 1 \text{ GeV}^2 \\ \text{region in-between resonances} \Rightarrow Q'^2 \in (2.25, 9) \text{ GeV}^2 \end{cases}$$

- Low Q^2 + high Q'^2... as for TCS* $\rightarrow$ potential with CLAS12 (Hall B, JLab) & SoLID† (Hall A, JLab) $\Leftrightarrow$ muon detector or reconstruction trajectories.

*Chatagnon et al., PRL 127, 262501.

†Arrington et al., J. Phys. G 50 (2023) 11, 110501. Also, **G. Huber's, M. Boër's talks.**

Starting point: OPE + CFT (Braun-Ji-Manashov)

$$T^{\mu\nu} = i \int d^4 z \, e^{iq'z} \langle p' | \mathcal{T}\{ j^\nu(z) j^\mu(0)\} | p \rangle =$$

$$\frac{1}{i\pi^2} i \int d^4 z \, e^{iq'z} \Big\{ \frac{1}{(-z^2+i0)^2} \left[g^{\nu\mu} \mathscr{O}(1,0) - z^\nu \partial^\mu \int_0^1 du \; \mathscr{O}(\bar{u},0) - z^\mu (\partial^\nu - i\Delta^\nu) \int_0^1 dv \; \mathscr{O}(1,v) \right]$$

$$- \frac{1}{-z^2+i0} \left[\frac{i}{2} (\Delta^\mu \partial^\nu - (\nu \leftrightarrow \mu)) \int_0^1 du \int_0^{\bar{u}} dv \; \mathscr{O}(\bar{u},v) - \frac{t}{4} z^\nu \partial^\mu \int_0^1 du \; u \int_0^{\bar{u}} dv \; \mathscr{O}(\bar{u},v) \right]$$

$$+ \cdots$$

Operators $\mathscr{O}$ above are understood as matrix elements, that is:

$$\langle p' | \mathscr{O}(\lambda_1,\lambda_2) | p \rangle = \frac{2i}{\lambda_{12}} \iint_{\mathbb{D}} d\beta d\alpha \; \left[e^{-i\ell_{\lambda_1,\lambda_2} z} \right]_{\mathrm{LT}} \Phi^{(+)}(\beta,\alpha,t) \,,$$

where

$$\ell_{\lambda_1,\lambda_2} = -\lambda_1 \Delta - \lambda_{12} \left[\beta \bar{p} - \tfrac{1}{2}(\alpha+1)\Delta \right]$$

and $\Phi^{(+)}$ is given by the usual DDs h_f, g_f as

$$\Phi^{(+)}(\beta,\alpha,t) = \sum_f \left(\frac{e_f}{e} \right)^2 \Phi_f^{(+)}(\beta,\alpha,t) \,, \quad \Phi_f^{(+)}(\beta,\alpha,t) = \partial_\beta h_f + \partial_\alpha g_f$$

More details on: Braun, Ji & Manashov, JHEP 03 (2021) 051; JHEP 01 (2023) 078; and **V. Braun's talk.**

Scalar and pseudo-scalar target

- Spin-0 target $\Rightarrow$ vector component of $T^{\mu\nu}$ is enough.
- Parameterization of $T^{\mu\nu} \rightarrow$ **helicity amplitudes, $\mathcal{A}^{AB}$.**
- Spin-0 $\Rightarrow \mathcal{A}^{AB} \equiv \mathcal{H}^{AB}/2$ **(5 CFFs in total).**

$$T^{\mu\nu} = \mathcal{A}^{00} \frac{-i}{QQ'R^2} \left[(qq')(Q'^2 q^\mu q^\nu - Q^2 q'^\mu q'^\nu) + Q^2 Q'^2 q^\mu q'^\nu - (qq')^2 q'^\mu q^\nu\right]$$
$$+ \mathcal{A}^{+0} \frac{i\sqrt{2}}{R|\bar{p}_\perp|} \left[Q' q^\mu - \frac{qq'}{Q'} q'^\mu\right] \bar{p}_\perp^\nu - \mathcal{A}^{0+} \frac{\sqrt{2}}{R|\bar{p}_\perp|} \bar{p}_\perp^\mu \left[\frac{qq'}{Q} q^\nu + Q q'^\nu\right]$$
$$+ \mathcal{A}^{+-} \frac{1}{|\bar{p}_\perp|^2} \left[\bar{p}_\perp^\mu \bar{p}_\perp^\nu - \widetilde{\bar{p}}_\perp^\mu \widetilde{\bar{p}}_\perp^\nu\right] - \mathcal{A}^{++} g_\perp^{\mu\nu} \, ,$$

- Read out projectors $\rightarrow \mathcal{A}^{AB} = \boldsymbol{\Pi}_{\mu\nu}^{(AB)} T^{\mu\nu}$.

$\mathcal{A}^{++} = \mathrm{LT} + O(\mathrm{tw\text{-}4})$

$$
\begin{aligned}
\mathcal{A}^{++} = \frac{1}{2}\int_{-1}^{1} dx \Bigg\{ &- \left(1 - \frac{t}{2\mathbb{Q}^2} + \frac{t(\xi-\rho)}{\mathbb{Q}^2}\partial_\xi\right) \frac{H^{(+)}}{x-\rho+i0} \\
&+ \frac{t}{\xi\mathbb{Q}^2}\Bigg[\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\widetilde{\mathbb{P}}_{(\mathrm{i})} - \widetilde{\mathbb{P}}_{(\mathrm{iii})}}{2} - \frac{\xi(J+L)}{2} \\
&\qquad - \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\Bigg] H^{(+)} \\
&- \frac{t}{\mathbb{Q}^2}\partial_\xi\Bigg[\Bigg(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} - \frac{\xi}{x+\xi}\Bigg(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) \\
&\qquad - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\Bigg)\Bigg) H^{(+)}\Bigg] \\
&+ \frac{\bar{p}_\perp^2}{\mathbb{Q}^2} 2\xi^3 \partial_\xi^2 \Bigg[\Bigg(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\widetilde{\mathbb{P}}_{(\mathrm{i})} - \widetilde{\mathbb{P}}_{(\mathrm{iii})}}{2} - \frac{\xi L}{2} + \ln\left(\frac{x-\rho+i0}{x-\xi+i0}\right)\Bigg) H^{(+)}\Bigg]\Bigg\} \\
&+ O(\mathrm{tw\text{-}6})\,.
\end{aligned}
$$

$$\mathbb{P}_{(\mathrm{i})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right),$$

$$\widetilde{\mathbb{P}}_{(\mathrm{i})}(x/\xi,\rho/\xi) = -\frac{\xi-\rho}{x-\xi}\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right),$$

$$\mathbb{P}_{(\mathrm{ii})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x+\xi}\left[\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right)-(x\to-\xi)\right],$$

$$\widetilde{\mathbb{P}}_{(\mathrm{iii})}(x/\xi,\rho/\xi) = -\frac{\xi+\rho}{x+\xi}\ln\left(\frac{x-\rho+i0}{-\xi-\rho+i0}\right),$$

$$L = \int_0^1 dw\ \frac{-4}{x-\xi-w(x+\xi)}\int_0^1 du\ \ln\left(\xi-\rho+i0+\bar{u}[x-\xi-w(x+\xi)]\right)C_{\bar{u},\bar{u}w}\,,$$

$$C_{\bar{u},v} = \ln\left(\frac{\bar{u}-v}{1-v}\right)+\frac{1}{1-v}\,.$$

$$
\begin{aligned}
J = \frac{2(x+\rho)}{(x-\xi)(x+\xi)} \Bigg[& -\mathrm{Li}_2\left(-\frac{x+\rho}{-\xi-\rho+i0}\right) - \mathrm{Li}_2\left(\frac{2\xi}{x+\xi}\right) + \mathrm{Li}_2\left(\frac{x+\rho}{-\xi-\rho+i0}\frac{-2\xi}{x+\xi}\right) \\
& + \mathrm{Li}_2\left(\frac{x-\rho}{\xi-\rho+i0}\right) + \mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right) + \mathrm{Li}_2\left(\frac{x-\rho}{x+\xi}\right) \\
& - \mathrm{Li}_2\left(\frac{\xi-\rho+i0}{x+\xi}\right) - \mathrm{Li}_2\left(\frac{x+\rho}{x+\xi}\right) + \mathrm{Li}_2\left(-\frac{-\xi-\rho+i0}{x+\xi}\right) \\
& + \mathrm{Li}_2\left(-\frac{x-\xi}{-x-\rho+i0}\right) + \frac{1}{2}\ln^2\left(\frac{-\xi-\rho+i0}{-x-\rho+i0}\right) \Bigg] \\
+ \frac{\xi-\rho}{\xi(x+\xi)} \Bigg[& -\mathrm{Li}_2\left(\frac{2\xi}{\xi-\rho+i0}\right) - \mathrm{Li}_2\left(-\frac{2\xi}{-\xi-\rho+i0}\right) + 2\mathrm{Li}_2(1) \\
& - \mathrm{Li}_2\left(\frac{x-\xi}{x+\xi}\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \mathrm{Li}_2\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) + \mathrm{Li}_2\left(\frac{x-\xi}{x+\xi}\frac{\xi-\rho+i0}{-\xi-\rho+i0}\right) \\
& - \mathrm{Li}_2\left(\frac{\xi-\rho+i0}{-\xi-\rho+i0}\right) - \ln\left(\frac{2\xi}{x+\xi}\right)\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) \\
& - \ln\left(-\frac{2\xi}{x+\xi}\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right)\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) \\
& - \mathrm{Li}_2\left(\frac{2\xi}{x+\xi}\frac{x-\rho+i0}{\xi-\rho+i0}\right) + \mathrm{Li}_2\left(\frac{2\xi}{x+\xi}\right) \Bigg] \\
- \frac{\xi+\rho}{\xi(x-\xi)} \Bigg[& \mathrm{Li}_2\left(\frac{x-\xi}{x+\xi}\right) - \mathrm{Li}_2\left(\frac{x-\xi}{x+\xi}\frac{\xi-\rho+i0}{-\xi-\rho+i0}\right) \\
& + \ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right)\ln\left(\frac{2\xi}{x+\xi}\right) - \ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right)\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\frac{x-\xi}{x+\xi}\right) \Bigg].
\end{aligned}
$$

TCS limit

$$
\begin{aligned}
\mathcal{A}^{++}_{\rm TCS} &= \lim_{Q^2\to 0} \mathcal{A}^{++} \\
&= \frac{1}{2}\int_{-1}^{1} dx \, \Bigg\{ - \left(1 - \frac{5}{2}\frac{t}{\mathbb{Q}^2}\right) \frac{H^{(+)}}{x + \xi(1 - 2t/Q'^2) + i0} \\
&\quad + \frac{2t}{\mathbb{Q}^2}\Bigg[\frac{1}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right) + \frac{1}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right) - \mathrm{Li}_2\left(1\right)\right) \\
&\qquad - \frac{\xi}{4}(L_{\rm TCS} + J_{\rm TCS})\Bigg] H^{(+)} \\
&\quad - \frac{t}{\mathbb{Q}^2}\partial_\xi \Bigg[\Bigg(\frac{2\xi}{x+\xi+i0} + \frac{2\xi}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right) + \frac{2\xi}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right) - \mathrm{Li}_2\left(1\right)\right) \\
&\qquad - \frac{\xi L_{\rm TCS}}{2} - \frac{\xi}{x-\xi}\ln\left(\frac{x+\xi+i0}{2\xi}\right)\Bigg) H^{(+)}\Bigg] \\
&\quad + \frac{\bar{p}_\perp^2}{\mathbb{Q}^2} 2\xi^3 \partial_\xi^2 \Bigg[\Bigg(\frac{2\xi}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right) + \frac{2\xi}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi}\right) - \mathrm{Li}_2\left(1\right)\right) \\
&\qquad - \frac{\xi L_{\rm TCS}}{2} + \frac{\xi}{x-\xi}\ln\left(\frac{x+\xi+i0}{2\xi}\right) + \ln\left(\frac{x+\xi+i0}{x-\xi+i0}\right)\Bigg) H^{(+)}\Bigg]\Bigg\} \\
&\quad + O(\mathrm{tw\text{-}6}) \, .
\end{aligned}
$$

$\mathcal{A}^{+-}$

$\mathcal{A}^{+-}$ is of special interest as it enters the amplitude at:

- LO, kinematic twist-4,
- NLO, kinematic twist-2 through transversity GPDs.[$]
  For the spin-0 target: 1 (chiral-odd) transversity quark GPD
  + 1 gluon transversity GPD.

[$] Belitsky, Müller, PLB 486, 369–377, (2000).

$\mathcal{A}^{+-}$

For DDVCS:

$$\mathcal{A}^{+-} = \frac{\bar{p}_\perp^2}{\mathbb{Q}^2} 2(\xi^2 \partial_\xi)^2 \int_{-1}^{1} \frac{dx}{2\xi} \left(\widetilde{\mathbb{P}}_{(\mathrm{iii})} - \widetilde{\mathbb{P}}_{(\mathrm{i})} + 2 \ln\left(\frac{x - \rho + i0}{-2\xi} \right) \right) H^{(+)} + O(\text{tw-6}) \,.$$

and its TCS limit:

$$\mathcal{A}_{\mathrm{TCS}}^{+-} = \frac{\bar{p}_\perp^2}{\mathbb{Q}^2} 2(\xi^2 \partial_\xi)^2 \int_{-1}^{1} \frac{dx}{2\xi} \, \frac{2x}{x - \xi} \ln\left(\frac{x + \xi + i0}{2\xi} \right) H^{(+)} + O(\text{tw-6}) \,.$$

$\mathcal{A}^{+0,\,0+}$

For DDVCS:

$$\mathcal{A}^{+0} = \frac{-iQ'|\bar{p}_\perp|}{\sqrt{2}\mathbb{Q}^2} \int_{-1}^{1} dx\; 2\xi^2 \partial_\xi \left(\frac{1}{x-\xi} \ln\left(\frac{x-\rho+i0}{\xi-\rho+i0} \right) H^{(+)} \right) + O(\text{tw-5})\,,$$

$$\mathcal{A}^{0+} = \frac{-Q|\bar{p}_\perp|}{\sqrt{2}\mathbb{Q}^2} \int_{-1}^{1} dx\; 2\xi^2 \partial_\xi \left(\frac{H^{(+)}}{2\xi} \left[\frac{\xi+\rho}{x+\xi} \ln\left(\frac{x-\rho+i0}{-\xi-\rho+i0} \right) + \frac{\xi-\rho}{x+\xi} \ln\left(\frac{x(1-2t/\mathbb{Q}^2)-\rho+i0}{-\xi(1-2t/\mathbb{Q}^2)-\rho+i0} \right) \right] \right) + O(\text{tw-5})\,,$$

and their TCS limits:

$$\mathcal{A}^{+0}_{\text{TCS}} = \frac{-iQ'|\bar{p}_\perp|}{\sqrt{2}\mathbb{Q}^2} \int_{-1}^{1} dx\; 2\xi^2 \partial_\xi \left(\frac{1}{x-\xi} \ln\left(\frac{x+\xi+i0}{2\xi} \right) \right) + O(\text{tw-5})\,,$$

$\mathcal{A}^{0+}_{\text{TCS}} = 0 \;\rightarrow$ incoming photon is real in TCS.

$\mathcal{A}^{00}$: exclusive of DDVCS

$$\mathcal{A}^{00} = \mathcal{A}^{00}_{(0),\, z\partial} + \mathcal{A}^{00}_{(0),\, z(\partial - i\Delta)} + \mathcal{A}^{00}_{(0),\, \Delta\partial} + \mathcal{A}^{00}_{(0),\, tz\partial} + \mathcal{A}^{00}_{(1),\, z\Delta} + \mathcal{A}^{00}_{(2),\, z\Delta} + \mathcal{A}^{00}_{(1),\, z\partial} + \mathcal{A}^{00}_{(2),\, z\partial} \,.$$

For example:

$$\mathcal{A}^{00}_{(0),\, z\partial} = \frac{iQQ'}{\mathbb{Q}^2} \int_{-1}^{1} dx \int_0^1 du\, \frac{\xi - \rho}{2\xi}$$
$$\times \left\{ \frac{t}{\mathbb{Q}^2} \left[3N(\bar{u},0) - 4(\xi - \rho)(N(\bar{u},0))^2 + 2(\xi-\rho)^2 (N(\bar{u},0))^3 - 4\widetilde{N}(\bar{u},0) \right] H^{(+)} \right.$$
$$- \frac{t}{\mathbb{Q}^2} \partial_\xi \left(\xi \left[2N(\bar{u},0) - 4\widetilde{N}(\bar{u},0) \right] H^{(+)} \right)$$
$$\left. + \frac{\bar{p}_\perp^2}{\mathbb{Q}^2} 2\xi^3 \partial_\xi^2 \left(\xi \left[2N(\bar{u},0) - 4\widetilde{N}(\bar{u},0) \right] H^{(+)} \right) \right\} + O(\text{tw-6}) \,,$$

where

$$N(\lambda_1, \lambda_2) = \frac{1}{\lambda_1 (x - \xi) - \lambda_2 (x + \xi) + \xi - \rho + i0} \,,$$
$$\widetilde{N}(\lambda_1, \lambda_2) = \frac{1}{\lambda_1 (x - \xi) - \lambda_2 (x + \xi)} \ln \left(1 + \frac{\lambda_1 (x - \xi) - \lambda_2 (x + \xi)}{\xi - \rho + i0} \right) .$$

Results for the spin-0 target in DVCS were already published by Braun, Manashov & Pirnay in PRD 86, 014003 (2012); and with an alternative method by Braun, Ji & Manashov in JHEP 01 (2023) 078.

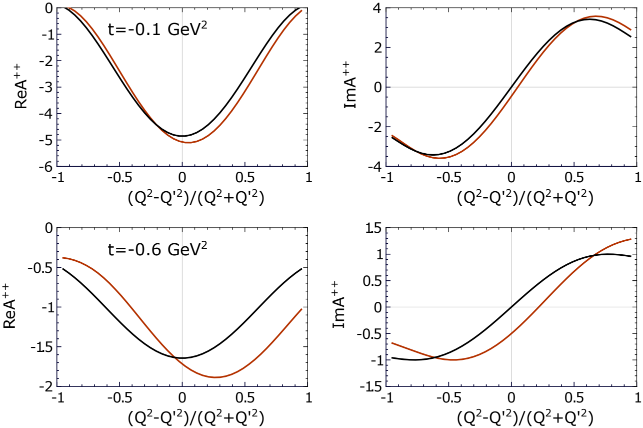
Those results can be read out as the DVCS limit of our DDVCS formulas.

For spin-1/2 in DVCS: Braun, Manashov, Müller, Pirnay, PRD 89 (2014) 7, 074022.

Phenomenology for pion target

PRELIMINARY RESULTS

π-GPD model: J. M. Morgado-Chávez et al., PRD 105, 094012 (2022)



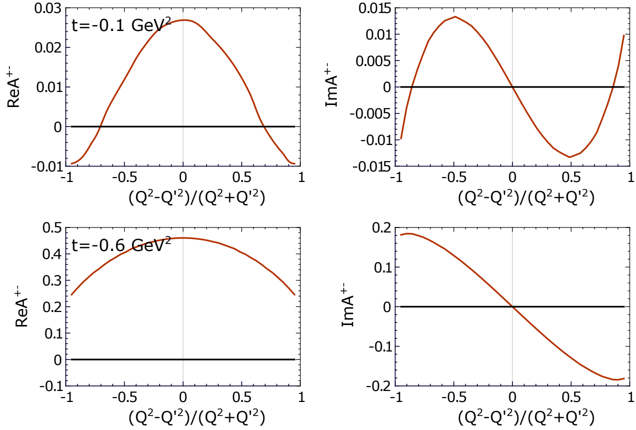
$\xi = 0.2$, $\mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$.

Black line: LT. Red line: LT + tw-4.

Phenomenology for pion target

PRELIMINARY RESULTS

π-GPD model: J. M. Morgado-Chávez et al., PRD 105, 094012 (2022)

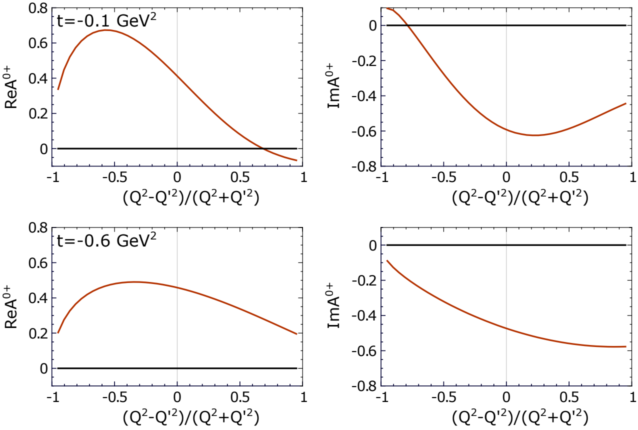


$\xi = 0.2$, $\mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$

Phenomenology for pion target

PRELIMINARY RESULTS

π-GPD model: J. M. Morgado-Chávez et al., PRD 105, 094012 (2022)

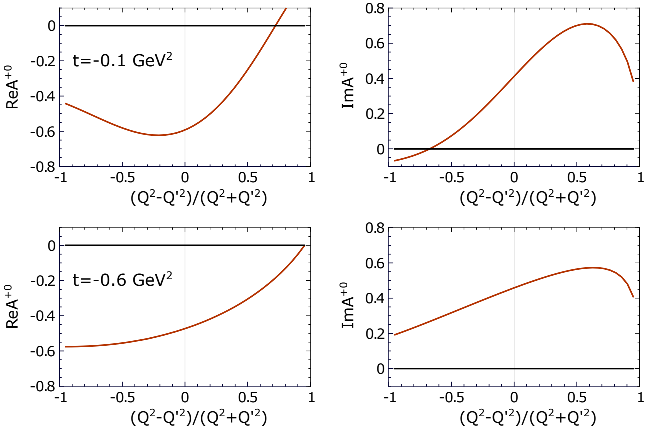


$$\xi = 0.2,\ \mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$$

Phenomenology for pion target

PRELIMINARY RESULTS

π-GPD model: J. M. Morgado-Chávez et al., PRD 105, 094012 (2022)

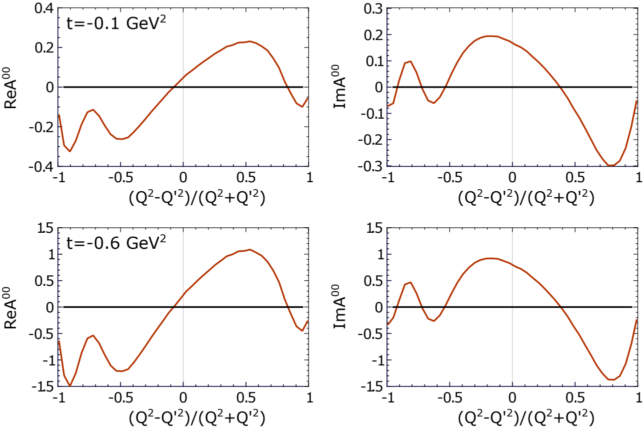


$$\xi = 0.2,\ \mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$$

Phenomenology for pion target

PRELIMINARY RESULTS

π-GPD model: $H(x, \xi) = A\,x(1 - x^2)(1 + B\xi^2)$, A & B fixed values for a given t.



$\xi = 0.2,\ \mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$

Summary and conclusions

- Our DDVCS formulation renders back DVCS & TCS in the appropriate small-virtuality limit.
- LO + LT DDVCS off the nucleon $\rightarrow$ fairly large asymmetries.
- LO + kinematic twist corrections in DDVCS off the (pseudo-)scalar target:

1. $\mathcal{A}^{++} \sim \text{LT} + \text{tw-4} + \cdots$
2. $\mathcal{A}^{+-,\,00} \sim \text{tw-4} + \cdots$
3. $\mathcal{A}^{0+,\,+0} \sim \text{tw-3} + \cdots$
4. All amplitudes have been computed. Numerics for $\mathcal{A}^{00}$ with a more realistic pion-GPD model are in progress.

Grazie!

Complementary slides

Kinematic vs geometric twist

- **Kinematic** twist $\to$ power expansion of observables $\sim \frac{|t|}{\mathbb{Q}^2}, \frac{M^2}{\mathbb{Q}^2} \Rightarrow$ **frame dependent.**

- **Geometric** twist $\to$ expansion of operators $\mathscr{O}(z)$ in components belonging to irreps of the Lorentz group $\Rightarrow$ **Lorentz invariant.**

- $\boldsymbol{\mathscr{O}(z)}$ **such that** $\boldsymbol{z^2 \neq 0 \Rightarrow}$ **relaxation of the Björken limit in** $\boldsymbol{\int d^4z\ e^{-irz} \langle p' | \mathscr{O}(z) | p \rangle}$ **(in observables).**

Energy scale of two-photon processes

- Compton tensor:

$(\ell = \ell(\Delta, \bar{p})\,,\, \Delta^2 = t\,,\, \bar{p} = (p + p')/2\,,\, p^2 = p'^{\,2} = M^2)$

$$T^{\mu\nu}_{s_2 s_1} = i \int d^4z\, e^{iq'z} \langle p', s_2 | \mathcal{T}\left\{ j^\nu(z) j^\mu(0) \right\} | p, s_1 \rangle$$

$$\sim i \int d^4z\, e^{iq'z} \frac{f^{\mu\nu}(z,\partial)}{(-z^2 + i0)^J} \underbrace{\left[e^{-i\ell z} \right]_{\mathrm{LT}}}_{\text{geom. LT}} \quad \longrightarrow \quad \text{geom. LT to connect to the usual GPDs}$$

$$\sim i \int d^4z \int_0^1 dw\, e^{i(q' - w\ell)z} \frac{\widetilde{f}^{\mu\nu}(z,w)}{(-z^2 + i0)^K}$$

$$\sim \sum_{n,m} f^{\mu\nu}_{n,m}(\Delta, \bar{p}, q') \times I_{n,m} \qquad \longrightarrow \qquad I_{n,m} = \int_0^1 dw\, \frac{w^n}{(\ell^2 w^2 - 2q'\ell w + Q'^2 + i0)^m}$$

$$\sim \underbrace{O(1)}_{\text{kin. LT}} + \left(\text{powers of } \frac{|t|}{-2q'\Delta}\,,\, \frac{M^2}{-2q'\Delta} \right)\,.$$

- Scale is **in general:** $-2q'\Delta = Q^2 + Q'^{\,2} + t \equiv \mathbb{Q}^2\,.$

More details on conformal operator-product expansion: Braun, Ji & Manashov, JHEP 03 (2021) 051; and JHEP 01 (2023) 078.

Why DDVCS? / State of art

Why DDVCS?

1. Access to $x \neq \pm\xi \leftarrow$ restriction in DVCS & TCS (at lowest order).
2. Single framework to describe DVCS, TCS & DDVCS:

$$\mathrm{DVCS:}\ Q'^2 \to 0,\ \rho \to \xi\,; \quad \mathrm{TCS:}\ Q^2 \to 0,\ \rho \to -\xi\ \mathrm{(at\ LT)}$$

State of art:

- Original papers in DDVCS **(LO LT)**:
  1. Belitsky & Müller, PRL 90, 022001 (2003).
  2. Guidal & Vanderhaeghen, PRL 90, 012001 (2003).
  3. Belitsky & Müller, PRD 68, 116005 (2003).
- Alternative derivation of amplitudes and cross-section, and JLab and EIC's phenomenology **(LO LT)** are discussed in: K. Deja, VMF, B. Pire, P. Sznajder & J. Wagner, PRD 107, 094035 (2023).
- **NLO** is known for DVCS, TCS & DDVCS.
- **New:** kinematic twist corrections for TCS & DDVCS.

Kinematic twist

- LT: Q^2, $Q'^2 \to \infty$, Björken limit.
- Kinematic power corrections = kinematic higher-twist corrections:

$$\sim \left(\frac{|t|}{Q^2}\right)^P, \quad \sim \left(\frac{M^2}{Q^2}\right)^P,$$

$$P = \frac{\tau_{\text{kin}} - 2}{2}.$$

- Similarly for Q'^2.
- LT = kinematic leading twist = kinematic twist-2, $\tau_{\text{kin}} = 2$.

Higher twists corrections on spin-0 target, why spin 0?

1. It is the simplest case (spin-1/2 and spin-1 targets to be studied later) + least number of CFFs $\Rightarrow$ least number of measurements.

2. Study of DVCS on scalar and pseudo-scalar nuclei, e.g. $^4\mathrm{He}$:

   S. Fucini, S. Scopetta, & M. Viviani, PRC 98, 015203 (2018).

3. Study of meson GPDs (π) through the Sullivan process: $\gamma^* p \to \gamma \pi^+ n$

   J. D. Sullivan, PRD 5, 1732 (1972).

   Pion GPDs from J. M. Morgado-Chavez et al., PRD 105, 094012 (2022).

   D. Amrath, M. Diehl, J.-P. Lansberg, EPJC 58, 179–192 (2008).

   J. M. Morgado Chávez, PRL 128, 202501 (2022).

4. Kinematical higher twist contributions recently studied in related process $\gamma^*\gamma \to \pi^+\pi^-$: access to pion GDAs:

   C. Lorcé, B. Pire, Q.-T. Song, PRD 106, 094030 (2022); B. Pire, Q.-T. Song, PRD 107, 114014 (2023) .

Conformal twist expansion

- **Breakthrough:** OPE based on conformal field theory calculation by Braun, Ji & Manashov, JHEP 2021, 51 (2021). **OPE to all (kinematic) twists!!**
- Use of conformal symmetry to constrain the coefficients of the expansion around the light-cone.
- Kinematic power expansion. No higher-twist GPDs.
- Natchman-like corrections are more involved as

$$\langle p'|\partial^{\mu}\mathcal{O}|p\rangle = i\Delta^{\mu}\langle p'|\mathcal{O}|p\rangle \, .$$

- This formalism satisfies QED gauge and translation invariance.