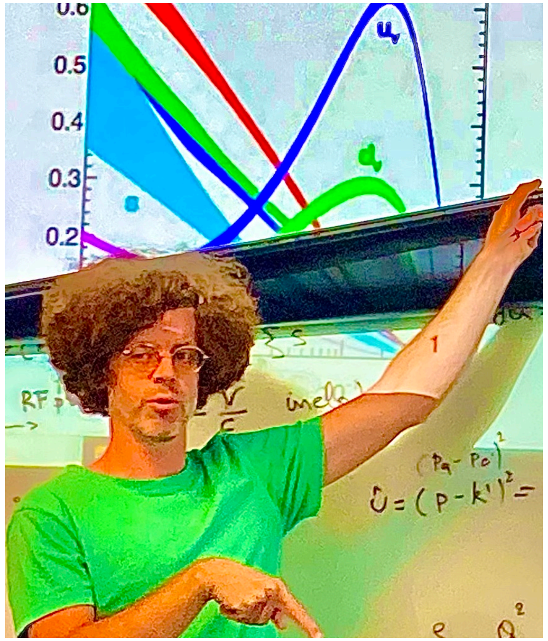
Polarized Deuteron GPD via coherent DVCS and BH



Wim Cosyn

Kazuki Makino

FLORIDA INTERNATIONAL UNIVERSITY

Work In progress

Before I begin…

(Hi! to all Jlab people)

γ

Due to very high luminosity photons…in Miami

FIU
FLORIDA INTERNATIONAL UNIVERSITY

Kazuki Makino

OLD DOMINION UNIVERSITY

Jefferson Lab

We have very simple Desire

We want to know…

Distribution of quarks & gluons inside a **polarized spin-1 hadron**

-> Structure of the cross section for **DVCS + BH** for a **polarized deuteron** to allow extraction of **GPDs** and do theory prediction of cross section

Outline.

1. We want to catalog independent structures in cross section.
   - -polarization/geometry
     - - spin-1 has tensor polarization

Spin 1/2 ->Spin 1 = more FFs & more GPDs & more SF = much more involved

2. Provide Expressions for SF using

Spin1 EM FF (BH)

Spin 1 CFF/ GPD (DVCS)

Outline

1. Cataloging independent structures in cross section
   - polarization/geometry
     - spin-1 has tensor polarization

2. Provide Expressions for SF using

Spin1 EM FF

Spin 1 CFF/ GPD

Expressions exist but,
Kirchner, Mueller, EPJC32 (2003)

- Not intuitive for spin 1
- & the BH expressions does not fully reflect geometry

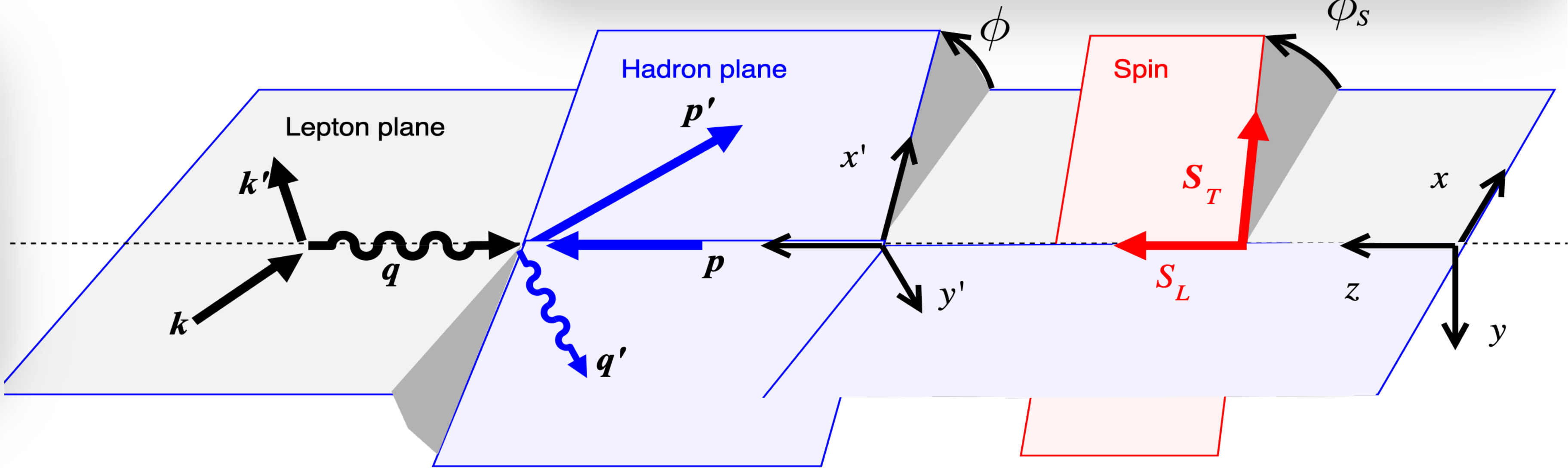
General approach
-to calculate DVCS structures

We start from…

$$\frac{L^{\mu\nu}W_{\mu\nu}}{Q^4}$$

Decompose hadronic tensor on a basis constructed with momenta + polarization

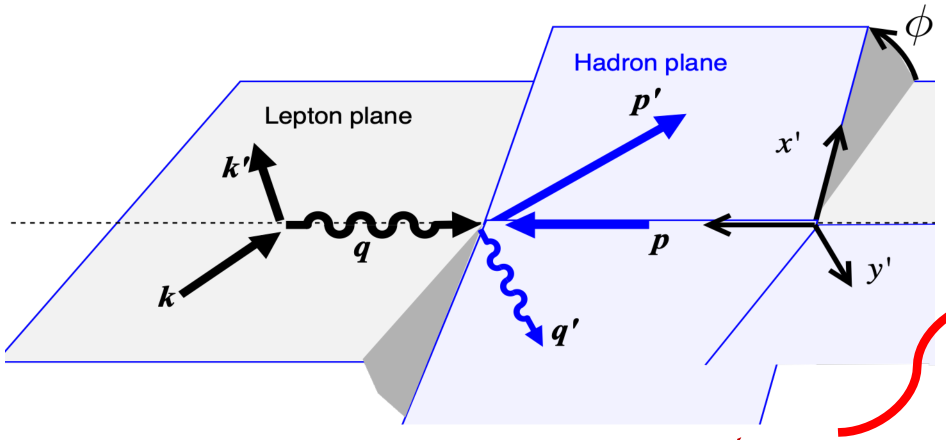
(massless lepton)
Only one non-zero basis tensor



We can construct full basis from three vectors, p, p’,q

Now, Construct DVCS hadron basis *explicitly!* EXPLICIT CONTENT

In particular, we construct collinear basis where incoming hadron and exchanged photon are collinear



DVCS

Transverse tensors

$$q^\mu, \quad L_q^\mu = p^\mu - (p\cdot q)\frac{q^\mu}{q^2} \Rightarrow e_q^\mu = \frac{q^\mu}{\sqrt{-q^2}} \quad e_{qL}^\mu = \frac{L_q^\mu}{\sqrt{L_q^2}}$$

$$p' g_{\perp \mathrm{DVCS}}^{\mu\nu} = p' \left(g^{\mu\nu} + e_q^\mu e_q^\nu - e_{qL}^\mu e_{qL}^\nu \right) \Rightarrow e_{qT}^\mu \quad \textit{normal}$$

$$\epsilon_{\perp \mathrm{DVCS}}^{\mu\nu} e_{qT,\nu} = \epsilon^{\mu\nu\rho\sigma} e_{qL,\rho} e_{q,\sigma} e_{qT,\nu} \Rightarrow e_{qN}^\mu \quad \textit{pseudo}$$

$$\mathcal{A}_h^{\mu\nu} = e_{qL}^\mu e_{qT}^\mu$$

etc

This is equivalent to introducing virtual photon basis

Polarization is described by Spin 1 density matrix

We consider deuteron polarization as ensemble and use spin-1 density matrix

The cross-section is weighted by polarization $$d\sigma = \sum_{\lambda,\lambda'} \rho(\lambda,\lambda') d\sigma(\lambda',\lambda)$$

Spin1 density matrix

$$\rho(\lambda,\lambda') = \begin{bmatrix} \frac{1}{3} + \frac{1}{2}S_L + \frac{1}{2}T_{LL} & \frac{1}{2\sqrt{2}}S_T e^{-i(\phi_h - \phi S)} + \frac{1}{\sqrt{2}}T_{LT}e^{-i(\phi_h - \phi_{T_L})} & \frac{1}{2}T_{TT}e^{-i(2\phi_h - 2\phi_{T_T})} \\ \frac{1}{2\sqrt{2}}S_T e^{i(\phi_h - \phi_S)} + \frac{1}{\sqrt{2}}T_{LT}e^{i(\phi_h - \phi_{T_L})} & \frac{1}{3} - T_{LL} & \frac{1}{2\sqrt{2}}S_T e^{-i(\phi_h - \phi_S)} - \frac{1}{\sqrt{2}}T_{LT}e^{-i(\phi_h - \phi_{T_L})} \\ \frac{1}{2}T_{TT}e^{i(2\phi_h - 2\phi_{T_T})} & \frac{1}{2\sqrt{2}}S_T e^{i(\phi_h - \phi_S)} & \frac{1}{3} - \frac{1}{2}S_L + \frac{1}{2}T_{LL} \end{bmatrix}.$$

Unpolarized, Vector & Tensor pol

-> Polarization sums in terms of Covariant density matrix

Weighted polarization sums

Initial deuteron is polarized final deuteron is unpolarized

We define

Covariant density matrix

$$\rho^{\alpha\beta} \equiv \sum_{\lambda,\lambda'} \rho\left(\lambda, \lambda'\right) \epsilon^{\alpha}(\lambda) \epsilon^{\beta *}\left(\lambda'\right)$$

Polarizations can be decomposed covariantly

$$\rho^{\alpha\beta} = \frac{1}{3}\left(-g^{\alpha\beta} + \frac{p^{\alpha}p^{\beta}}{M^2}\right) + \frac{i}{2M}\epsilon^{\alpha\beta\gamma\delta}p_{\gamma}S_{\delta} - T^{\alpha\beta}$$

$$\equiv \rho^{\alpha\beta}[\,\text{unpol}\,] + \rho^{\alpha\beta}[\,\text{vector}\,] + \rho^{\alpha\beta}[\,\text{tensor}\,].$$

Hadronic tensor is shown here weighted by density matrix

$$\langle W^{\mu\nu}\rangle \equiv \sum_{\alpha,\beta} \rho^{\alpha\beta} W^{\mu\nu}_{\alpha\beta}$$

Polarization and geometry

The polarizations and geometry determines the number of structure functions in DVCS

Spin1 density matrix

$$
\rho(\lambda,\lambda') = \begin{bmatrix} \frac{1}{3}+\frac{1}{2}S_L+\frac{1}{2}T_{LL} & \frac{1}{2\sqrt{2}}S_T e^{-i(\phi_h-\phi S)} + \frac{1}{\sqrt{2}}T_{LT}e^{-i(\phi_h-\phi_{T_L})} & \frac{1}{2}T_{TT}e^{-i(2\phi_h-2\phi_{T_T})} \\ \frac{1}{2\sqrt{2}}S_T e^{i(\phi_h-\phi_S)} + \frac{1}{\sqrt{2}}T_{LT}e^{i(\phi_h-\phi_{T_L})} & \frac{1}{3}-T_{LL} & \frac{1}{2\sqrt{2}}S_T e^{-i(\phi_h-\phi_S)} - \frac{1}{\sqrt{2}}T_{LT}e^{-i(\phi_h-\phi_{T_L})} \\ \frac{1}{2}T_{TT}e^{i(2\phi_h-2\phi_{T_T})} & \frac{1}{2\sqrt{2}}S_T e^{i(\phi_h-\phi_S)} & \frac{1}{3}-\frac{1}{2}S_L+\frac{1}{2}T_{LL} \end{bmatrix}.
$$

Hadron Basis

$$
W^{\mu\nu} = (\text{Angular modulation})\{S_L, S_T, T_{LL}\ldots\}\times \mathcal{A}_h^{\mu\nu} \rightarrow F
$$

41 Structure functions

Geometry of Lepton/Hadron plane -Two plane rotated by an angle

Contraction of Leptonic tensor (basis)/hadron basis is parametrized by Rosenbluth variable

Hadron Basis

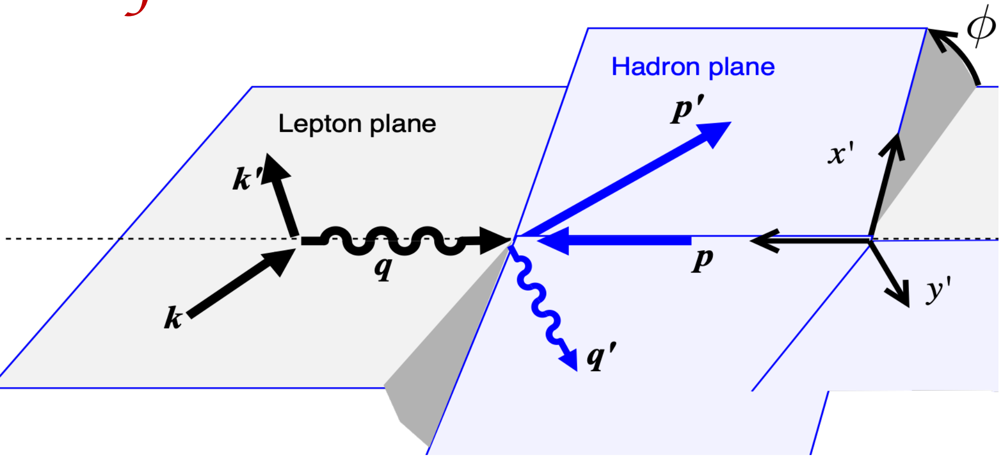
$$L^{\mu\nu} A_h^{\mu\nu}$$

Leptonic tensor

$$\epsilon \equiv \frac{e^{\mu}_{qL} e^{\nu}_{qL} L_{\mu\nu}[\text{ unpol }]}{g^{\mu\nu}_{\perp\text{DVCS}} \, L_{\mu\nu} \, [\text{unpol }]} = \frac{1 - y - \gamma^2 y^2/4}{1 - y + y^2/2 + \gamma^2 y^2/4}$$

"Inelasticity" y

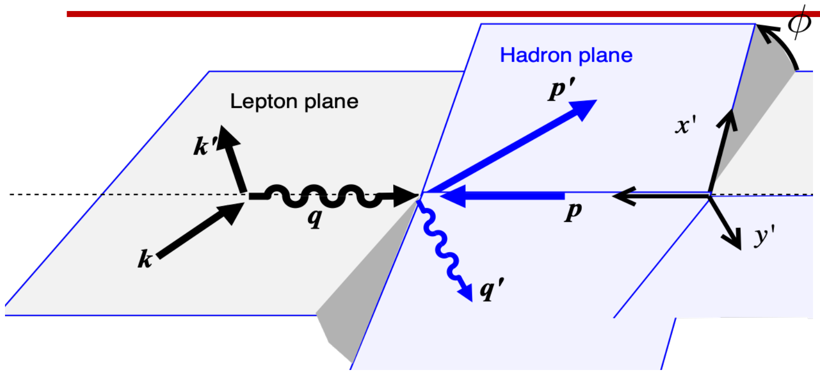
$$\gamma = \frac{2Mx}{Q}$$



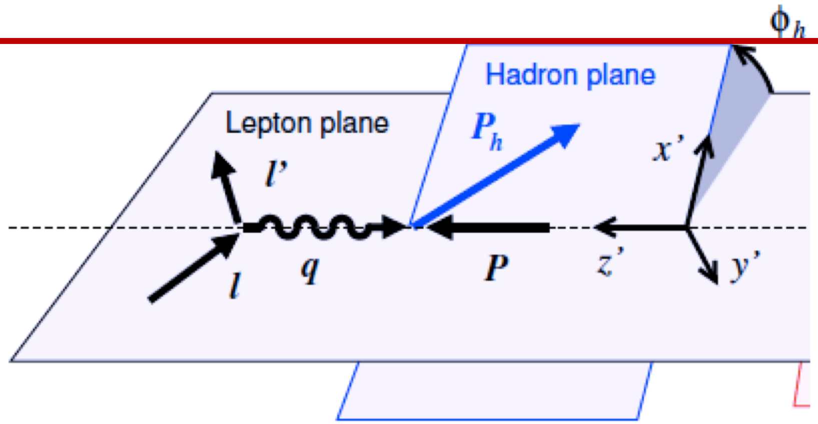
DVCS

Cross section expression for DVCS/SIDIS

DVCS geometry is same as SIDIS geometry



DVCS

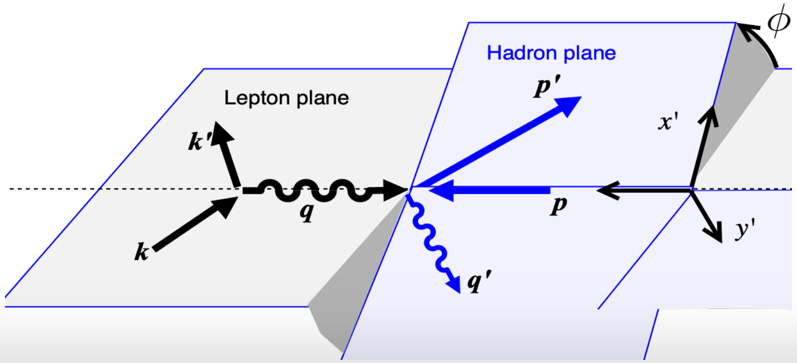


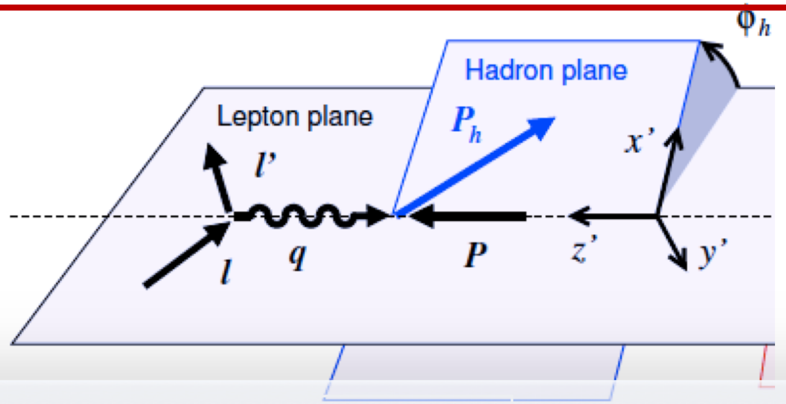
SIDIS

Cross section expression for DVCS/SIDIS

Bacchetta et al. JHEP02 (2007) Diehl, Sapeta, EPJC41 (2005) Liuti, Kriesten et al. PRD101

DVCS geometry is same as SIDIS geometry
-> Cross sections decompose in the same way





Unpolarized/vector polarized have same structures as spin ½ case - with more CFFs/GPDs

$$\begin{aligned}\mathcal{F}_U &= F_{UU,T} + \epsilon F_{UU,L} \\ &+ \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} \\ &+ \epsilon\cos 2\phi_h F_{UU}^{\cos 2\phi_h} \\ &+ (2\lambda_e)\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}\end{aligned}$$

$$\begin{aligned}\mathcal{F}_S &= S_L\left[\sqrt{2\epsilon(1+\epsilon)}\sin\phi_h F_{US_L}^{\sin\phi_h} + \epsilon\sin 2\phi_h F_{US_L}^{\sin 2\phi_h}\right] \\ &+ S_L\,(2\lambda_e)\left[\sqrt{1-\epsilon^2}F_{LS_L} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_h F_{LS_L}^{\cos\phi_h}\right] \\ &+ S_T\left[\sin(\phi_h-\phi_S)\left(F_{US_T,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{US_T,L}^{\sin(\phi_h-\phi_S)}\right)\right. \\ &\quad + \epsilon\sin(\phi_h+\phi_S)\,F_{US_T}^{\sin(\phi_h+\phi_S)}\end{aligned}$$

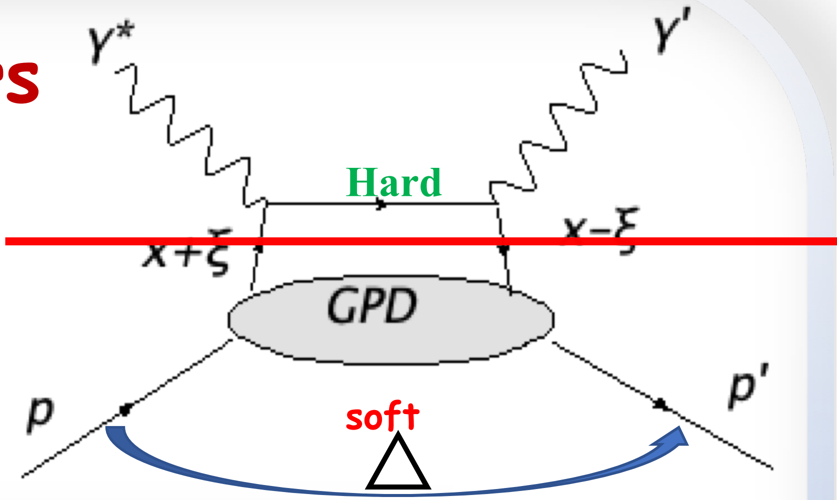
Cosyn, Weiss PRC (2020)

Tensor polarized 23 SF

In terms of $\{T_{LL}, T_{TL}, T_{LL}\}$

$$\begin{aligned}\mathcal{F}_T &= T_{LL}\left[F_{UT_{LL},T} + \epsilon F_{UT_{LL},L}\right. \\ &+ \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h F_{UT_{LL}}^{\cos\phi_h} \\ &\left.+\epsilon\cos 2\phi_h F_{UT_{LL}}^{\cos 2\phi_h}\right] \\ &+ T_{LL}\,(2\lambda_e)\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h F_{LT_{LL}}^{\sin\phi_h}\end{aligned}$$

Quick Overview of how SPIN-1GPD enters in DVCS



Five Spin-1 vector and four axial GPDs enter in DVCS

$$\mathcal{M}_{\text{DVCS}} = \int dx \mathcal{M}_{\text{hard}}(x) \otimes \mathcal{M}_{\text{soft}}(x)$$

$$\frac{1}{x-\xi+i\varepsilon} + \frac{1}{x+\xi-i\varepsilon}$$

$$-(\epsilon'^{*}\cdot\epsilon)\mathcal{H}_1 + \ldots + (\ldots)\mathcal{H}_5$$

$$\int \frac{dz^+}{2\pi} \left\langle p' \left| e^{iz^+\cdot k^-} \bar{q}(0)\gamma^- q(z) \right| p \right\rangle \Big|_{z^-=0, z_\perp=0}$$

$$-(\epsilon'^{*}\cdot\epsilon)H_1 + \ldots + (\ldots)H_5$$

Compton Form Factors (CFFs) ⟷ GPDs

SPIN-1 GPD parametrizations

Berger, et al, PRL 2001

Spin 1 matrix elements are parametrized by spin1 GPDs as follows

Vector GPDs

$$V_{\lambda'\lambda} = -(\epsilon'^* \cdot \epsilon)\, H_1 + \frac{(\epsilon \cdot n)\,(\epsilon'^*.P) + (\epsilon'^* \cdot n)\,(\epsilon.P)}{P.n} H_2 - \frac{(\epsilon.P)\,(\epsilon'^*.P)}{2M^2} H_3$$
$$+ \frac{(\epsilon \cdot n)\,(\epsilon'^* \cdot P) - (\epsilon'^*.n)\,(\epsilon \cdot P)}{P.n} H_4 + \left\{ 4M^2 \frac{(\epsilon \cdot n)\,(\epsilon'^* \cdot n)}{(P.n)^2} + \frac{1}{3}\,(\epsilon'^* \cdot \epsilon) \right\} H_5,$$

Axial GPDs

$$A_{\lambda'\lambda} = -i\frac{\epsilon_{\mu\alpha\beta\gamma} n^\mu \epsilon'^{*\alpha} \epsilon^\beta P^\gamma}{P.n} \tilde{H}_1 + i\frac{\epsilon_{\mu\alpha\beta\gamma} n^\mu \Delta^\alpha P^\beta}{P.n} \frac{\epsilon^\gamma\,(\epsilon'^*.P) + \epsilon'^{*\gamma}(\epsilon.P)}{M^2} \tilde{H}_2$$
$$+ i\frac{\epsilon_{\mu\alpha\beta\gamma} n^\mu \Delta^\alpha P^\beta}{P.n} \frac{\epsilon^\gamma\,(\epsilon'^*.P) - \epsilon'^{*\gamma}(\epsilon.P)}{M^2} \tilde{H}_3 + i\frac{\epsilon_{\mu\alpha\beta\gamma} n^\mu \Delta^\alpha P^\beta}{P.n} \frac{\epsilon^\gamma\,(\epsilon'^*.n) + \epsilon'^{*\gamma}(\epsilon.n)}{P.n} \tilde{H}_4$$

GPD Structure in DVCS

Expression of DVCS Hadronic tensor are in terms of the matrix elements (parametrized by CFFs) & geometric structures

Axial *Vector*

$$T^{\nu\rho} = V^*_{\rm CFF} V_{\rm CFF}\, g_\perp^{\nu\rho} - i A^*_{\rm CFF} V_{\rm CFF}\, \epsilon^{\nu\rho+-} + i V^*_{\rm CFF} A_{\rm CFF}\, \epsilon^{\rho\nu+-} + A^*_{\rm CFF} A_{\rm CFF}\, \epsilon^{\alpha\nu+-}\epsilon^{\alpha\rho+-}$$

GPD Structure in DVCS

Matrix elements (parametrized by CFFs) and geometric structures are completely separated in DVCS hadronic tensor

Axial *Vector*

Leading order/leading twist

$$T^{\nu\rho} = V^*_{\mathrm{CFF}} V_{\mathrm{CFF}}\, g_\perp^{\nu\rho} - i A^*_{\mathrm{CFF}} V_{\mathrm{CFF}}\, \epsilon^{\nu\rho+-} + i V^*_{\mathrm{CFF}} A_{\mathrm{CFF}}\, \epsilon^{\rho\nu+-} + A^*_{\mathrm{CFF}} A_{\mathrm{CFF}}\, \epsilon^{\alpha\nu+-}\epsilon^{\alpha\rho+-}$$

$$\rho^{\alpha\beta}(S,T)$$

Polarizations appears in the spin -1 polarization sum of CFF parametrization

GPD Structure in DVCS

Axial Vector

$$T^{\nu\rho} = V^*_{\mathrm{CFF}} V_{\mathrm{CFF}} g_\perp^{\nu\rho} - i A^*_{\mathrm{CFF}} V_{\mathrm{CFF}} \epsilon^{\nu\rho+-} + i V^*_{\mathrm{CFF}} A_{\mathrm{CFF}} \epsilon^{\rho\nu+-} + A^*_{\mathrm{CFF}} A_{\mathrm{CFF}} \epsilon^{\alpha\nu+-} \epsilon^{\alpha\rho+-}$$

Only transverse Structure Functions are nonzero

$$\mathcal{A}_T^{\mu\nu} = \frac{1}{2}\left(e_T^\mu e_T^\nu + e_N^\mu e_N^\nu\right) \qquad \mathcal{A}_{TT''}^{\mu\nu} = -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma} e_{q,\rho} e_{L,\sigma}$$

Non-zero structure functions for spin 1 DVCS

$$F_{UU,T} \quad S_T \sin(\phi_h - \phi_S) F_{US_T,T}^{\sin(\phi_h-\phi_S)}$$

$$T_{LT} \cos(\phi_h - \phi_{T_L}) F_{UT_{LT},T}^{\cos(\phi_h-\phi_{T_L})}$$

$$T_{LL} F_{UT_{LL},T} \quad T_{TT} \cos(2\phi_h - 2\phi_{T_T}) F_{UT_{TT},T}^{\cos(2\phi_h-2\phi_{T_T})}$$

Single polarization (unpolarized electron)

$$S_T \cos(\phi_h - \phi_S) F_{LS_T}^{\cos(\phi_h-\phi_S)} \quad T_{LT} \sin(\phi_h - \phi_{T_L}) F_{LT_{LT}}^{\sin(\phi_h-\phi_{T_L})}$$

$$S_L F_{LS_L} \quad T_{TT} \sin(2\phi_h - 2\phi_{T_T}) F_{LT_{TT}}^{\sin(2\phi_h-2\phi_{T_T})}$$

double polarization (polarized electron)

GPD Structures

The square of matrix elements are calculated for unpolarized/vector/tensor polarized cases.

$$T^{\nu\rho} = V^*_{\rm CFF} V_{\rm CFF} g_\perp^{\nu\rho} - i A^*_{\rm CFF} V_{\rm CFF} \epsilon^{\nu\rho+-} + i V^*_{\rm CFF} A_{\rm CFF} \epsilon^{\rho\nu+-} + A^*_{\rm CFF} A_{\rm CFF} \epsilon^{\alpha\nu+-} \epsilon^{\alpha\rho+-}$$

Example, result for axial CFFs matrix elements for vector polarized case

$$\tau = -\frac{t}{4M^2}$$

Preliminary

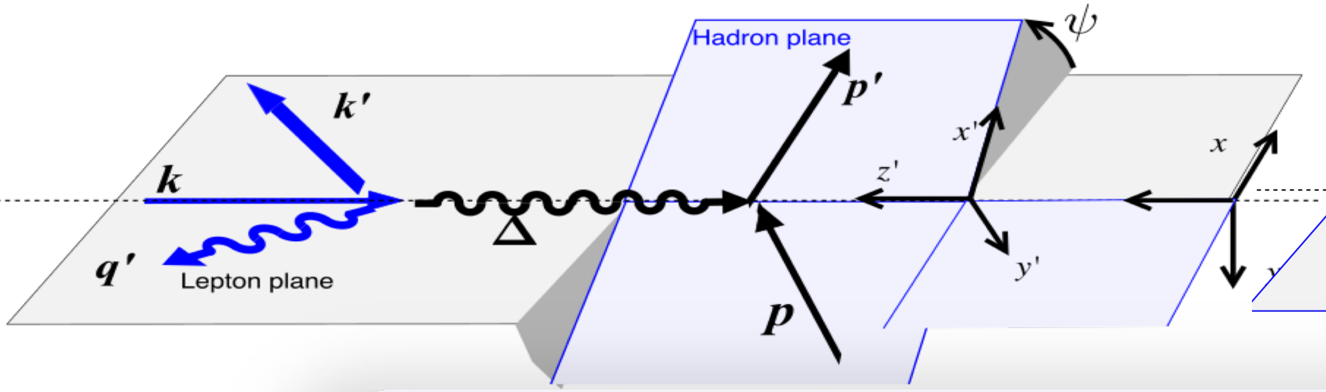
$$\sim (1+\tau)\left(\mathrm{Re}\left\{\tilde{\mathcal{H}}_1 \tilde{\mathcal{H}}_2^*\right\}\right) + \left(1 - \frac{1}{\xi}\right) \mathrm{Re}\left\{\tilde{\mathcal{H}}_1 \tilde{\mathcal{H}}_4^*\right\} + \left(\xi + \frac{\tau}{\xi} - \xi\tau\right)\left(\mathrm{Re}\left\{\tilde{\mathcal{H}}_3 \tilde{\mathcal{H}}_4^*\right\}\right)$$

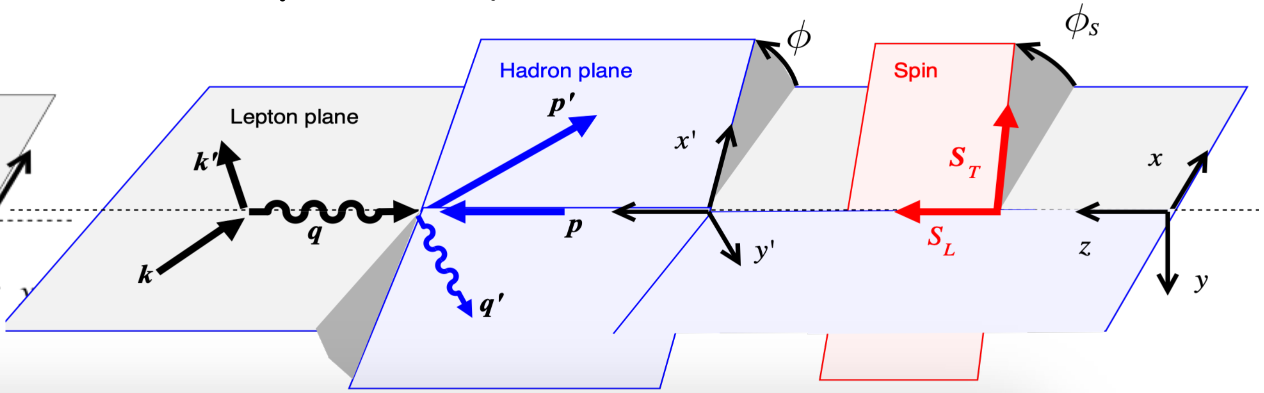
Analogous Kinematics of BH and DVCS

BH ⟷ **DVCS**

k and Δ are collinear (BH)

p and q are collinear (DVCS)





BH and DVCS geometries are analogous

->Express DVCS and BH in analogous kinematic invariants

Analogous kinematics variables ~ illustrated

relation between ψ ↔ φ in terms of invariants

$$t, y, x, Q^2, \phi \leftrightarrow Q^2, y_h, x_e, t, \psi$$

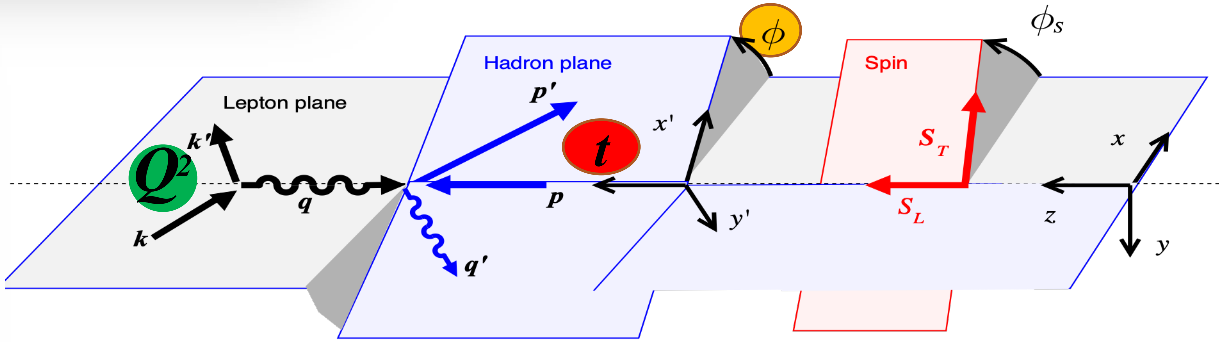
We define analogous BH kinematics variables

$y \quad \frac{p \cdot q}{p \cdot k}$

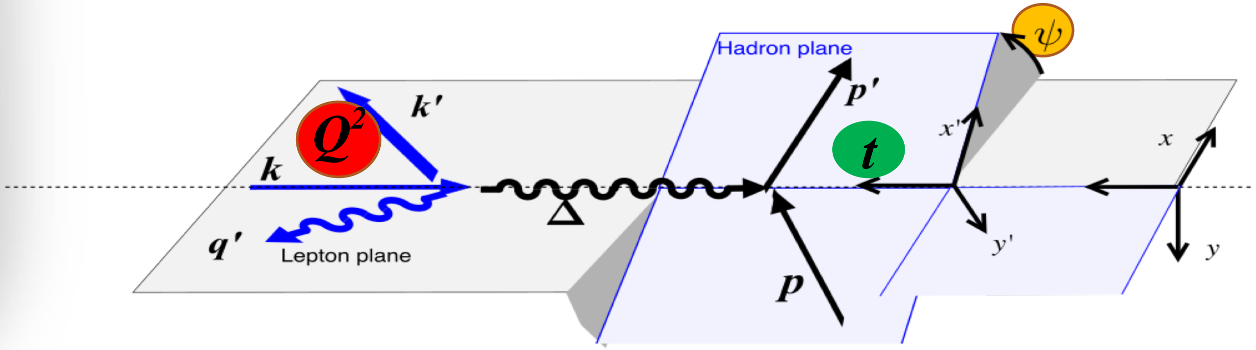
$W^2 = M^2 + Q^2\left(\frac{1}{x} - 1\right) \quad x \quad \frac{Q^2}{2(pq)}$

$y_h \quad \frac{k \cdot \Delta}{k \cdot p'}$

$W_e^2 = m_e^2 - t\left(\frac{1}{x_e} - 1\right) \quad x_e \quad \frac{t}{2(k\Delta)}$



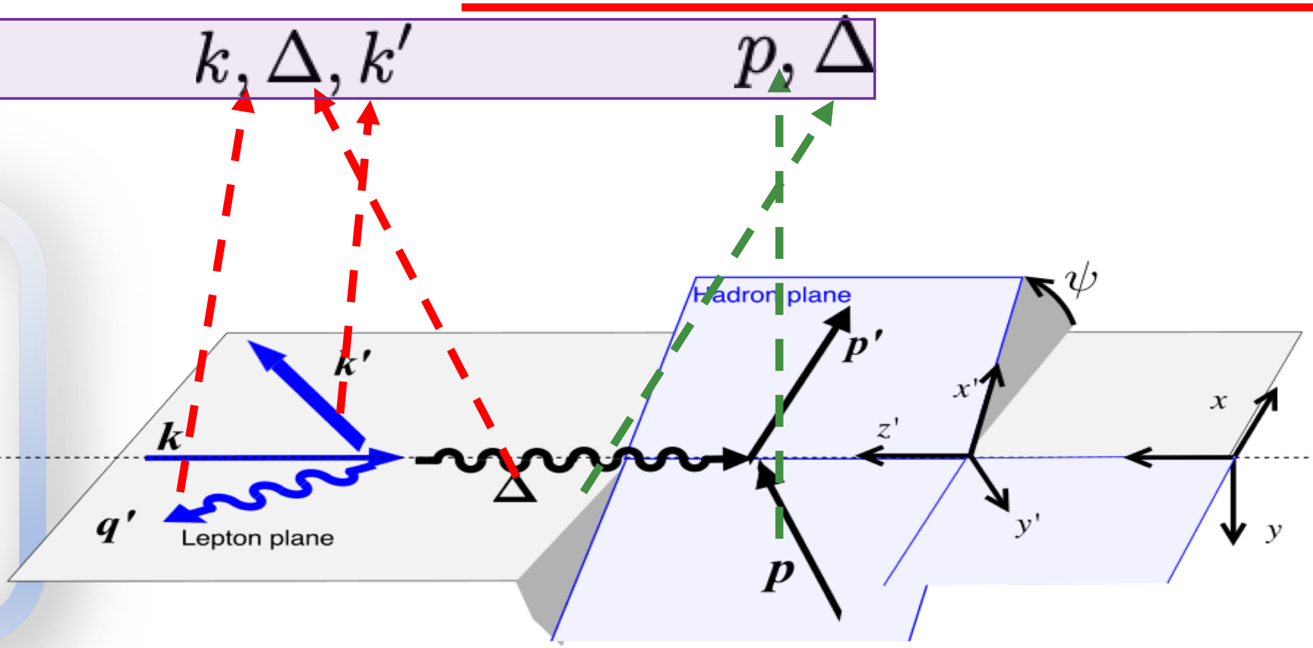
p and q are collinear (DVCS)



k and △ are collinear (BH)

Basis Construction – BH -analogous to DVCS

Decompose both **Lepton and hadron** on a basis constructed with momenta + polarization



Analogous construction as DVCS,
but different basis and polarization parametrization

BH Leptonic tensor decomposed in BH lepton basis

$$L^{\mu\nu}_{\rm BH} = -\frac{8Q^2x_e^2}{t}\left(\mathbf{e}^{\mu}_{\Delta L}\mathbf{e}^{\nu}_{\Delta L}\right) - \frac{4Qx_e\left(x_e\left(t-Q^2\right)+2Q^2x_e^2-t\right)}{t\sqrt{\left(x_e-1\right)\left(Q^2x_e+t\right)}}\left(\mathbf{g}^{\mu\nu}_{\perp\rm BH}\right)$$

G_{UL} G_{UT}

$$-\frac{4Q^2x_e^2}{t}\left(\mathbf{e}^{\mu}_{\Delta T}\mathbf{e}^{\nu}_{\Delta T}-\mathbf{e}^{\mu}_{\Delta N}\mathbf{e}^{\nu}_{\Delta N}\right)$$

G_{ULT}

Leptonic structures (unpolarized)

$$-\frac{2\left(x_e\left(x_e\left(2Q^2x_e\left(Q^2x_e-Q^2+t\right)+Q^4-4Q^2t+t^2\right)+2t\left(Q^2-t\right)\right)+2t^2\right)}{t\left(x_e-1\right)\left(Q^2x_e+t\right)}\left(\mathbf{e}^{\mu}_{\Delta L}\mathbf{e}^{\nu}_{\Delta T}-\mathbf{e}^{\mu}_{\Delta T}\mathbf{e}^{\nu}_{\Delta L}\right)$$

G_{UTT}

$$+\frac{8\lambda Qx_e\left(x_e\left(Q^2-t\right)+t\right)}{t\sqrt{\left(x_e-1\right)\left(Q^2x_e+t\right)}}\left(-i\epsilon^{\mu\nu\rho\sigma}\mathbf{e}_{\Delta,\rho}\mathbf{e}_{\Delta N,\sigma}\right)$$

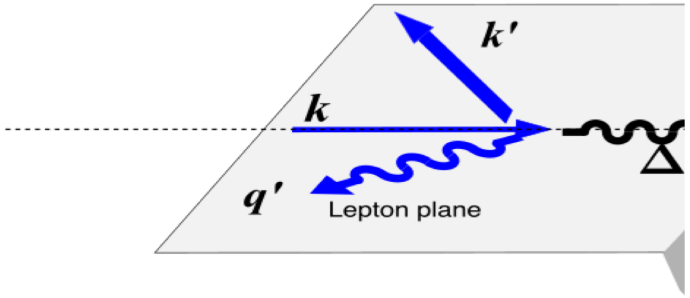
$G_{LT'''}$

Leptonic structures (polarized)

$$+\frac{4\lambda x_e\left(x_e\left(Q^4-4Q^2t+t^2\right)-2x_e^2\left(Q^4-Q^2t\right)+2t\left(Q^2-t\right)\right)}{t\left(x_e-1\right)\left(Q^2x_e+t\right)}\left(-i\epsilon^{\mu\nu\rho\sigma}\mathbf{e}_{\Delta L,\rho}\mathbf{e}_{\Delta,\sigma}\right)$$

$G_{TT''}$

Helicity $\lambda = \pm 1$ (Massless lepton)

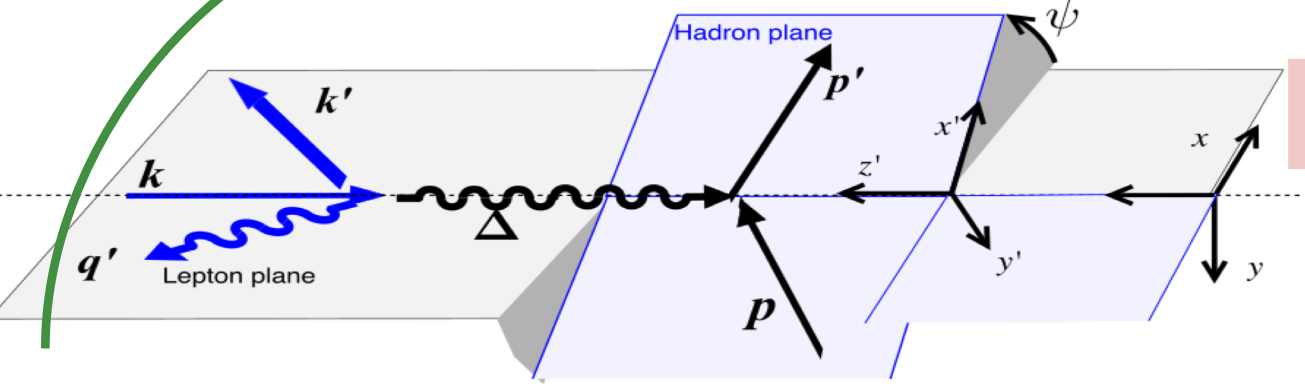


Geometry of Lepton/Hadron plane

Contraction of Lepton/hadron basis is parametrized by BH Rosenbluth variable

Analogous to elastic leptonic tensor (DVCS)
Transverse basis is used to define ϵ_{BH}

$$\mathcal{A}_l^{\prime\mu\nu}\,\mathcal{A}^{h\prime}_{\mu\nu}$$



$$\epsilon_{\rm BH} \equiv \frac{e^{\mu}_{\Delta L} e^{\nu}_{\Delta L} L^{T,\rm BH}_{\mu\nu}[\text{ unpol }]}{-g^{\mu\nu}_{BH} L^{T,\rm BH}_{\mu\nu}[\text{ unpol }]} = \frac{1 - y_h - \frac{y_h^2}{4\tau}}{1 - y_h + (1 + \frac{1}{2\tau})\frac{y_h^2}{2}}$$

For BH kinematics, BH leptonic tensor also can be decomposed in the lepton basis

BH Structure functions, eg

Example of expressions that would appear in cross section

$$d\sigma \sim \sum_{ij} f(\varepsilon) G_i F_j$$

$$\left(\mathcal{T}_{LL}\frac{1+\varepsilon_{\rm BH}}{2(1-\varepsilon_{\rm BH})}\right)\left(-\frac{8Q^2x_e^2}{t}\right)\left(-\frac{32}{3}M^2\tau(\tau+1)G_Q\left(3G_C+\tau G_Q\right)\right)$$

G_{UL} $\quad F_{UT_{LL,L}}$

$$\left(-\sqrt{\frac{1+\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}S_T\cos(\psi)\cos(\omega_S)-S_T\sin(\psi)\sin(\omega_S)\right)\left(\frac{8\lambda Q x_e\left(x_e\left(Q^2-t\right)+t\right)}{t\sqrt{(x_e-1)\left(Q^2x_e+t\right)}}\right)\left(-\frac{8}{3}M^2\sqrt{\tau}(\tau+1)G_M\left(3G_C+\tau G_Q\right)\right)$$

$G_{LT'''}$ $\quad F_{S_T}$

Azimuthal dependence is separated out from structure functions.

BH Structure functions and geometric factors

$$d\sigma \sim \sum_{ij} f(\varepsilon) G_i F_j$$

Reflecting on BH geometry leads to intuitive structure functions

sym & sym, unpolarized

	$Q^{\{\mu\nu\}}_{\Delta L}$	$Q^{\{\mu\nu\}}_{\Delta T}$	$Q^{\{\mu\nu\}}_{\Delta LT}$	$Q^{\{\mu\nu\}}_{\Delta TT}$
$P^{\{\mu\nu\}}_{\Delta L}$	$\frac{1+\epsilon_{\rm BH}}{2(1-\epsilon_{\rm BH})}$	$\frac{\epsilon_{\rm BH}}{(1-\epsilon_{\rm BH})}$	$\frac{\sqrt{2\epsilon_{\rm BH}(1+\epsilon_{\rm BH})}}{(1-\epsilon_{\rm BH})}\cos(\psi)$	$\frac{\epsilon_{\rm BH}}{(1-\epsilon_{\rm BH})}\cos(2\psi)$
$P^{\{\mu\nu\}}_{\Delta T}$	$\frac{\epsilon_{\rm BH}}{(1-\epsilon_{\rm BH})}$	$\frac{1}{(1-\epsilon_{\rm BH})}$	$\frac{\sqrt{\epsilon_{\rm BH}(1+\epsilon_{\rm BH})}}{2(1-\epsilon_{\rm BH})}\cos(\psi)$	$\frac{\epsilon_{\rm BH}}{(1-\epsilon_{\rm BH})}\cos(2\psi)$

anti-sym & anti-sym, vector polarized

	$Q^{[\mu\nu]}_{\Delta LT'''}$	$Q^{[\mu\nu]}_{\Delta TT''}$
$P^{[\mu}_{\Delta T}S^{\nu]}_{L}$	$\sqrt{\frac{2\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}\cos(\psi)$	$-\sqrt{\frac{1+\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}$
$P^{[\mu}_{\Delta T}S^{\nu]}_{T}$	$-\sqrt{\frac{1+\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}S_T\cos\psi\cos\omega_S - S_T\sin\psi\sin\omega_S$	$\sqrt{\frac{2\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}S_T\cos[\omega_S]$

$$F_{UL} = \frac{8}{9}M^2(\tau+1)\left(9G_C^2 + 8\tau^2 G_Q^2\right)$$
$$F_{UT} = \frac{16}{3}M^2\tau(\tau+1)G_M^2$$
$$F_{S_L} = 4M^2\tau(\tau+1)G_M^2$$
$$F_{S_T} = -\frac{8}{3}M^2\sqrt{\tau}(\tau+1)G_M\left(3G_C + \tau G_Q\right)$$
$$F_{T_{LL,L}} = -\frac{32}{3}M^2\tau(\tau+1)G_Q\left(3G_C + \tau G_Q\right)$$
$$F_{T_{LL,T}} = -4M^2\tau(\tau+1)G_M^2$$
$$F_{T_{LT}} = -16M^2\tau^{3/2}(\tau+1)G_M G_Q$$
$$F_{T_{TT}} = -4M^2\tau(\tau+1)G_M^2$$

Lastly, number of structures is… 28

$$d\sigma \sim \sum_{ij} f(\varepsilon) G_i F_j$$

sym & sym basis $4\times2=8$

	$Q^{\{\mu\nu\}}_{\Delta L}$	$Q^{\{\mu\nu\}}_{\Delta T}$	$Q^{\{\mu\nu\}}_{\Delta LT}$	$Q^{\{\mu\nu\}}_{\Delta TT}$
$P^{\{\mu\nu\}}_{\Delta L}$	$\frac{1+\epsilon_{\rm BH}}{2(1-\epsilon_{\rm BH})}$	$\frac{\epsilon_{\rm BH}}{(1-\epsilon_{\rm BH})}$	$\frac{\sqrt{2\epsilon_{\rm BH}(1+\epsilon_{\rm BH})}}{(1-\epsilon_{\rm BH})}\cos(\psi)$	$\frac{\epsilon_{\rm BH}}{(1-\epsilon_{\rm BH})}\cos(2\psi)$
$P^{\{\mu\nu\}}_{\Delta T}$	$\frac{\epsilon_{\rm BH}}{(1-\epsilon_{\rm BH})}$	$\frac{1}{(1-\epsilon_{\rm BH})}$	$\frac{\sqrt{\epsilon_{\rm BH}(1+\epsilon_{\rm BH})}}{2(1-\epsilon_{\rm BH})}\cos(\psi)$	$\frac{\epsilon_{\rm BH}}{(1-\epsilon_{\rm BH})}\cos(2\psi)$

Anti-sym & anti-sym basis $2\times2=4$

	$Q^{[\mu\nu]}_{\Delta LT'''}$	$Q^{[\mu\nu]}_{\Delta TT''}$
$P^{[\mu}_{\Delta T}S^{\nu]}_{L}$	$\sqrt{\frac{2\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}\cos(\psi)$	$-\sqrt{\frac{1+\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}$
$P^{[\mu}_{\Delta T}S^{\nu]}_{T}$	$-\sqrt{\frac{1+\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}S_T\cos\psi\cos\omega_S - S_T\sin\psi\sin\omega_S$	$\sqrt{\frac{2\epsilon_{\rm BH}}{1-\epsilon_{\rm BH}}}S_T\cos[\omega_S]$

sym & sym (tensor basis)
$4\times4=16$

$$F_{UL} = \frac{8}{9}M^2(\tau+1)\left(9G_C^2+8\tau^2G_Q^2\right)$$
$$F_{UT} = \frac{16}{3}M^2\tau(\tau+1)G_M^2$$
$$F_{S_L} = 4M^2\tau(\tau+1)G_M^2$$
$$F_{S_T} = -\frac{8}{3}M^2\sqrt{\tau}(\tau+1)G_M\left(3G_C+\tau G_Q\right)$$
$$F_{T_{LL,L}} = -\frac{32}{3}M^2\tau(\tau+1)G_Q\left(3G_C+\tau G_Q\right)$$
$$F_{T_{LL,T}} = -4M^2\tau(\tau+1)G_M^2$$
$$F_{T_{LT}} = -16M^2\tau^{3/2}(\tau+1)G_MG_Q$$
$$F_{T_{TT}} = -4M^2\tau(\tau+1)G_M^2$$

Summary

We now have...

- Intuitive spin 1 structure function for DVCS and BH that reflects geometry, by suitable basis decomposition

Outlook

BH+DVCS Interference term

Provide numerical estimates