

Nucleon tomography from lattice QCD data

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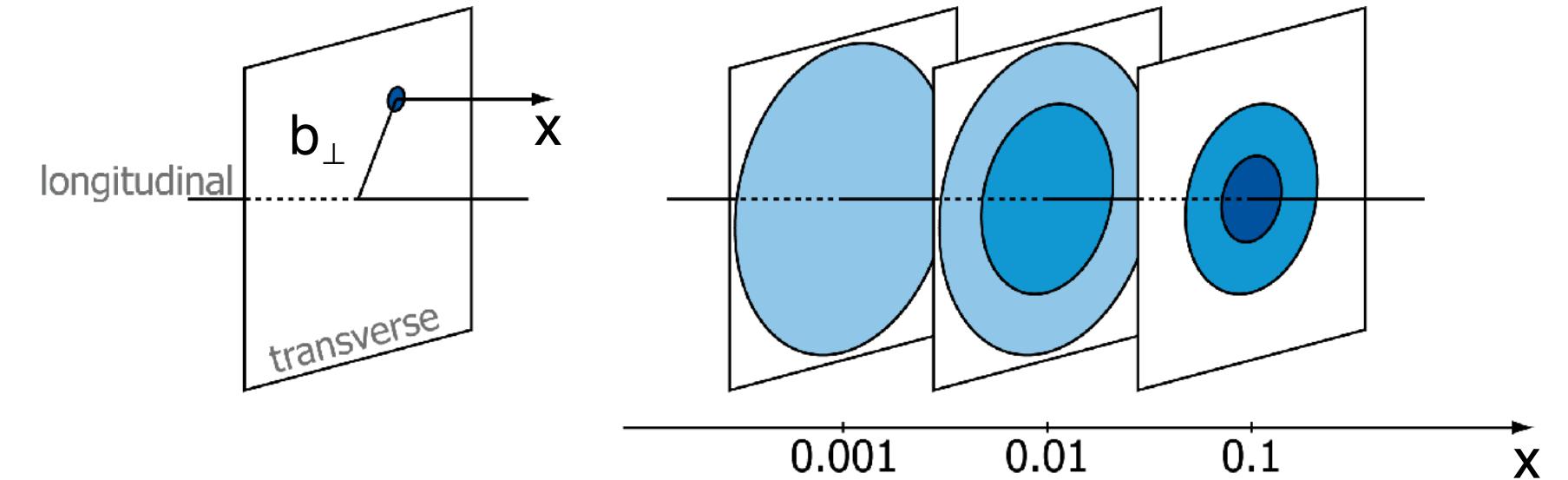
in collaboration with M. Constantinou, K. Cichy and J. Wagner



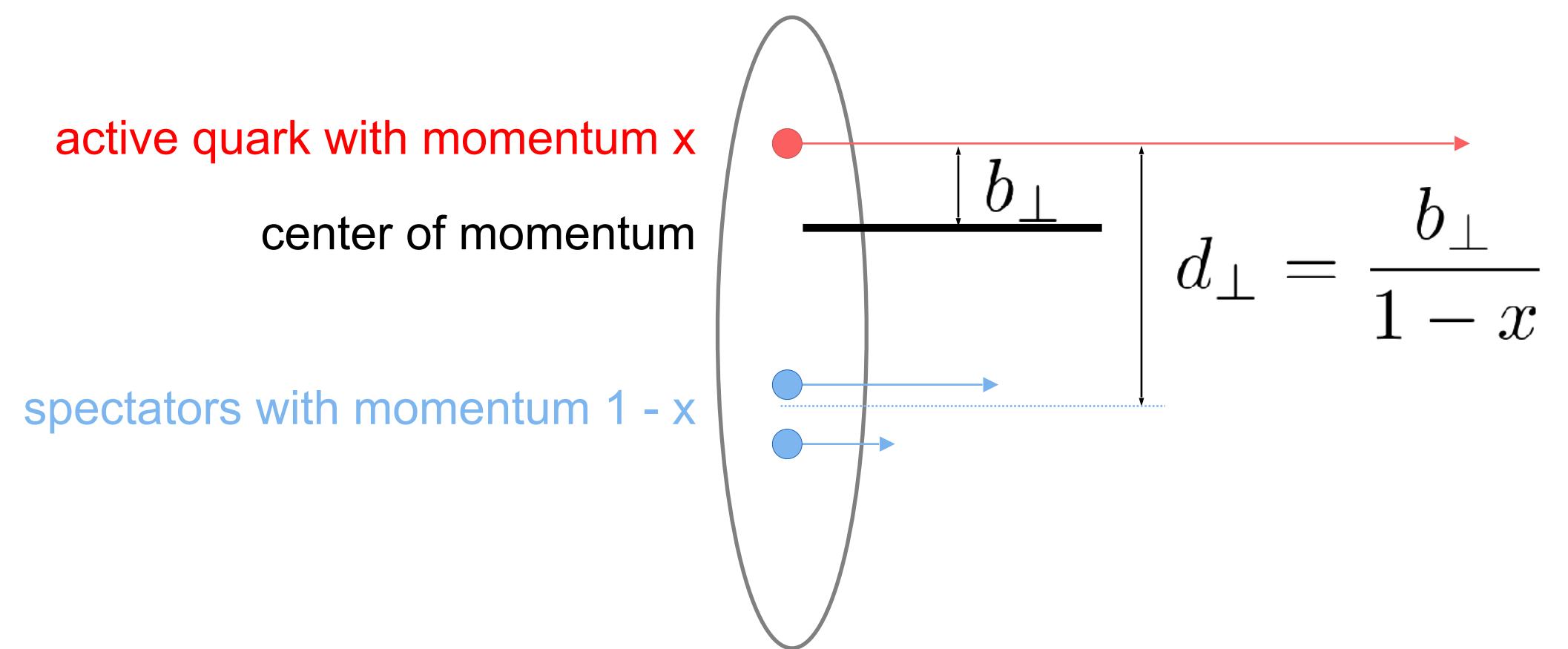
Towards improved hadron tomography with hard exclusive reactions,
Trento, Italy, August 7th, 2024

- Nucleon tomography

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \Delta} H^q(x, 0, t = -\Delta^2)$$



- Study of long. polarization with GPD \tilde{H}
- Study of distortion in transv. polarized nucleon with GPD E
- Impact parameter \mathbf{b}_\perp defined w.r.t. center of momentum, such as $\sum x \mathbf{b}_\perp = 0$

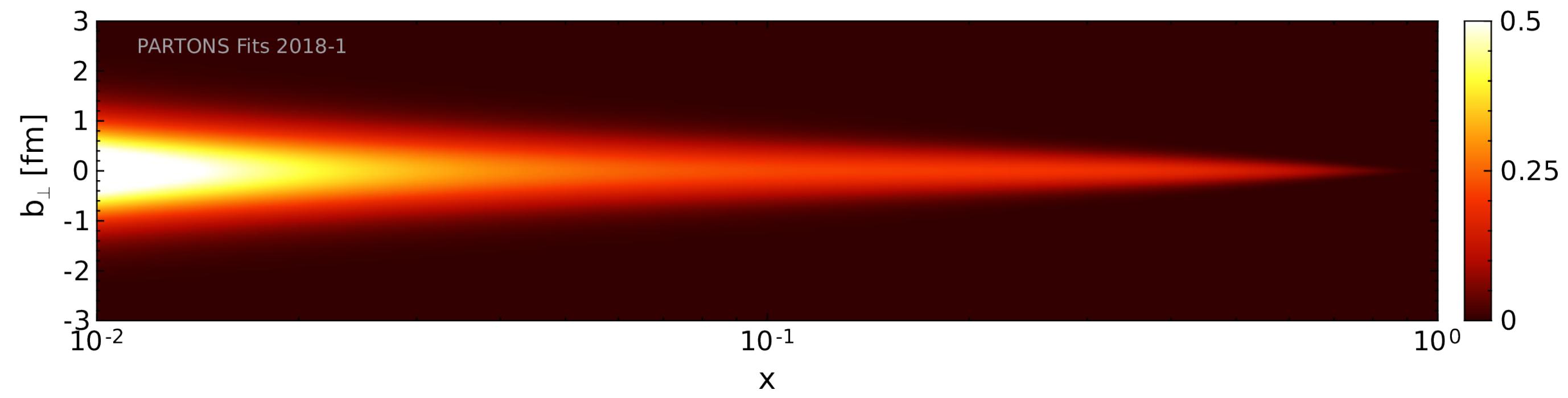


What do we know about nucleon tomography now?

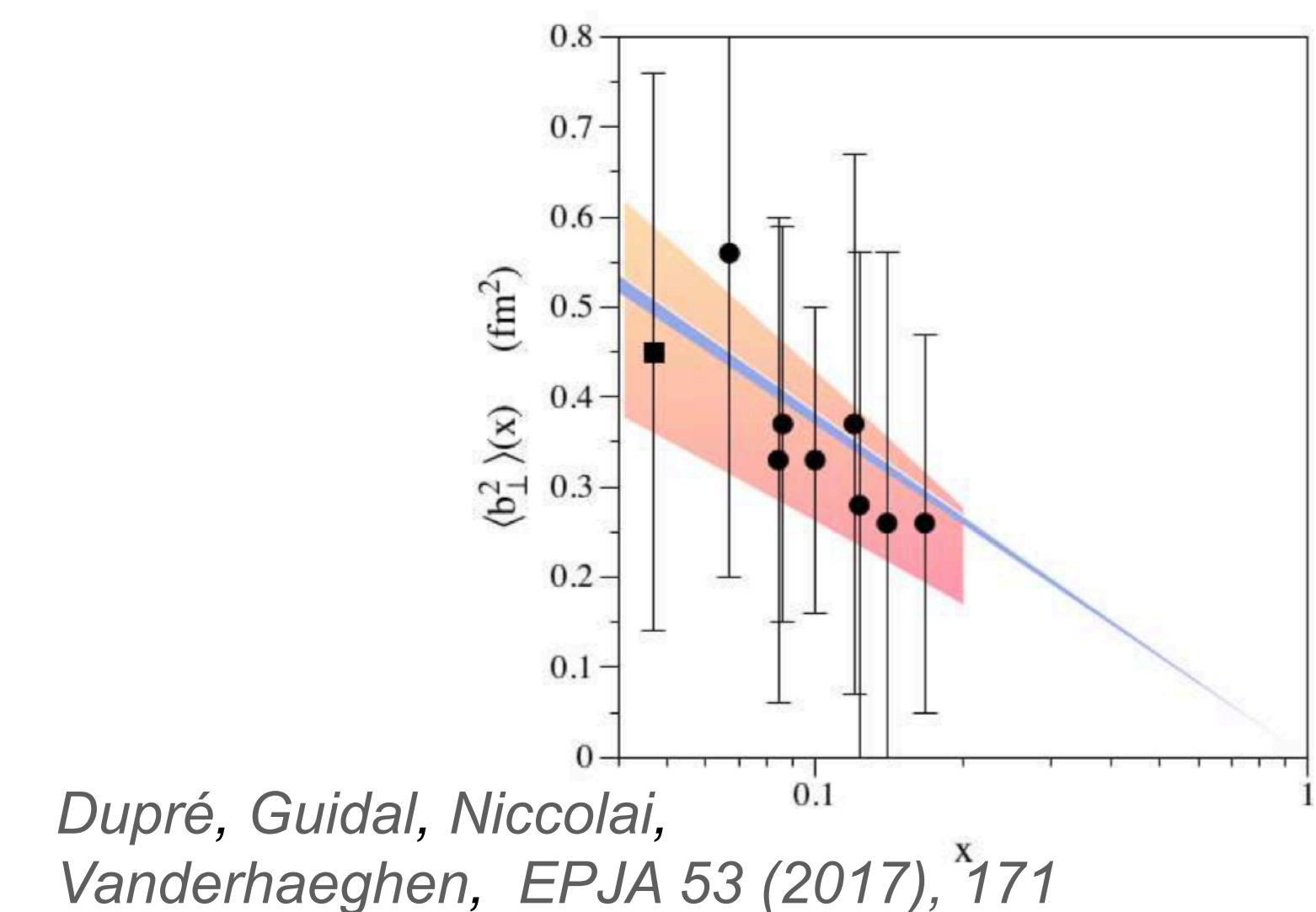
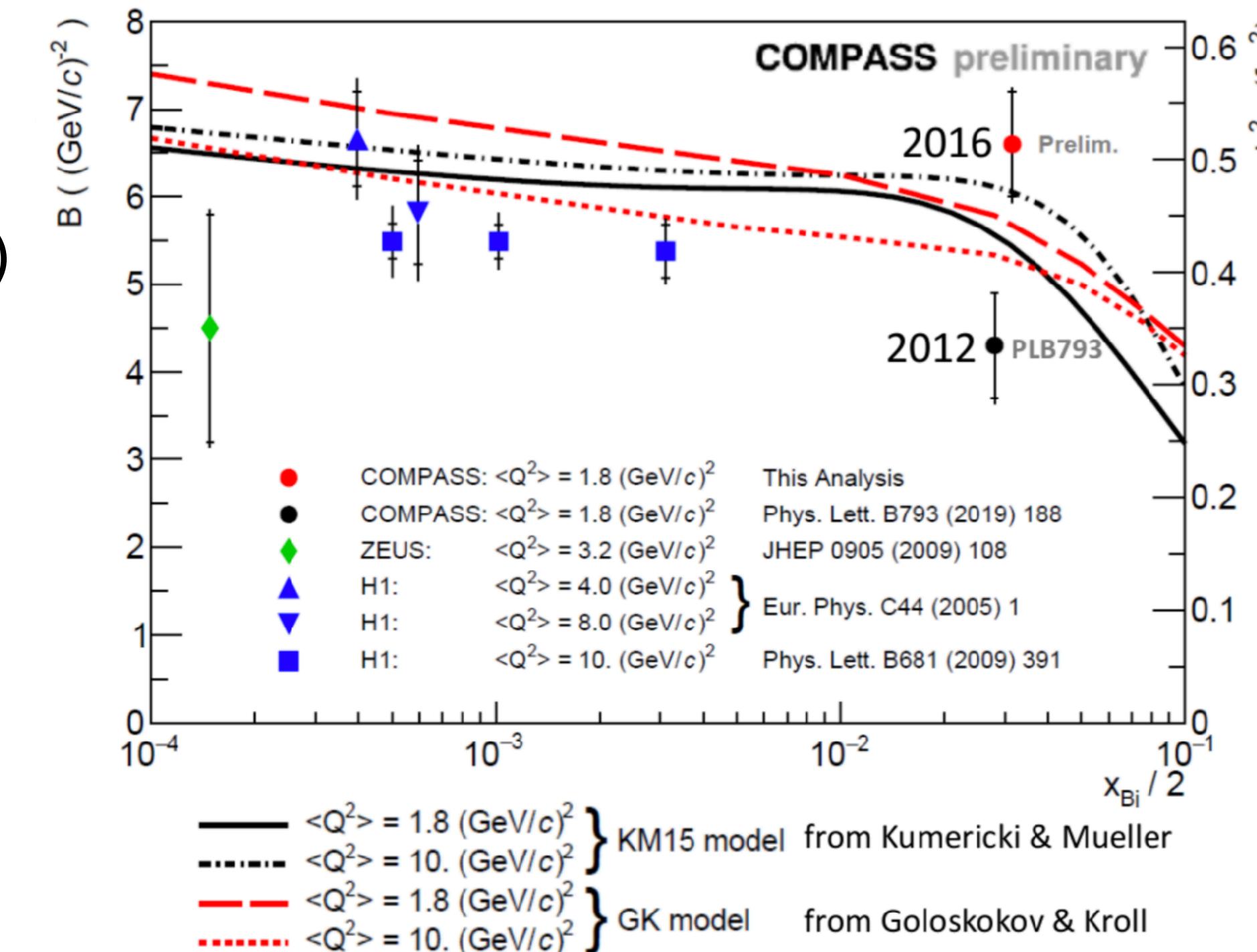
- "direct" extraction of nucleon tomography information
(i.e. directly from DVCS cross-section, only applicable at small x_B)

$$\begin{aligned} d^3\sigma/(dx_{Bj} dQ^2 dt) &\propto (\text{Im}\mathcal{H}(\xi, t))^2 \propto \left(\sum_q e_q^2 H^{q(+)}(\xi, \xi, t) \right)^2 \\ &\propto \left(\sum_q e_q^2 H^{q(+)}(\xi, 0, t) \right)^2 \end{aligned}$$

- extraction involving all types of contributing CFFs
(requires "de-skewness", allows to use high x_B data)



H. Moutarde, PS, J. Wagner, EPJC 78 (2018), 890



Dupr , Guidal, Niccolai,
Vanderhaeghen, EPJA 53 (2017), 171

- GPDs in Ioffe time:

$$\hat{H}(\nu, \xi, t) = \int_{-1}^1 dx e^{ix\nu} H(x, \xi, t)$$

- Single and non-singlet combinations of GPDs

$$H^{(+)}(x, \xi, t) = H(x, \xi, t) - H(-x, \xi, t)$$

$$H^{(-)}(x, \xi, t) = H(x, \xi, t) + H(-x, \xi, t)$$

at $\xi=0$:

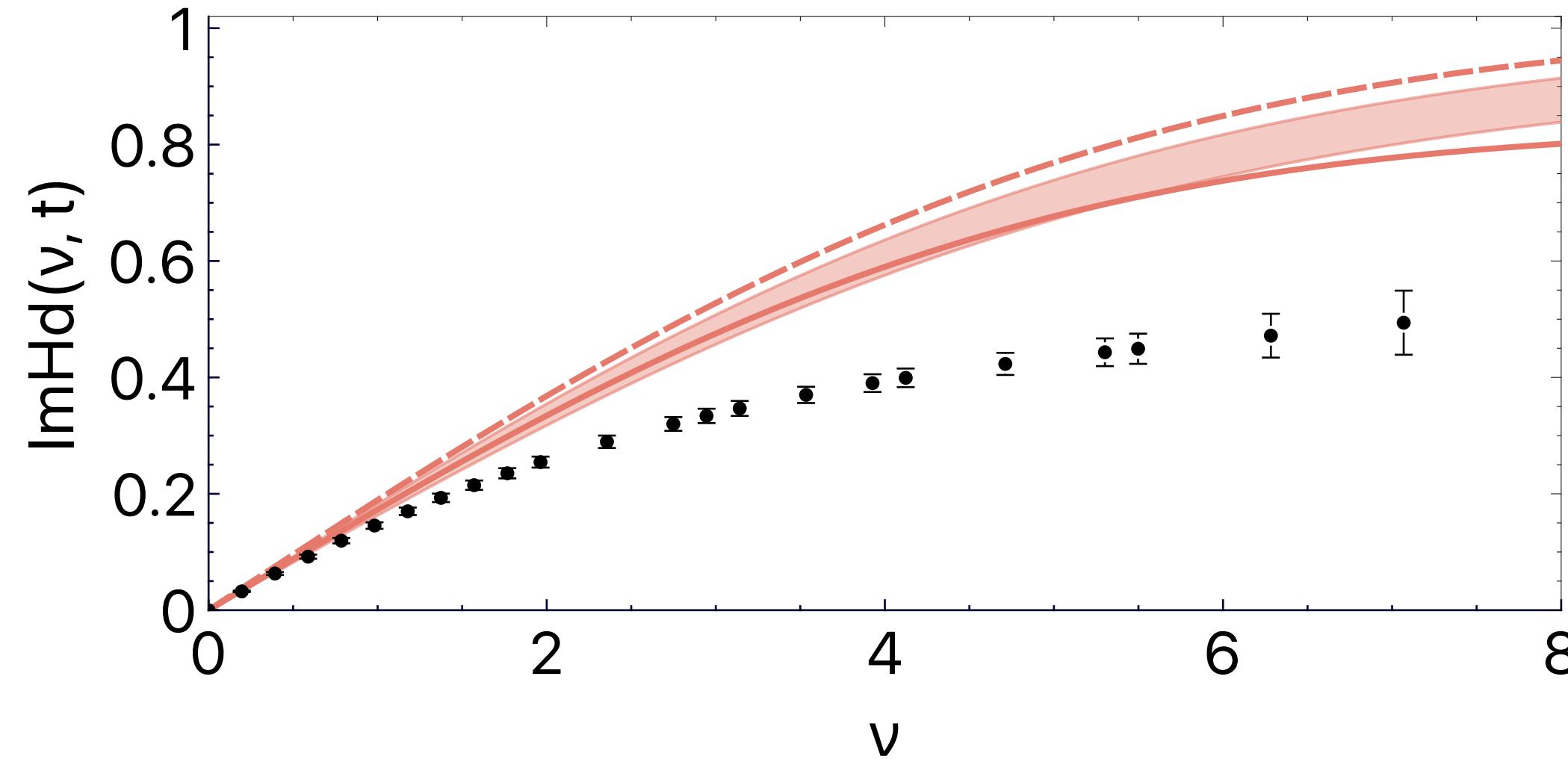
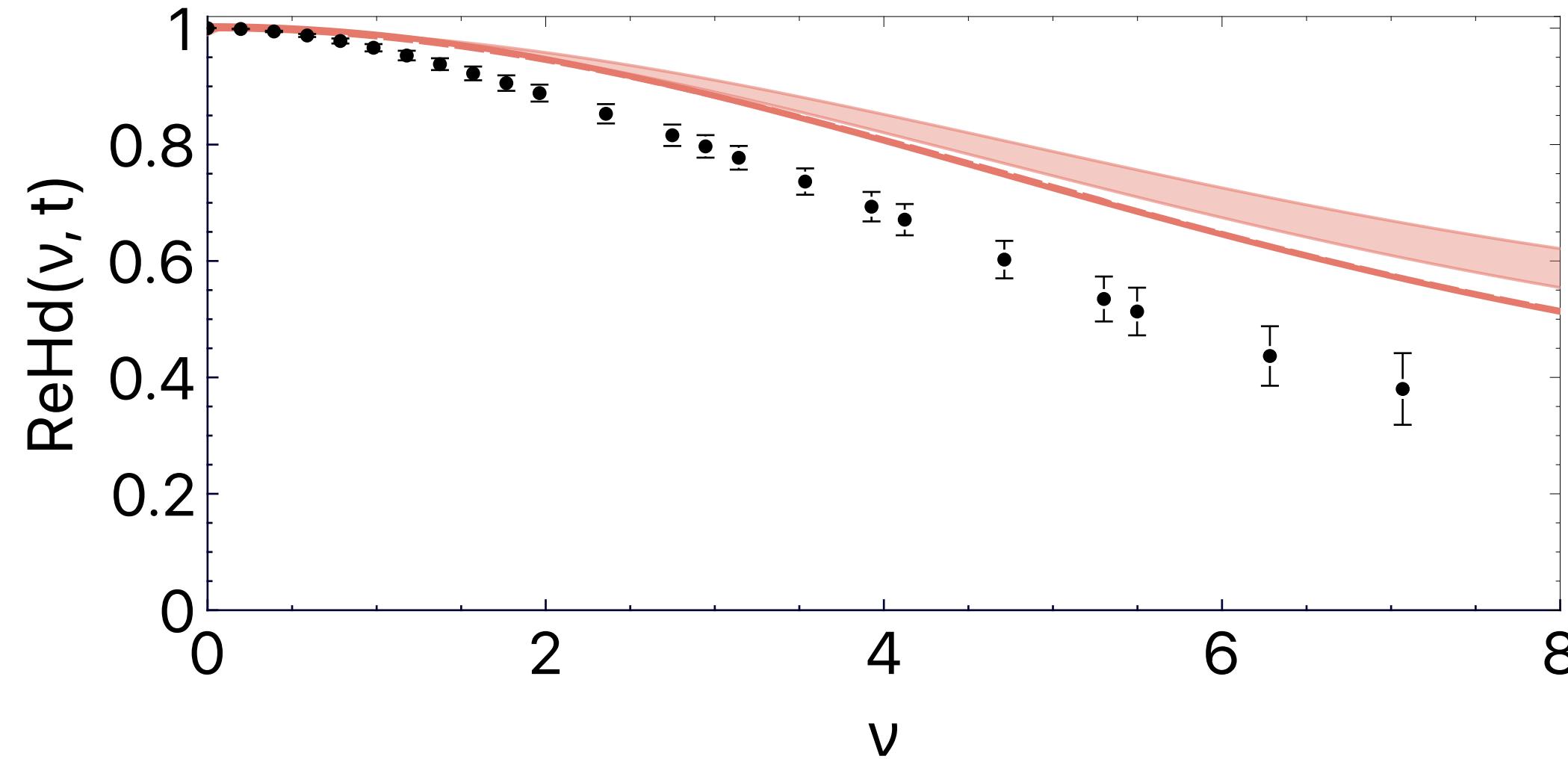
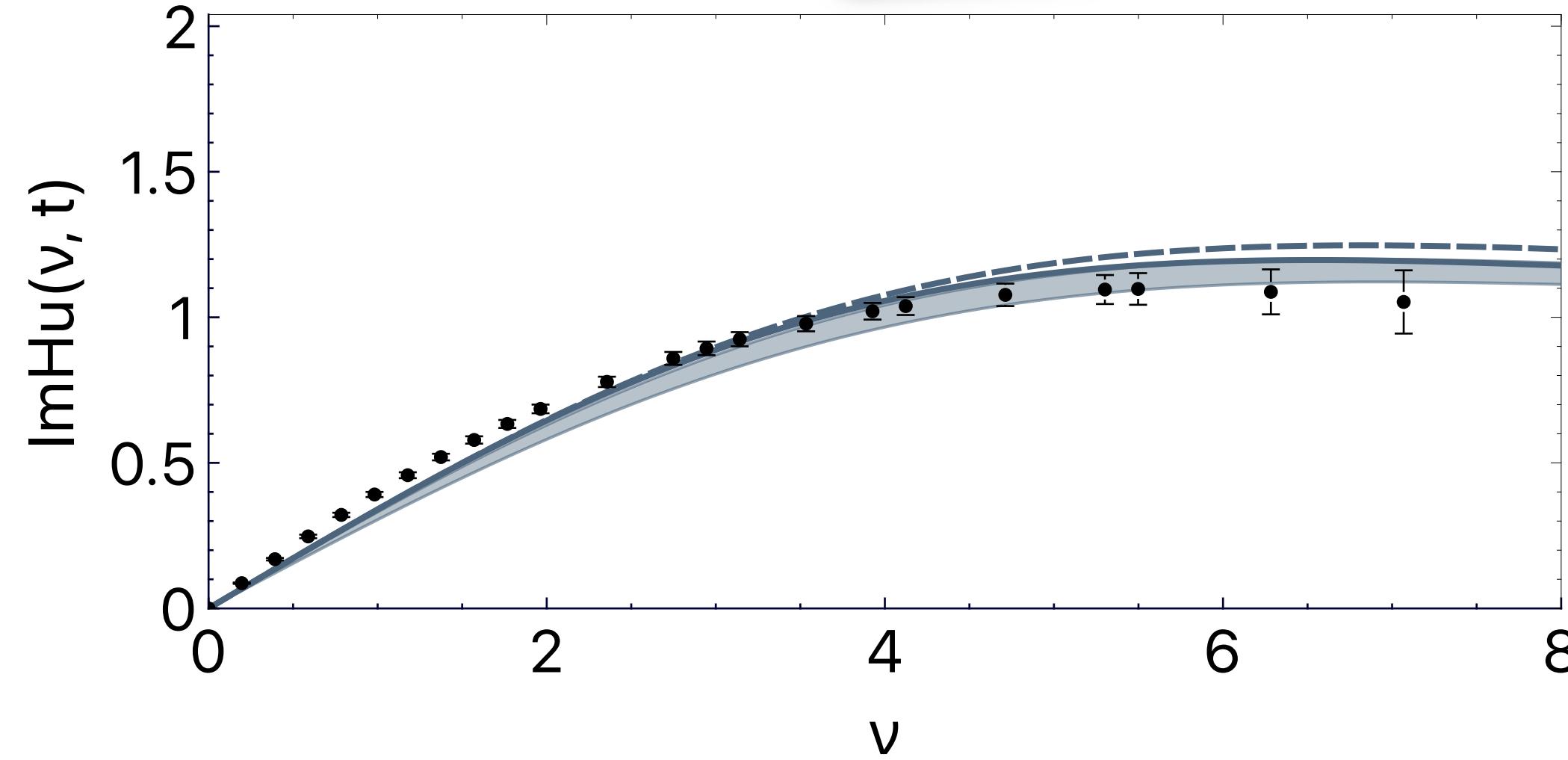
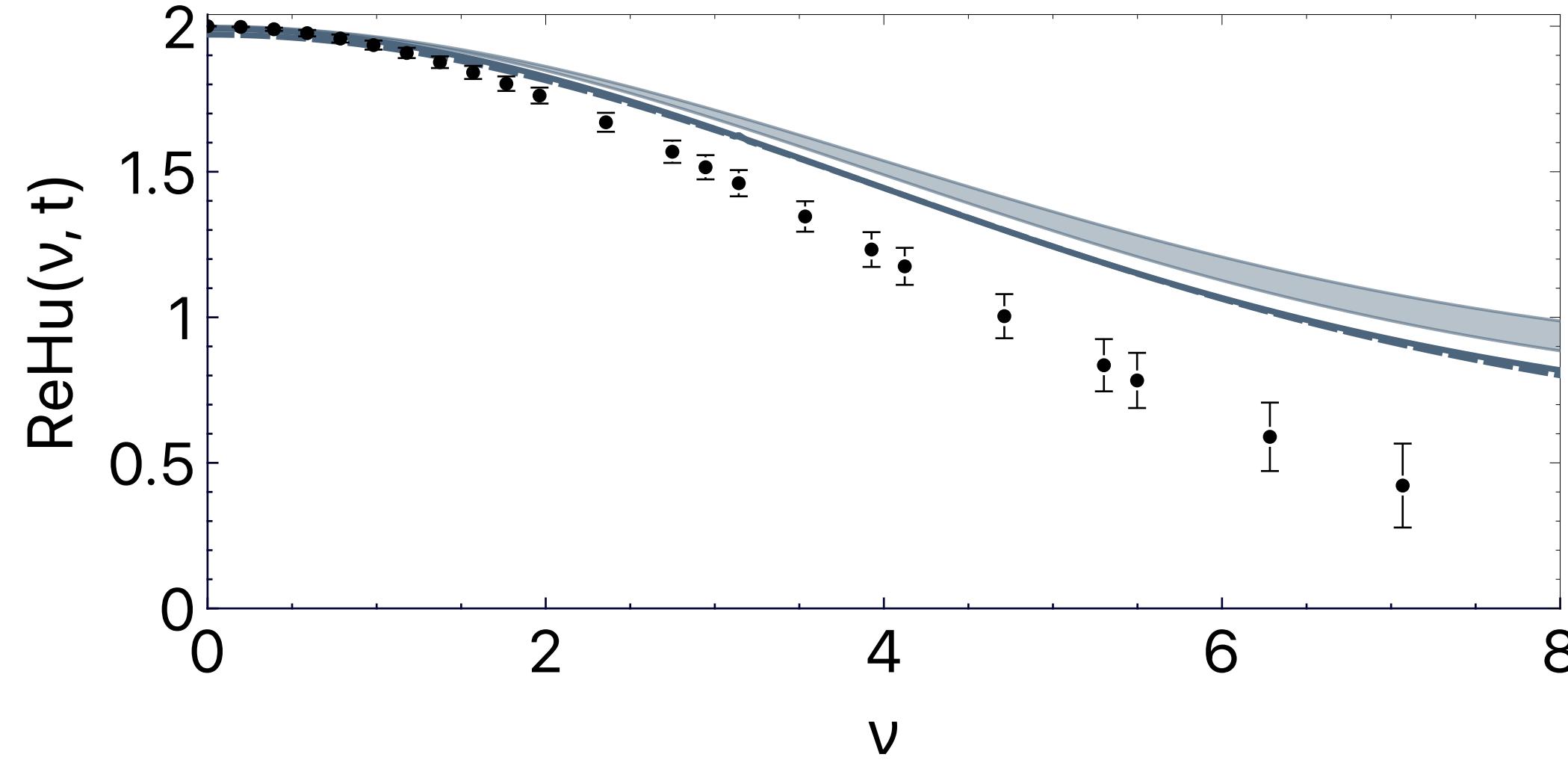
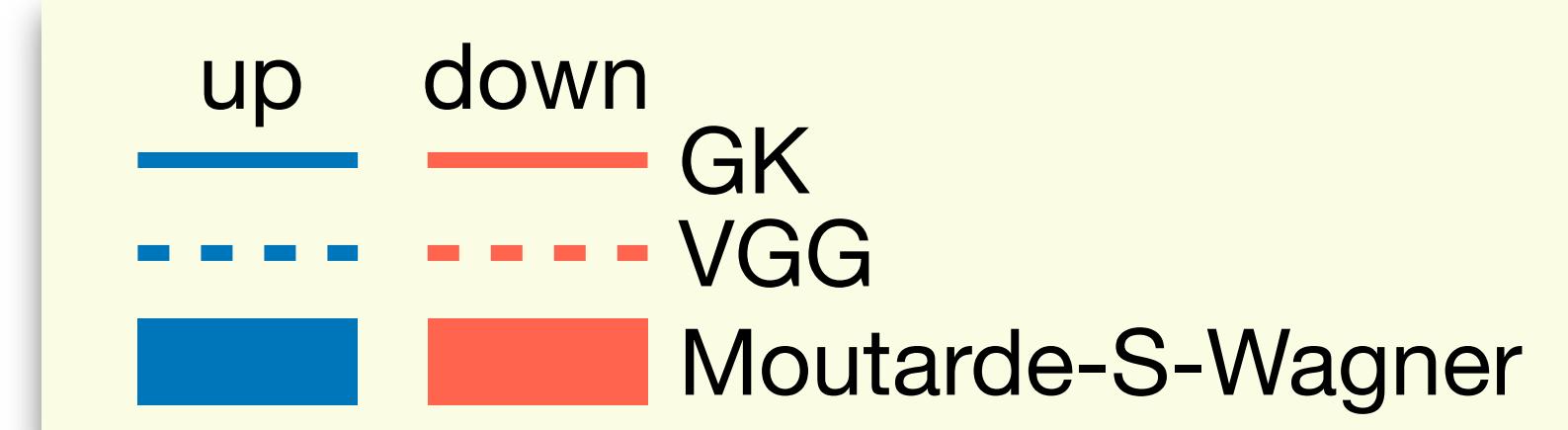
$$H^{(+)}(x, 0, t) = H_{\text{val}}(x, 0, t) + 2H_{\text{sea}}(x, 0, t)$$

$$H^{(-)}(x, 0, t) = H_{\text{val}}(x, 0, t)$$

- therefore:

$$\text{Re}\hat{H}(\nu, \xi, t) = \int_0^1 dx \cos(x\nu) H^{(-)}(x, \xi, t)$$

$$\text{Im}\hat{H}(\nu, \xi, t) = \int_0^1 dx \sin(x\nu) H^{(+)}(x, \xi, t)$$

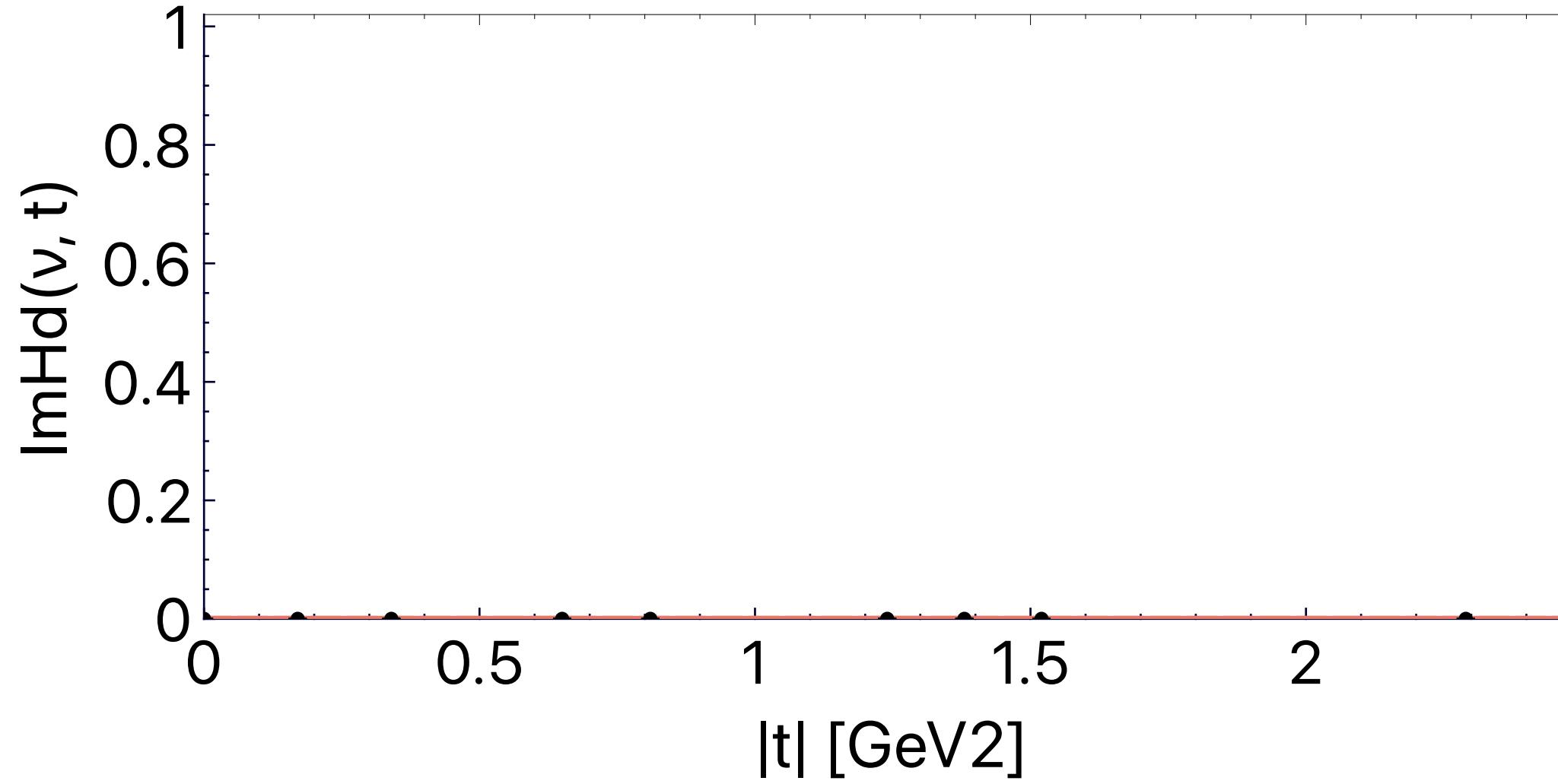
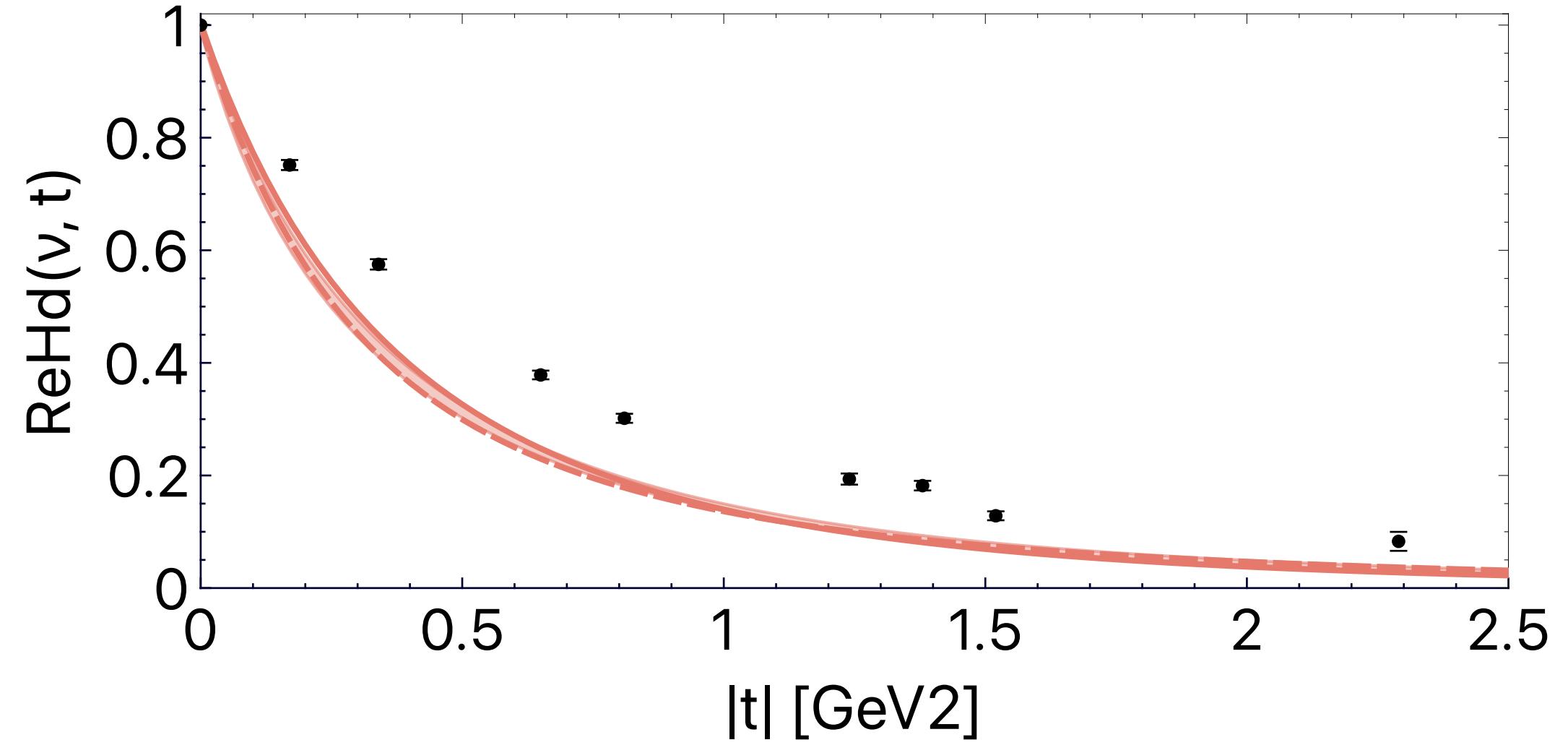
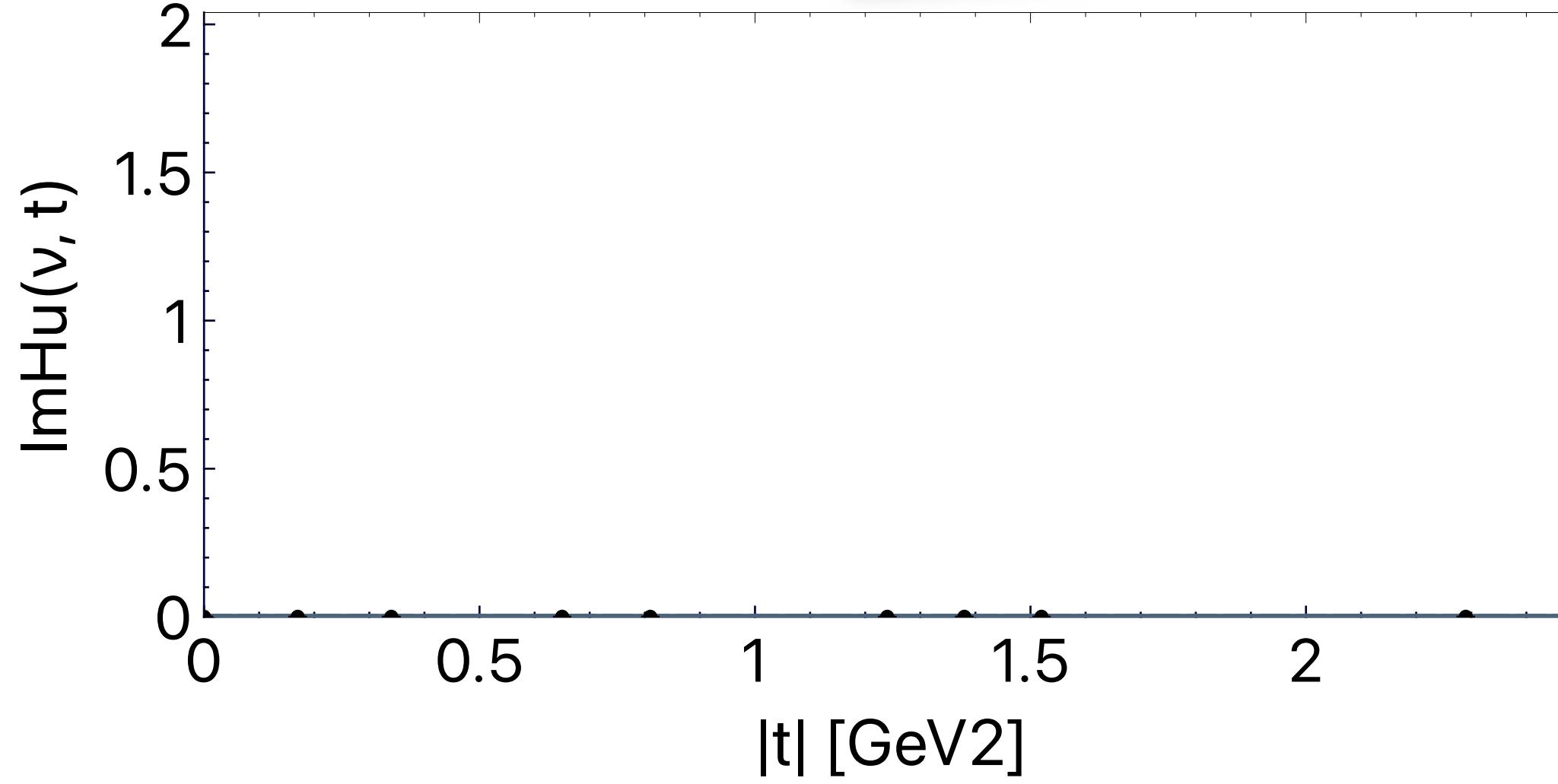
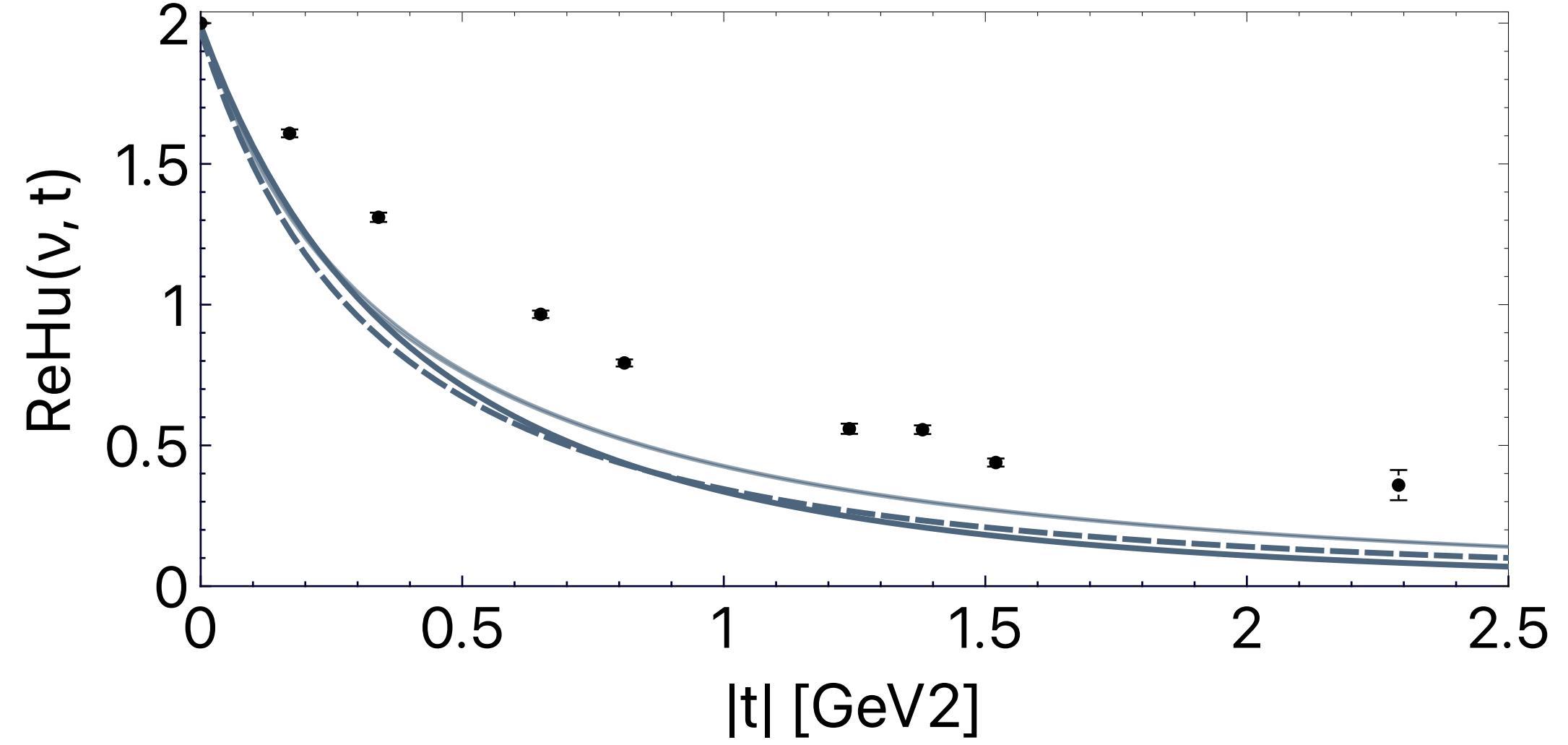
GPD H, $t=0$ 

- For details on these latticeQCD data see Krzysztof's talk

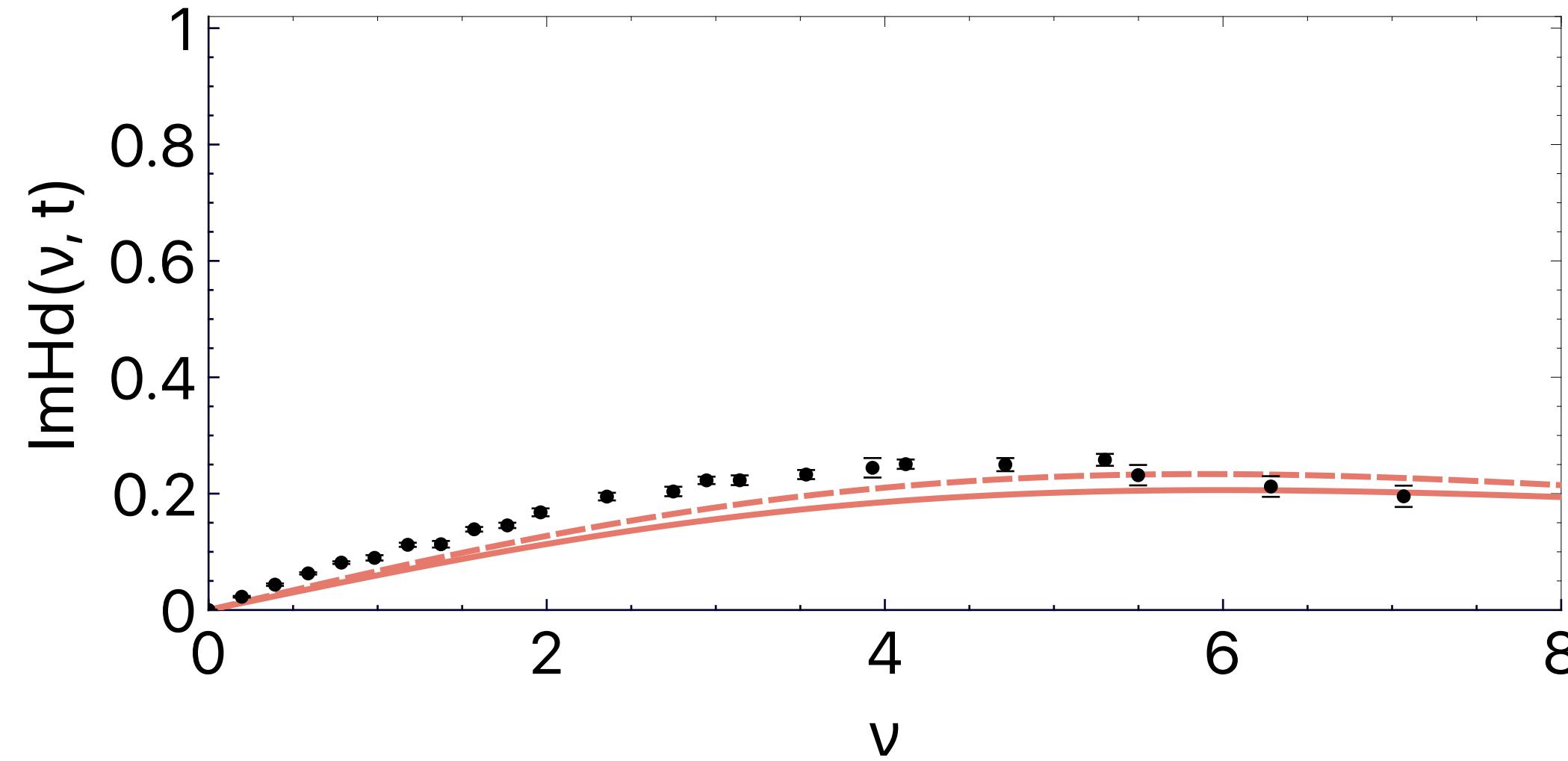
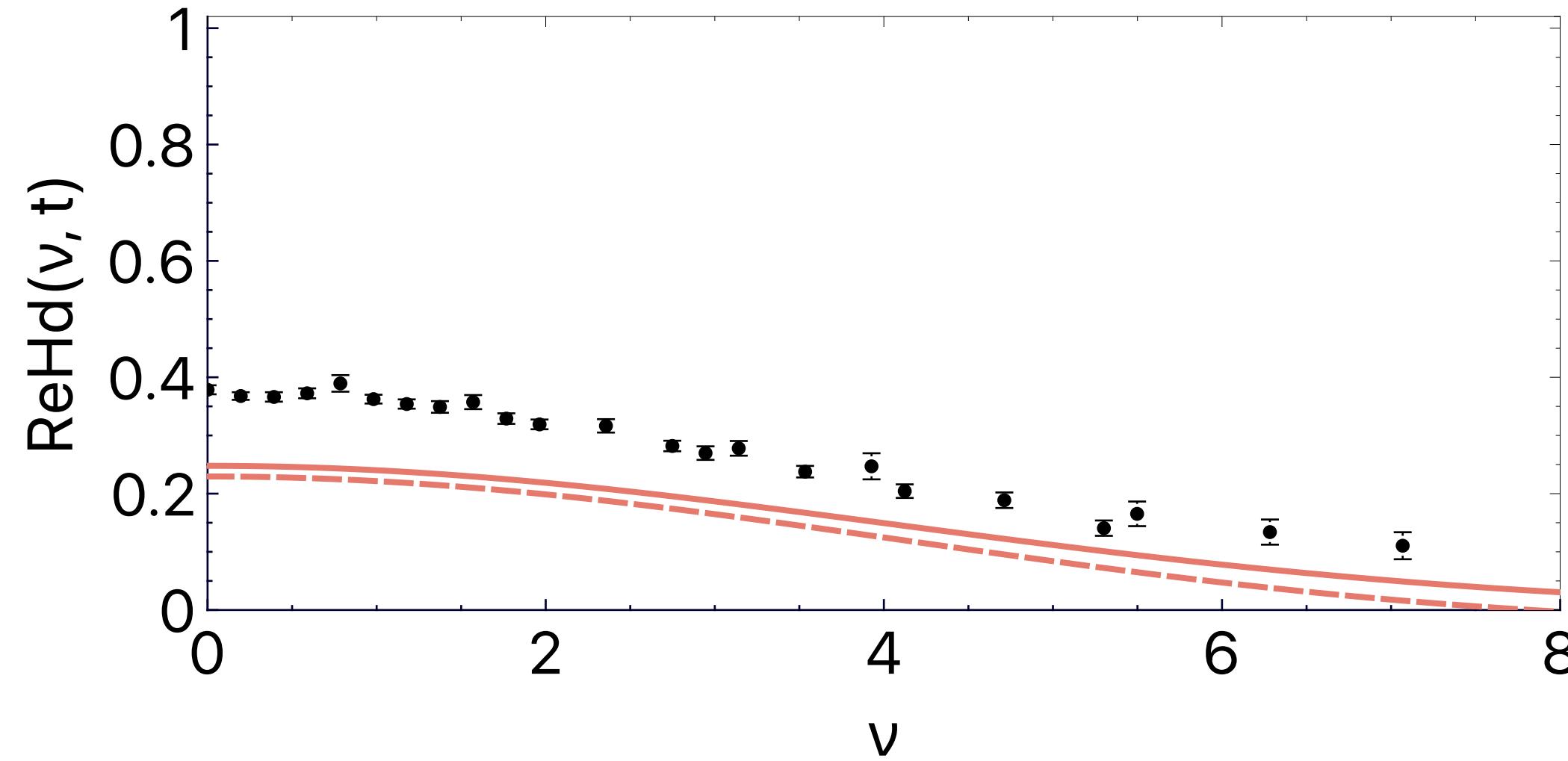
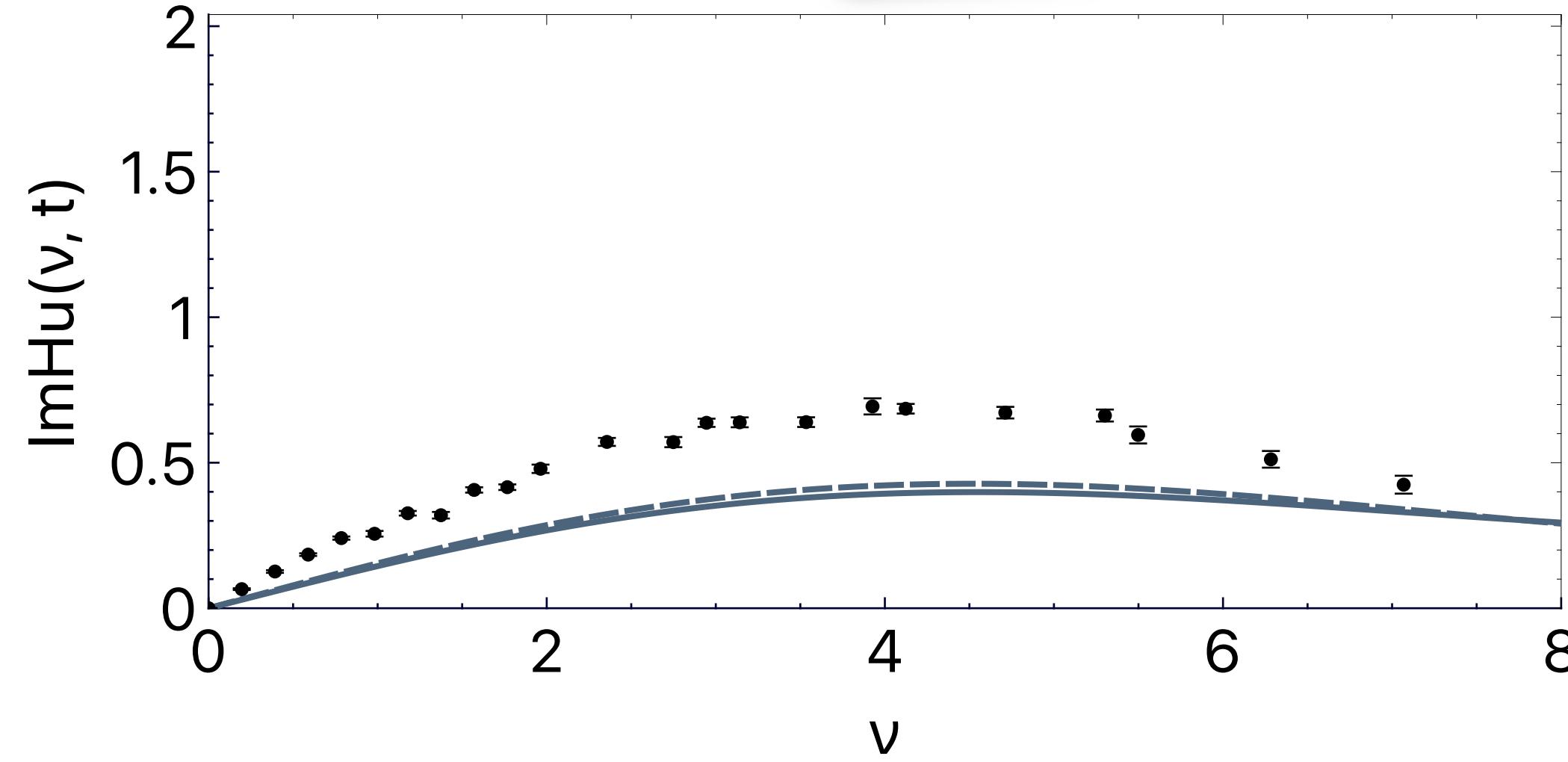
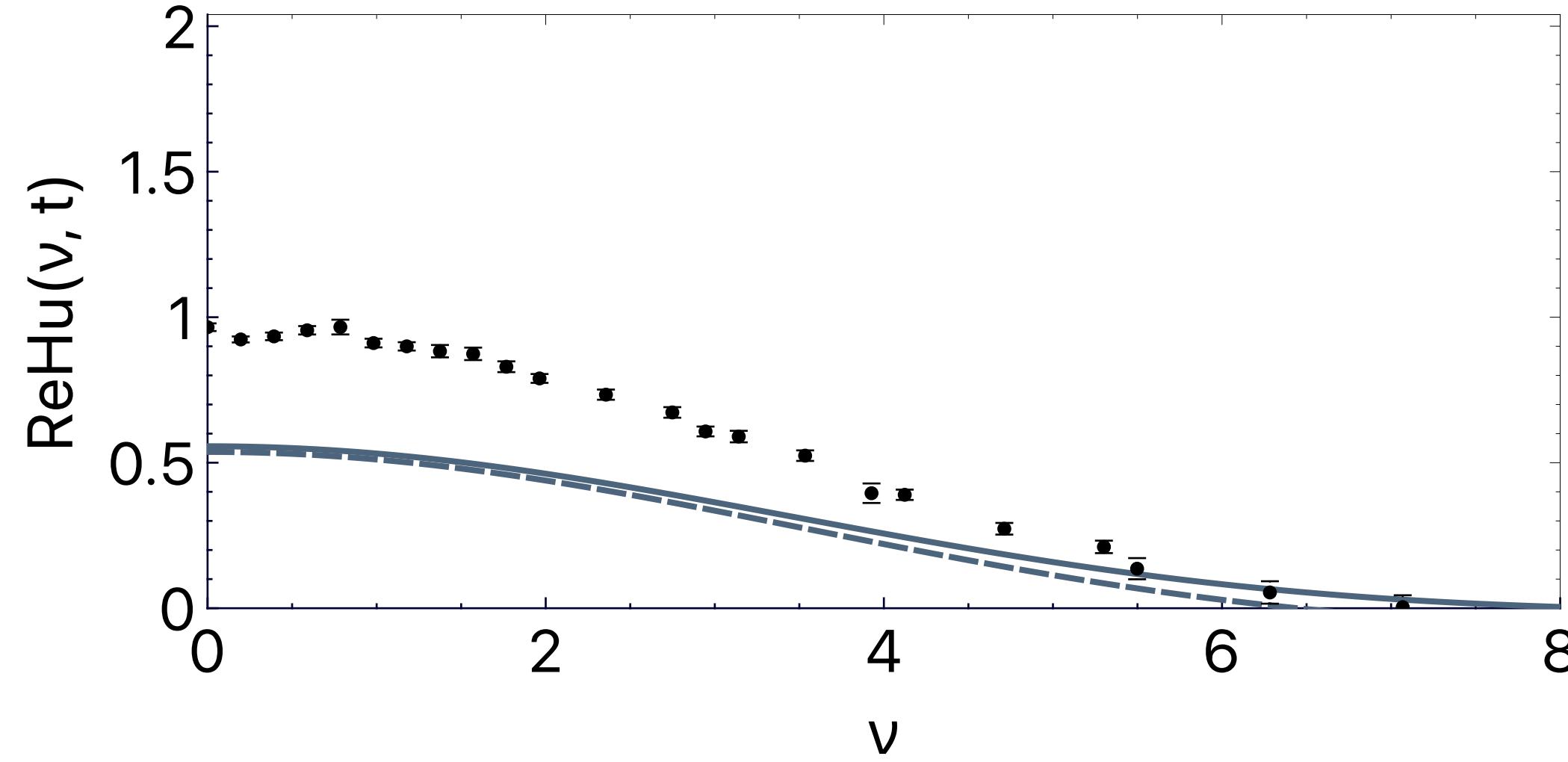
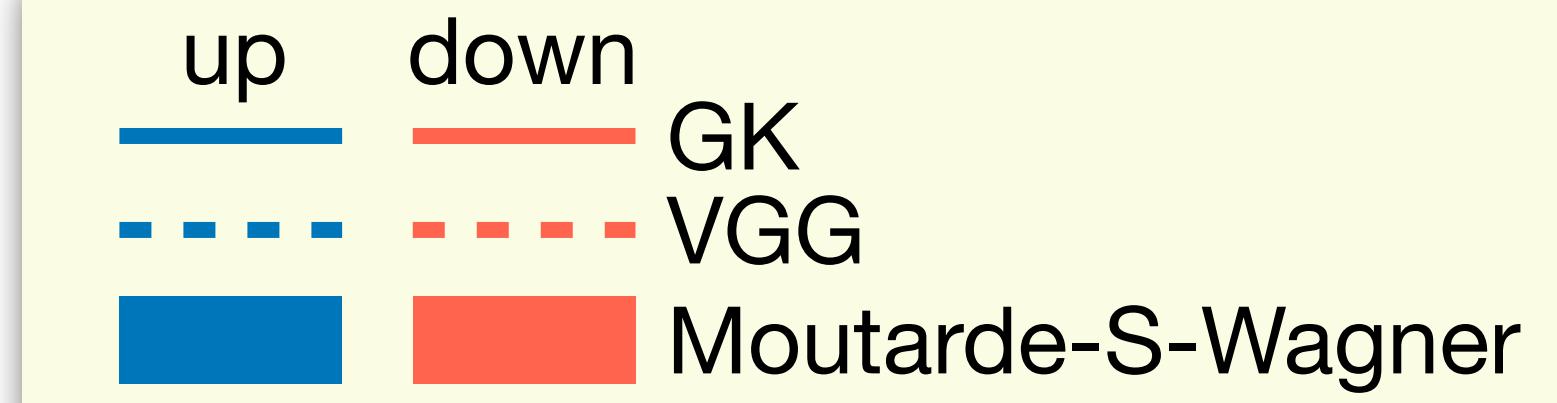
Lattice-QCD data

GPD H, $\nu=0$

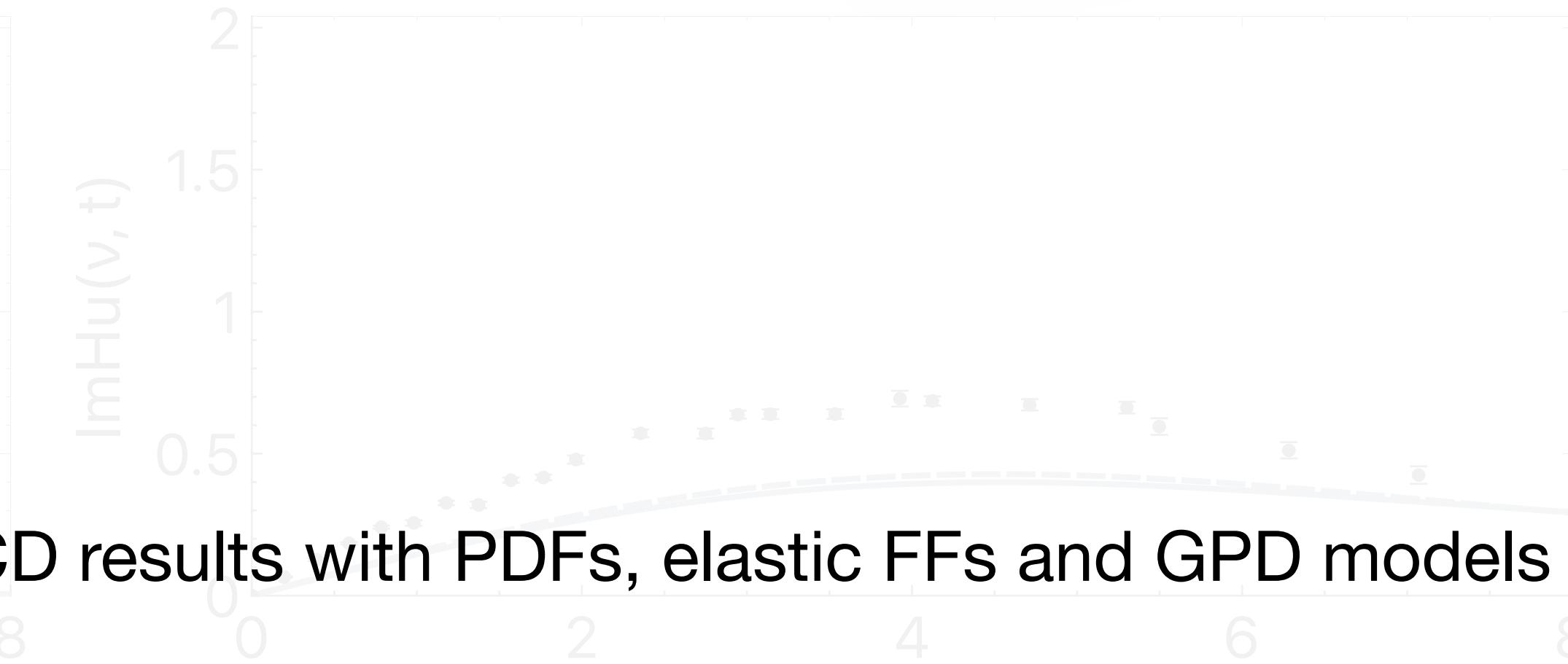
up	down	
		GK
		VGG
		Moutarde-S-Wagner



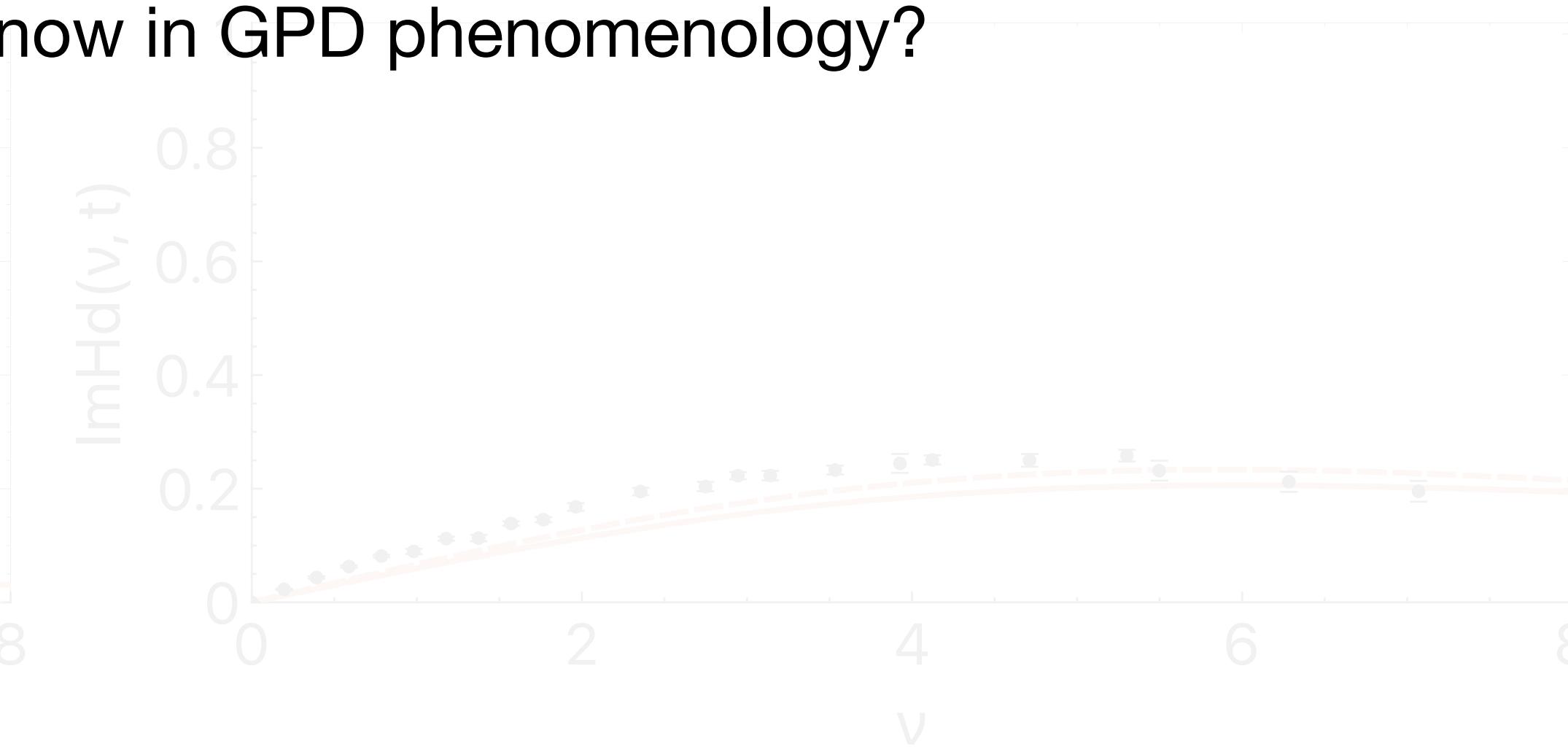
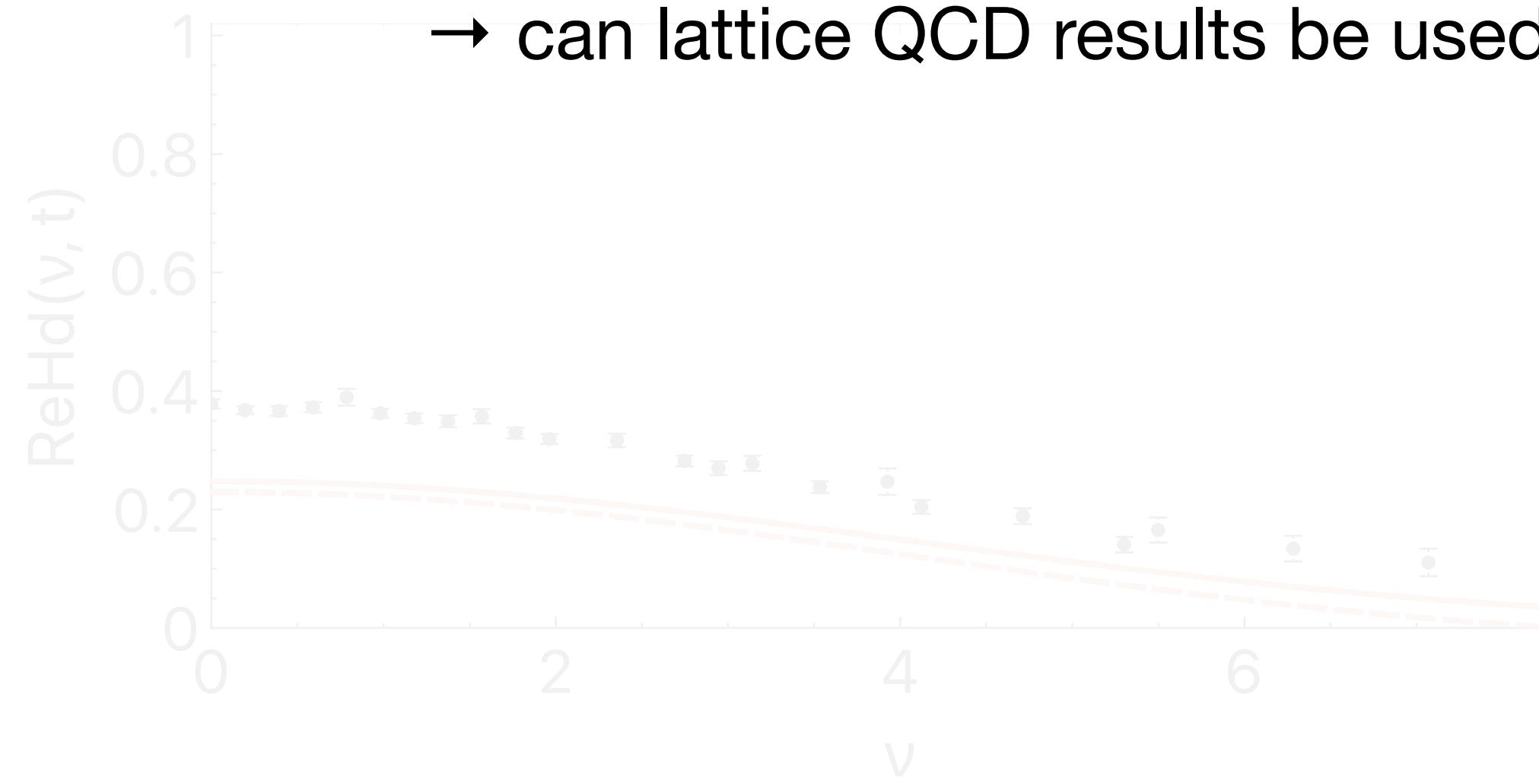
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GPD H, $|t|=0.65 \text{ GeV}^2$ 

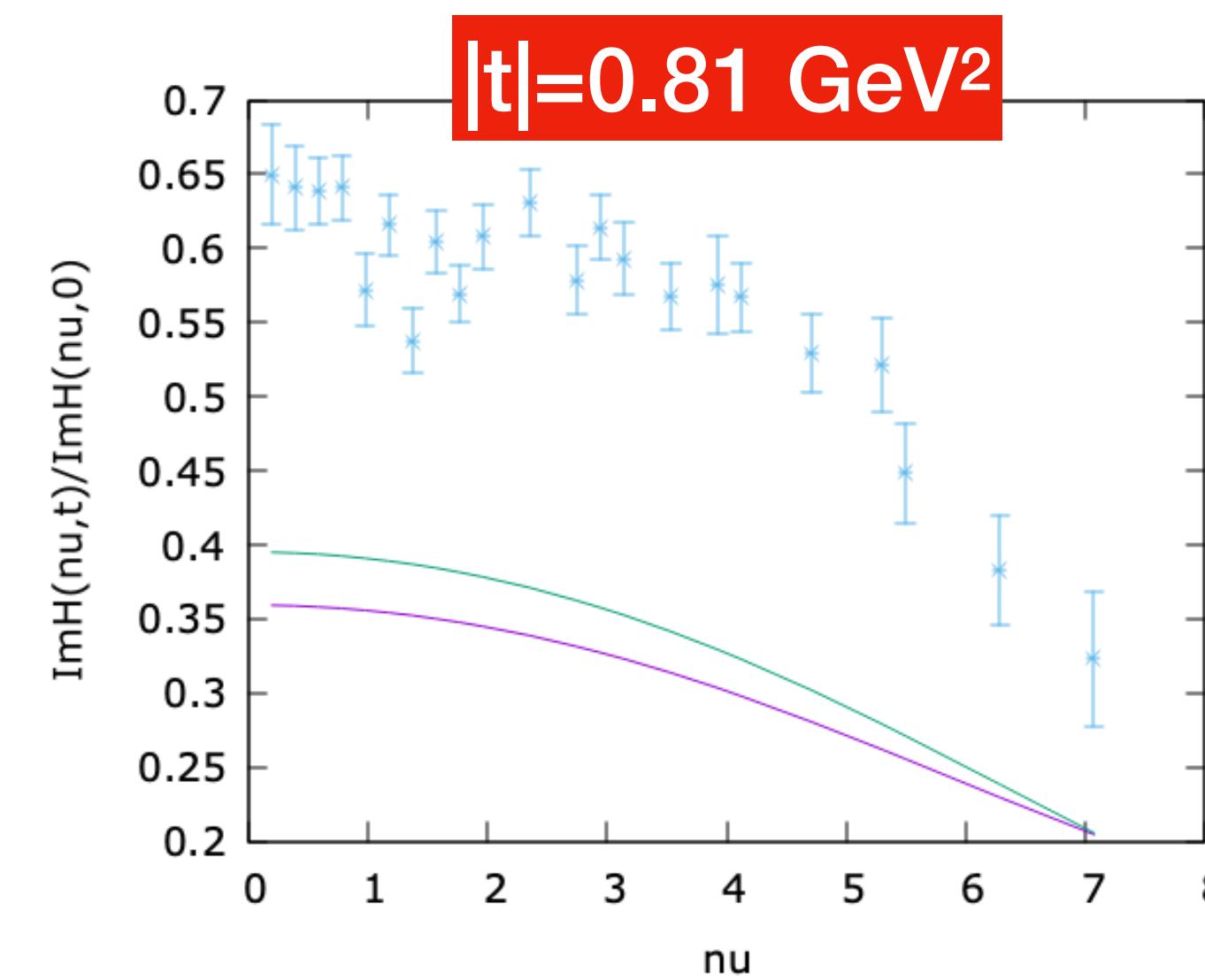
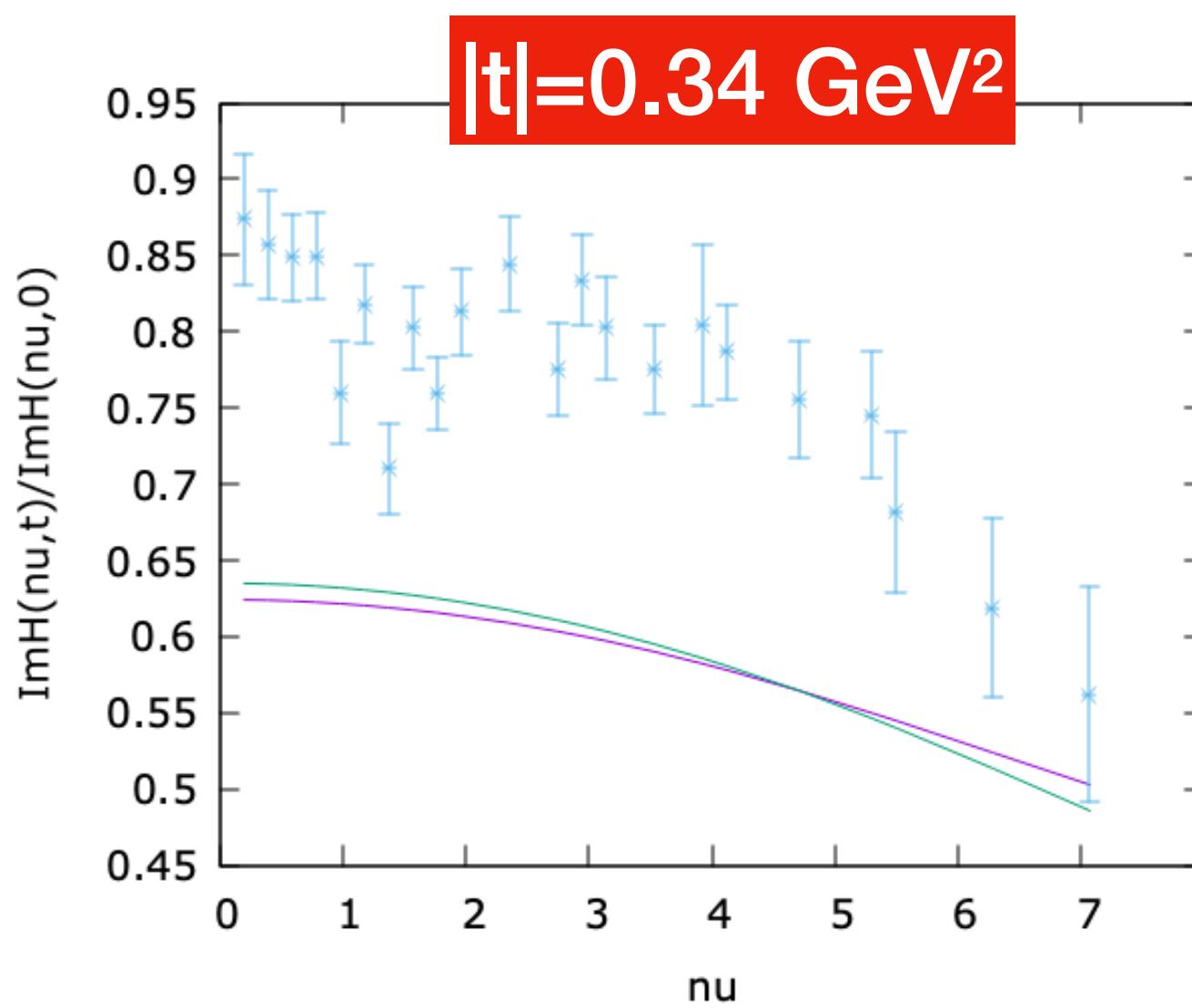
- Rather poor agreement of latticeQCD results with PDFs, elastic FFs and GPD models
- This will improve over time!
- Should we just wait, or is there something that lattice QCD results already get right?
→ can lattice QCD results be used now in GPD phenomenology?



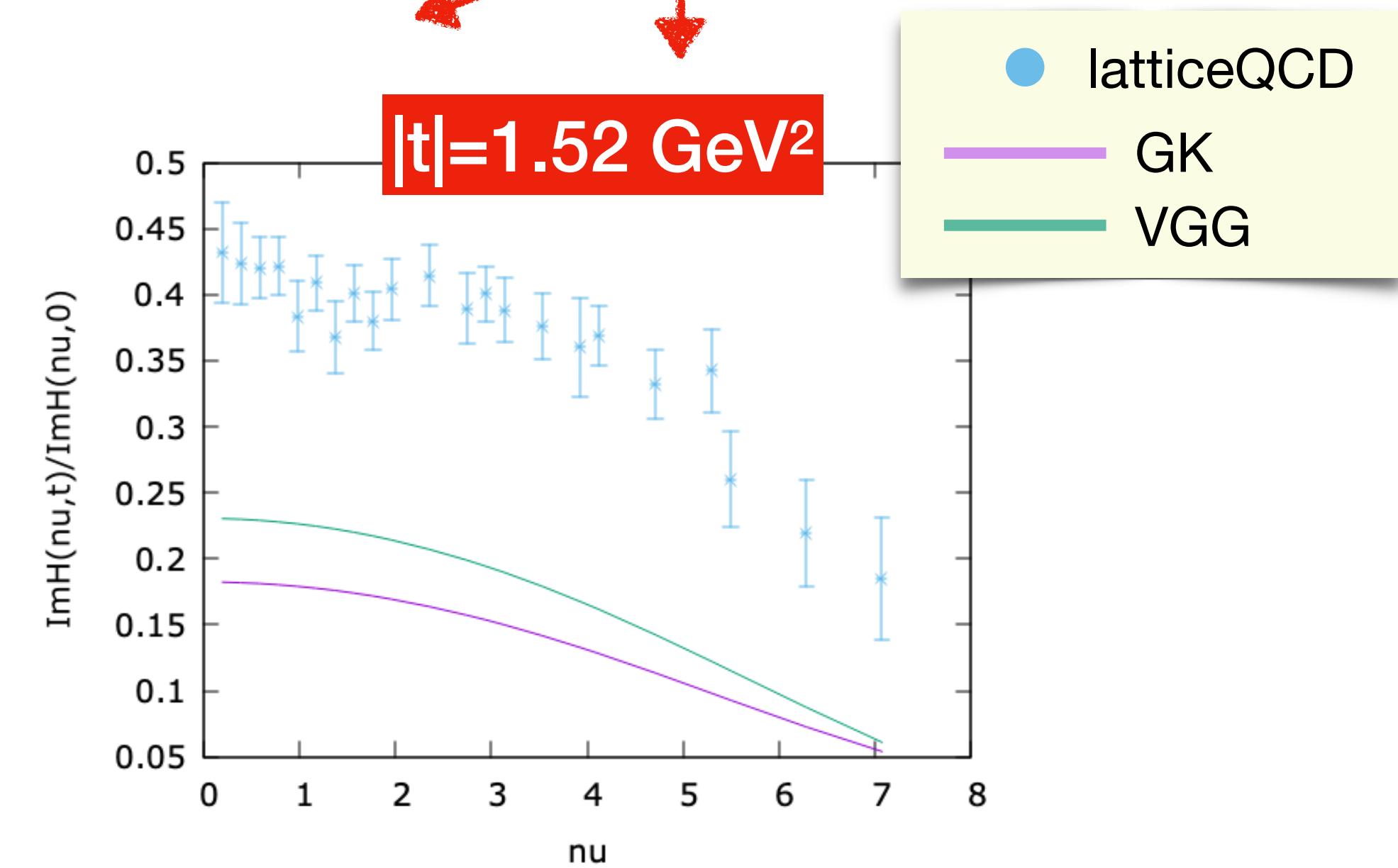
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Double ratio

$$DR_{Re}(\nu, t) = \frac{\text{Re}H(\nu, t)}{\text{Re}H(\nu, 0)} \frac{\text{Re}H(0, 0)}{\text{Re}H(0, t)}$$



$$DR_{Im}(\nu, t) = \lim_{\nu' \rightarrow 0} \frac{\text{Im}H(\nu, t)}{\text{Im}H(\nu, 0)} \frac{\text{Im}H(\nu', 0)}{\text{Im}H(\nu', t)}$$



- plateau of $\text{Im}H(\nu, t)/\text{Im}H(\nu, 0)$ at $\nu = 0$ makes the definition of DR_{Im} robust
- this plateau (also seen for $\text{Re}H(\nu, t)$) is a consequence of the restricted range of x in Fourier transform
- single ratios, here $\text{Im}H(\nu, t)/\text{Im}H(\nu, 0)$, do not provide a good comparison with models

$$\text{DR}_{\text{Re}}(\nu, t) = \frac{\text{Re}H(\nu, t)}{\text{Re}H(\nu, 0)} \frac{\text{Re}H(0, 0)}{\text{Re}H(0, t)}$$

$$\text{DR}_{\text{Im}}(\nu, t) = \lim_{\nu' \rightarrow 0} \frac{\text{Im}H(\nu, t)}{\text{Im}H(\nu, 0)} \frac{\text{Im}H(\nu', 0)}{\text{Im}H(\nu', t)}$$

$$\text{Re}H(\nu, t) = \text{Re}H(0, 0) \times \frac{\text{Re}H(\nu, 0)}{\text{Re}H(0, 0)} \times \frac{\text{Re}H(0, t)}{\text{Re}H(0, 0)} \times \text{DR}_{\text{Re}}(\nu, t)$$

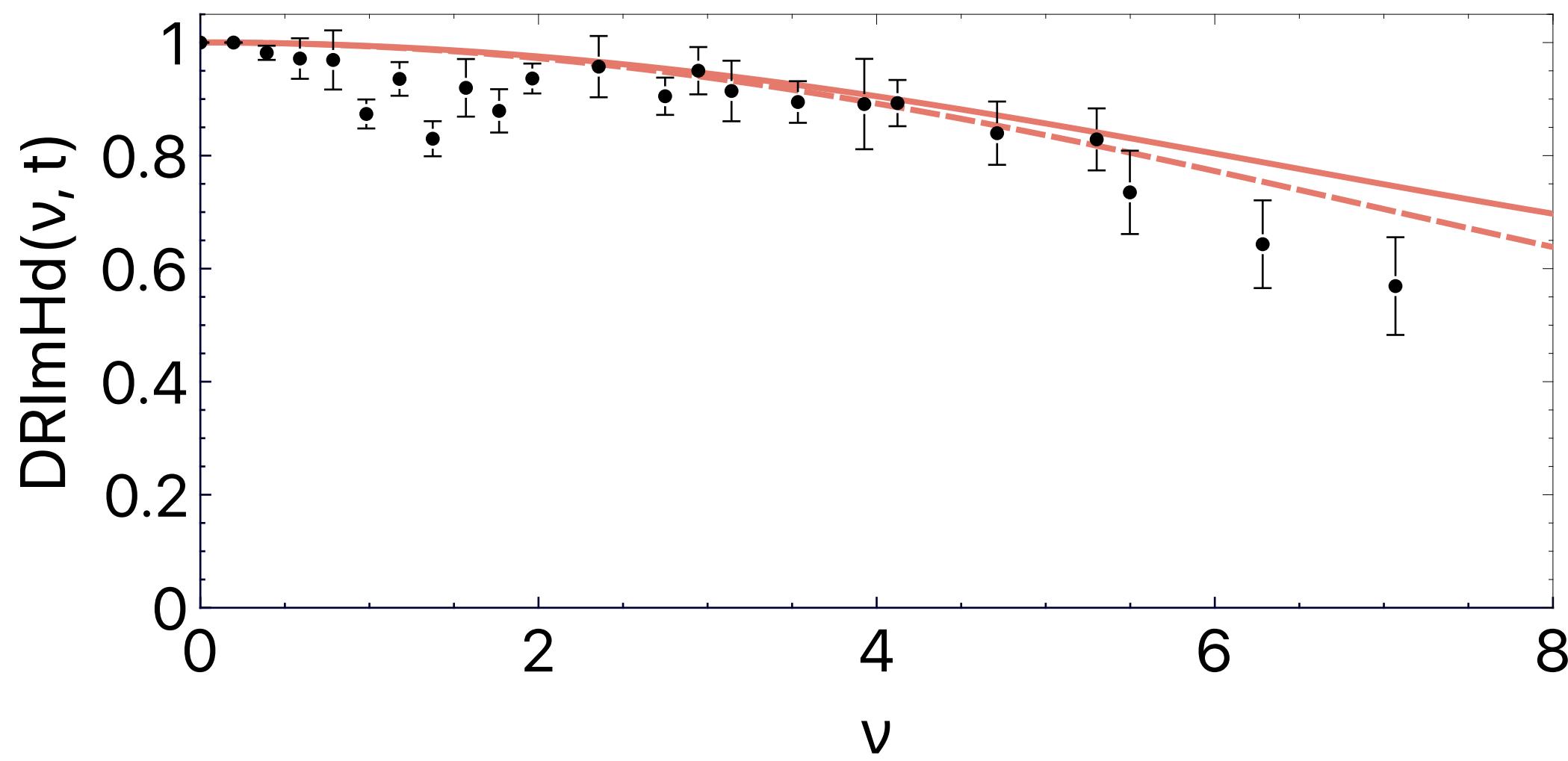
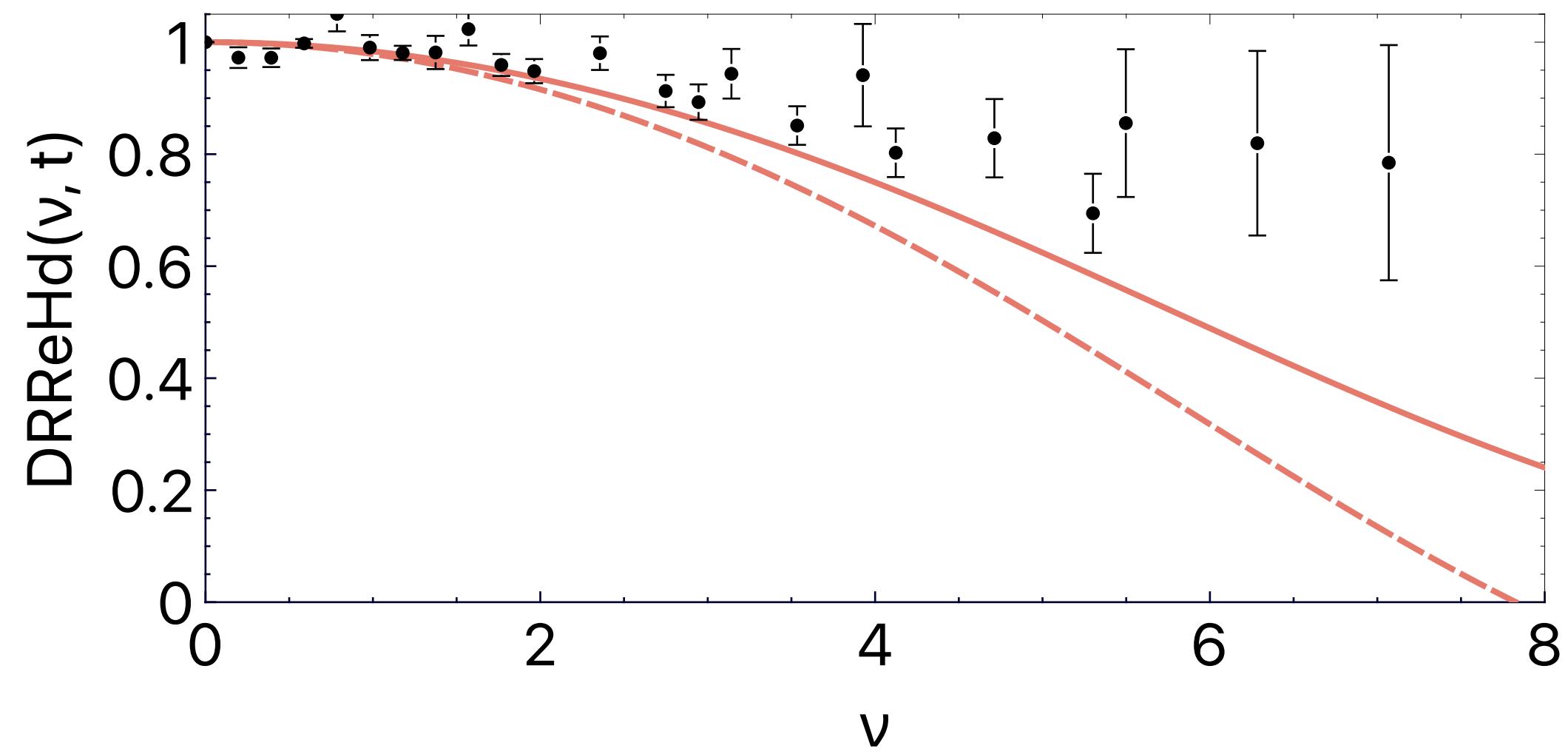
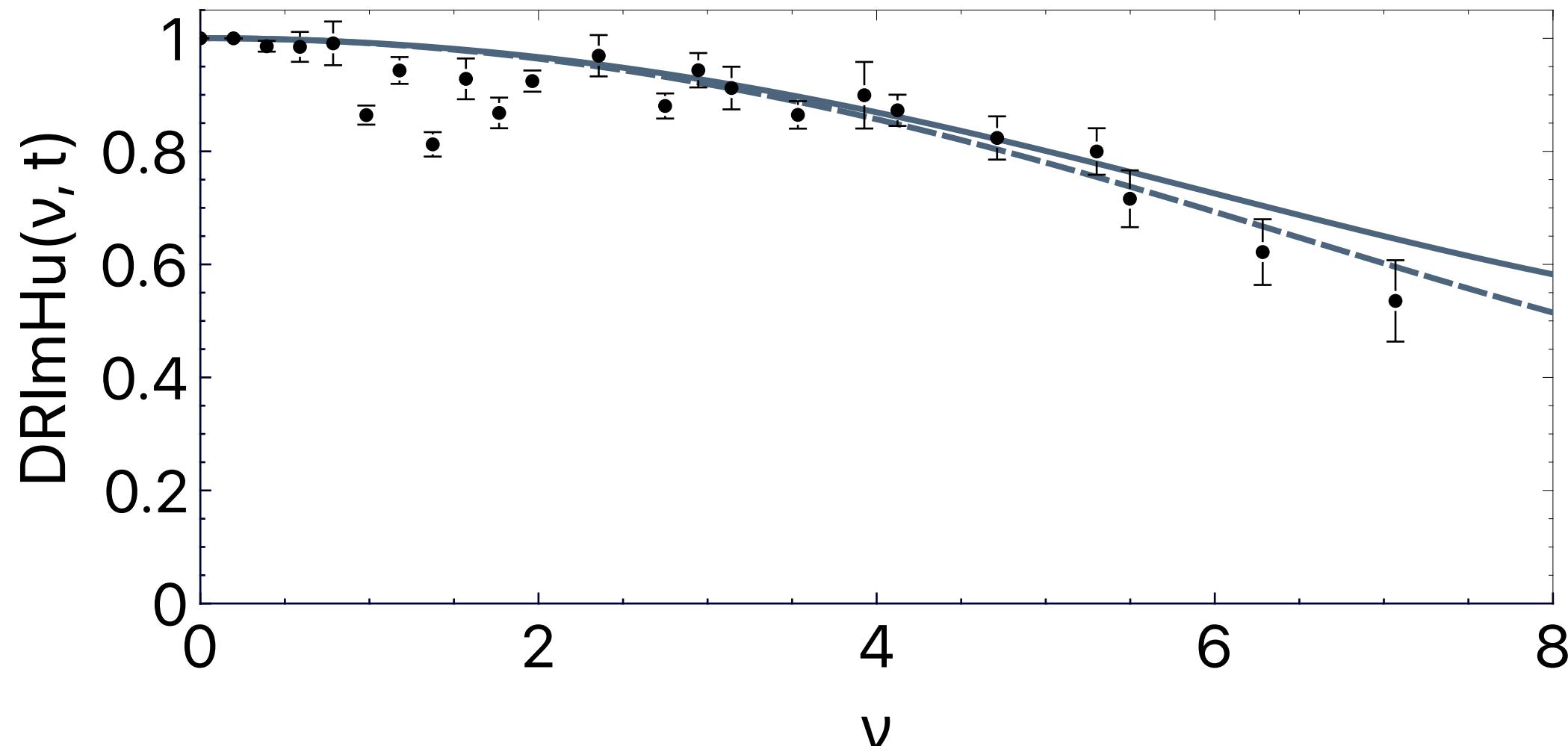
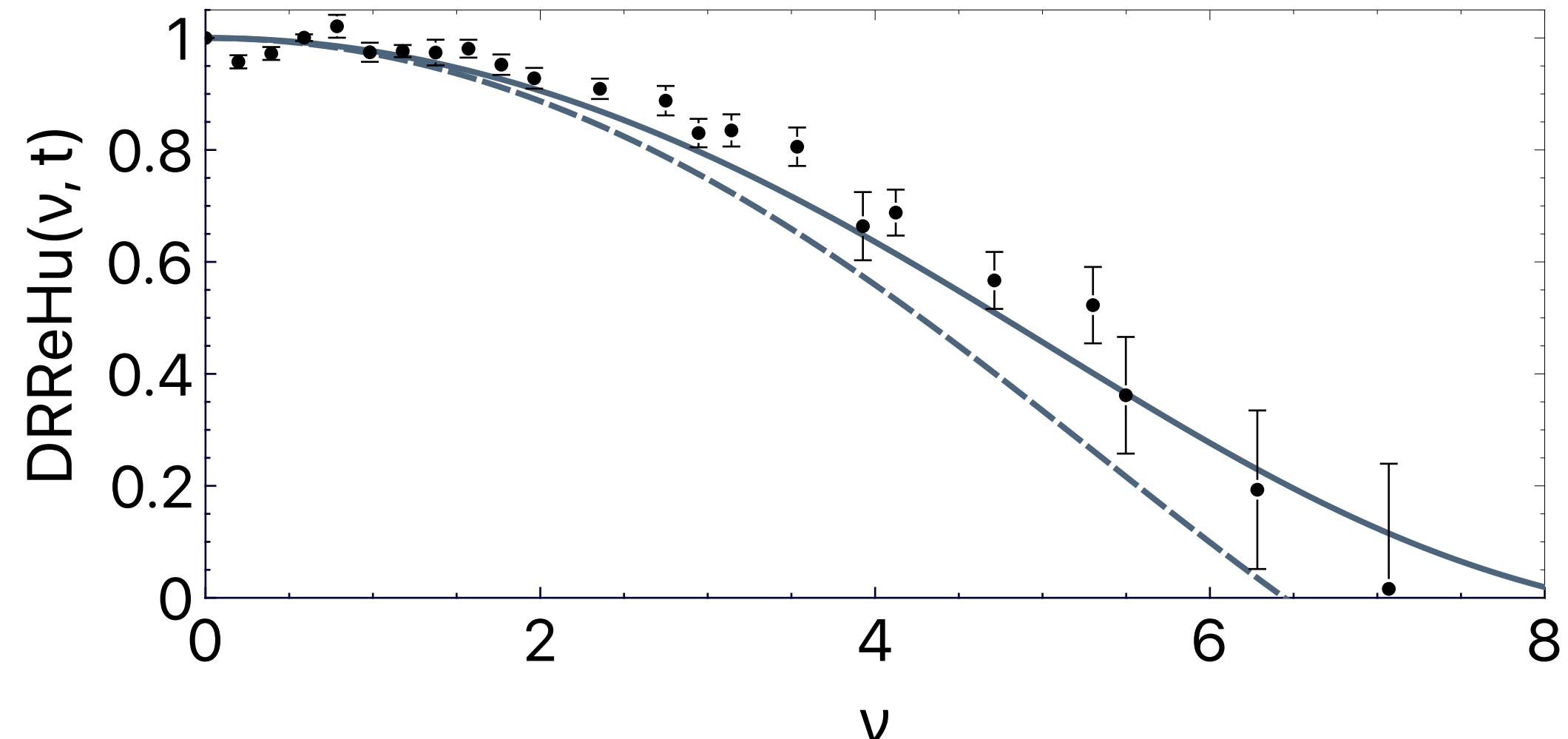
norm:
 2 for H^u
 1 for H^d
normalised
PDF
normalised
elastic FF
correlation

- double ratio describes deviation of $H(\nu, t)$ from factorised Ansatz: $H(\nu, t) = q(\nu)F(t)/\text{norm}$
- elastic FF not defined for imaginary part
- forward limit of GPD E not probed by (semi-)inclusive scattering

Double ratio

GPD H, $|t|=0.65 \text{ GeV}^2$

up	down	GK
		GK
		VGG

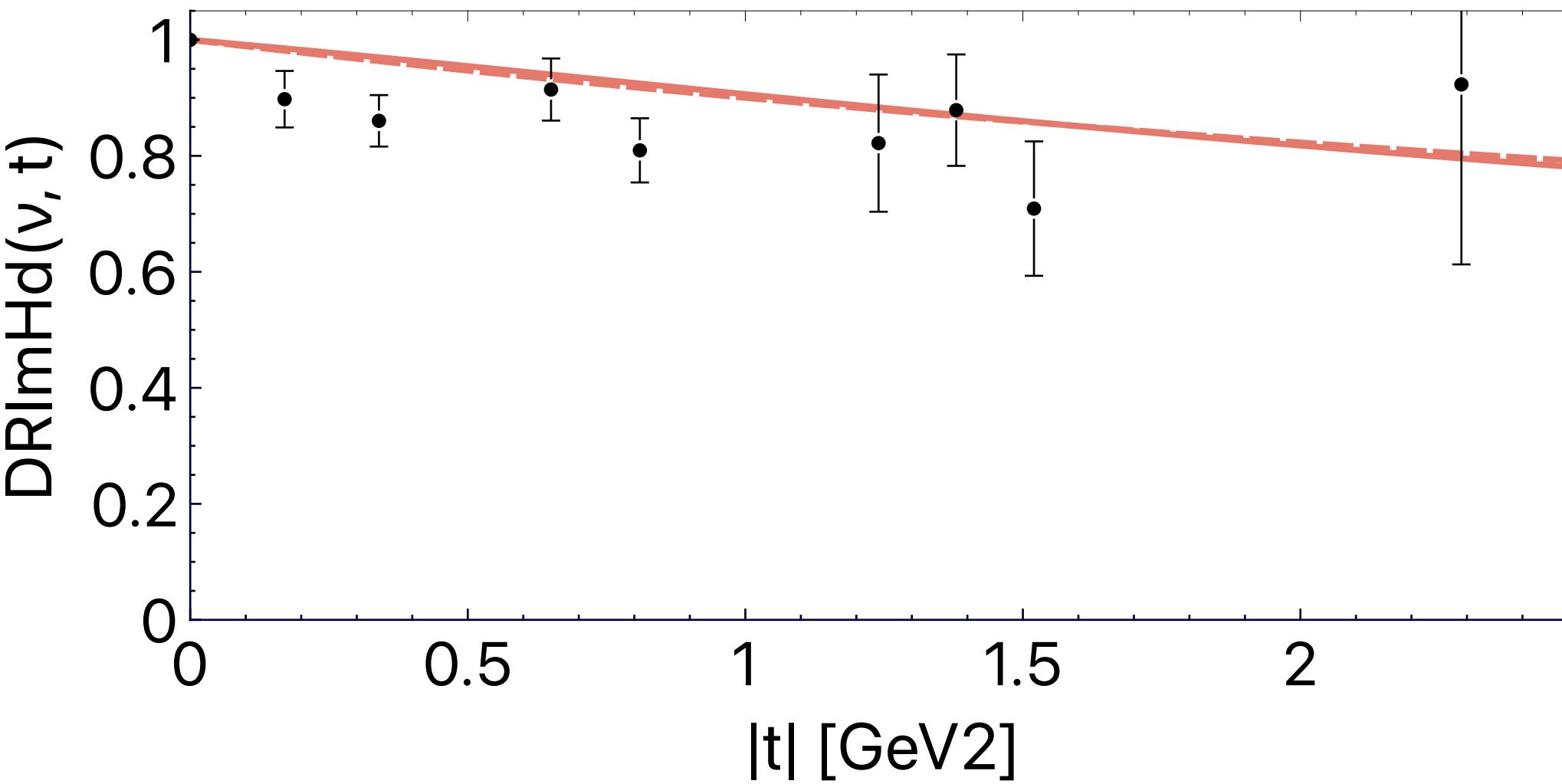
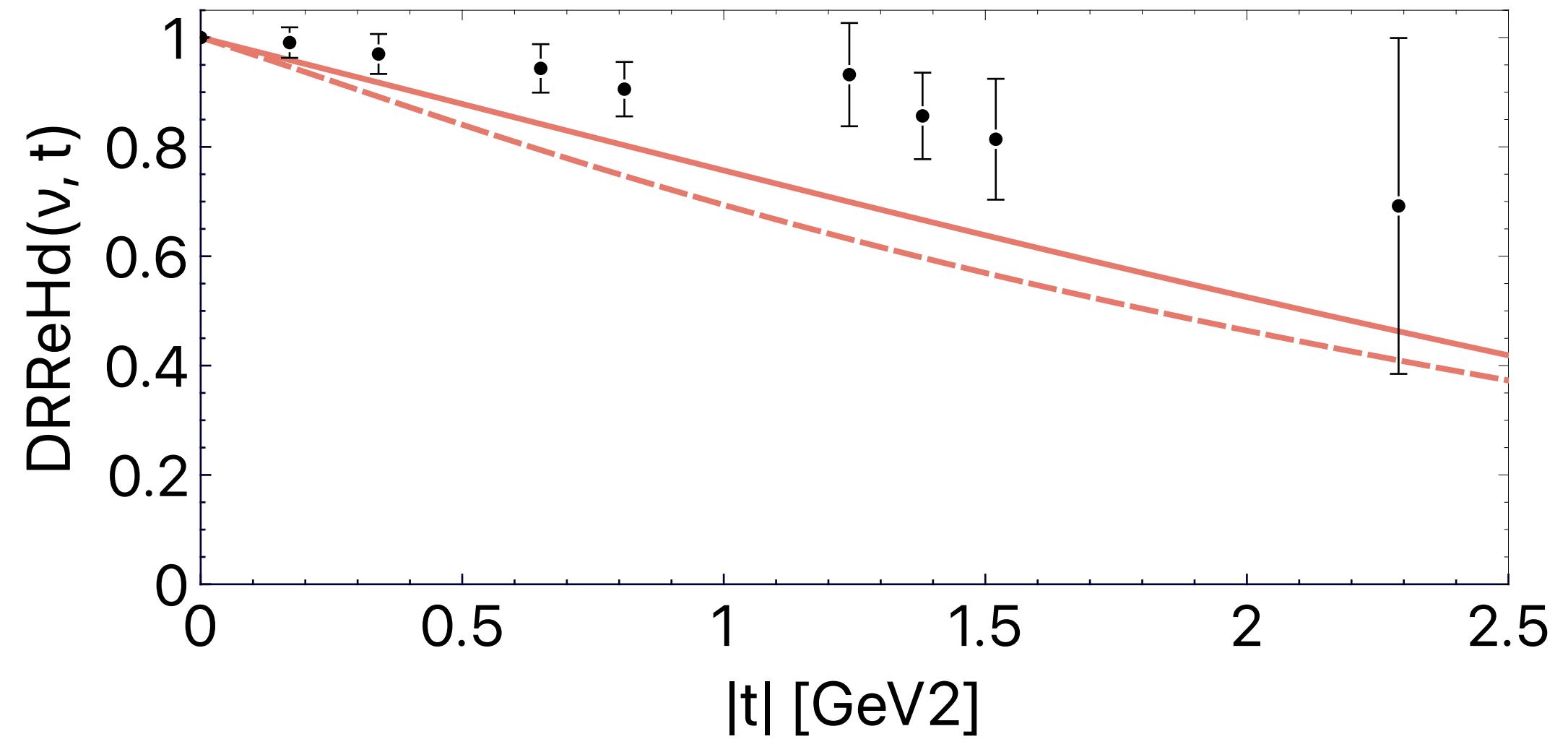
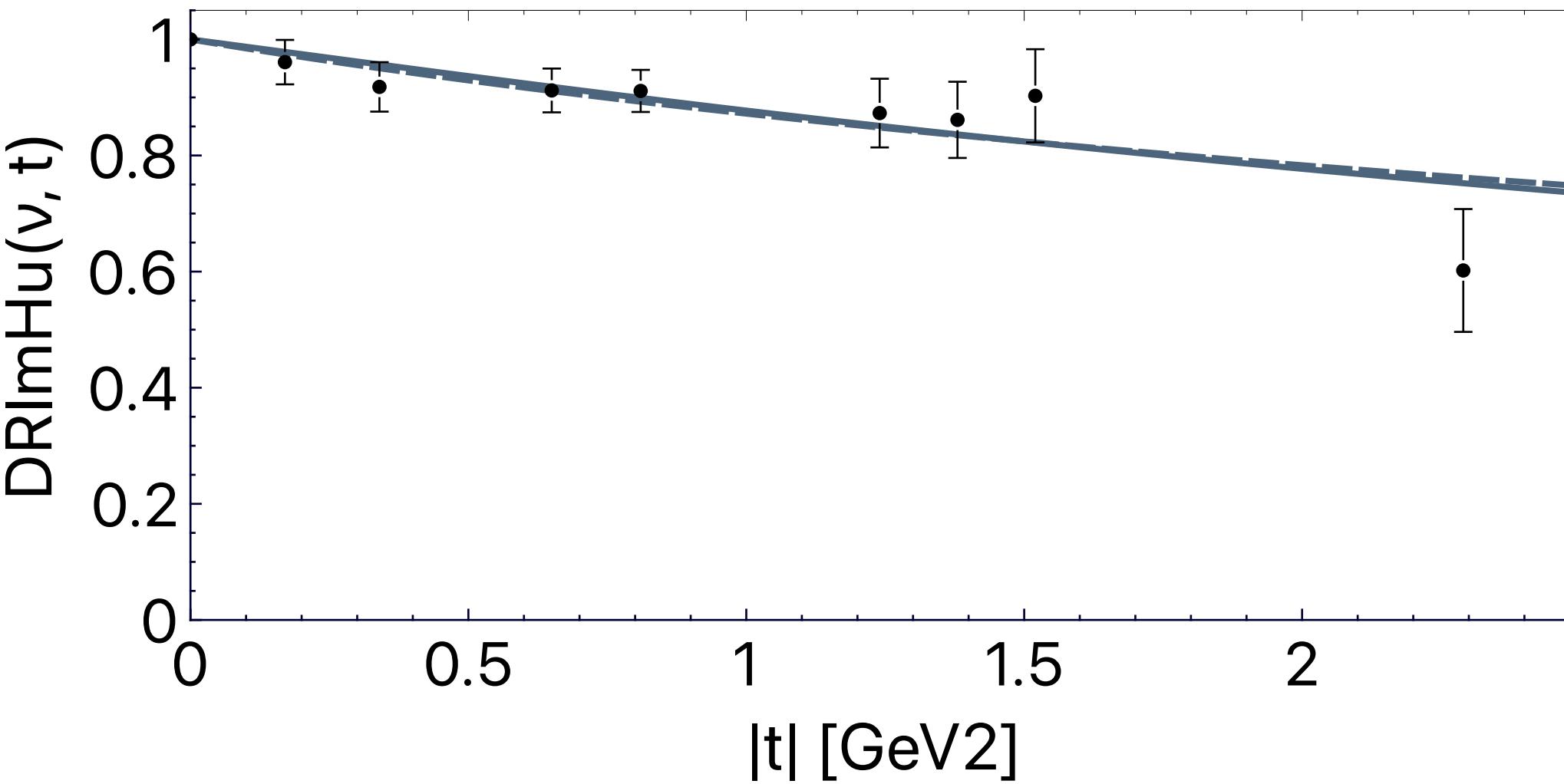
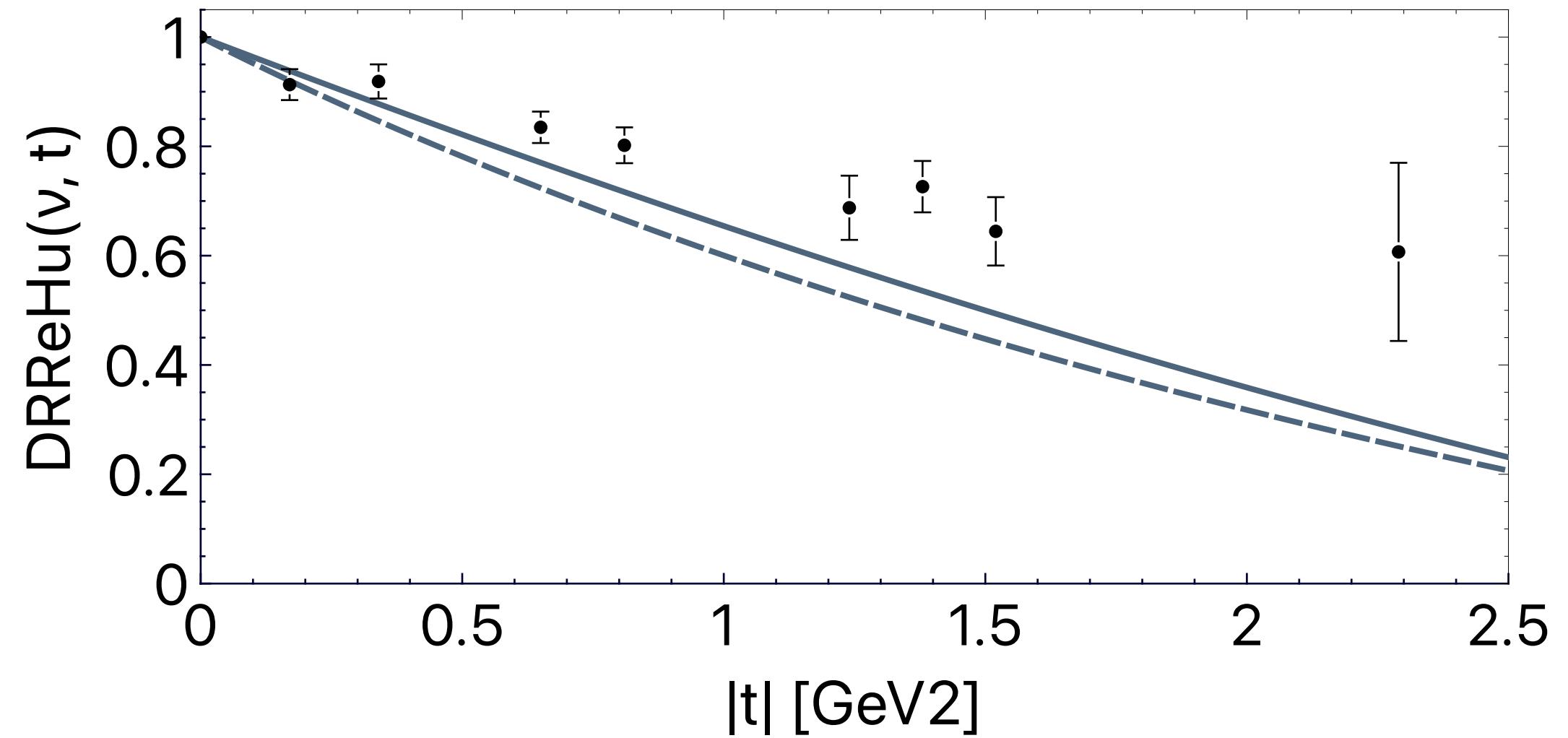


- double ratios evaluated from latticeQCD results comparable with models!

Double ratio

GPD H, $\nu=3.14$

up	down	GK
		GK
		VGG



- double ratios evaluated from latticeQCD results comparable with models!

Constraints:

$$H(x,0) = q(x)$$

$$\int_{-1}^1 dx H(x, t) = F(t)$$

$$\lim_{x \rightarrow 1} \frac{d}{dt} H(x, t) = 0$$

$$\lim_{x \rightarrow 1} \frac{d}{dt} \int_0^\infty d\nu H(b, t) \cos(x b) = 0$$

Strategy A:

Ansatz: $H(\nu, t) = q(\nu)F(t)\text{DR}(\nu, t)/\text{norm}$

1. Choose PDF and elastic FF parameterisations
2. Fit $\text{DR}(\nu, t)$ to latticeQCD data
3. evaluate $H(x,t)$
4. evaluate $H(x,b)$

Strategy B:

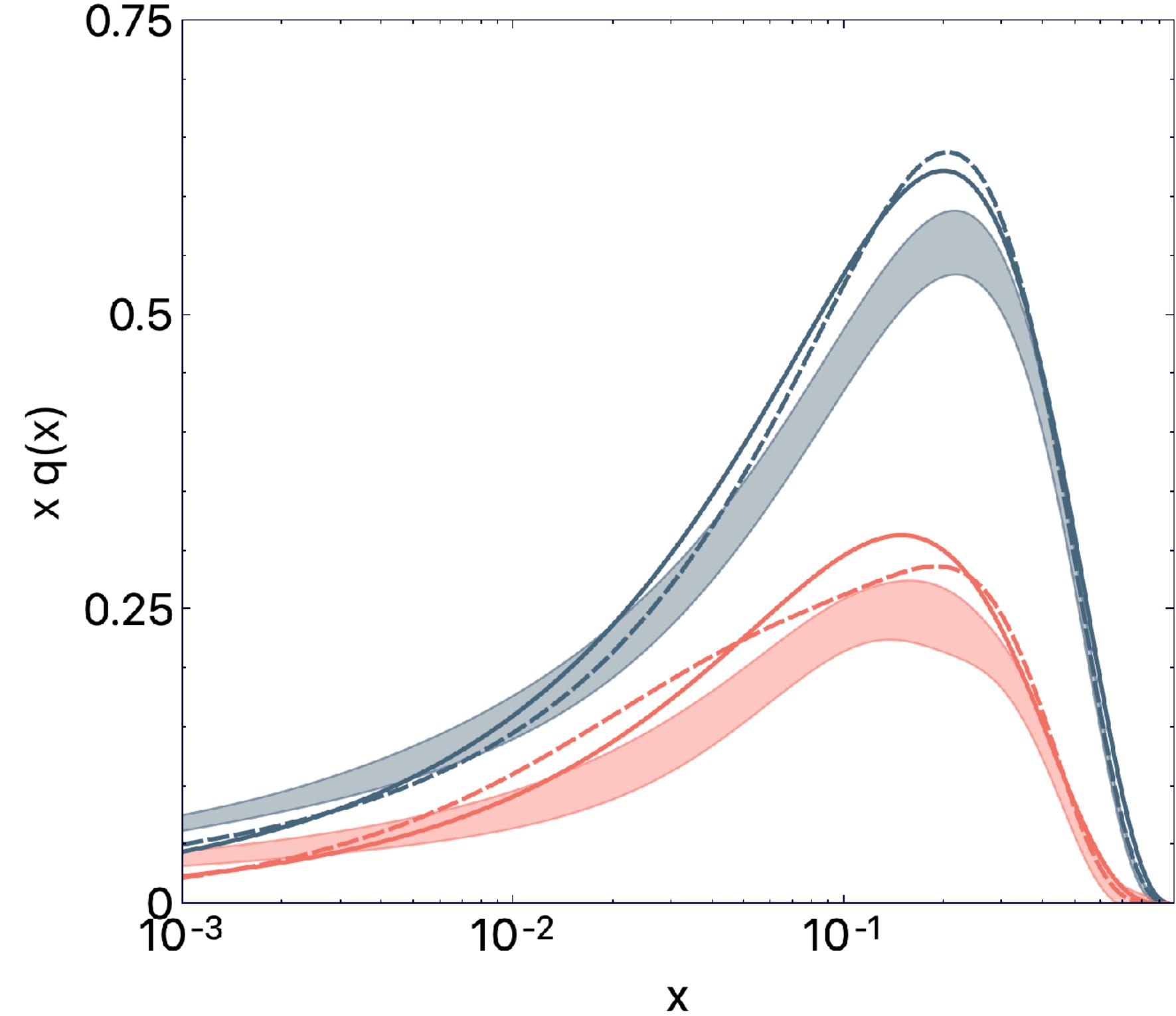
Ansatz: $H(x, t) = q(x)f(x, t)$

1. Choose PDF parameterisation
2. Fit $f(x,t)$ to latticeQCD and elastic FF data
3. evaluate $H(x,b)$

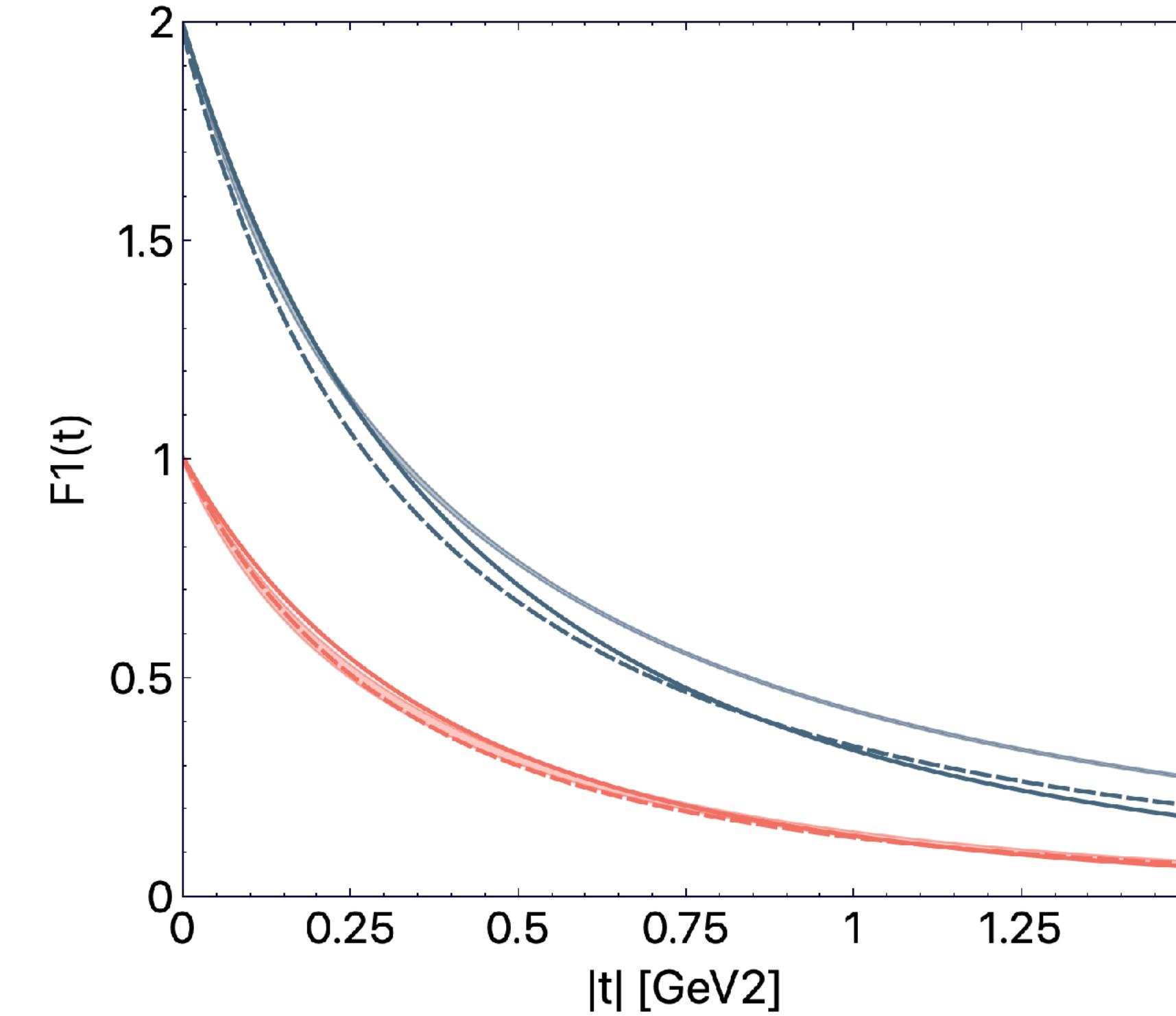
Input PDF and elastic FF

up	down	
		GK
		VGG
		Moutarde-S-Wagner

forward limit of GPD H (PDF, at $\mu = 2$ GeV)



Dirac FF

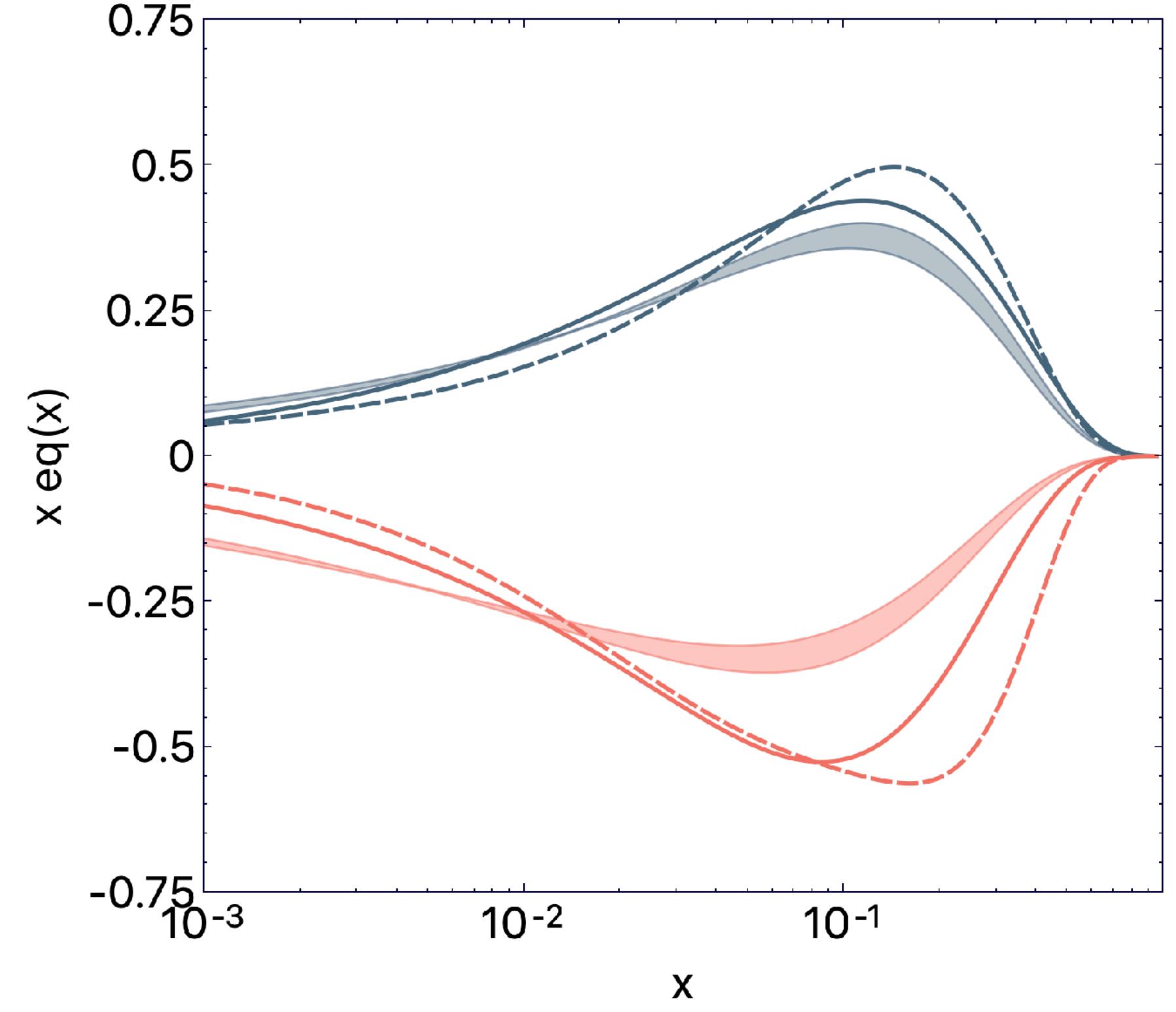


- input to the following analysis are Moutarde-S-Wagner parameterisations (EPJC 78 (2018), 890)
- these parameterisations come in a form of replica sets
- forward limit of GPD E available too

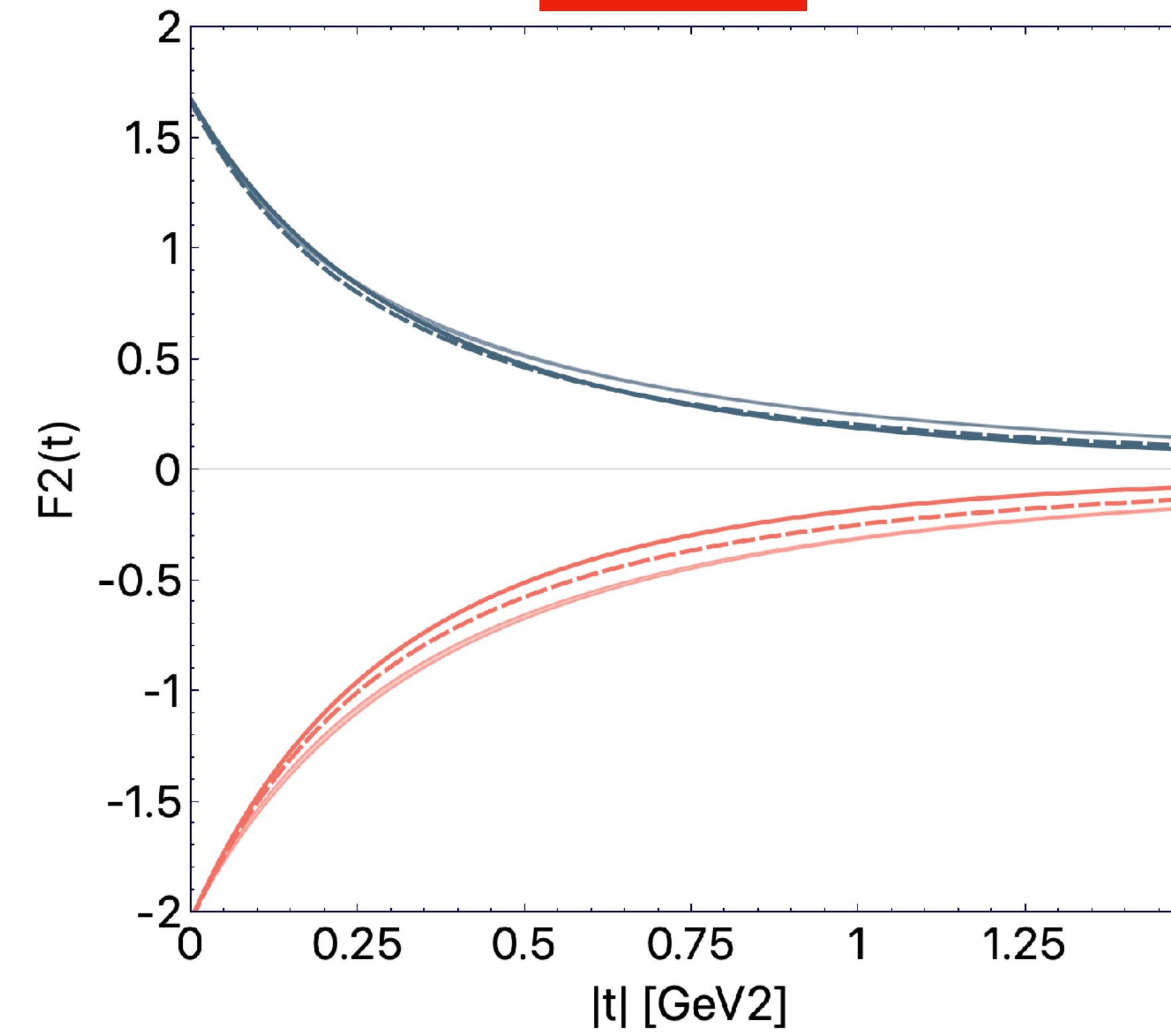
Input PDF and elastic FF

up	down	GK
		VGG
		Moutarde-S-Wagner

forward limit of GPD E (at $\mu = 2$ GeV)

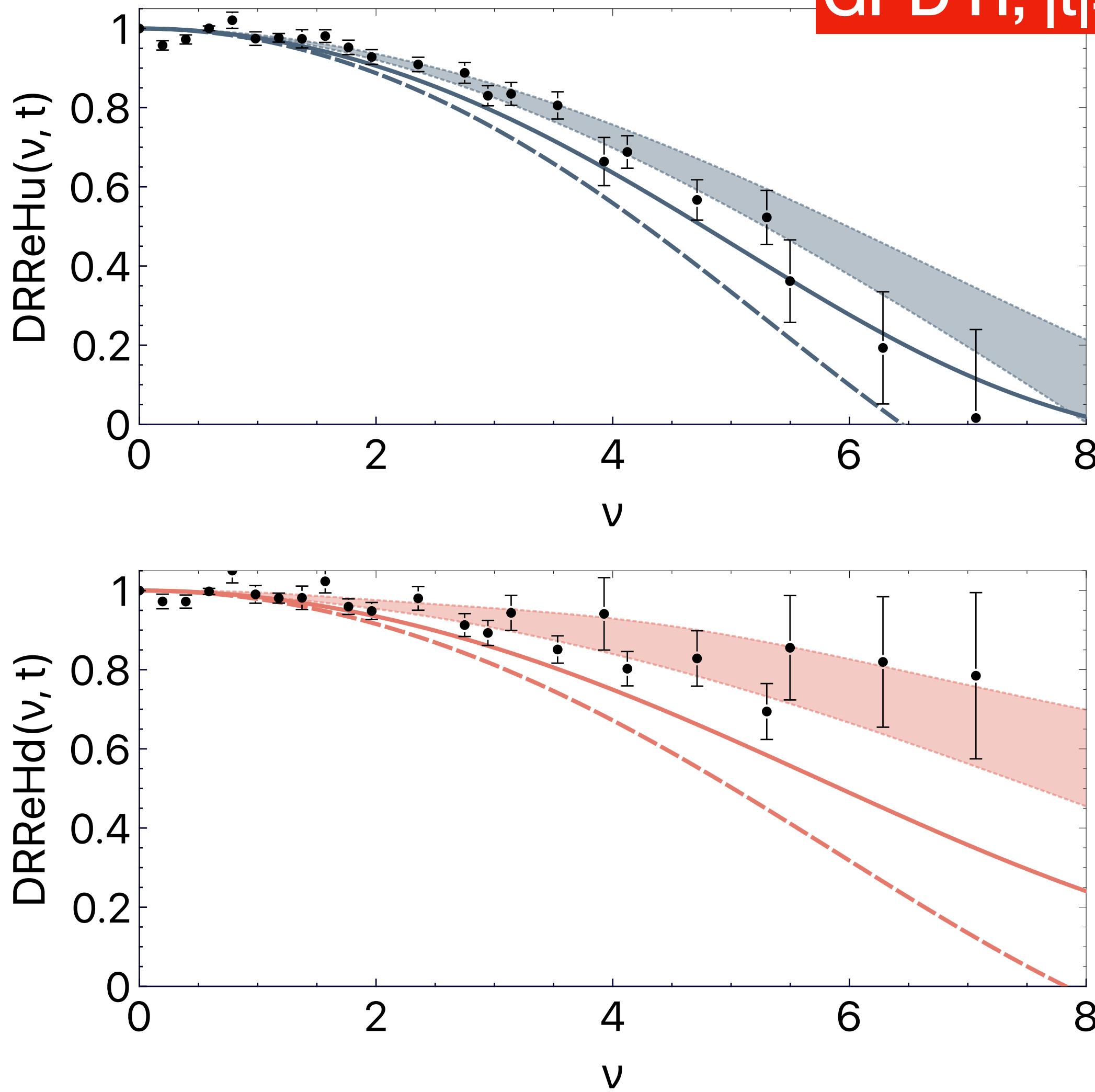


Pauli FF

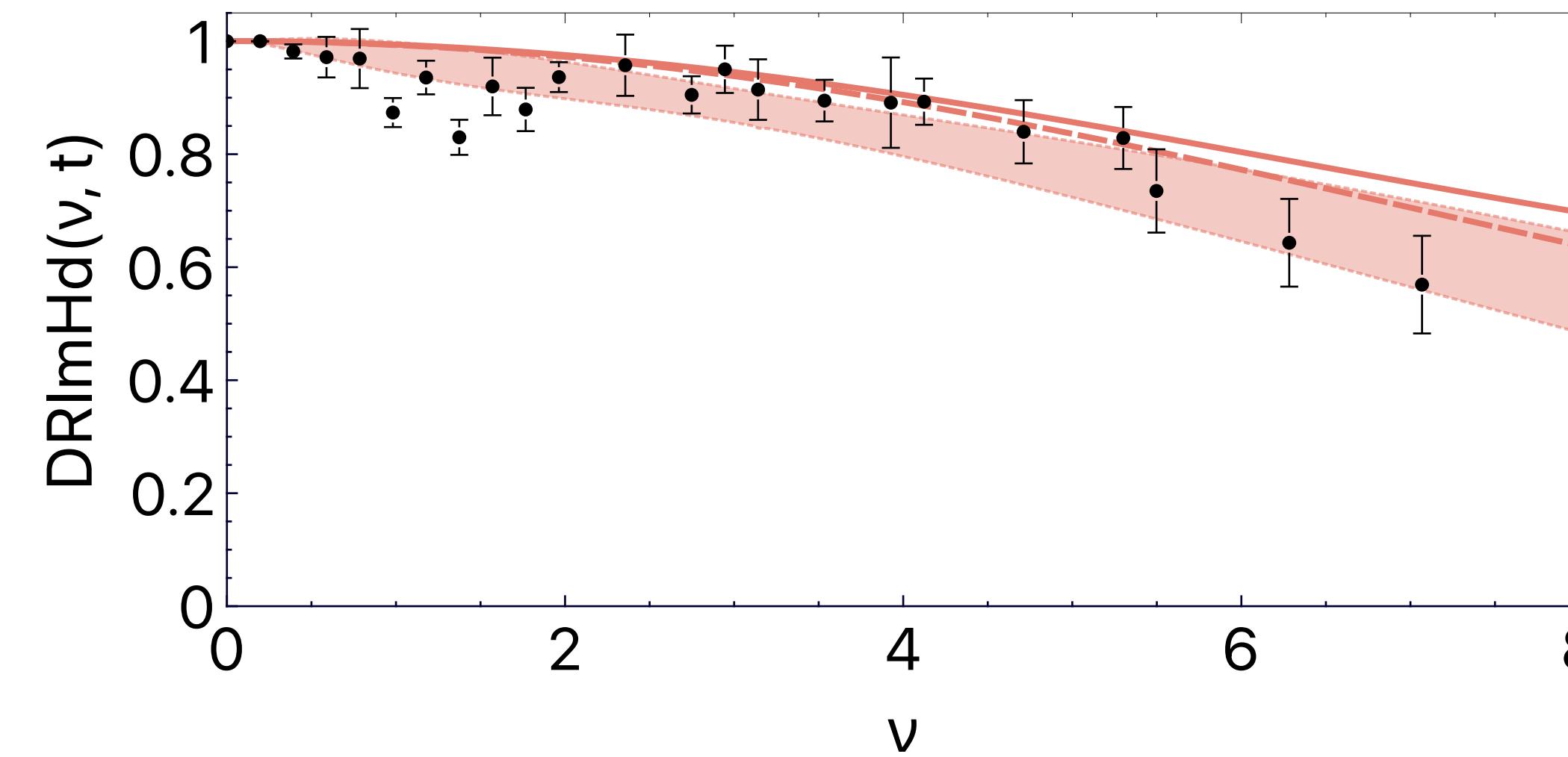
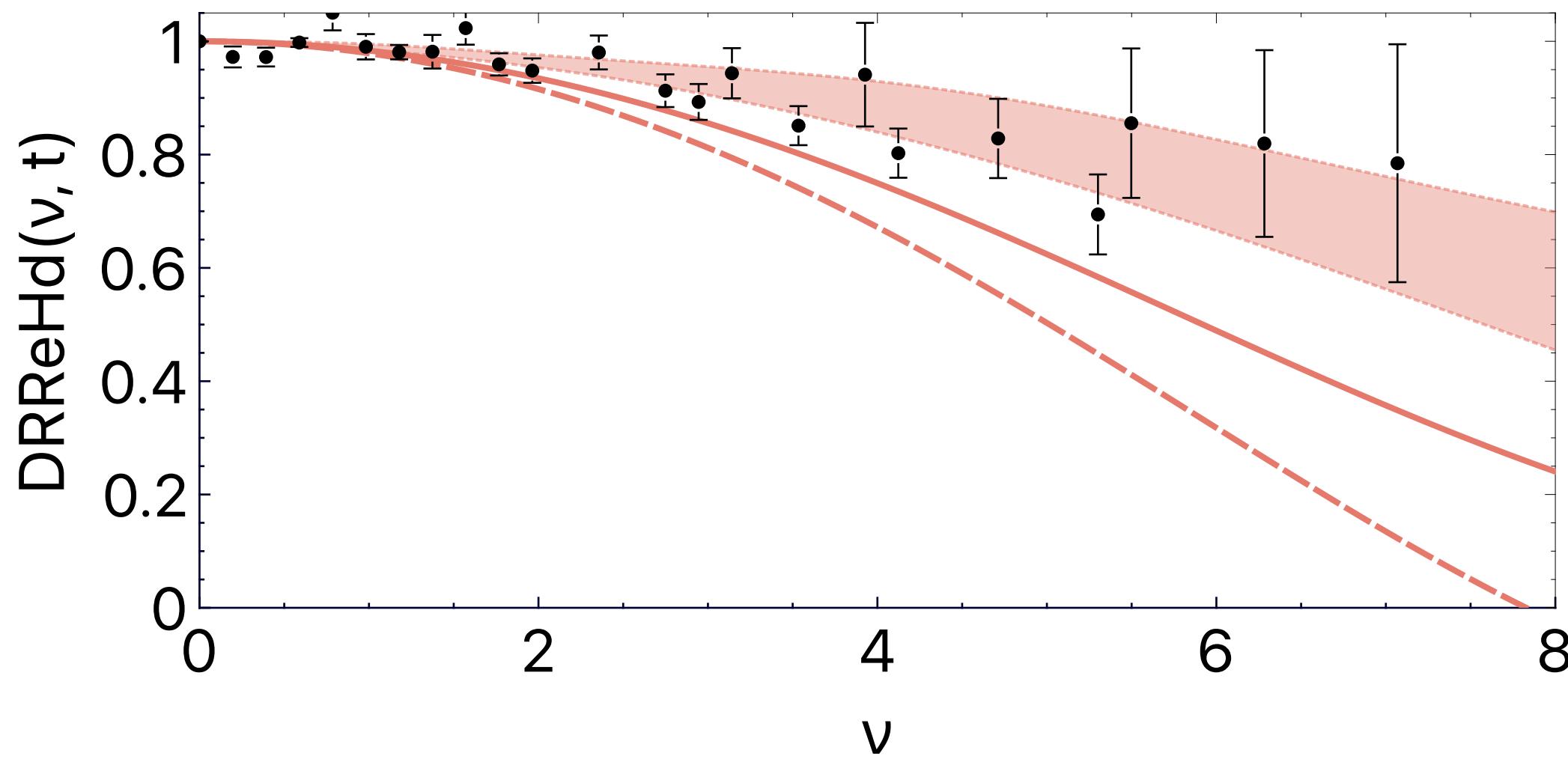
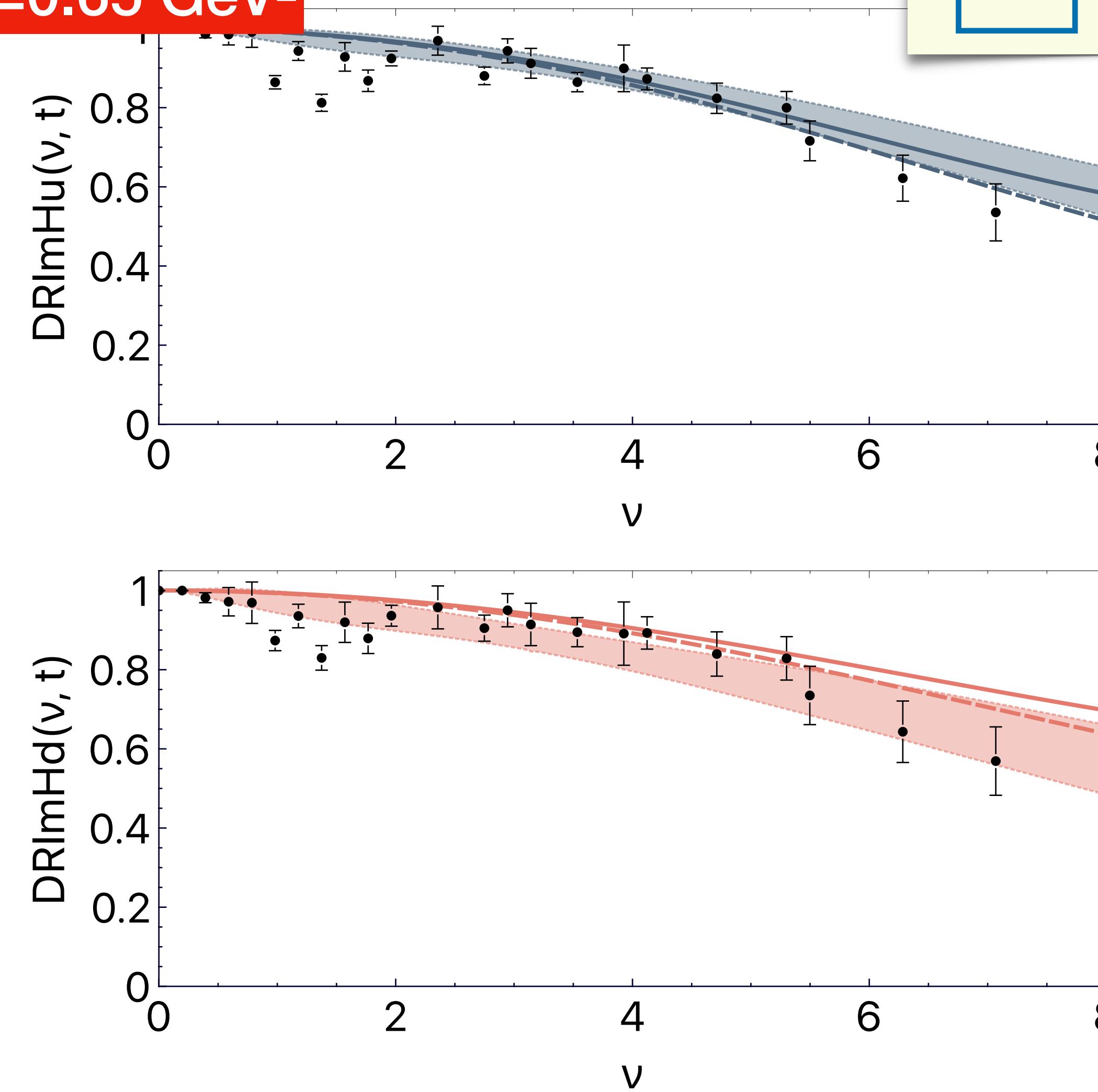


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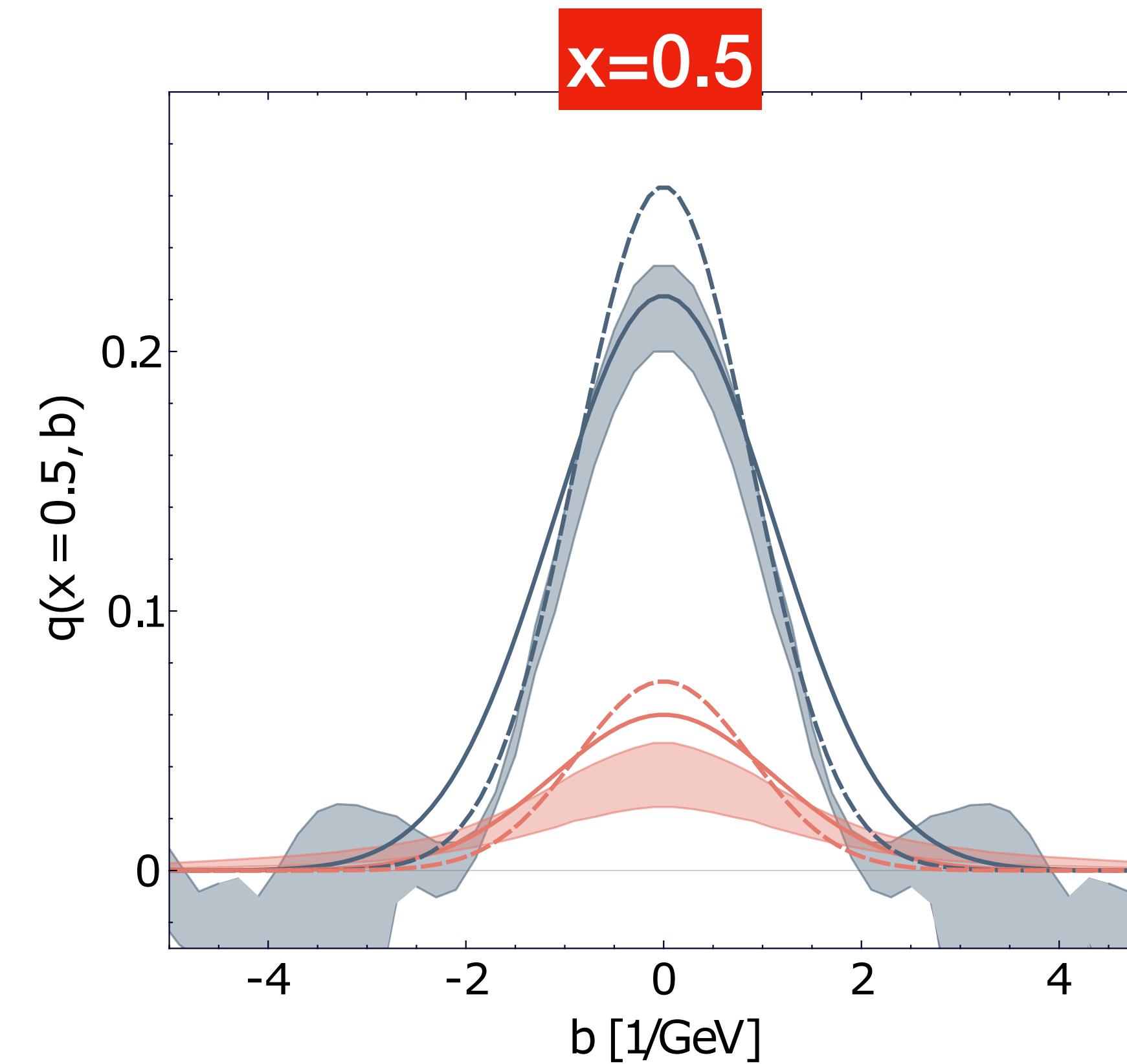
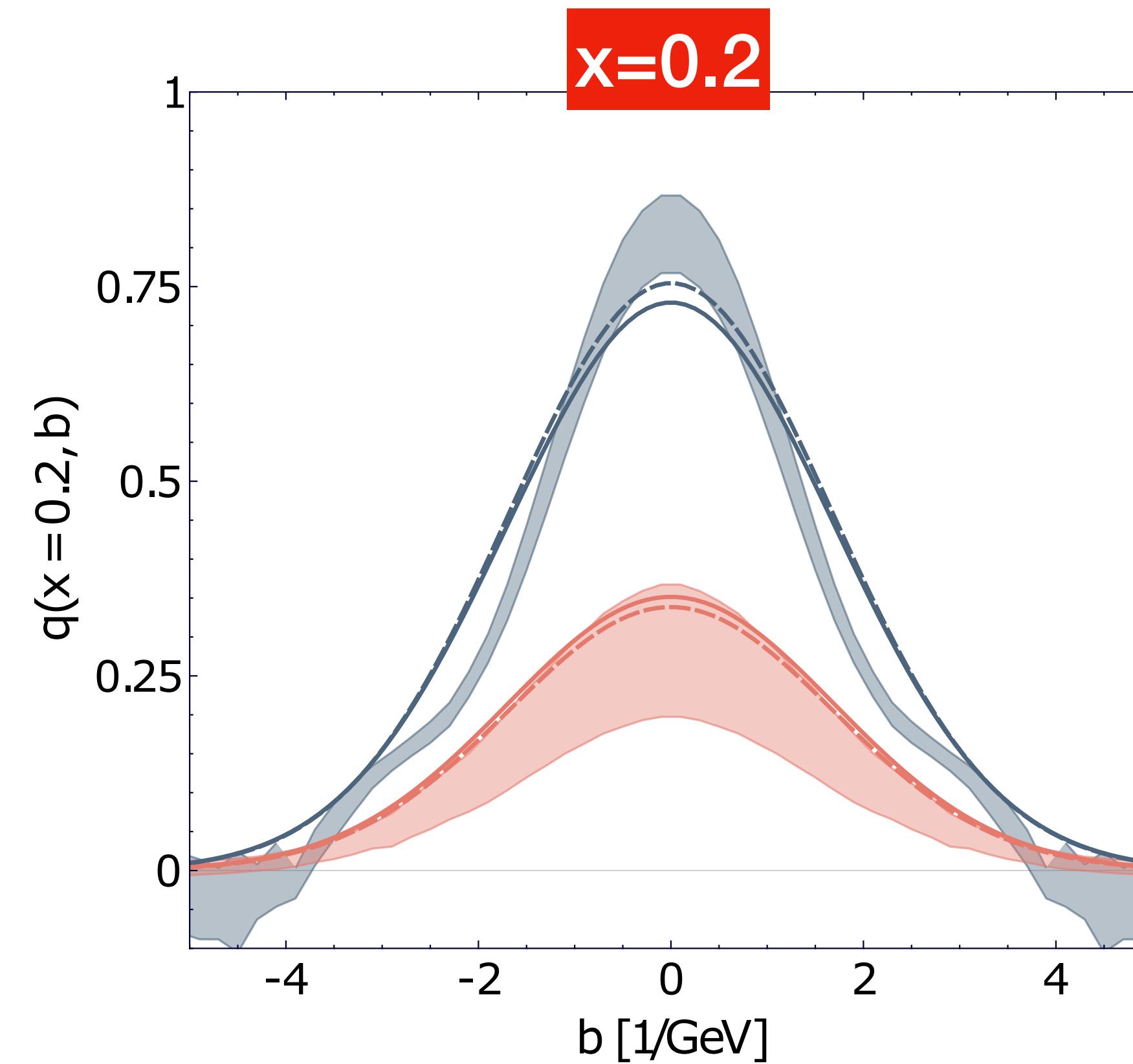
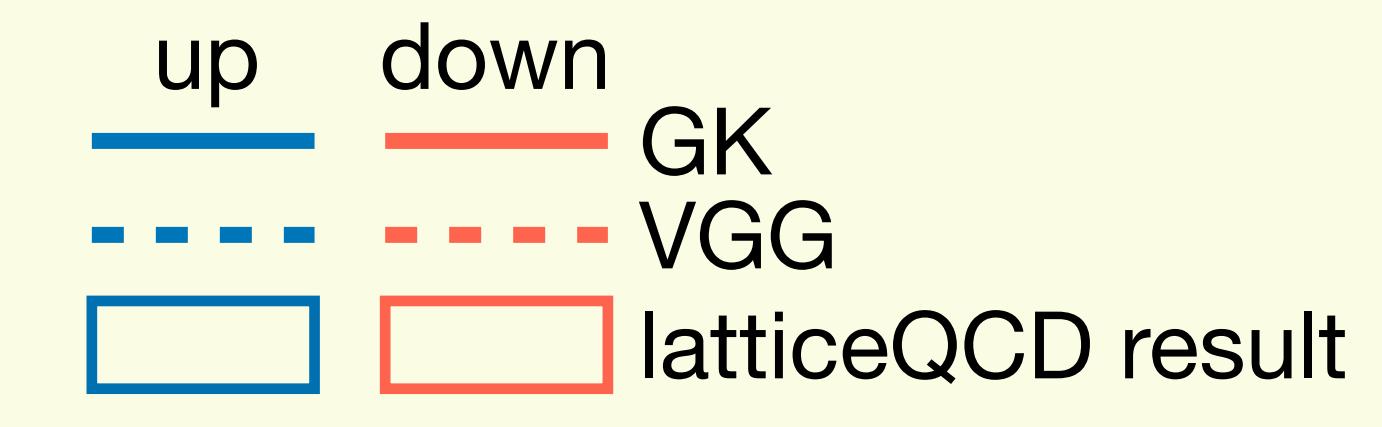
Fit to double ratios



GPD H, $|t|=0.65 \text{ GeV}^2$

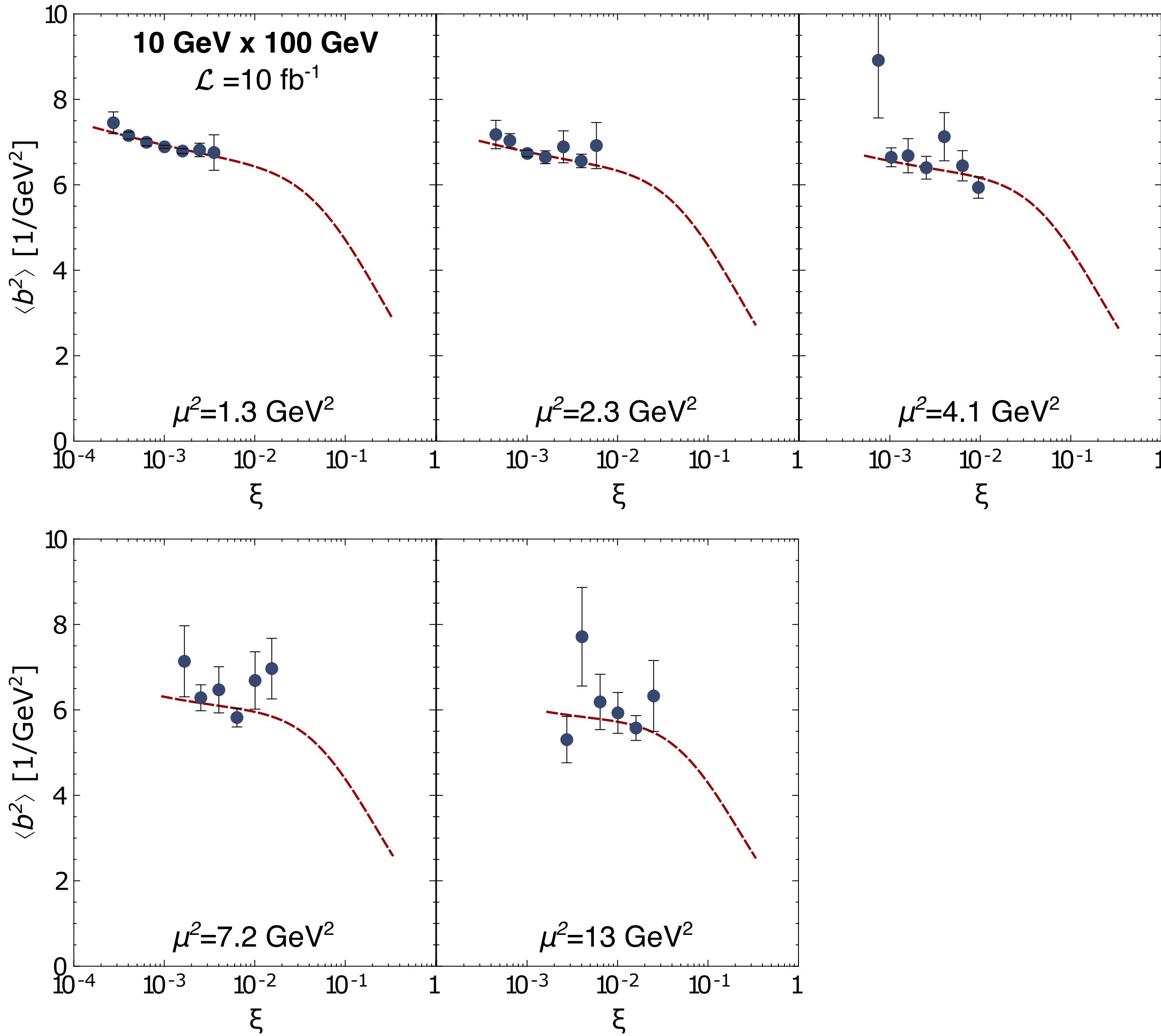


- fitting Ansatz: $\text{DR}_{\text{Re}}(\nu, t) = \frac{1}{\sum_i p_i} \sum_i p_i \exp(p'_i \nu^i t)$ with counter term proportional to νt to fulfil theory constraints



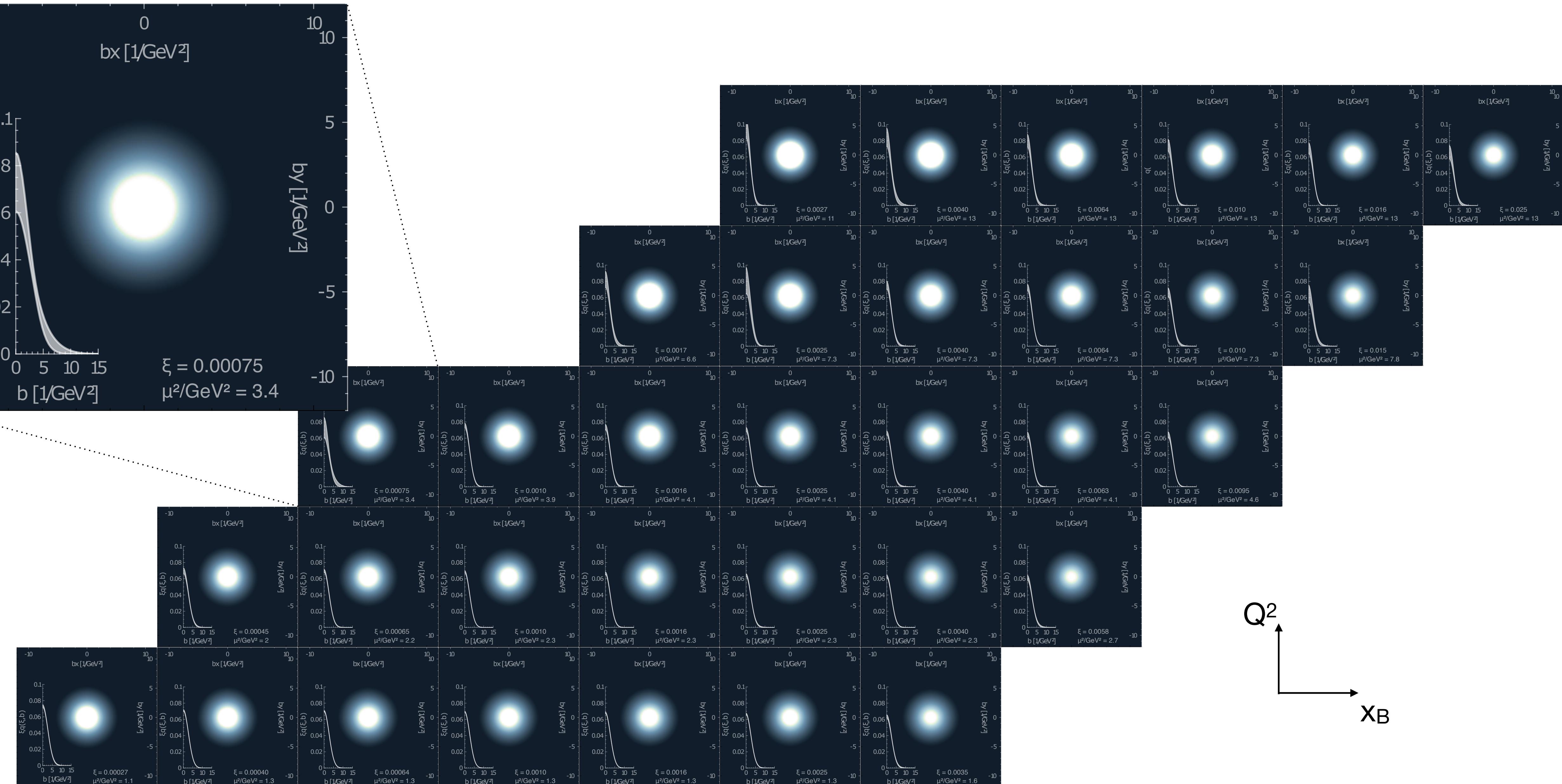
- uncertainties evaluated from latticeQCD, PDF and elasticFF replicas

What about the sea?



- new projections for DVCS at EIC (ePIC) by E. C. Aschenauer, V. Batozskaya, S. Fazio, A. Jentsch, K. Kumerički, H. Moutarde, K. Passek-K., D. Sokhan, H. Spiesberger, PS, K. Tezgin
- analysis based on EpIC MC generator and state-of-the-art description of ePIC apparatus
- includes projections for basic kinematic distributions, estimation of pi0 background and RC effects, extraction of nucleon tomography and Compton FFs
- analysis to be released soon
- see also Alex and Krešimir's talks

What about the sea?



- double ratio seems to be a good quantity to be used in global fits
→ a lot of systematics cancels out
- first (still preliminary) extraction of nucleon tomography information using this quantity ready
- complementary to the extraction from data!