Twist-3 GPDs from Lattice QCD

Martha Constantinou

Temple University

ECT* Workshop:
Towards improved hadron tomography with hard exclusive reactions

August 5 - 9, 2024

Recent developments in twist-3 PDFs/GPDs

S. Bhattacharya, K. Cichy, J. Dodson, A. Metz, J. Miller, A. Scapellato, F. Steffens

1

PHYSICAL REVIEW D **102,** 111501(R) (2020)

Rapid Communications | Editors' Suggestion

Insights on proton structure from lattice QCD: The twist-3 parton distribution function $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

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PHYSICAL REVIEW D **102,** 114025 (2020)

The role of zero-mode contributions in the matching for the twist-3 PDFs $e(x)$ and $h_L(x)$

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

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PHYSICAL REVIEW D **102,** 034005 (2020)

One-loop matching for the twist-3 parton distribution $g_T(x)$

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PHYSICAL REVIEW D **108**, 054501 (2023)

This talk

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

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J. Miller
(Grad student, Temple)

Outline

- ★ Twist classification (see [V. Braun, EPJ Web of Conferences 274, 01012 (2022)])

- ★ Approaches to access information on GPDs from lattice QCD

- ★ Definition of light-cone GPDs vs Euclidean lattice definition (quasi/pseudo GPDs)

- ★ New lattice results on axial twist-3 GPDs

- ★ Synergies with theory and phenomenology

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"it is no longer the dimension alone that determines the importance of an operator near the light-cone, but rather the difference between the dimension and spin"

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K. Cichy, this Workshop

- ★ New lattice results on axial twist-3 GPDs
- ★ Synergies with theory and phenomenology

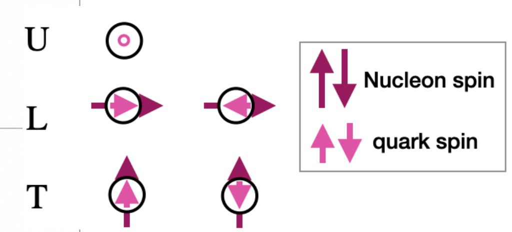
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
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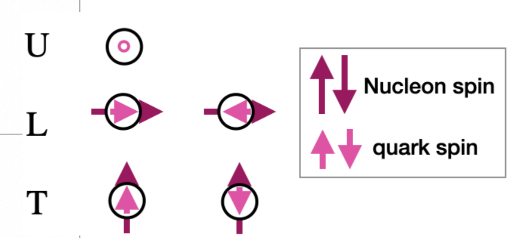

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★ Twist-2: probabilistic densities - a wealth of information exists (mostly on PDFs)

★ Twist-3: poorly known, but very important:
- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

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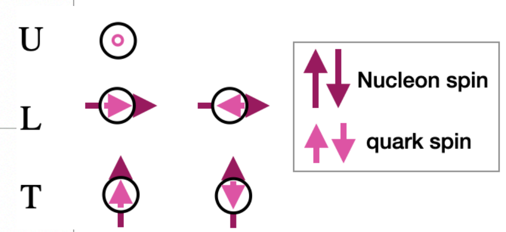

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While twist-3 $f_i^{(1)}$ share some similarities with twist-2 $f_i^{(0)}$ in their extraction, there are several challenges both experimentally and theoretically

Twist-3 PDFs, GPDs

★ **Certain observables require the use of twist-3 correlators**

★ **Proton collinear twist-3 PDFs:** $g_T(x),\ e(x),\ h_L(x)$

- chiral-even $g_T(x)$ couples to inclusive DIS
- $e(x),\ h_L(x)$: chiral-odd (need e.g. chirality flip process)
- $h_L(x)$: double-polarized Drell-Yan process, single-inclusive particle production in proton-proton collisions

★ **Twist-3 GPDs practically unknown; several challenges**

- inverse problem - shadow GPDs [Phys.Rev.D 103 (2021) 11, 114019, Phys.Rev.D 108 (2023) 3, 036027]

★ **Twist-3 GPDs contain physical information**

- $\widetilde{H} + \widetilde{G}_2$ related to tomography of F⊥ acting on the active q in DIS off a transversely polarized N right after the virtual photon absorbing [Phys.Rev.D 88 (2013) 114502, Phys.Rev.D 100 (2019) 9, 096021]
- Related to certain spin-orbit correlations [Phys.Lett.B 735 (2014) 344, Phys.Lett.B 774 (2017) 435]
- $G_2(x, \xi, t)$ related to $L_q^{\rm kin}$ [Phys.Lett.B 491 (2000) 96]

$$L_q^{\rm kin} = -\int_{-1}^{1} dx\, x\, G_2^q(x, \xi, t=0)$$

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) = \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma\overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Bigg\}$$

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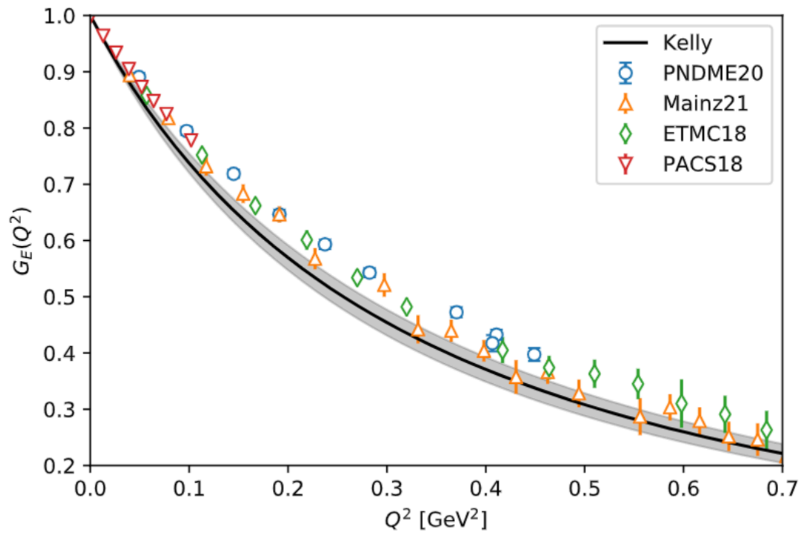

Wide -t range that comes at the cost of 1 (in the majority of cases)

Accessing information on GPDs

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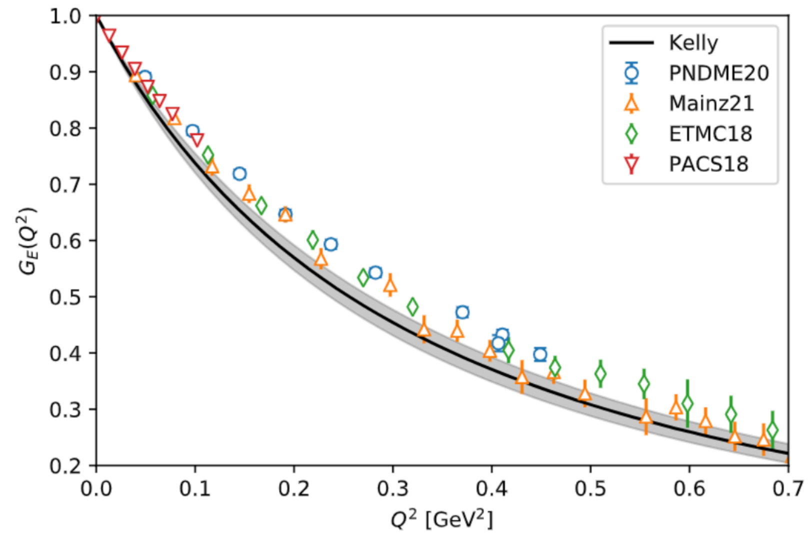

Wide -t range that comes at the cost of 1 (in the majority of cases)

★ Matrix elements of non-local operators
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

GPDs

Through non-local matrix elements

of momentum-boosted hadrons

Access of PDFs/GPDs on a Euclidean Lattice

Light-Cone:

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

Euclidean lattice:

- ★ Matrix elements of mom.-boosted states and nonlocal operators
- ★ Connection to light-cone GPDs through LaMET [X. Ji, PRL 110 (2013) 262002], SDF [A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}_\Gamma^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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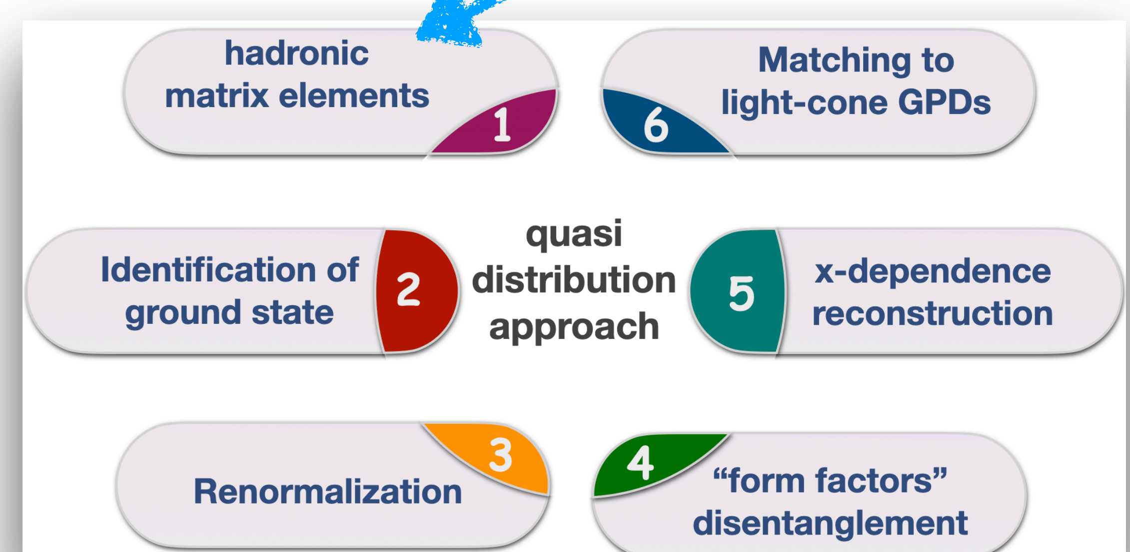
Computationally intensive

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$



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Aurora Scapellato,[1] and Fernanda Steffens[4]

Also Josh Miller

(Graduate student at Temple)

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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$$\begin{aligned}F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar u(p_f,\lambda')\Big[&P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde H}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde E}(x,\xi,t;P^3)\\ &+\Delta^\mu_\perp\frac{\gamma_5}{2m}F_{\widetilde E+\widetilde G_1}(x,\xi,t;P^3) + \gamma^\mu_\perp\gamma_5 F_{\widetilde H+\widetilde G_2}(x,\xi,t;P^3)\\ &+\Delta^\mu_\perp\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde G_3}(x,\xi,t;P^3) + i\varepsilon^{\mu\nu}_\perp\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde G_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)\end{aligned}$$

★ Twist-3 contributions to helicity GPDs: $\gamma^{1,2}\gamma_5$

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

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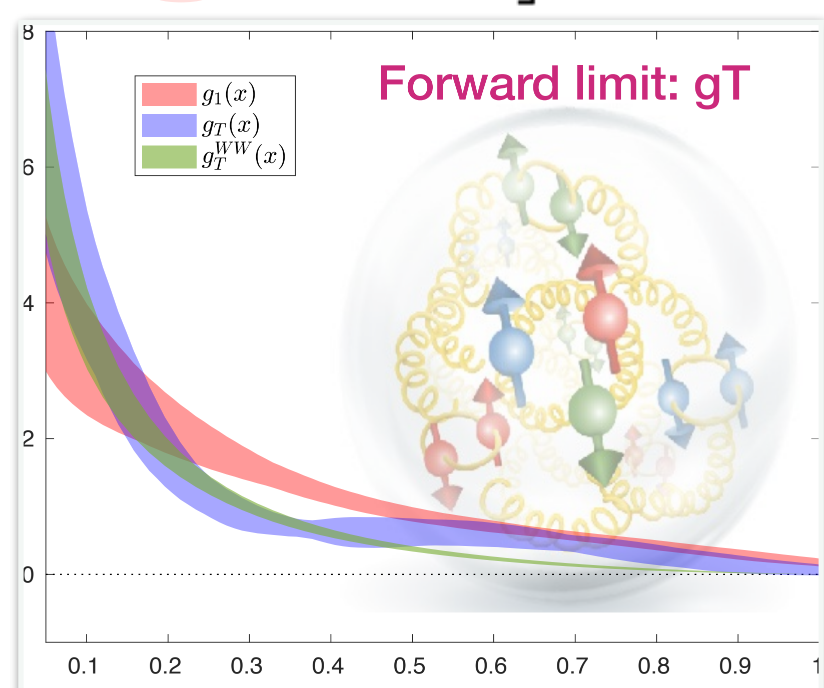

[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

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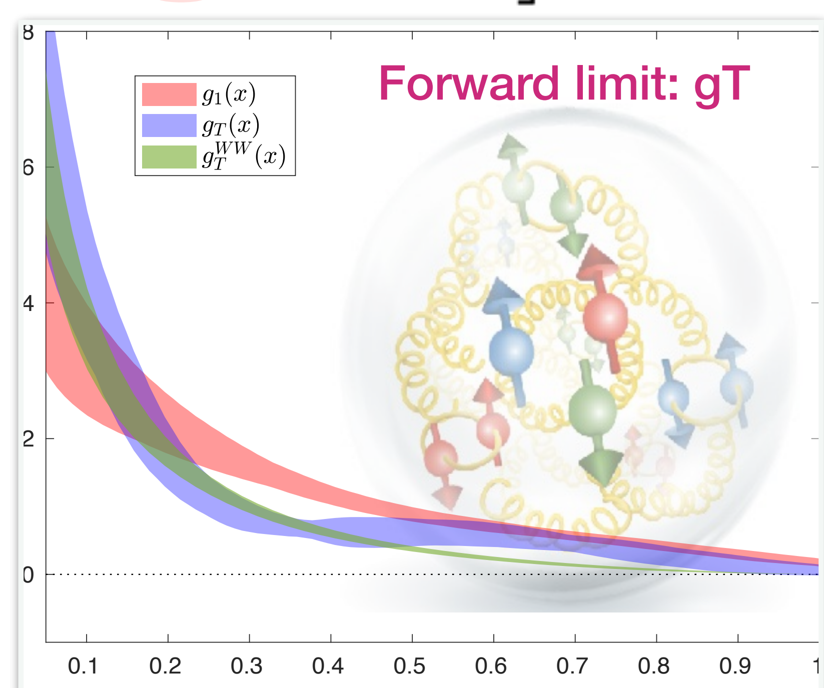
★ Twist-3 contributions to helicity GPDs: $\gamma^{1,2}\gamma_5$

★ Kinematic twist-3 contributions to pseudo- and quasi-GPDs to restore translation invariance

[V. Braun et al., JHEP 10 (2023) 134]



[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

★ Calculation of connected diagram

P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
±0.83	(0, 0, 0)	0	2	194	8	3104
±1.25	(0, 0, 0)	0	2	731	16	23392
±1.67	(0, 0, 0)	0	2	1644	64	210432
±0.83	(±2, 0, 0)	0.69	8	67	8	4288
±1.25	(±2, 0, 0)	0.69	8	249	8	15936
±1.67	(±2, 0, 0)	0.69	8	294	32	75264
±1.25	(±2, ±2, 0)	1.38	16	224	8	28672
±1.25	(±4, 0, 0)	2.76	8	329	32	84224

Symmetric frame computationally expensive

Zero skewness calculation

Decomposition

★ Requirement: four independent matrix elements

★ Average kinematically equivalent matrix elements

$$\Pi^1(\Gamma_0) = C\left(- F_{\widetilde{H}+\widetilde{G}_2} \frac{P_3 \Delta_y}{4m^2} - F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_y (E+m)}{2m^2} \right),$$

$$\Pi^1(\Gamma_1) = i\, C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{(4m(E+m) + \Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_x^2 (E+m)}{8m^3} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_y^2 (E+m)}{4m^2 P_3} \right)$$

$$\Pi^1(\Gamma_2) = i\, C\left(- F_{\widetilde{H}+\widetilde{G}_2} \frac{\Delta_x \Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_x \Delta_y (E+m)}{8m^3} - F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_x \Delta_y (E+m)}{4m^2 P_3} \right),$$

$$\Pi^1(\Gamma_3) = C\left(- F_{\widetilde{G}_3} \frac{E \Delta_x (E+m)}{2m^2 P_3} \right),$$

$$\Pi^2(\Gamma_0) = C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{P_3 \Delta_x}{4m^2} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_x (E+m)}{2m^2} \right),$$

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$$\Pi^2(\Gamma_2) = i\, C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{(4m(E+m) + \Delta_x^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_y^2 (E+m)}{8m^3} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_x^2 (E+m)}{4m^2 P_3} \right)$$

$$\Pi^2(\Gamma_3) = C\left(- F_{\widetilde{G}_3} \frac{E \Delta_y (E+m)}{2m^2 P_3} \right),$$

Consistency Checks

★ Sum Rules (generalization of Burkhardt-Cottingham)

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

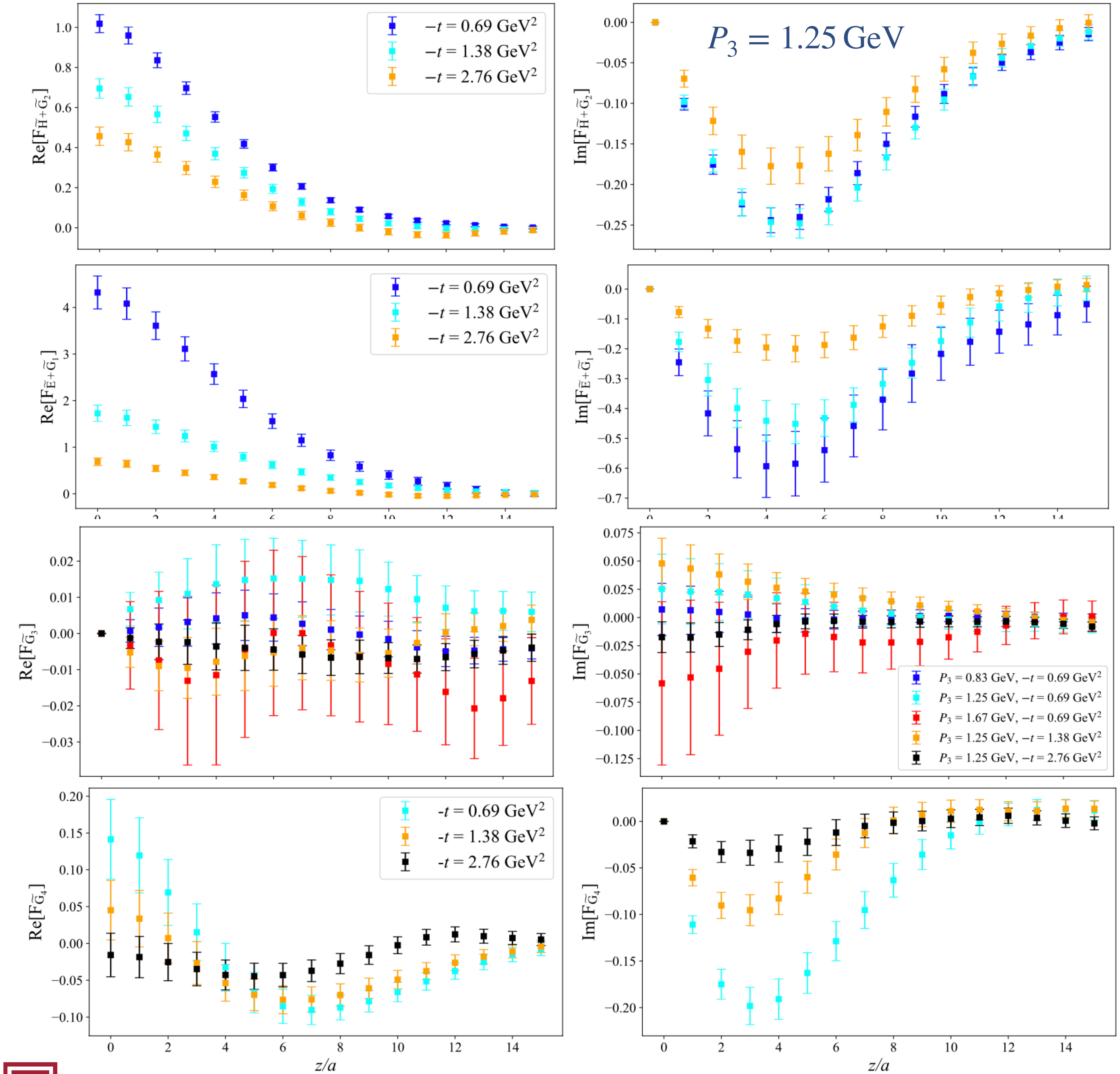
★ Sum Rules (generalization of Efremov-Leader-Teryaev)

[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = \frac{\xi}{4} G_E \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4} G_E(t)$$

G_E : electric FF

Selected Results - quasi-GPDs



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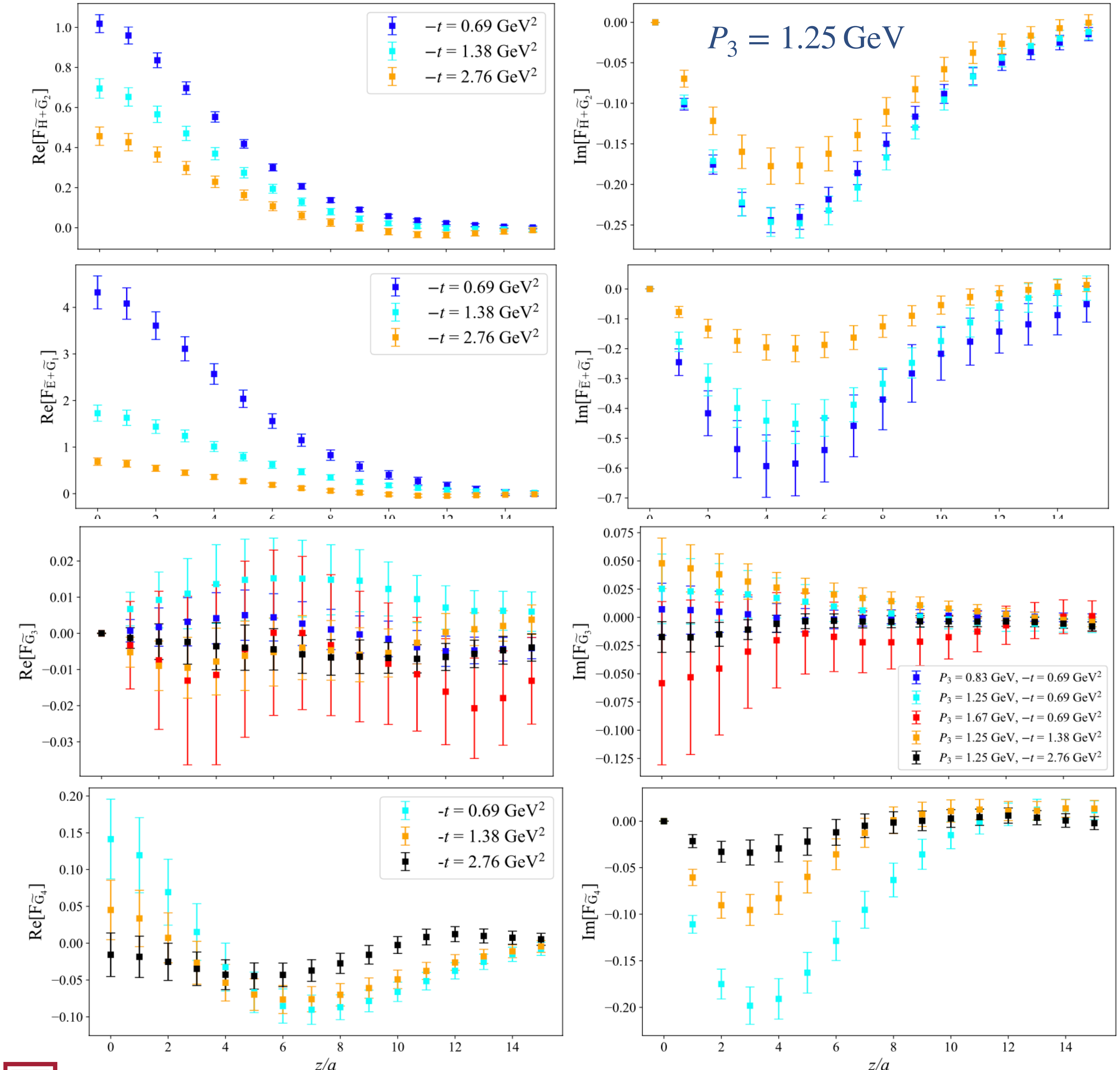
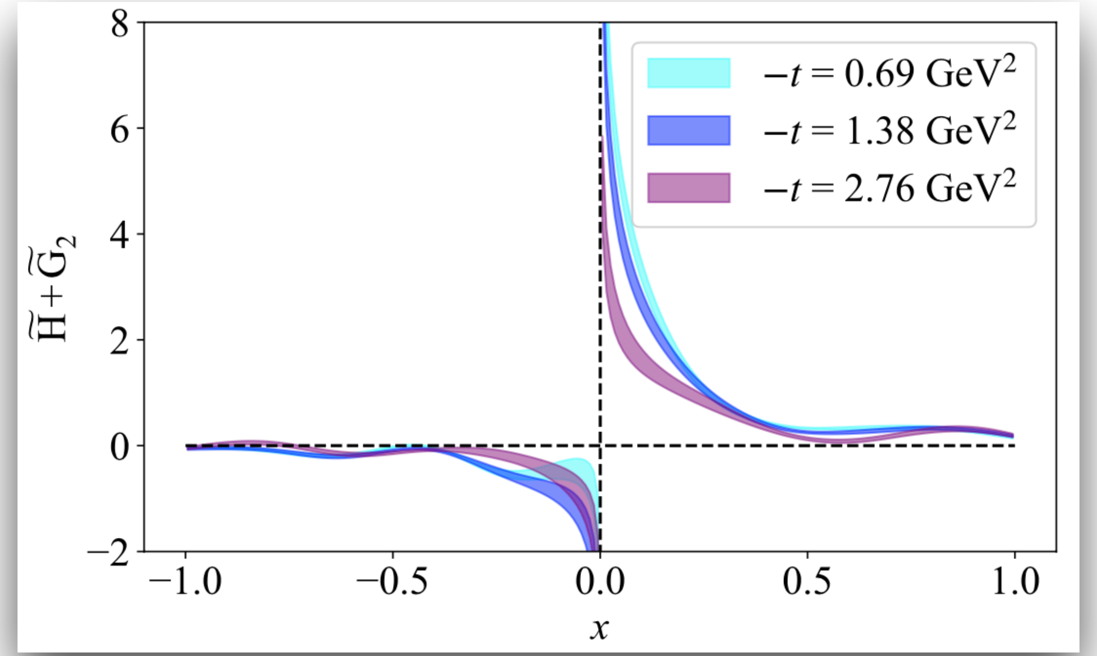


$$\int dx\, x\, \widetilde{G}_3 = \frac{\xi}{4} G_E(t)$$

Indeed, numerically found to be zero within uncertainties at ξ=0

Lattice Results - light-cone GPDs



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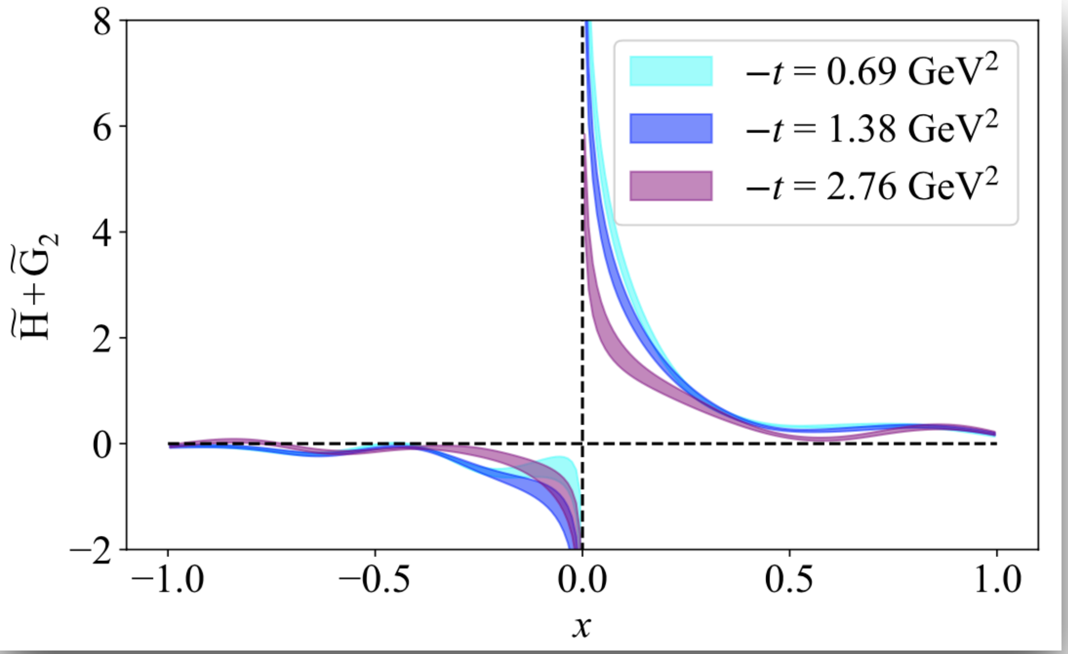
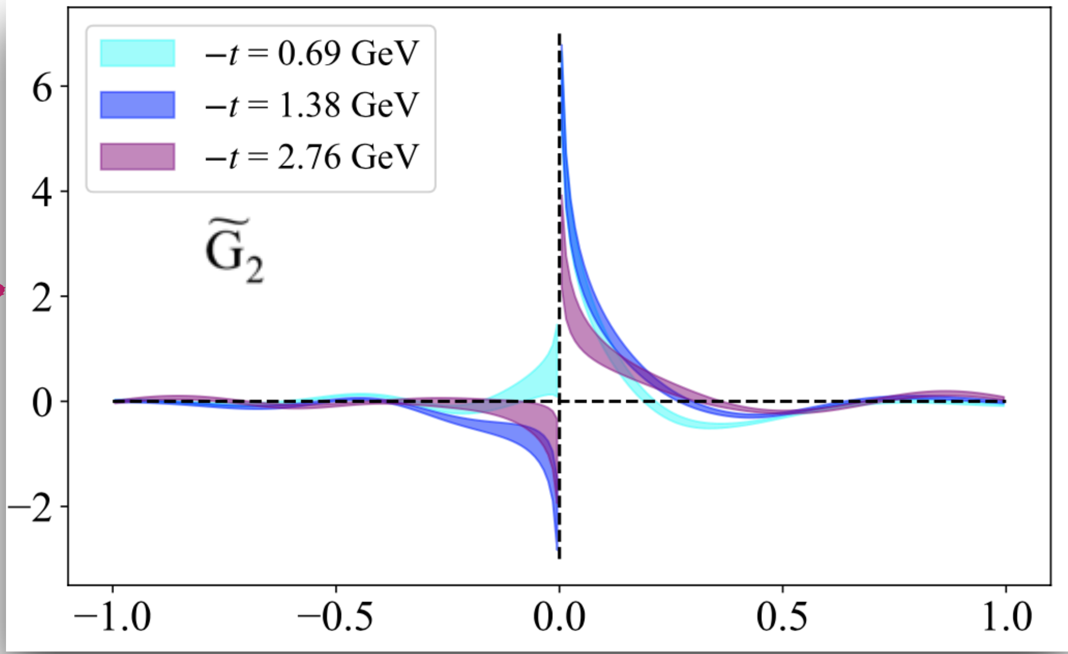


Isolating $\widetilde{G}_2$

using $\widetilde{H}$



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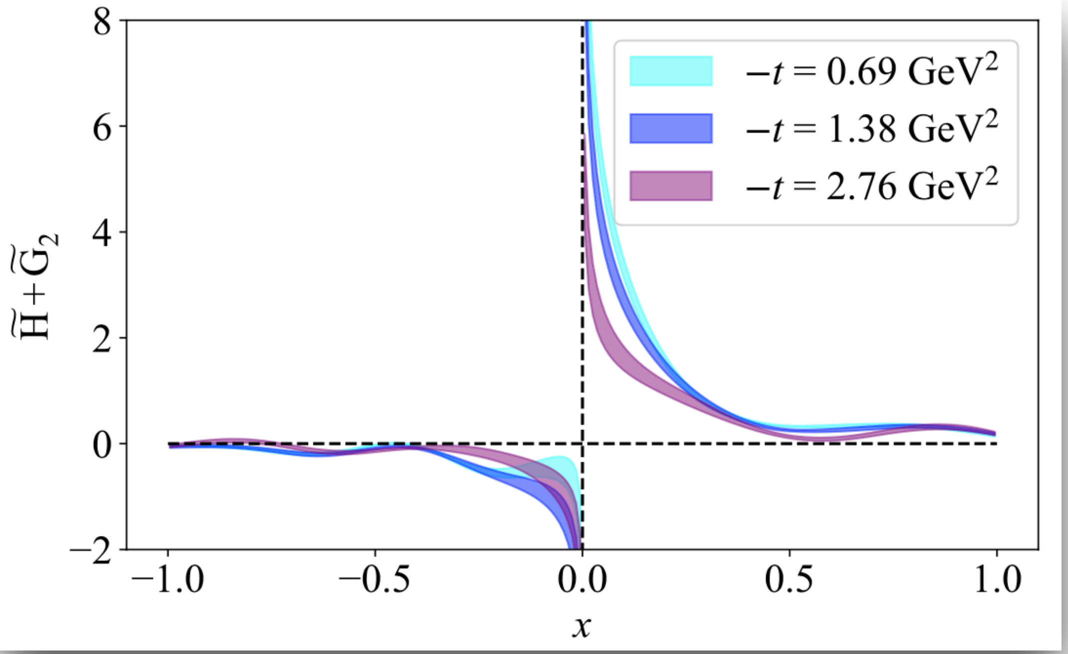

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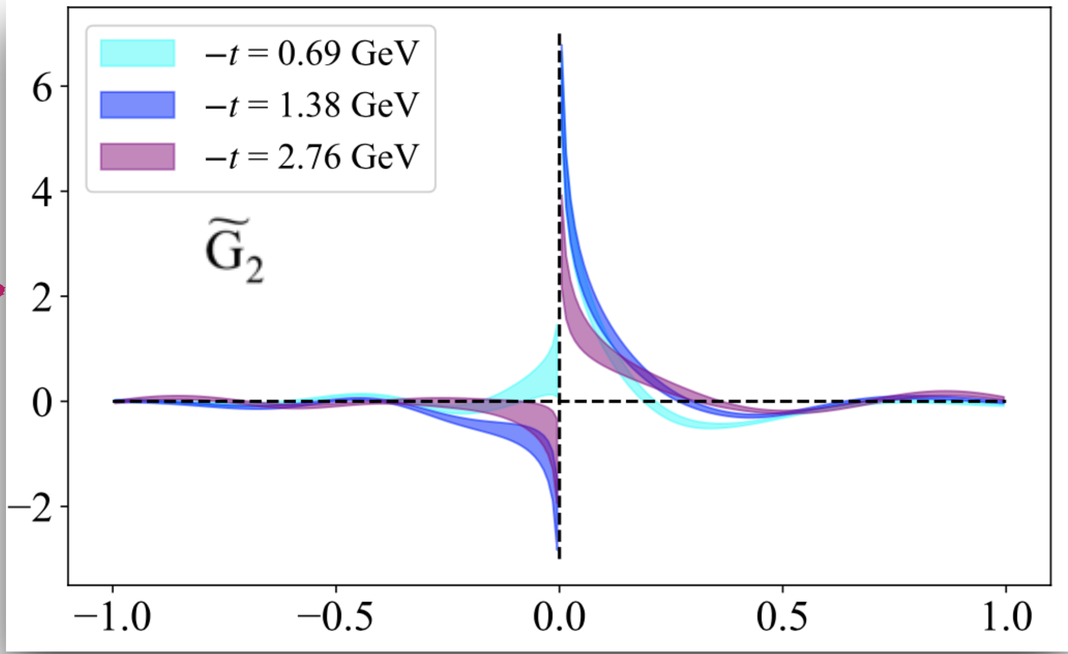

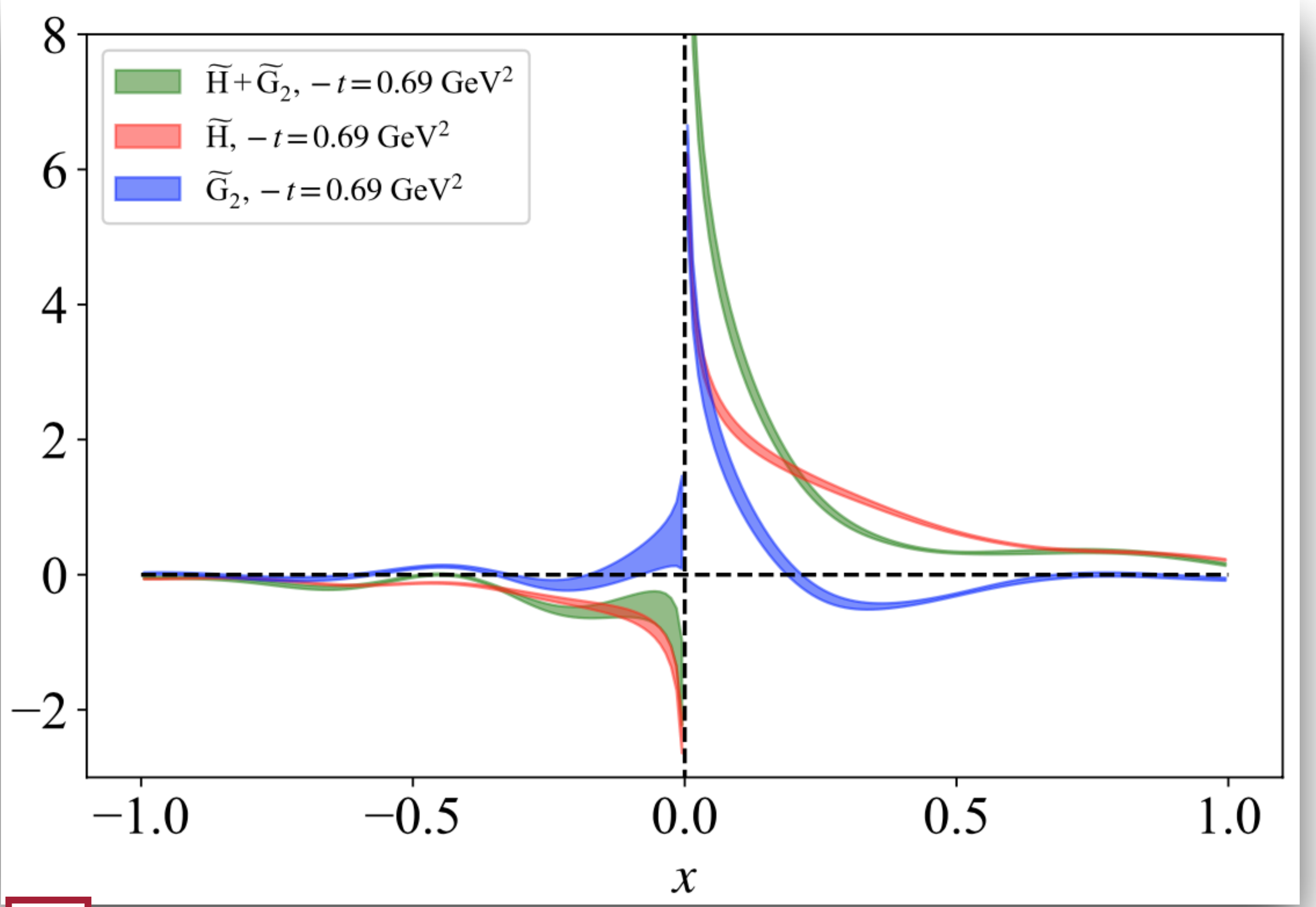

Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

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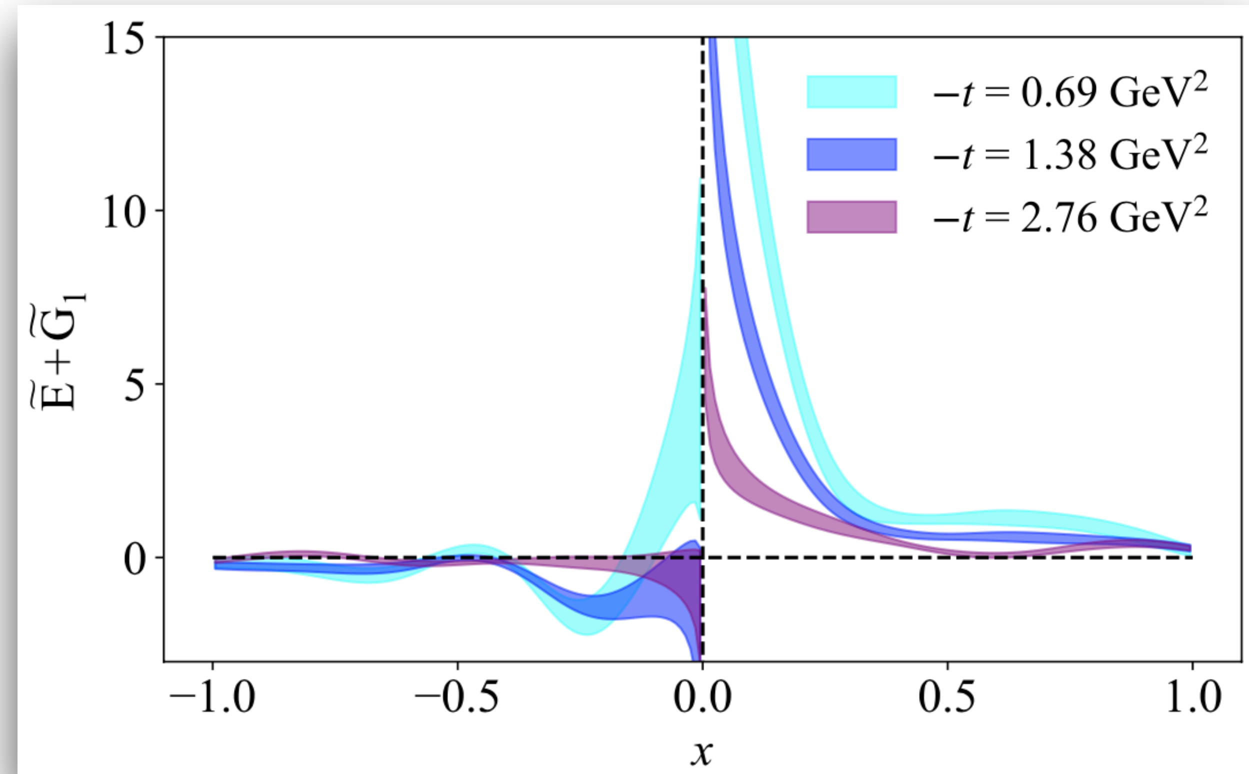

★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

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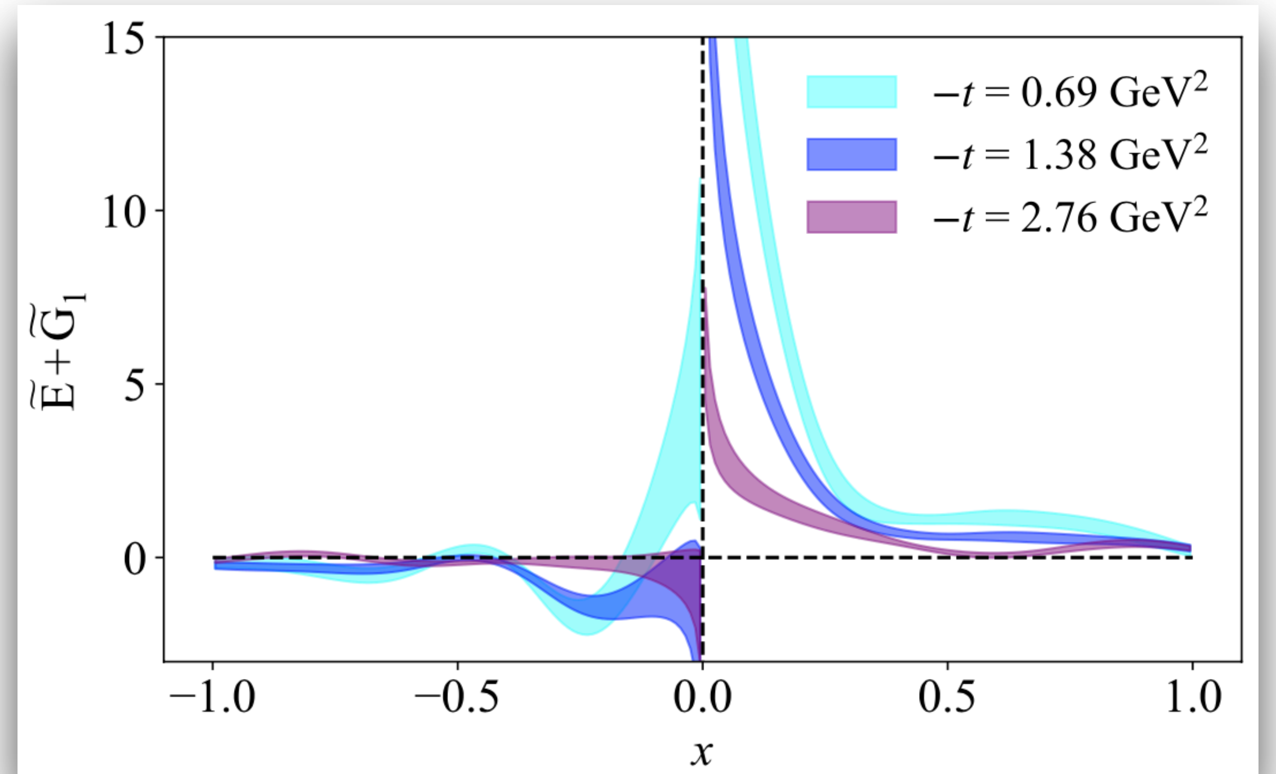

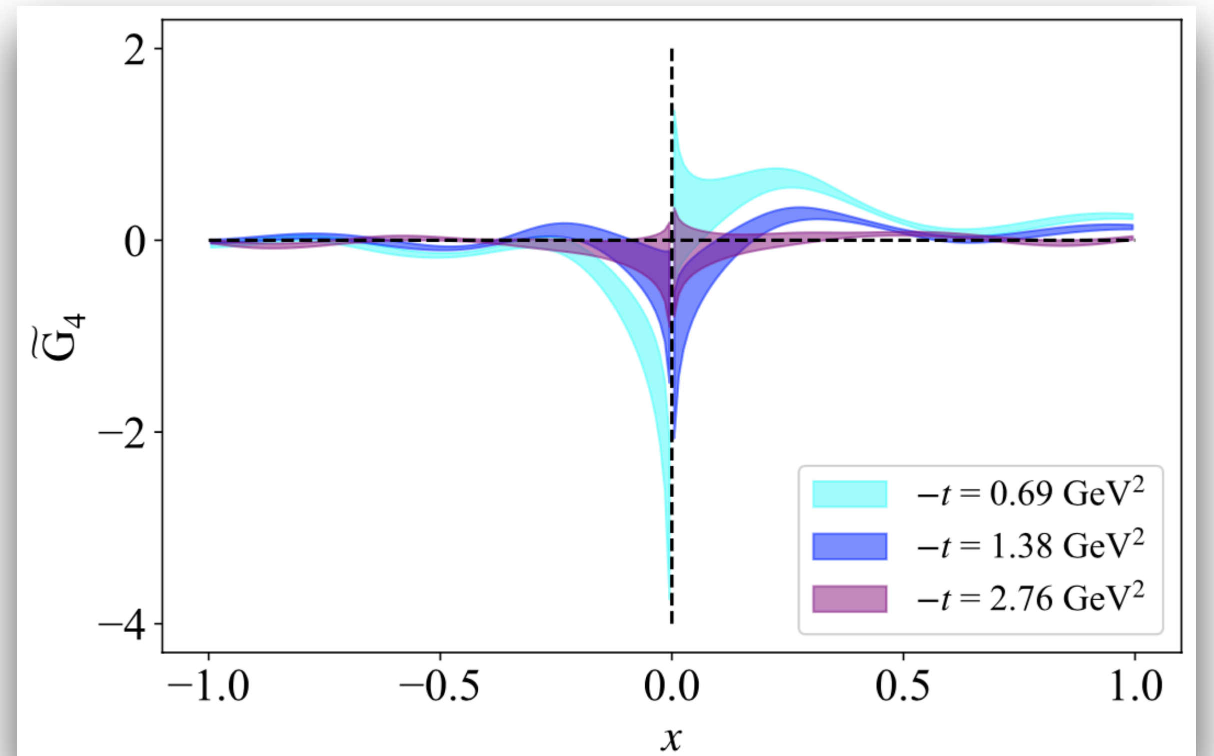

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★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

Consistency checks

★ Norms satisfied

encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
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★ Alternative kinematic setup can be

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz
Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2]
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$$\widetilde{F}^\mu(z,P,\Delta) \equiv \langle p_f;\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\gamma^\mu\gamma_5\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i;\lambda\rangle$$
$$= \bar{u}(p_f,\lambda')\Big[\frac{i\epsilon^{\mu Pz\Delta}}{m}\widetilde{A}_1 + \gamma^\mu\gamma_5\widetilde{A}_2 + \gamma_5\Big(\frac{P^\mu}{m}\widetilde{A}_3 + mz^\mu\widetilde{A}_4 + \frac{\Delta^\mu}{m}\widetilde{A}_5\Big) + m\not{z}\gamma_5\Big(\frac{P^\mu}{m}\widetilde{A}_6 + mz^\mu\widetilde{A}_7 + \frac{\Delta^\mu}{m}\widetilde{A}_8\Big)\Big]u(p_i,\lambda)\,,$$

Axial twist-3 GPDs with asymmetric frame

$$F^s_{\widetilde{E}+\widetilde{G}_1} = \frac{-2E^2}{P_3} z\tilde{A}_1 + 2\tilde{A}_5$$

$$F^s_{\widetilde{H}+\widetilde{G}_2} = \frac{-E^2(\Delta_x^2 + \Delta_y^2)}{2m^2 P_3} z\tilde{A}_1 + \tilde{A}_2$$

$$F^s_{\widetilde{G}_3} = zP_3\tilde{A}_8$$

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- ★ **Kinematic coefficients defined in symmetric frame**
- ★ **Amplitudes extracted from any frame.**
  Asymmetric frame calculations give $\tilde{A}_i$ at t^a, but F_i defined in t^s

Axial twist-3 GPDs with asymmetric frame

$$F^s_{\widetilde{E}+\widetilde{G}_1} = \frac{-2E^2}{P_3} z\tilde{A}_1 + 2\tilde{A}_5$$

$$F^s_{\widetilde{H}+\widetilde{G}_2} = \frac{-E^2(\Delta_x^2 + \Delta_y^2)}{2m^2 P_3} z\tilde{A}_1 + \tilde{A}_2$$

$$F^s_{\widetilde{G}_3} = zP_3\tilde{A}_8$$

$$F^s_{\widetilde{G}_4} = \frac{-EP_3}{m^2}\left(\frac{-E^2}{P_3} + P_3\right) z\tilde{A}_1$$

★ Kinematic coefficients defined in symmetric frame

★ Amplitudes extracted from any frame.
Asymmetric frame calculations give $\tilde{A}_i$ at t^a, but F_i defined in t^s

Lorentz transformation of kinematic factors

$$F^a_{\widetilde{E}+\widetilde{G}_1} = \frac{-E_f(E_f + E_i)}{P_3} z\tilde{A}_1 + 2\tilde{A}_5$$

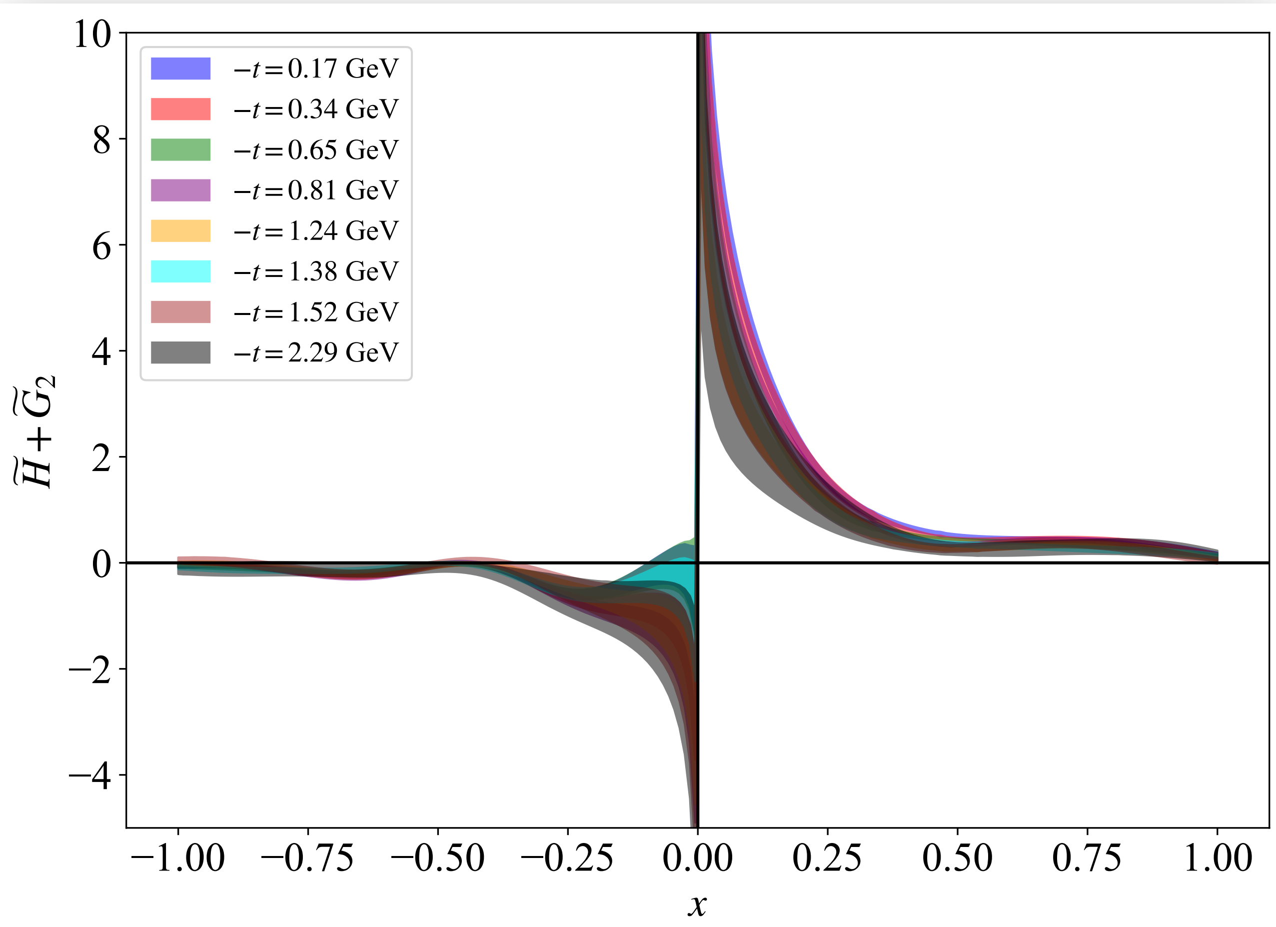
$$F^a_{\widetilde{H}+\widetilde{G}_2} = \frac{-E_f^2(\Delta_x^2 + \Delta_y^2)}{2m^2 P_3} z\tilde{A}_1 + \tilde{A}_2$$

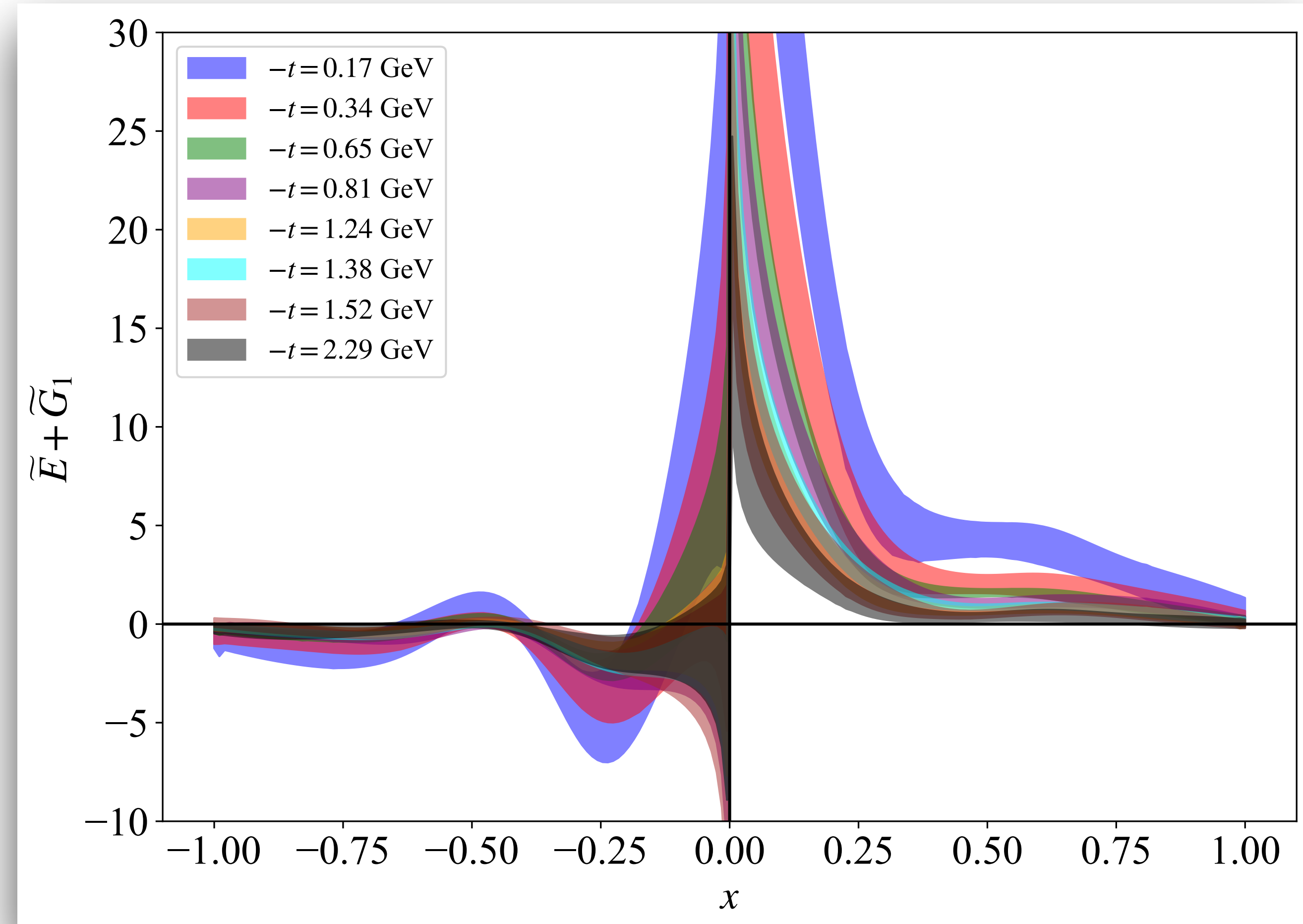
$$F^a_{\widetilde{G}_3} = zP_3\tilde{A}_8$$

$$F^a_{\widetilde{G}_4} = -\sqrt{\frac{E_f(E_f + E_i)}{2}}\frac{P_3}{m^2}\left(\frac{-E_f(E_f + E_i)}{2P_3} + P_3\right) z\tilde{A}_1$$

Preliminary Results

Axial twist-3 GPDs via asymmetric frame calculation

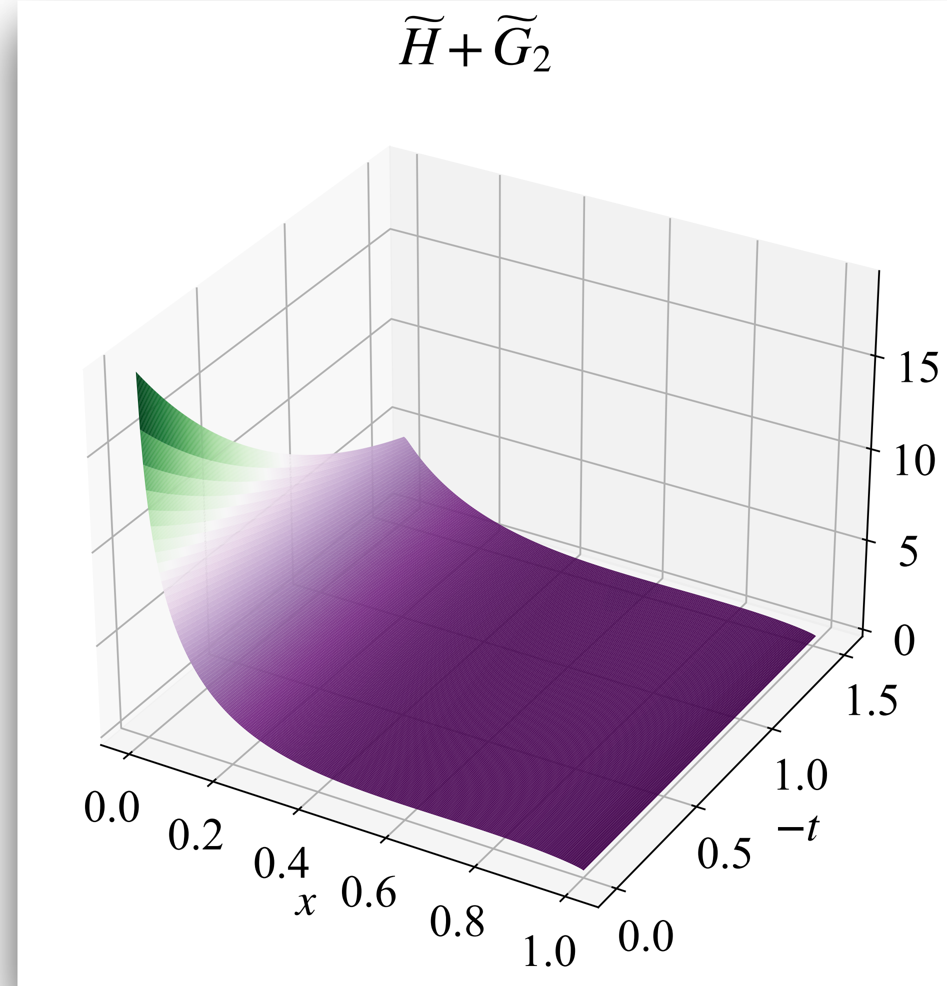


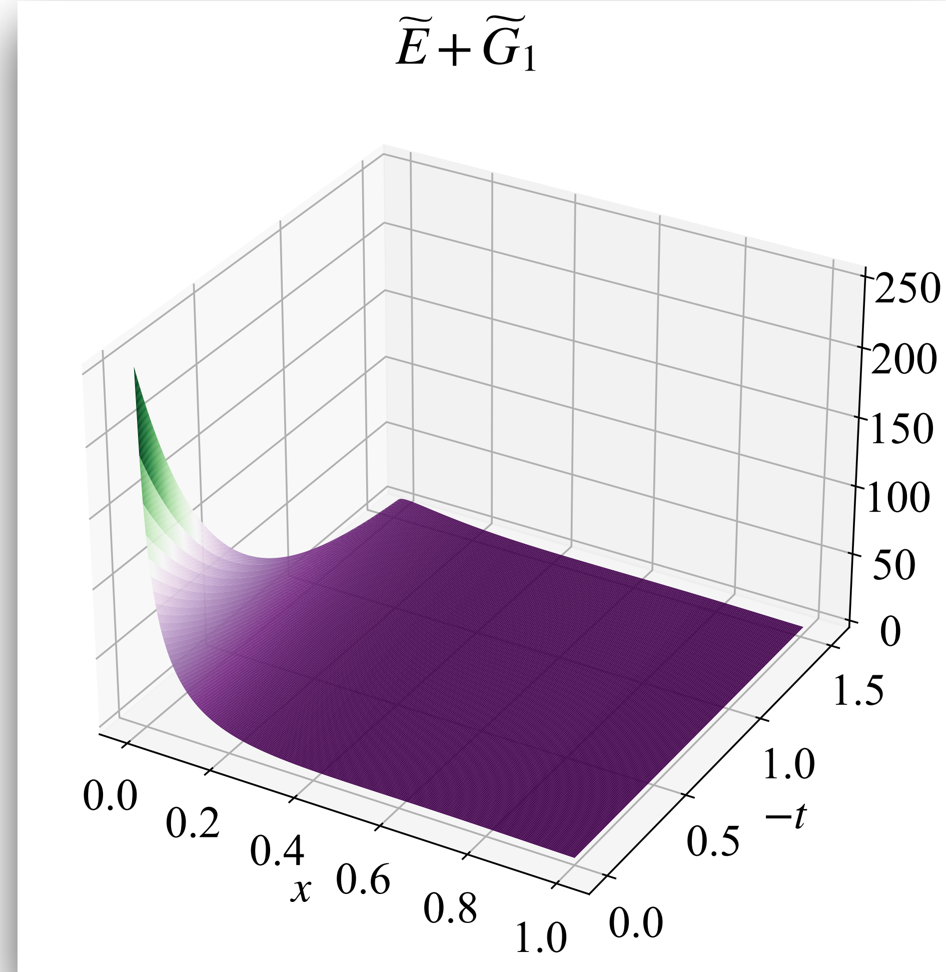


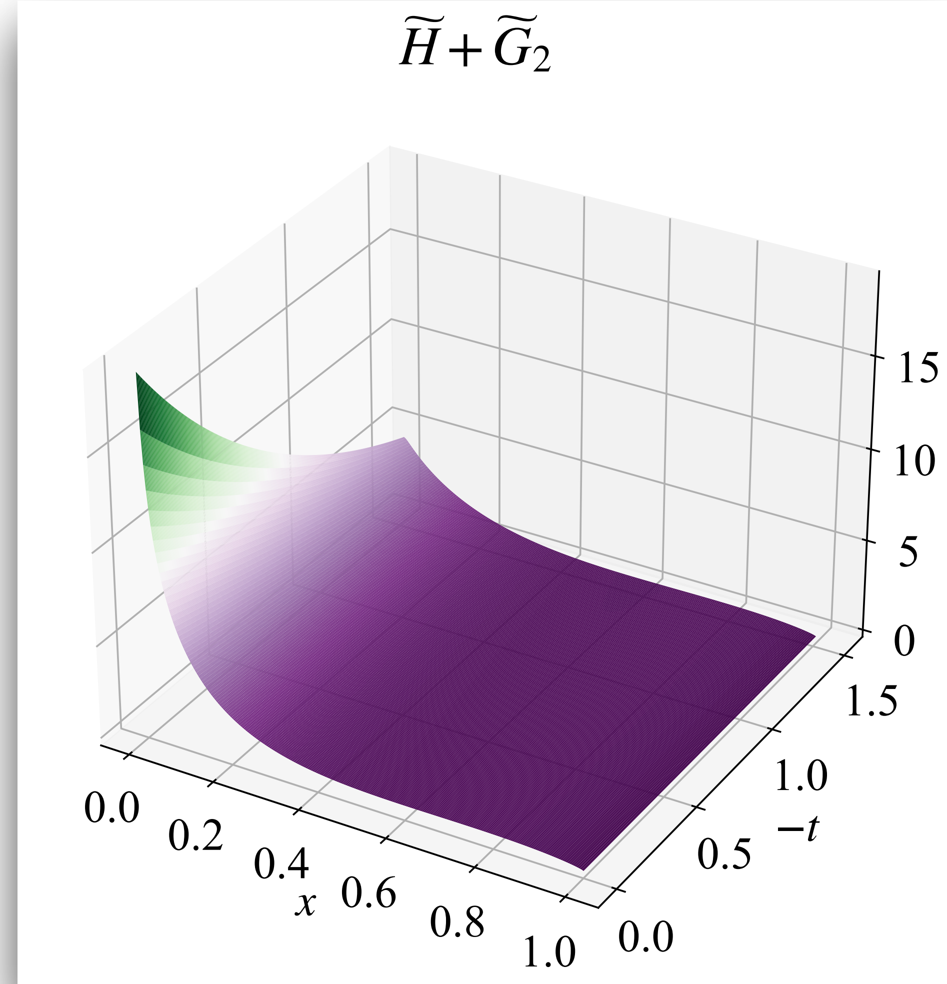
★ Parametrization of -t dependence

$$\text{GPD}(x,-t,0) = A x^{\alpha_0 - \alpha_1 t} (1-x)^{\beta}$$

Ademollo & Del Giudice Gatto & Preparata


$\widetilde{H}+\widetilde{G}_2$
15
10
5
0
1.5
1.0
0.5
0.0
$-t$
0.0
0.2
0.4
0.6
0.8
1.0
x


$\widetilde{E}+\widetilde{G}_1$
250
200
150
100
50
0
1.5
1.0
0.5
0.0
$-t$
0.0
0.2
0.4
0.6
0.8
1.0
x


$\widetilde{H}+\widetilde{G}_2$
15
10
5
0
0.0
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0.4
0.6
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1.0
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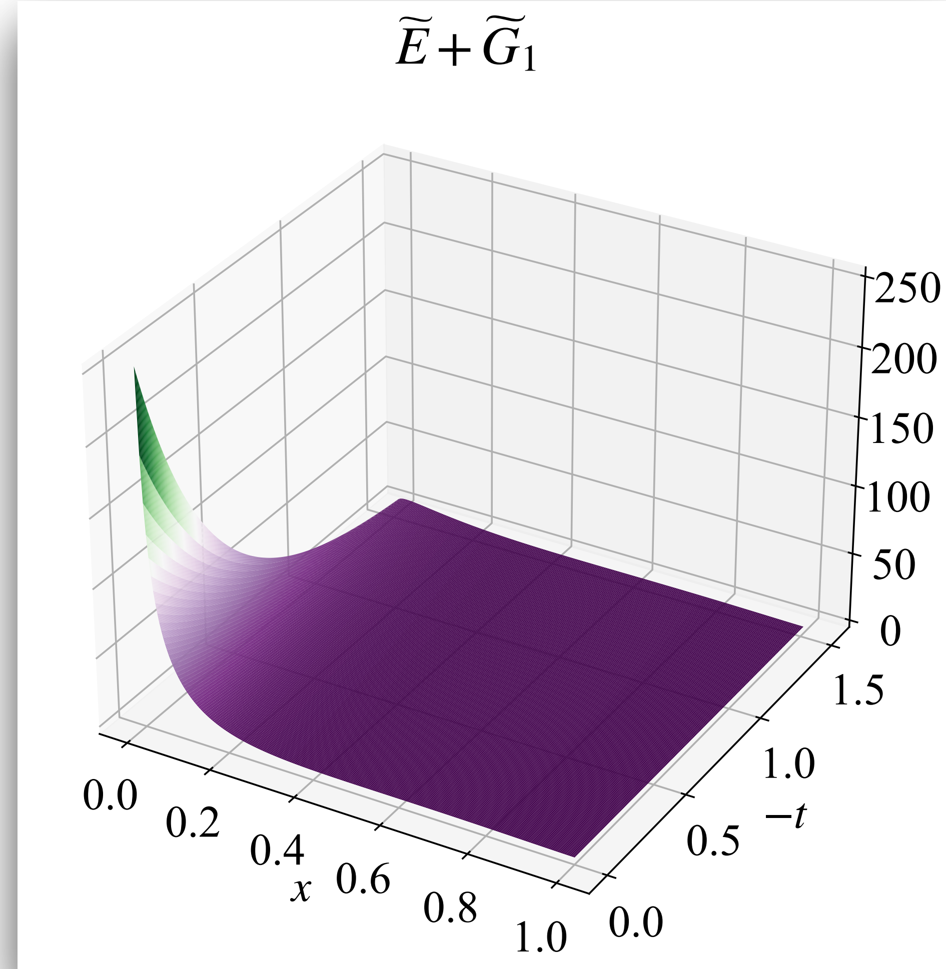
0.0
0.5
1.0
1.5
$-t$


$\widetilde{E}+\widetilde{G}_1$
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200
150
100
50
0
0.0
0.2
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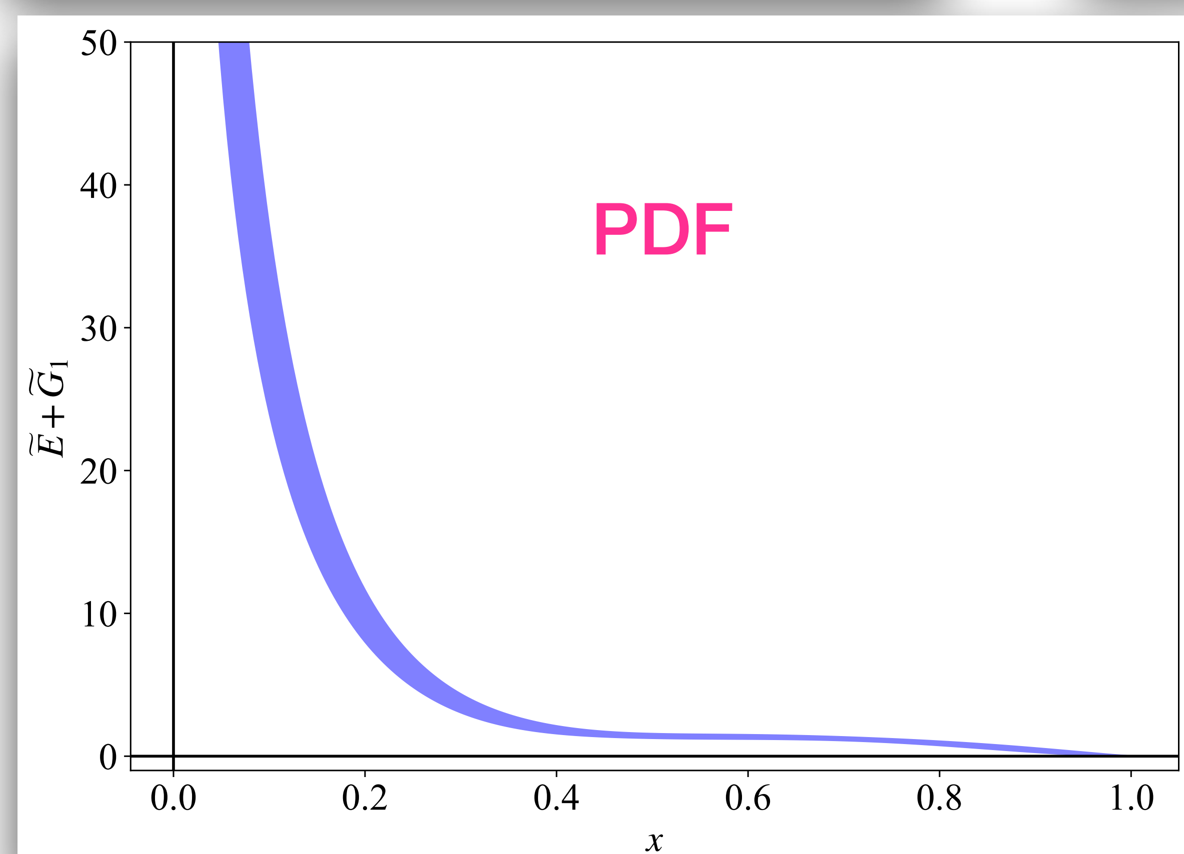
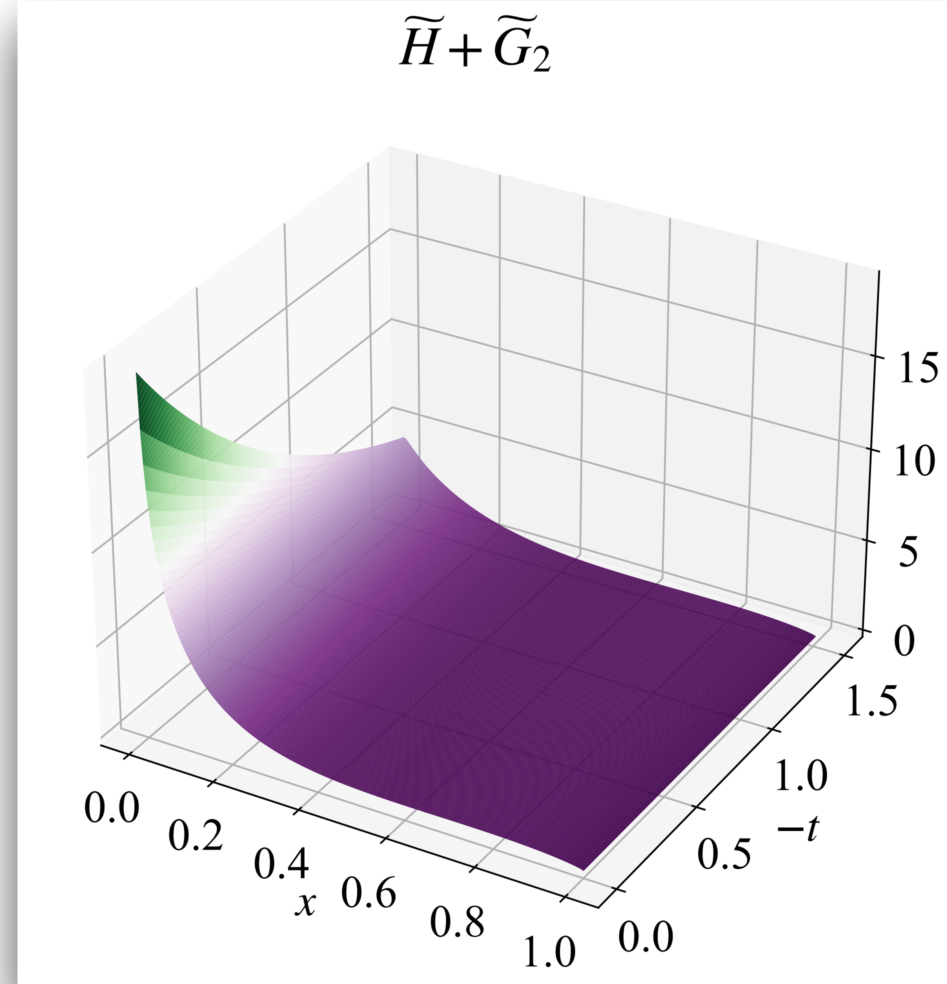
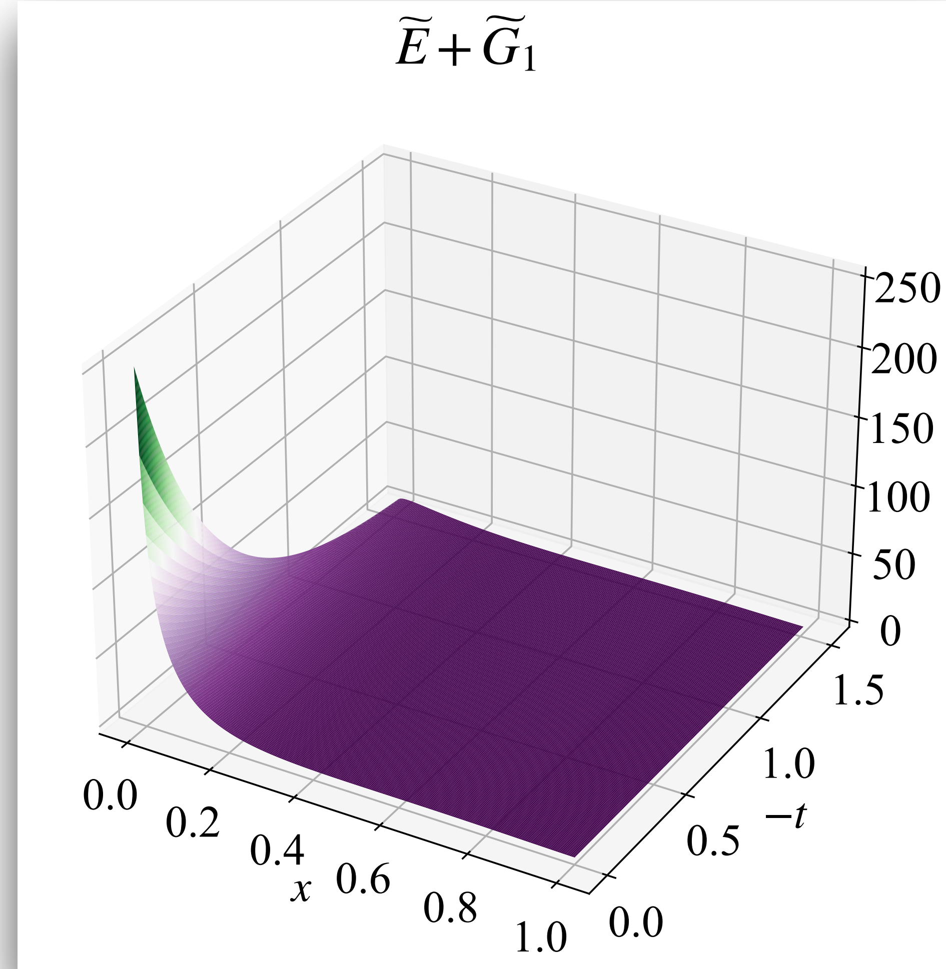
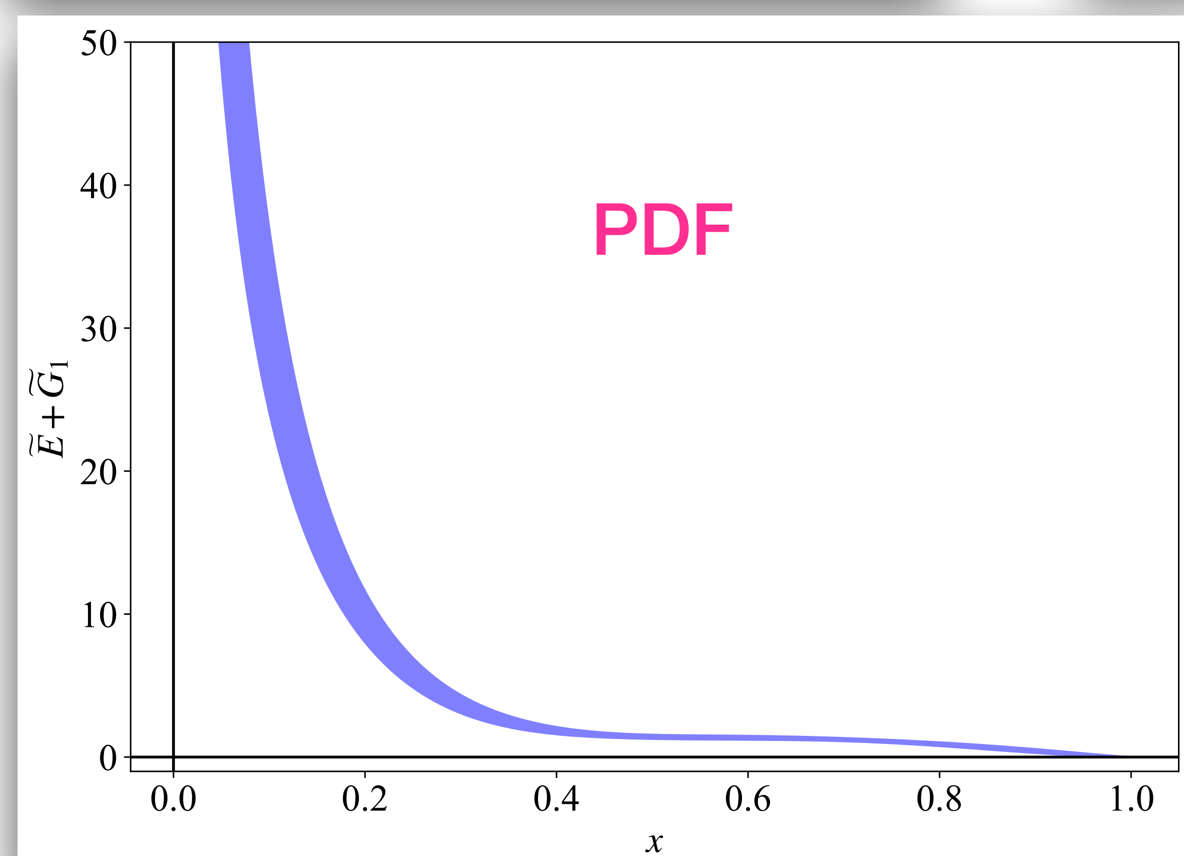
0.6
0.8
1.0
x
0.0
0.5
1.0
1.5
$-t$


PDF
$\widetilde{E}+\widetilde{G}_1$
0
10
20
30
40
50
0.0
0.2
0.4
0.6
0.8
1.0
x







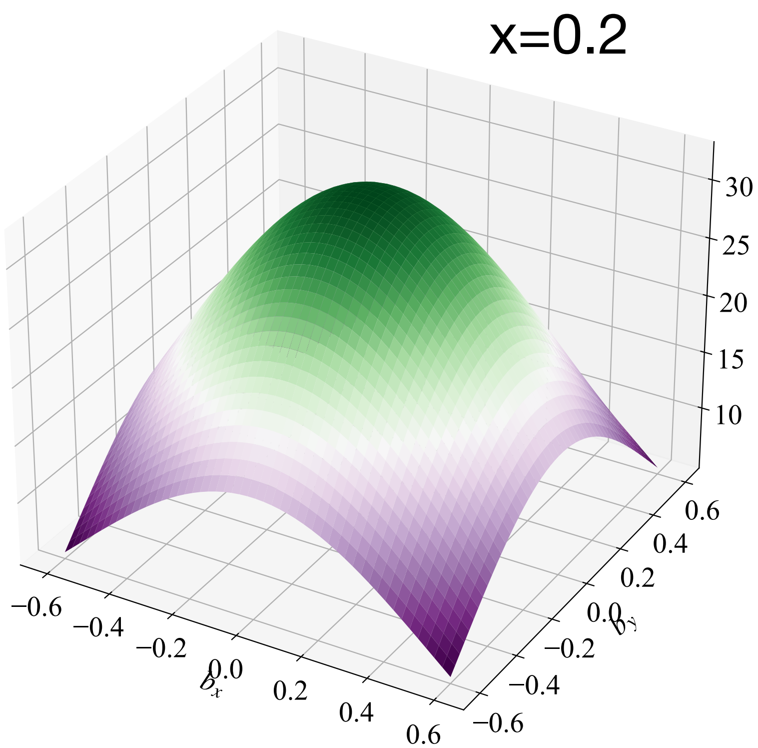
★ **At ξ=0 we obtain GPDs in transverse plane via Fourier transform**

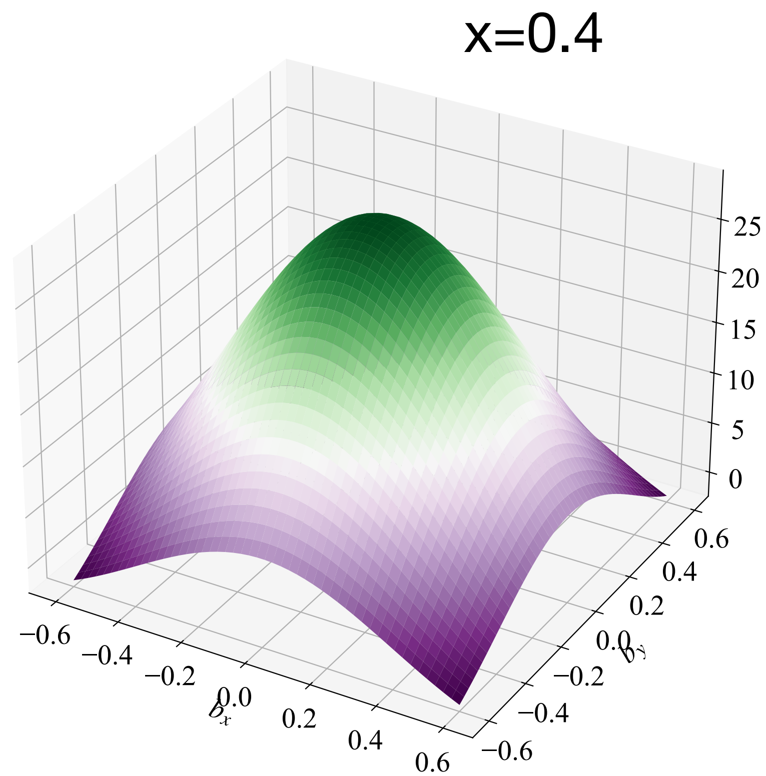
$$q(x,\mathbf{b}_\perp) = |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} H_q(x, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

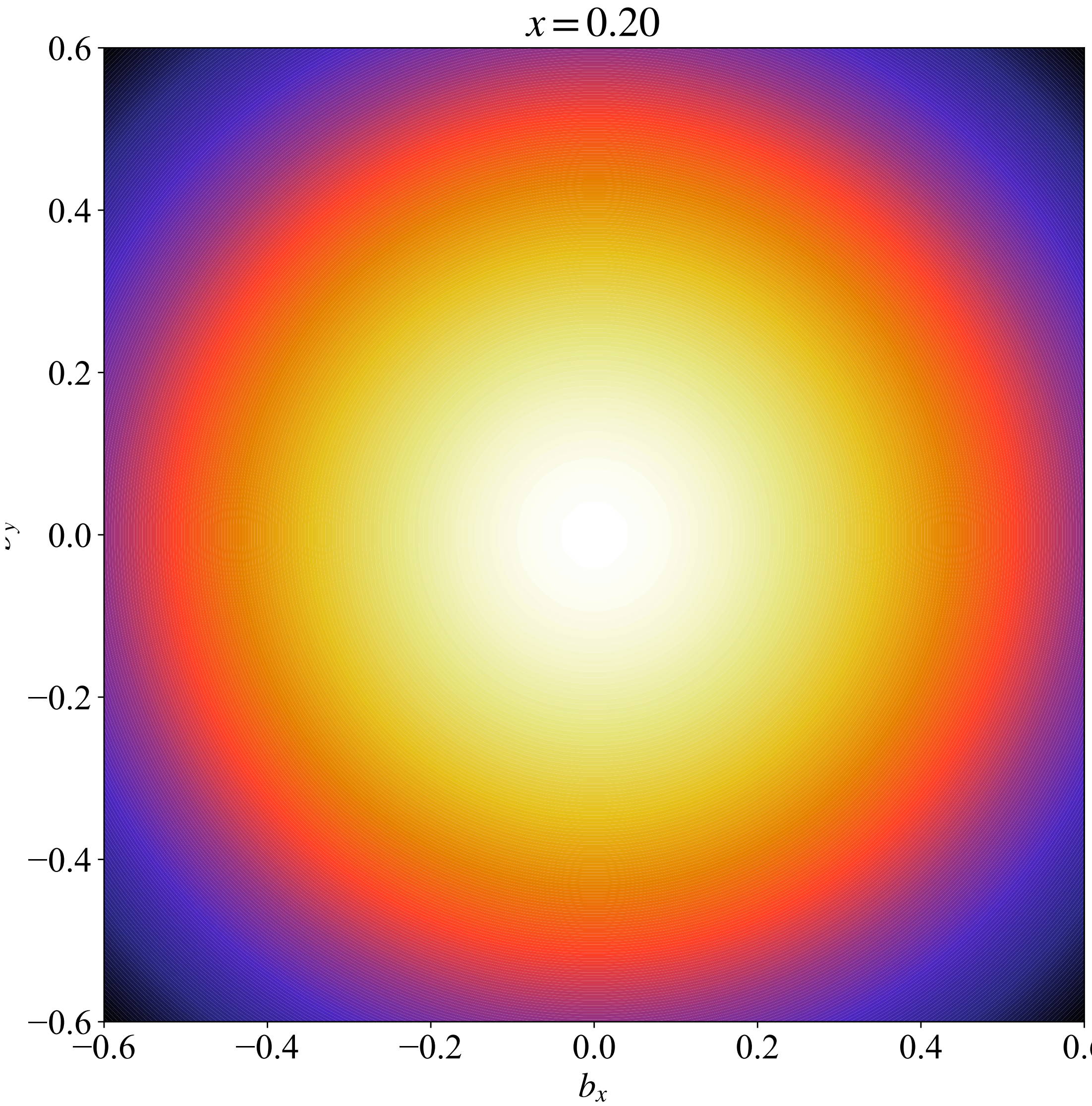
$$= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H_q(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp},$$

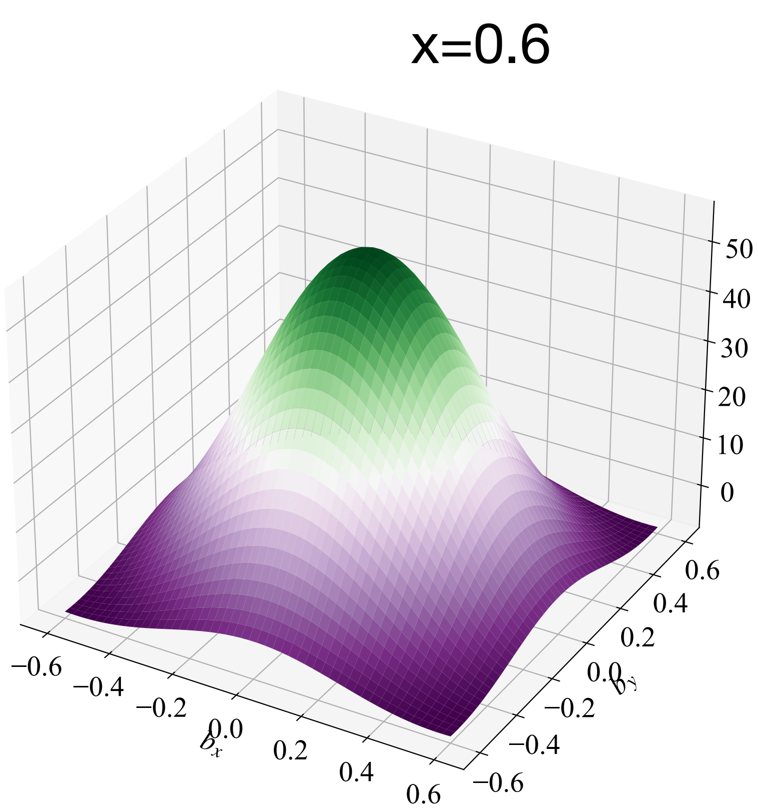
$b_\perp$: transverse distance from the (transverse) center of momentum

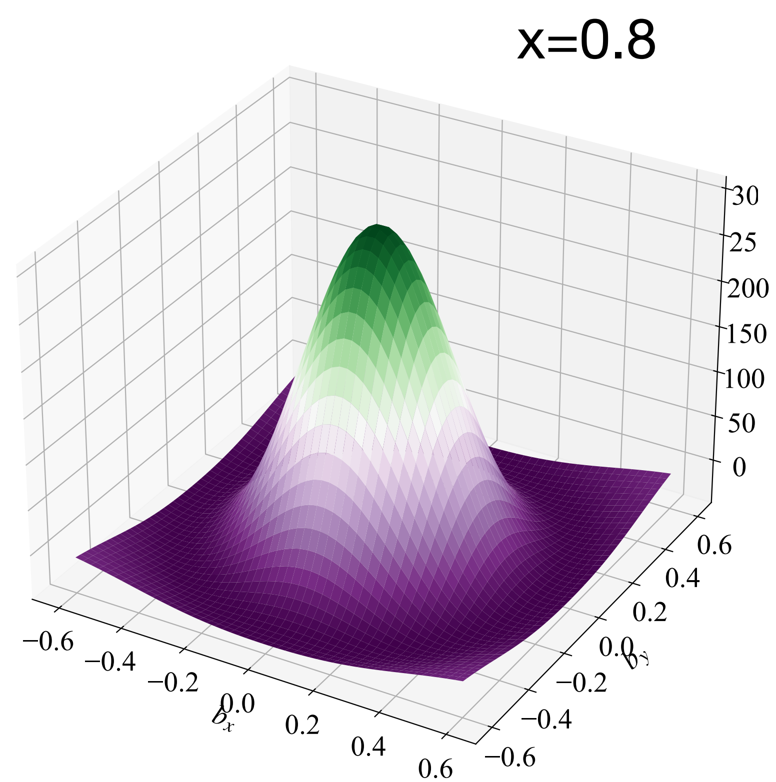
Impact parameter space $\widetilde{H} + \widetilde{G}_2$



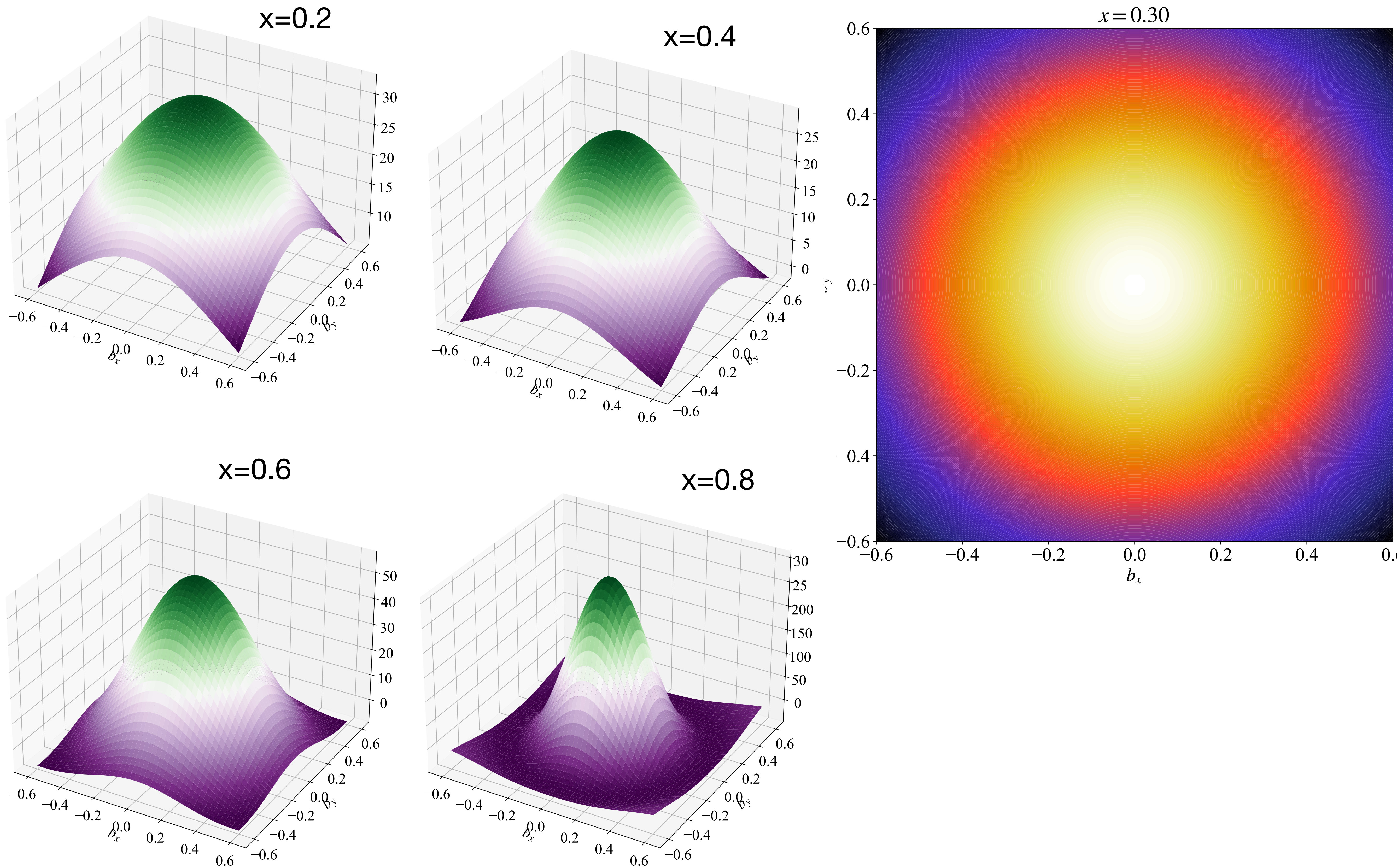




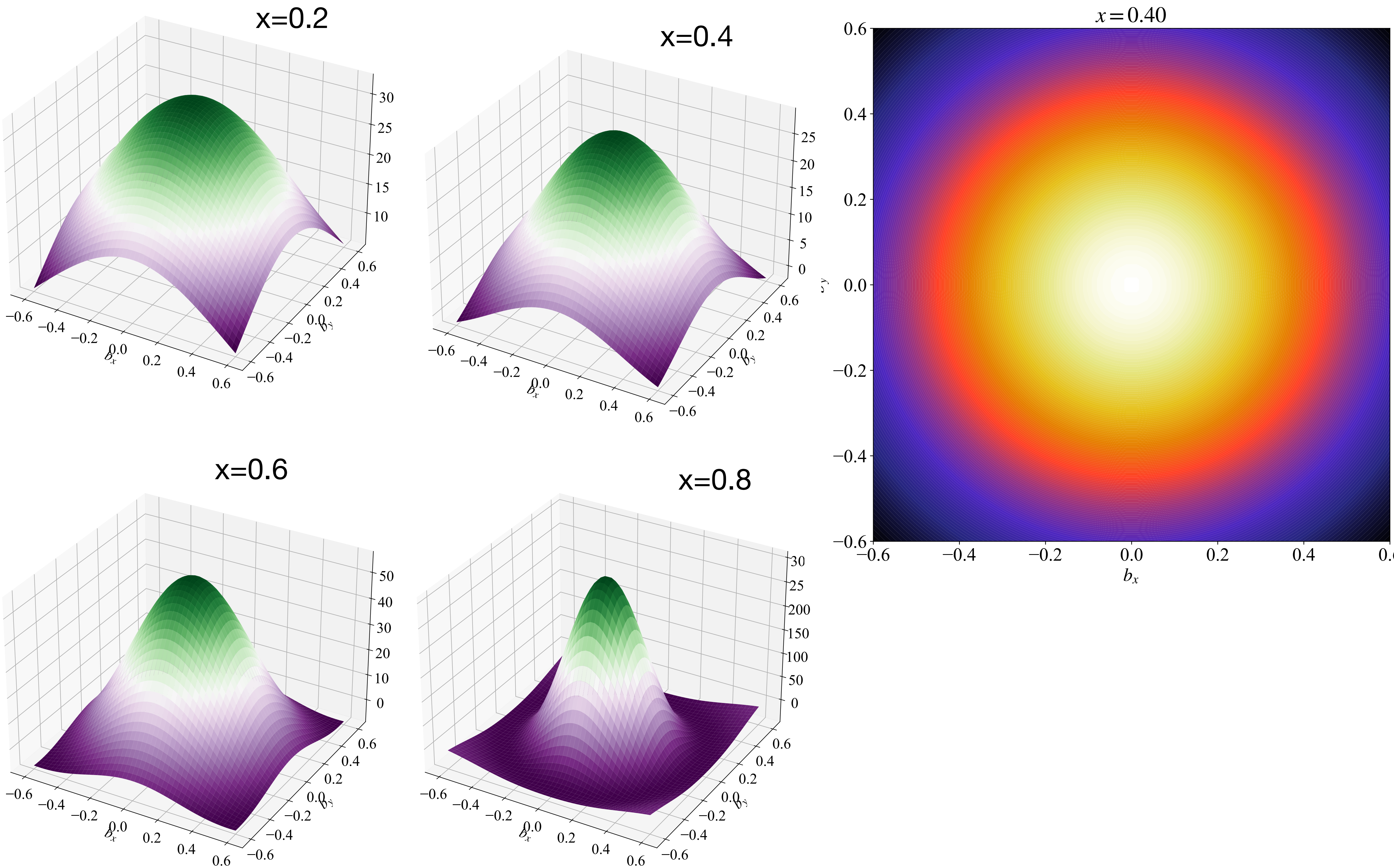




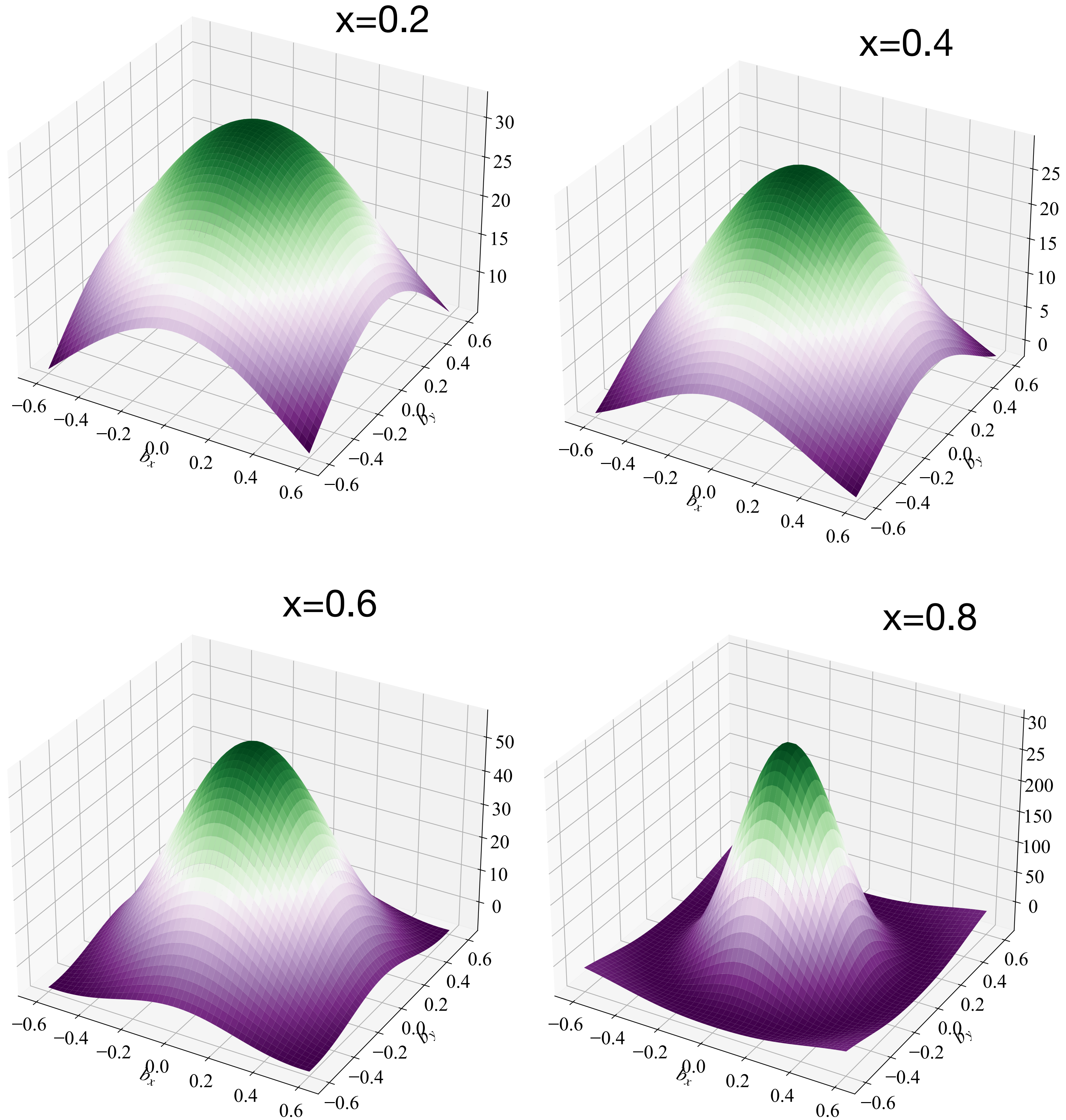
Impact parameter space $\widetilde{H} + \widetilde{G}_2$

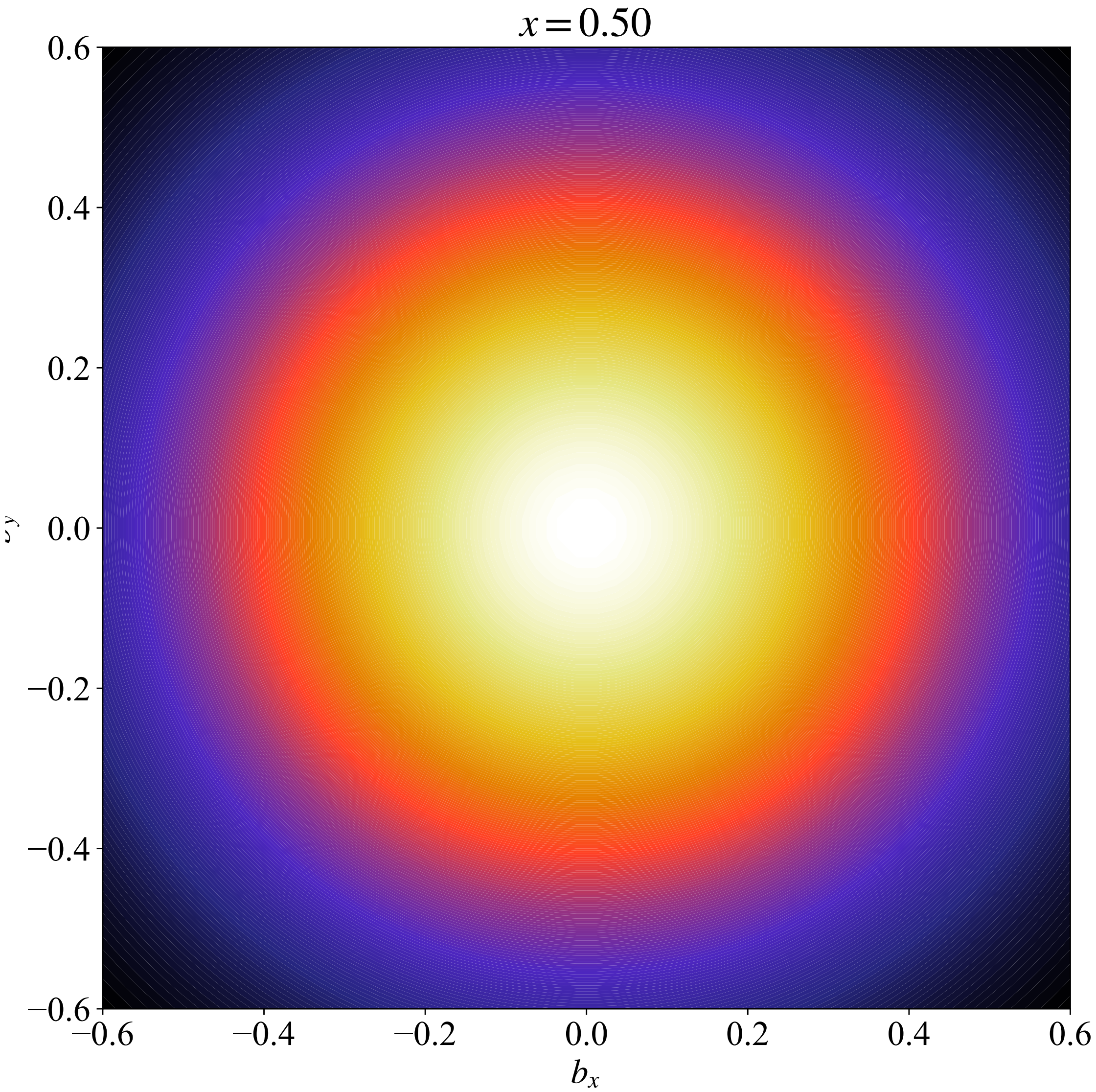


Impact parameter space $\widetilde{H} + \widetilde{G}_2$



Impact parameter space $\widetilde{H} + \widetilde{G}_2$





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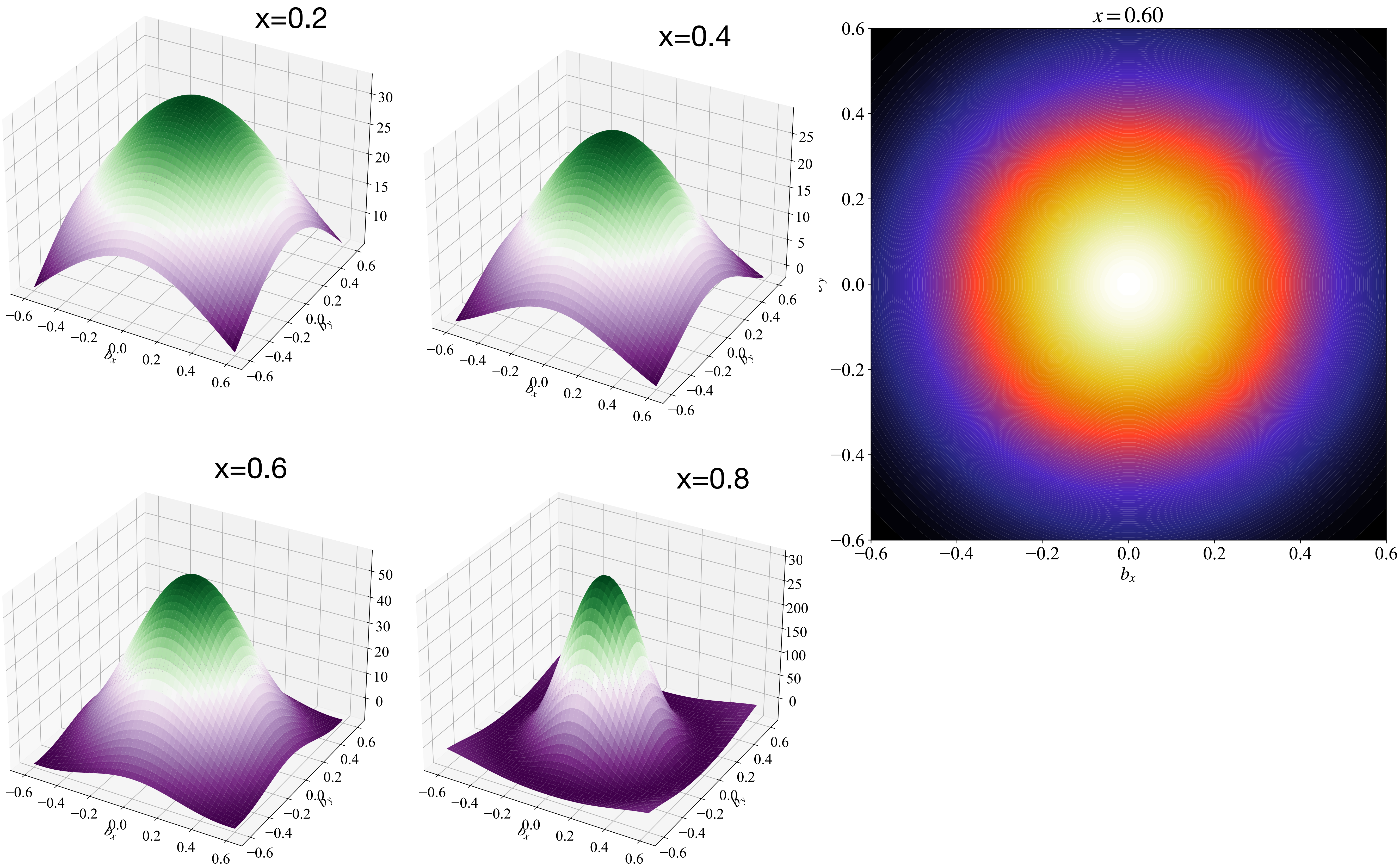

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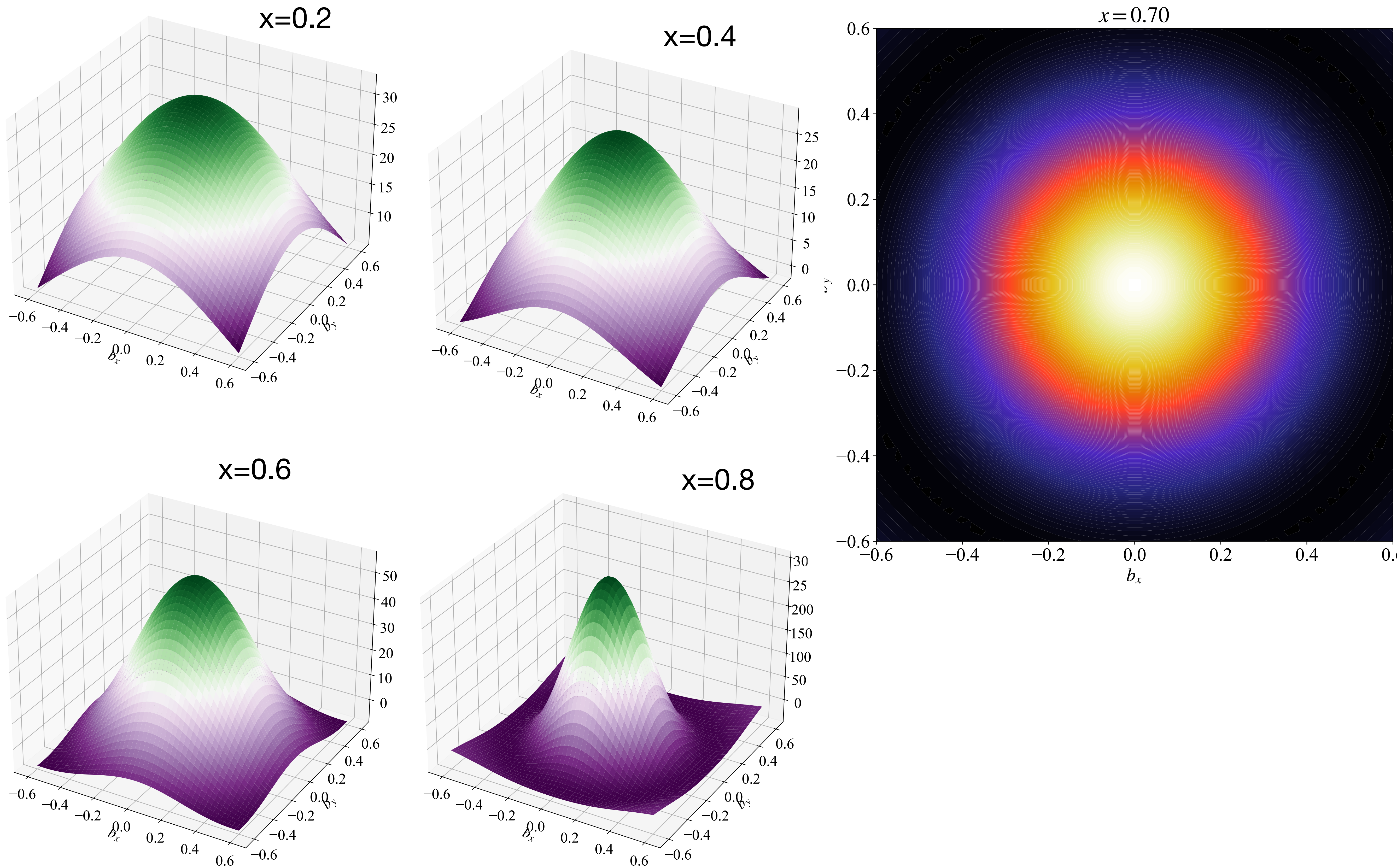
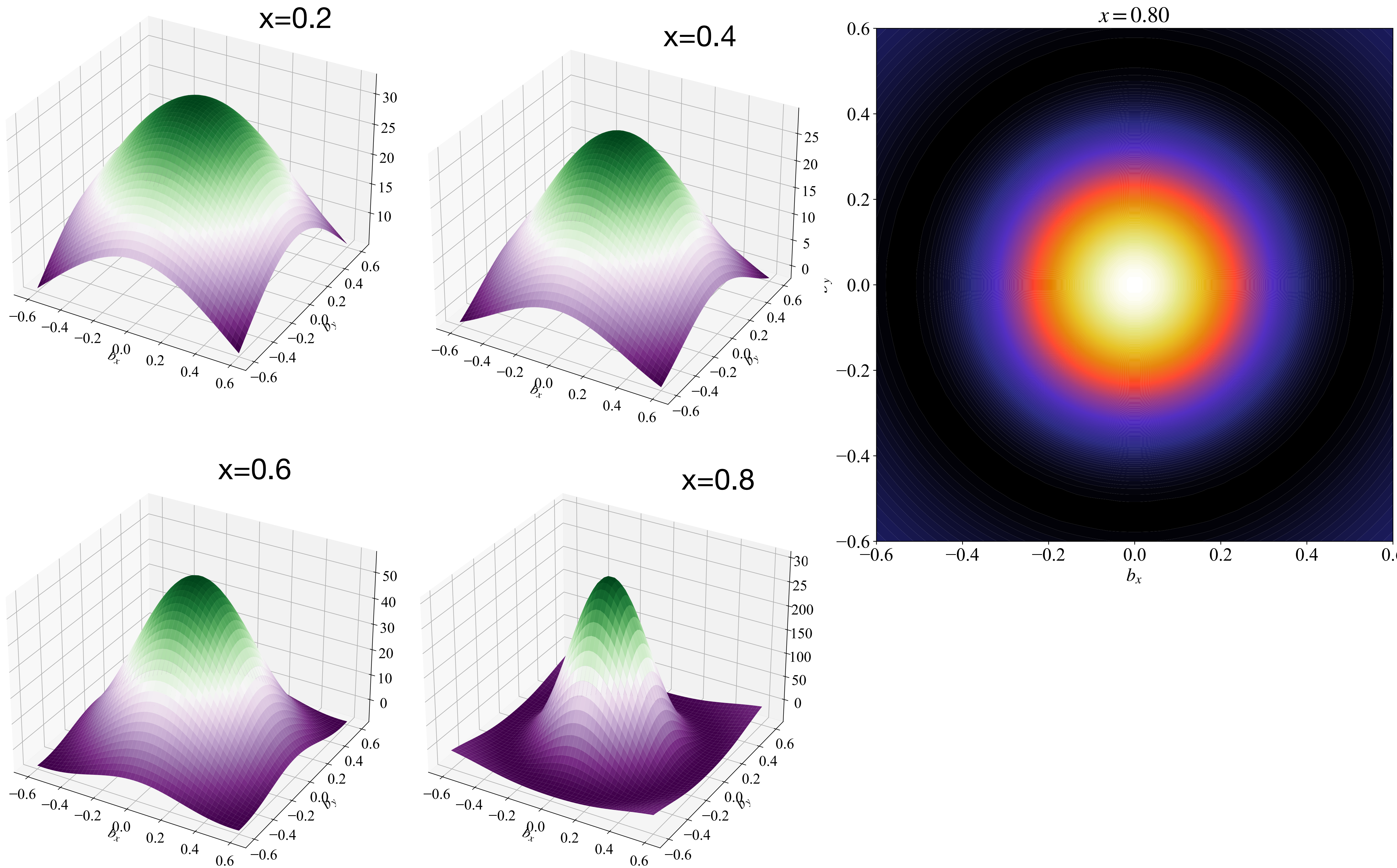
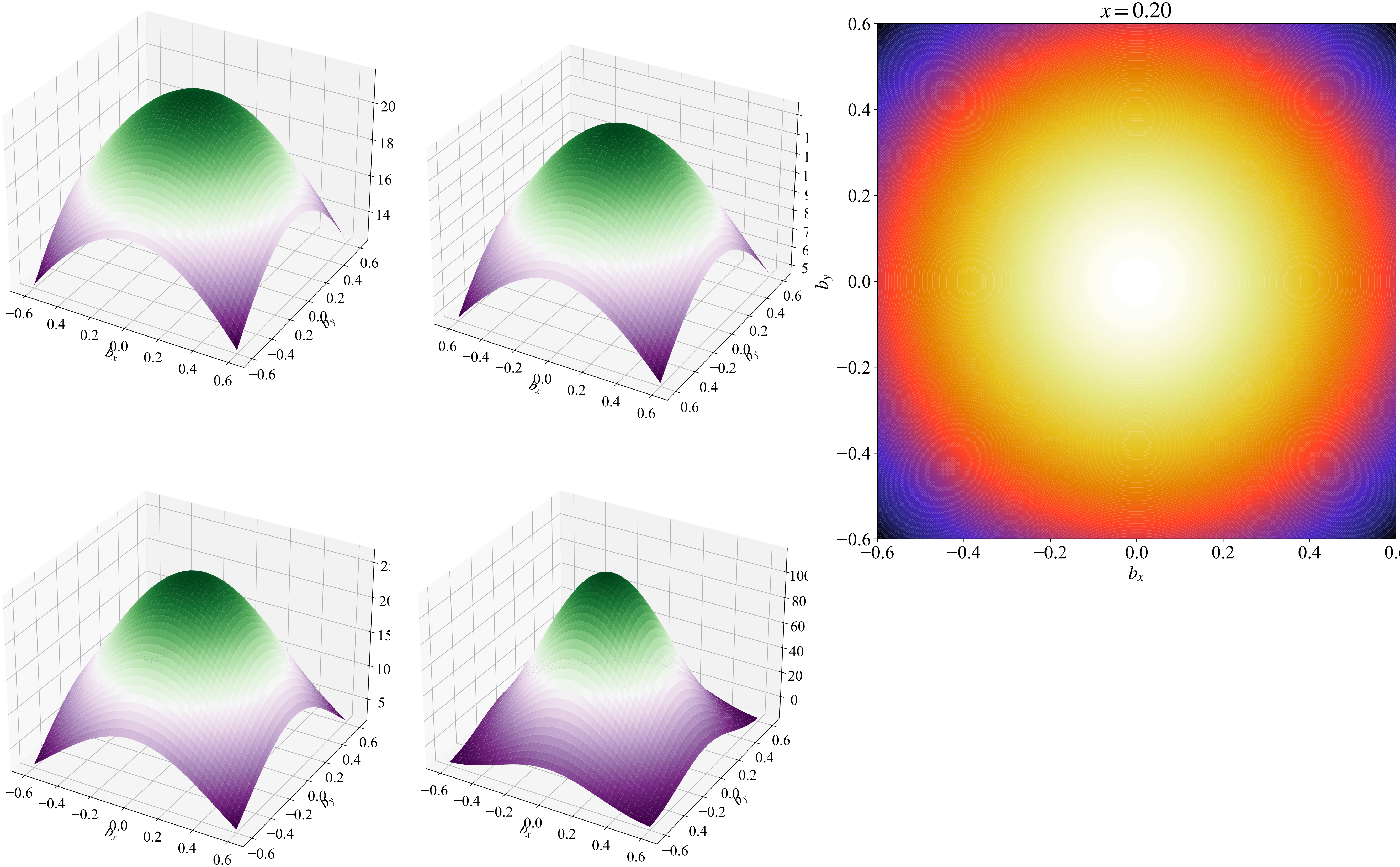


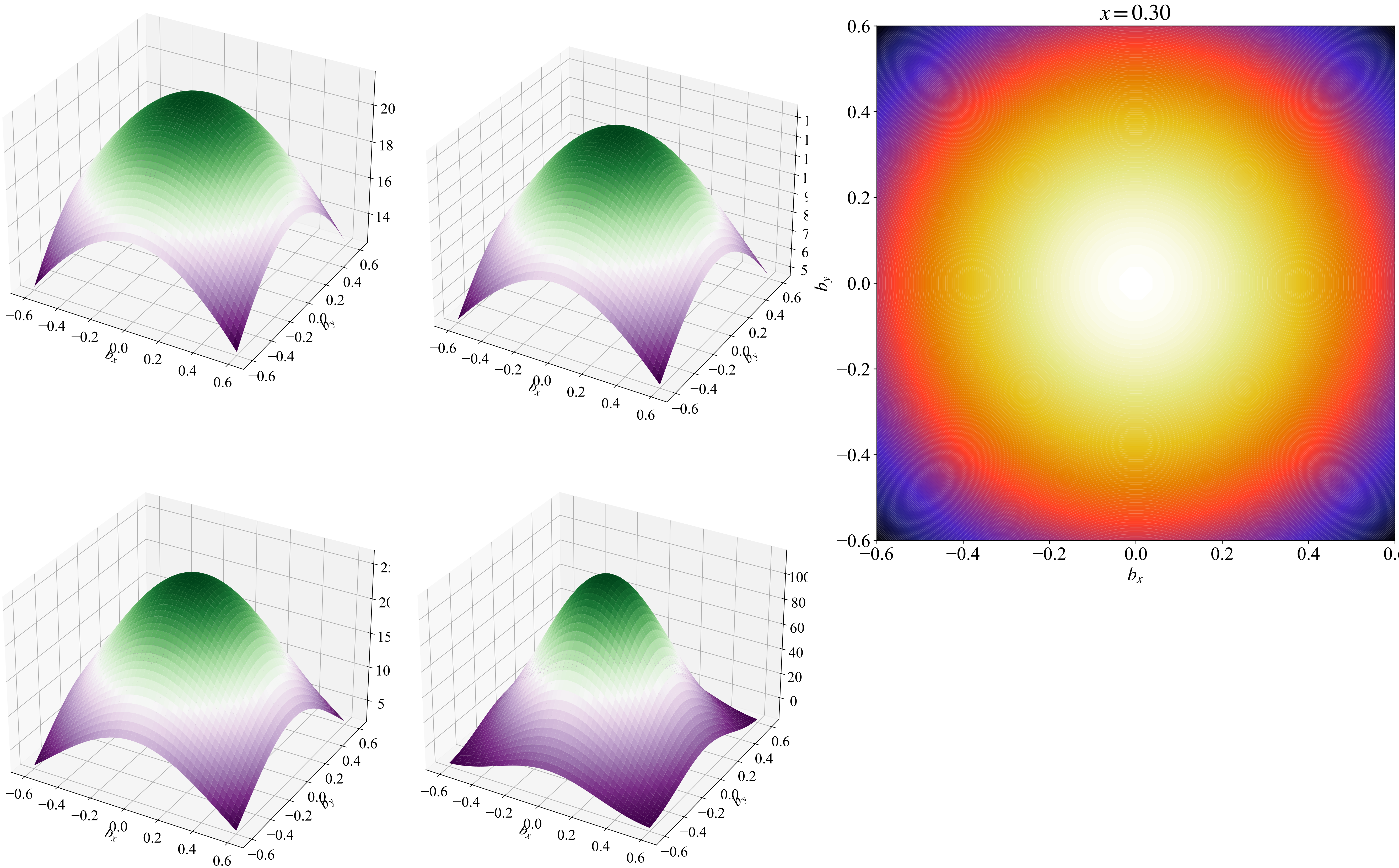
Impact parameter space $\widetilde{H} + \widetilde{G}_2$



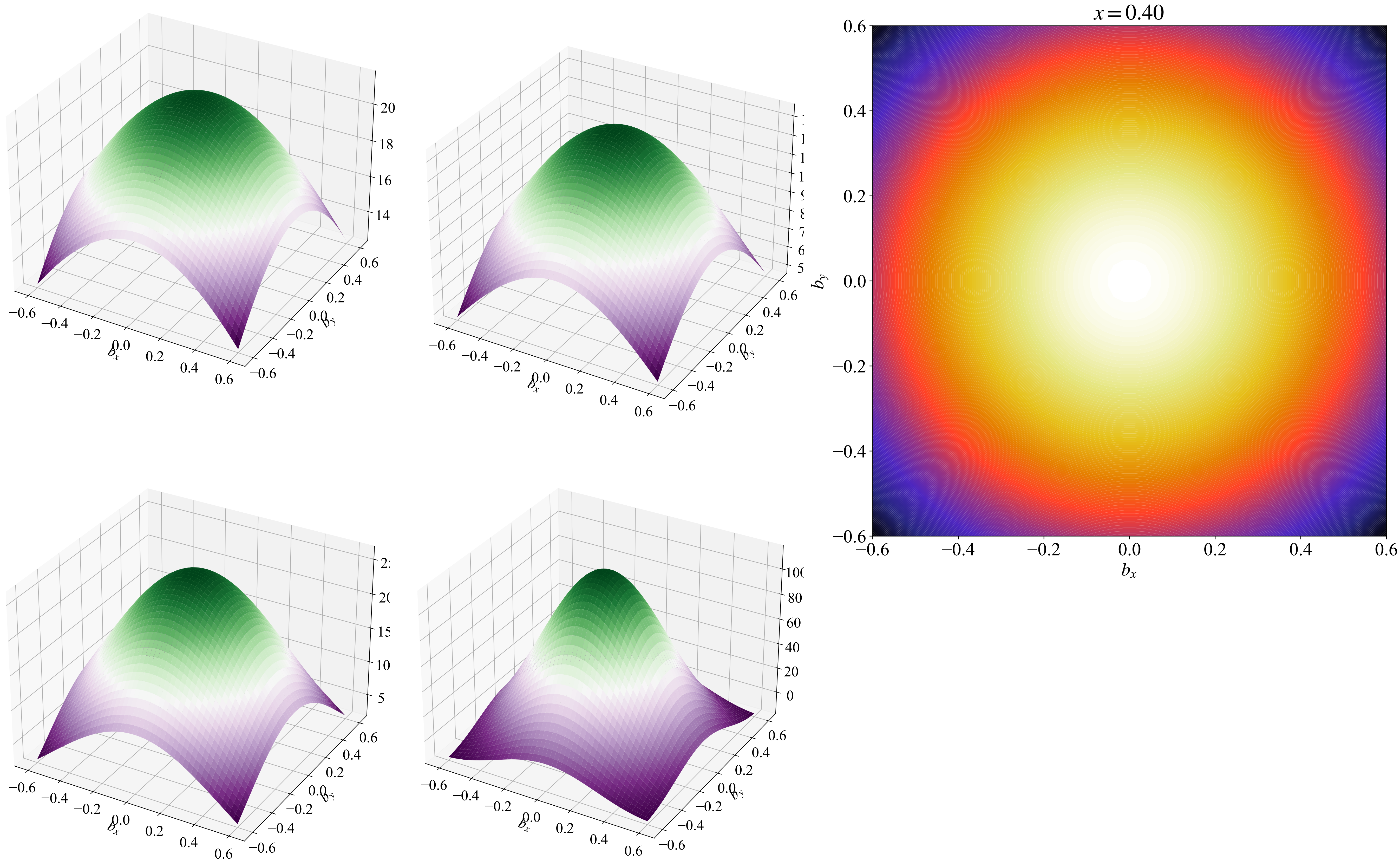
Impact parameter space $\widetilde{E} + \widetilde{G}_1$



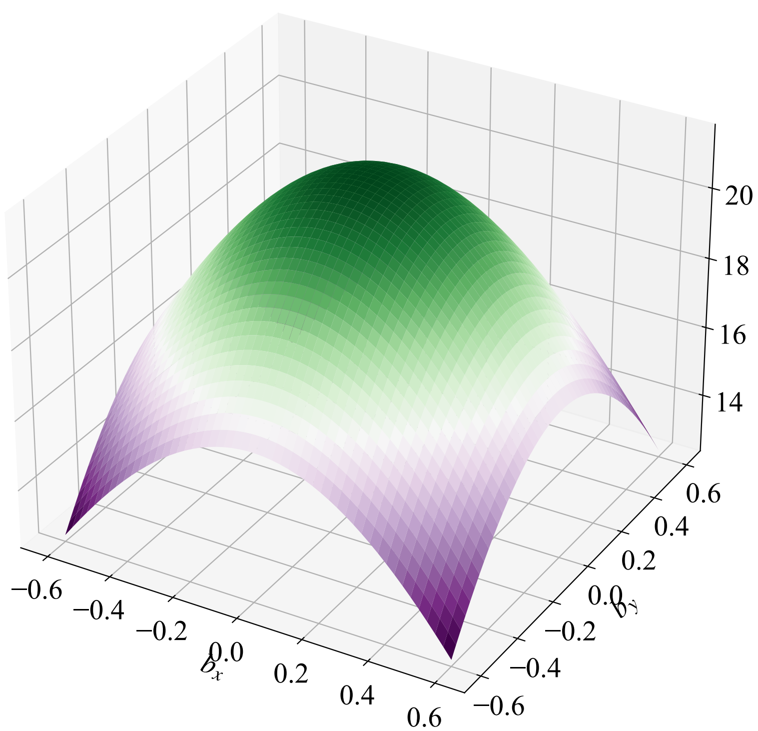
Impact parameter space $\widetilde{E} + \widetilde{G}_1$

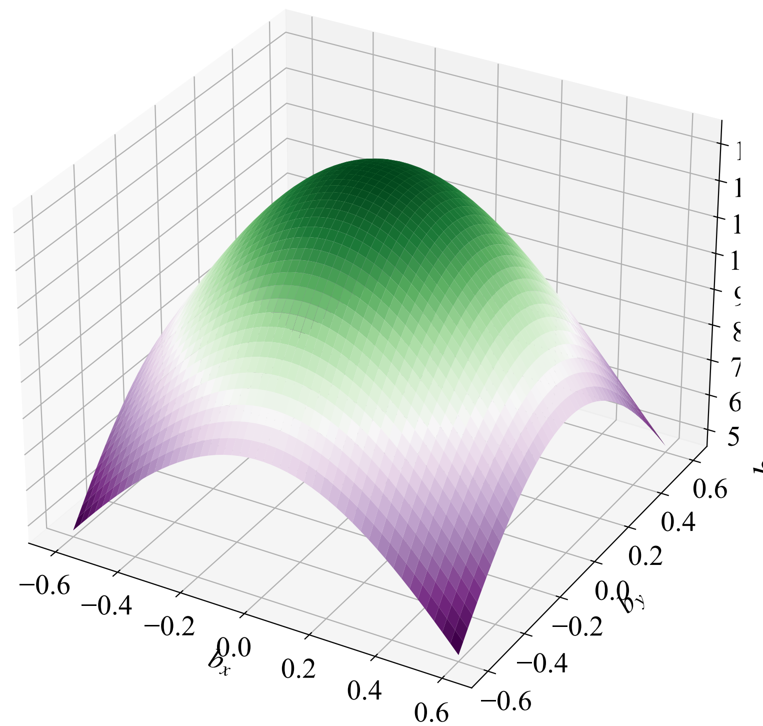


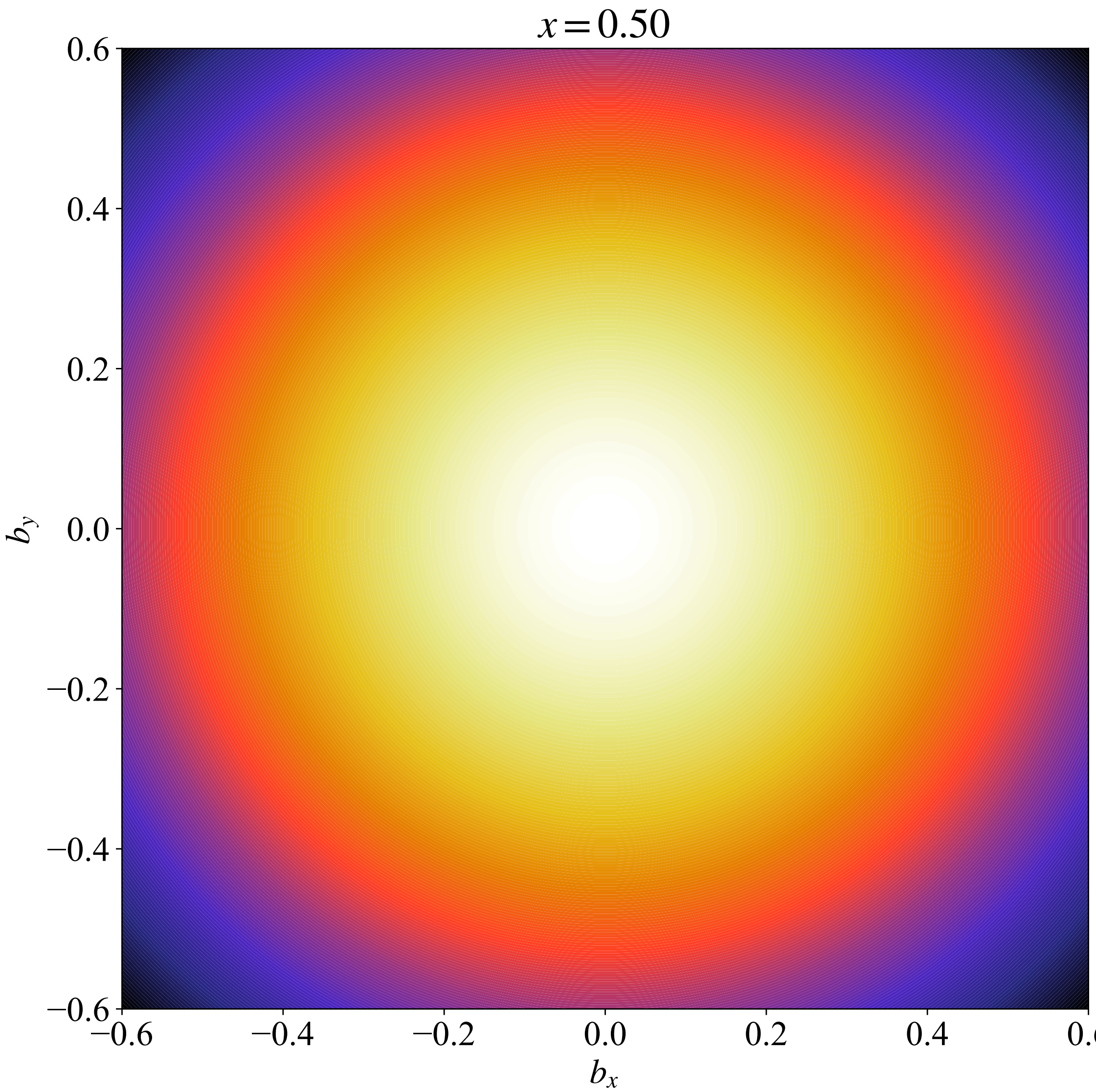
Impact parameter space $\widetilde{E}+\widetilde{G}_1$

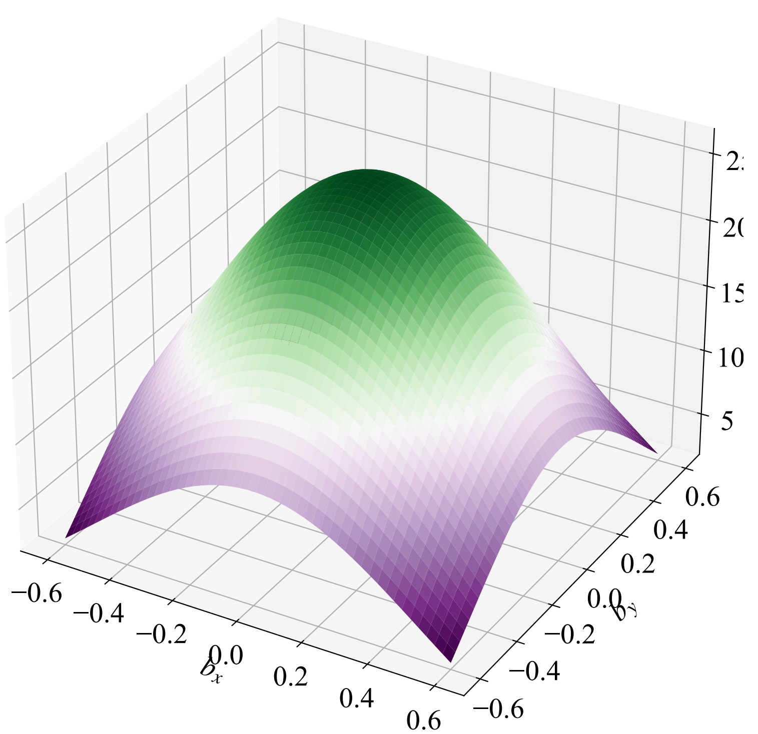


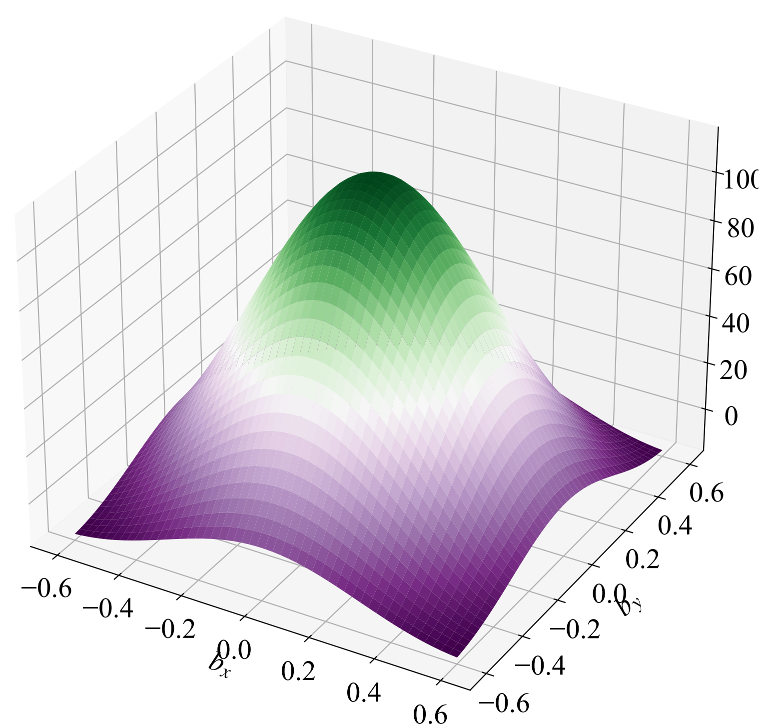
Impact parameter space $\widetilde{E}+\widetilde{G}_1$



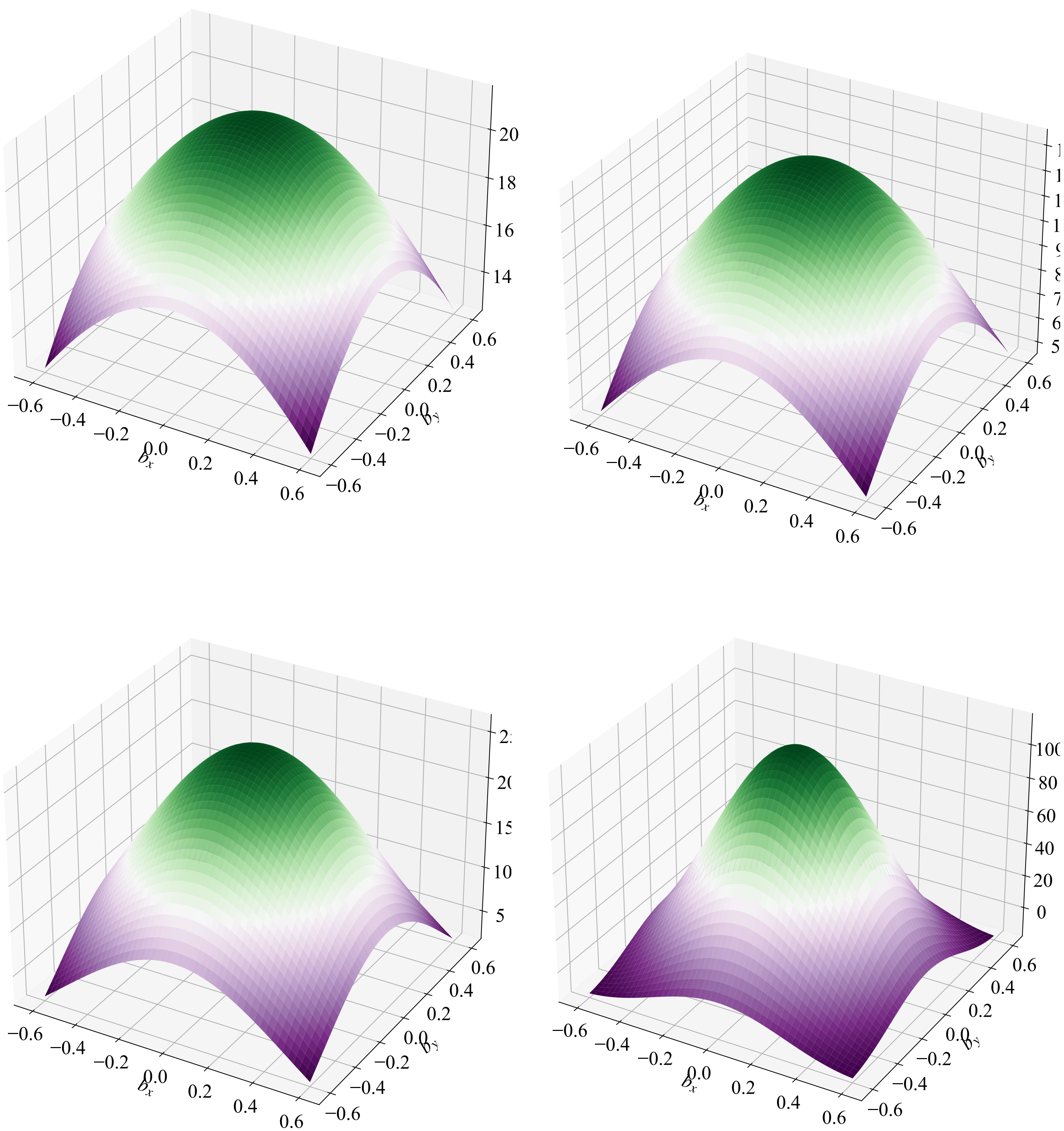


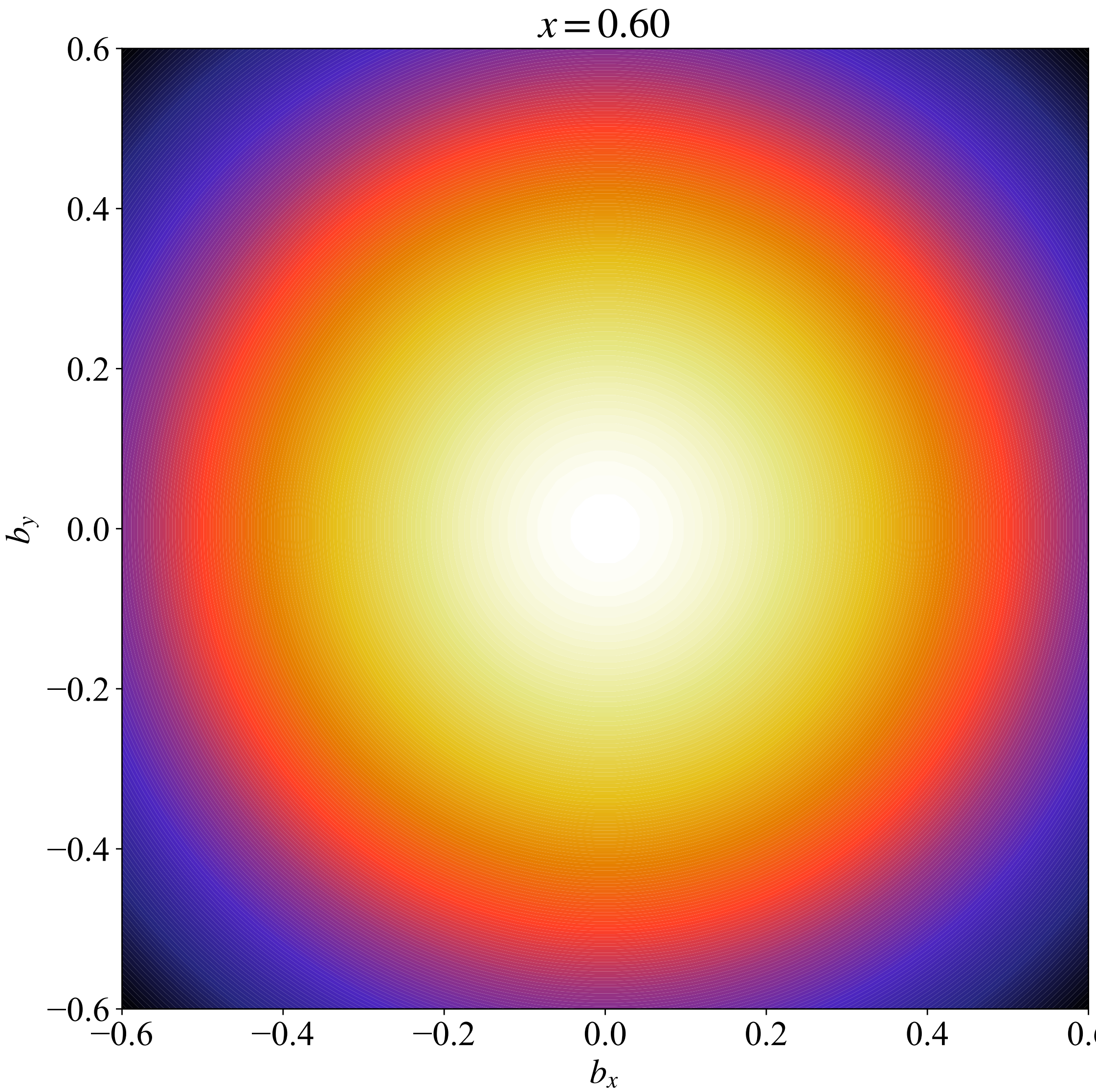




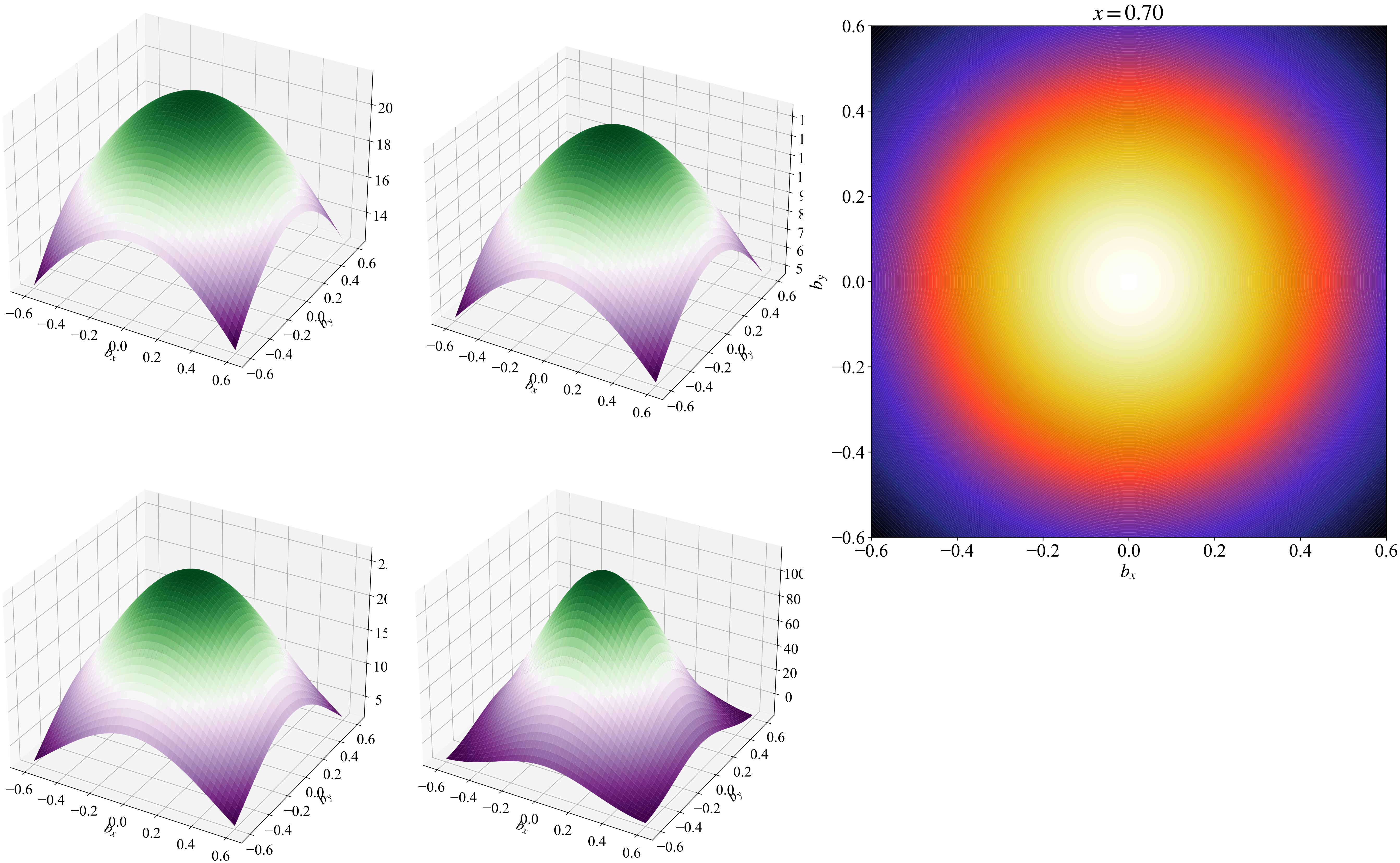


Impact parameter space $\widetilde{E} + \widetilde{G}_1$

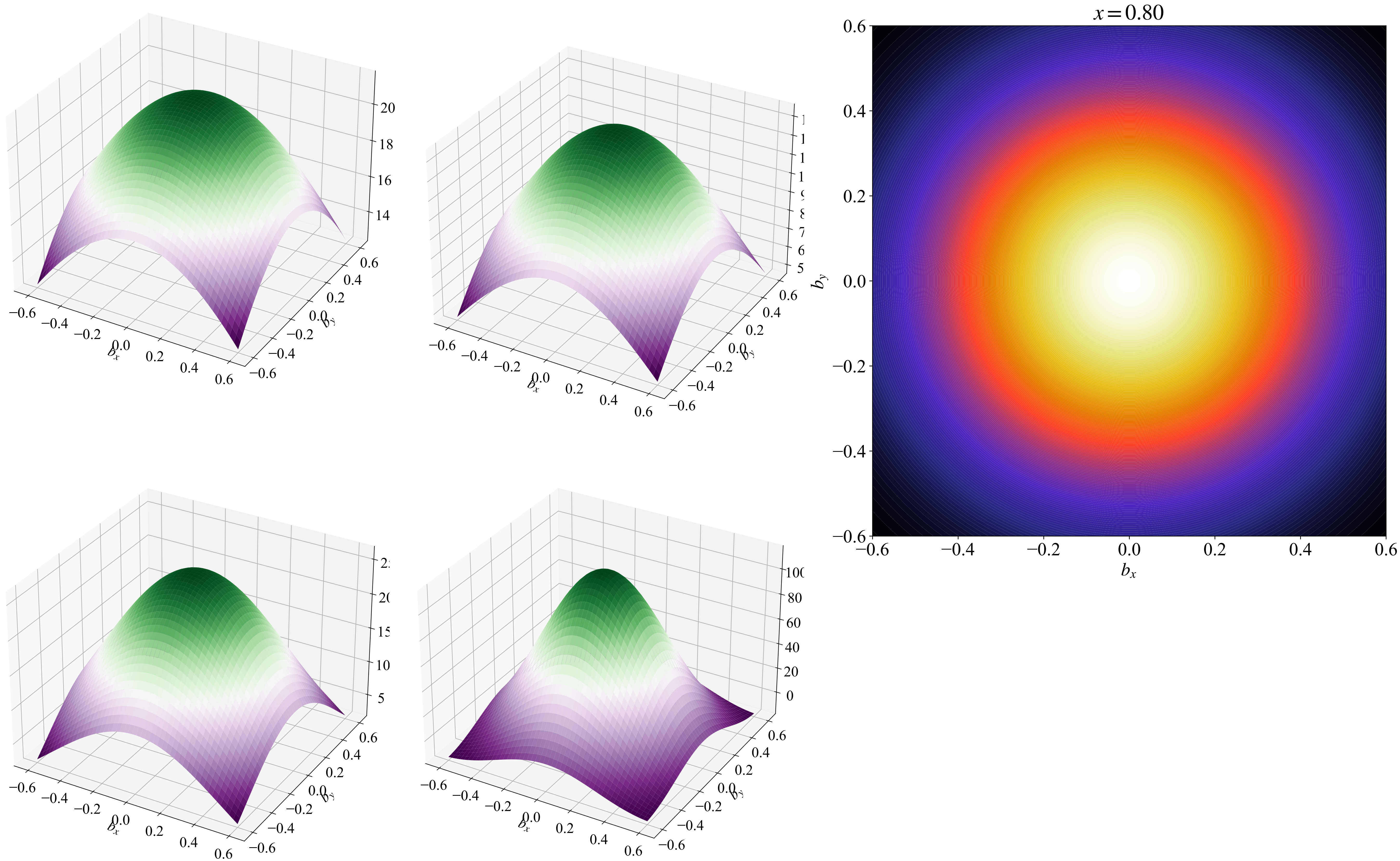




Impact parameter space $\widetilde{E} + \widetilde{G}_1$



Impact parameter space $\widetilde{E} + \widetilde{G}_1$



Synergistic efforts

How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science
Award Number: DE-SC0023646

1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. Lattice QCD calculations of GPDs and related structures

3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

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PHENOMENOLOGY
LATTICE QCD
THEORY

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★ Three bridge faculty positions will be created in nuclear theory

Stony Brook & Temple: Faculty positions in Fall 2024

QGT TC Publications

https://qgtcollab.github.io/publications.html

- ★ 24 publications (PRL, PRC, PRD, PLB, JHEP, Rev. Mod. Phys.)
- ★ 16 preprints
- ★ also proceedings

- **Fangcheng He, Ismail Zahed**, *Gravitational form factors of light nuclei: Impulse approximation*, Phys.Rev.C 109 (Apr 2024)
- **Florian Hechenberger, Kiminad A. Mamo, Ismail Zahed**, *Threshold photoproduction of $|eta_c$ and $|eta_b$ using holographic QCD*, Phys.Rev.D 109 (Apr 2024)
- **Kiminad A. Mamo, Ismail Zahed**, *String-based parametrization of nucleon GPDs at any skewness: a comparison to lattice QCD*, Unpublished (Apr 2024)
- **Kemal Tezgin, Brean Maynard, Peter Schweitzer**, *Chiral-odd GPDs in the bag model*, Unpublished (Apr 2024)
- **Sebastian Grieninger, Kazuki Ikeda, Ismail Zahed**, *Quasi-parton distributions in massive QED2: Towards quantum computation*, Unpublished (Apr 2024)
- **Wei-Yang Liu, Ismail Zahed**, *Photo-production of $|eta_{c,b}$ near Threshold*, Unpublished (Apr 2024)
- **Wei-Yang Liu, Edward Shuryak, Ismail Zahed**, *Glue in hadrons at medium resolution and the QCD instanton vacuum*, Unpublished (Apr 2024)
- **H. Dutrieux, J. Karpie, C. Monahan, K. Orginos, S. Zafieropoulos**, *Evolution of Parton Distribution Functions in the Short-Distance Factorization Scheme*, JHEP 04, 61 (Apr 2024)
- **Peng-Xiang Ma, Xu Feng, Mikhail Gorchtein, Lu-Chang Jin, Keh-Fei Liu, Chien-Yeah Seng, Bi-Geng Wang, Zhao-Long Zhang**, *Lattice QCD Calculation of Electroweak Box Contributions to Superallowed Nuclear and Neutron Beta Decays*, acceped in Phys. Rev. Lett. (Apr 2024)
- **Xiang Gao, Wei-Yang Liu, Yong Zhao**, *Parton Distributions from Boosted Fields in the Coulomb Gauge*, accepted in Phys. Rev. D (Apr 2024)
- **X. Gao, A. D. Hanlon, S. Mukherjee, P. Petreczky, Q. Shi, S. Syritsyn and Y. Zhao**, *Transversity PDFs of the proton from lattice QCD with physical quark masses*, Phys.Rev.D 109 (Mar 2024)
- **Yoshitaka Hatta, Feng Yuan**, *Angular dependence in transverse momentum dependent diffractive parton distributions at small-x*, Unpublished (Mar 2024)
- **Mary Alberg, Gerald A. Miller**, *Quark Counting, Drell-Yan West, and the Pion Wave Function*, Unpublished (Mar 2024)
- **Nicholas Miesch, Edward Shuryak, Ismail Zahed**, *Bridging hadronic and vacuum structure by heavy quarkonia*, Unpublished (Mar 2024)
- **Joe Karpie, Richard Whitehill, Wally Melnitchouk, Chris Monahan, Kostas Orginos, Jian-Wei Qiu, David Richards, Nobuo Sato, Savvas Zafeiropoulos**, *Gluon helicity from global analysis of experimental data and lattice QCD Ioffe time distributions*, Phys. Rev. D 109, 036031 (Feb 2024)
- **Florian Hechenberger, Kiminad A. Mamo, Ismail Zahed**, *Holographic odderon at TOTEM?*, Phys.Rev.D 109 (Feb 2024)
- **Shohini Bhattacharya , Krzysztof Cichy , Martha Constantinou , Jack Dodson , Xiang Gao , Andreas Metz , Joshua Miller , Swagato Mukherjee , Peter Petreczky , Fernanda Steffens , Yong Zhao**, *Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case*, Phys.Rev.D 109 (Feb 2024)
- **Fangcheng He, Ismail Zahed**, *Deuteron gravitational form factors: exchange currents*, Unpublished (Jan 2024)
- **Bigeng Wang, Fangcheng He, Gen Wang, Terrence Draper, Jian Liang, Keh-Fei Liu, Yi-Bo Yang**, *Trace anomaly form factors from lattice QCD*, Accepted in Phys. Rev. D (Jan 2024)
- **Yuxun Guo, Xiangdong Ji, Feng Yuan**, *Proton's gluon GPDs at large skewness and gravitational form factors from near threshold heavy quarkonium photo-production*, Phys.Rev.D 109 (Jan 2024)
- **F. Aslan , M. Boglione , J.O. Gonzalez-Hernandez , T. Rainaldi , T.C. Rogers , A. Simonelli**, *Phenomenology of TMD parton distributions in Drell-Yan and Z^0 boson production in a hadron structure oriented approach*, Unpublished (Jan 2024)
- **Yuxun Guo, Feng Yuan**, *Explore the Nucleon Tomography through Di-hadron Correlation in Opposite Hemisphere in Deep Inelastic Scattering*, Unpublished (Dec 2023)
- **Keh-Fei Liu**, *Hadrons, superconductor vortices, and cosmological constant*, Phys. Lett. B 849, 138418 (Dec 2023)
- **Daniel C. Hackett, Patrick R. Oare, Dimitra A. Pefkou, Phiala E. Shanahan**, *Gravitational form factors of the pion from lattice QCD*, Phys. Rev. D 108 (Dec 2023)
- **Yong Zhao**, *Transverse Momentum Distributions from Lattice QCD without Wilson Lines*, Unpublished (Nov 2023)
- **Jian Liang , Raza Sabbir Sufian , Bigeng Wang , Terrence Draper , Tanjib Khan , Keh-Fei Liu , Yi-Bo Yang**, *Elastic and resonance structures of the nucleon from hadronic tensor in lattice QCD: implications for neutrino-nucleon scattering and hadron physics*, Unpublished (Nov 2023)
- **Ho-Yeon Won, Hyun-Chul Kim, June-Young Kim**, *Role of strange quarks in the D-term and cosmological constant term of the proton*, Phys. Rev. D 108 (Nov 2023)
- **Tanjib Khan, Tianbo Liu, Raza Sabbir Sufian**, *Gluon helicity in the nucleon from lattice QCD and machine learning*, Phys. Rev. D 108, 074502 (Oct 2023)
- **V.D. Burkert, L. Elouadrhiri, F.X. Girod, C. Lorcé, P. Schweitzer, P.E. Shanahan**, *Colloquium: Gravitational Form Factors of the Proton*, Rev. Mod. Phys. 95 (Oct 2023)
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- **Shohini Bhattacharya, Krzysztof Cichy, Martha Constantinou, Jack Dodson, Andreas Metz, Aurora Scapellato, Fernanda Steffens**, *Chiral-even axial twist-3 GPDs of the proton from lattice QCD*, Phys. Rev. D 108 (Sep 2023)
- **Eric Moffat, Adam Freese, Ian Cloët, Thomas Donohoe, Leonard Gamberg, W. Melnitchouk, Andreas Metz, Alexei Prokudin, Nobuo Sato**, *Shedding light on shadow generalized parton distributions*, Phys. Rev. D 108 (Aug 2023)
- **June-Young Kim**, *Quark distribution functions and spin-flavor structures in $N|to |Delta$ transitions*, Phys.Rev.D 108 (Aug 2023)
- **Shohini Bhattacharya Krzysztof Cichy, Martha Constantinou, Xiang Gao, Andreas Metz, Joshua Miller, Swagato Mukherjee, Peter Petreczky, Fernanda Steffens, Yong Zhao**, *Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO*, Phy. Rev. D, 108, 014507 (Jul 2023)
- **Adam Freese, Gerald Miller**, *Synchronization effects on rest frame energy and momentum densities in the proton*, Phys Rev D 108 (Jul 2023)
- **Xiang Gao, Wei-Yang Liu, Yong Zhao**, *Parton Distributions from Boosted Fields in the Coulomb Gauge*, Unpublished (Jun 2023)
- **Edward Shuryak, Ismail Zahed**, *Hadronic structure on the light-front VI. Generalized parton distributions of unpolarized hadrons*, Phys. Rev. D 107 (May 2023)
- **Tom Dodge, Peter Schweitzer**, *Exactly solvable models of nonlinear extensions of the Schrödinger equation*, Unpublished (Apr 2023)
- **X. Gao, A. D. Hanlon, J. Holligan, N. Karthik, S. Mukherjee, P. Petreczky, S. Syritsyn and Y. Zhao**, *Unpolarized proton PDF at NNLO from lattice QCD with physical quark masses*, Phys. Rev. D 107 (Apr 2023)
- **Yuxun Guo, Xiangdong Ji, M. Gabriel Santiago, Kyle Shiells, Jinghong Yang**, *Generalized parton distributions through universal moment parameterization: non-zero skewness case*, JHEP 05 150 (Feb 2023)

Summary

★ New proposal for Lorentz invariant decomposition has great advantages:
- significant reduction of computational cost
- access to a broad range of t and ξ

★ LaMET formalism is applicable beyond leading twist.
However, several improvements needed, e.g.,
mixing with quark-gluon-quark correlator

★ Future calculations have the potential to transform the field of GPDs

★ Extension to other hadrons

★ Synergy with phenomenology is an exciting prospect!
QGT Collaboration will be instrumental in such effort

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Thank you

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Award Number: DE-SC0023646

DOE Early Career Award (NP)
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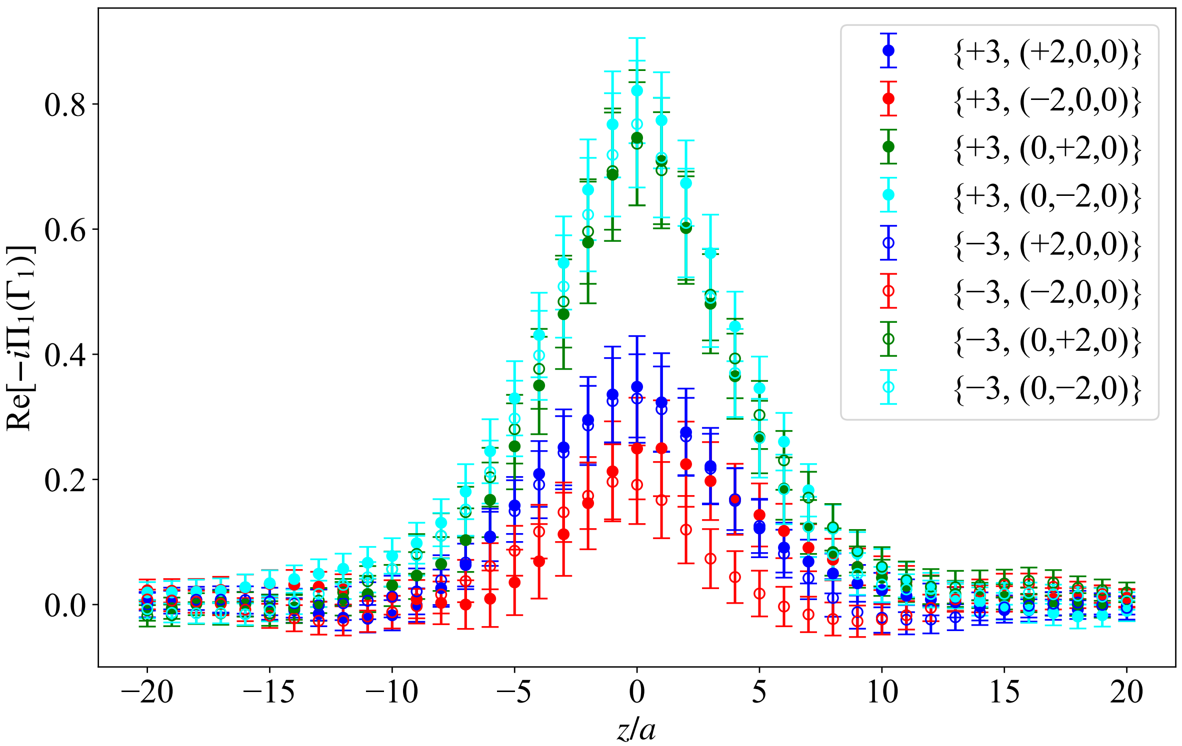
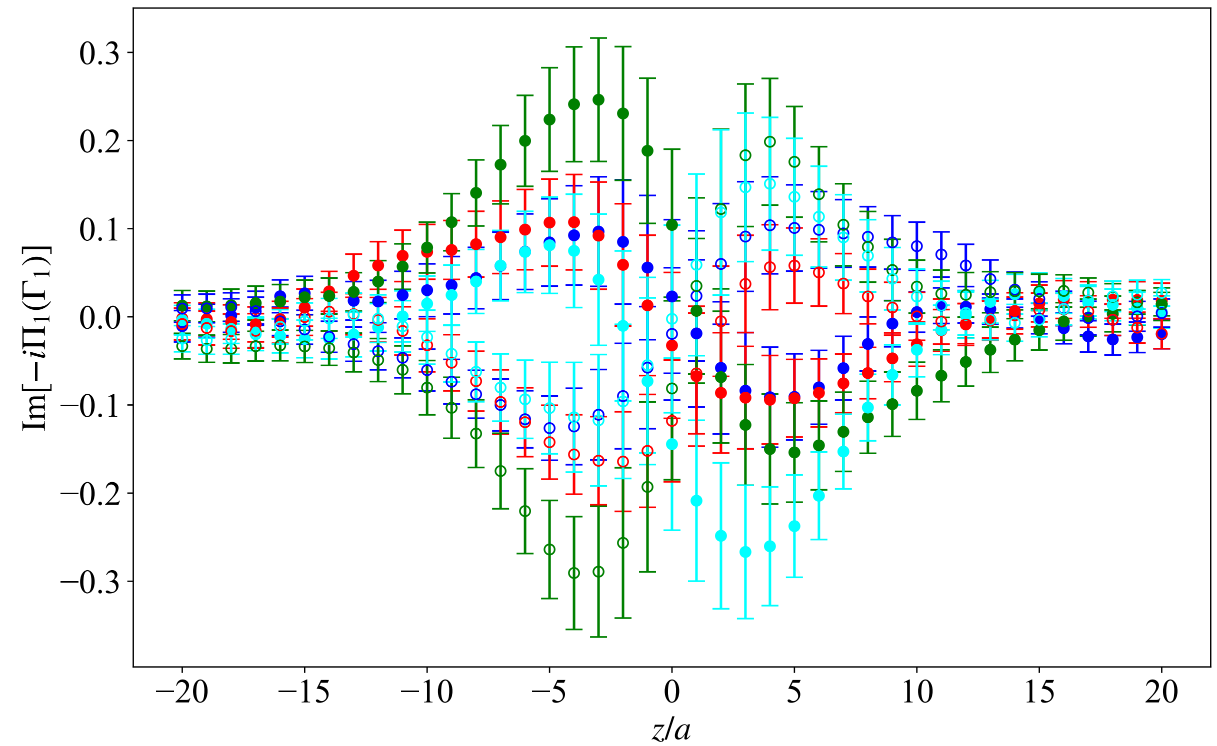
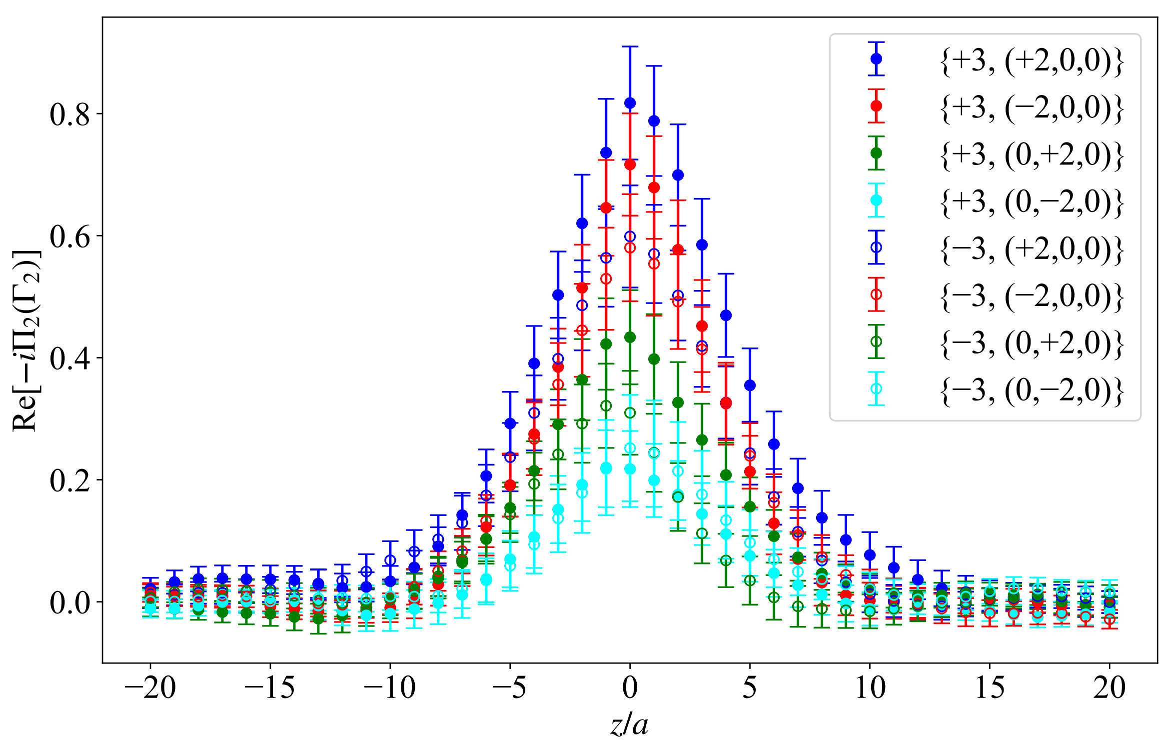
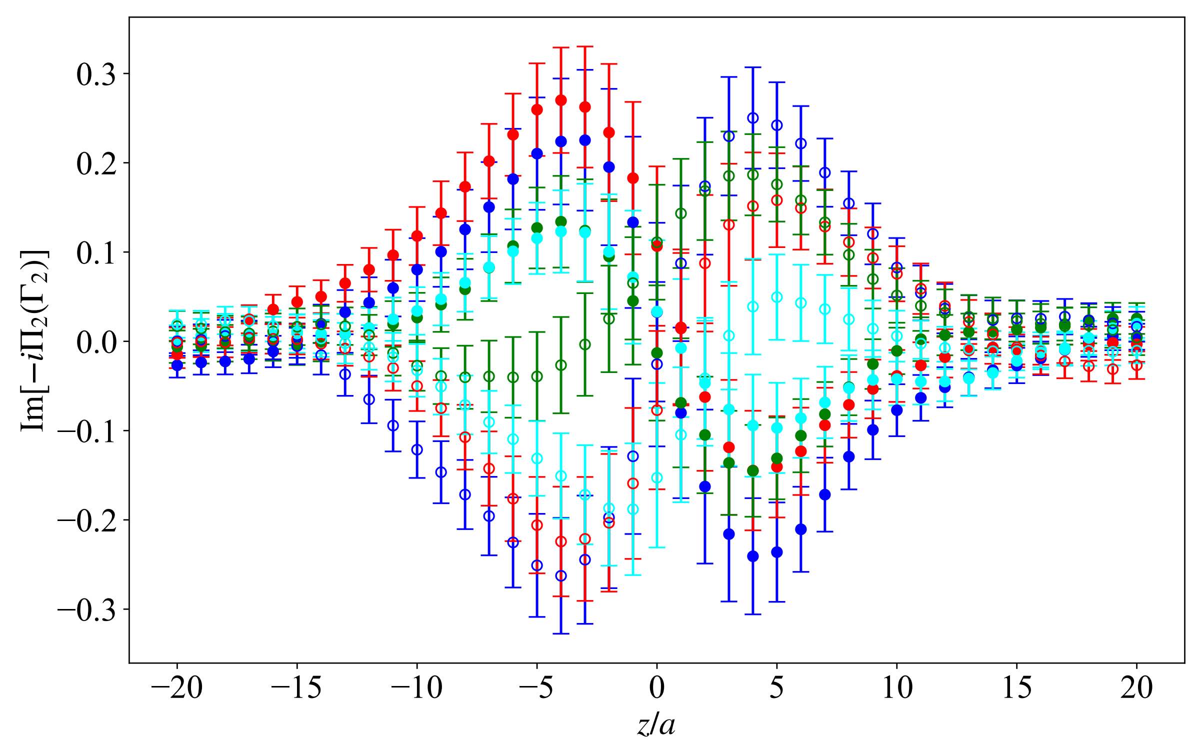
Miscellaneous

Axial twist-3 GPDs with asymmetric frame





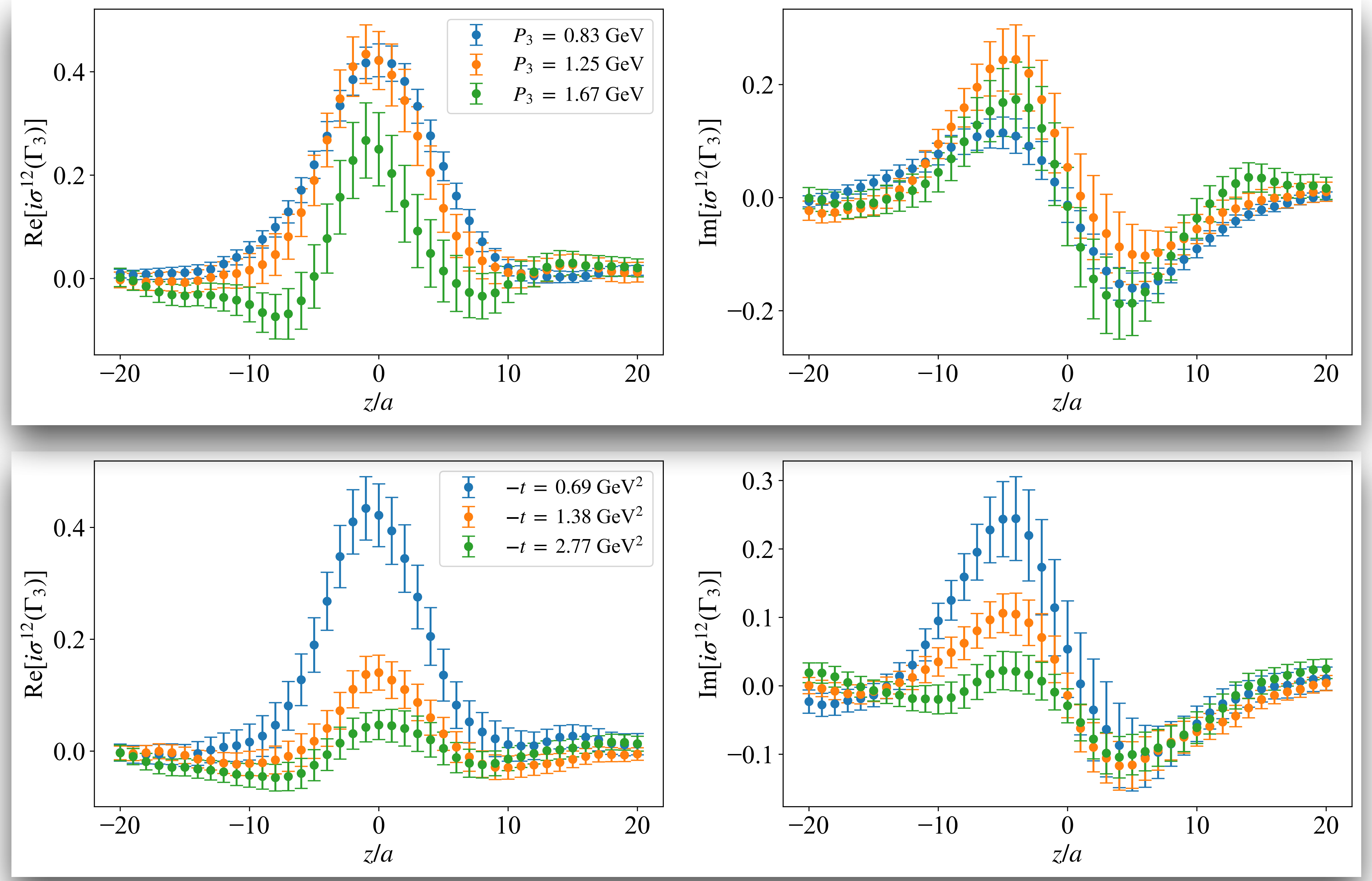




Extension to twist-3 tensor GPDs

★ Parametrization [Meissner et al., *JHEP* 08 (2009) 056]

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p') \left(\gamma^+\gamma_5\, \widetilde{H}_2' + \frac{P^+\gamma_5}{M}\, \widetilde{E}_2' \right) u(p)$$



Reconstruction of x-dependence & matching

★ quasi-GPDs transformed to momentum space using Backus Gilbert
[G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)]

★ Matching formalism to 1 loop accuracy level

$$F_X^{\mathrm{M\overline{MS}}}(x,t,P_3,\mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C_{\gamma_j\gamma_5}^{\mathrm{M\overline{MS}},\overline{\mathrm{MS}}}\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) G_X^{\overline{\mathrm{MS}}}(y,t,\mu) \; + \mathcal{O}\left(\frac{m^2}{P_3^2}, \frac{t}{P_3^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{x^2 P_3^2}\right)$$

★ Operator dependent kernel

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One-loop matching for the twist-3 parton distribution $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

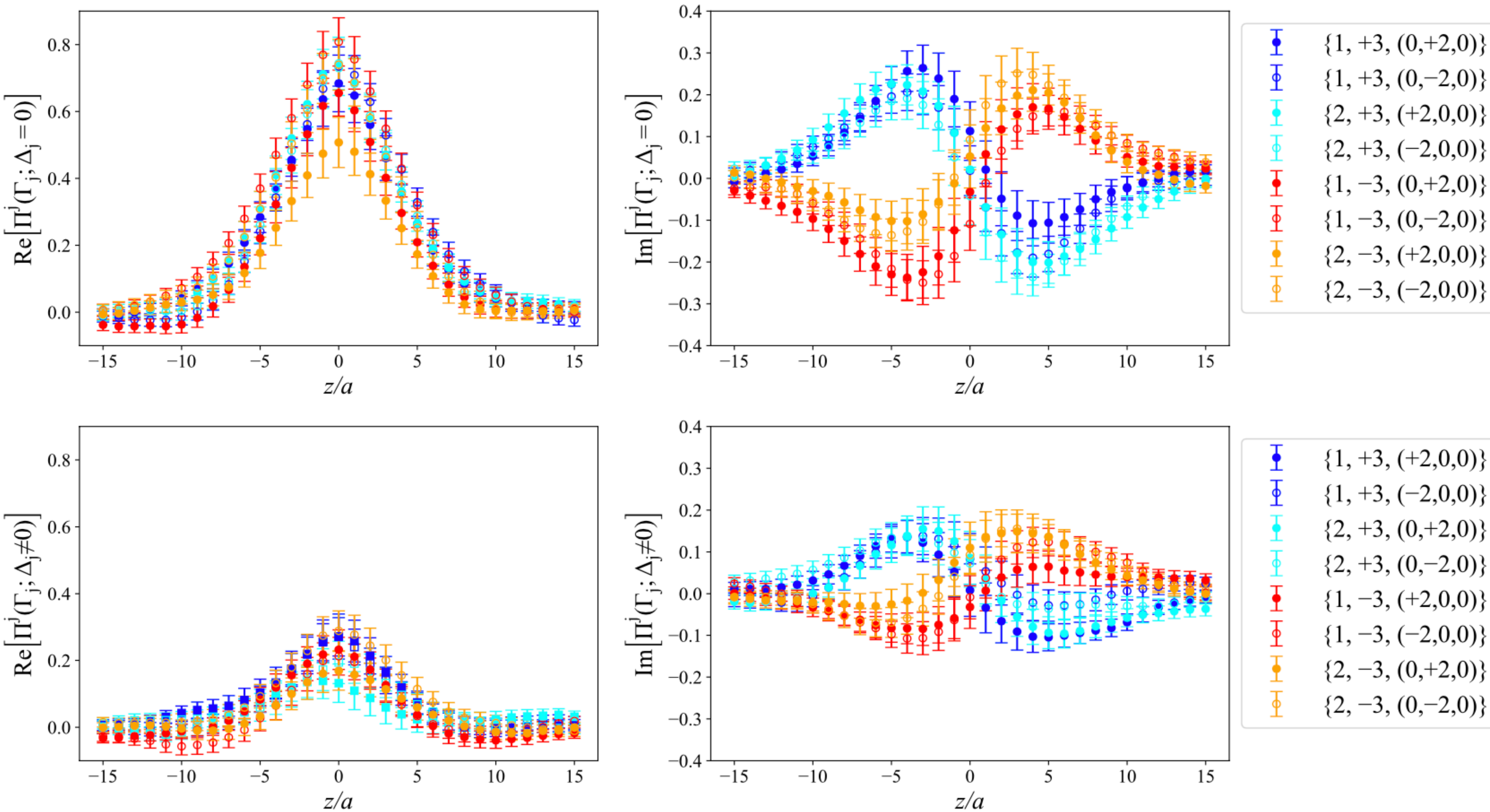
$$C^{(1)}_{\mathrm{M\overline{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} 0 \\ \delta(\xi) \\ 0 \end{cases} + \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ & \xi > 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0\,, \end{cases}$$

★ Matching does not consider mixing with q-g-q correlators
[V. Braun et al., JHEP 05 (2021) 086]

Lattice Results - Matrix Elements

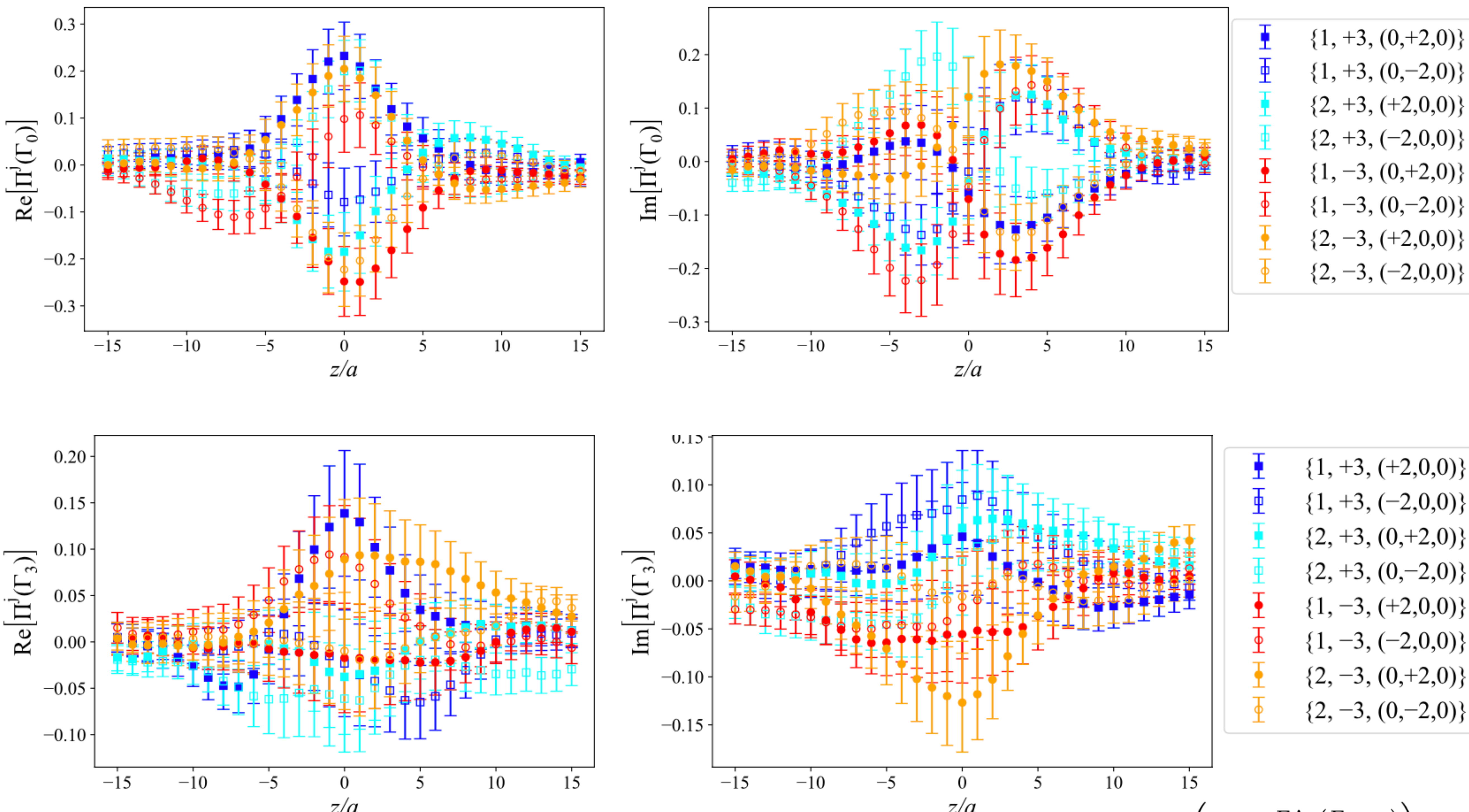
★ Bare matrix elements

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\,\frac{\left(4m(E+m)+\Delta_y^2\right)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\,\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\,\frac{\mathrm{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2 P_3}\right)$$



Lattice Results - Matrix Elements

★ Bare matrix elements



$$\Pi^1(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_x(E+m)}{2m^2P_3}\right)$$

★ Suppressed signal compared to $\gamma_+\gamma_5$ operators

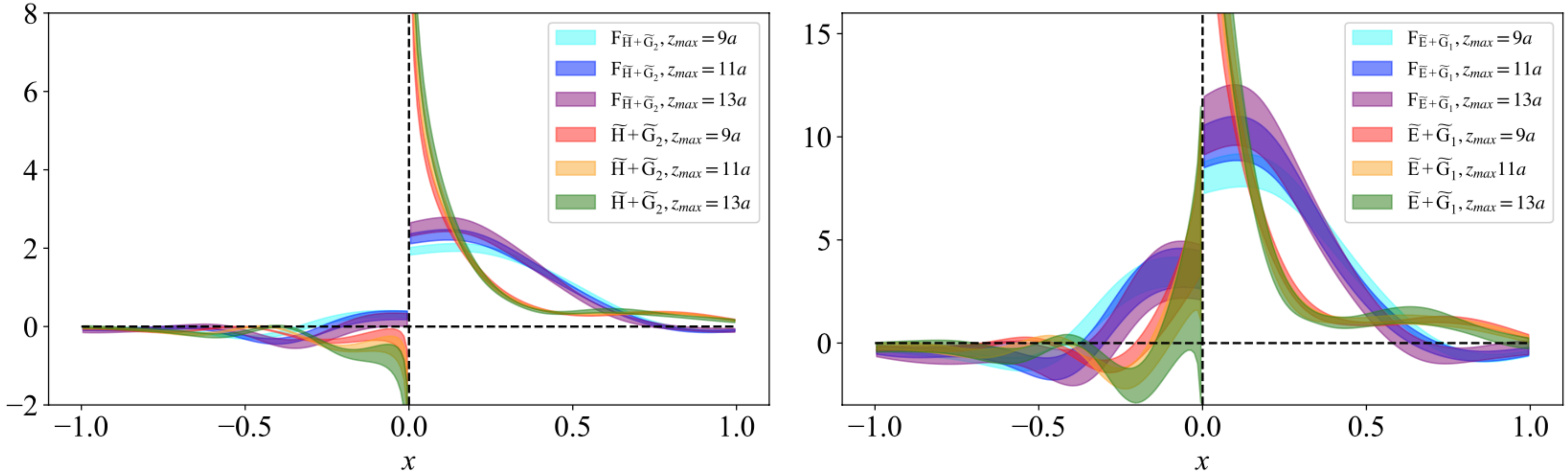


FIG. 10. $z_{\max}$ dependence of $F_{\widetilde{H}+\widetilde{G}_2}$ and $\widetilde{H}+\widetilde{G}_2$ (left), as well as $F_{\widetilde{E}+\widetilde{G}_1}$ and $\widetilde{E}+\widetilde{G}_1$ (right) at $-t=0.69$ GeV^2 and $P_3=1.25$ GeV. Results are given in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

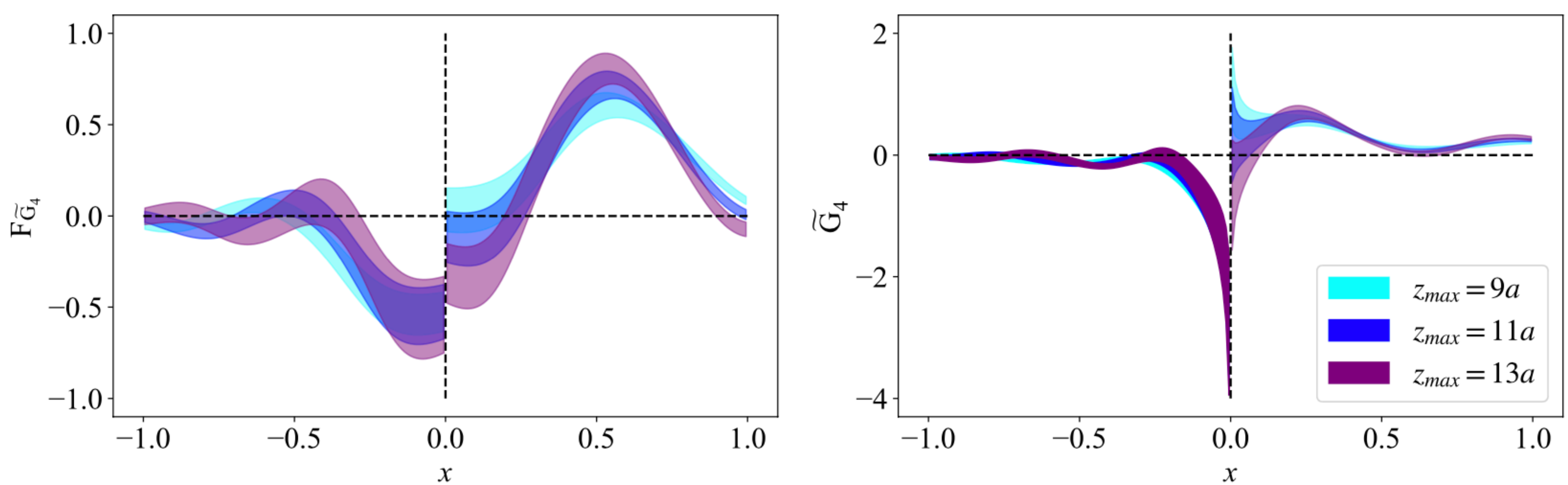


FIG. 11. $z_{\max}$ dependence of $F_{\widetilde{G}_4}$ and $\widetilde{G}_4$ at $-t=0.69$ GeV^2 and $P_3=1.25$ GeV. Results are given in $\overline{\text{MS}}$ scheme at a scale of 2 GeV.