Leading-twist GPDs from Lattice QCD

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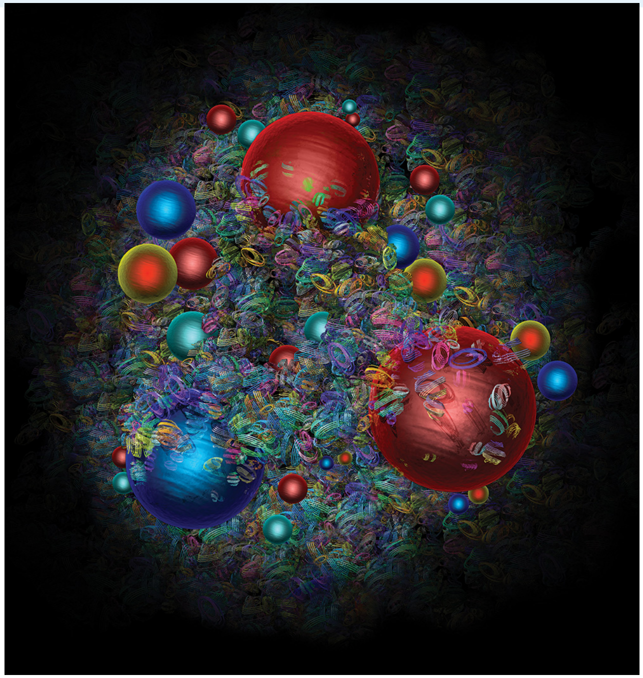
Outline:

Introduction
GPDs from lattice:
– how to access
– reference frames
– quasi and pseudo
– results
Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, S. Bhattacharya, M. Constantinou, J. Dodson, X. Gao
K. Hadjiyiannakou, K. Jansen, A. Metz, J. Miller, S. Mukherjee
N. Nurminen, P. Petreczky, A. Scapellato, F. Steffens, Y. Zhao

Nucleon structure and GPDs

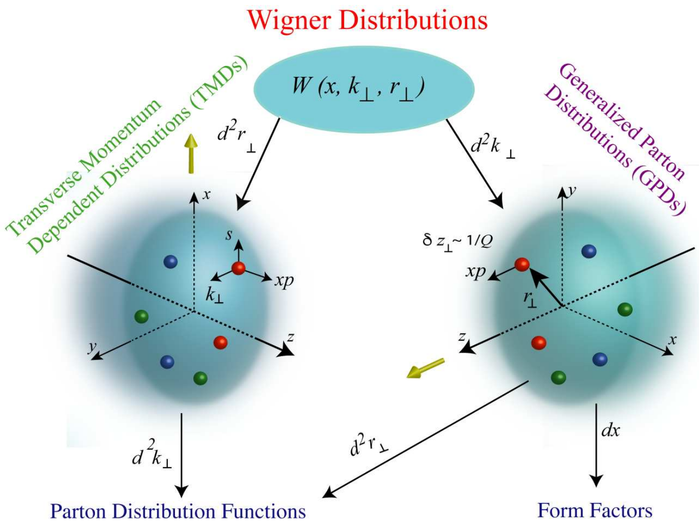


One of the central aims of hadron physics:
to understand better nucleon's 3D structure.

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise? NAS report 2018
- What are the emergent properties of dense systems of gluons?
- Answering these questions is one of the crucial expectations for the upcoming years!
- For this, we need to probe the 3D structure.
- Transverse position of quarks: GPDs.
- Twist-2 GPDs as first aim, but higher-twist of growing importance.
- Both theoretical and experimental input needed.

Generalized parton distributions (GPDs):

- much more difficult to extract than PDFs,
- but they provide a wealth of information:
  - ⋆ spatial distribution of partons in the transverse plane,
  - ⋆ mechanical properties of hadrons,
  - ⋆ hadron's spin decomposition,
- reduce to PDFs in the forward limit, e.g. $H(x,0,0) = q(x)$,
- their moments are form factors, e.g. $\int dx\, H(x,\xi,t) = F_1(t)$.



- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.
- Way out: similar as experimental access to these distributions – **factorization**
  (experiment) cross-section = perturbative-part $*$ partonic-distribution
  (lattice) lattice-observable = perturbative-part $*$ partonic-distribution

Partonic structure from Lattice QCD

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  - ⋆ **quasi-distributions** – Ji, 2013
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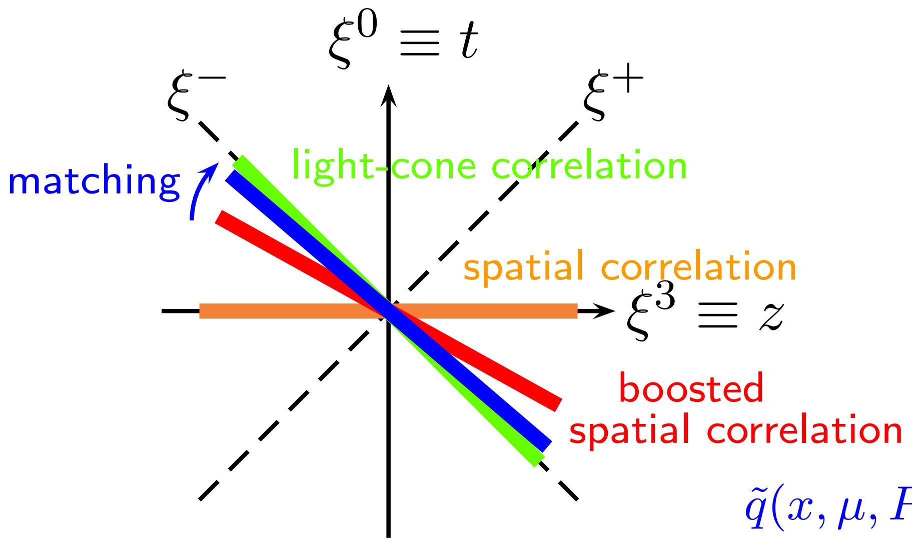

Euclidean matrix element:
$\langle P_f|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P_i\rangle$

Its Fourier transform (quasi-distribution)
can be matched onto the light-cone distribution:
(Large Momentum Effective Theory (LaMET))

$$\underbrace{\tilde{q}(x,\mu,P_3)}_{\text{quasi-PDF}} = \int_{-1}^{1}\frac{dy}{|y|}\underbrace{C\left(\frac{x}{y},\frac{\mu}{P_3}\right)}_{\text{pert.kernel}}\underbrace{q(y,\mu)}_{\text{PDF}} + \underbrace{\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)}_{\text{higher-twist effects}}$$

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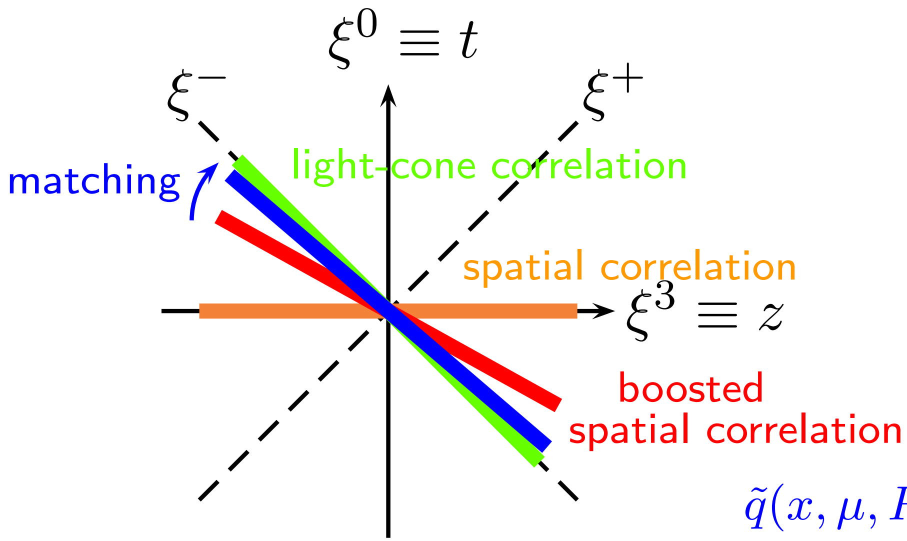
Dirac structures Γ for different GPDs:

VECTOR: γ_0, γ_3: H, E (unpolarized twist-2)
γ_1, γ_2: G_1, G_2, G_3, G_4 (vector twist-3)

AXIAL VECTOR: $\gamma_5\gamma_0, \gamma_5\gamma_3$: $\tilde{H}, \tilde{E}$ (helicity twist-2)
$\gamma_5\gamma_1, \gamma_5\gamma_2$: $\tilde{G}_1, \tilde{G}_2, \tilde{G}_3, \tilde{G}_4$ (axial vector twist-3)

TENSOR: $\gamma_1\gamma_3, \gamma_2\gamma_3$: $H_T, E_T, \tilde{H}_T, \tilde{E}_T$ (transversity twist-2)
$\gamma_1\gamma_2$: H_2', E_2' (tensor twist-3)



Parity projectors to disentangle 2/4 GPDs:

UNPOL: $\mathcal{P} = \frac{1+\gamma_0}{4}$, POL-$k$: $\mathcal{P} = \frac{1+\gamma_0}{4} i\gamma_5\gamma_k$

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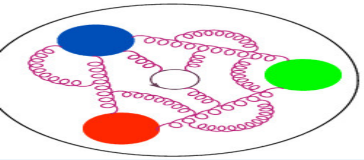


Setup

Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L \approx 3$ fm,
- $m_\pi \approx 260$ MeV.

Kinematics:

- nucleon boosts up to $P_3 = 1.67$ GeV,
- momentum transfers: $-t \leq 2.76$ GeV^2, most data: $-t = 0.65,\ 0.69$ GeV^2,
- skewness: $\xi = 0,\ 1/3$.

up to $\mathcal{O}$(250K) measurements (≈ 500 confs, 32 src positions, 16 permut. of $\vec{\Delta}$).

Twist-2 unpolarized+helicity GPDs C. Alexandrou et al. (ETMC), PRL 125(2020)262001

Twist-2 transversity GPDs C. Alexandrou et al. (ETMC), PRD 105(2022)034501

Twist-2 unpolarized GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 106(2022)114512

Twist-2 unpolarized GPDs (OPE) S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507

Twist-3 axial GPDs S. Bhattacharya et al. (ETMC/Temple), PRD 108(2023)054501

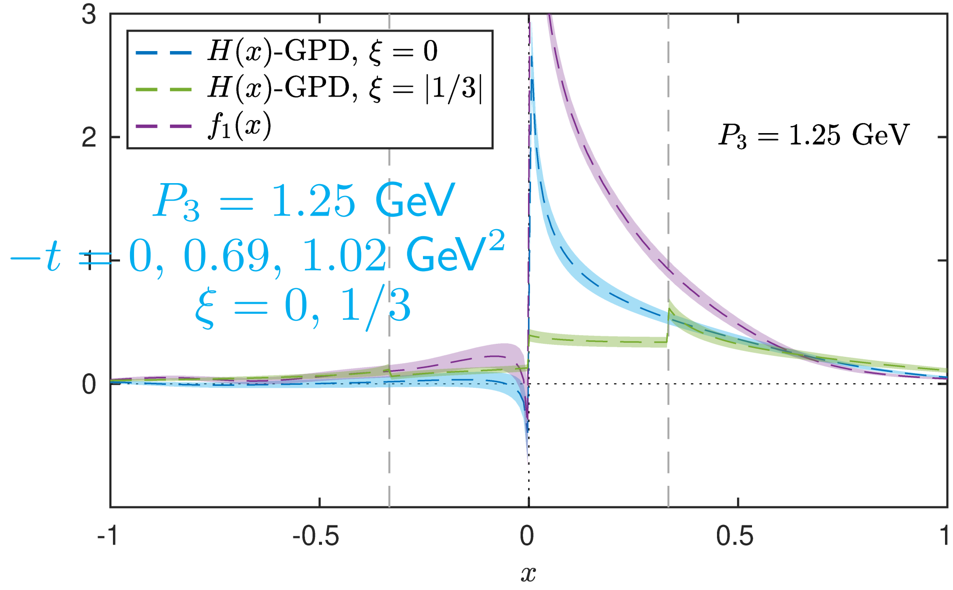
Twist-2 helicity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 109(2024)034508

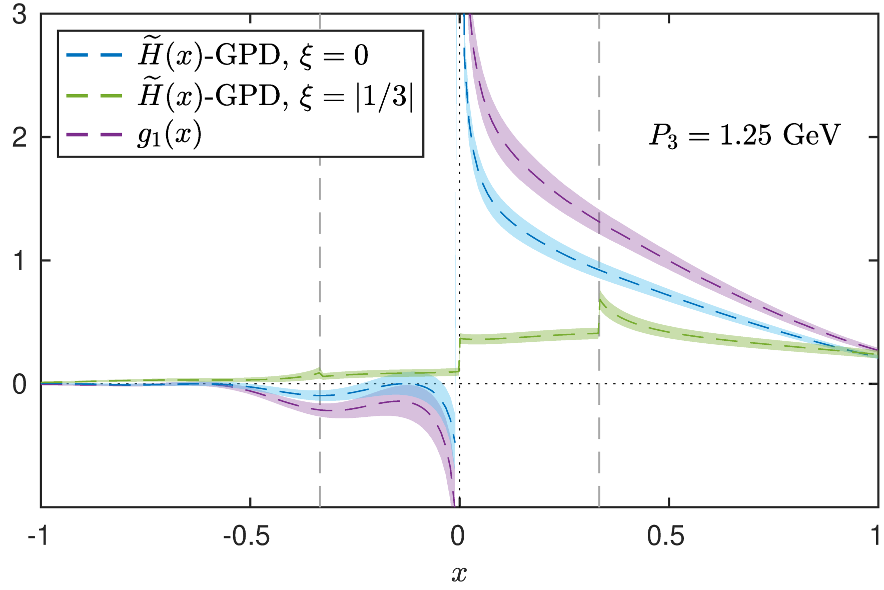
Twist-2 unpolarized GPDs (pseudo-GPDs) S. Bhattacharya et al. (ETMC/Temple) arXiv:2405.04414

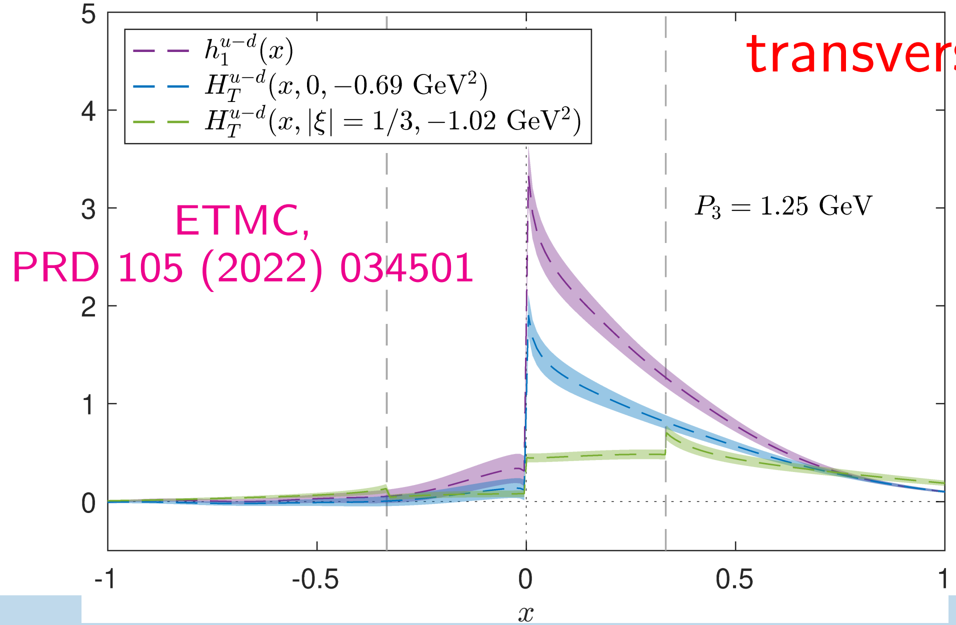
Twist-2 transversity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) in preparation

First extractions of x-dependent GPDs

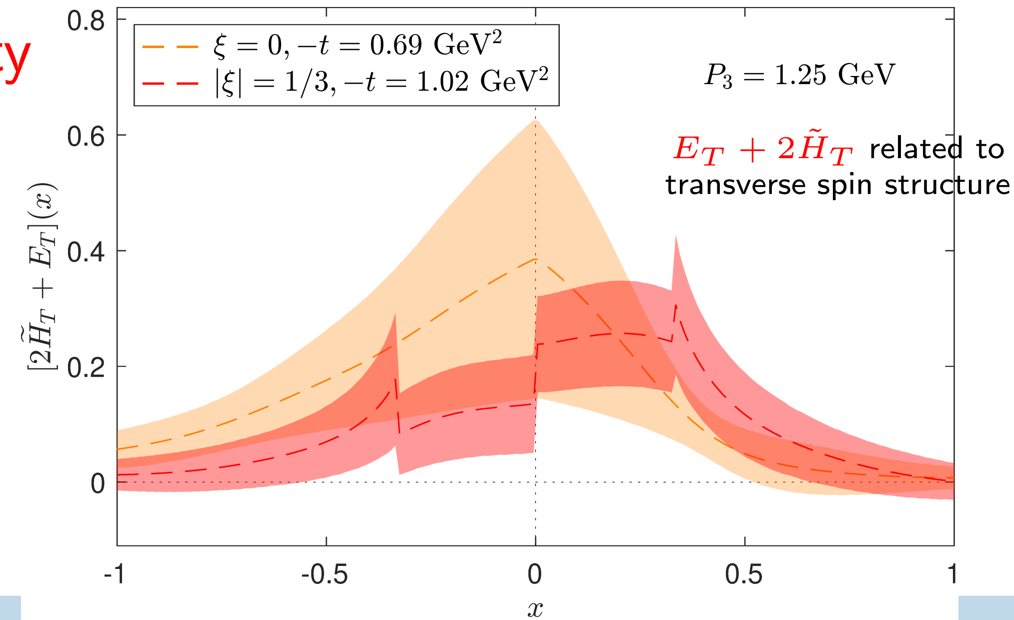
unpolarized | ETMC, Phys. Rev. Lett. 125 (2020) 262001 | helicity







transversity



Standard symmetric (Breit) frame:

source momentum: $P_i = (E, \vec{P} - \vec{\Delta}/2)$,
sink momentum: $P_f = (E, \vec{P} + \vec{\Delta}/2)$.

GPDs in different frames of reference

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Lattice perspective:

construction of the 3-point correlation functions required for the MEs needs the calculation of the all-to-all propagator

preferred way: "sequential propagator" – implies separate inversions (most costly part!) for each P_f.

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Lattice perspective:

Several momentum transfer vectors $\vec{\Delta}$ can be obtained within a single calculation!

Lorentz-covariant parametrization

Main theoretical tool: unpolarized: S. Bhattacharya et al., PRD106(2022)114512

Lorentz-covariant parametrization of matrix elements:

$$F^{\mu}(z,P,\Delta)=\bar{u}(p',\lambda')\left[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+\frac{z^{\mu}i\sigma^{z\Delta}}{m}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p,\lambda),$$

$$F^{[\gamma^{\mu}\gamma_5]}=\bar{u}(p',\lambda')\left[\frac{i\epsilon^{\mu P z\Delta}}{m}\widetilde{A_1}+\gamma^{\mu}\gamma_5\widetilde{A_2}+\gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A_3}+mz^{\mu}\widetilde{A_4}+\frac{\Delta^{\mu}}{m}\widetilde{A_5}\right)+m\not{z}\gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A_6}+mz^{\mu}\widetilde{A_7}+\frac{\Delta^{\mu}}{m}\widetilde{A_8}\right)\right]u(p,\lambda)$$

helicity: S. Bhattacharya et al., PRD109(2024)034508

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$ or $\widetilde{A_i}(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

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Example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0)=C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1+\frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5+\frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

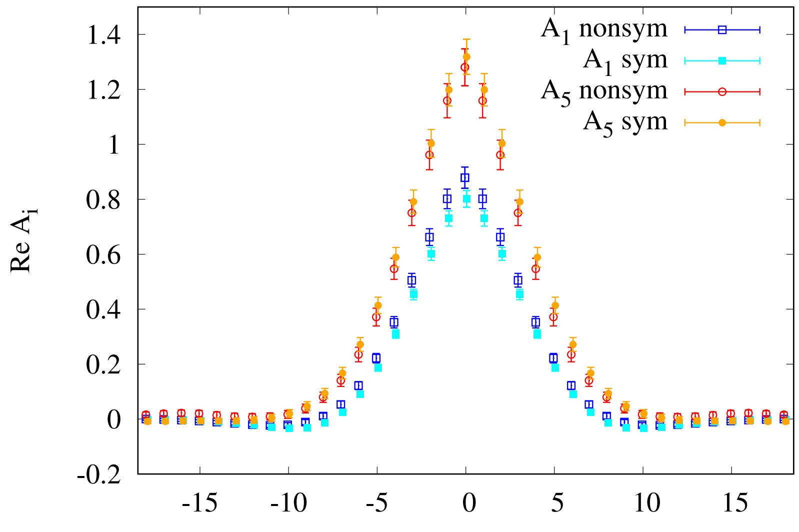
asymmetric frame:

$$\Pi_0^a(\Gamma_0)=C\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1-\frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3+\frac{(E_i-E_f)P_3z}{4m}A_4$$
$$+\frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5+\frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6+\frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\Bigg).$$

- matrix elements $\Pi_{\mu}(\Gamma_{\nu})$ or $\Pi_{\mu 5}(\Gamma_{\nu})$ are **frame-dependent**,
- but the amplitudes A_i or $\widetilde{A}_i$ are **frame-invariant**.

Proof of concept (comparison between frames)

A_1, A_5 (unpolarized leading ones) $\widetilde{A_2}$, $\widetilde{A_5}$ (helicity leading ones)



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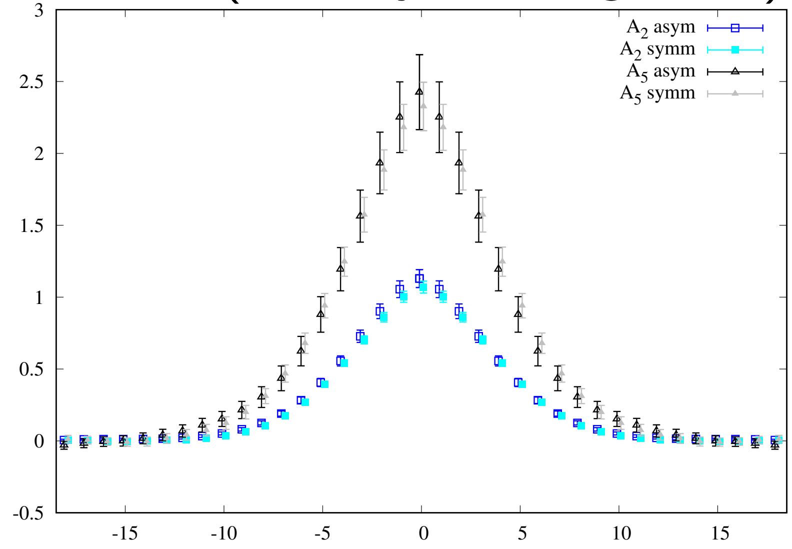
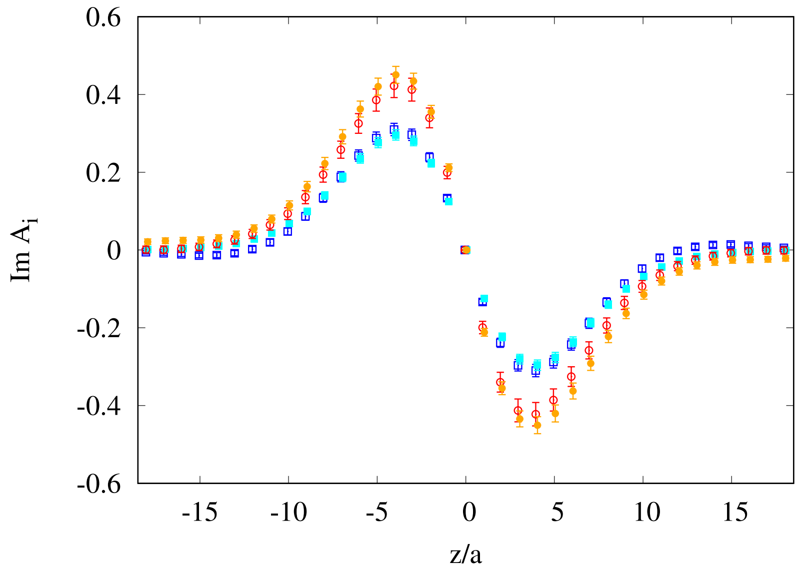


PRD106(2022)114512 S. Bhattacharya et al. PRD109(2024)034508



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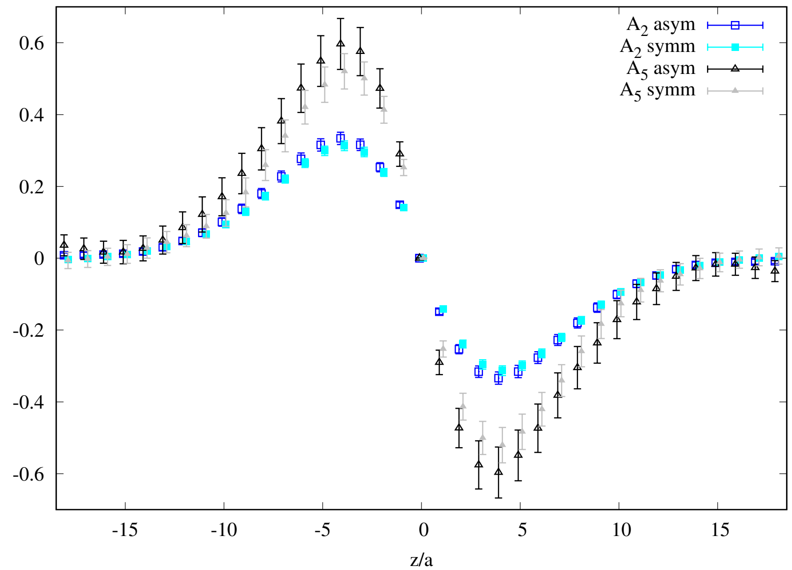

GPDs – possible definitions

Defining H and E GPDs in the standard way, expressions are frame-dependent:

SYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3} A_6 \,, \qquad F_{E(0)} = -A_1 + 2A_5 + \frac{z\left(4E^2 - \Delta_1^2 - \Delta_2^2\right)}{2P_3} A_6 \,.$$

ASYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{\Delta_0}{P_0} A_3 + \frac{m^2 z \Delta_0}{2P_0 P_3} A_4 + \frac{z(\Delta_0^2 + \Delta_\perp^2)}{2P_3} A_6 + \frac{z(\Delta_0^3 + \Delta_0 \Delta_\perp^2)}{2P_0 P_3} A_8 \,,$$

$$F_{E(0)} = -A_1 - \frac{\Delta_0}{P_0} A_3 - \frac{m^2 z(\Delta_0 + 2P_0)}{2P_0 P_3} A_4 + 2A_5 - \frac{z\left(\Delta_0^2 + 2P_0\Delta_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_3} A_6 - \frac{z\Delta_0\left(\Delta_0^2 + 2\Delta_0 P_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_0 P_3} A_8 \,.$$

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One can also modify the definition to make it Lorentz-invariant and arrive at:

ANY frame:

$$F_H = A_1\,, \qquad F_E = -A_1 + 2A_5 + 2zP_3A_6\,.$$

With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

In terms of matrix elements: standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,

LI definition – additionally: $\Pi_{1/2}(\Gamma_3)$ (both frames), $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$ (asym.).

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Two definitions of $\widetilde{H}$:

standard ($\gamma_5\gamma_3$ operator): $F_{\tilde{H}} = \widetilde{A_2} + zP_3\widetilde{A_6} - m^2 z^2 \widetilde{A_7}\,,$

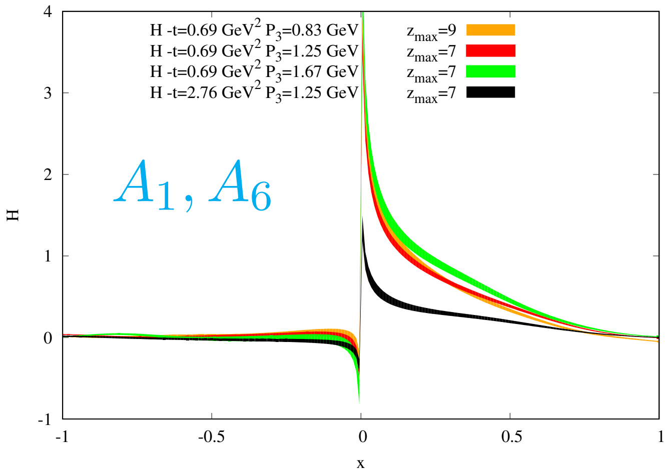
another ($\gamma_5\gamma_i$ operators, $i = 0, 1, 2$): $F_{\tilde{H}} = \widetilde{A_2} + zP_3\widetilde{A_6}\,.$

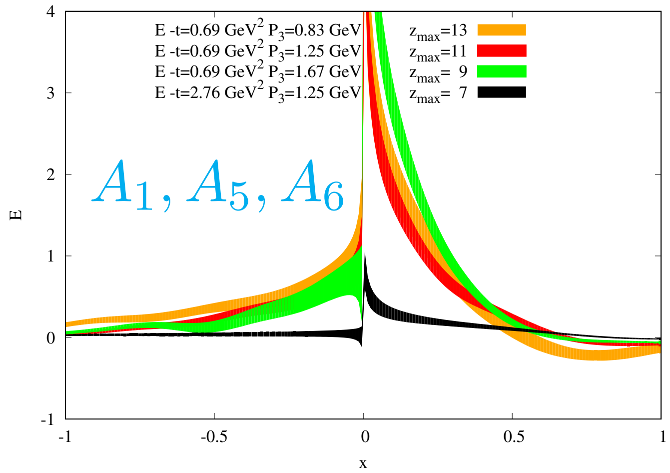
$\widetilde{E}$ impossible to extract at zero skewness: $F_{\tilde{E}} = 2\,\frac{P\cdot z}{\Delta\cdot z}\widetilde{A_3} + 2\widetilde{A_5}\,.$

Convergence of alternative definitions of $\tilde{H}/H/E$

STANDARD

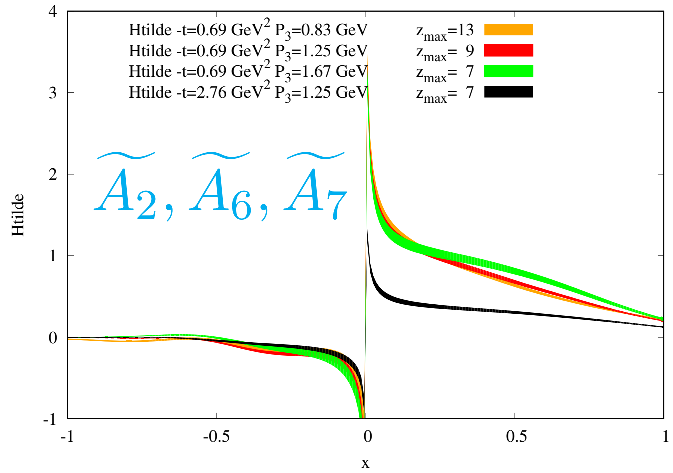
UNPOLARIZED





γ_0 operator (non-LI)

HELICITY



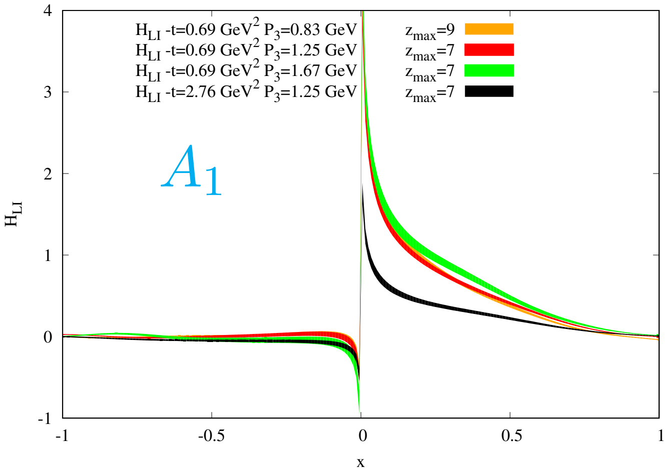
$\gamma_5\gamma_3$ operator (LI)

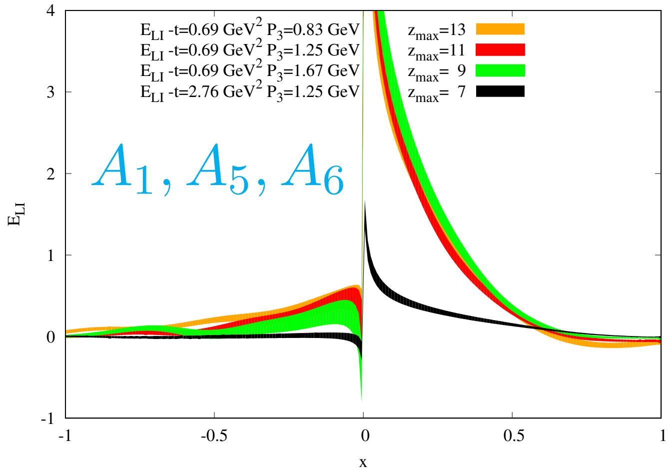
ALTERNATIVE

H-GPD

E-GPD

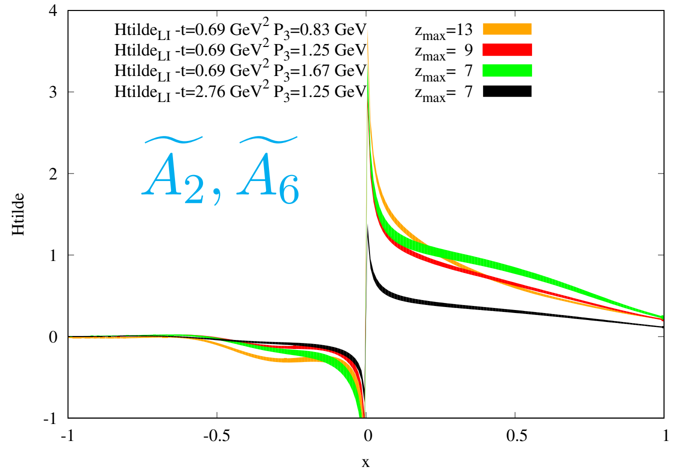
γ_0, γ_T operators (LI)





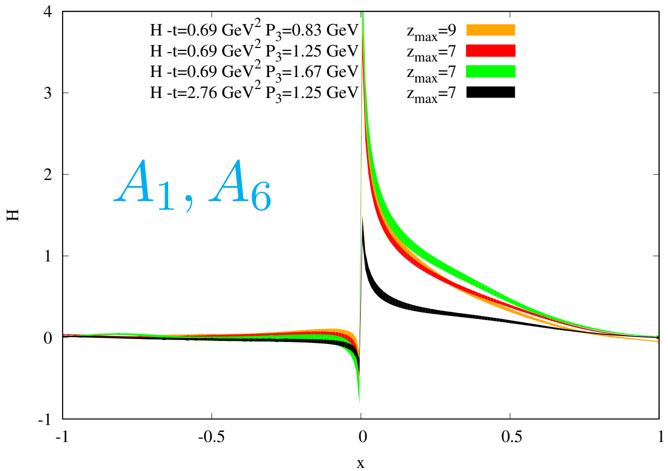
$\widetilde{H}$-GPD

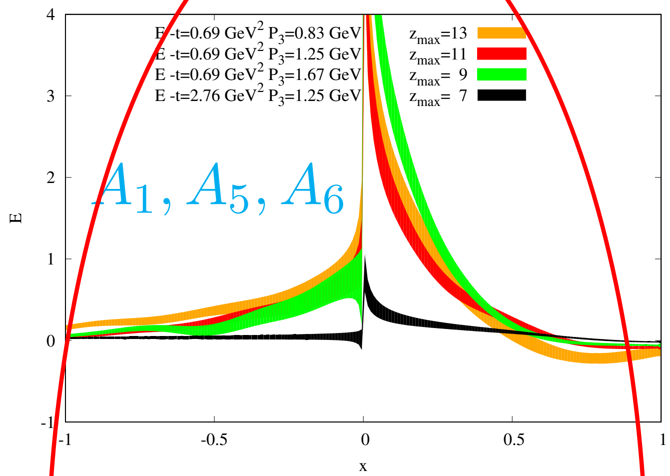
$\gamma_5\gamma_0$, $\gamma_5\gamma_T$ operators (LI)

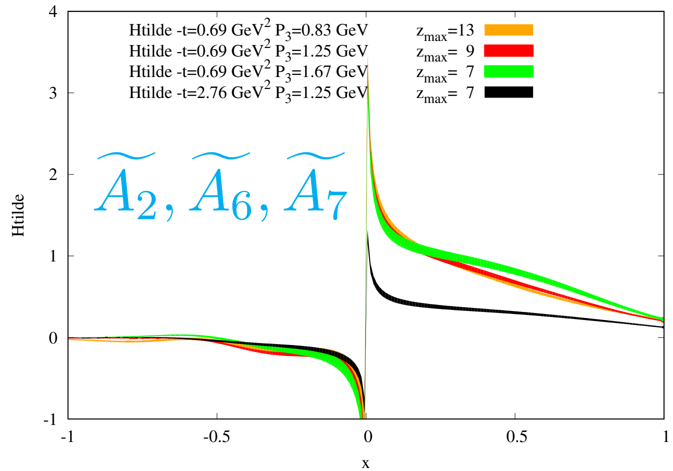


UNPOLARIZED | HELICITY

STANDARD





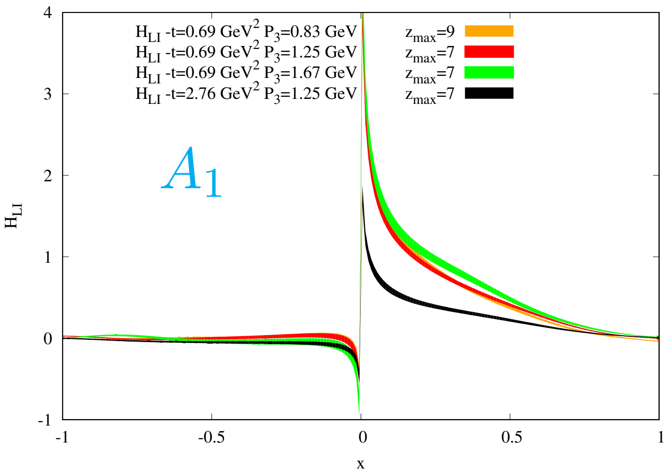


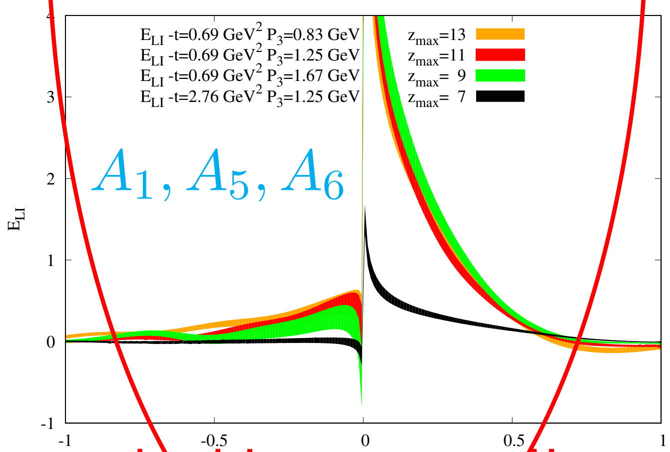
γ_0 operator (non-LI) | $\gamma_5\gamma_3$ operator (LI)

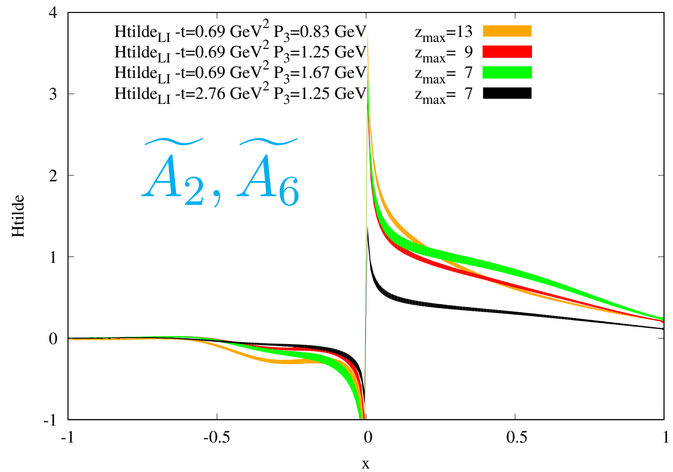
H-GPD | E-GPD | $\widetilde{H}$-GPD

γ_0, γ_T operators (LI) | $\gamma_5\gamma_0$, $\gamma_5\gamma_T$ operators (LI)

ALTERNATIVE







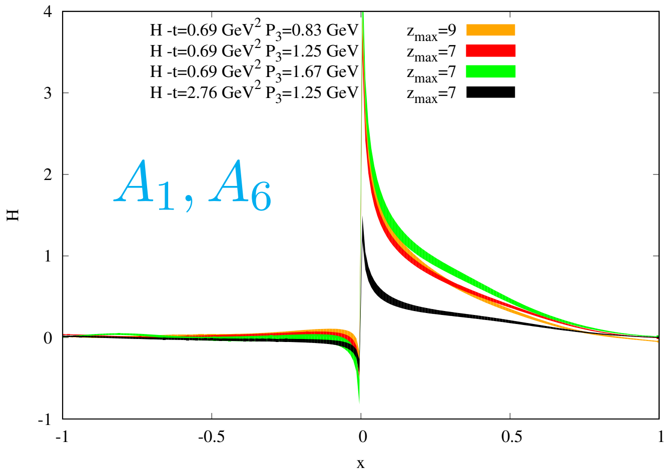
highly-improved!

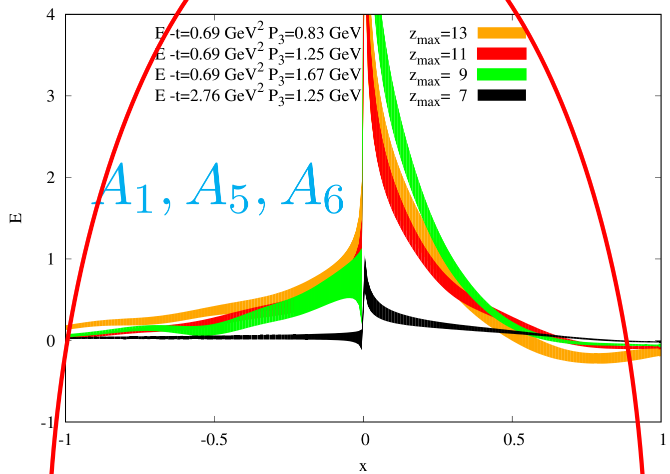
Convergence of alternative definitions of $\tilde{H}/H/E$

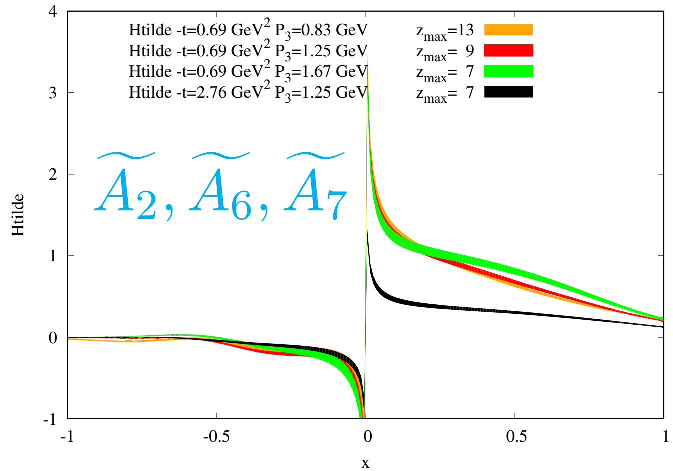
UNPOLARIZED

HELICITY

STANDARD







γ_0 operator (non-LI)

$\gamma_5\gamma_3$ operator (LI)

H-GPD

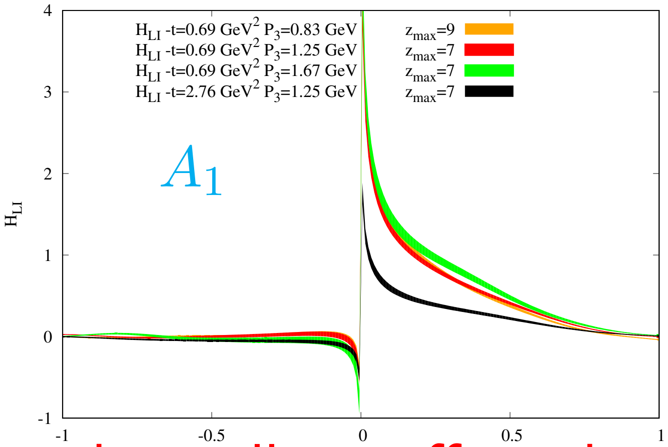
E-GPD

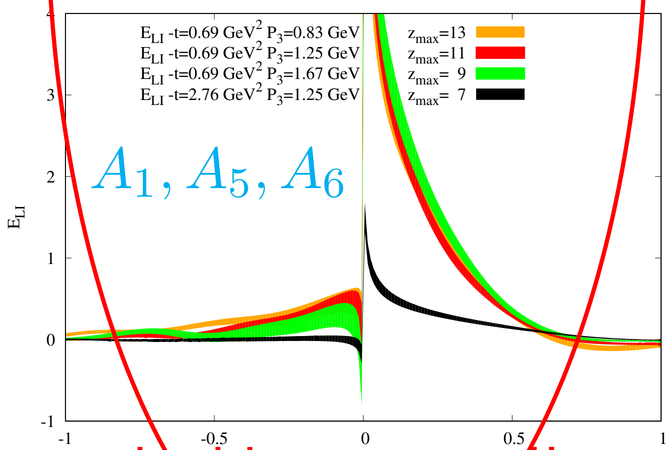
$\widetilde{H}$-GPD

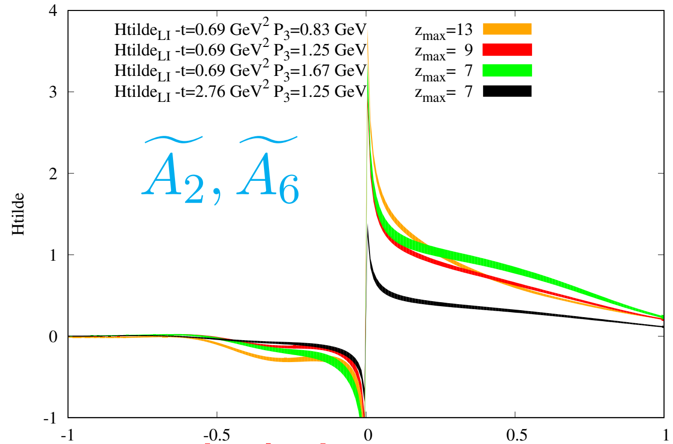
γ_0, γ_T operators (LI)

$\gamma_5\gamma_0$, $\gamma_5\gamma_T$ operators (LI)

ALTERNATIVE







basically unaffected

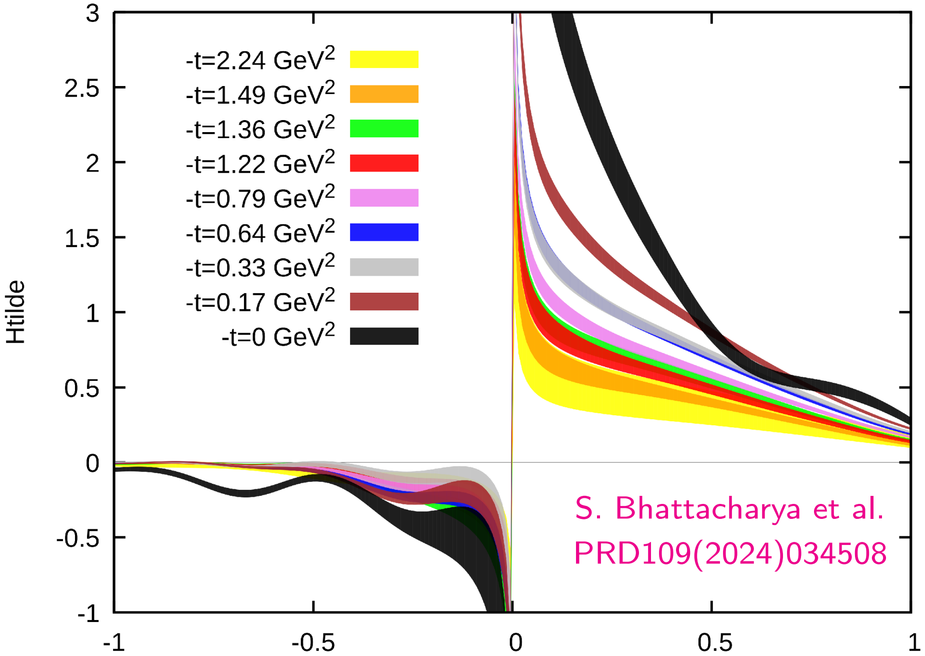
highly-improved!

slightly worse

t-dependence of $\tilde{H}/H/E$ GPDs (quasi)

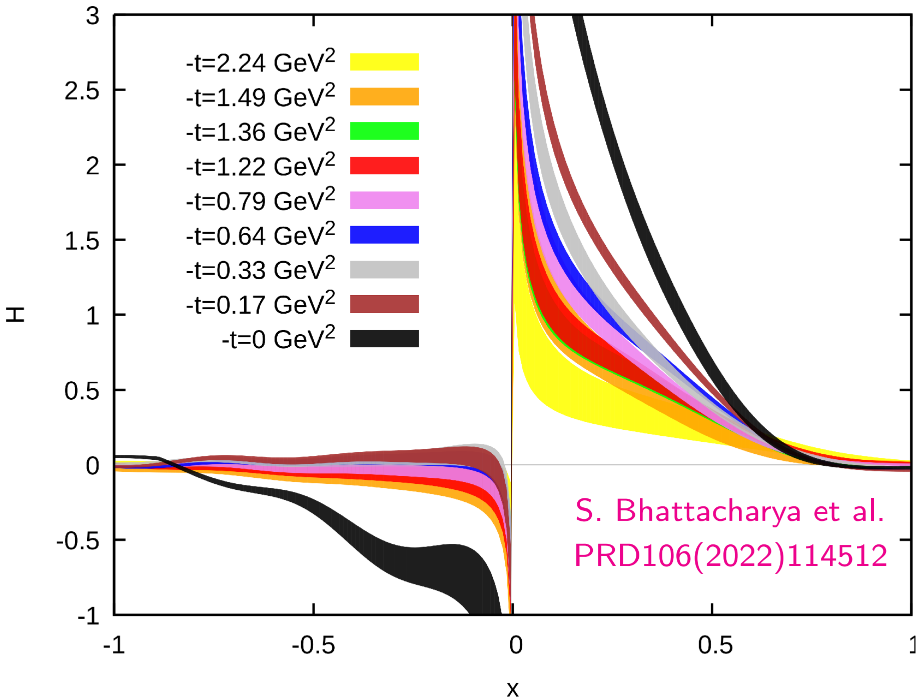
Nucleon structure
Partonic structure in LQCD
Setup
First extraction
Reference frames
GPDs definitions
Quasi-GPDs
Quasi vs. pseudo
Pseudo-GPDs
GPDs moments
Summary

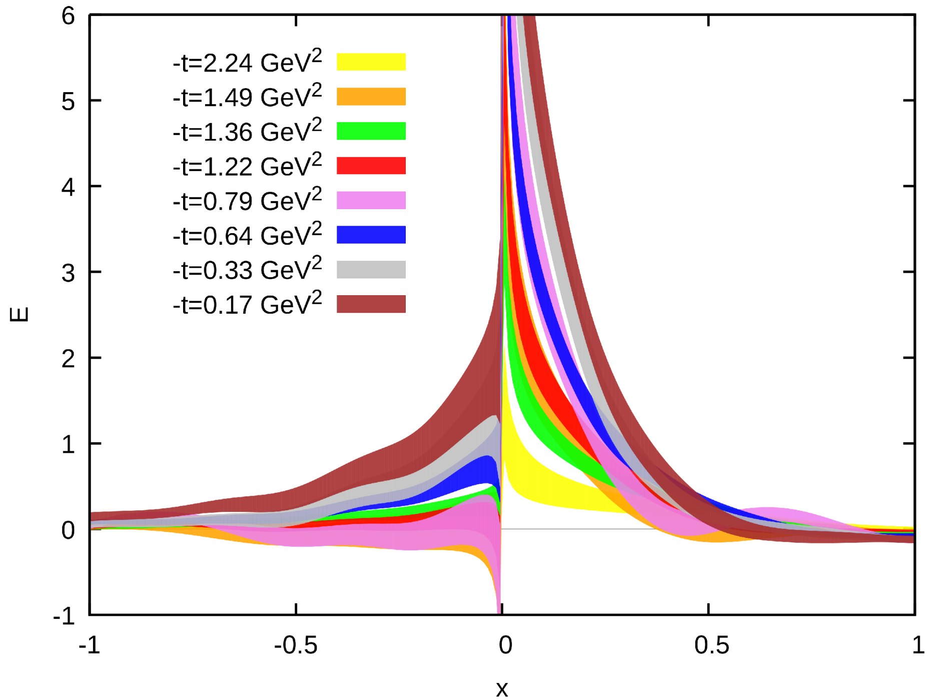
$\Delta = (1,0,0) \Rightarrow -t = 0.17$ GeV^2
$\Delta = (1,1,0) \Rightarrow -t = 0.33$ GeV^2
$\Delta = (2,0,0) \Rightarrow -t = 0.64$ GeV^2
$\Delta = (2,1,0) \Rightarrow -t = 0.79$ GeV^2
$\Delta = (2,2,0) \Rightarrow -t = 1.22$ GeV^2
$\Delta = (3,0,0) \Rightarrow -t = 1.36$ GeV^2
$\Delta = (3,1,0) \Rightarrow -t = 1.49$ GeV^2
$\Delta = (4,0,0) \Rightarrow -t = 2.24$ GeV^2

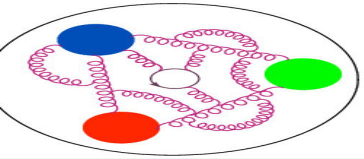


Impact parameter distribution:

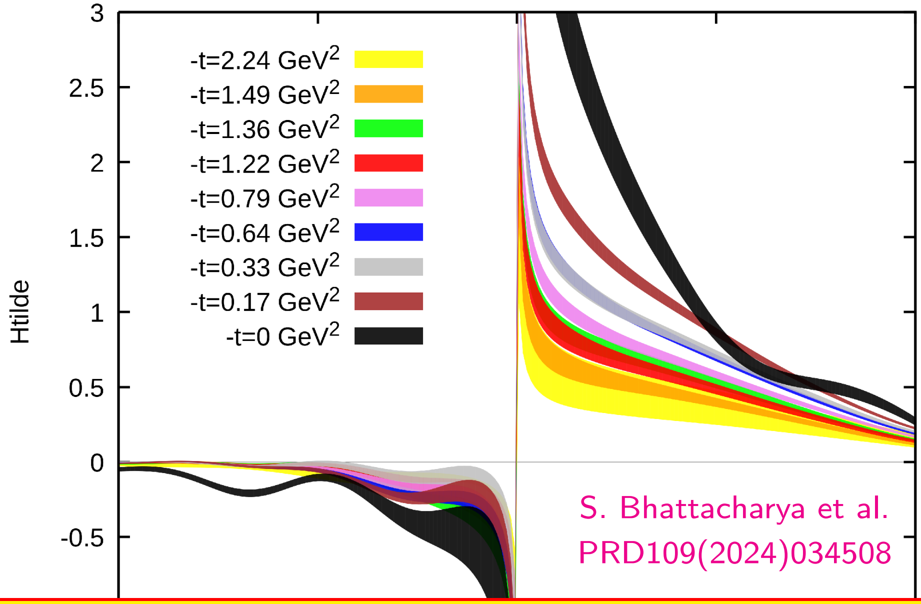
$$GPD(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} GPD(x,t)$$







t-dependence of $\tilde{H}/H/E$ GPDs (quasi)



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Impact parameter distribution:

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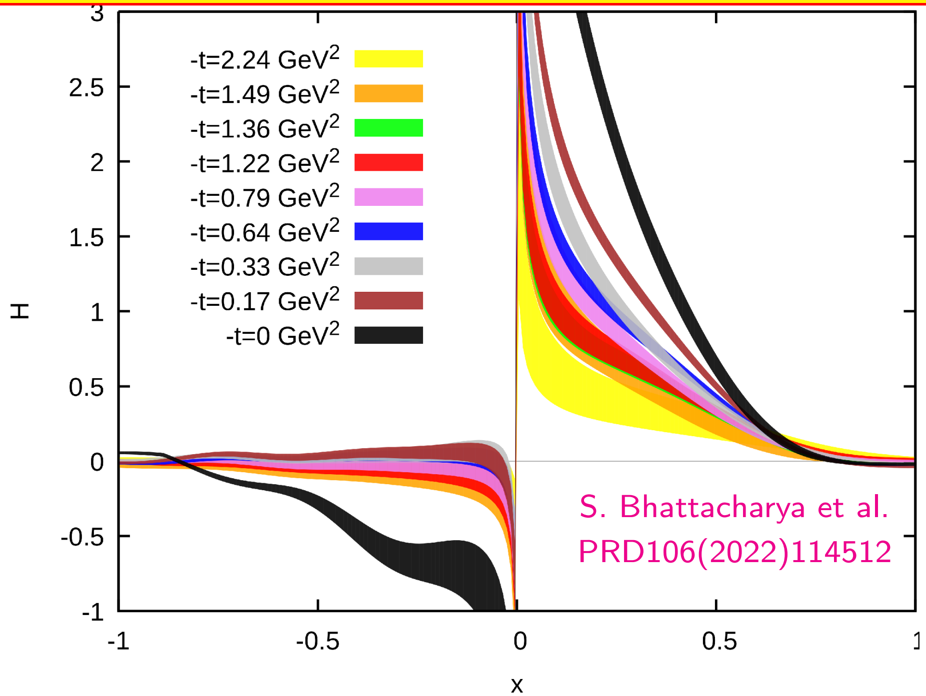
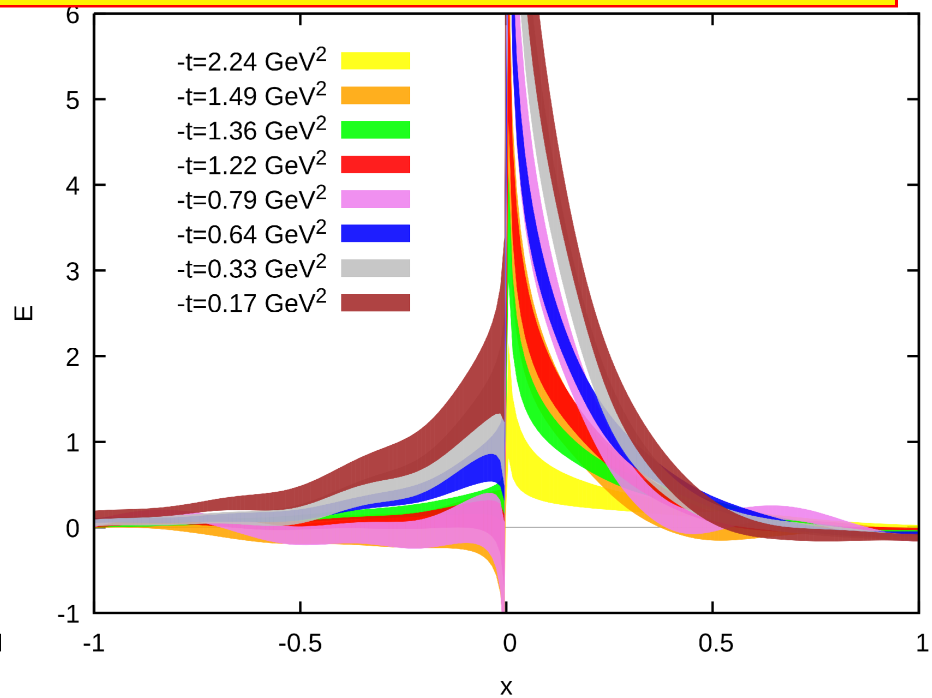
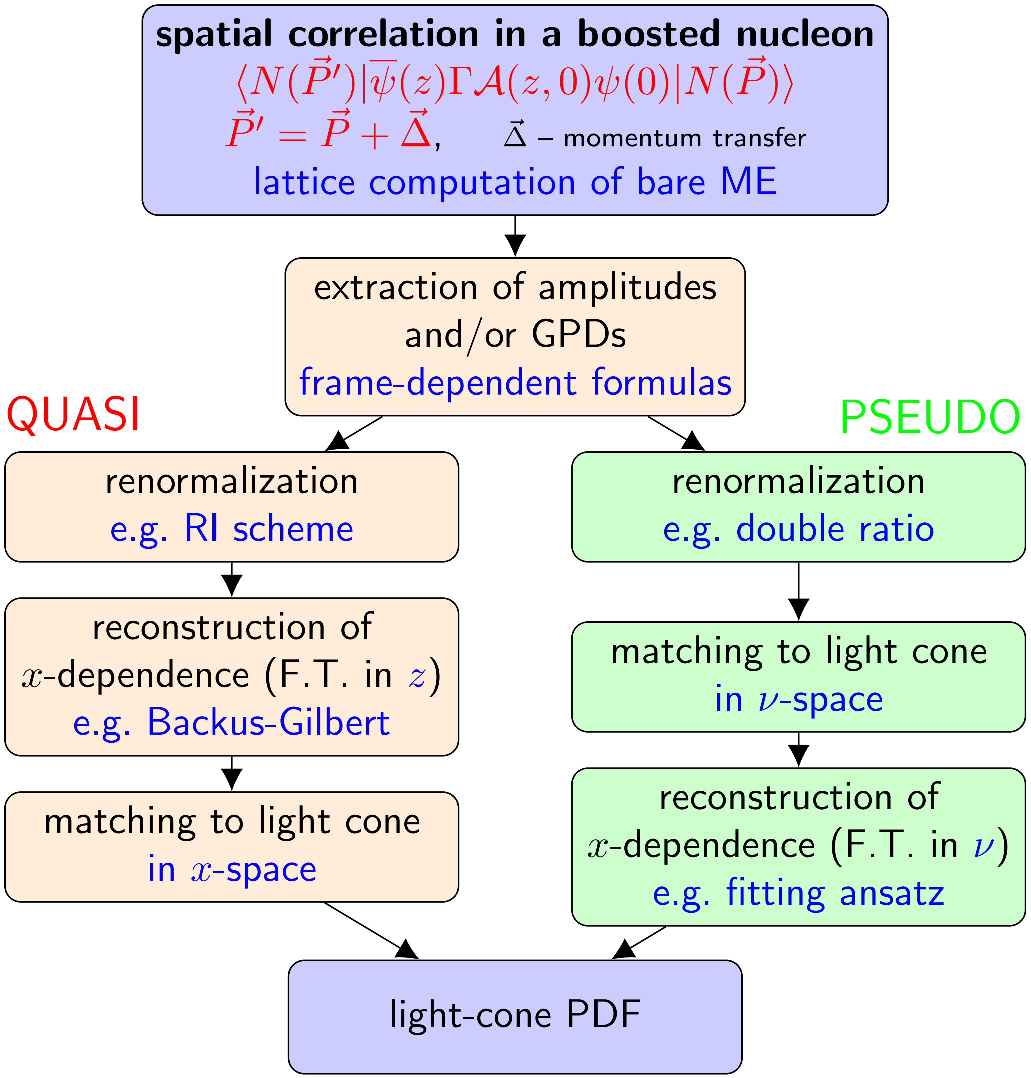
For prelim. results on transversity GPDs, see talk by Josh Miller next week @ LaMET workshop!

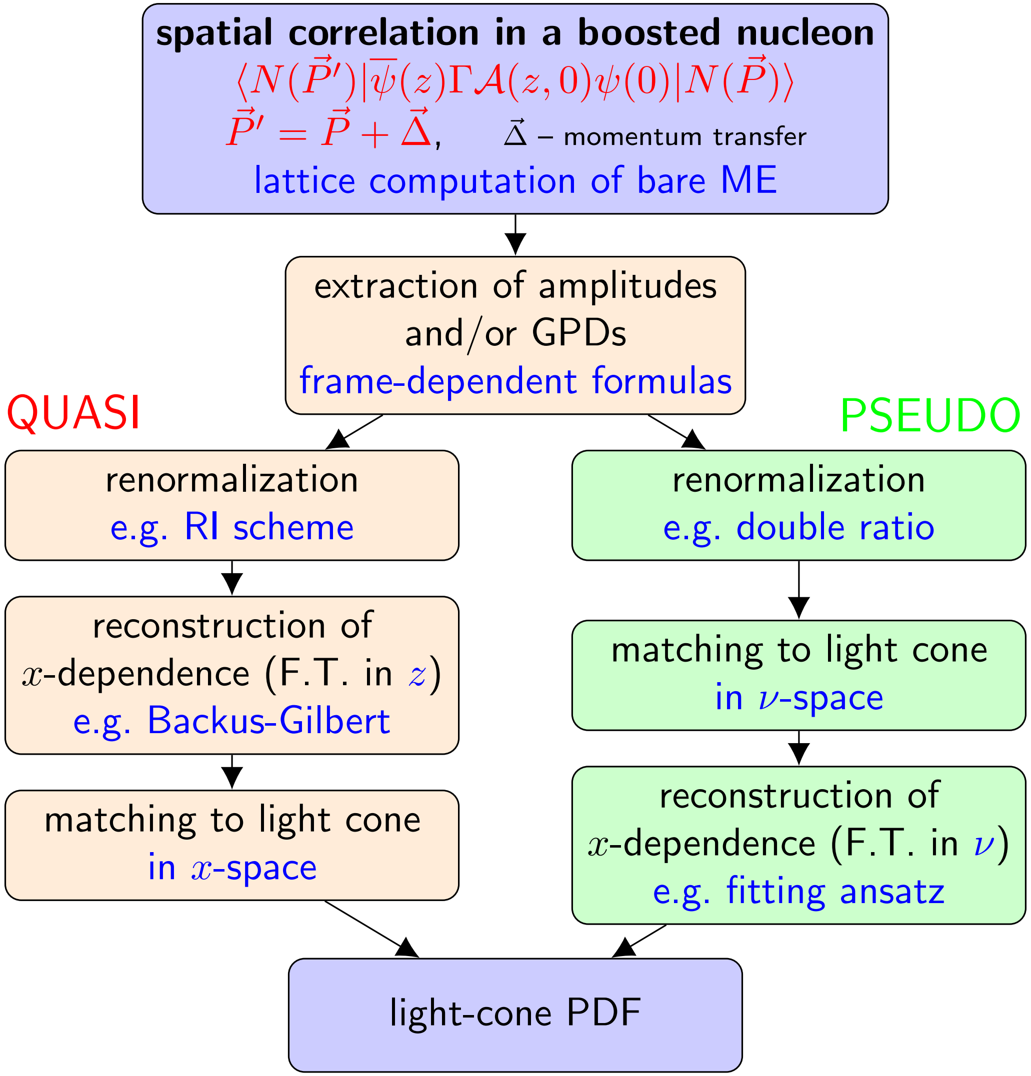




Quasi- and pseudo-GPDs lattice procedures



Quasi- and pseudo-GPDs lattice procedures



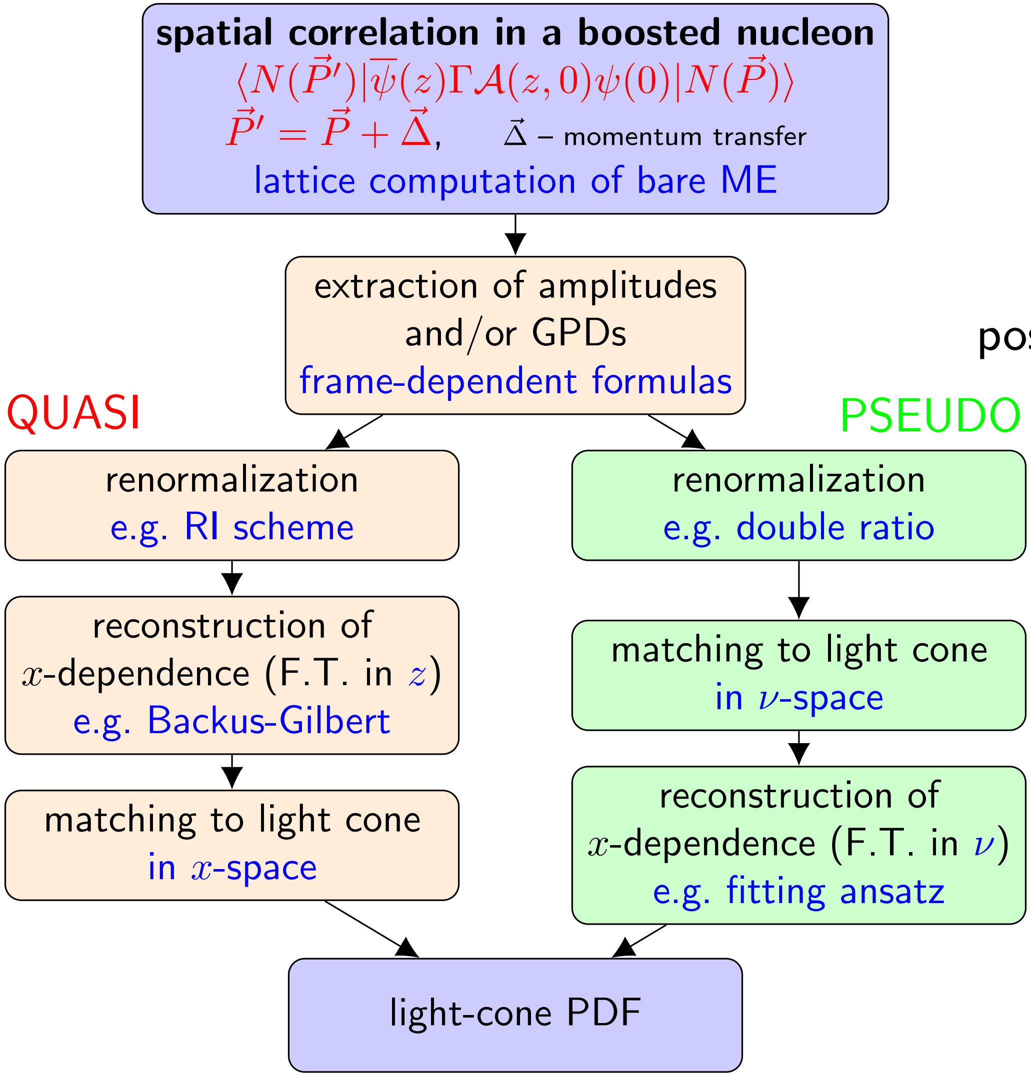
different insertions and projectors

several $\vec{\Delta}$ vectors

symmetric: each $\vec{\Delta}$ separate calc.

asymmetric: many $\vec{\Delta}$ at once!

Quasi- and pseudo-GPDs lattice procedures



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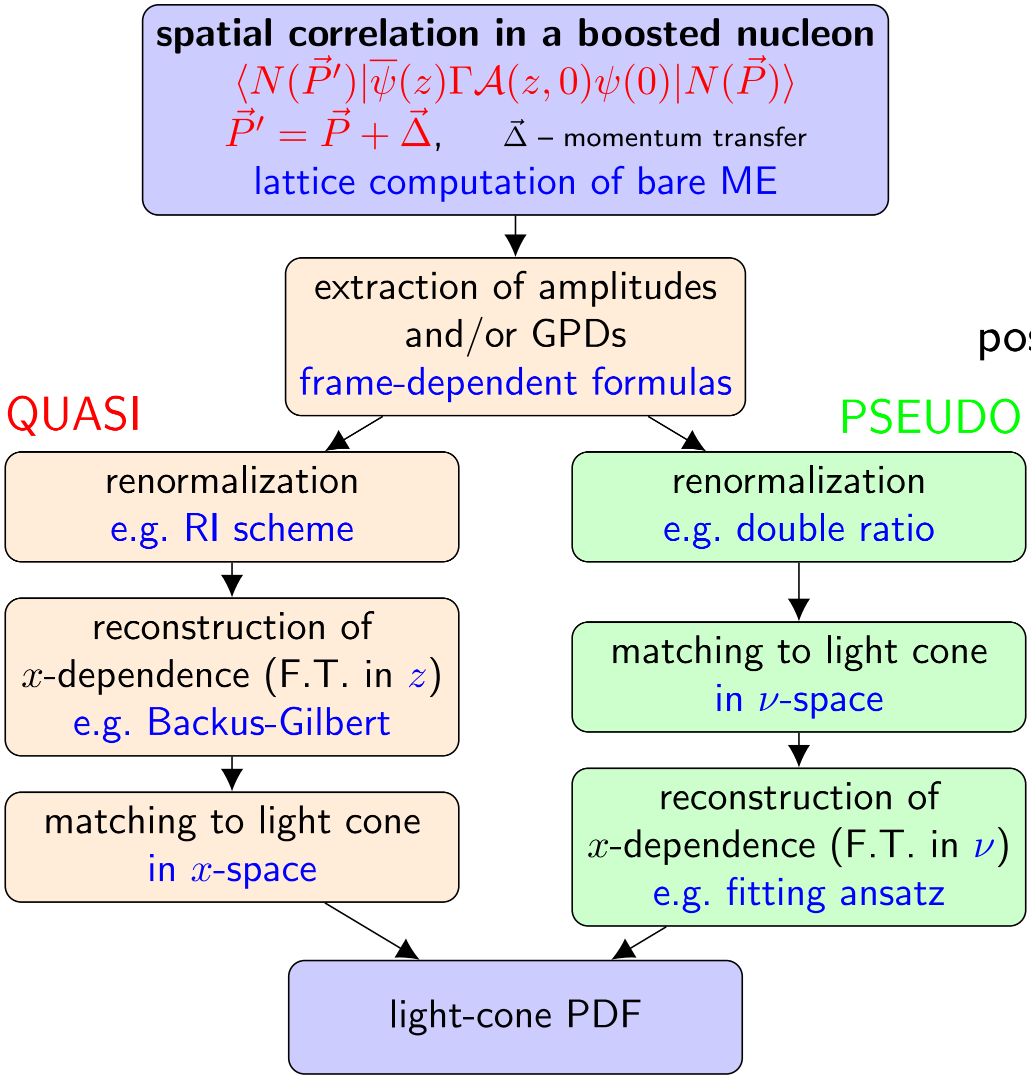
amplitudes frame-invariant

possible different definitions of GPDs

Quasi- and pseudo-GPDs lattice procedures



different insertions and projectors
several $\vec{\Delta}$ vectors
symmetric: each $\vec{\Delta}$ separate calc.
asymmetric: many $\vec{\Delta}$ at once!

amplitudes frame-invariant
possible different definitions of GPDs

logarithmic and power divergences
in bare MEs/GPDs

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{\Delta}$, $\vec{\Delta}$ – momentum transfer
lattice computation of bare ME

↓

extraction of amplitudes and/or GPDs
frame-dependent formulas

QUASI

renormalization
e.g. RI scheme

↓

reconstruction of x-dependence (F.T. in z)
e.g. Backus-Gilbert

↓

matching to light cone
in x-space

PSEUDO

renormalization
e.g. double ratio

↓

matching to light cone
in ν-space

↓

reconstruction of x-dependence (F.T. in ν)
e.g. fitting ansatz

↓

light-cone PDF

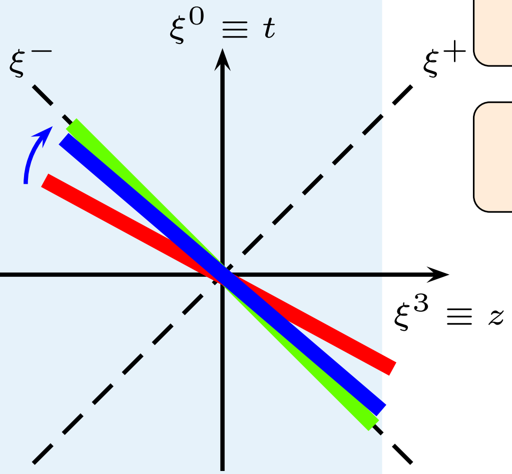
different insertions and projectors
several $\vec{\Delta}$ vectors
symmetric: each $\vec{\Delta}$ separate calc.
asymmetric: many $\vec{\Delta}$ at once!

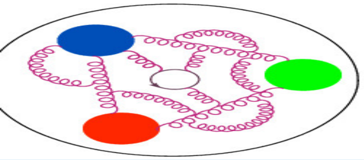
amplitudes frame-invariant
possible different definitions of GPDs

logarithmic and power divergences
in bare MEs/GPDs

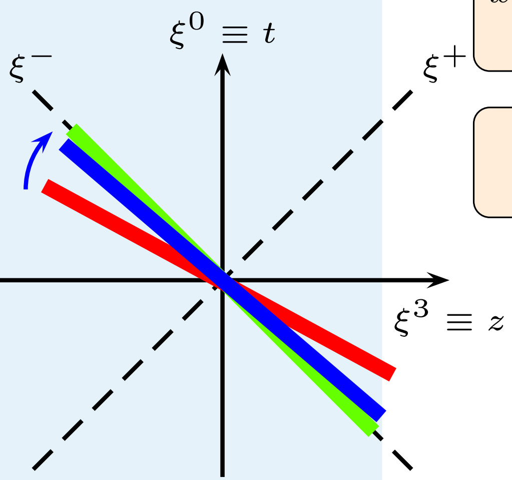
reconstruction:
non-trivial ("inverse problem")

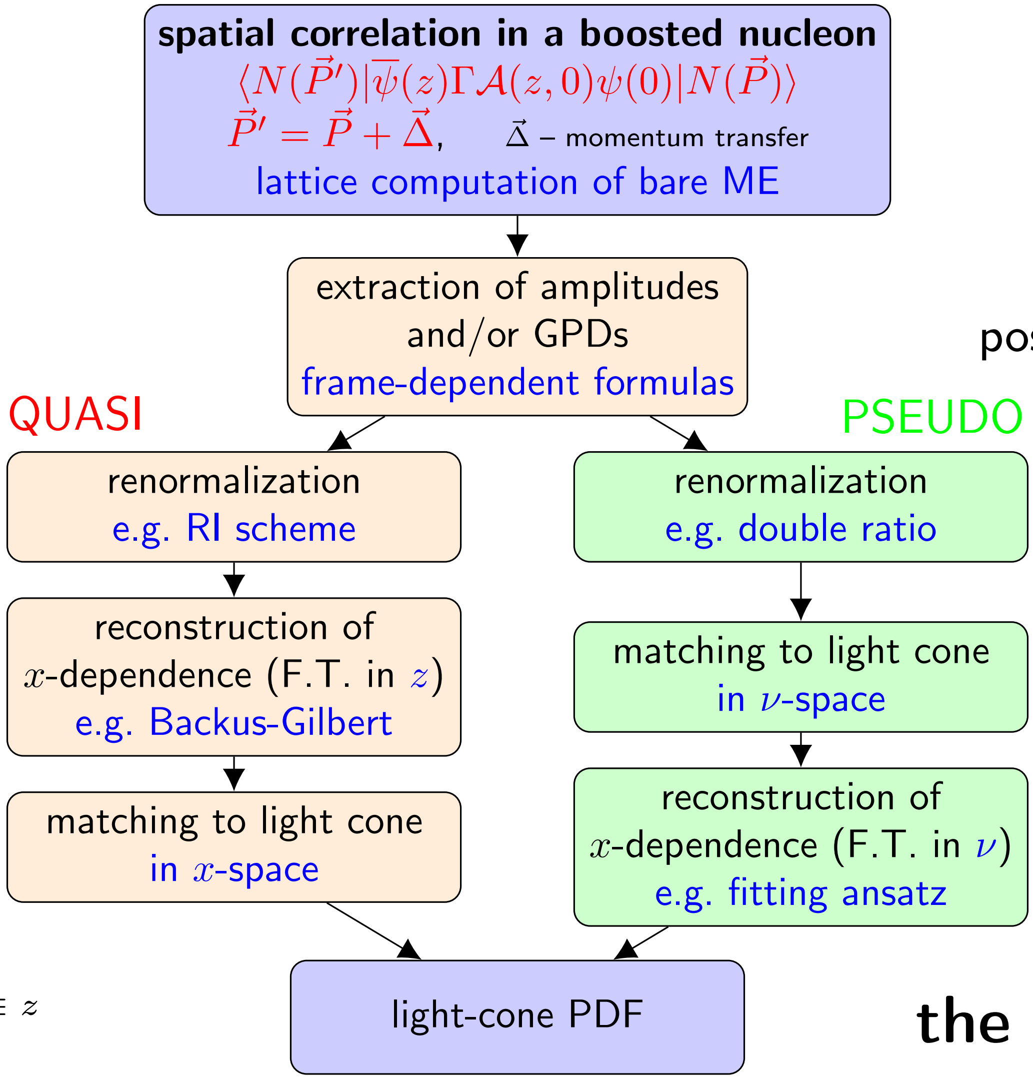
matching:
needs large boosts
valid up to HTEs





Quasi- and pseudo-GPDs lattice procedures





different insertions and projectors
several $\vec{\Delta}$ vectors
symmetric: each $\vec{\Delta}$ separate calc.
asymmetric: many $\vec{\Delta}$ at once!

amplitudes frame-invariant
possible different definitions of GPDs

logarithmic and power divergences
in bare MEs/GPDs

reconstruction:
non-trivial ("inverse problem")

matching:
needs large boosts
valid up to HTEs

the final desired object!

Pseudo-ITDs (double-ratio renormalization):

A. Radyushkin, Phys. Rev. D100 (2019) 116011

$$\mathcal{F}(P_3, z) = \frac{F(P_3, z)}{f(0, z)} \frac{f(0,0)}{f(P_3, 0)}$$

$\mathcal{F} = \{\mathcal{H}, \mathcal{E}\}$ – MEs of GPDs $(-t > 0)$

f – MEs of PDFs $(t = 0)$

Pseudo-GPDs

Pseudo-ITDs (double-ratio renormalization): A. Radyushkin, Phys. Rev. D100 (2019) 116011

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$\mathcal{F} = \{\mathcal{H}, \mathcal{E}\}$ – MEs of GPDs ($-t > 0$)
f – MEs of PDFs ($t = 0$)

can be matched to light-cone ITDs at short distances:

$$\overline{\mathcal{F}}(P_3, z) = \mathcal{F}(P_3, z) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du\, C(u) \left(\mathcal{F}(uP_3, z) - \mathcal{F}(P_3, z)\right),$$

$$C(u) = \frac{1+u^2}{u-1} \ln \frac{z^2 \mu^2 e^{2\gamma_E + 1}}{4} + 4\frac{\ln(1-u)}{u-1} - 2(u-1).$$

Pseudo-GPDs

Pseudo-ITDs (double-ratio renormalization): A. Radyushkin, Phys. Rev. D100 (2019) 116011

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$$C(u) = \frac{1+u^2}{u-1}\ln\frac{z^2\mu^2 e^{2\gamma_E+1}}{4} + 4\frac{\ln(1-u)}{u-1} - 2(u-1).$$

Given matched ITDs one can reconstruct x-dependent GPDs:

$$\mathrm{Re}\,\overline{\mathcal{F}}(\nu,\mu) = \int_0^1 dx\,\cos(\nu x)\overline{\mathcal{F}}_v(x,\mu),$$

$$\mathrm{Im}\,\overline{\mathcal{F}}(\nu,\mu) = \int_0^1 dx\,\sin(\nu x)\overline{\mathcal{F}}_{v2s}(x,\mu)$$

Pseudo-GPDs

Pseudo-ITDs (double-ratio renormalization): A. Radyushkin, Phys. Rev. D100 (2019) 116011

$$\mathcal{F}(P_3, z) = \frac{F(P_3, z)}{f(0, z)} \frac{f(0,0)}{f(P_3, 0)}$$

$\mathcal{F} = \{\mathcal{H}, \mathcal{E}\}$ – MEs of GPDs ($-t > 0$)
f – MEs of PDFs ($t = 0$)

can be matched to light-cone ITDs at short distances:

$$\overline{\mathcal{F}}(P_3, z) = \mathcal{F}(P_3, z) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du\, C(u) \left(\mathcal{F}(uP_3, z) - \mathcal{F}(P_3, z)\right),$$

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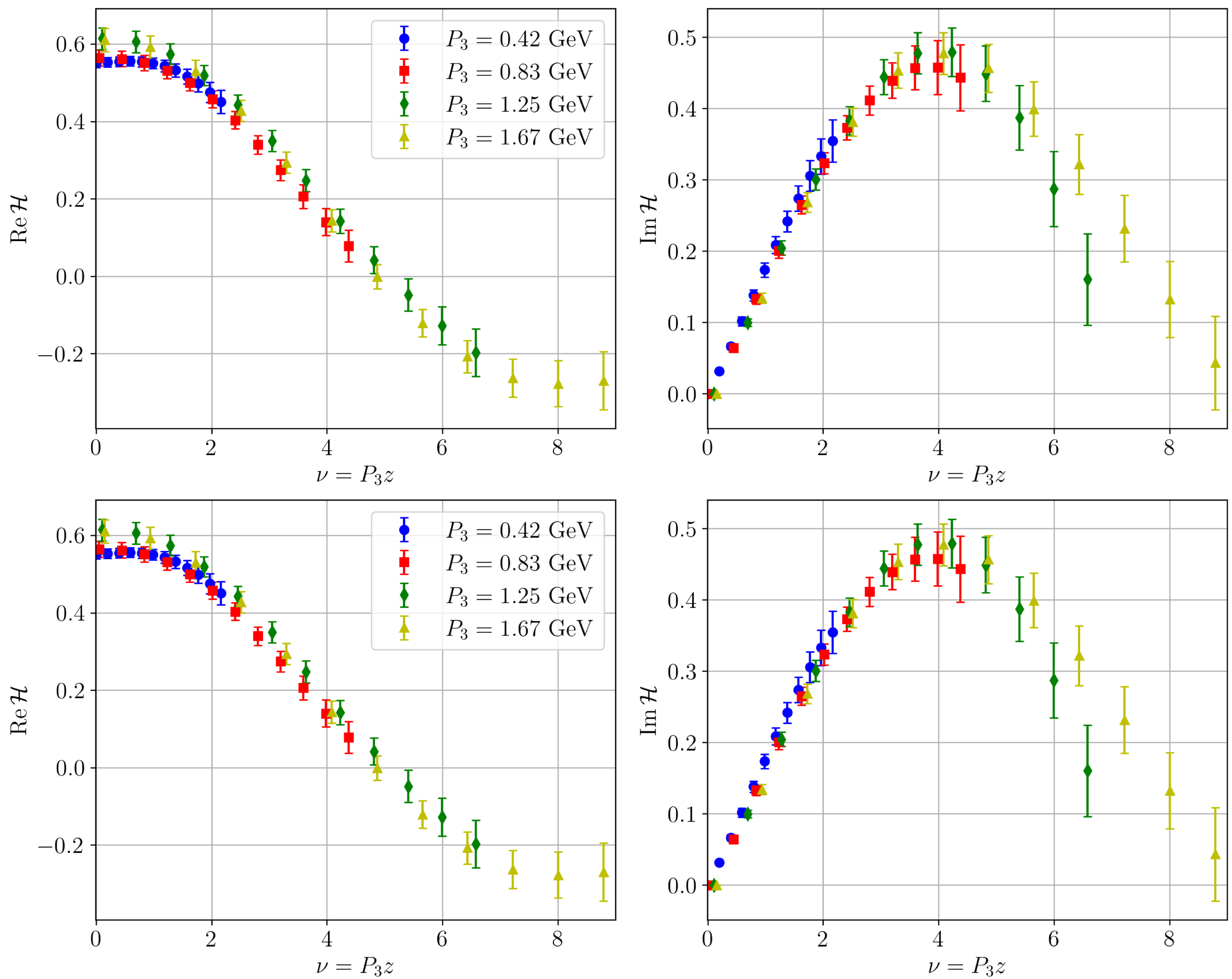
$$\mathrm{Im}\, \overline{\mathcal{F}}(\nu, \mu) = \int_0^1 dx\, \sin(\nu x) \overline{\mathcal{F}}_{v2s}(x, \mu)$$

using a fitting ansatz:

$$\overline{\mathcal{F}}(x) = N x^a (1-x)^b \left(1 + c x^{d_1} (1-x)^{d_2}\right).$$

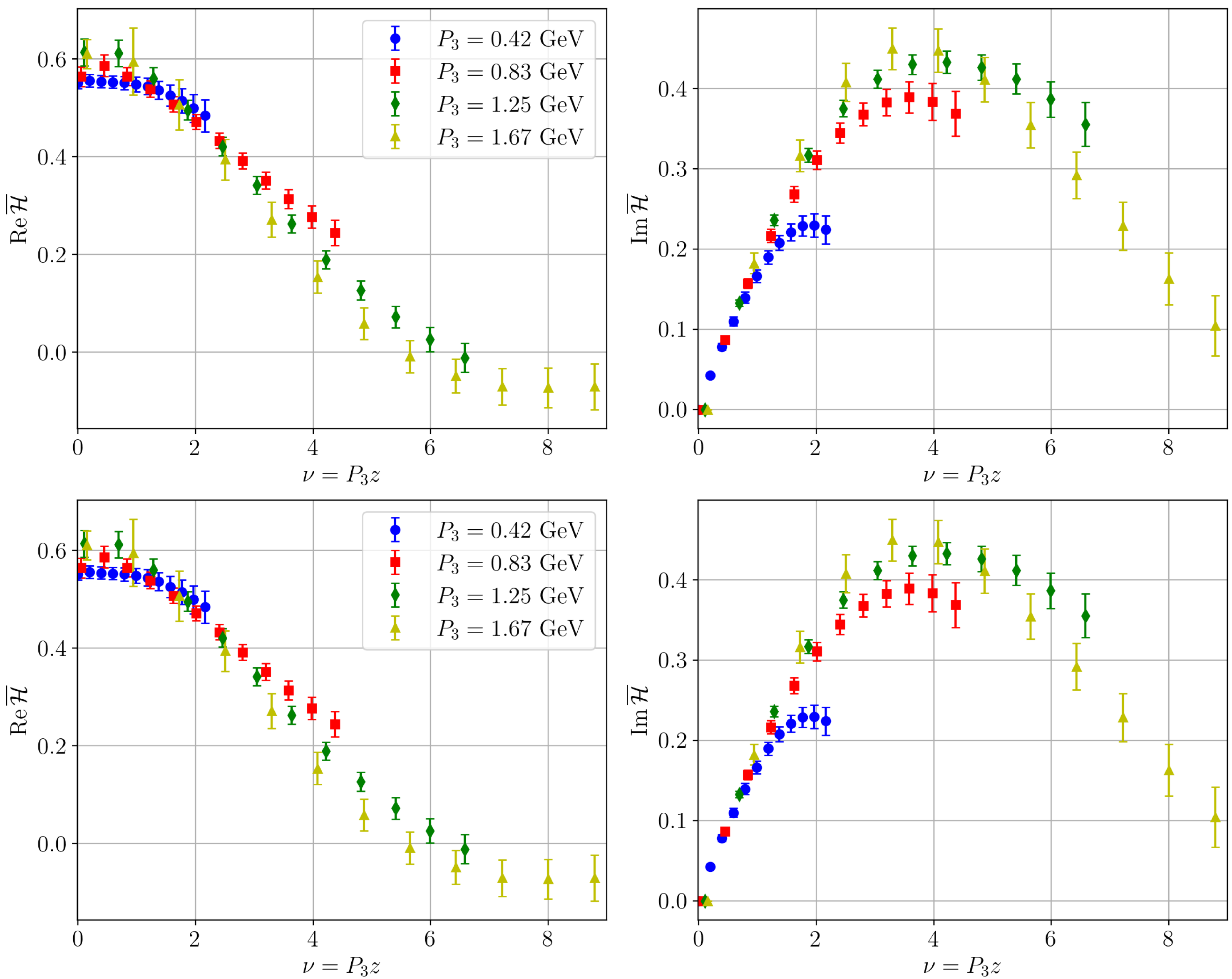
Reduced ITDs



S. Bhattacharya et al., 2405.04414

$-t$ [GeV^2]	P_3 [GeV]	(Δ_1, Δ_2) [$2\pi/L$]	N_Δ	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm meas}$
0	±0.42	(0,0)	2	100	8	1600
0	±0.83	(0,0)	2	100	8	1600
0	±1.25	(0,0)	2	269	16	8608
0	±1.67	(0,0)	2	506	32	32384
0.17	±0.42	(±1,0), (0,±1)	8	100	8	6400
0.17	±0.83	(±1,0), (0,±1)	8	100	8	6400
0.17	±1.25	(±1,0), (0,±1)	8	269	8	17216
0.17	±1.67	(±1,0), (0,±1)	8	506	32	129536
0.34	±0.42	(±1, ±1)	8	100	8	6400
0.34	±0.83	(±1, ±1)	8	100	8	6400
0.34	±1.25	(±1, ±1)	8	195	8	12480
0.34	±1.67	(±1, ±1)	8	506	32	129536
0.65	±0.42	(±2,0), (0,±2)	8	100	8	6400
0.65	±0.83	(±2,0), (0,±2)	8	100	8	6400
0.65	±1.25	(±2,0), (0,±2)	8	269	8	17216
0.65	±1.67	(±2,0), (0,±2)	8	506	32	129536
0.81	±0.42	(±1,±2), (±2,±1)	16	100	8	12800
0.81	±0.83	(±1,±2), (±2,±1)	16	100	8	12800
0.81	±1.25	(±1,±2), (±2,±1)	16	195	8	24960
0.81	±1.67	(±1,±2), (±2,±1)	16	506	32	259072
1.24	±0.42	(±2, ±2)	8	100	8	6400
1.24	±0.83	(±2, ±2)	8	100	8	6400
1.24	±1.25	(±2, ±2)	8	195	8	12480
1.24	±1.67	(±2, ±2)	8	506	32	129536
1.38	±0.42	(±3,0), (0,±3)	8	100	8	6400
1.38	±0.83	(±3,0), (0,±3)	8	100	8	6400
1.38	±1.25	(±3,0), (0,±3)	8	269	8	17216
1.38	±1.67	(±3,0), (0,±3)	8	506	32	129536
1.52	±0.42	(±1,±3), (±3,±1)	16	100	8	12800
1.52	±0.83	(±1,±3), (±3,±1)	16	100	8	12800
1.52	±1.25	(±1,±3), (±3,±1)	16	195	8	24960
1.52	±1.67	(±1,±3), (±3,±1)	16	506	32	259072
2.29	±0.42	(±4,0), (0,±4)	8	100	8	6400
2.29	±0.83	(±4,0), (0,±4)	8	100	8	6400
2.29	±1.25	(±4,0), (0,±4)	8	269	8	17216
2.29	±1.67	(±4,0), (0,±4)	8	506	32	129536

Light-cone (matched) ITDs



S. Bhattacharya et al., 2405.04414

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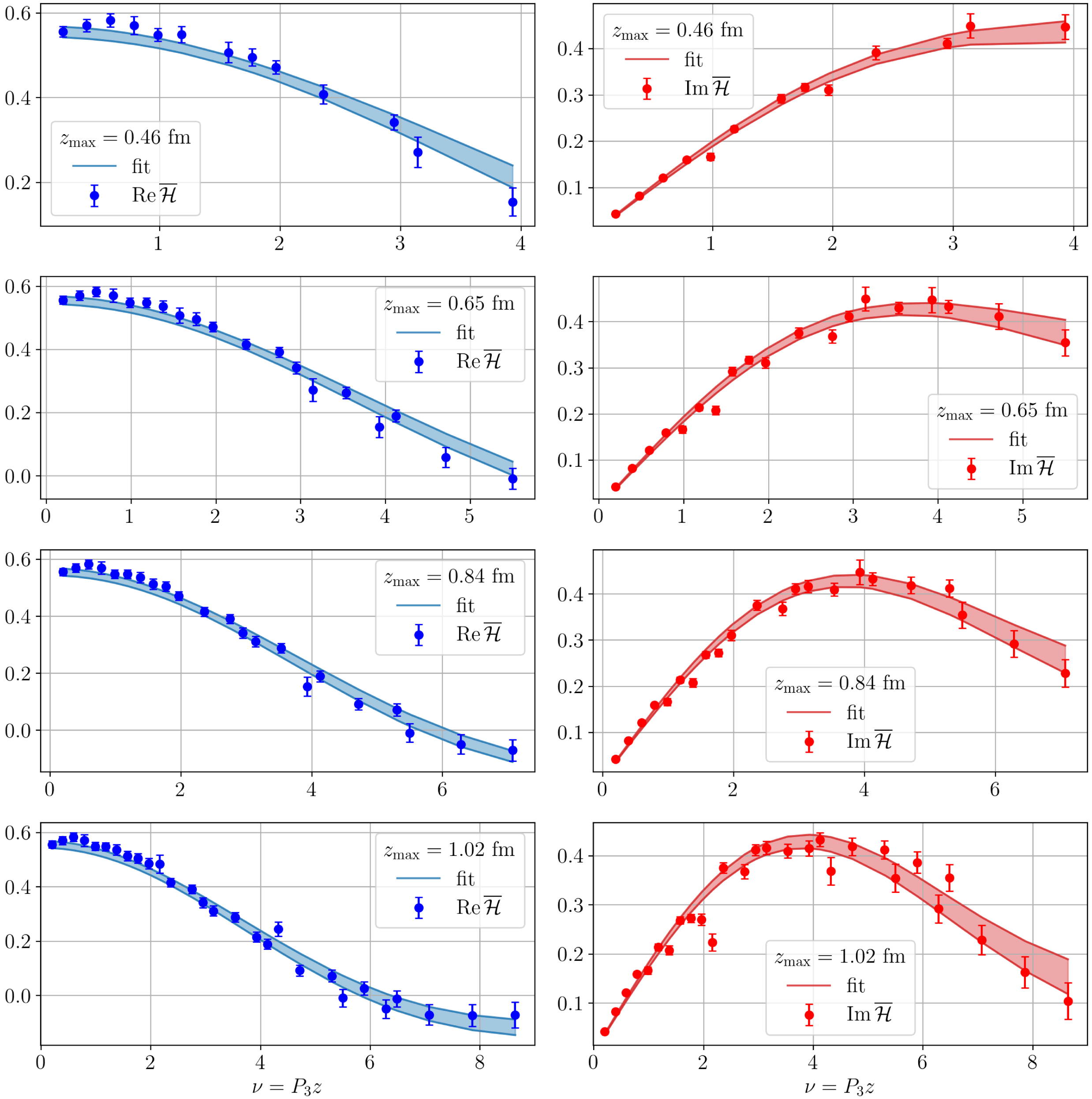
0	±0.42	(0,0)	2	100	8	1600
0	±0.83	(0,0)	2	100	8	1600
0	±1.25	(0,0)	2	269	16	8608
0	±1.67	(0,0)	2	506	32	32384
0.17	±0.42	(±1,0), (0,±1)	8	100	8	6400
0.17	±0.83	(±1,0), (0,±1)	8	100	8	6400
0.17	±1.25	(±1,0), (0,±1)	8	269	8	17216
0.17	±1.67	(±1,0), (0,±1)	8	506	32	129536
0.34	±0.42	(±1, ±1)	8	100	8	6400
0.34	±0.83	(±1, ±1)	8	100	8	6400
0.34	±1.25	(±1, ±1)	8	195	8	12480
0.34	±1.67	(±1, ±1)	8	506	32	129536
0.65	±0.42	(±2,0), (0,±2)	8	100	8	6400
0.65	±0.83	(±2,0), (0,±2)	8	100	8	6400
0.65	±1.25	(±2,0), (0,±2)	8	269	8	17216
0.65	±1.67	(±2,0), (0,±2)	8	506	32	129536
0.81	±0.42	(±1,±2), (±2,±1)	16	100	8	12800
0.81	±0.83	(±1,±2), (±2,±1)	16	100	8	12800
0.81	±1.25	(±1,±2), (±2,±1)	16	195	8	24960
0.81	±1.67	(±1,±2), (±2,±1)	16	506	32	259072
1.24	±0.42	(±2, ±2)	8	100	8	6400
1.24	±0.83	(±2, ±2)	8	100	8	6400
1.24	±1.25	(±2, ±2)	8	195	8	12480
1.24	±1.67	(±2, ±2)	8	506	32	129536
1.38	±0.42	(±3,0), (0,±3)	8	100	8	6400
1.38	±0.83	(±3,0), (0,±3)	8	100	8	6400
1.38	±1.25	(±3,0), (0,±3)	8	269	8	17216
1.38	±1.67	(±3,0), (0,±3)	8	506	32	129536
1.52	±0.42	(±1,±3), (±3,±1)	16	100	8	12800
1.52	±0.83	(±1,±3), (±3,±1)	16	100	8	12800
1.52	±1.25	(±1,±3), (±3,±1)	16	195	8	24960
1.52	±1.67	(±1,±3), (±3,±1)	16	506	32	259072
2.29	±0.42	(±4,0), (0,±4)	8	100	8	6400
2.29	±0.83	(±4,0), (0,±4)	8	100	8	6400
2.29	±1.25	(±4,0), (0,±4)	8	269	8	17216
2.29	±1.67	(±4,0), (0,±4)	8	506	32	129536

Fitted ITDs

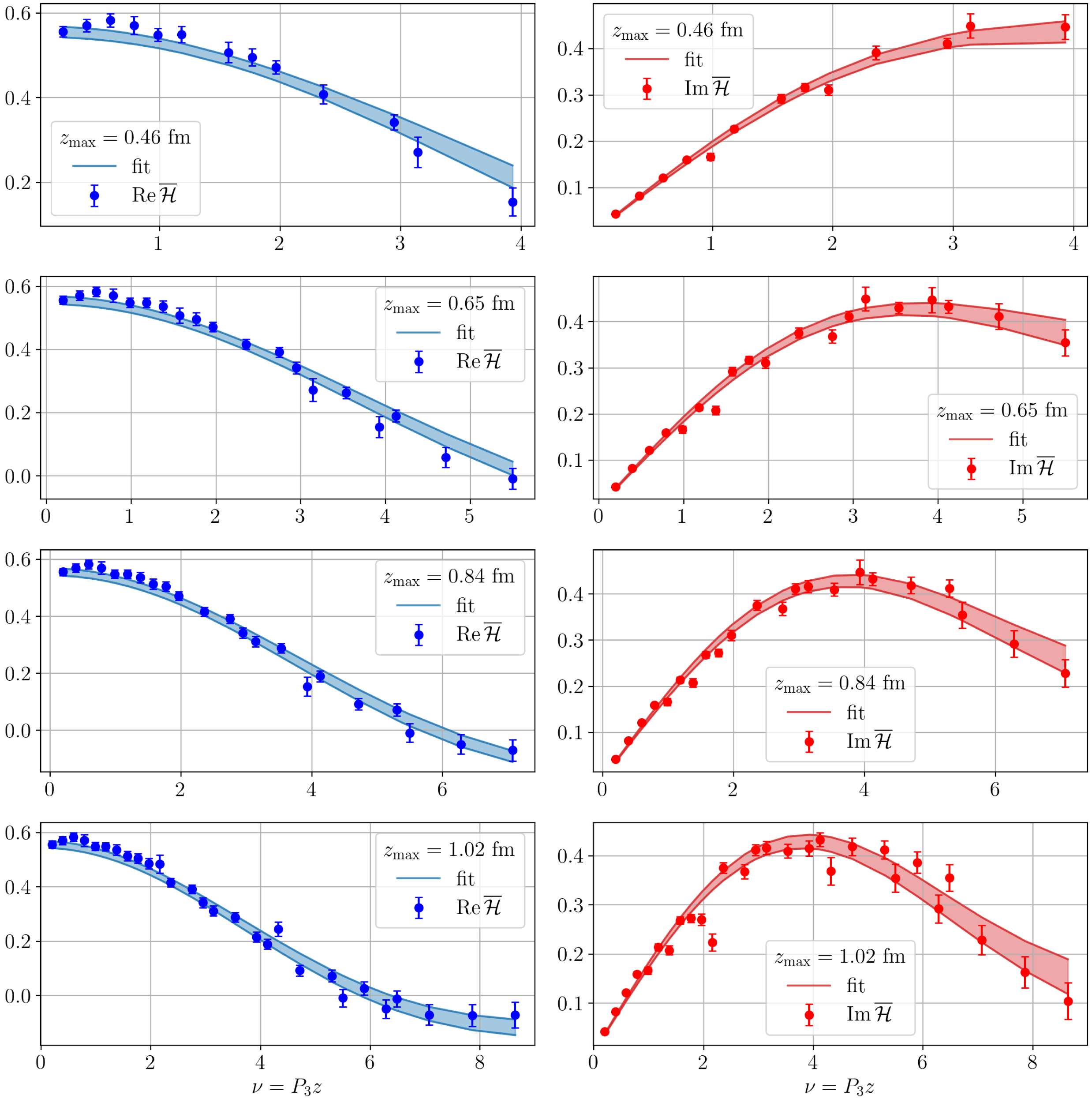


$$\overline{\mathcal{F}}(x) = N x^a (1-x)^b$$

marginal or absent sensitivity to additional fit parameters

S. Bhattacharya et al., 2405.04414

Fitted ITDs



$$\overline{\mathcal{F}}(x) = N x^a (1-x)^b$$

marginal or absent sensitivity to additional fit parameters

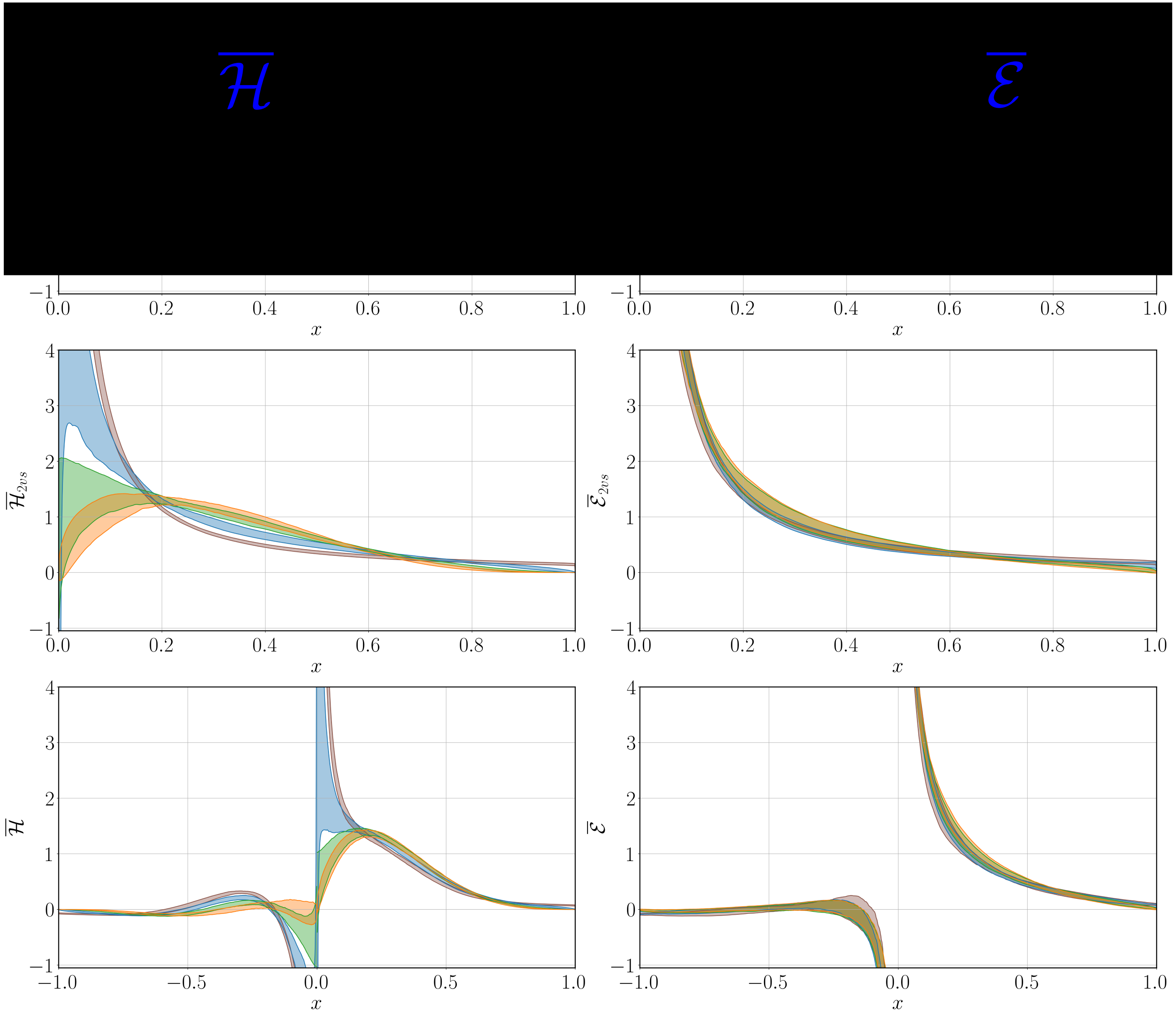
$z_{\max} = 7a \approx 0.65$ fm – rather conservative choice

$z_{\max} = 9a \approx 0.84$ fm – also plausible (particularly for valence)

S. Bhattacharya et al., 2405.04414

Fitting-reconstructed GPDs in x-space

S. Bhattacharya et al., 2405.04414



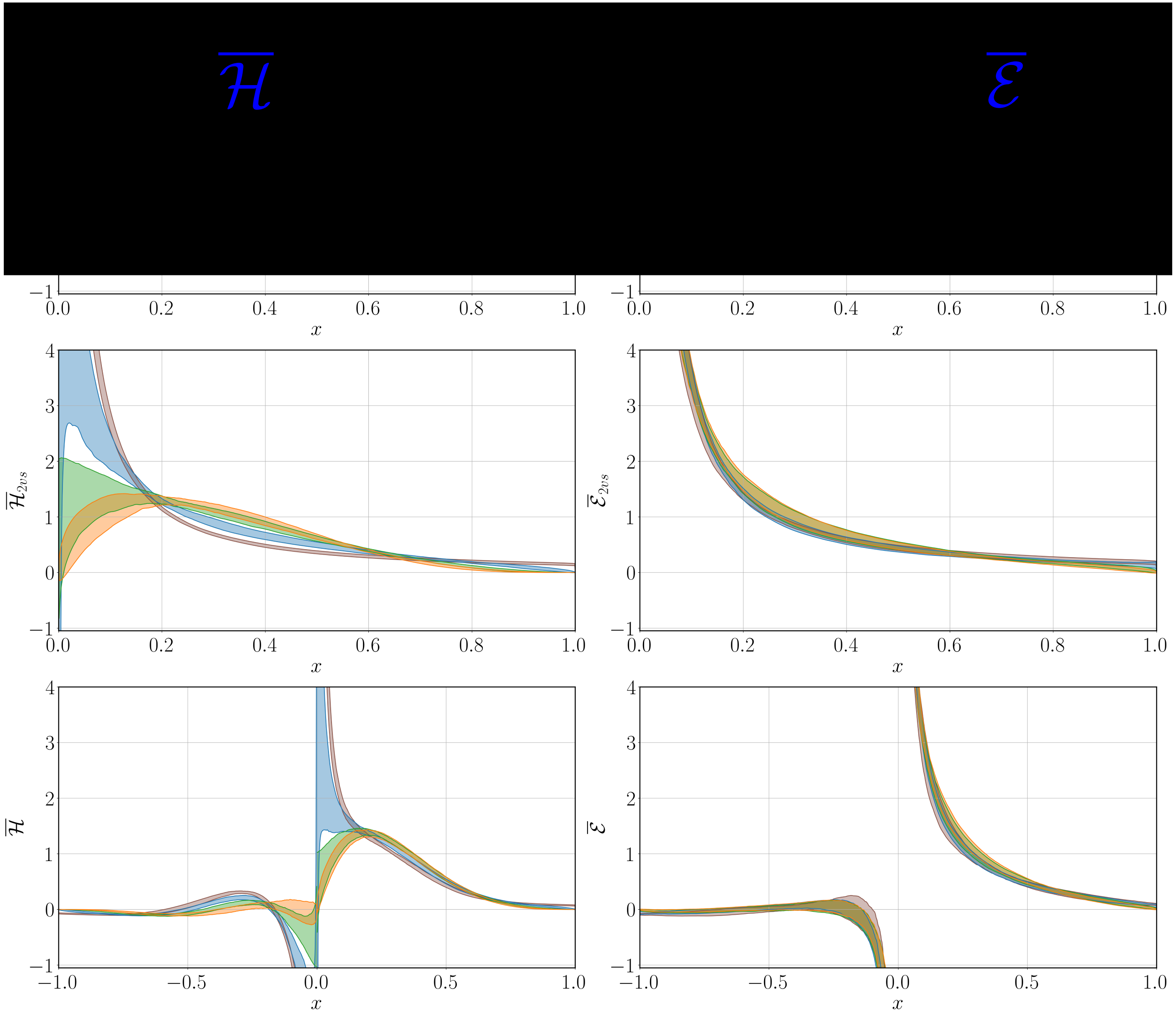
VALENCE

VALENCE + 2*SEA

VALENCE + SEA

Fitting-reconstructed GPDs in x-space

S. Bhattacharya et al., 2405.04414



VALENCE

VALENCE + 2*SEA

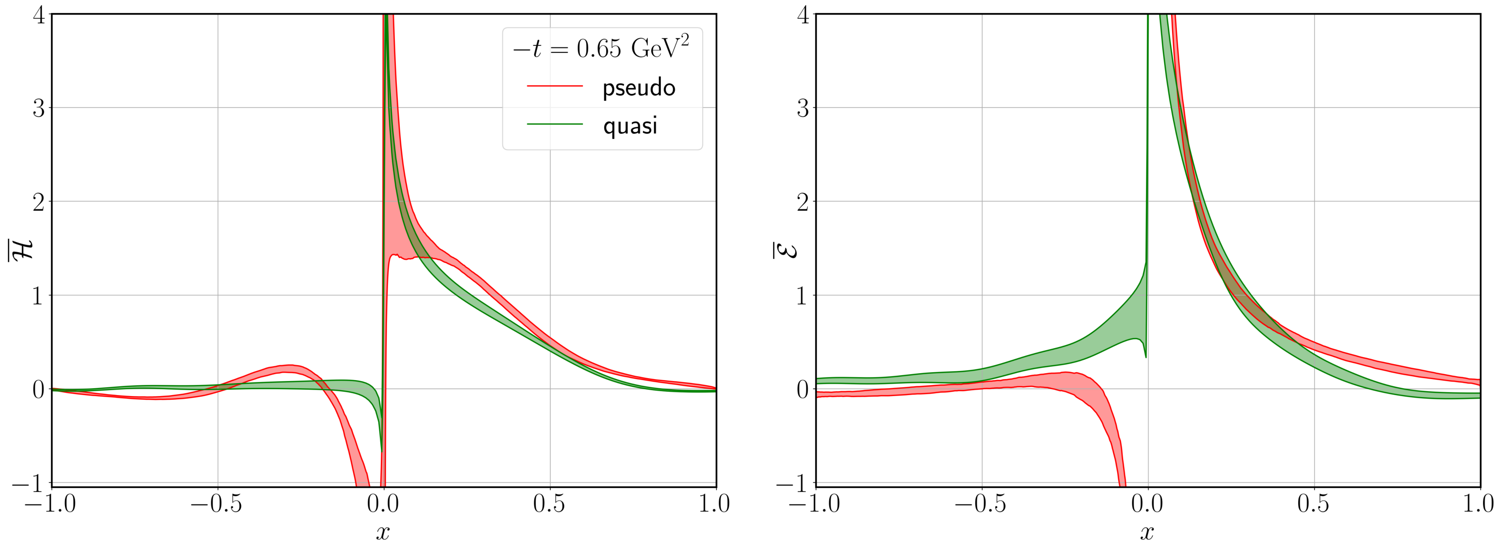
VALENCE + SEA

In all cases $z_{\text{max}} = 7a$ and $9a$ nicely compatible

Interesting:
$\overline{\mathcal{E}}$ obviously better-behaved!

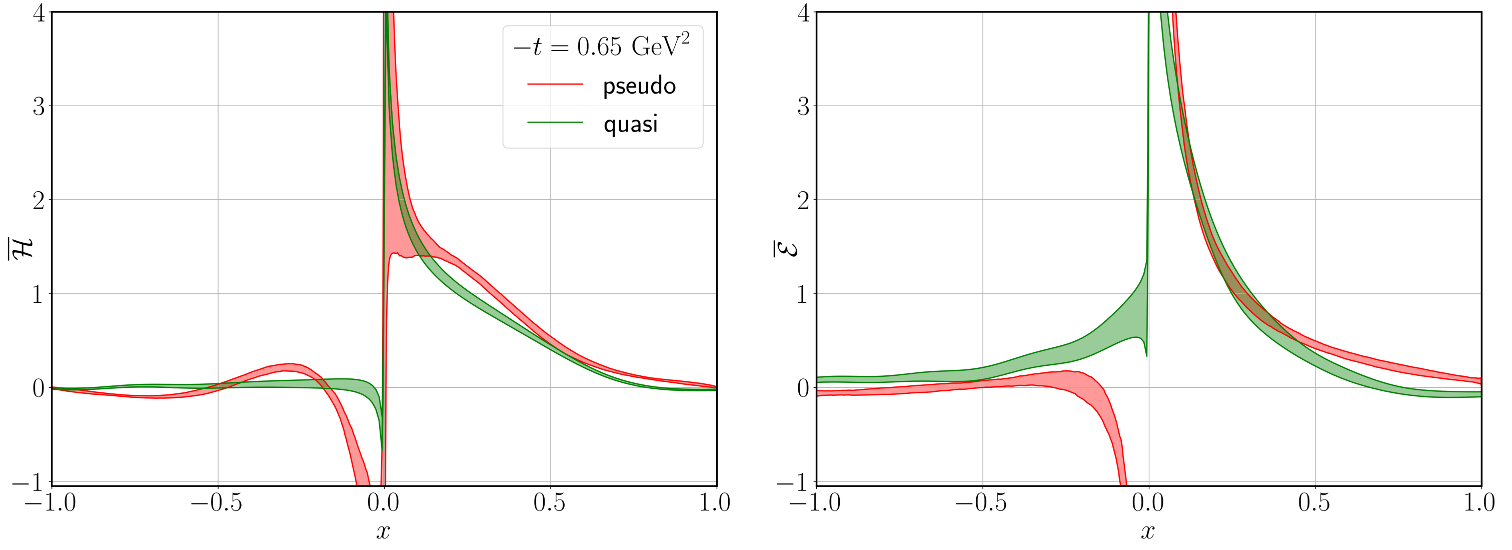
H GPD from quasi vs. pseudo, $-t = 0.65$ GeV^2

S. Bhattacharya et al., 2405.04414



H GPD from quasi vs. pseudo, $-t = 0.65$ GeV^2

S. Bhattacharya et al., 2405.04414



Qualitative agreement between pseudo and quasi.

Evinced difference as measure of unquantified systematic effects.

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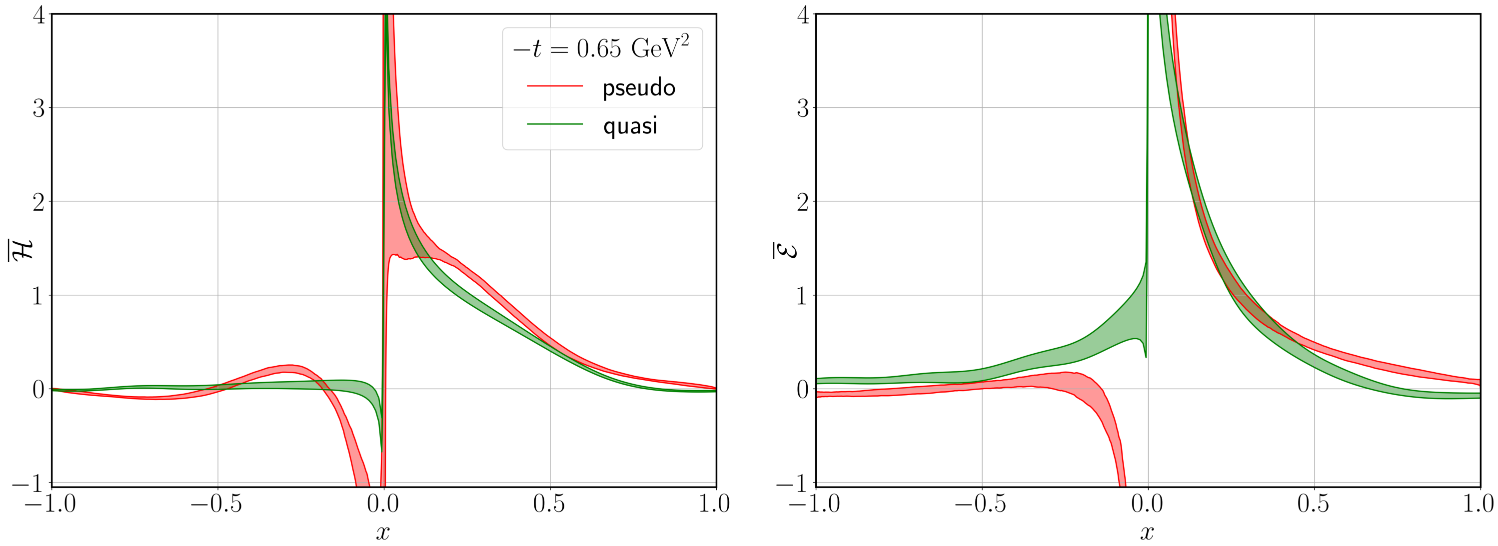
S. Bhattacharya et al., 2405.04414



Qualitative agreement between pseudo and quasi.

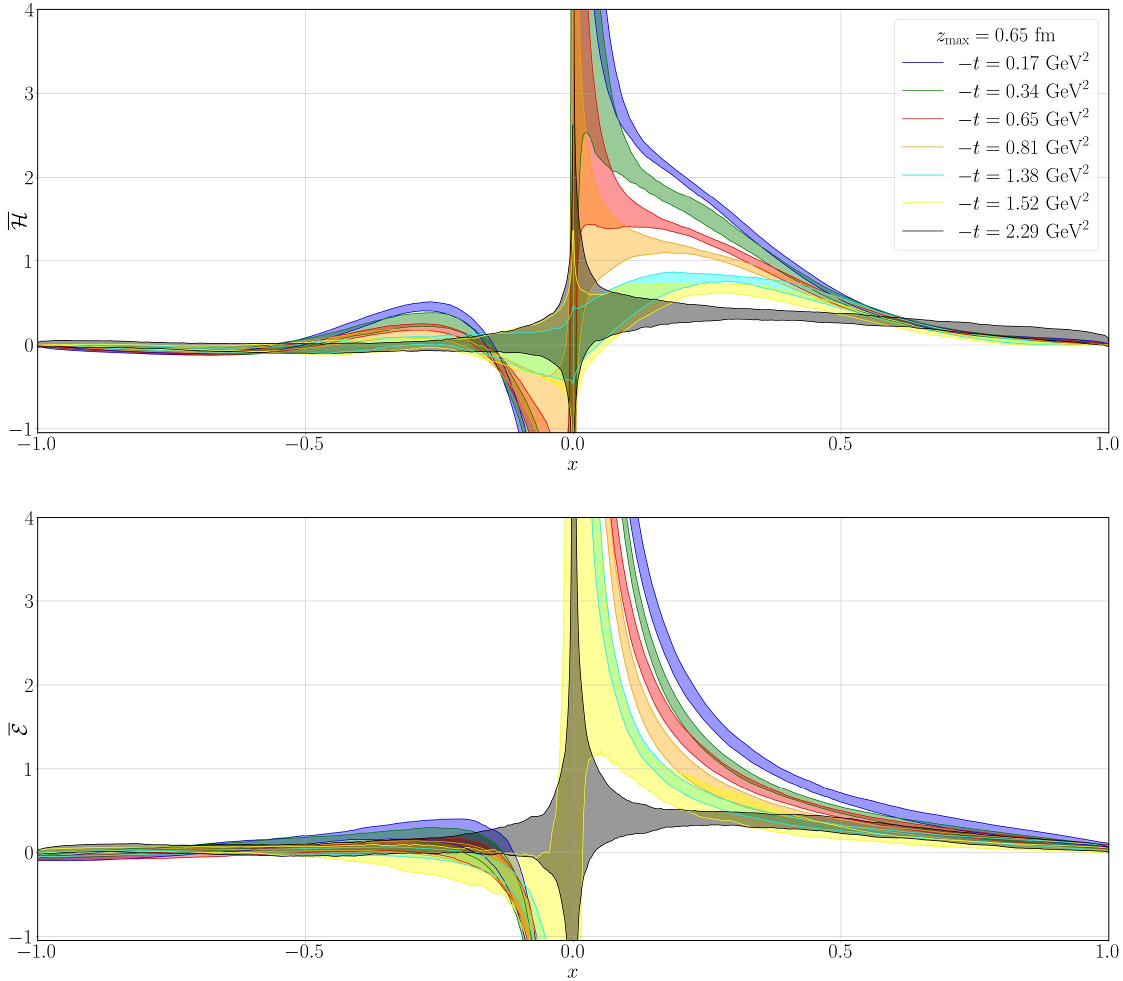
Evinced difference as measure of unquantified systematic effects.

Reminder:

- Main difference:
  quasi = factorization in x-space (LaMET),
  pseudo = short-distance factorization (SDF) in ν-space.
- Practical difference: reconstruction of x-dependence
  quasi = Backus-Gilbert,
  pseudo = fitting ansatz.

t-dependence of H/E GPDs (pseudo)

S. Bhattacharya et al., 2405.04414



Qualitatively similar picture to the one from quasi-GPDs.
Quantitative conclusions after careful estimation of systematics!

Short-distance factorization (SDF) can also be used to extract moments of GPDs.

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For ratio-renormalized H/E: $\mathcal{F}^{\overline{\rm MS}}(z,P,\Delta)=\sum_{n=0}\frac{(-izP)^n}{n!}C_n^{\overline{\rm MS}}(\mu^2z^2)\langle x^n\rangle+\mathcal{O}(\Lambda^2_{\rm QCD}z^2)$,

$C_n^{\overline{\rm MS}}(\mu^2z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)

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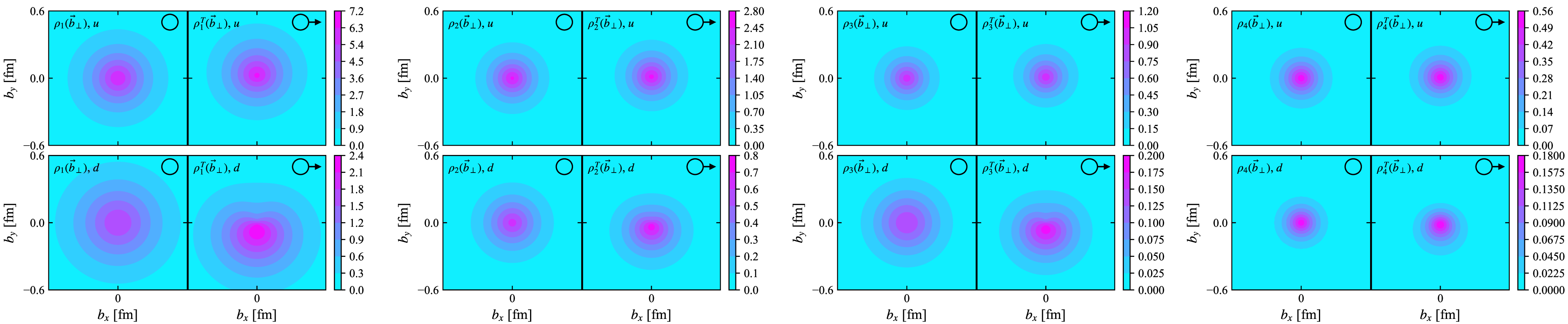
Moments of impact parameter parton distributions in the transverse plane:

$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\vec{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp},$$

$$\rho_{n+1}^T(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} [A_{n+1,0}(-\vec{\Delta}_\perp^2) + i\frac{\Delta_y}{2M} B_{n+1,0}(-\vec{\Delta}_\perp^2)] e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}.$$

1st moment | 2nd moment | 3rd moment | 4th moment



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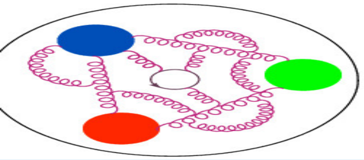
Conclusions and prospects

- Main message: **probing nucleon's 3D structure with LQCD becomes feasible!**
- Recent breakthrough for GPDs: **computationally more efficient calculations in non-symmetric frames**.
- A lot of follow-up work in progress: transversity GPDs, pion and kaon GPDs, other twist-3 GPDs, extension of kinematics.
- Obviously, GPDs much more challenging than PDFs.
- Several challenges have to be overcome – control of lattice and other systematics.
- Quantification of systematics very laborious, but crucial.
- Consistent progress will ensure complementary role to pheno!



Conclusions and prospects

- Main message: **probing nucleon's 3D structure with LQCD becomes feasible!**
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Thank you for your attention!

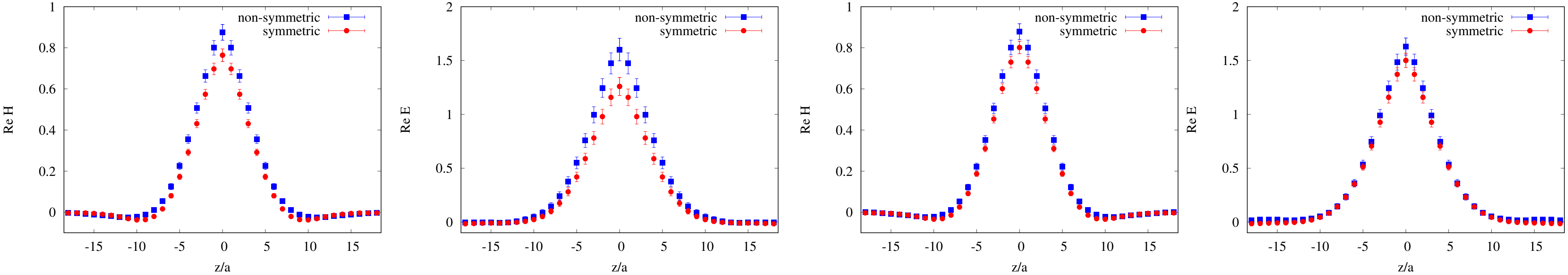
See the talk by Martha for twist-3 GPDs!

Backup slides

H and E GPDs – comparison of definitions

STANDARD DEFINITION | LORENTZ-INVARIANT DEFINITION

H-GPD | E-GPD | H-GPD | E-GPD



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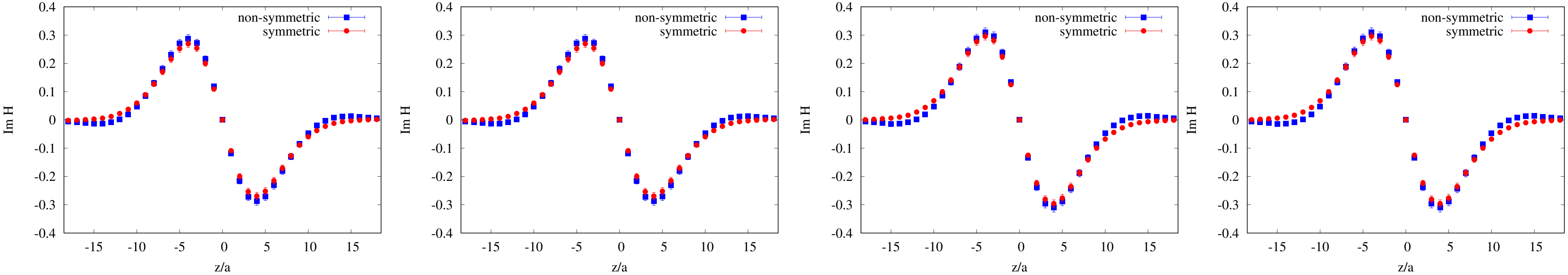
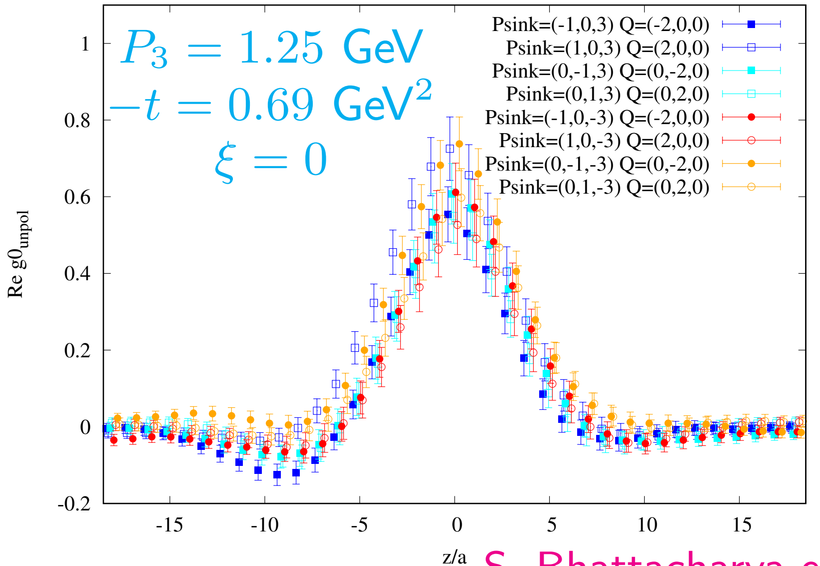
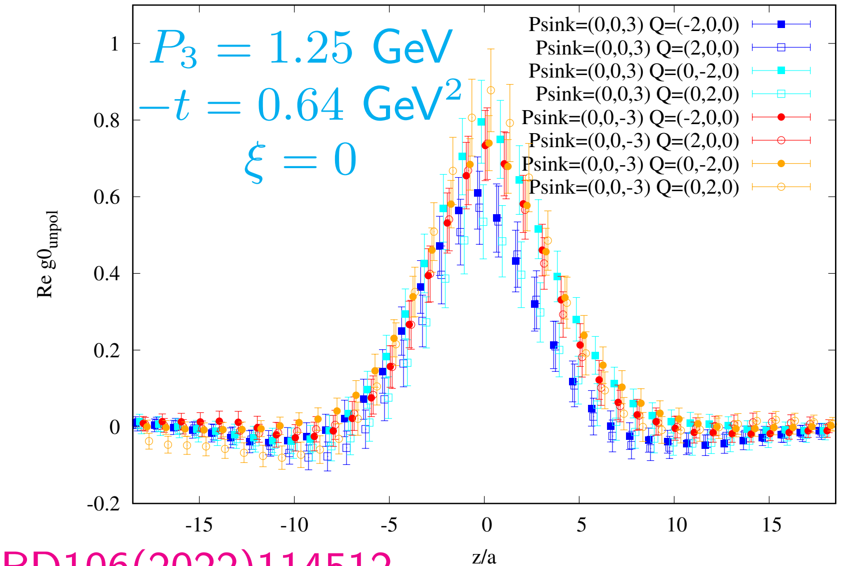


Bare matrix elements of $\Pi_0(\Gamma_0)$

symmetric frame | **non-symmetric frame**

Re





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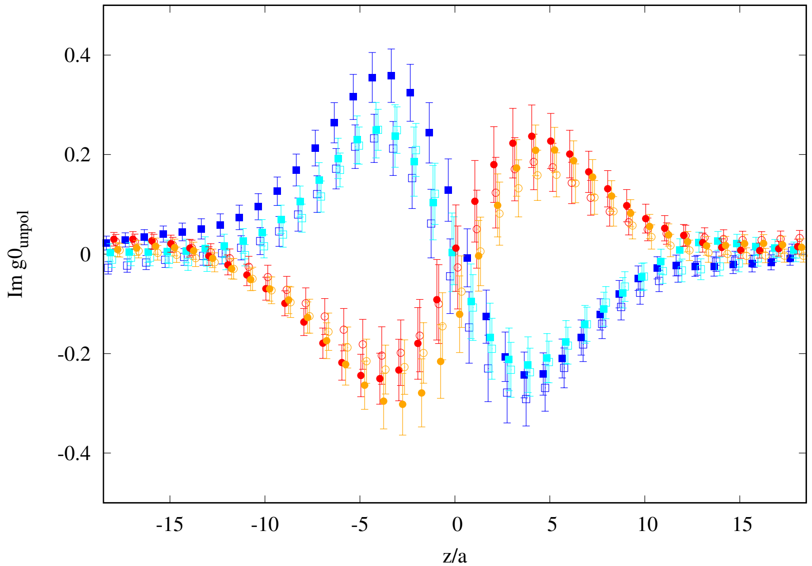
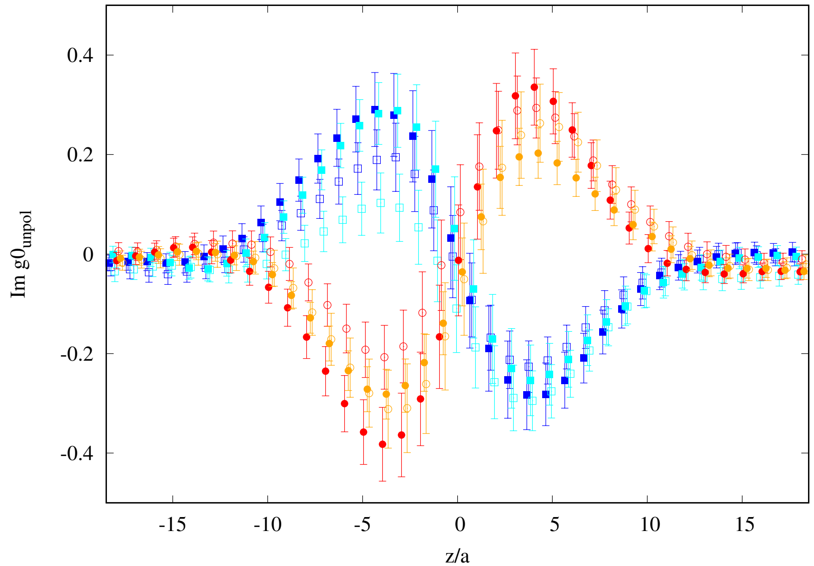
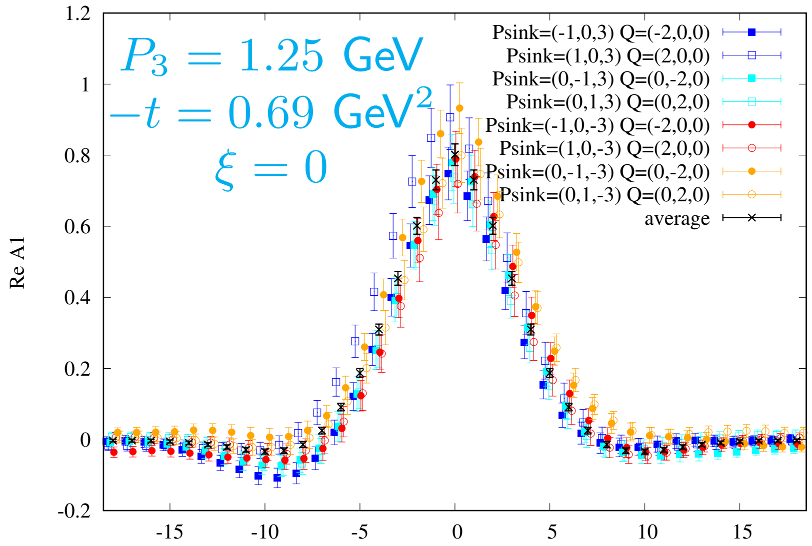
Im



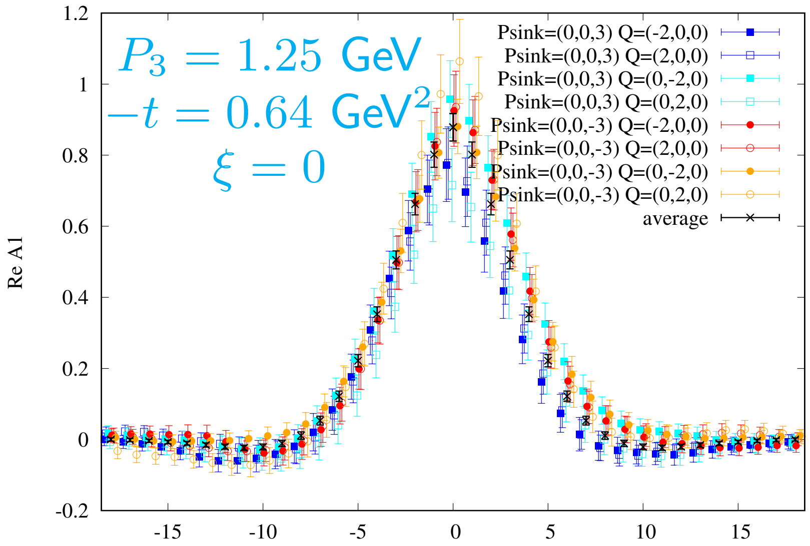


Example amplitude A_1

symmetric frame

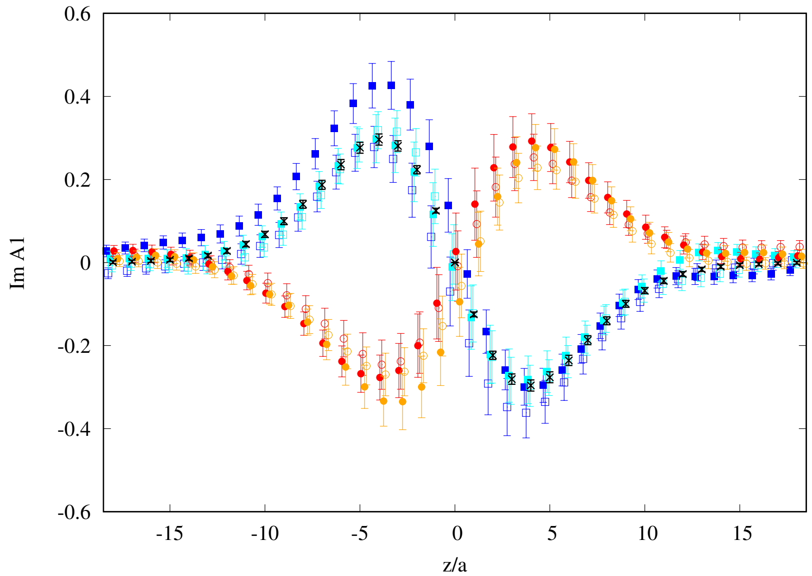


non-symmetric frame



Re

z/a S. Bhattacharya et al., PRD106(2022)114512 z/a



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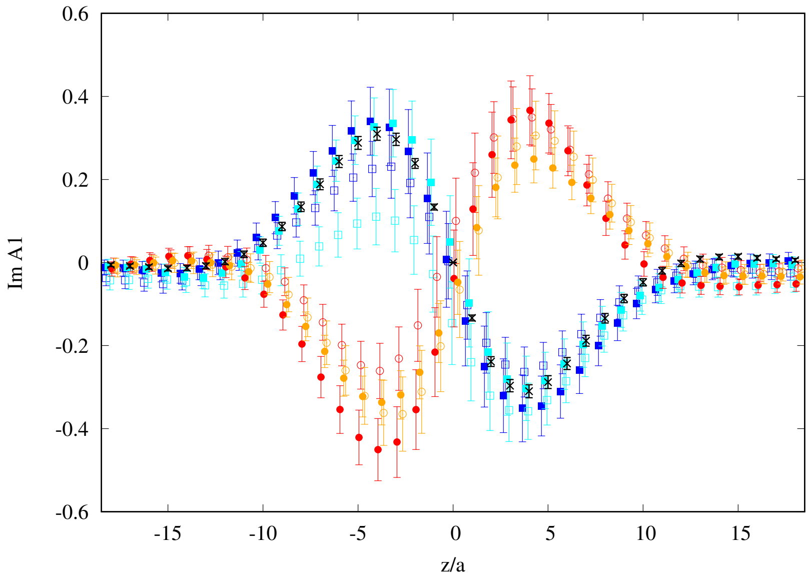
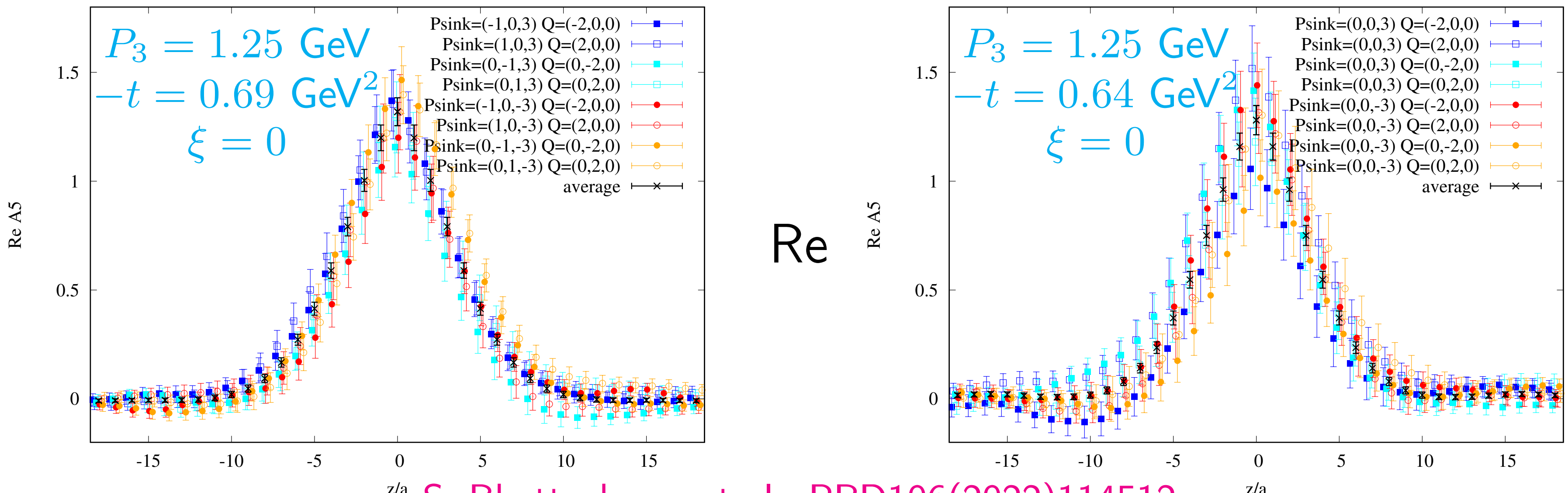


Example amplitude A_5

symmetric frame | non-symmetric frame



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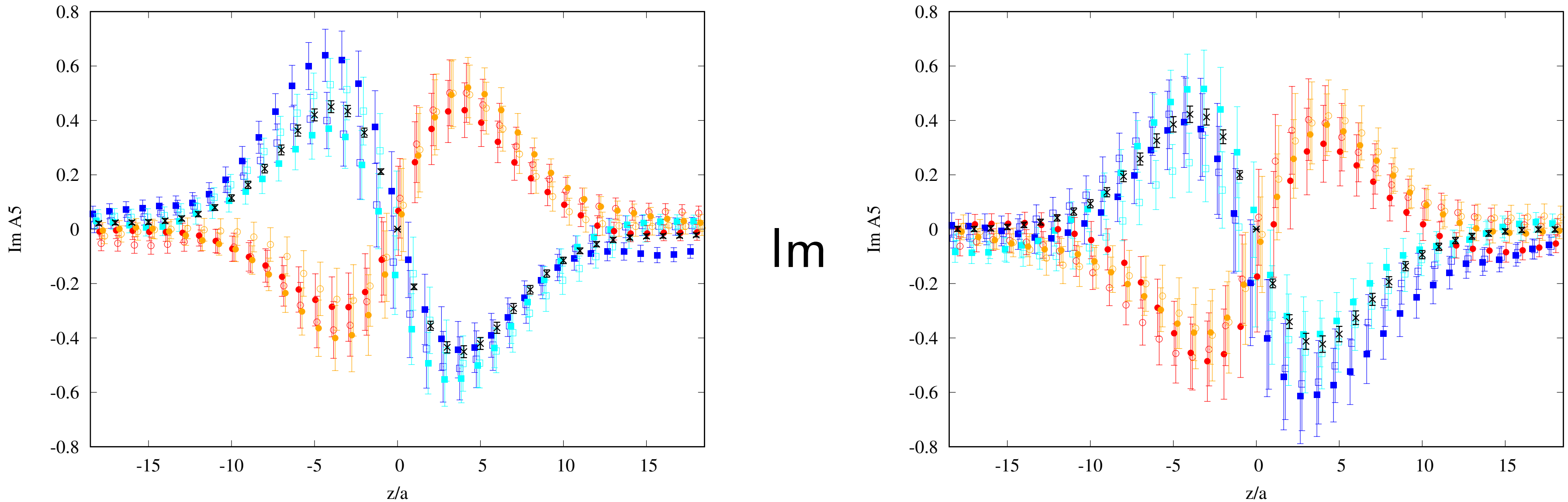
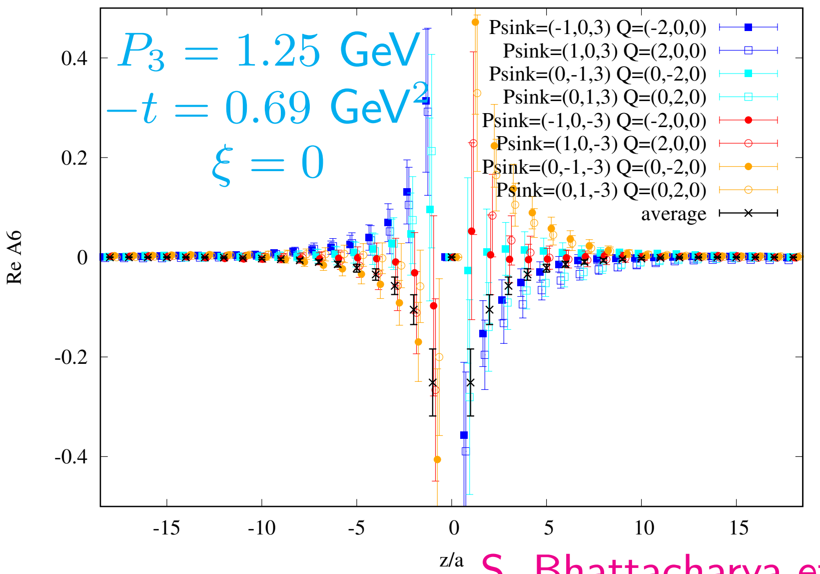
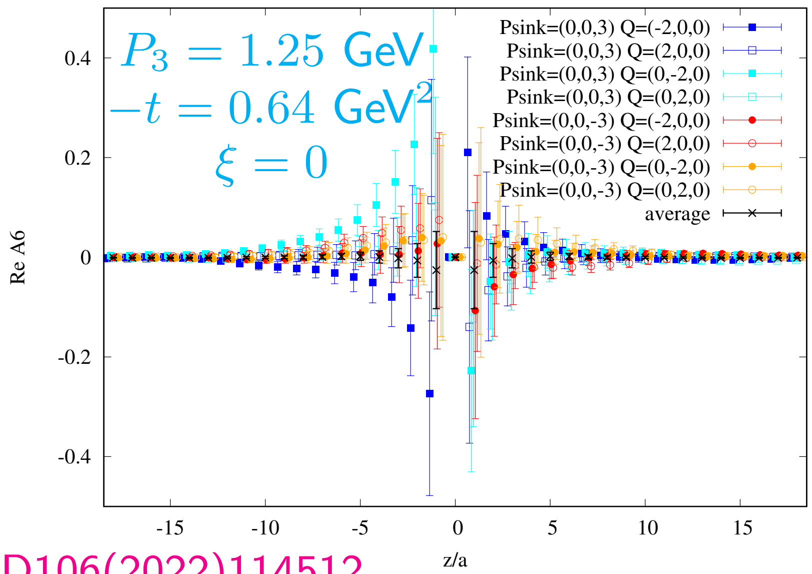


Example amplitude A_6

symmetric frame | **non-symmetric frame**



Re



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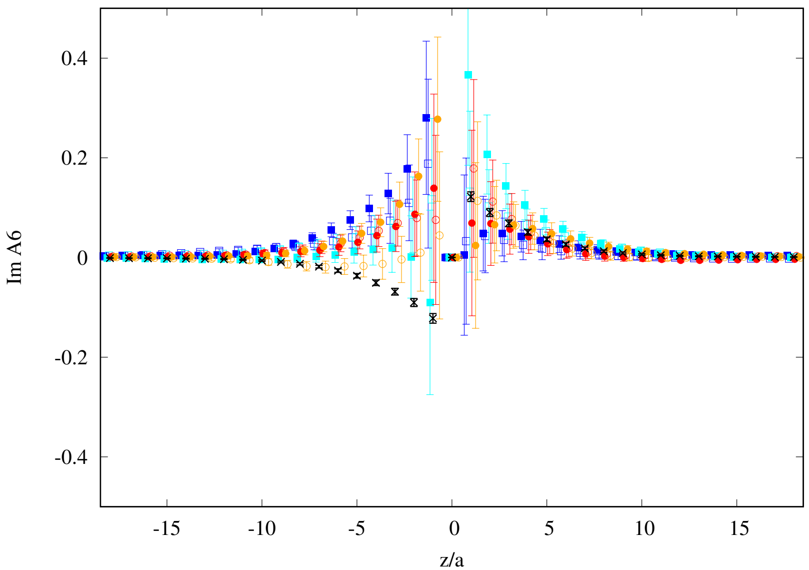
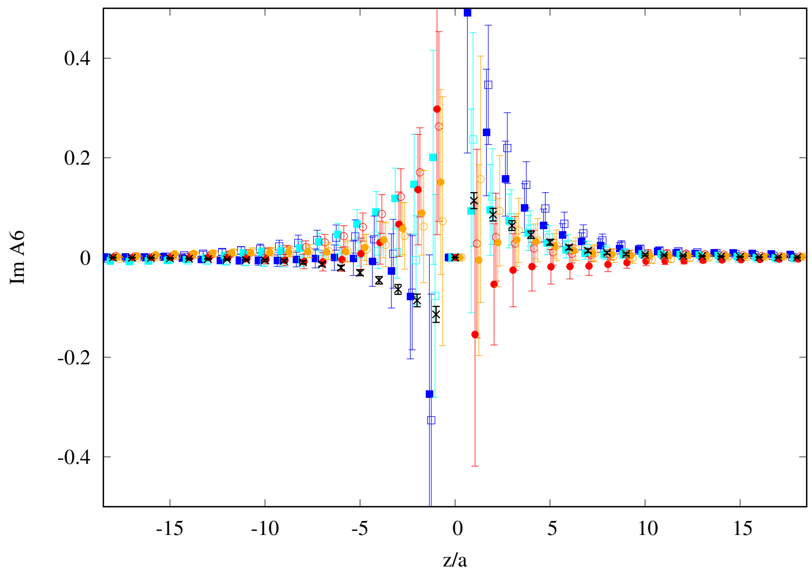
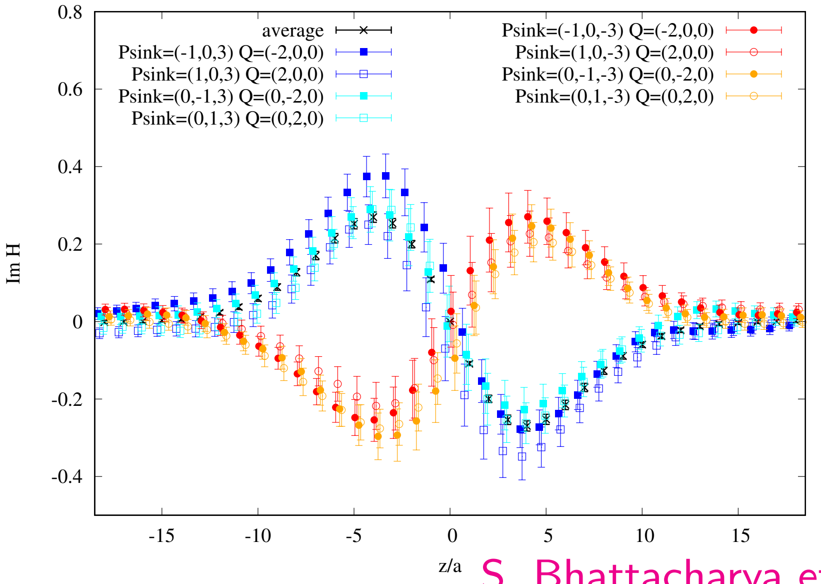


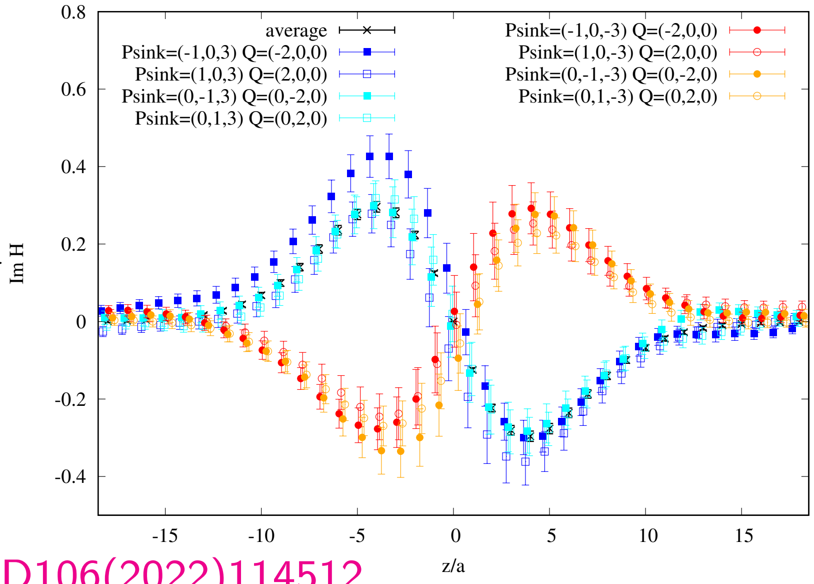
Im



H and E GPDs – signal improvement

standard | Lorentz-invariant





Im H

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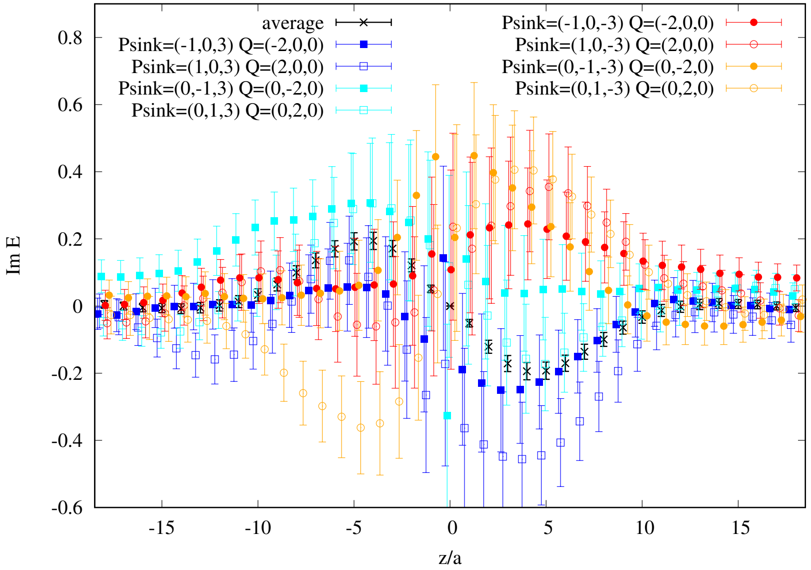
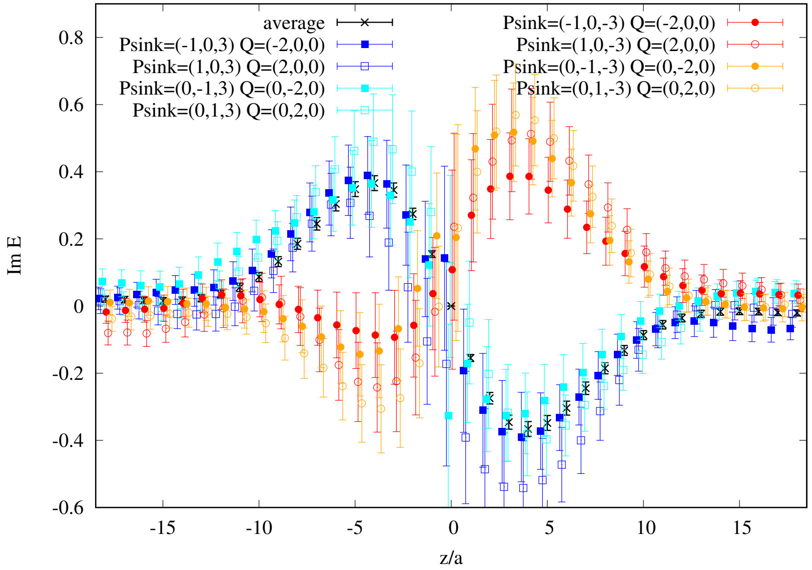
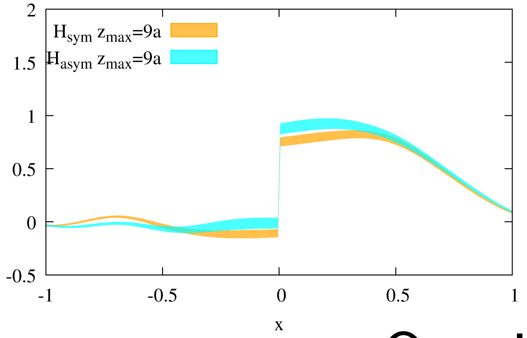


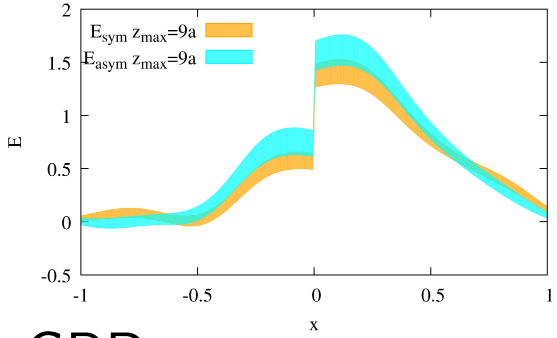


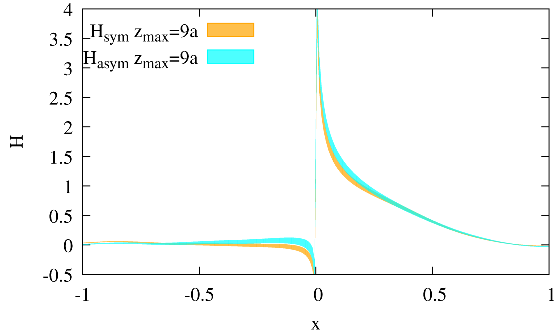
Im E

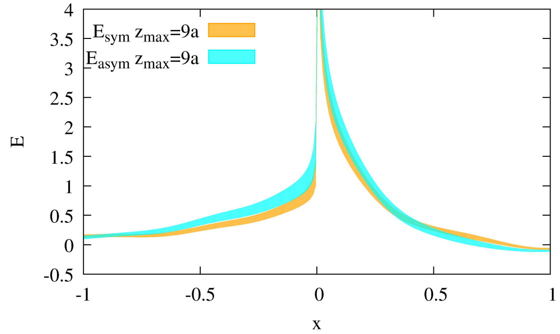
Quasi- and matched H and E GPDs

STANDARD DEFINITION





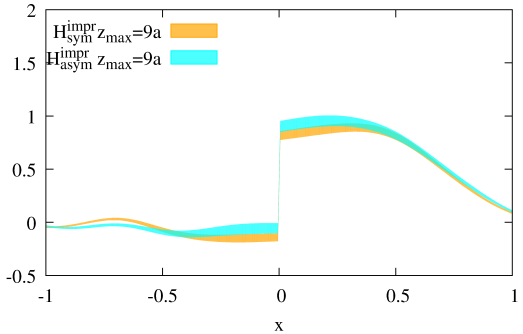


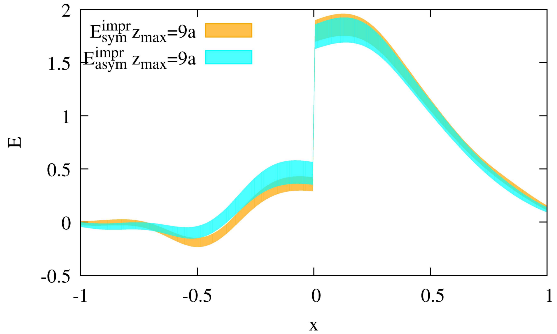


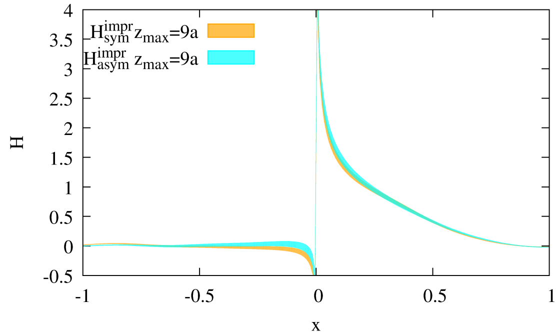
Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

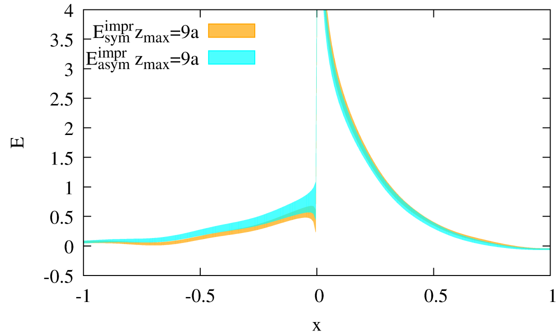
H-GPD E-GPD H-GPD E-GPD

LORENTZ-INVARIANT DEFINITION









GPDs moments from OPE of non-local operators

Short-distance factorization of ratio-renormalized H/E:

$$\mathcal{F}^{\overline{\mathrm{MS}}}(z,P,\Delta)=\sum_{n=0}\frac{(-izP)^n}{n!}C_n^{\overline{\mathrm{MS}}}(\mu^2z^2)\langle x^n\rangle+\mathcal{O}(\Lambda_{\mathrm{QCD}}^2z^2),$$

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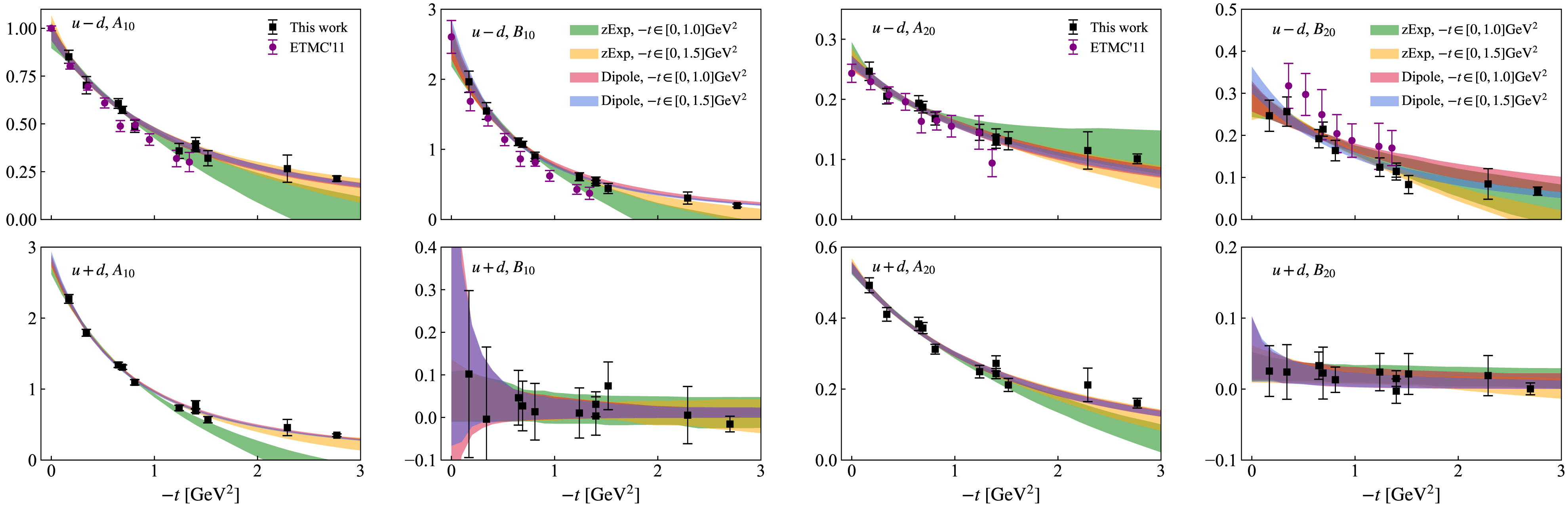

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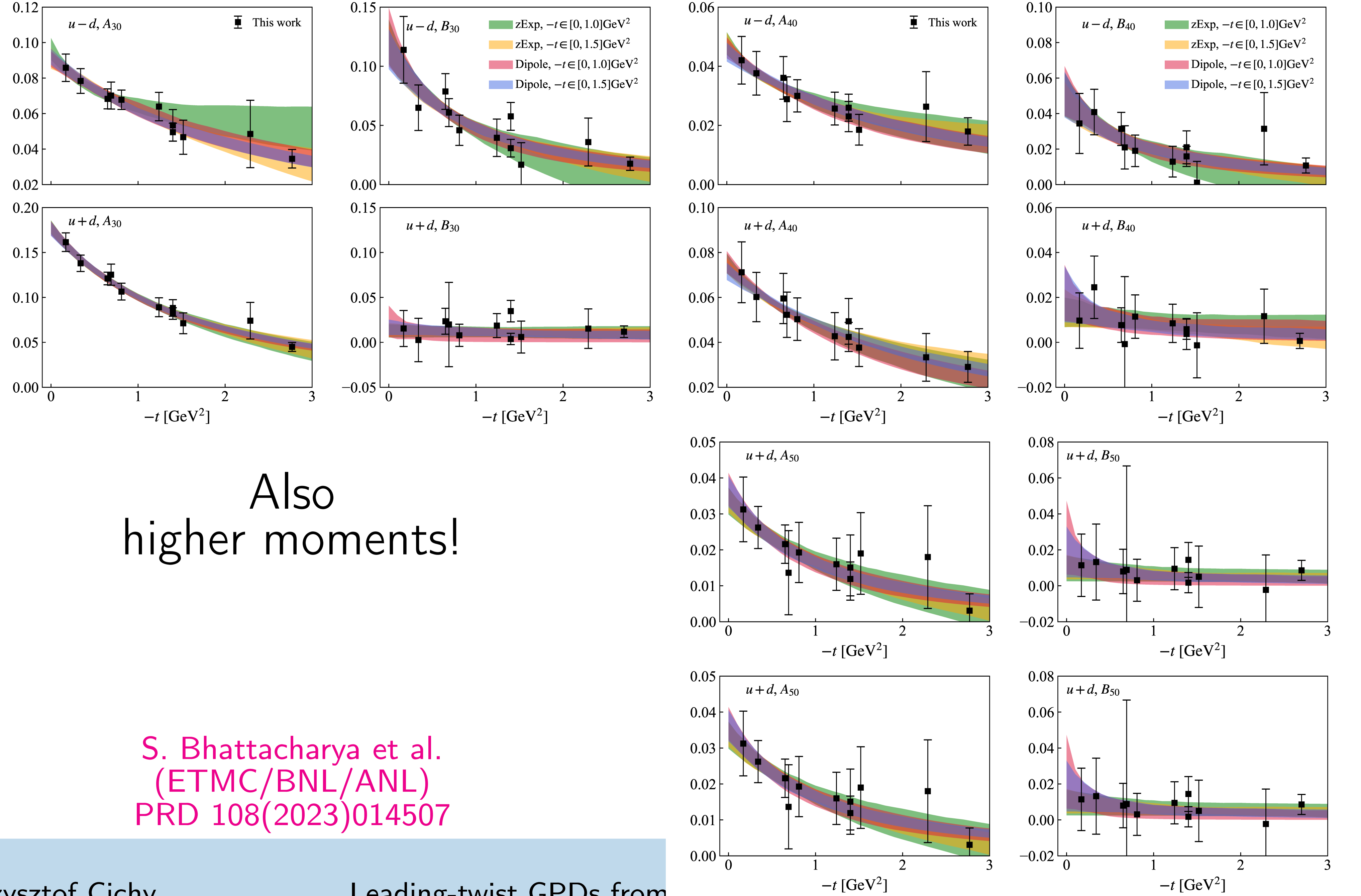
GPDs moments from OPE of non-local operators



Also
higher moments!

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Moments of impact parameter parton distributions in the transverse plane:

$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\vec{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp},$$

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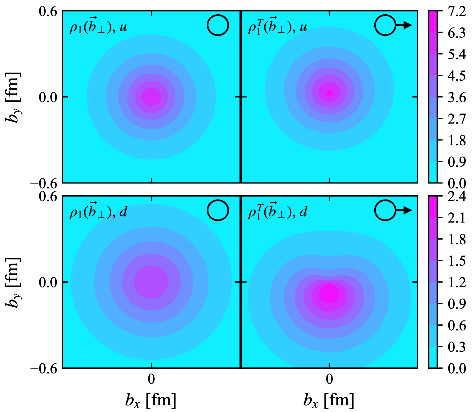
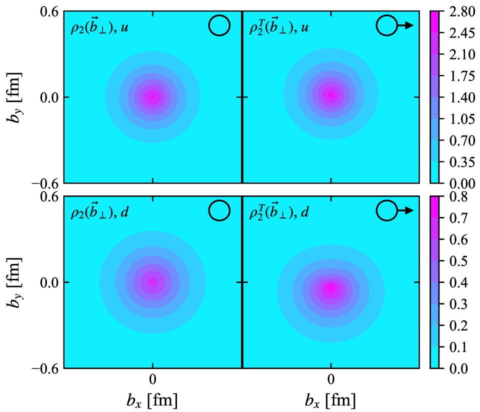
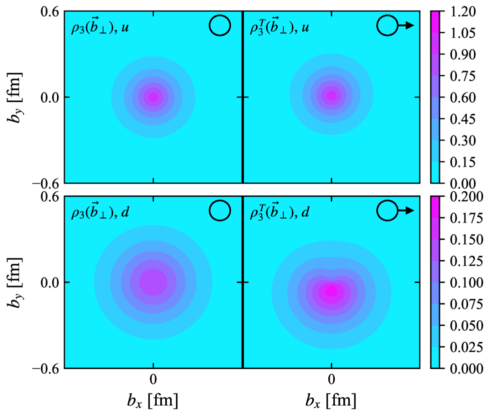
1st moment



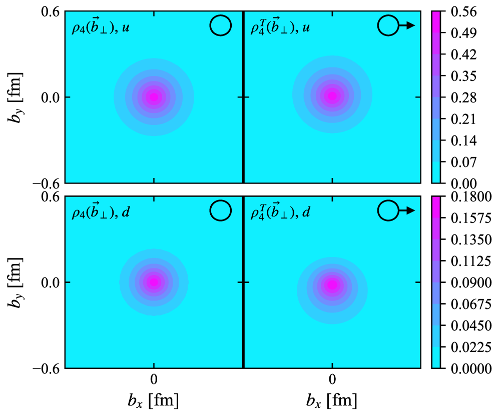
2nd moment



3rd moment



4th moment



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