Nucleon Structure using Pseudo-Distributions

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Jefferson Lab

Hadstruc Collaboration

ECT*, 7th August, 2024

“Towards Improved Hadron Tomography with Hard Exclusive Reactions”

Outline

- Pseudo-Distributions
- 1D Nucleon Structure: PDFs, Transversity, Gluon
- Generalized Parton Distributions and Generalized Form Factors
- Summary and Future Prospects

Hadron Structure: No-go Theorem?

PDFs, GPDs, TMDs

- First Challenge:
  - Euclidean lattice precludes calculation of *light-cone/time-dependent* correlation functions

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P \rangle$$

So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

$$\longrightarrow \quad \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\dots D_{\mu_n}\psi \mid P \rangle \to P_{\mu_1}\dots P_{\mu_n} a^{(n)}$$

- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz' A^z(z')} \psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

Large-Momentum Effective Theory (LaMET)
Quasi-PDF (qPDF)

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

PDFs, GPDs and TMDs

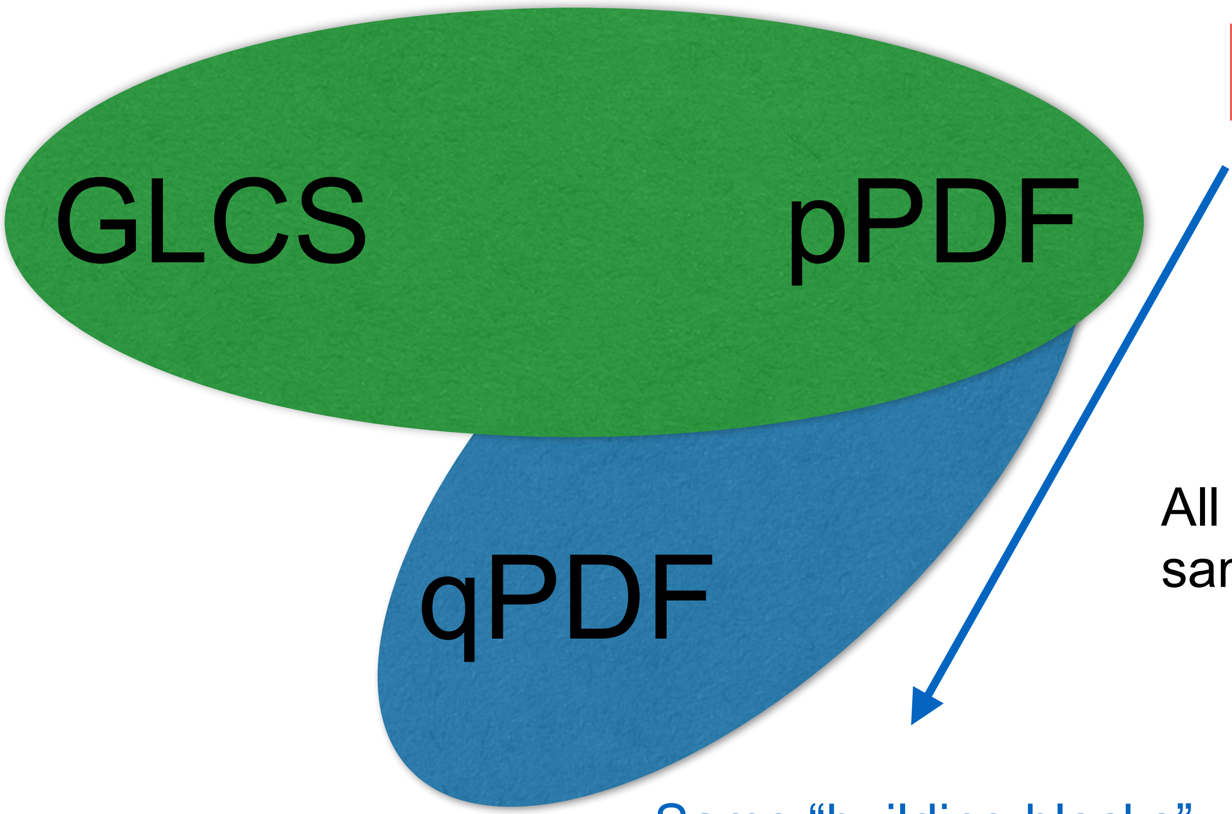
Ma and Qiu, Phys. Rev. Lett. 120 022003

Characterized by *short-distance factorization*

"Good lattice cross sections"

Pseudo-Distributions

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)



Same "building blocks"

All approaches should give same after:

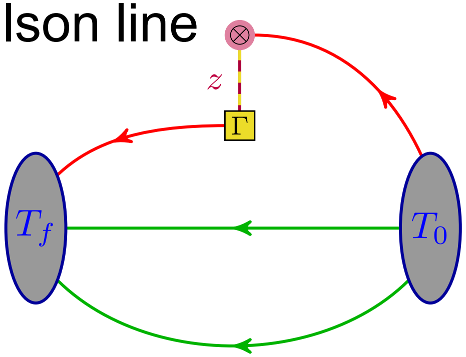
- Finite volume
- Discretization Uncertainties
- *Infinite momentum*

Pseudo-PDFs

Lattice "building blocks" that of quasi-PDF approach.

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Wilson line



A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et a*l, PRD51, 6036 (1995)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$$p = (p^+, m^2/2p^+, 0_T) \qquad z = (0, z_-, 0_T)$$

$$\Downarrow$$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

$$\mathfrak{M} = \frac{\mathscr{M}(\nu, z^2)}{\mathscr{M}(0,z^2)} \equiv \left(\frac{\mathscr{M}(\nu, z^2)}{\mathscr{M}(\nu,0)}\right) / \left(\frac{\mathscr{M}(0,z^2)}{\mathscr{M}(0,0)}\right)$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu, \mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$\Downarrow$

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Inverse problem

Need data for all ν, *or additional physics input*

ITD ↔ PDF

Distillation and Hadron Structure

To control systematic uncertainties, need precise computations over a wide range of momentum. M.Peardon *et al* (Hadspec), Phys.Rev.D 80 (2009) 054506

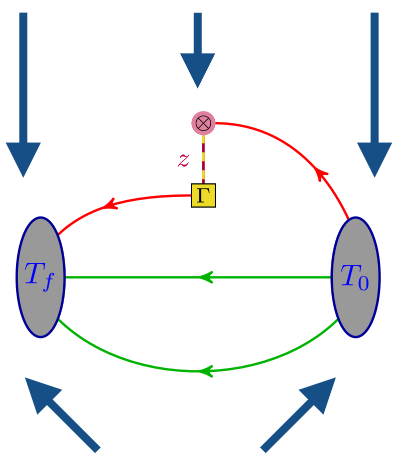
- Use a low-mode projector to capture states of interest "distillation"
- Enables momentum projection at each temporal point.

$$\tau_{\alpha\beta}^{(l,k)}(t',t) = \xi^{(l)\dagger}(t')\, M_{\alpha\beta}^{-1}(t',t)\, \xi^{(k)}(t)$$ *Perambulators* → quark propagation

$$\Phi_{\alpha\beta\gamma}^{(i,j,k)}(t) = \epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a \left(\mathcal{D}_2\xi^{(j)}\right)^b \left(\mathcal{D}_3\xi^{(k)}\right)^c (t)\, S_{\alpha\beta\gamma}$$ *Elementals* → (baryon) operators

$$C_{rs}(t) = \sum_{\vec{x},\vec{y}} \langle 0 \mid \mathcal{O}_r(t,\vec{x})\mathcal{O}_s^\dagger(0,\vec{y}) \mid 0\rangle \equiv \mathrm{Tr}\left[\Phi_r(t) \otimes \tau(t,0)\tau(t,0)\tau(t,0) \otimes \Phi_s(0)\right]$$

Momentum projection

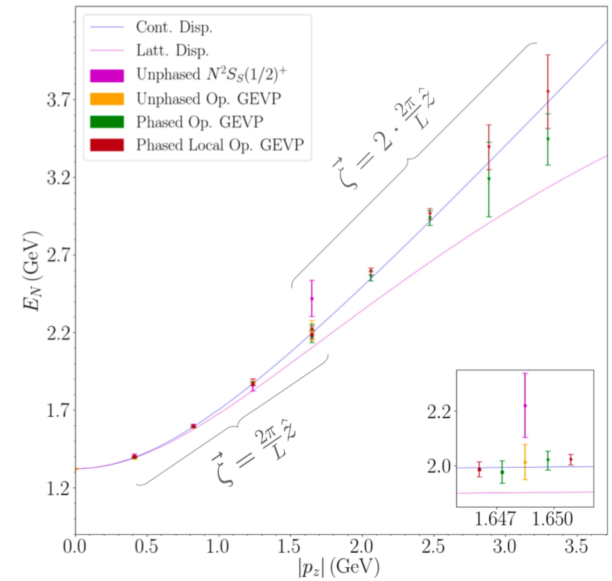


Variational basis

+ momentum smearing

G.Bali *et al*, Phys.Rev.D 93 (2016) 9, 094515

C.Egerer *et al* (Hadstruc), Phys. Rev. D 103, 034502 (2021)



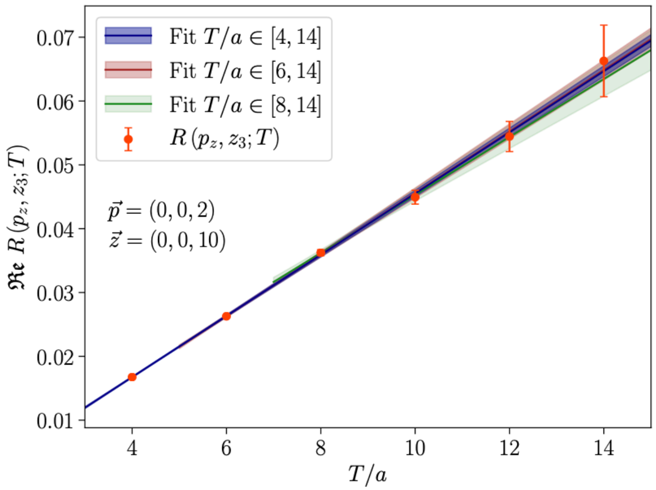
Nucleon Isovector PDF

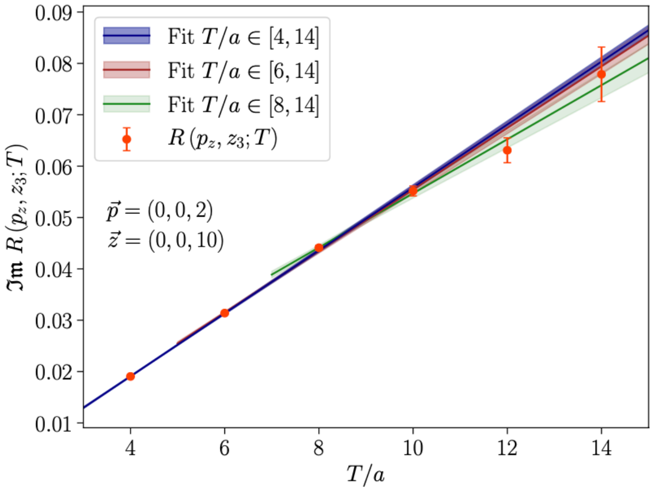
C.Egerer *et al.* (hadstruc), JHEP 11 (2021) 148

ID	a_s (fm)	m_π (MeV)	$L_s^3 \times N_t$	$N_{\rm cfg}$	$N_{\rm srcs}$	$R_{\mathcal{D}}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	4	64

JLab/WM/LANL/MIT 2+1 clover - used throughout rest of this talk

Matrix elements extracted using summation method - *reduced excited-state contributions*





Expand the x-dependence in terms of (shifted) Jacobi Polynomials

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \mathfrak{Re}\int_0^1 dx\, \mathcal{K}_{\rm v}\left(x\nu, z^2\mu^2\right) x^\alpha (1-x)^\beta\, \Omega_n^{(\alpha,\beta)}(x)$$

Matching kernel

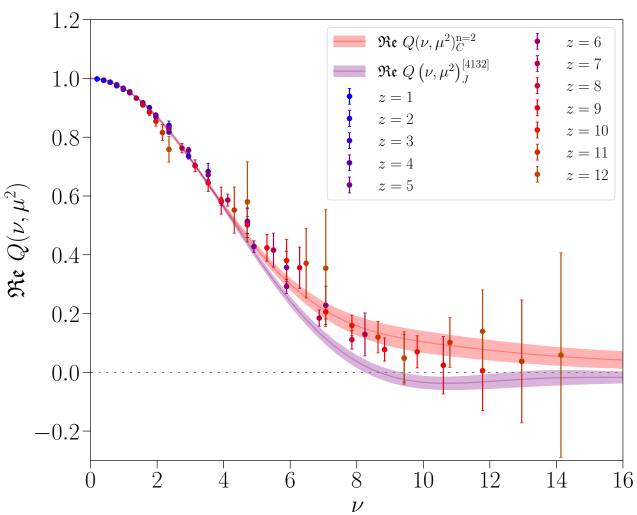
J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos, arXiv:2105.13313

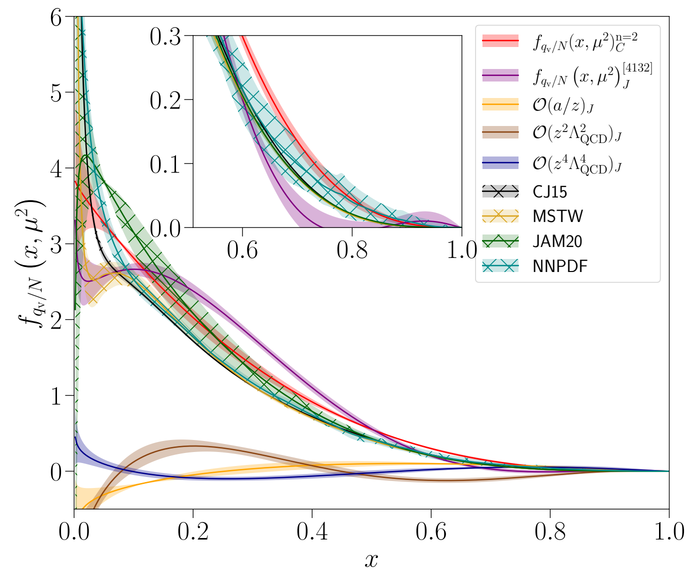
$$\mathfrak{Re}\,\mathfrak{M}_{\rm fit}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{{\rm v},n}^{lt\,(\alpha,\beta)} + \left(\frac{a}{z}\right) \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{az\,(\alpha,\beta)} + z^2\Lambda_{\rm QCD}^2 \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{t4\,(\alpha,\beta)}$$

Discretization

Higher twist

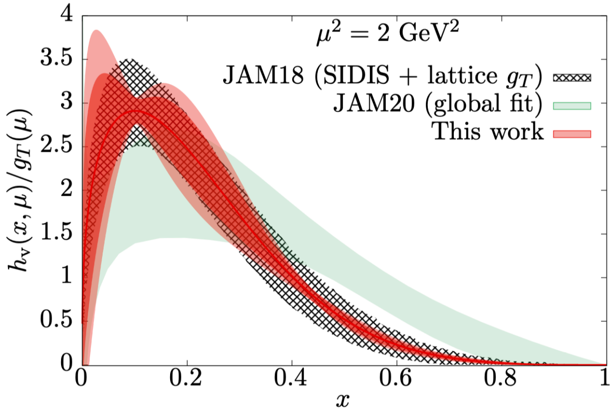
$m_\pi \simeq 358$ MeV

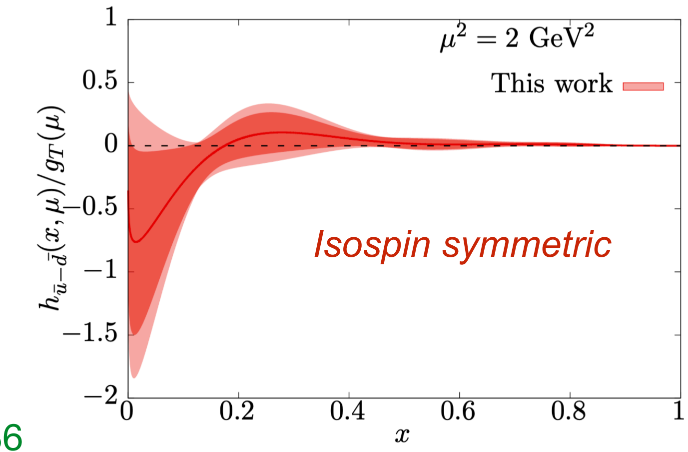




Transversity and Helicity

Phys.Rev.D 105 (2022) 3, 034507, Hadstruc Collaboration, (C.Egerer et al).

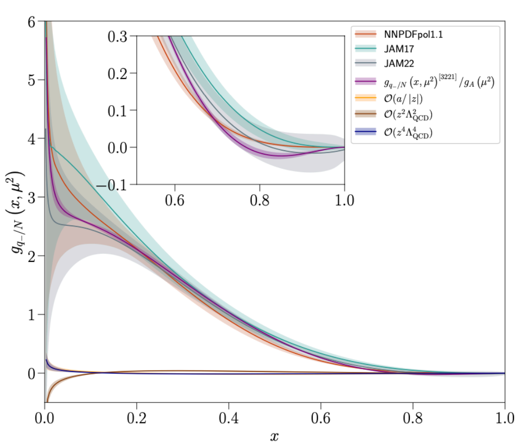


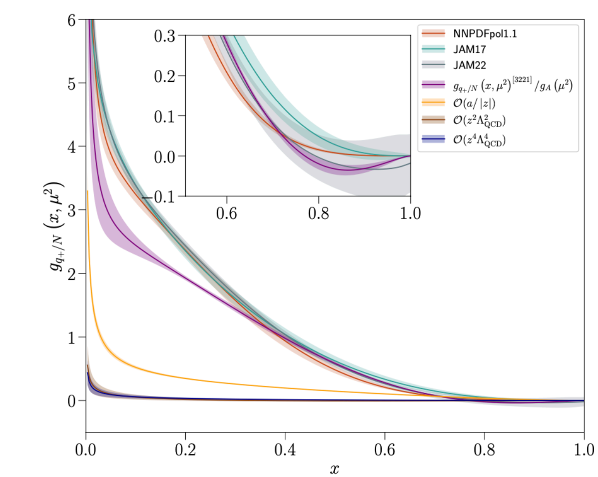


HadStruc Collab (R.Edwards et al), *JHEP* 03 (2023) 086

Valence quark helicity distribution, together with contamination terms

CP-odd helicity distribution, together with contamination terms





3D Hadron Structure

Generalized Parton Distributions (GPDs) provide 3D description in terms of longitudinal momentum fraction and (2D) transverse displacement

– Orbital Angular Moment
– Integrated Generalized Form Factors: distribution of mass, charge, pressure

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Towards Unpolarized GPDs from Pseudo-Distributions

HadStruc Collaboration • Hervé Dutrieux (William-Mary Coll.) Show All(10)

May 16, 2024

JHEP (to appear)

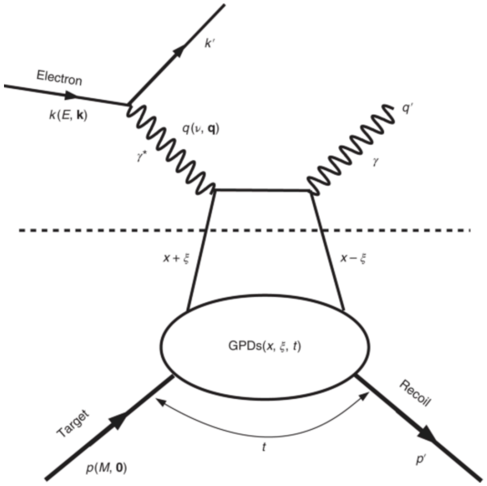
e-Print: 2405.10304 [hep-lat]

Report number: JLAB-THY-24-4059, JLAB-THY-24-4059

View in: HAL Science Ouverte, ADS Abstract Service

PHENOMENOLOGY LATTICE QCD THEORY

GPD Formalism



Kinematic variables

$$F^q(x, p_f, p_i) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \langle N(p_f,\lambda_f)|\bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\hat{W}\left(-\frac{z}{2},\frac{z}{2};A\right)\psi^q\left(\frac{z}{2}\right)|N(p_i,\lambda_i)\rangle|_{z^+=0,\mathbf{z}_\perp=\mathbf{0}_\perp}$$
$$= \frac{1}{2P^+}\bar{u}(p_f,\lambda_f)\left[\gamma^+ H^q(x,\xi,t) + \frac{i\sigma^{+\nu}q_\nu}{2m}E^q(x,\xi,t)\right]u(p_i,\lambda_i)$$

$$P \equiv \frac{1}{2}(p+p')\,,\quad q \equiv p'-p\,,\quad t \equiv q^2\,,\quad \xi \equiv -\frac{q^+}{2P^+}\,.$$

Total Momentum Momentum transfer Skewness

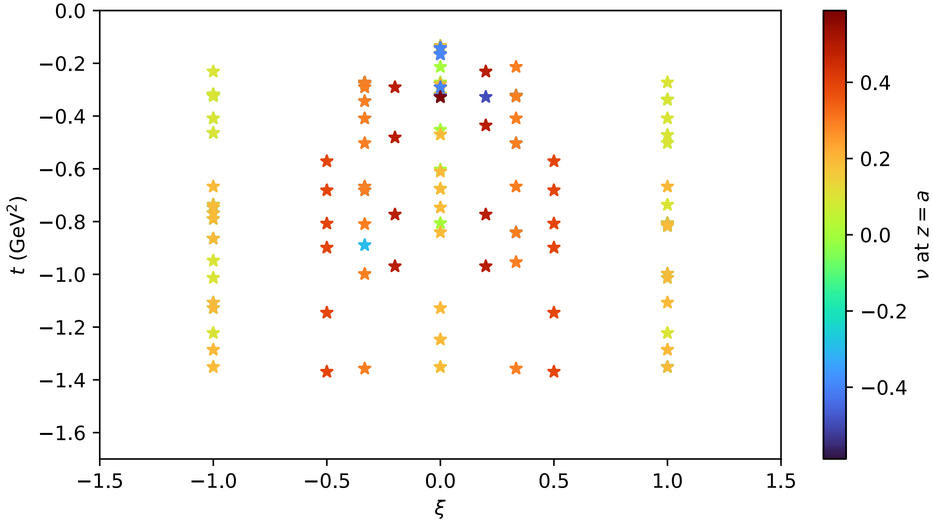
As for the case of the PDFs, we generalize and calculate matrix elements at *space-like separations z.*

S.Bhattacharya *et al., PRD106, 114512*

$$\mathcal{M}^\mu(p_f,p_i,z) = \langle\langle\gamma^\mu\rangle\rangle\mathcal{A}_1(\nu,\xi,t,z^2) + z^\mu\langle\langle\mathbb{1}\rangle\rangle\mathcal{A}_2(\nu,\xi,t,z^2) + i\langle\langle\sigma^{\mu z}\rangle\rangle\mathcal{A}_3(\nu,\xi,t,z^2)$$
$$+ \frac{i}{2m}\langle\langle\sigma^{\mu q}\rangle\rangle\mathcal{A}_4(\nu,\xi,t,z^2) + \frac{q^\mu}{2m}\langle\langle\mathbb{1}\rangle\rangle\mathcal{A}_5(\nu,\xi,t,z^2)$$
$$+ \frac{i}{2m}\langle\langle\sigma^{zq}\rangle\rangle\left[P^\mu\mathcal{A}_6(\nu,\xi,t,z^2) + q^\mu\mathcal{A}_7(\nu,\xi,t,z^2) + z^\mu\mathcal{A}_8(\nu,\xi,t,z^2)\right]$$

$$\xi = -\frac{q.z}{2P.z} = -\frac{q.z}{2\nu}$$

- In contrast to DVCS and DVMP, lattice QCD admits the calculation of GPDs at discrete points in 3D - DVCS along $x \simeq \xi$. Essentially 2D



IGPDs and GPDs related through transform

$$\begin{pmatrix} H^q \\ E^q \end{pmatrix}(x,\xi,t) = \int \frac{d\nu}{2\pi} e^{ix\nu} \begin{pmatrix} H^q \\ E^q \end{pmatrix}(\nu,\xi,t)\,,$$

As before, this involves tackling *inverse problem*

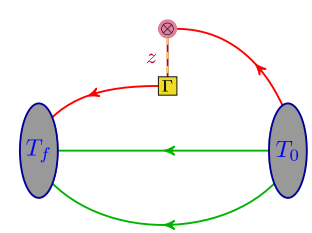
$$H^q(\nu,\xi,t) = \lim_{z^2\to 0}\left[\mathcal{A}_1 - \xi \mathcal{A}_5\right]$$

$$E^q(\nu,\xi,t) = \lim_{z^2\to 0}\left[\mathcal{A}_4 + \nu\mathcal{A}_6 - 2\xi\nu\mathcal{A}_7 + \xi \mathcal{A}_5\right]$$

Very Computationally and Data Demanding

$$\Xi^{\mathbf{\Gamma}(i,j)}_{\alpha\beta;ab}(T_f, T_i;\tau,z) = \sum_{\vec{y}} \xi^{(i)\dagger}_a(T_f)\, D^{-1}_{\alpha\sigma;ac}(T_f;\tau,\vec{y})\,\mathbf{\Gamma}(\tau)\, D^{-1}_{\rho\beta;db}(\tau,\vec{x};T_i)\,\xi^{(j)}_b(T_i)$$

Do calculations "on the fly"…



Moments of GPDs

For this first study, we will focus on calculations of the moments of GPDs

$$\int_{-1}^{1} \mathrm{d}x\, x^{n-1} \begin{pmatrix} H^{u-d} \\ E^{u-d} \end{pmatrix} (x, \xi, t) = \sum_{k=0 \text{ even}}^{n-1} \begin{pmatrix} A_{n,k}(t) \\ B_{n,k}(t) \end{pmatrix} \xi^k \pm \mathrm{mod}(n+1, 2) \xi^n C_n(t) \,,$$

Moments of D term

Moments can be obtained by ν expansion of the Ioffe-time distribution

$$F(\nu, \xi, t, z^2) = \sum_{n=0}^{\infty} \frac{(-i\nu)^n}{n!} F_{n+1}(\xi, t, z^2) \quad \text{where} \quad F_n(\xi, t, z^2) \equiv \int_{-1}^{1} dx\, x^{n-1} F(x, \xi, t, z^2)$$

We fit the resulting GFFs to a *dipole*

$$A_{n,k}(t) = A_{n,k}(t=0) \left(1 - \frac{t}{\Lambda_{n,k}^2}\right)^{-2}$$

More rigorously, use the so-called *z-expansion.*

$$F(t) = \sum_{k=0}^{2} a_k z(t)^k \,, \quad z(t) = \frac{\sqrt{t_{\mathrm{cut}} - t} - \sqrt{t_{\mathrm{cut}} - t_0}}{\sqrt{t_{\mathrm{cut}} - t} + \sqrt{t_{\mathrm{cut}} - t_0}}$$

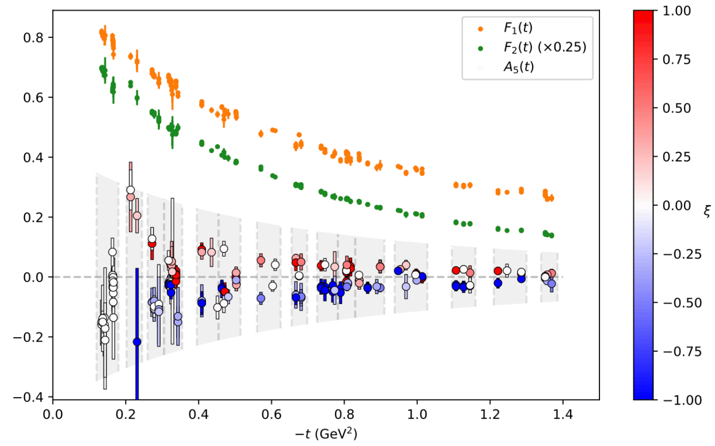
↑ $4m_\pi^2$

EM Form Factors

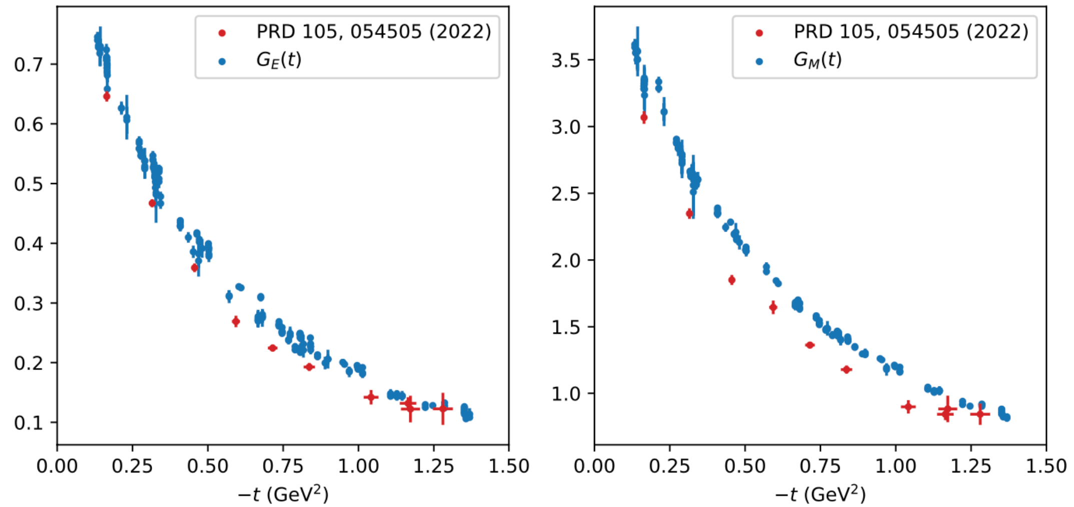
Can calculate EM form factors from local matrix elements, i.e. *z=0*

$A_5 = 0$; Ward identity

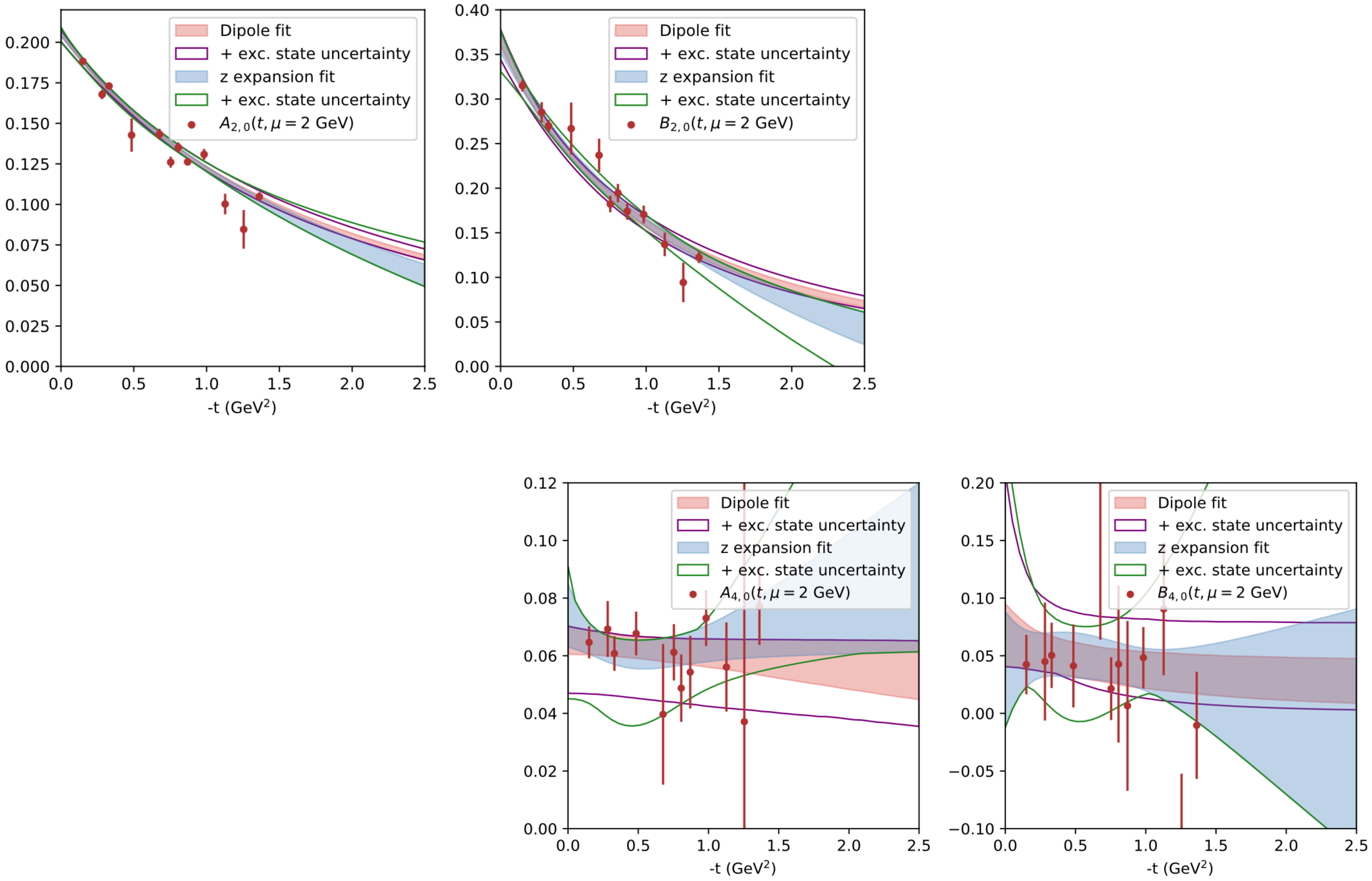
Non-zero - discretization errors, sensitivity to excited states



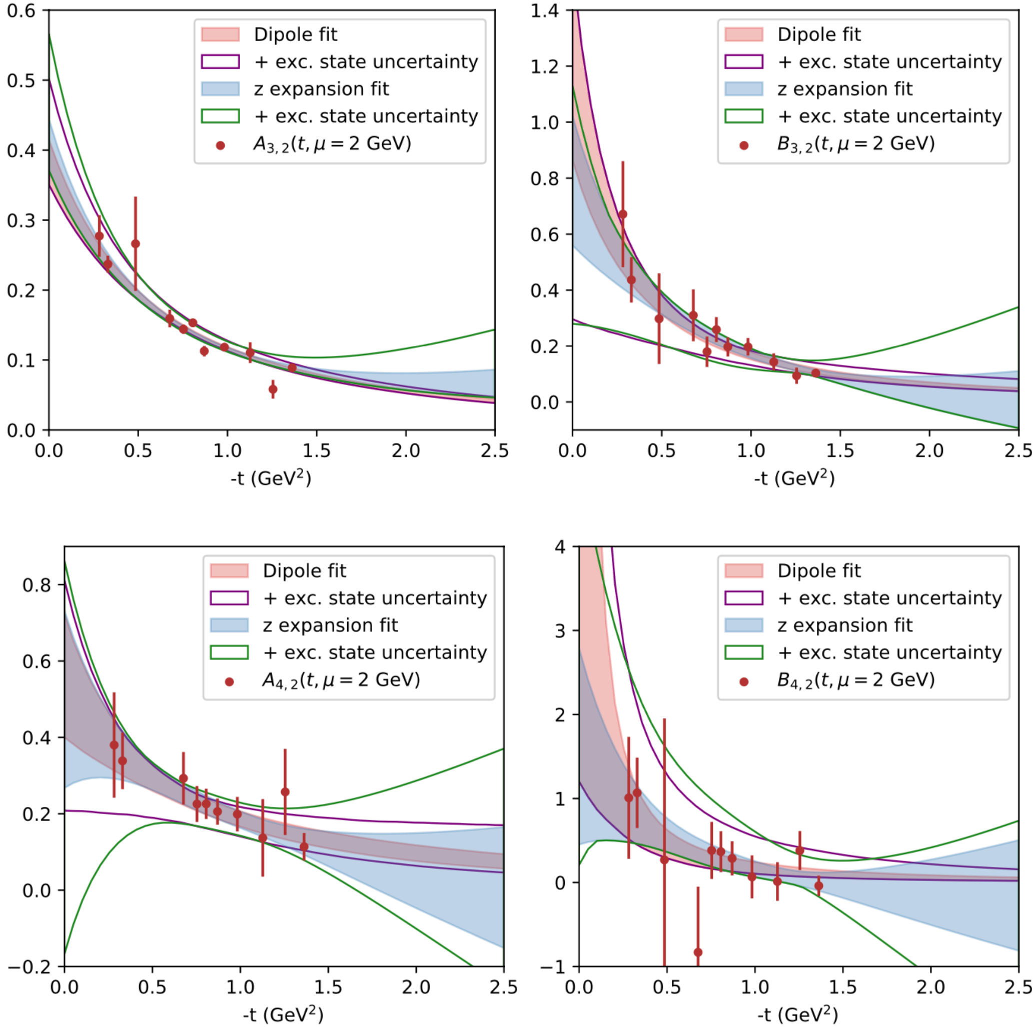
c.f. non-“distilled” calculation



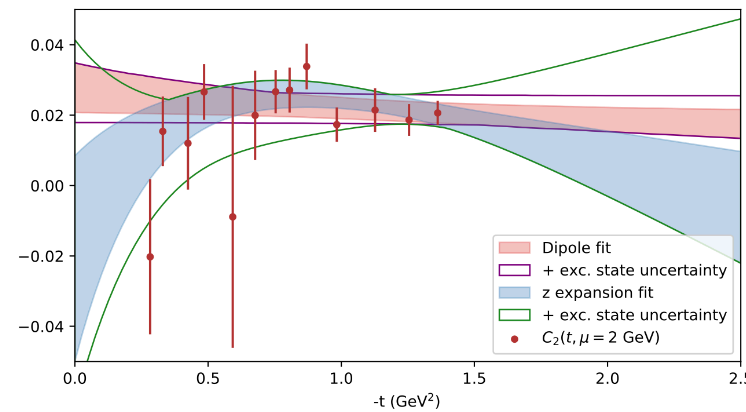
Generalized Form Factors - ξ^0



Generalized Form Factors - ξ^2



(Small) Isovector D term



Summary

Pion mass = 0.36 GeV - Proton mass = 1.12 GeV
No continuum limit - signs of discretization errors / light-cone uncertainty
Matching at 2 GeV with leading logarithmic accuracy

Value at t = 0

GPD H^{u-d}		GPD E^{u-d}	
$A_{1,0}$: 0.974^{+12}_{-5}		$B_{1,0}$: 3.40^{+7}_{-1}	
$A_{2,0}$: 0.206^{+2}_{-6}		$B_{2,0}$: 0.370^{+9}_{-24}	
$A_{3,0}$: 0.064^{+2}_{-6}	$A_{3,2}$: 0.39^{+11}_{-3}	$B_{3,0}$: 0.063^{+24}_{-8}	$B_{3,2}$: 1.1^{+4}_{-8}
$A_{4,0}$: 0.065^{+5}_{-19}	$A_{4,2}$: 0.5^{+3}_{-3}	$B_{4,0}$: 0.06^{+16}_{-2}	$B_{4,2}$: > 1.1

Dipole mass (GeV)

GPD H^{u-d}		GPD E^{u-d}	
$A_{1,0}$: 1.255^{+3}_{-29}		$B_{1,0}$: 0.987^{+2}_{-6}	
$A_{2,0}$: 1.83^{+9}_{-3}		$B_{2,0}$: 1.39^{+11}_{-5}	
$A_{3,0}$: 2.3^{+2}_{-5}	$A_{3,2}$: 1.10^{+7}_{-11}	$B_{3,0}$: 2.2^{+36}_{-5}	$B_{3,2}$: 0.78^{+77}_{-9}
$A_{4,0}$: > 3.5	$A_{4,2}$: > 0.9	$B_{4,0}$: > 0.6	$B_{4,2}$: 0.5^{+5}_{-2}

D-term^{u-d}

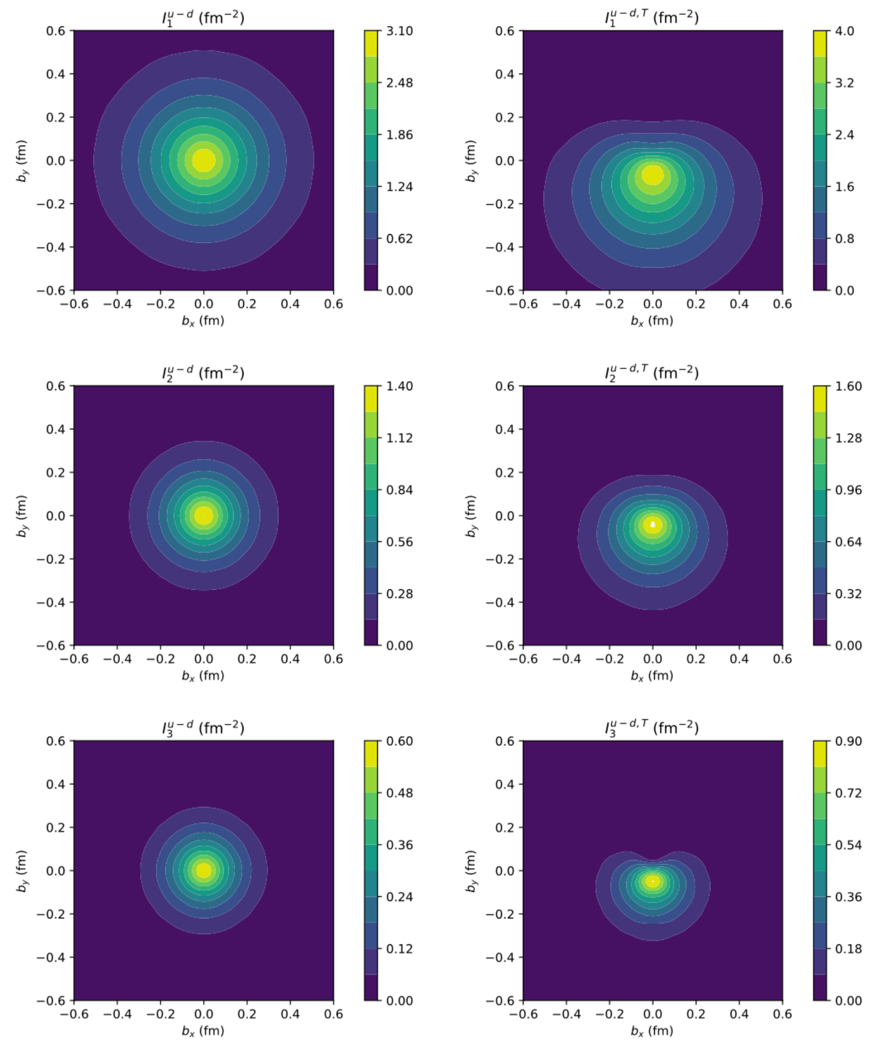
	Value at t = 0	Dipole mass (GeV)
C_2	0.025^{+8}_{-8}	> 2.2

Translate to Impact-Parameter Space

Transform to impact-parameter space:
narrowing of distribution with increasing moment

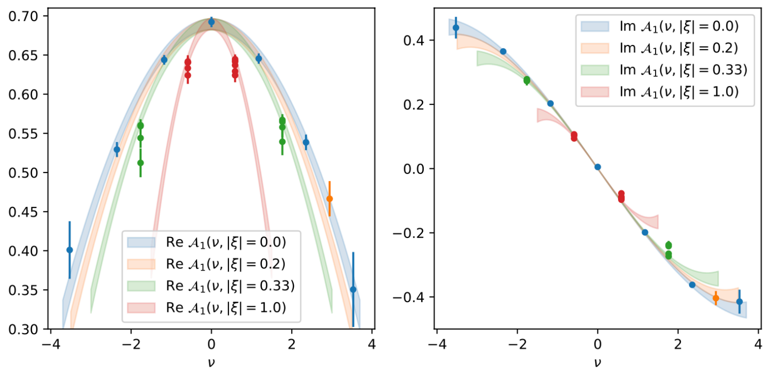
$$I^{u-d}_{n+1}(\vec{b}_\perp) = \int \frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}\, A^{u-d}_{n+1,0}(-\vec{\Delta}^2_\perp)$$

$$I^{u-d,T}_{n+1}(\vec{b}_\perp) = \int \frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}\left[A^{u-d}_{n+1,0}(-\vec{\Delta}^2_\perp) + i\frac{\Delta_y}{2m}B^{u-d}_{n+1,0}(-\vec{\Delta}^2_\perp)\right]$$



Summary

- Realistic calculation of light-cone distributions from LQCD now available
- Focus on understanding systematic contributions in pseudo-PDF framework
- Distillation + boosting enables both far increased reach in momentum, and improved sampling of lattice
- Are able to isolate leading twist from higher-twist and discretization contamination
- 3D Hadron Structure through GPDs
  - Moment calculation allows higher moments than from local operators
  - "Towards GPDs" - *calculation of x dependence in progress*



  - Extend to higher ν; pion masses and lattice spacings
  - Next frontier - flavor singlet.
  - Incorporate in Global Analysis

FIRST INTERNATIONAL SCHOOL OF HADRON FEMTOGRAPHY

Jefferson Lab | September 16 - 25, 2024

The Center for Nuclear Femtography (CNF) and the Quark and Gluon Tomography (QGT) collaboration have joined forces to launch the First International School of Hadron Femtography. The school will take place at Jefferson Lab September 16-25, 2024. The program is designed to offer comprehensive lectures aimed at early-career experimental and theoretical scientists, including graduate students and post-doctoral researchers.

Acceptance to the program is through competitive application. Support will be provided for accepted participants, funded by CNF, supported by the Commonwealth of Virginia, and QGT, supported by the US Department of Energy. Participants will be housed on site at Jefferson Lab with ample opportunity for interactions with lecturers, and with other participants. Applications are now open, and for full consideration applications must be received by June 24, 2024.

Topics:
QCD Analysis - Theory & Experiment
Processes, DVCS, DVMP and multiparticle final states
Lattice QCD
Imaging Structure & Dynamics
GPD analysis as an Inverse problem
Experimental methodologies
AI for nuclear femtography

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