Rescuing collinear factorisation at high energy for heavy quarkonium photoproduction

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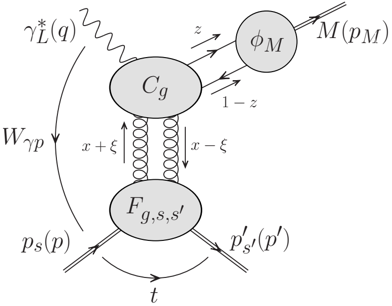
Exclusive photoproduction of vector quarkonia



+ similar diagram with quark GPDs, starting from NLO.

- Hard exclusive reaction, similar to DVMP, but not "deeply virtual" ($q^2 \simeq 0$, $\perp$ photon). The quarkonium (J/ψ, Υ) mass $M_Q^2 \gg \Lambda_{\mathrm{QCD}}^2$ provides the hard scale
- Experimental data on $\sigma(W_{\gamma p})$ and $d\sigma/dt$ are available from *ep*-collisions (JLAB, HERA, COMPASS) and UPCs (ALICE, CMS, LHCb)
- Collinear Factorisation(CF) is not proven to all orders for the case when $q^2 \simeq 0$, but **complete** NLO computation [Ivanov, Schaefer, Szymanowsky, Krasnikov, 2004] in CF was done and it formally works.

Quarkonium is treated **non-relativistically**, either using $\phi(z, k_T)$ obtained from Schrödinger wavefunction (only at LO and usually in the high-energy regime) or even resorting to the "static" approximation $\phi(z) \propto R(0)\delta(z-1/2)$, which corresponds to the strict LO in relative velocity of $Q\bar{Q}$ in the bound state (v^2).

Collinear factorisation

$$\mathcal{A} = -(\varepsilon_\mu^{*(\mathcal{Q})}\varepsilon_\nu^{(\gamma)} g_\perp^{\mu\nu}) \sum_{i=q,g} \int_{-1}^{1} \frac{dx}{x^{1+\delta_{ig}}} C_i\,(x,\xi)\, F_i(x,\xi,t,\mu_F),$$

CF coefficient function: $C_i = C_i^{(0)} + (\alpha_s(\mu_R)/\pi) C_i^{(1)} + \ldots$, with LO:

$$C_g^{(0)}\,(x,\xi) = \frac{x^2 c}{[x+\xi-i\varepsilon][x-\xi+i\varepsilon]},$$

where $c = (4\pi\alpha_s e e_Q R(0))/(m_Q^{3/2}\sqrt{2\pi N_c})$. $R_{J/\psi}(0) = 1\ \mathrm{GeV}^{3/2}$ and $R_\Upsilon(0) = 3\ \mathrm{GeV}^{3/2}$ from potential models and NLO decay widths.
In our calculation we use the complete NLO result for coefficient functions [Ivanov, Schafer, Szymanowsky, Krasnikov, 2004] .

GPDs:

$$F_{q,ss'} = \frac{1}{2}\int \frac{\mathrm{d}y^-}{2\pi} e^{ixP^+y^-} \langle p', s'|\bar{\psi}^q\left(\tfrac{-y}{2}\right)\gamma^+\psi^q\left(\tfrac{y}{2}\right)|p,s\rangle|_{y^+=y_\perp=0},$$

$$F_{g,ss'} = \frac{1}{P^+}\int \frac{\mathrm{d}y^-}{2\pi} e^{ixP^+y^-} \langle p', s'|F^{+\mu}\left(\tfrac{-y}{2}\right) F_\mu{}^{+}\left(\tfrac{y}{2}\right)|p,s\rangle|_{y^+=y_\perp=0},$$

are parametrised as $(j = g, q)$:

$$F_{j,ss'} = \frac{1}{2P^+}\left[\bar{u}_{s'}(p')\left(H_j\gamma^+ + E_j\frac{i\sigma^{+\Delta}}{2m_p}\right)u_s(p)\right].$$

GPD input

For numerical calcuations we use GPDs obtained as the result of **full LO GPD evolution** w.r.t. μ_F with initial condition at $\mu_0 = 2$ GeV, given by the double-distribution ansatz (without D-term):

$$H_i(x,\xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \, \delta(\beta + \xi\alpha - x) \, f_i(\beta,\alpha) \,,$$

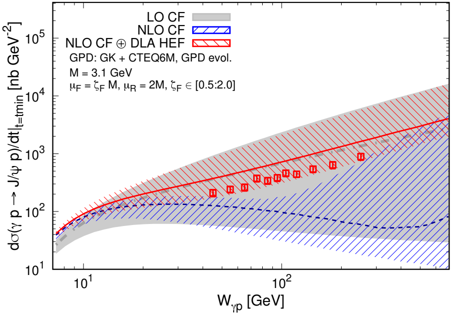
with the following model for DDs:

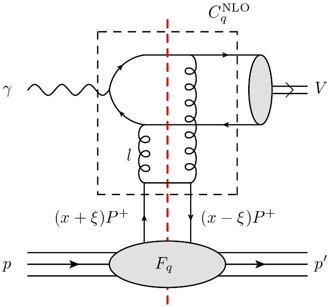
$$f_i(\beta,\alpha) = h_i(\beta,\alpha) \times \begin{cases} |\beta| g(|\beta|) & \text{for } i = g, \\ \theta(\beta) q_{\text{val}}(|\beta|) & \text{for valence } q, \\ \text{sgn}(\beta) q_{\text{sea}}(|\beta|) & \text{for sea } q. \end{cases}$$

where the profile function $h_i(\beta,\alpha) = \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)} \frac{\left((1-|\beta|)^2-\alpha^2\right)^{n_i}}{(1-|\beta|)^{2n_i+1}}$, with $n_g = n_q^{\text{sea}} = 2$ and $n_q^{\text{val}} = 1$ as in GK model. A range of values for n_g was tried with very small (few %) numerical effects on the cross section.

High-energy instability of NLO CF

The μ_F-dependence of the **LO** vs. **NLO** CF calculation:





The instability is caused by the high-partonic-energy ($\xi \ll |x| \lesssim 1$) DGLAP region [Ivanov, 2007] :

$$\int\limits_{\xi}^{1} \frac{dx}{x^2} F_g(x,\xi,\mu_F) C^{(1)}(x,\xi) \sim \int\limits_{\xi}^{1} \frac{dx}{x} = \ln\frac{1}{\xi}, \text{ if } F_g(x) \sim \text{const. and } C^{(1)} \sim x.$$

And for $\xi \ll x$ we actually have:

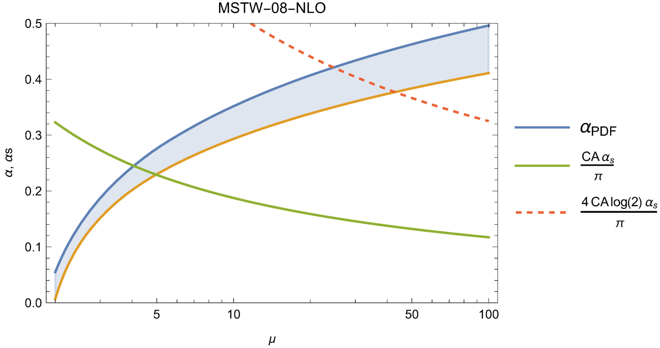
$$C^{(1)}_{g,q}(x,\xi) \sim -\frac{i\pi|x|}{2\xi} \ln\left(\frac{M_{\mathcal{Q}}^2}{4\mu_F^2}\right) \times \left\{ C_A, 2C_F \right\} \equiv C^{(1,\text{ asy.})}_{\{g,q\}}(x,\xi).$$

Partonic high-energy logarithms for low-scale processes

$$\sigma(x) \propto \int_0^1 \frac{dz}{z} C(z) \tilde{f}_g(x/z, \mu^2),$$

where $\tilde{f}_g(x,\mu^2) = x f_g(x,\mu^2)$.
Suppose for $z \ll 1$: $C(z) \sim \alpha_s^n(\mu) \ln^{n-1}(1/z)$ and $\tilde{f}_g(x,\mu^2) \sim x^{-\alpha(\mu)}$. Then for $x \ll 1$:

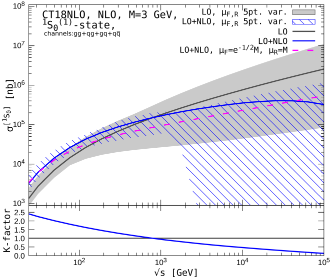
$$\sigma(x) \sim x^{-\alpha} \left(\frac{\alpha_s(\mu)}{\alpha(\mu)}\right)^n,$$



Digression: quarkonium total *inclusive* cross sections

Inclusive η_c-hadroproduction (CSM)

[Mangano *et.al.*, '97, ..., Lansberg, Ozcelik, '20]



$$p+p \to c\bar{c}\left[{}^1S_0^{[1]}\right]+X,\ \text{LO: } g(p_1)+g(p_2) \to c\bar{c}\left[{}^1S_0^{[1]}\right],$$

$$\sigma(\sqrt{s_{pp}}) = f_i(x_1,\mu_F) \otimes f_j(x_2,\mu_F) \otimes \hat{\sigma}(z),$$

where $z = \frac{M^2}{\hat{s}}$ with $\hat{s} = (p_1+p_2)^2$.

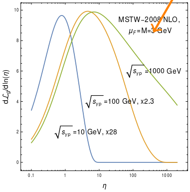
Inclusive J/ψ-photoproduction (CSM)

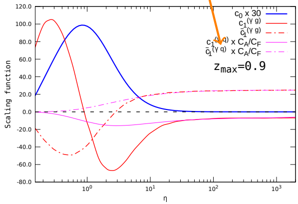
[Krämer, '96, ...,Colpani Serri *et.al*, '21]

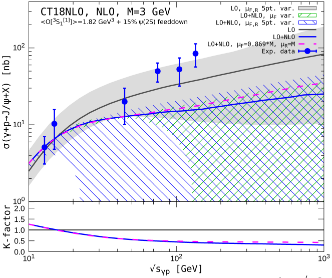
$$\gamma+p \to c\bar{c}\left[{}^3S_1^{[1]}\right]+X,\ \text{LO: } \gamma(q)+g(p_1) \to c\bar{c}\left[{}^3S_1^{[1]}\right]+g,$$

$$\sigma(\sqrt{s_{\gamma p}}) = f_i(x_1,\mu_F) \otimes \hat{\sigma}(\eta),$$

where $\eta = \frac{\hat{s}-M^2}{M^2}$ with $\hat{s} = (q+p_1)^2$, $z = \frac{pP}{qP}$.



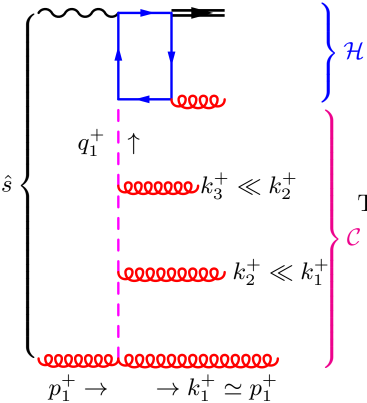




Digression: High-Energy Factorization (*inclusive* J/ψ photoproduction)

The **LLA** ($\sum_n \alpha_s^n \ln^{n-1}$) formalism [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91,'94]

Physical picture in the **LLA** for photoproduction:



The LLA in $\ln \frac{p_1^+}{q_1^+} \sim \ln(1+\eta)$:

$$\hat{\sigma}_{\rm HEF}^{\ln(1/\xi)}(\eta) \propto \int\limits_{1/z}^{1+\eta} \frac{dy}{y} \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(y, \mathbf{q}_{T1}^2),$$

The **strict LLA** in $\ln(1+\eta) = \ln \frac{\hat{s}}{M^2}$:

$$\hat{\sigma}_{\rm HEF}^{\ln(1+\eta)}(\eta) \propto \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{1}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \int\limits_{1/z}^\infty \frac{dy}{y} \mathcal{H}(y, \mathbf{q}_{T1}^2).$$

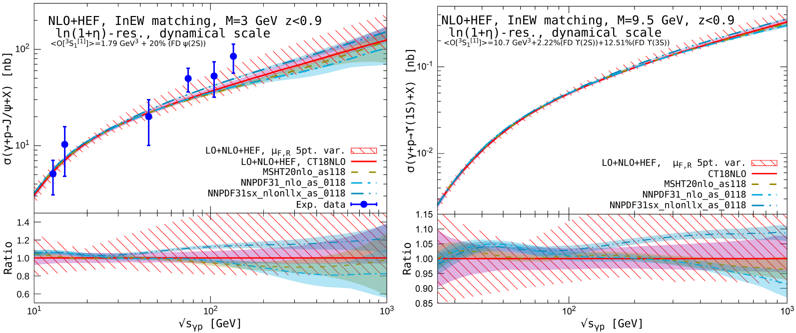
The coefficient function $\mathcal{H}$ has been calculated at LO [Kniehl, Vasin, Saleev, '06] and decreases as $1/y^2$ for $y \gg 1$.

Inclusive J/ψ photoproduction: NLO CF$\oplus$DLA HEF

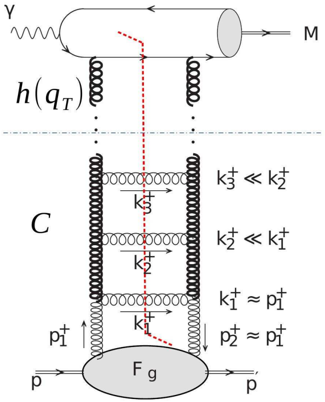
Matched results for J/ψ photoproduction can be further improved by noticing that in the LO process:

$$\gamma(q) + g(p_1) \to Q\bar{Q}\left[{}^3S_1^{[1]}\right] + g,$$

the emitted gluon can not be soft, so that $\langle\hat{s}\rangle_{\rm LO}$ (~ 25 GeV^2 at high $\sqrt{s_{\gamma p}}$ for J/ψ) rather than M^2 can be taken as a default value of μ_F^2 and μ_R^2:



Back to *exclusive* case: HEF for imaginary part



HEF-resummed result for the imaginary part in the DGLAP region [Ivanov, 2007] :

$$C_i^{(\mathrm{HEF})}(\rho) = \frac{-i\pi}{2}\,\frac{c}{|\rho|}\int\limits_0^\infty d\mathbf{q}_T^2\,\mathcal{C}_{gi}(|\rho|,\mathbf{q}_T^2)h(\mathbf{q}_T^2),$$

where $\rho = \xi/x$ and (in the LO in v^2 and α_s):

$$h(\mathbf{q}_T^2) = \frac{M_{\mathcal{Q}}^2}{M_{\mathcal{Q}}^2 + 4\mathbf{q}_T^2}.$$

LLA evolution w.r.t. $\ln 1/\rho$

In the LL($\ln 1/\rho$)-approximation, the $Y = \ln 1/\rho$-evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$-factor has the form:

$$\tilde{\mathcal{C}}_{gg}(\rho, \mathbf{q}_T) = \delta(1-\rho)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int\limits_{\xi}^{1} \frac{dz}{z} \int d^{2-2\epsilon}\mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2)\tilde{\mathcal{C}}_{gg}\left(\frac{\rho}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

$$K(\mathbf{k}_T^2, \mathbf{q}_T^2) = \frac{1}{\pi(2\pi)^{-2\epsilon}\mathbf{k}_T^2} + \delta^{(2-2\epsilon)}(\mathbf{k}_T)\ 2\omega_g(\mathbf{q}_T^2),$$

where $\omega_g(\mathbf{q}_T^2)$ – 1-loop Regge trajectory of a gluon. It is convenient to go from $(z, \mathbf{q}_T)$-space to $(N, \mathbf{x}_T)$-space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon}\mathbf{q}_T\ e^{i\mathbf{x}_T\mathbf{q}_T} \int\limits_0^1 dx\ x^{N-1}\ \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- Mellin convolutions over x turn into products
- Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^k \frac{x}{\xi} \to \frac{\alpha_s^{k+1}}{N^{k+1}}}$
- All *collinear divergences* are contained inside $\mathcal{C}$ in $\mathbf{x}_T$-space.

Exact LL solution and the DLA

In $(N, \mathbf{q}_T)$-space, subtracted $\mathcal{C}$, which resums all terms $\propto (\hat{\alpha}_s/N)^n$ (complete LLA) has the form [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91,'94]:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}(N, \alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, '82]:

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma),$$

where $\psi(\gamma) = d \ln \Gamma(\gamma)/d\gamma$ – Euler's ψ-function. The first few terms:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA [Blümlein, '95]}} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \ldots}_{\text{LLA}}$$

$\frac{\hat{\alpha}_s}{N} \leftrightarrow P_{gg}(z \to 0) = \frac{2C_A}{z} + \ldots$

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).$$

HEF-resummed coefficient function

Resummed coefficient function in N-space ($\gamma_N = \hat{\alpha}_s(\mu_R)/N$):

$$C_g^{(\mathrm{HEF})}(N) = \frac{-i\pi c}{2}\left(\frac{M_{\mathcal{Q}}^2}{4\mu_F^2}\right)^{\gamma_N}\frac{\pi\gamma_N}{\sin(\pi\gamma_N)}.$$

Resummed coefficient function in ρ-space:

$$\check{C}_g^{(\mathrm{HEF})}(\rho) = \frac{-i\pi c}{2}\frac{\hat{\alpha}_s}{|\rho|}\sqrt{\frac{L_\mu}{L_\rho}}\left\{I_1\left(2\sqrt{L_\rho L_\mu}\right) - 2\sum_{k=1}^{\infty}\mathrm{Li}_{2k}(-1)\left(\frac{L_\rho}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_\rho L_\mu}\right)\right\},$$

where $L_\rho = \hat{\alpha}_s \ln 1/|\rho|$ and $L_\mu = \ln[M_{\mathcal{Q}}^2/(4\mu_F^2)]$.
The $\rho \ll 1$-behaviour is governed by the singularity at $N = \hat{\alpha}_s$:

$$\check{C}_g^{(\mathrm{HEF})}(\rho) \sim \rho^{-\hat{\alpha}_s},$$

so the *hard Pomeron intercept* in DLA is $\hat{\alpha}_s$, not $4\hat{\alpha}_s \ln 2$ like in the full LLA.

Matching of the CF NLO and HEF-resummed coefficient functions

The $C_g^{(\mathrm{HEF})}(\rho)$ can be expanded in α_s (up to overall factor $-i\pi c/2$):

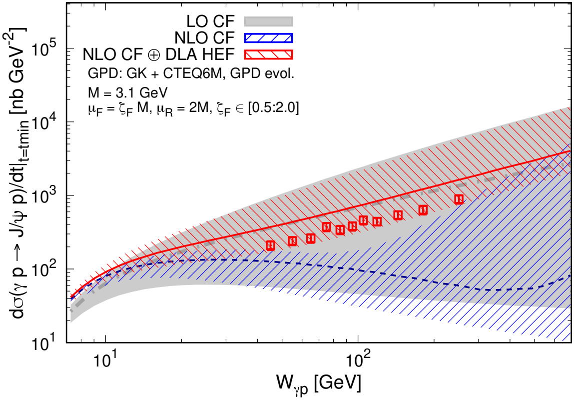
$$\underbrace{\delta(|\rho|-1)}_{\mathrm{LO}} + \underbrace{\frac{\hat{\alpha}_s}{|\rho|}\ln\left(\frac{M_{\mathcal{Q}}^2}{4\mu_F^2}\right)}_{=\frac{\alpha_s}{\pi}C_g^{(1,\ \mathrm{asy.})}(x,\xi)} + \frac{\hat{\alpha}_s^2}{|\rho|}\ln\frac{1}{|\rho|}\left[\frac{\pi^2}{6}+\frac{1}{2}\ln^2\left(\frac{M_{\mathcal{Q}}^2}{4\mu_F^2}\right)\right] + O(\alpha_s^3),$$

To avoid double-counting with NLO, we use the following *subtractive matching prescription*:

$$C_{g,q}^{(\mathrm{match.})}(x,\xi) = C_{g,q}^{(0)}(x,\xi) + \frac{\alpha_s(\mu_R)}{\pi}C_{g,q}^{(1)}(x,\xi)$$
$$+\left[\check{C}_{g,q}^{(\mathrm{HEF})}(\xi/|x|) - \frac{\alpha_s(\mu_R)}{\pi}C_{g,q}^{(1,\ \mathrm{asy.})}(x,\xi)\right]\theta(|x|-\xi).$$

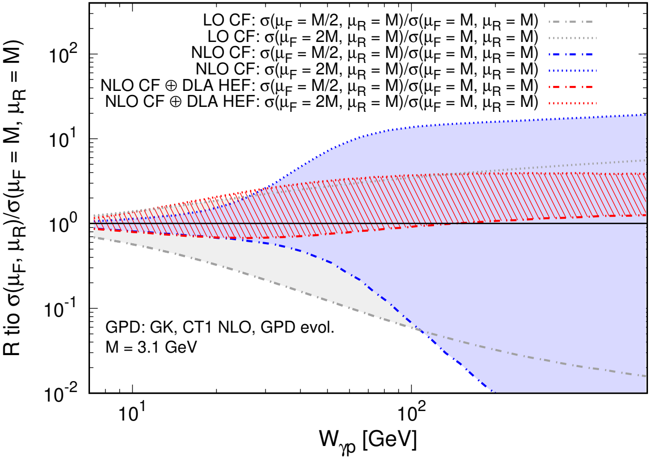
Numerical results, $\mu_R = 2M$, μ_F-variation

The μ_F-dependence of the **LO** vs. **NLO** CF and **NLO CF⊕DLA HEF matched** calculation:

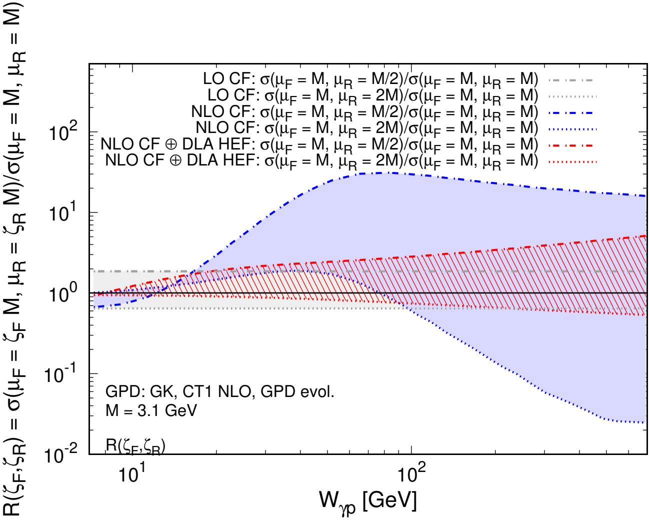


Points – H1 data on $d\sigma/dt$ at $t \simeq 0$.

μ_F-variation



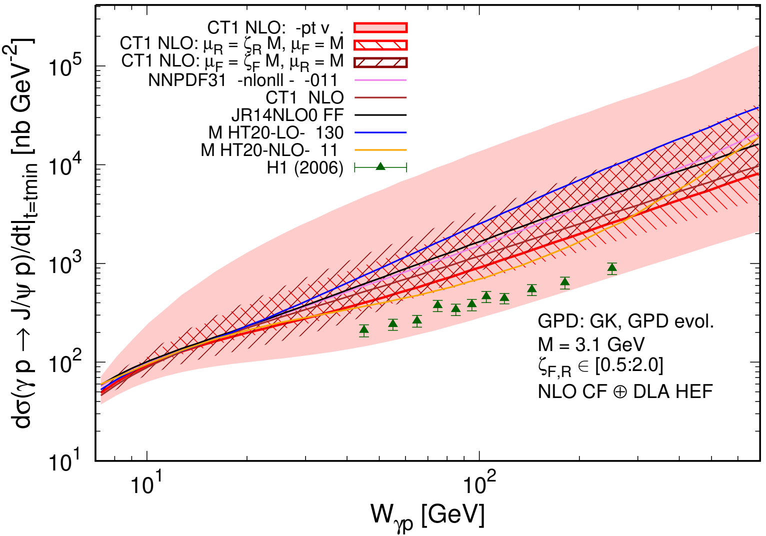
μ_R-variation



The increase of μ_R-variation of the matched result with energy is due to the μ_R-dependence of the $C_i^{(\mathrm{HEF})}(\rho) \sim \rho^{-\hat{\alpha}_s(\mu_R)}$ for $\rho \to 0$.

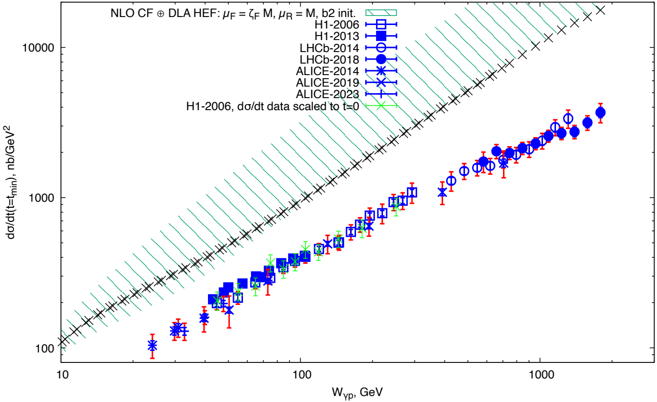
Exclusive J/ψ photoproduction in CF⊕HEF

9-point μ_F and μ_R variation:

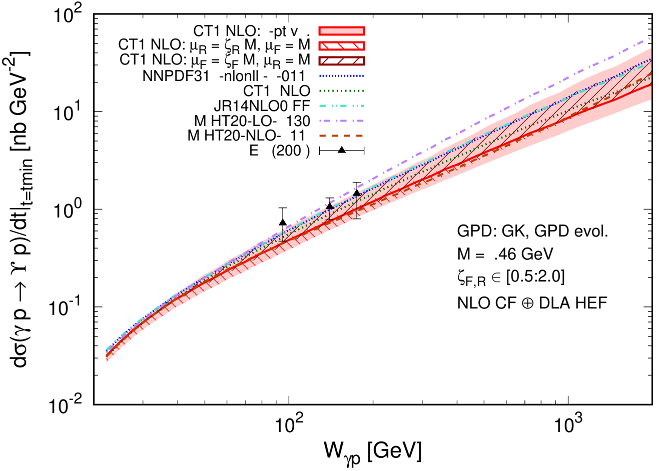


Exclusive J/ψ photoproduction in CF⊕HEF

Comparison to data on $d\sigma/dt(t_{\min})$, extrapolated from total cross section data at various energies:

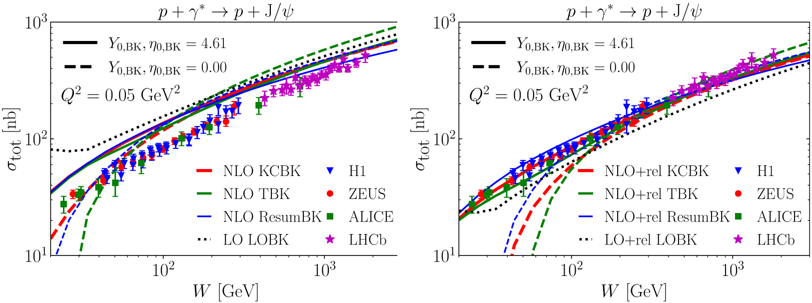


Results for $\Upsilon(1S)$



v^2-corrections to exclusive J/ψ photoproduction CGC

Plots from hep-ph/2204.14031 CGC calculation without and with $O(v^2)$-correction:



v^2-corrections in NRQCD

In NRQCD, the production amplitude is expanded in a series in relative momentum $\mathbf{k}^2 \sim v^2$, which is perturbatively matched on a series in terms of NRQCD operators:

$$\mathcal{A}_{\rm QCD}\left[\gamma + p \to Q\bar{Q}(\mathbf{k}) + p\right] = c_0 \langle Q\bar{Q}(\mathbf{k})|\psi^\dagger \sigma_i \chi|0\rangle + c_2 \langle Q\bar{Q}(\mathbf{k})|\psi^\dagger \sigma_i \left(-\mathbf{D}^2\right)\chi|0\rangle + \ldots,$$

in this way the coefficients c_0, c_2 are determined and then the state $|Q\bar{Q}(\mathbf{k})\rangle \to |\mathcal{Q}\rangle$:

$$\mathcal{A}\left[\gamma + p \to \mathcal{Q} + p\right] = c_0 \underbrace{\langle \mathcal{Q}|\psi^\dagger \sigma_i \chi|0\rangle}_{\propto R(0)+O(v^2)} + c_2 \underbrace{\langle \mathcal{Q}|\psi^\dagger \sigma_i \left(-\mathbf{D}^2\right)\chi|0\rangle}_{\propto \nabla_r^2 R(0)+O(v^2)} + \ldots,$$

Genuine many-body operators, such as: $\psi^\dagger \sigma_i \mathbf{E}\cdot\mathbf{D}\chi$, can also be included into this expansion.
The quantity:

$$\langle v^2\rangle = \frac{\langle \mathcal{Q}|\psi^\dagger \sigma_i \left(-\mathbf{D}^2\right)\chi|0\rangle}{m_Q \langle \mathcal{Q}|\psi^\dagger \sigma_i \chi|0\rangle} \simeq \frac{\nabla_r R(0)}{m_Q^2 R(0)},$$

is estimated [Bodwin, Kang, Lee, 2006] to be $\simeq 0.25$ for J/ψ and $\simeq 0.1$ for Υ.

Resummation and v^2-corrections

It makes sense to consider $\langle v^2\rangle$-corrections to the resummed coefficient function, because on the level of the full amplitude (C⊗GPDs):

$$\mathcal{A}(\xi) = \mathcal{A}^{(0,0)} + \alpha_s \mathcal{A}^{(1,0)} + \langle v^2\rangle \mathcal{A}^{(0,1)} + \sum_{k=1}^{\infty} \left[\alpha_s \ln \frac{1}{\xi}\right]^k \left(\mathcal{A}_k^{(\text{res.},0)} + \langle v^2\rangle \mathcal{A}_k^{(\text{res.},1)}\right) + \ldots,$$

and we have $\alpha_s(M_{J/\psi}) \simeq \langle v^2\rangle \simeq 0.25$, while $\alpha_s \ln(1/\xi) \sim 1$.

The HEF coefficient function can be expanded in $\langle v^2\rangle$:

$$h(\mathbf{q}_T^2) = h^{(0)}(\mathbf{q}_T^2) + \langle v^2\rangle h^{(1)}(\mathbf{q}_T^2) + \ldots,$$

$$h^{(0)}(\mathbf{q}_T^2) = \frac{M^2}{M^2 + 4\mathbf{q}_T^2},$$

$$h^{(1)}(\mathbf{q}_T^2) = \frac{M^2\left(31M^4 + 104M^2\mathbf{q}_T^2 + 176(\mathbf{q}_T^2)^2\right)}{12\left(M^2 + 4\mathbf{q}_T^2\right)^3},$$

and the $O(v^2)$ correction to the resummed CF coefficient function in Mellin space is:

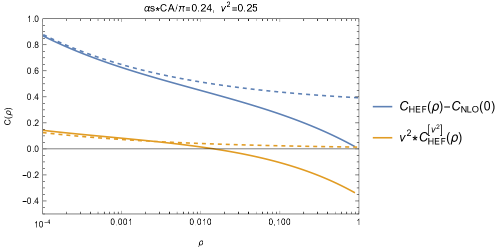
$$C_i^{(\text{HEF},\ v^2)}(N) = \frac{1}{12}\left(4\gamma_N(2\gamma_N - 7) + 31\right) \times C_i^{(\text{HEF},\ v^0)}(N).$$

Resummation and v^2-corrections

$$\frac{2}{-i\pi c} C_i^{(\mathrm{HEF},\ v^0)}(\rho) = \delta(|\rho|-1) + \frac{\hat{\alpha}_s}{|\rho|} L_\mu + \frac{\hat{\alpha}_s^2}{|\rho|} \ln \frac{1}{|\rho|} \left(\frac{L_\mu^2}{2} + \frac{\pi^2}{6} \right) + O(\alpha_s^3),$$

$$\frac{2}{-i\pi c} C_i^{(\mathrm{HEF},\ v^2)}(\rho) = \frac{31}{12} \delta(|\rho|-1) + \frac{\hat{\alpha}_s}{|\rho|} \left(\frac{31}{12} L_\mu - \frac{7}{3} \right)$$
$$+ \frac{\hat{\alpha}_s^2}{|\rho|} \ln \frac{1}{|\rho|} \left(\frac{31}{24} L_\mu^2 - \frac{7}{3} L_\mu + \frac{2}{3} + \frac{31\pi^2}{72} \right) + O(\alpha_s^3),$$

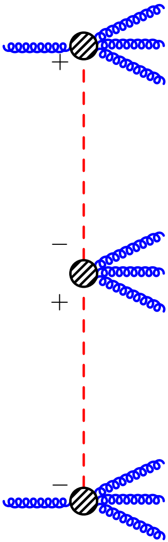
where $L_\mu = \ln[M^2/(4\mu_F^2)]$. Correction to the LO in α_s is $+31/12\langle v^2 \rangle$, however the correction to the resummed coefficient funciton has a negative part:



So the relativistic correction does not behave as just the overall factor.

Towards the Next-to-DLA

The Gauge-Invariant EFT for Multi-Regge processes in QCD



- Reggeized gluon fields $R_\pm$ carry $(k_\pm, \mathbf{k}_T, k_\mp = 0)$: $\partial_\mp R_\pm = 0$.
- **Induced interactions** of particles and Reggeons [Lipatov '95, '97; Bondarenko, Zubkov '18]:

$$L = \frac{i}{g_s} \mathrm{tr}\left[R_+ \partial_\perp^2 \partial_- \left(W[A_-] - W^\dagger [A_-] \right) + (+ \leftrightarrow -) \right],$$

with $W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] = P \exp\left[\frac{-ig_s}{2} \int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T) \right] = \left(1 + ig_s \partial_\pm^{-1} A_\pm\right)^{-1}$.

- Expansion of the Wilson line generates **induced vertices**:

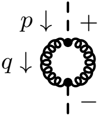
$$\mathrm{tr}\left[R_+ \partial_\perp^2 A_- + (-ig_s)(\partial_\perp^2 R_+)(A_- \partial_-^{-1} A_-) \right.$$
$$\left. + (-ig_s)^2 (\partial_\perp^2 R_+)(A_- \partial_-^{-1} A_- \partial_-^{-1} A_-) + O(g_s^3) + (+ \leftrightarrow -) \right].$$

- The *Eikonal propagators* $\partial_\pm^{-1} \to -i/(k^\pm)$ lead to **rapidity divergences**, which are regularized by tilting the Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis *et. al.*, '12-'13; M.N. '19]:

$$n_\pm^\mu \to \tilde{n}_\pm^\mu = n_\pm^\mu + r n_\mp^\mu,\ r \ll 1:\ \tilde{k}^\pm = \tilde{n}^\pm k.$$

The terms for conversion of the result into any other regularisation scheme for RDs can be easily computed.

Rapidity divergences and regularization.

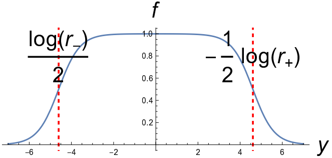


$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}, \quad \int \frac{dq^+ dq^- (\dots)}{q^+ q^-} = \int\limits_{y_1}^{y_2} dy \int \frac{dq^2 (\dots)}{q^2 + \mathbf{q}_T^2}$$

the regularization by explicit cutoff in rapidity was originally proposed [Lipatov, '95] ($q^\pm = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

The square of regularized Lipatov vertex:



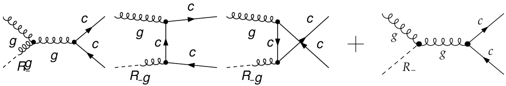
$$\Gamma_{+\mu-} \Gamma_{+\nu-} P^{\mu\nu} = \frac{16 \mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y} + e^{y})(re^{y} + e^{-y})},$$

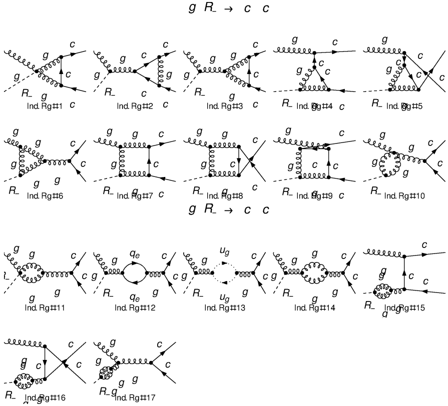
$$\int\limits_{-\infty}^{+\infty} dy \; f(y) = -\ln r + O(r)$$

$Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ and $c\bar{c}\left[{}^3S_1^{[8]}\right]$ @ 1 loop

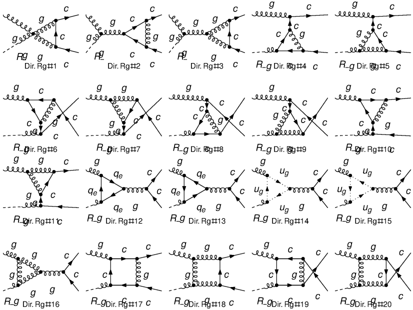
Interference with LO:



Induced Rgg coupling diagrams:



Some Rg-coupling diagrams:



and so on...

- Diagrams had been generated using custom **`FeynArts`** model-file, projector on the $c\bar{c}\left[{}^1S_0^{[1]}\right]$-state is inserted
- heavy-quark momenta $= p_{\mathcal{Q}}/2 \Rightarrow$ need to resolve linear dependence of quadratic denominators in some diagrams before IBP
- IBP reduction to master integrals has been performed using **`FIRE`**
- Master integrals with linear and massless quadratic denominators are expanded in $r \ll 1$ using Mellin-Barnes representation. The differential equations technique is used when the integral depends on more than one scale of virtuality.
- In presence of the linear denominator the massive propagator can be converted to the massless one:

$$\frac{1}{((\tilde{n}_+ l)+k_+)(l^2-m^2)} = \frac{1}{((\tilde{n}_+ l)+k_+)(l+\kappa\tilde{n}_+)^2} + \frac{2\kappa\left[(\tilde{n}_+ l) + \frac{m^2+\tilde{n}_+^2\kappa^2}{2\kappa}\right]}{((\tilde{n}_+ l)+k_+)(l+\kappa\tilde{n}_+)^2(l^2-m^2)}$$

$\Rightarrow$ all the masses can be moved to integrals with **only quadratic propagators**.

Result: $Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ @ 1 loop

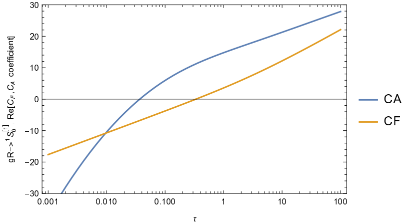
Result [MN, '23] for $2\Re\left[\frac{H_{\mathbf{1L\ x\ LO}}(\mathbf{q}_T) - \mathbf{(On\text{-}shell\ mass\ CT)}}{(\alpha_s/(2\pi))H_{\rm LO}(\mathbf{q}_T)}\right]$:

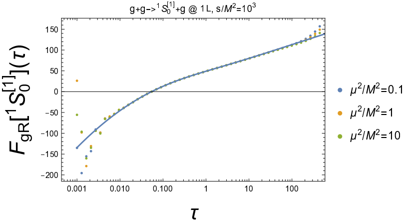
$$ {}^1S_0^{[1]} : \left(\frac{\mu^2}{\mathbf{q}_T^2}\right)^{\epsilon}\left\{-\frac{N_c}{\epsilon^2} + \frac{1}{\epsilon}\left[N_c\left(\ln\frac{q_-^2}{\mathbf{q}_T^2 r} + \frac{25}{6}\right) - \frac{2n_F}{3} - \frac{3}{2N_c}\right]\right\} + F_{{}^1S_0^{[1]}}(\mathbf{q}_T^2/M^2) $$

$$ F_{{}^1S_0^{[1]}}(\tau) = -\frac{10}{9}n_F + \Re[C_F F^{(C_F)}_{{}^1S_0^{[1]}}(\tau) + C_A F^{(C_A)}_{{}^1S_0^{[1]}}(\tau)], $$

$$ F^{(C_F)}_{{}^1S_0^{[1]}}(\tau) = F^{(C_F)}_{{}^1S_0^{[8]}}(\tau), $$

while $F^{(C_A)}_{{}^1S_0^{[1]}}(\tau) \neq F^{(C_A)}_{{}^1S_0^{[8]}}(\tau)$.





Role of $\ln \mathbf{q}_T^2$-corrections in the matching

Next-to-DLA coefficient function contains:

$$C_i^{\rm HEF}(\rho) \supset \int\limits_0^\infty d\mathbf{q}_T^2 \mathcal{C}_{gi}^{(\rm DLA)}(\rho, \mathbf{q}_T^2, \mu_F^2, \mu_R^2) \left[h^{(\rm LO)}(\mathbf{q}_T^2) + \frac{\alpha_s}{2\pi} h^{(\rm NLO)}(\mathbf{q}_T^2) + \ldots \right],$$

suppose $h^{(\rm NLO)}(\mathbf{q}_T^2) \sim \ln^n(\mathbf{q}_T^2)$ for $\mathbf{q}_T^2 \ll M_{\mathcal{Q}}^2$ with $n = 1, 2$. In N-space ($\gamma_N = \hat{\alpha}_s / N$):

$$\int\limits_0^{\mu_F^2} d\mathbf{q}_T^2\, \mathcal{C}^{(\rm DLA)}(N, \mathbf{q}_T^2, \mu_F^2) \times \hat{\alpha}_s \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2} = \hat{\alpha}_s \gamma_N \int\limits_0^{\mu_F^2} \frac{d\mathbf{q}_T^2}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_N} \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2}$$

$$= \hat{\alpha}_s \frac{(-1)^n n!}{\gamma_N^n} = \begin{cases} -N & \text{for } n = 1 \\ \frac{2N^2}{\hat{\alpha}_s} & \text{for } n = 2 \end{cases} \xrightarrow[\text{Mellin transform}]{} \begin{cases} -\delta'(|\rho| - 1) & \text{for } n = 1 \\ \frac{2}{\hat{\alpha}_s} \delta''(|\rho| - 1) & \text{for } n = 2 \end{cases}$$

So these contributions *do not belong to NLA and will be removed in the matching procedure!*

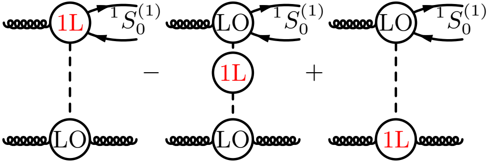
Conclusions and outlook

- The perturbative instability of NLO CF computation for vector quarkonium photoproduction comes from the high partonic energy region $\xi \ll x \lesssim 1$
- It can be resolved by the resummation of higher-order corrections $\sim \alpha_s^n \ln^{n-1} |x/\xi|$ (LLA) in the coefficient function, using High-Energy Factorisation(HEF)
- The DLA is the truncation of LLA, appropriate for the use together with fixed-order GPD evolution
- The matched NLO CF$\oplus$DLA HEF results agrees with data within very large scale uncertainty
- **This results confirm that there is no pathology in the leading-twist CF computation and it can be safely used e.g. in the low-energy region**
- Study of $O(v^2)$ (“relativistic”) corrections and Next-to-DLA corrections is underway

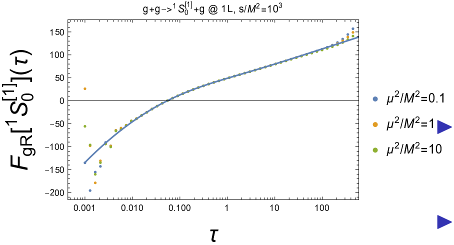
Thank you for your attention!

$Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ @ 1 loop, cross-check

In the combination of 1-loop results in the EFT:



the $\ln r$ cancels and it should reproduce the the Regge limit($s \gg -t$) of the *real part* of the $2 \to 2$ 1-loop QCD amplitude:



$\tau = -t/M^2$

$$g + g \to c\bar{c}\left[{}^1S_0^{(1)}\right] + g.$$

- The $2 \to 2$ QCD 1-loop amplitude can be computed numerically using `FormCalc` (with some tricks, due to Coulomb divergence)
- The Regge limit of $1/\epsilon$ divergent part agrees with the EFT result
- For the finite part agreement within few % is reached, need to push to higher s

Non-relativistic QCD

The velocity-expansion for quarkonium eigenstate is carbon-copy of corresponding arguments from atomic physics (hierarchy of E-dipole/M-dipole with ΔS/M-dipole transitions):

$$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots, \end{aligned}$$

for validity of this arguments, we should work in *non-relativistic EFT*, dynamics of which conserves number of heavy quarks. In such EFT, $Q\bar{Q}$-pair is produced in a point, by local operator:

$$\mathcal{A}_{\rm NRQCD} = \langle J/\psi + X|\, \chi^\dagger(0)\kappa_n\psi(0)\,|0\rangle\,,$$

Different operators "couple" to different Fock states:

$$\chi^\dagger(0)\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1S_0^{(1)}\right]\right\rangle,\ \chi^\dagger(0)\sigma_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle,$$

$$\chi^\dagger(0)\sigma_i T^a\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(8)}\right]\right\rangle,\ \chi^\dagger(0)D_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1P_1^{(8)}\right]\right\rangle, \ldots$$

squared NRQCD amplitude (=LDME):

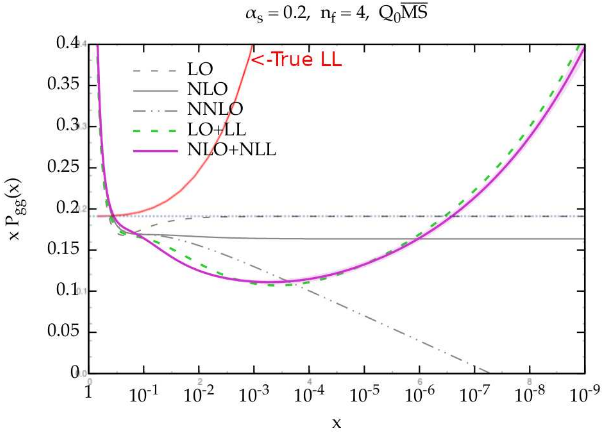
$$\sum_X |\mathcal{A}|^2 = \langle 0|\underbrace{\psi^\dagger\kappa_n^\dagger\chi a^\dagger_{J/\psi}a_{J/\psi}\chi^\dagger\kappa_n\psi}_{\mathcal{O}_n^{J/\psi}}|0\rangle = \left\langle \mathcal{O}_n^{J/\psi}\right\rangle,$$

Backup: DGLAP P_{gg} at small z

LO: $P_{gg}(z) = \frac{2C_A}{z} + \ldots \Leftrightarrow \gamma_N = \frac{\hat{\alpha}_s}{N}$

Plot from hep-ph/1607.02153 with my curve (in red) for the **strict LLA:**

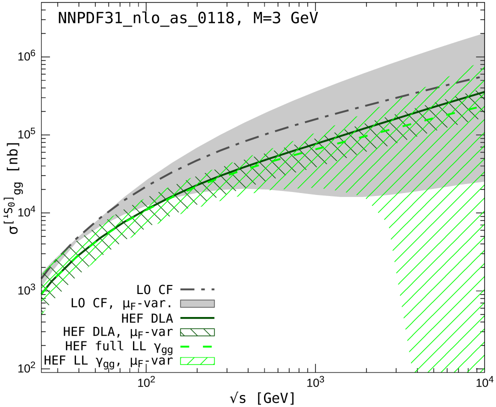
$\frac{\hat{\alpha}_s}{N}\chi_{LO}(\gamma_{gg}(N)) = 1 \Rightarrow \gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$



The "LO+LL" and "NLO+NLL" curves represent a form of matching between DGLAP and BFKL expansions, in a scheme by Altarelli, Ball and Forte which is more complicated than the **strict LL or NLL approximation**.

Effect of anomalous dimension beyond LO

Effect of taking **full LLA** for $\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$ together with NLO PDF.



Scale-fixing solution

Studied in [Lansberg, Ozcelik, 20'], [Lansberg *et.al*, 21']. For J/ψ **photoproduction**:

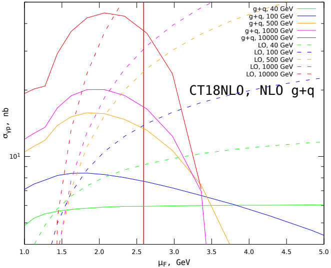
$$\frac{d\sigma_{\gamma p}^{(\mathrm{LO+NLO})}}{d\ln\mu_F^2} \propto \left(\frac{\alpha_s}{2\pi}\right)^2 \int\limits_0^{\eta_{\max}} d\eta \left\{ \ln(1+\eta) \left[c_1(\eta\to\infty) + \bar{c}_1(\eta\to\infty)\ln\frac{M^2}{\mu_F^2} \right] \right.$$
$$\left. \times \left(f_g(x_\eta,\mu_F^2) + \frac{C_F}{C_A} f_q(x_\eta,\mu_F^2) \right) + \text{non-singular terms at } \eta \gg 1 \right\}$$

"principle of minimal scale-sensitivity" $\Rightarrow$ for J/ψ **photoproduction**:

$$\hat{\mu}_F = M \exp\left[\frac{c_1(\eta\to\infty)}{2\bar{c}_1(\eta\to\infty)}\right] \simeq 0.87M,$$

for η_c-hadroproduction:

$$\hat{\mu}_F = M \exp\left[\frac{A_1}{2}\right] = \frac{M}{\sqrt{e}} \simeq 0.61M.$$



The $\hat{\mu}_F$-scale removes corrections $\propto \alpha_s^n \ln^{n-1}(1+\eta)$ from $\hat{\sigma}_i(\eta)$ and resums them into PDFs. But is such resummation complete?

Quarkonium in the potential model

Cornell potential:

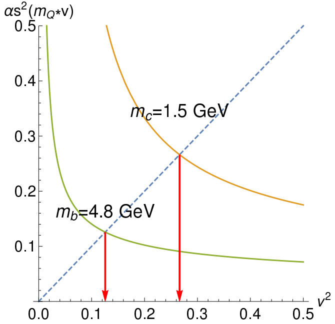
$$V(r) = -C_F \frac{\alpha_s(1/r)}{r} + \sigma r,$$

neglect linear part, because quarkonium is "small" (~ 0.3 fm) $\rightarrow$ Coulomb wavefunction (for effective mass $\frac{m_1 m_2}{m_1+m_2} = \frac{m_Q}{2}$):

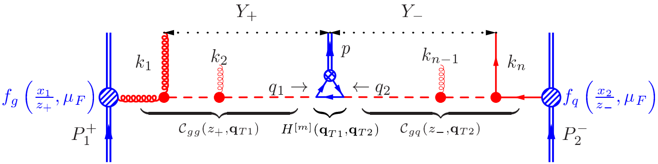
$$R(r) = \frac{\sqrt{m_Q^3 \alpha_s^3 C_F^3}}{2} e^{-\frac{\alpha_s C_F}{2} m_Q r}$$

$$\langle v^2 \rangle = \frac{C_F^2 \alpha_s^2}{2}, \langle r \rangle = \frac{3}{2C_F} \frac{1}{m_Q v}$$

$$\Rightarrow \boxed{\alpha_s^2(m_Q v) \simeq v^2}$$



High-Energy Factorization (η_c hadroproduction)



Small parameter: $z = \frac{M^2}{\hat{s}}$, LLA in $\alpha_s^n \ln^{n-1} \frac{1}{z}$:

$$\hat{\sigma}_{ij}^{[m],\ \mathrm{HEF}}(z, \mu_F, \mu_R) = \int\limits_{-\infty}^{\infty} d\eta \int\limits_{0}^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2\, \mathcal{C}_{gi}\left(\frac{M_T}{M}\sqrt{z}e^{\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right)$$

$$\times \mathcal{C}_{gj}\left(\frac{M_T}{M}\sqrt{z}e^{-\eta}, \mathbf{q}_{T2}^2, \mu_F, \mu_R\right) \int\limits_{0}^{2\pi} \frac{d\phi}{2} \frac{H^{[m]}(\mathbf{q}_{T1}^2, \mathbf{q}_{T2}^2, \phi)}{M_T^4}$$

The coefficient functions $H^{[m]}$ are known at LO in α_s [Hagler *et.al*, 2000; Kniehl, Vasin, Saleev 2006] for $m = {}^1S_0^{(1,8)}, {}^3P_J^{(1,8)}, {}^3S_1^{(8)}$.
The $H^{[m]}$ is a tree-level "squared matrix element" of the $2 \rightarrow 1$-type process:

$$R_+(\mathbf{q}_{T1}, q_1^+) + R_-(\mathbf{q}_{T2}, q_2^-) \rightarrow c\bar{c}[m].$$

Fixed-order asymptotics from HEF

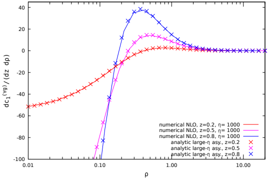
When expanded up to $O(\alpha_s)$ the HEF resummation should predict the $\hat{s} \gg M^2$ asymptotics of the CF coefficient function $\hat{\sigma}$

For the $g + g \to c\bar{c}\left[{}^1S_0^{(1)}, {}^3P_0^{(1)}, {}^3P_2^{(1)}\right]$ the NLO and NNLO($\alpha_s^2 \ln(1/z)$) terms in $\hat{\sigma}$ are predicted [M.N., Lansberg, Ozcelik '22]:

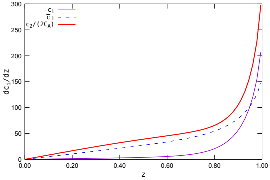
State	$A_0^{[m]}$	$A_1^{[m]}$	$A_2^{[m]}$	$B_2^{[m]}$
${}^1S_0$	1	$\mathbf{-1}$	$\frac{\pi^2}{6}$	$\frac{\pi^2}{6}$
${}^3S_1$	0	1	0	$\frac{\pi^2}{6}$
${}^3P_0$	1	$\mathbf{-\frac{43}{27}}$	$\frac{\pi^2}{6} + \frac{2}{3}$	$\frac{\pi^2}{6} + \frac{40}{27}$
${}^3P_1$	0	$\frac{5}{54}$	$-\frac{1}{9}$	$-\frac{2}{9}$
${}^3P_2$	1	$\mathbf{-\frac{53}{36}}$	$\frac{\pi^2}{6} + \frac{1}{2}$	$\frac{\pi^2}{6} + \frac{11}{9}$

$$\hat{\sigma}_{gg}^{[m]}(z \to 0) = \sigma_{\rm LO}^{[m]} \left\{ A_0^{[m]} \delta(1-z) + \frac{\alpha_s}{\pi} 2C_A \left[A_1^{[m]} + A_0^{[m]} \ln \frac{M^2}{\mu_F^2} \right] + \left(\frac{\alpha_s}{\pi}\right)^2 \ln\frac{1}{z} \cdot C_A^2 \left[2A_2^{[m]} + B_2^{[m]} + 4A_1^{[m]} \ln\frac{M^2}{\mu_F^2} + 2A_0^{[m]} \ln^2 \frac{M^2}{\mu_F^2} \right] + O(\alpha_s^3) \right\},$$

For the $\gamma + g \to c\bar{c}\left[{}^3S_1^{(1)}\right] + g$ we have computed $\eta \to \infty$ limit of the z and $\rho = \mathbf{p}_T^2/M^2$-differential NLO "scaling functions" in closed analytic form,



and obtained numerical results for NNLO "scaling function" c_2 in front of $\alpha_s \ln(1+\eta)$.



Inverse Error Weighting (InEW) matching

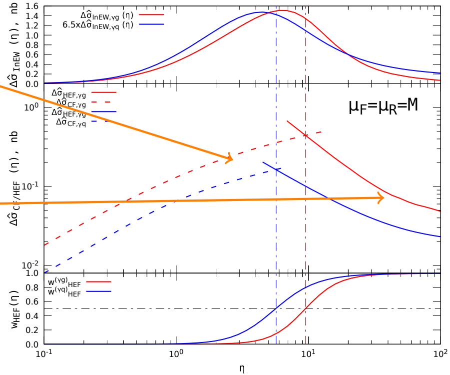
Development of an idea from [Echevarria *et al.*, 18'] :

$$\hat{\sigma}(\eta) = w_{\rm CF}(\eta)\hat{\sigma}_{\rm CF}(\eta) + (1 - w_{\rm CF}(\eta))\hat{\sigma}_{\rm HEF}(\eta),$$

the weights are determined through the estimates of "errors":

$$w_{\rm CF}(\eta) = \frac{\Delta\hat{\sigma}^{-2}_{\rm CF}(\eta)}{\Delta\hat{\sigma}^{-2}_{\rm CF}(\eta) + \Delta\hat{\sigma}^{-2}_{\rm HEF}(\eta)}, \quad w_{\rm HEF}(\eta) = 1 - w_{\rm CF}(\eta).$$

- $\Delta\hat{\sigma}_{\rm CF}(\eta)$ is due to **missing higher orders and large logarithms,** it can be estimated from the α_s expansion of $\hat{\sigma}_{\rm HEF}(\eta)$.
- $\Delta\hat{\sigma}_{\rm HEF}(\eta)$ is (mostly) due to **missing power corrections in** $1/\eta$: $\Delta\hat{\sigma}_{\rm HEF}(\eta) \sim A\eta^{-\alpha_{\rm HEF}}$. We determine A and $\alpha_{\rm HEF}$ from behaviour of $\hat{\sigma}_{\rm CF}(\eta) - \hat{\sigma}_{\rm CF}(\infty)$ at $\eta \gg 1$.

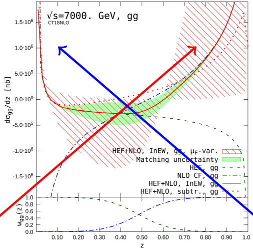


Matching with NLO

The HEF is valid in the **leading-power** in $M^2/\hat{s}$, so for $\hat{s} \sim M^2$ we match it with NLO CF by the *Inverse-Error Weighting Method* [Echevarria *et.al.*, 18'].

η_c**-hadroproduction**,

$z = M^2/\hat{s}$:



NLO **HEF**

J/ψ**-photoproduction**,

$\eta = (\hat{s} - M^2)/M^2$: