HELMHOLTZ
Helmholtz-Institut Mainz

Proton electromagnetic FF from lattice QCD

Dalibor Djukanovic

HELMHOLTZ
Helmholtz-Institut Mainz

Proton electromagnetic FF from lattice QCD

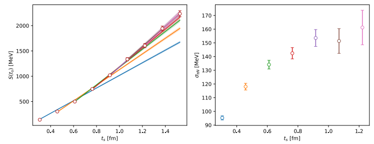
Mainz

Dalibor Djukanovic

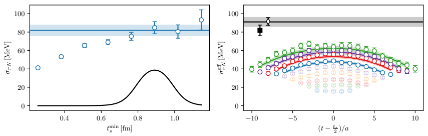
Outline

- Basically summary of the results published this year

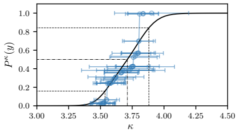
  D.D. et al. *Phys.Rev.Lett.* 132 (2024) 21, 21 e-Print: 2309.07491
  D.D. et al. *Phys.Rev.D* 109 (2024) 9, 9 e-Print: 2309.06590
  D.D. et al. *Phys.Rev.D* 110 (2024) 1, 1 e-Print: 2309.17232



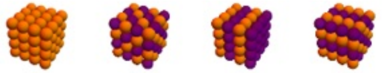
- Which draws on Methods/Results established earlier



  Agadjanov, D.D. et al. *Phys.Rev.Lett.* 131 (2023) 26, 26 e-Print: 2303.08741
  D.D. et al. *Phys.Rev.D* 106 (2022) 7, 074503 e-Print: 2207.03440
  D.D. et al. *Phys.Rev.D* 103 (2021) 9, 094522 e-Print: 2102.07460
  D.D. et al. *Phys.Rev.Lett.* 123 (2019) 21, 212001 e-Print: 1903.12566



- Which started much earlier (Nf=2)



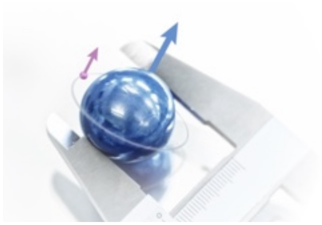
  Capitani, *D.D.* et al. *Phys.Rev.D* 92 (2015) 5, 054511 e-Print: 1504.04628

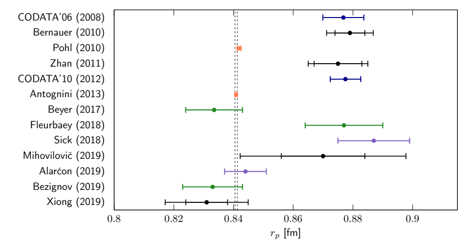
- Based on ensemble (start) generation in

  Bruno, *D.D.* et al. *JHEP* 02 (2015) 043 e-Print: 1411.3982

Impact of LQCD for Form Factors

- Proton Radius Puzzle

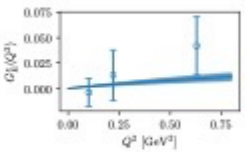




Taken from Bernauer,
EPJ Web Conf. 234 (2020)

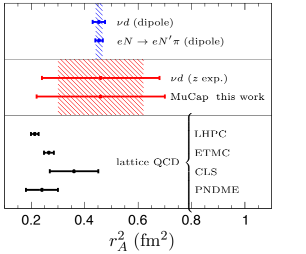
  - Provide ab-initio calculation

- Precision Tests of SM

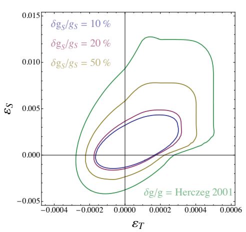


Taken from D.D. et al.
Phys.Rev.Lett. 123 (2019) 21, 212001

  - Via strangeness FF → Parity Violation Experiments
    - Lattice determinations of strange FF very precise
  - Via axial FF → Vital input to neutrino-nucleus scattering
    - Lattice competitive to z-exp extractions of experiments



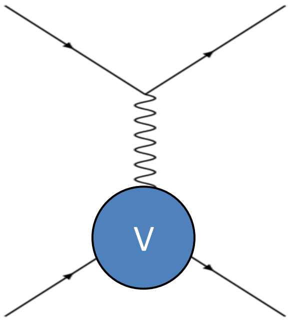
Taken from A. Kronfeld, et al.
Eur. Phys. J. A **55,** 196 (2019)



Taken from T. Bhattacharya et al.,
Phys. Rev. D85, 054512 (2012)

  - Via Charges → Constraining BSM EFT couplings

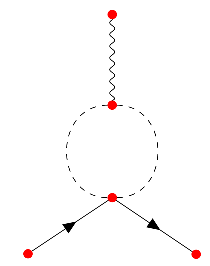
Nucleon Form Factors

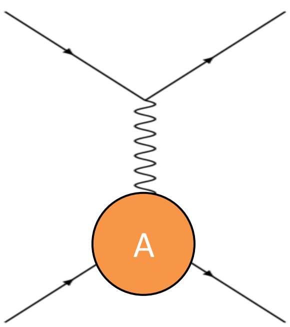


$$\langle N(p')|\bar{q}\gamma_\mu q|N(p)\rangle$$
$$= \bar{u}(p')\left[\gamma_\mu F_1^q(Q^2=-q^2) + i\,\frac{\sigma_{\mu\nu}}{2m_N}\,q^\nu F_2^q(Q^2)\right]u(p)$$
$$G_E(Q^2) = F_1^q(Q^2) - \frac{Q^2}{2m_N}F_2^q(Q^2) = \left(1 - \frac{1}{6}\langle r_E^2\rangle\, Q^2 + \cdots\right)$$
$$G_M(Q^2) = F_1^q(Q^2) + F_2^q(Q^2) = \mu\left(1 - \frac{1}{6}\langle r_M^2\rangle\, Q^2 + \cdots\right)$$

Dirac Pauli

$$\langle r_{E/M}^2\rangle \xrightarrow{M_\pi\to 0} \{\ln M_\pi, \frac{1}{M_\pi}\}$$

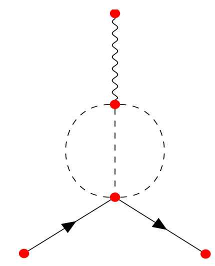




$$\langle N(p')|\bar{q}\gamma_\mu\gamma_5 q|N(p)\rangle$$
$$= \bar{u}(p')\left[\gamma_\mu G_A(Q^2) + i\,\frac{q^\mu}{2m_N}\,G_P(Q^2)\right]\gamma_5\frac{\tau^3}{2}u(p)$$
$$G_A(Q^2) = g_A\left(1 - \tfrac{1}{6}\langle r_A^2\rangle\, Q^2 + \cdots\right)$$

$$2m_N G_A(Q^2) - \frac{Q^2}{2\,m_N}G_P(Q^2) = \frac{2\,M_\pi^2 F_\pi}{M_\pi^2+Q^2}G_{\pi N}(Q^2)$$ (PCAC)

$$\langle r_A^2\rangle \xrightarrow{M_\pi\to 0} const.$$



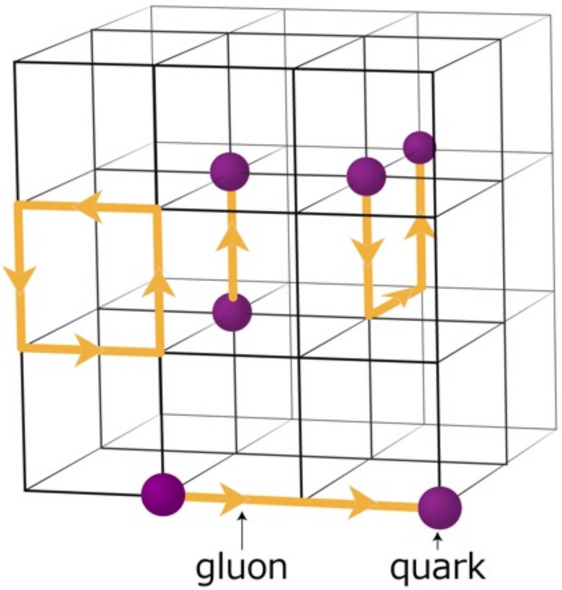
Lattice – Oneslide Intro

- Discretize Space Time
- Pathintgeral rotated to Euclidean time
- Lattice action

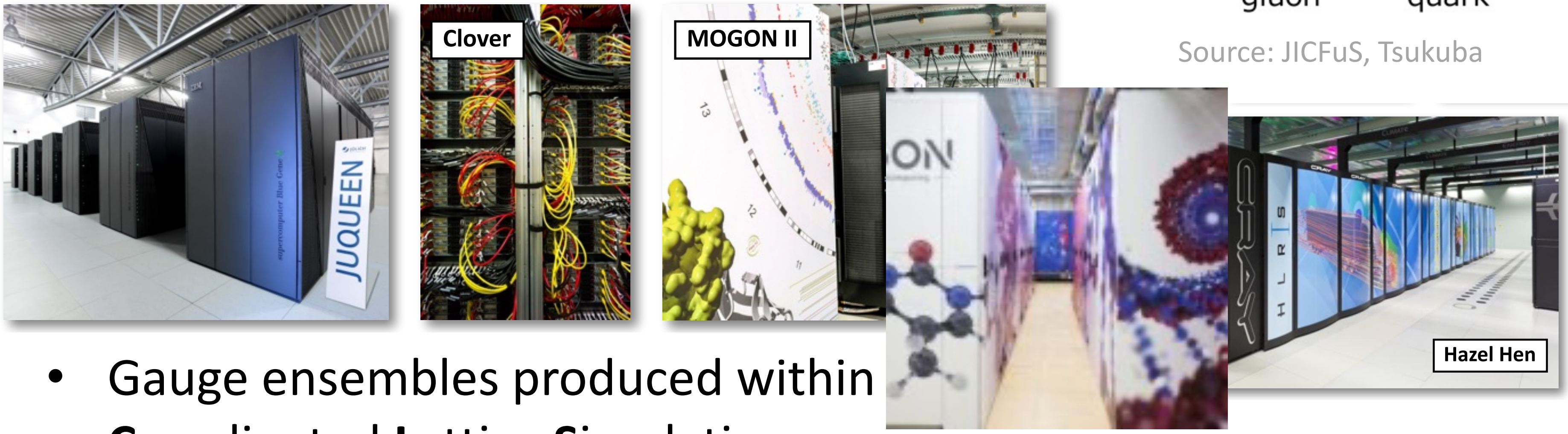
$$S^{Lat}[U,\Psi,\bar{\Psi}] = S_G^{Lat}[U] + S_F^{Lat}[U,\Psi,\bar{\Psi}]$$

$$\langle\Omega\rangle = \frac{1}{Z}\int \prod_{x,\mu} dU_\mu(x)\,\Omega \prod_{f=u,d,s}\det[D+m_f]\,e^{-S_G}$$

- $\langle\Omega\rangle$ evaluated statistically (MC-HMC)



Source: JICFuS, Tsukuba



- Gauge ensembles produced within **C**oordinated **L**attice **S**imulations

O(a)-improved Wilson-Fermions, Nf=2+1

Sources of Error

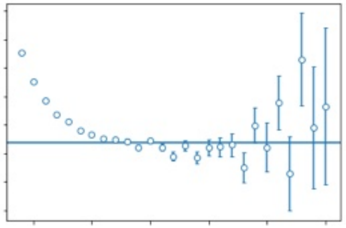
[…] there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns - the ones we don't know we don't know.“

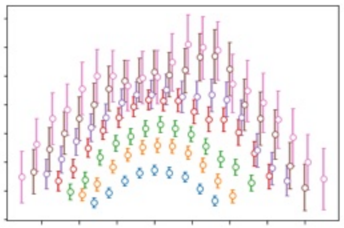
Sources of Error

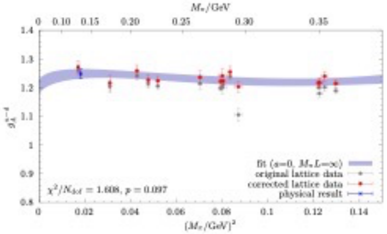
[…] there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns - the ones we don't know we don't know.“

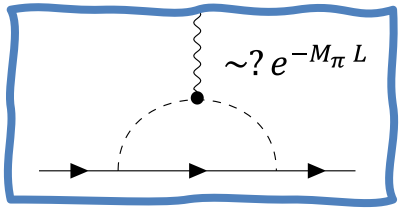
Rumsfeld during a Pentagon news briefing in February 2002

Rumsfeld-Classification	Knowns	Unknowns
Known	ME can be calculated using Lattice - Pathintegral	Statistical Errors, Dealing with Correlations
Unkknown	Systematics – excited states, finite lattice spacing, finite volume, chiral, continuum	?









Lattice Setup

ID	a [fm]	T/a	L/a	M_π[MeV]	$M_\pi L$	t_{sep}[fm]	N_{cfg}
H102	0.086	96	32	354	4.96	0.35, 0.43, 0.52, 0.6, 0.69, 0.78, 0.86, 0.95, 1.04, 1.12, 1.21, 1.3, 1.38, 1.47	2005
H105		96	32	280	3.93		1027
C101		96	48	225	4.73		2000
N101		128	48	281	5.91		1596
S400	0.076	128	32	350	4.33	0.31, 0.46, 0.61, 0.76, 0.92, 1.07, 1.22, 1.37, 1.53	2873
N451		128	48	286	5.31		1011
D450		128	64	216	5.35		500
D452		128	64	153	3.79		1000
N203	0.064	128	48	346	5.41	0.26, 0.39, 0.51, 0.64, 0.77, 0.9, 1.03, 1.16, 1.29, 1.41	1543
N200		128	48	281	4.39		1712
D200		128	64	203	4.22		2000
E250		192	96	129	4.04		400
S201		128	32	293	3.05		2093
N302	0.050	128	48	348	4.22	0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1., 1.1, 1.2, 1.3, 1.39	2201
J303		192	64	260	4.19		1073
E300		192	64	174	4.21		570

- Enlarged range in t_{sep}
  → Monitor excited state contribution
- Roughly same statistics at every t_{sep}
  → Number of sources adapted to t_{sep}
- Chiral/Continuum/Finite-Size extrapolation possible

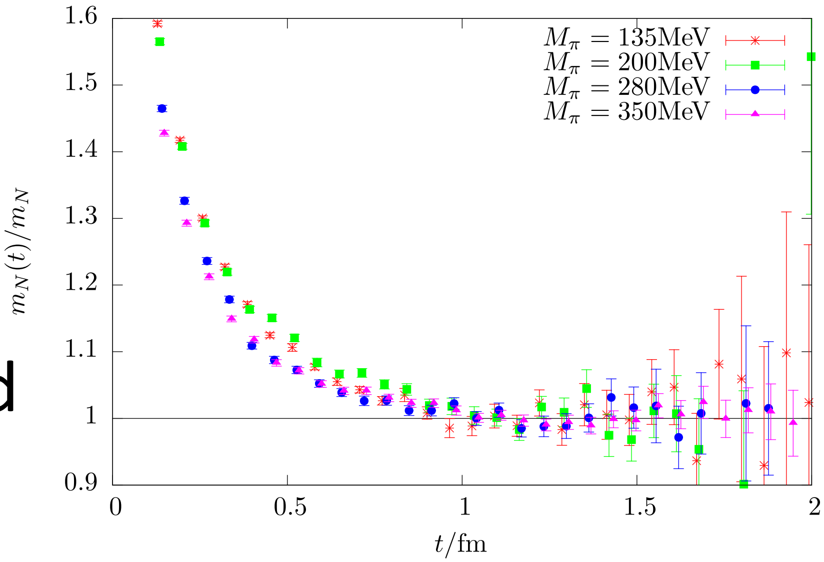
2pt Functions

- Physics contained in correlation functions

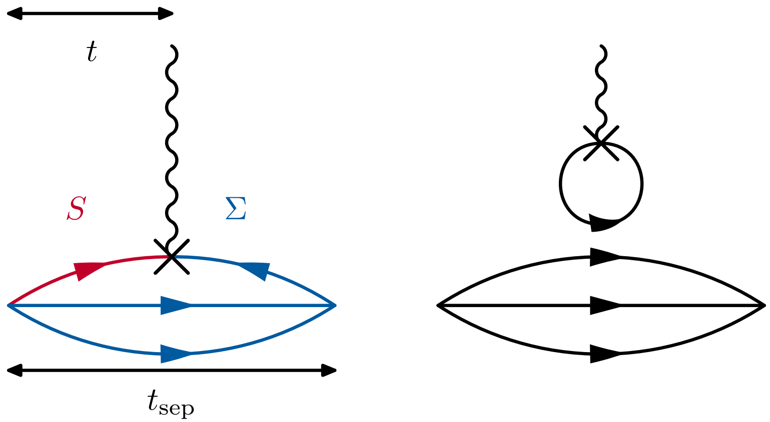
$$\sum e^{i\boldsymbol{p}(\boldsymbol{y}-\boldsymbol{x})} \langle \mathcal{O}_N(x)\mathcal{O}_N(y)^\dagger \rangle = \sum a_n(\boldsymbol{p}) e^{-E_n(p)(y_0-x_0)}$$

$$\xrightarrow{(y_0-x_0)\to\infty} a_0(\boldsymbol{p}) e^{-E_0(y_0-x_0)}$$

- $\mathcal{O}_N$: Nucleon interpolating operator
- Ground state dominates for large Euclidean time
- Challenges:
  - Signal to noise deteriorates for large times
  - Need to control excited states



3pt Functions 2 Distances



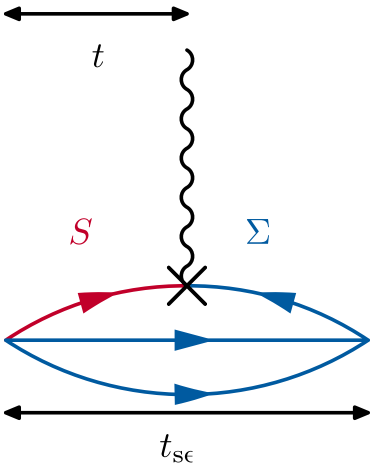
- In 3pt-functions we have two distances between
  - Source and current insertion time (t)
  - Source and sink time (or source-sink separation) (t_{sep})
- Very hard to make both large at the same time
- Excited-state problem is exacerbated
- Additional problem from Quark Disconnected Diagrams (notoriously hard to evaluate)

Direct Determination

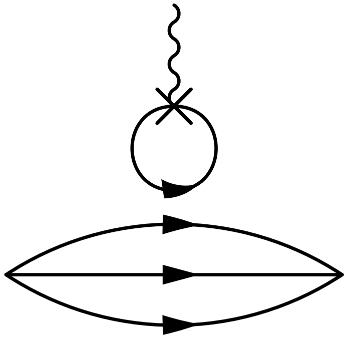
- Connected part
  - Sequential Source
  - Zero Momentum at sink

$$C_2(t;\mathbf{p}) = \Gamma_{\alpha\beta} \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} \left\langle \Psi_\beta(\mathbf{x},t)\overline{\Psi}_\alpha(0)\right\rangle,$$

$$C_3(t,t_s;\mathbf{q}) = \Gamma'_{\alpha\beta} \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{q}\mathbf{y}} \left\langle \Psi_\beta(\mathbf{x},t_s)\mathcal{O}_{\mathrm{S}}(\mathbf{y},t)\overline{\Psi}_\alpha(0)\right\rangle,$$



- Disconnected part
  - Loops All-to-All: OET+HPE+HP
  - Still Noisy:
    → Additional two-point functions



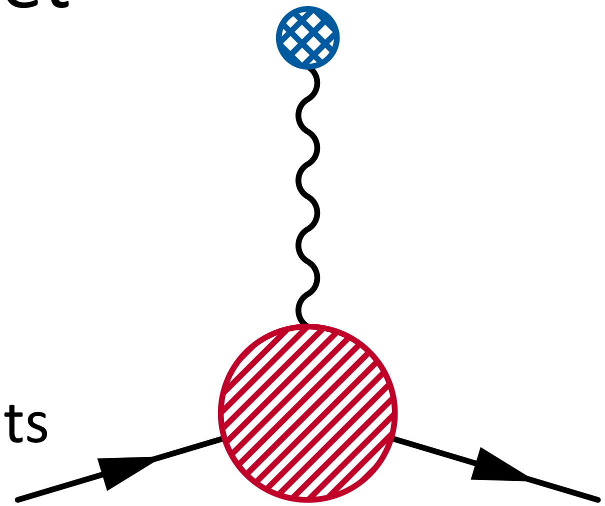
$$C_3^{\mathrm{disc}}(t,t_s;\mathbf{0}) = \left\langle L_S(\mathbf{0},z_0)\cdot C_2(\mathbf{p}',y_0,x;\Gamma')\right\rangle - \left\langle L_S(\mathbf{0},z_0)\right\rangle \cdot \left\langle C_2(\mathbf{p}',y_0,x;\Gamma')\right\rangle$$

Form Factors on the Lattice

- Computational Frame Work is set
- One needs to
  - Plugin the desired current
  - Deal with the result
    - Extract ground-state matrix elements

$$R^s_{V_\mu}(z_0,\boldsymbol{q};y_0,\boldsymbol{p}';\Gamma_\nu) = \frac{C^s_{3,V_\mu}(\boldsymbol{q},z_0;\boldsymbol{p}',y_0;\Gamma_\nu)}{C_2(\boldsymbol{p}',y_0)} \times\sqrt{\frac{C_2(\boldsymbol{p}',y_0)C_2(\boldsymbol{p}',z_0)C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},y_0\text{-}z_0)}{C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},y_0)C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},z_0)C_2(\boldsymbol{p}',y_0\text{-}z_0)}}.$$

    - Perform CCF extrapolation
    - Give best estimate of the error from the above

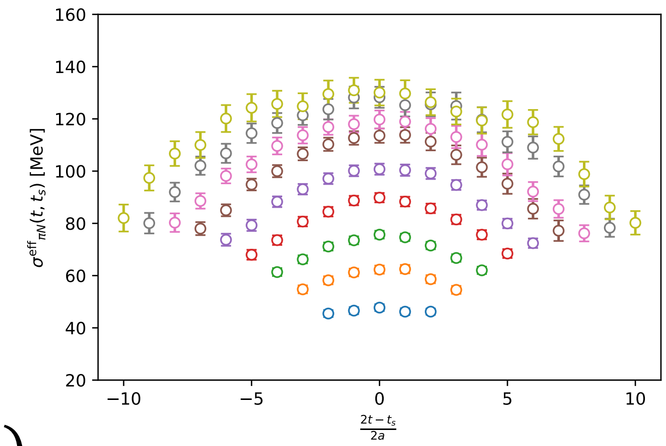


Excited States – Summation

- Usual Ratio (forward limit):

$$R(t,t_s) = \frac{C_3(t,t_s)}{C_2(t_s)} \qquad \mathrm{Re}\,R(t,t_s) \xrightarrow{t,(t_s-t)\gg 0} G_\mathrm{S}$$

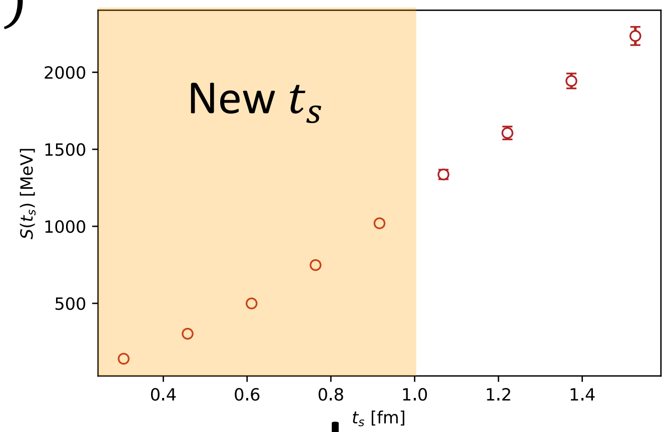
$$G_\mathrm{S}^\mathrm{eff}(t,t_s) = \mathrm{Re}\,R(t,t_s)$$



Excited states $\sim e^{-\Delta t}, e^{-\Delta(t_s-t)}$

- Summed correlator:

$$S(t_s) = \sum_{t=t_c}^{t_s-t_c} \sigma^{\mathrm{eff}}_{\pi N}(t,t_s)$$



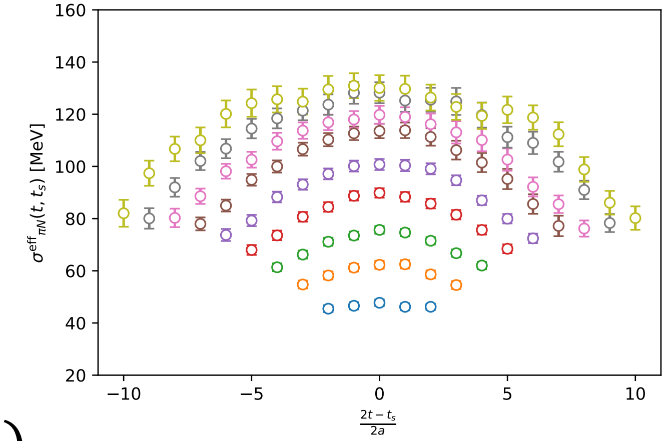
Excited states parametrically suppressed

Excited States – Summation Example scalar FF

- Usual Ratio (forward limit):

$$R(t,t_s) = \frac{C_3(t,t_s)}{C_2(t_s)} \qquad \mathrm{Re}\,R(t,t_s) \xrightarrow{t,(t_s-t)\gg 0} G_\mathrm{S}$$

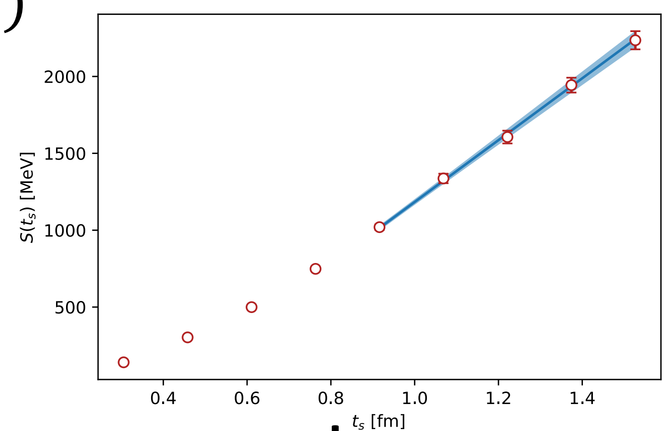
$$G_\mathrm{S}^\mathrm{eff}(t,t_s) = \mathrm{Re}\,R(t,t_s)$$



Excited states $\sim e^{-\Delta t}, e^{-\Delta(t_s-t)}$

- Summed correlator:

$$S(t_s) = \sum_{t=t_c}^{t_s-t_c} \sigma_{\pi N}^\mathrm{eff}(t,t_s)$$



Excited states parametrically suppressed

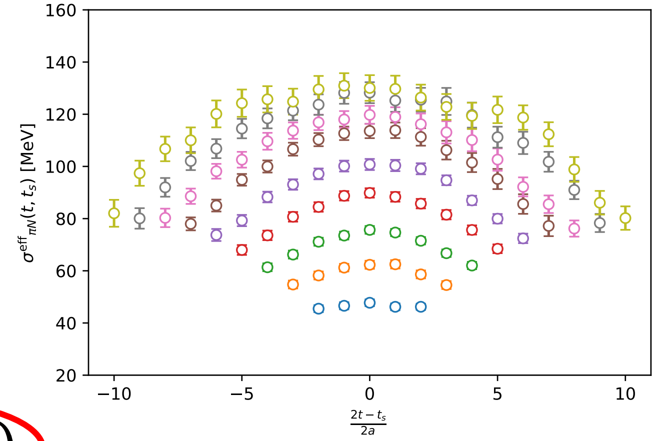
$$S(t_s) = (\sigma_{\pi N} \qquad\qquad) \, (1 + t_s - 2t_c) \qquad - \ldots$$

Excited States – Summation Example scalar FF

- Usual Ratio (forward limit):

$$R(t,t_s) = \frac{C_3(t,t_s)}{C_2(t_s)} \qquad \mathrm{Re}\, R(t,t_s) \xrightarrow{t,(t_s-t)\gg 0} G_\mathrm{S}$$

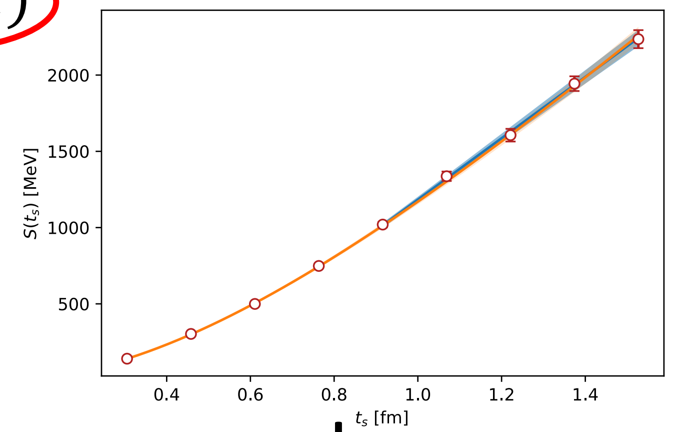
$$G_\mathrm{S}^\mathrm{eff}(t,t_s) = \mathrm{Re}\, R(t,t_s)$$



Excited states $\sim e^{-\Delta t}, e^{-\Delta(t_s - t)}$

- Summed correlator:

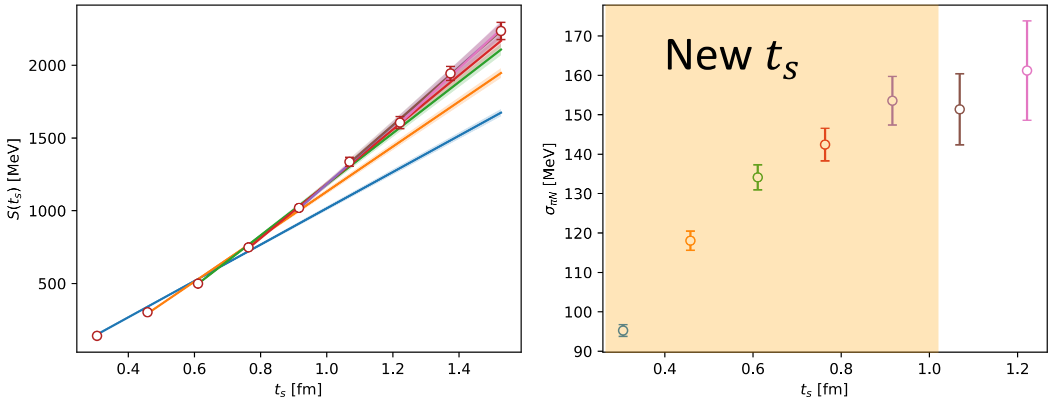
$$S(t_s) = \sum_{t=t_c}^{t_s-t_c} \sigma_{\pi N}^\mathrm{eff}(t,t_s)$$



Excited states parametrically suppressed

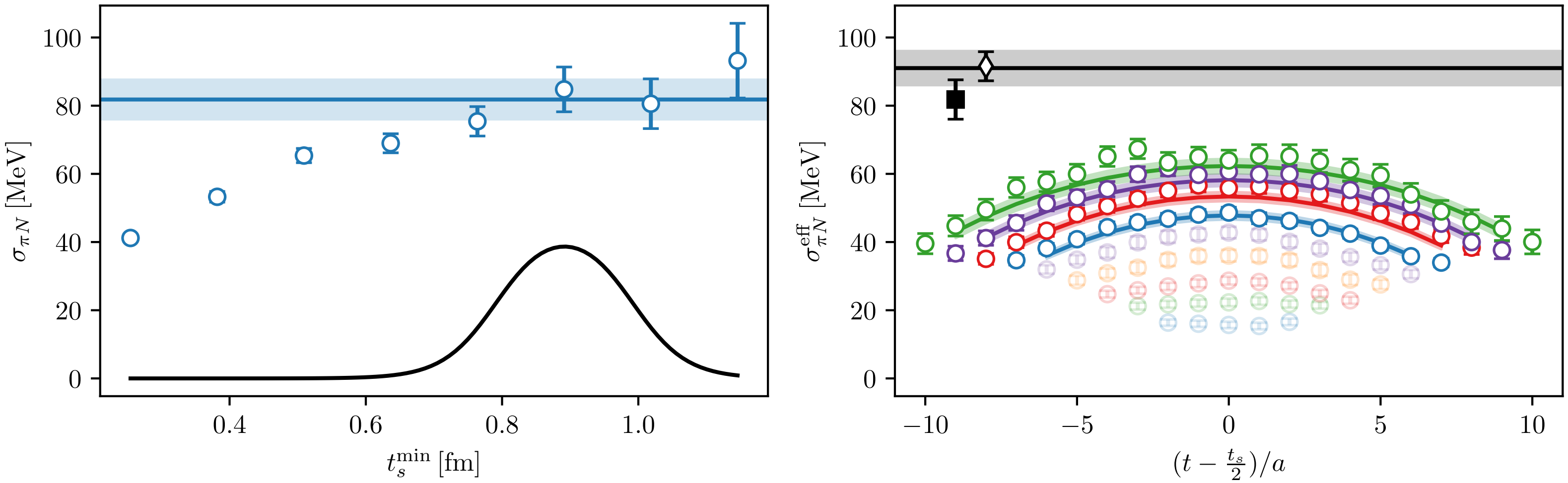
$$S(t_s) = (\sigma_{\pi N} + m_{11} e^{-\Delta t_s})(1 + t_s - 2t_c) + e^{-\Delta t_s} \frac{2m_{10}\left(e^{\Delta(1-t_c+t_s)} - e^{\Delta t_c}\right)}{e^{\Delta} - 1} + \ldots$$

Excited States – Summation Example scalar FF



- Excited State Fits need priors for gap Δ (like explicit 2-state-Fit)
- Linear Fits:
  - Not trustworthy for small t_s
  - Error increases with larger starting t_s
  - Several possibilities
    - Choose one or use weights e.g. AIC, p-values, …
    - Define a window in physical units and average

Excited States – Contamination Example scalar FF



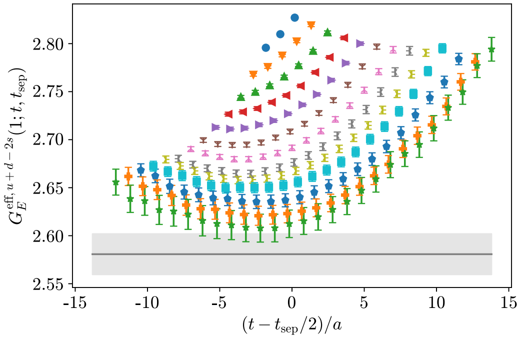
- Left: Blue data is linear fits to summation data at starting at t indicated on x-axis
  Black profile is window function

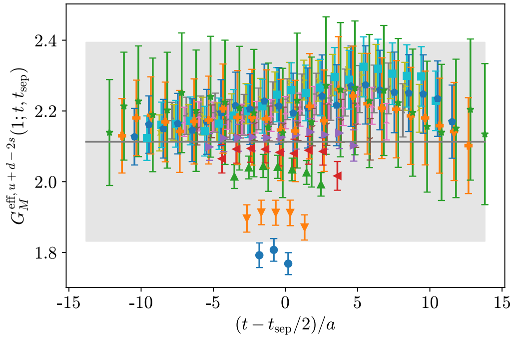
$$w_i = \frac{1}{2}\tanh\frac{t_s - t_{\mathrm{lo}}}{\Delta t} - \frac{1}{2}\tanh\frac{t_s - t_{\mathrm{up}}}{\Delta t}$$

  Blue band is weighted average of profile and data point
- Right: Effective FF data for different source-sink separations
  Black band is explicit two-state fit
- Have two separate ways for extracting the matrix element

EM FF of the Proton

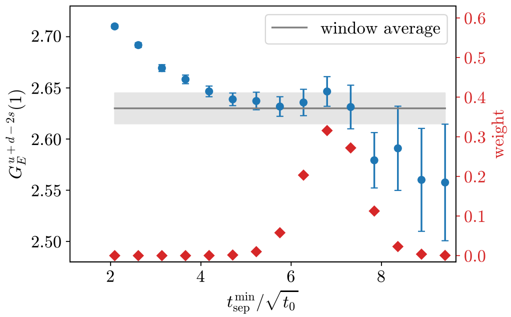
- Effective FF (isoscalar E300)

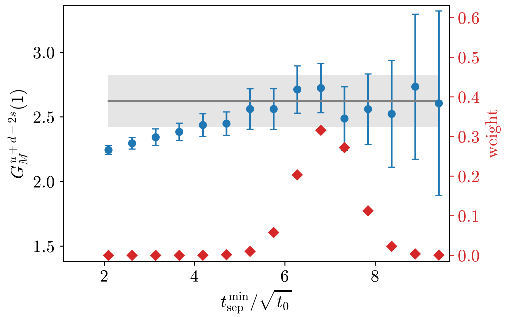




$$R^s_{V_\mu}(z_0,\boldsymbol{q};y_0,\boldsymbol{p}';\Gamma_\nu) = \frac{C^s_{3,V_\mu}(\boldsymbol{q},z_0;\boldsymbol{p}',y_0;\Gamma_\nu)}{C_2(\boldsymbol{p}',y_0)} \times \sqrt{\frac{C_2(\boldsymbol{p}',y_0)C_2(\boldsymbol{p}',z_0)C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},y_0\text{-}z_0)}{C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},y_0)C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},z_0)C_2(\boldsymbol{p}',y_0\text{-}z_0)}}.$$

- Use window-average of summed correlator





$$w_i = \tanh\frac{t_i - t_w^{\mathrm{low}}}{\Delta t_w} - \tanh\frac{t_i - t_w^{\mathrm{up}}}{\Delta t_w}.$$

CCF Fits

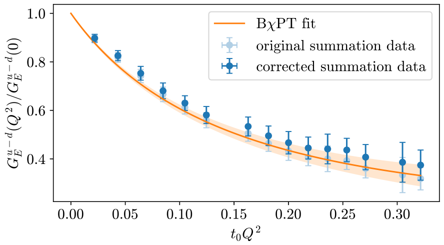
- We perform simultaneous fits to all FF using BChPT including Vector mesons T. Bauer, J. C. Bernauer, and S. Scherer, Phys. Rev. C86, 065206 (2012),
- We add lattice artifacts

$$G_E^{\mathrm{add}}(Q^2) = G_E^{\chi}(Q^2) + G_E^a a^2 Q^2 + G_E^L t_0 Q^2 e^{-M_\pi L},$$

$$G_M^{\mathrm{add}}(Q^2) = G_M^{\chi}(Q^2) + G_M^a \frac{a^2}{t_0} + \kappa_L M_\pi \left(1 - \frac{2}{M_\pi L}\right) e^{-M_\pi L} + G_M^L t_0 Q^2 e^{-M_\pi L},$$

$$G_E^{\mathrm{mult}}(Q^2) = G_E^{\chi}(Q^2) + \frac{G_E^a a^2 Q^2 + G_E^L t_0 Q^2 e^{-M_\pi L}}{t_0(M_\rho^2 + Q^2)},$$

$$G_M^{\mathrm{mult}}(Q^2) = G_M^{\chi}(Q^2) + \frac{G_M^a a^2/t_0 + G_M^L t_0 Q^2 e^{-M_\pi L}}{t_0(M_\rho^2 + Q^2)} + \kappa_L M_\pi \left(1 - \frac{2}{M_\pi L}\right) e^{-M_\pi L}.$$



- Apply cuts in pion mass, and momentum
- Perform model averages of these variations
- Note: Consistent results between BChPT and z-expansion

Akaike Information Criterion

- IC based on Kullback-Leibler Divergence

$$\mathrm{AIC} = -2\ln\hat{L} + 2k$$

- For least-square-fitting

$$\mathrm{AIC} = \chi^2(\hat{a}) + 2k$$

Fit parameters

- Including data selection

$$\mathrm{AIC} = \chi^2(\hat{a}) + 2k + 2d_c$$

Cut datapoints

$$w_i^{\mathrm{AIC}} = \frac{e^{-\frac{1}{2}\mathrm{AIC}_i}}{\sum_j e^{-\frac{1}{2}\mathrm{AIC}_j}},$$

- Weights are higher for
  - Better fits
  - Less fit parameters
  - More Data used

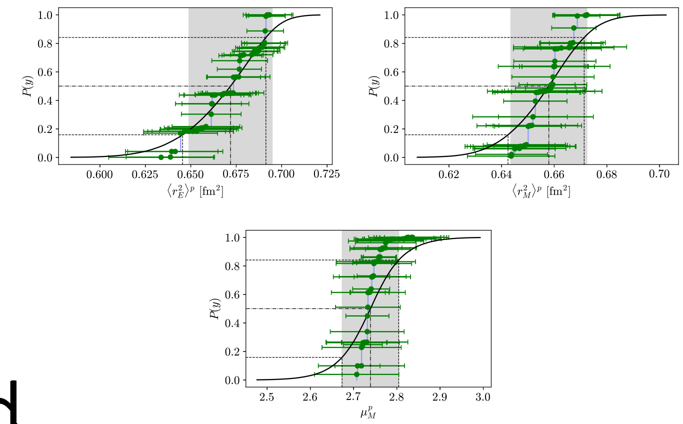
AIC Averages

- Once weights are calculated build CDF

$$P^x(y) = \int_{-\infty}^{y} \sum_i^n w_i \mathcal{N}(y'; x_i, \sigma_i^2) dy'$$

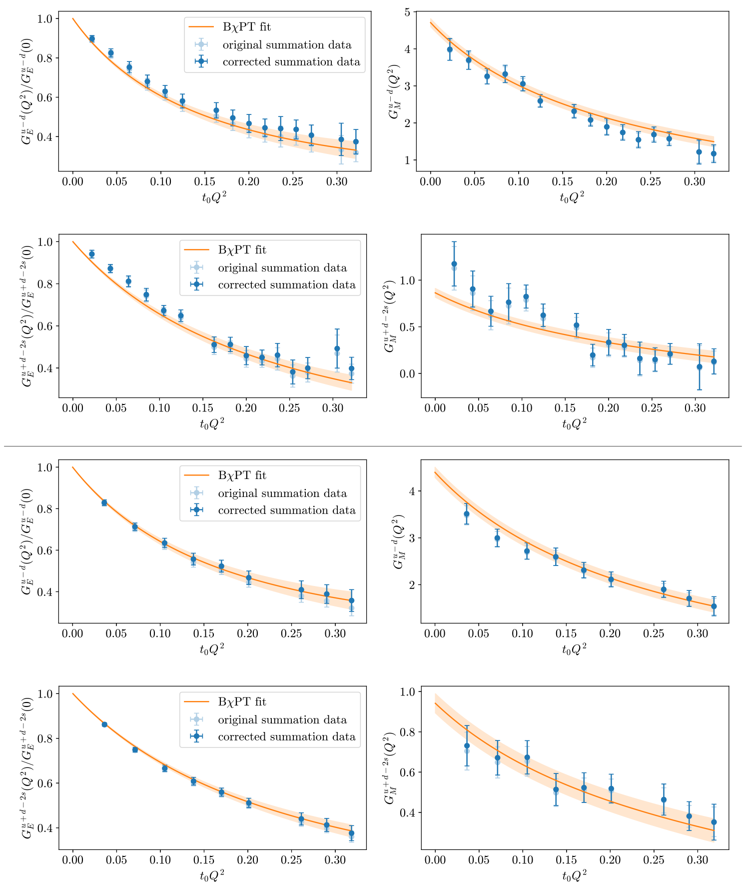
  with AIC weighted sum over Gaussians for different variations

- Median of CDF is the model-averaged result
- Percentile-errors quoted as model-averaged Errors



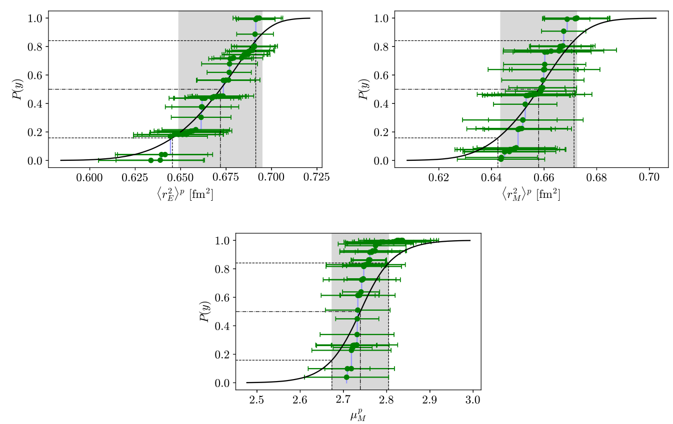
Method from: S. Borsányi et al., Nature 593, 51

FF of the Nucleon



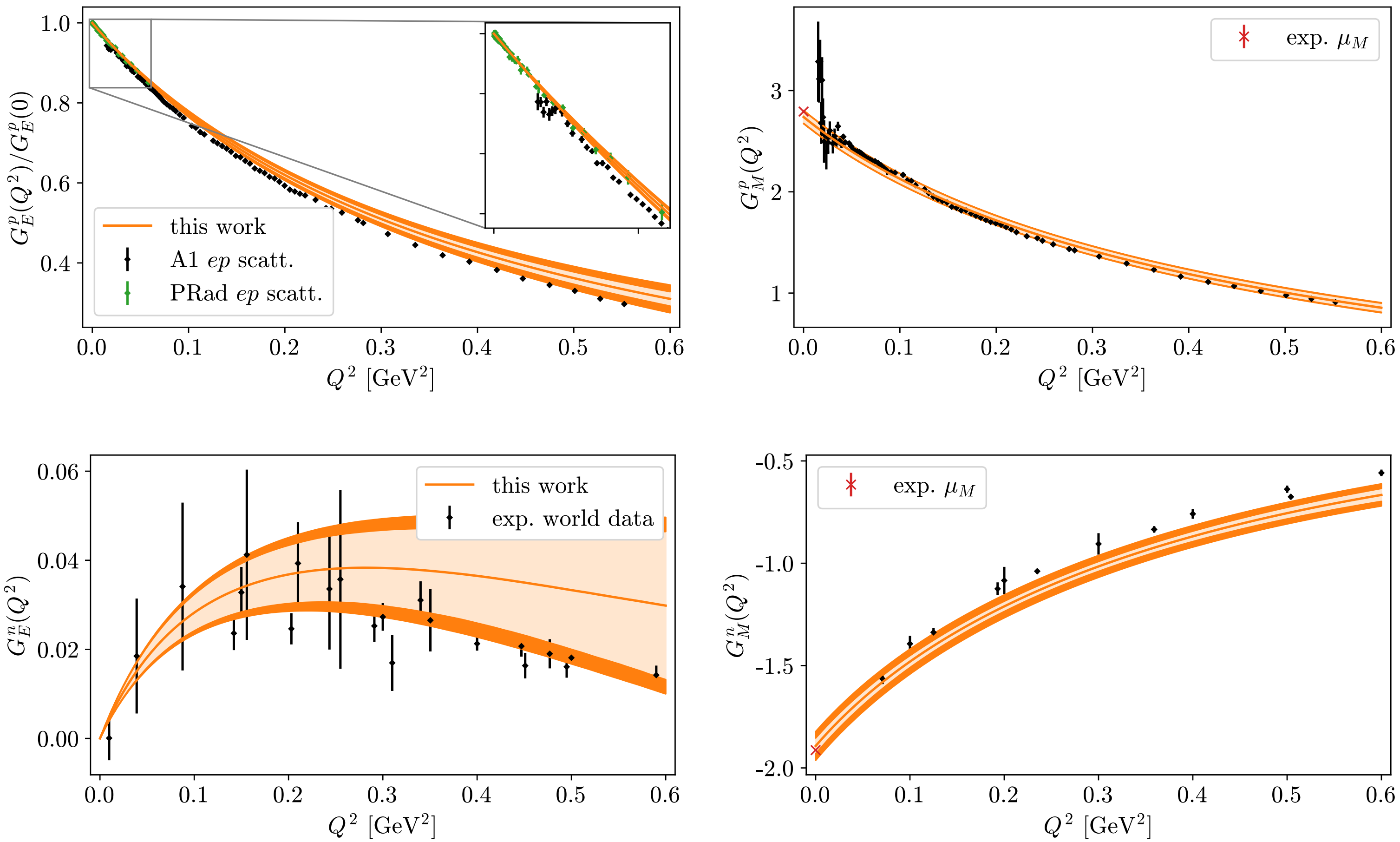
- Simultaneous fit to all ensembles using BChPT ammended with terms for continuum and finite volume T. Bauer, J. C. Bernauer, and S. Scherer, Phys. Rev. C86, 065206 (2012),
- Perform variations of the fits, e.g. cuts in momentum transfer, pion mass, etc.
- Calculate radii and magnetic moment

$$\langle r^2 \rangle = -\frac{6}{G(0)} \left. \frac{\partial G(Q^2)}{\partial Q^2} \right|_{Q^2=0} .$$

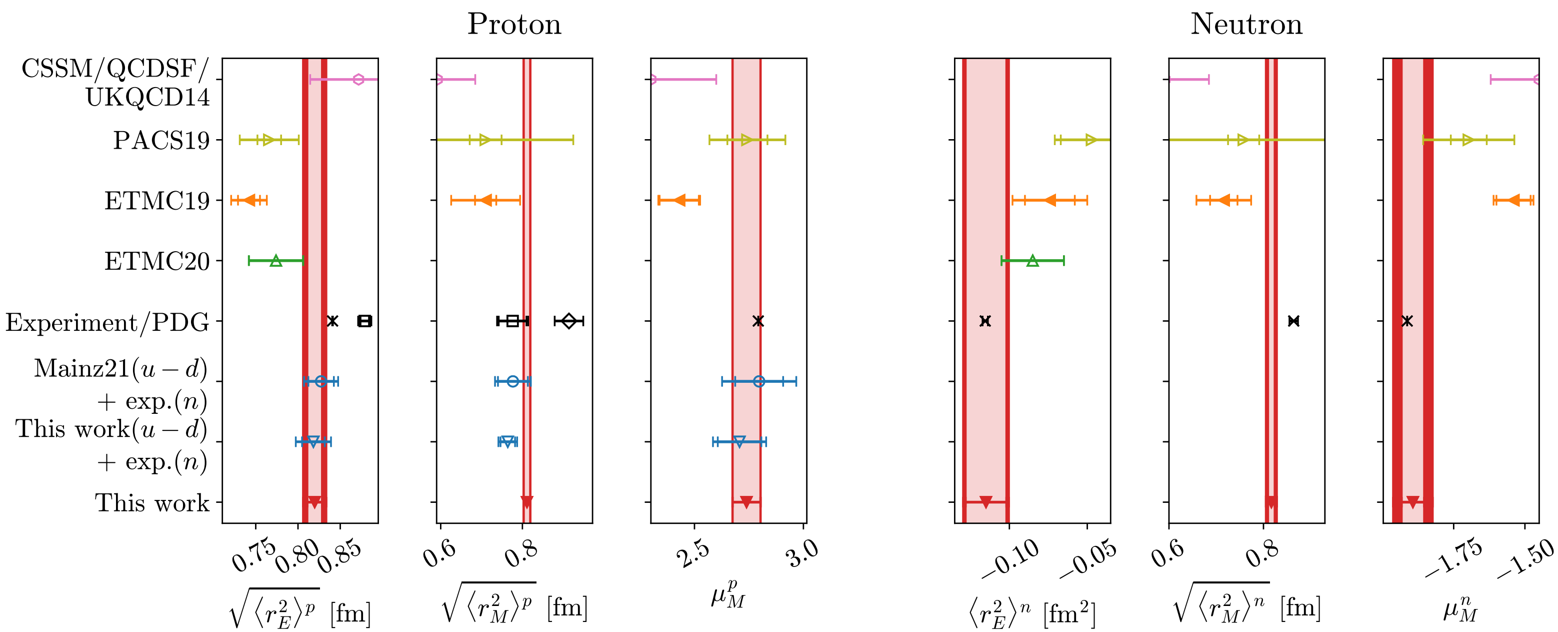


$$P^x(y) = \sum_{i=1}^{N_M} \frac{w_i^{\overline{\mathrm{BAIC}}}}{N_B} \sum_{n=1}^{N_B} \Theta(y - x_{i,n}).$$

Model Averaged FF @ physical pt



Comparison to others em FF

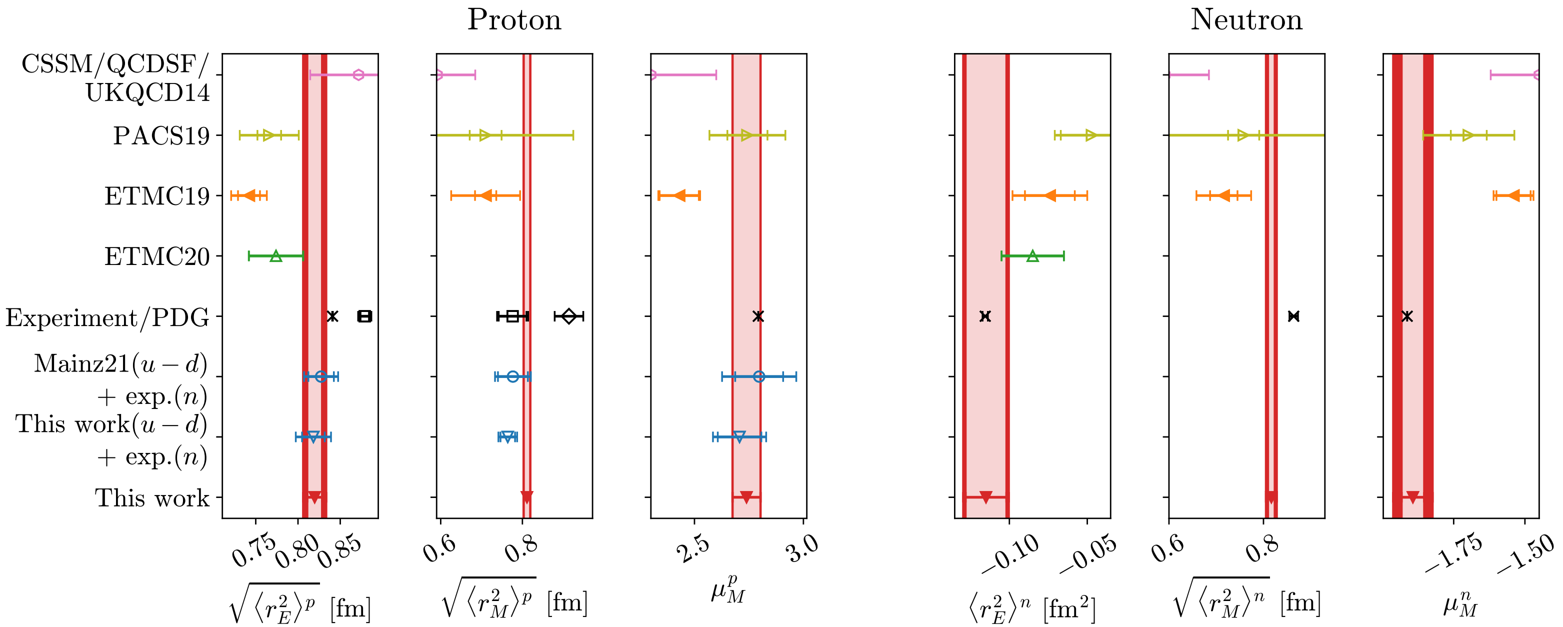


$$\langle r_E^2\rangle^p = (0.672 \pm 0.014\ (\text{stat}) \pm 0.018\ (\text{syst}))\ \text{fm}^2,$$
$$\langle r_M^2\rangle^p = (0.658 \pm 0.012\ (\text{stat}) \pm 0.008\ (\text{syst}))\ \text{fm}^2,$$
$$\mu_M^p = 2.739 \pm 0.063\ (\text{stat}) \pm 0.018\ (\text{syst}),$$

$$\langle r_E^2\rangle^n = (-0.115 \pm 0.013\ (\text{stat}) \pm 0.007\ (\text{syst}))\ \text{fm}^2$$

$$\langle r_M^2\rangle^n = (0.667 \pm 0.011\ (\text{stat}) \pm 0.016\ (\text{syst}))\ \text{fm}^2,$$
$$\mu_M^n = -1.893 \pm 0.039\ (\text{stat}) \pm 0.058\ (\text{syst}).$$

Comparison to others FF



- Electric radius and magnetic moment consistent with experiment and others
- Some tension for magnetic radius with experiment
- Magnetic properties accessible in spectroscopy measurement of HFS

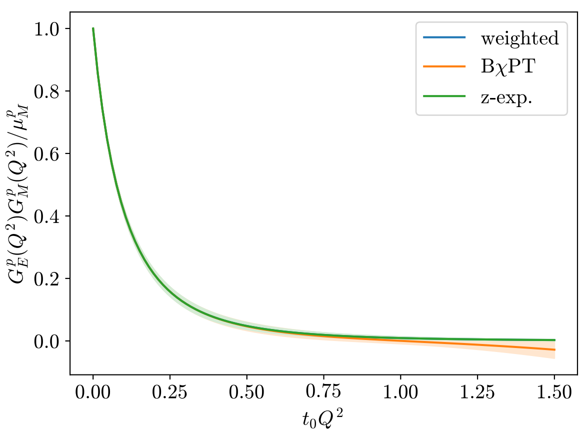
$$r_Z^p = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[\frac{G_E^p(Q^2)G_M^p(Q^2)}{\mu_M^p} - \frac{G_E^p(0)G_M^p(0)}{\mu_M^p}\right]$$
$$= -\frac{2}{\pi}\int_0^\infty \frac{dQ^2}{(Q^2)^{3/2}}\left[\frac{G_E^p(Q^2)G_M^p(Q^2)}{\mu_M^p} - 1\right].$$

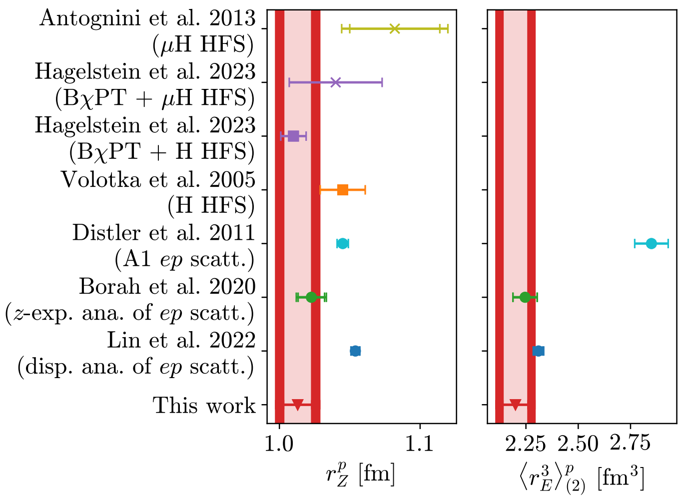
Zemachradius

Zemachradius

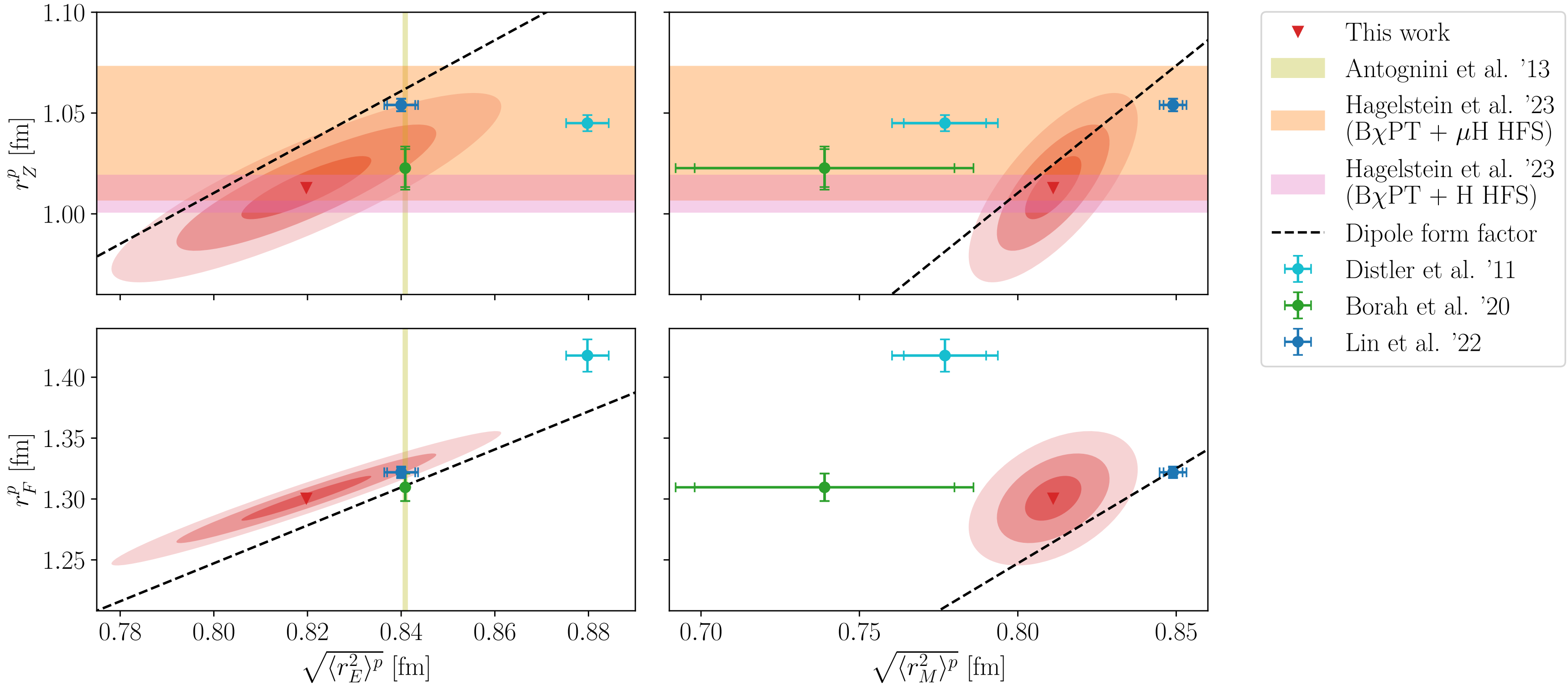
- Integral needs full range in momentum
- Smoothly interpolate between BChPT and z-expansion

$$F(Q^2) = \frac{1}{2}\left[1 - \tanh\left(\frac{Q^2 - Q^2_{\text{cut}}}{\Delta Q^2_w}\right)\right] F^{\chi}(Q^2) + \frac{1}{2}\left[1 + \tanh\left(\frac{Q^2 - Q^2_{\text{cut}}}{\Delta Q^2_w}\right)\right] F^{z}(Q^2),$$





Zemach and Friarradius



$$\langle r_E^3\rangle^p_{(2)} = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left[(G_E^p(Q^2))^2 - (G_E^p(0))^2 - \frac{\partial (G_E^p(Q^2))^2}{\partial Q^2}\bigg|_{Q^2=0} Q^2\right]$$

$$= \frac{24}{\pi}\int_0^\infty \frac{dQ^2}{(Q^2)^{5/2}}\left[(G_E^p(Q^2))^2 - 1 + \frac{1}{3}\langle r_E^2\rangle^p Q^2\right].$$

$$r_F^p = \sqrt[3]{\langle r_E^3\rangle^p_{(2)}}.$$

Summary

- Precision Calculation of nucleon FF from the lattice available, including error budget
- Puzzles:
  - Proton radius puzzle s(olv/urviv)ed?
    This time around a slight tension for the magnetic radius between lattice and dispersive approaches
  - Lattice reaching a precision for the pion-nucleon sigma term where a slight tension between lattice and dispersive approach is visible

Thank you!