Backward DVCS on Pion

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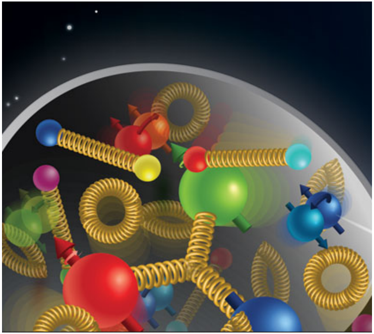
Outline

What are we looking for?

Hadron tomography



- How do the nucleonic properties emerge from partons and their underlying interactions?
- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?

What are we looking for?

Hadron tomography

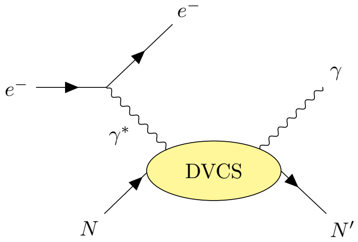
- How do quarks and gluons combine to make hadrons?
- Theory: PDFs, GPDs, TMDs, GDAs.
- Experimental: EIC, EicC, FCC, ...

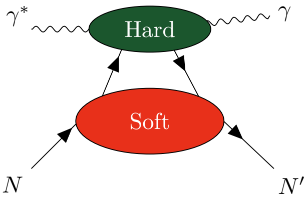
Backward DVCS

- Can we access DEMP and DVCS at Backward Angles?
- Can we measure in the backward region DVCS on pion?
- Theory tool: TDAs

Factorization

How to calculate high energy scattering cross-sections?





$Q^2 \to \infty$ with $Q^2 \gg t$
Perturbative QCD
Non-perturbative QCD

[Adv.Ser.Direct.High Energy Phys.5(1989)1-91, Phys.Rev.D59(1999)074009,]

Factorization

Factorization of the DVCS amplitude: Soft +Hard

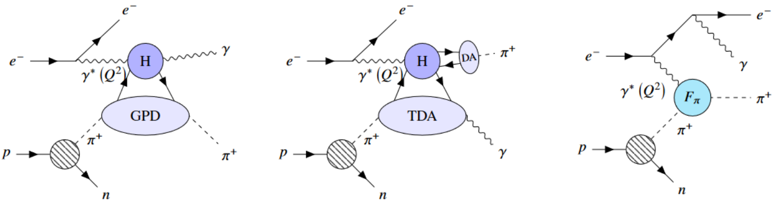
$$\mathcal{H}_\pi\left(\xi, t, Q^2\right) = \sum_{i=\{q\},g} \int_{-1}^{1} \frac{dx}{\xi} \mathcal{K}^i\left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}\right) H_\pi^i\left(x, \xi, t_\pi; \mu_F^2\right) \quad (1)$$

$$\mathcal{A}_\pi(\xi, u, Q^2) = \frac{16\pi\alpha_s e}{9Q} \int dx dz\, C_F^{ud}(x, z, \xi) \Phi^{\pi^+}(z) A^{\pi^+\gamma}(x, \xi, u)\,. \quad (2)$$

The factorization of the bDVCS amplitude is given as a convolution of a short distance coefficient function (C_F), a meson distribution amplitude (Φ^π) and a $\pi \to \gamma$ TDA ($A^{\pi\gamma}$).

[Phys.Rev.D 55 (1997) 7114-7125], [Phys.Rev.D56(1997)2982], [SciPost Phys. Proc.8(2022)165],[arXiv:2407.03011]

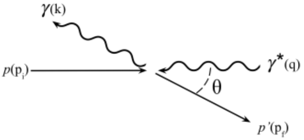
Sullivan Process on Pion



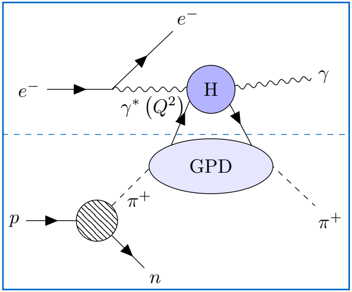
LEFT PANEL: forward DVCS; CENTRAL PANEL: backward DVCS; RIGHT PANEL: the Bethe-Heitler process, in the Sullivan reaction $e^- p \to e^- \gamma \pi^+ n$.

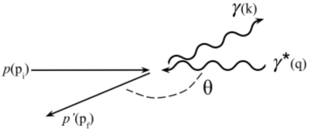
- **Goldstone boson**: insight on the mass generation phenomena quest.
- **Sullivan Process**:photon interaction with pion from the meson cloud inside of protons.
- "Can we obtain tomographic snapshots of the pion in the transverse plane?".
- "Measurement of DVCS off pion target as defined with Sullivan process." [EICYR:phys.ins-det/2103.05419]

What kind of DVCS process we have on different kinematic regions?

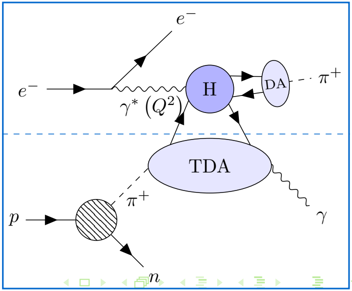


Forward





Backward



General Aspects

What they have in common?

- Both TDAs and GPDs are non-perturbative functions in QCD that provide insights into the partonic structure of hadrons.
- They are both defined in terms of matrix elements of non-local operators between hadronic states.
- Access to the longitudinal and transverse momentum distribution of quarks and gluons inside of hadrons;
- Both comes from the collinear factorization framework for hard exclusive reactions
- In the Sullivan case, the pion to photon TDAs are matrix elements of the same operator as in the pion (or nucleon) GPDs, since there is no baryonic number transfer.

What differences are between them?

GPD

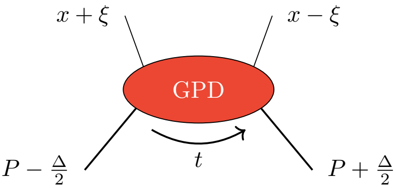
- Forward kinematics → t-channel;
- Non-diagonal matrix elements of the same operators;
- Tool to address the origin of the nucleon's spin;
- Link to PDFs and Form Factors;

TDA

- Backward kinematics → u-channel;
- Description of hard exclusive reactions with a non-zero baryon number exchange in the cross channel;
- Matrix elements of non-local light-cone operators between a baryon state and a meson state/photon state, or between meson state and a photon state.

[Few Body Syst.63(2022)3,62]; [Phys.Rept.940(2021)1-121]

Quark and Gluon GPDs



- x: average momentum fraction carried by the active parton along the lightcone;
- ξ: Fraction of momentum longitudinally transferred $\rightarrow$ $\xi \approx \frac{x_B}{2-x_B}$;
- t: Momentum transfer $\rightarrow$ $t = \Delta^2$.

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \langle p_2| \bar{\psi}^q \left(-\frac{z^-}{2}\right) \gamma^+ \psi^q \left(\frac{z^-}{2}\right) |p_1\rangle \tag{3}$$

$$H_\pi^g(x,\xi,t) = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|G^{+\mu} \left(-\frac{z^-}{2}\right) G_\mu^+ \left(\frac{z^-}{2}\right) |p_1\rangle \,, \tag{4}$$

with $z^+ = z_\perp = 0$. ψ_q is a quark field of flavour q and $G^{\mu\nu}$ is the gluon field strength. The average momentum of the nucleon is $P = \frac{p_1+p_2}{2}$ and the momentum transfer is $\Delta = p_2 - p_1$. [Few Body Syst.63(2022)3,62]

GPDs Connections

Connection with PDFs

When considering the forward limit $\Delta \to 0$ and equal helicities for the initial and final state hadrons, one has

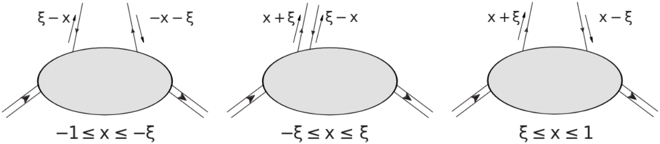
$$H_\pi^q(x) = q(x)\Theta(x) - \bar{q}(-x)\Theta(-x) \quad (5)$$
$$H_\pi^g(x) = xg(x)\Theta(x) - x\bar{g}(-x)\Theta(-x) \quad (6)$$

thus, we obtain the forward limits relating the pion GPDs with the quark q, antiquark $\bar{q}$ and gluon g PDFs. [Few Body Syst.63(2022)3,62]

Connection with EFF

$$\int dx\, H_\pi^q(x,\xi,t) = \frac{1}{2}\int \delta(P^+ z^-) dz^- \times \langle P + \frac{\Delta}{2}|\bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right)|P - \frac{\Delta}{2}\rangle\Big|_{z^+=0, z=0} \quad (7)$$
$$= \frac{1}{2P^+}\langle P + \frac{\Delta}{2}|\bar{\psi}^q(0)\gamma^+\psi^q(0)|P - \frac{\Delta}{2}\rangle = F^q(t) \quad (8)$$

GPDs Interpretation



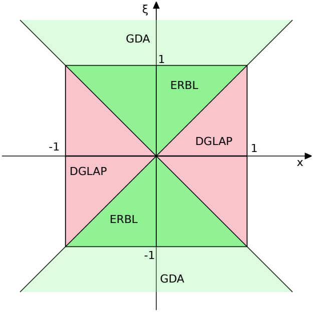
From left to right: 1. $\bar{q}$ interaction; 2. pair $\bar{q}q$ extraction; 3. q interaction.

- $-1 < x < -\xi$: an active antiquark being taken out and put back in the hadron.
- $-\xi < x < \xi$: extraction of a quark-antiquark pair from the hadron without breaking it.
- $\xi < x < 1$: an active quark being taken out and put back in the hadron.

[M.Diehl et al.-PLB:359(428)1998], [Few Body Syst.63(2022)3,62]

Support and Mellin Moments of GPDs

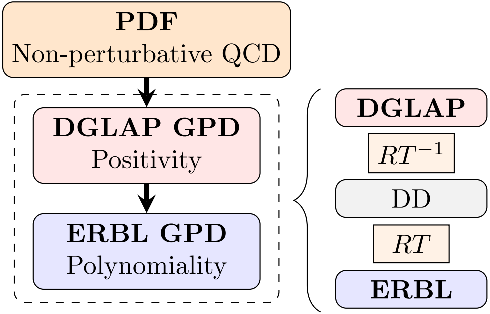
- **Physical region:** $(x, \xi) \in [-1, 1]^2$
  - $|x| \geq |\xi|$: DGLAP region ;
  - $|\xi| \geq |x|$: ERBL region;



- **Polynomiality:** $\int_{-1}^{1} dx\, x^m H^q(x, \xi, t) = \sum_{i=0}^{m+1} f_i^{m+1}(t) \xi^{2i}$
- **Positivity (in DGLAP):** $|H_\pi^q(x, \xi, t)| \leq \sqrt{q_\pi\left(\frac{x-\xi}{1-\xi}\right) q_\pi\left(\frac{x+\xi}{1+\xi}\right)}$

[M.Diehl et al.-PLB:359(428)1998; Few Body Syst.63(2022)3,62]
[X.Ji-JPG:1181(24)1998; A.Radyushkin-PLB:81(449)1999]
[P.V.Pobylitsa-PRD:114015(65)2002; B.Pire et al.-EPJC:103(8)1999]

Modelling GPDs



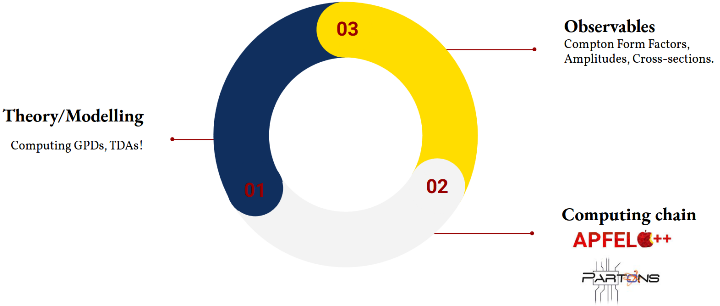
Modelling Strategies

By considering **Double Distribution** (DD) and **Radon Transform**, we can fulfil both polynomiality and positivity on GPDs.

$$H^q(x,\xi,t) = \int_\Omega d\beta\, d\alpha\, (F^q(\beta,\alpha,t) + \xi G^q(\beta,\alpha,t))\, \delta(x-\beta-\alpha\xi). \tag{9}$$

[EPJC:906(77)2017];[Phys.Rev.D.105(2022)094012]; [J.Morgado, 2022]
[Fortsch.Phys. 42 (1994) 101-141];[JLAB-THY-00-33]

From GPDs to Observables



$$\mathcal{M}_{e\pi} = \mathcal{M}_{DVCS} + \mathcal{M}_{BH} \quad (10)$$

$$\sigma_{e\pi}(\pm\lambda, e) \propto |M_{DVCS}|^2 + |M_{BH}|^2 \mp I \quad (11)$$

Can we probe pion GPDs in experiment? Yes!

[Rev.Mex.Fis.Suppl. 3 (2022) 3, 0308099]

[Comput.Phys.Commun. 185(2014)1647-1668]; [PoS-DIS2017(2018) 201]; [Eur.Phys.J.C 78(2018) 6,478]

What about TDAs?

- Guiding Experimental Design: Informed choice of experimental parameters for the backward scattering.
- Testing Factorization Hypotheses: Do TDA predictions agree with experimental data? $\rightarrow$ Validates the use of TDAs in the description of the DVCS.
- Understanding Nucleon Structure: Provides complementary information to GPDs.

TDAs are connected to the experimental and theoretical ongoing efforts about hadron tomography!

TDAs Definitions

There are four leading twist $\pi \to \gamma$ TDAs: one vector, one axial and two transversity. In our process, only the axial quark TDA A^{π} contributes. It is defined as

$$\frac{e}{f_\pi}\epsilon \cdot \Delta A^{\pi^+\gamma} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \left\langle \gamma, P+\frac{\Delta}{2} \middle| \overline{\psi}_{q'}\left(-\frac{z}{2}\right)\gamma^+\gamma_5\psi_q\left(\frac{z}{2}\right) \middle| \pi^+, P-\frac{\Delta}{2} \right\rangle |_{z^+=z_i^\perp=0} \quad (12)$$

where ϵ is the outgoing photon polarisation vector and f_π the pion decay constant. Recalling that the light cone field ϕ is given as a projection of the standard field ψ_q, in this case

$$\bar{\psi}_q\gamma^+\gamma_5\psi_q \quad = \quad \sqrt{2}\phi^\dagger\gamma_5\phi\,. \quad (13)$$

[Phys.Rept.388(2003)41]

[Phys.Rev.D 71 (2005) 111501]

TDAs Definitions

The incoming pion two mains states contribution and outgoing photon having three different states. Out of the six possible combinations, only two are non vanishing and are given as:

$$\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left(\langle\gamma|_{\gamma\cdot n} \right) \bar{\psi}(-\frac{z}{2})\gamma\cdot n\gamma_5\psi(\frac{z}{2}) \left|\pi^+,\uparrow\downarrow\right\rangle = \frac{e}{f_\pi}\epsilon\cdot\Delta A^{\pi^+}_{\uparrow\downarrow}\ , \tag{14}$$

$$\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left(\langle\gamma|_{\sigma^{n\perp}} \right) \bar{\psi}(-\frac{z}{2})\gamma\cdot n\gamma_5\psi(\frac{z}{2}) \left|\pi^+,\uparrow\uparrow\right\rangle = \frac{e}{f_\pi}\epsilon\cdot\Delta A^{\pi^+}_{\uparrow\uparrow}\ . \tag{15}$$

TDAs Modelling:Overlap representation

How to model TDAs?

- Overlap representation
  TDAs written as overlap of photon and pion LFWFs: $\psi^i(x, k_\perp)$.

For example, in our case, considering the following pion LFWF and the lowest Fock state description of a π^+ meson wave function:

$$\psi^{\pi^+}_{\uparrow\downarrow}(x, k_\perp) = 8\sqrt{15}\pi \frac{M^3}{(k_\perp^2 + M^2)^2} x(1-x), \quad (16)$$

$$|\pi^+, \uparrow\downarrow\rangle = \int \frac{\mathrm{d}k_\perp}{16\pi^3} \frac{x}{\sqrt{x(1-x)}} \psi_{\uparrow\downarrow} \Big[b^\dagger_{u,\uparrow}(x, k_\perp) d^\dagger_{d,\downarrow}(1-x, -k_\perp) - b^\dagger_{u,\downarrow}(x, k_\perp) d^\dagger_{d,\uparrow}(1-x, -k_\perp) \Big] |0\rangle \quad (17)$$

[Nucl.Phys.B 596 (2001)33-65]
[Phys.Rept.388(2003)41]
[Phys.Lett.B780(2018)287]

DGLAP pion-photon TDA

One obtain the contribution to the $\pi \to \gamma$ TDA generated by the $d\bar{d}$ Fock-space-expansion of the photon state in the DGLAP region

$$A^{\pi^+\gamma}_{d(\uparrow\downarrow)}(x,\xi,u)\Big|_{x\geq|\xi|} = \mathcal{N}_{\uparrow\downarrow}\frac{(1-x)^2(x+\xi)}{(1-\xi^2)^2(1+\xi)}\left[(\xi-x)+(1-x)\right] \times\frac{\tau(2\tau+1)-\sqrt{\frac{\tau}{\tau+1}}\tanh^{-1}\left(\sqrt{\frac{\tau}{\tau+1}}\right)}{\tau^2(1+\tau)}. \quad (18)$$

where $\tau = -\frac{(1-x)^2}{(1-\xi^2)}\frac{u}{4M^2}$. By the symmetry relation between

$$A^{\pi^+\gamma}_{u(\uparrow\downarrow)}(x,\xi,u) = A^{\pi^+\gamma}_{d(\uparrow\downarrow)}(-x,\xi,u), \quad (19)$$

one can have the $A^{\pi^+\gamma}_{u(\uparrow\downarrow)}$ contribution.

The pion presents a second independent LFWF, $\psi^{\pi}_{\uparrow\uparrow}(x,k_\perp)$ associated with the Fock state $|\pi^+,\uparrow\uparrow\rangle$.

DGLAP pion-photon TDA

The pion presents a second independent LFWF, $\psi^{\pi}_{\uparrow\uparrow}(x, k_\perp)$ associated with the Fock state $|\pi^+, \uparrow\uparrow\rangle$. This state gives the following contribution term on DGLAP region

$$A^{\pi^+\gamma}_{d(\uparrow\uparrow)}(x,\xi,u)\Big|_{x\geq|\xi|} = \mathcal{N}_{\uparrow\uparrow}\frac{(x+\xi)(1-x)^2}{(1+\xi)^3(1-\xi)} \times \frac{-\tau+(2\tau+1)\sqrt{\frac{\tau}{\tau+1}}\tanh^{-1}\left(\sqrt{\frac{\tau}{\tau+1}}\right)}{\tau^2(1+\tau)} . \quad (20)$$

where $\tau = -\frac{(1-x)^2}{(1-\xi^2)}\frac{u}{4M^2}$. This contribution has an antisymmetric relation between the quark and antiquark contributions:

$$A^{\pi^+\gamma}_{u(\uparrow\uparrow)}(x,\xi,u) = -A^{\pi^+\gamma}_{d(\uparrow\uparrow)}(-x,\xi,u)\,. \quad (21)$$

DGLAP pion-photon TDA

In such a two-body approach, the $\pi \to \gamma$ TDA can be written as

$$A^{\pi^+\gamma} = e\left(e_u A_u^{\pi^+\gamma} + e_d A_d^{\pi^+\gamma}\right), \tag{22}$$

where the labelling is related to the quark flavour involved in the formation of the outgoing photon. For example, the TDA in the DGLAP region $x \geq |\xi|$ in closed form would be

$$A_d^{\pi^+\gamma}(x,\xi,t)\Big|_{x\geq|\xi|} = A_{d(\uparrow\downarrow)}^{\pi^+\gamma}(x,\xi,t)\Big|_{x\geq|\xi|} + A_{d(\uparrow\uparrow)}^{\pi^+\gamma}(x,\xi,t)\Big|_{x\geq|\xi|}, \tag{23}$$

where $(\uparrow\downarrow)$ and $(\uparrow\uparrow)$ labels the quark helicity projections, and thus the different LFWFs contributions to the TDA.

TDAs Modelling:Covariant extension

Next step: Covariant extension!

- Relying on Radon transform $\mathcal{R}$ to map the ERBL region through our knowledge of DGLAP region.
  [N.Chouika et al.-EPJC:906(77)2017]

$$H(x,\xi,t) = \int_{\Omega} d\beta\, d\alpha\, \delta(x-\beta-\alpha\xi)\, h(\beta,\alpha,t) \tag{24}$$

- As GPDs, $\pi \to \gamma$ TDAs benefit from a representation as the Radon transform of double distributions.
- Provided that the solution to the inverse Radon transform problem exists and is unique when TDAs are known only on the DGLAP region, the associated double distribution can be found and employed afterward to reconstruct the ERBL domain.

TDAs Modelling:Covariant extension

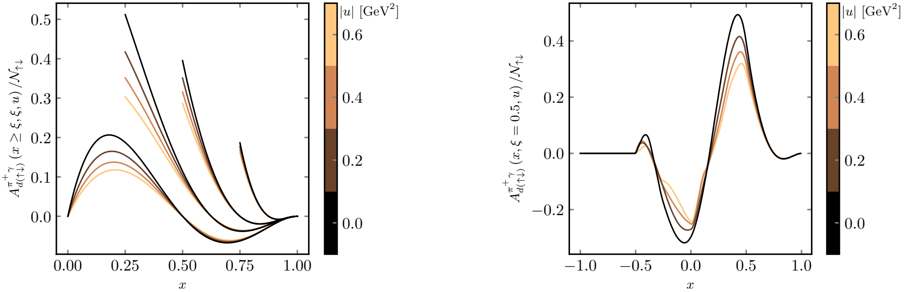
For the case above, Eq. (20), in the $u \to 0$, the double distribution, h, is found to be a polynomial in the kinematic variables (β, α)

$$h^{\pi^+\gamma}_{d(\uparrow\downarrow)}(\beta,\alpha,0) = -\mathcal{N}_{\uparrow\downarrow}\Big[\frac{1}{3}-\frac{10}{3}\alpha-\alpha^2+4\alpha^3-\frac{10}{3}\beta +6\alpha\beta+4\alpha^2\beta+7\beta^2-4\alpha\beta^2-4\beta^3\Big], \quad (25)$$

which yields

$$A^{\pi^+\gamma}_{d(\uparrow\downarrow)}(x,\xi,0)\,|_{x\le|\xi|} = \frac{\mathcal{N}_{\uparrow\downarrow}}{3\xi^4(1+\xi)^3}\Big[x^2\xi^2(5+\xi(20+3\xi)) -x^4(3+\xi(10+11\xi))-2\xi^4(1+\xi) -x\xi^3(1-\xi(8+5\xi))+x^3\xi\left(1-13\xi^2\right)\Big] \quad (26)$$

TDAs modelling:Covariant extension



Left panel: DGLAP region, Eq. (20), obtained from the overlap of pion and photon light-front wave functions. Curves correspond (from left to right) to $\xi \in \{0, 0.25, 0.5, 0.75\}$. Right panel: Full kinematic domain, including the ERBL region obtained through the covariant extension strategy, represented at $\xi = 0.5$.

Experimental Efforts on TDAs

- "Progress and Opportunities in Backward angle (u-channel)" Physics

  [Eur.Phys.J.A 57 (2021) 12, 342]

- "Accessing DEMP and DVCS at Backward Angles above the Resonance Region"

  [Li, Stevens, and Huber. Nucl-ex. 5(2022)]

- Adress different mesons $(\pi, \eta, \eta', \rho, \omega, \phi, K)$ backward electroproduction.
- Observe if the scaling behavior of the cross-section in Q^2 is as predicted by the collinear QCD factorization.
- Test the predictions of TDAs and establish the applicable kinematics range for a backward factorization scheme.
- Lay down a path to continue studying u-channel physics in the future EIC.

Conclusions & Future Work

Summary of we've done!

- The main purpose of this work is to see if it is feasible to measure the Sullivan process in the backward region.
- Based on the overlap LFWF for pion and photon, we are developing a phenomenological study on the backward pion-photon TDA.
- We obtained $\pi \to \gamma$ TDA expression in both DGLAP and ERBL regions.

Conclusions & Future Work

Summary of what we need to do!

- Expect to obtain the backward DVCS amplitude soon after evolving the TDA to relevant experimental scales.
- Evaluated the measurability of Sullivan Backward DVCS at existing and future facilities.
- Future refinements: NLO corrections and addressing other processes like TCS.
- Plan to test the vector $\pi \to \gamma$ TDA by replacing the produced π^+ meson with a longitudinally polarized ρ^+ meson.

Thank you!

Overlap Representation Kinematics

In the DGLAP region ($\xi < x < 1$), the momenta kinematics [2] is given by

$$\begin{aligned} \hat{x}_1 &= \frac{\bar{x}_1 - \xi}{1-\xi}, \quad \hat{x}_2 = \frac{\bar{x}_2}{1-\xi}, \\ \tilde{x}_1 &= \frac{\bar{x}_1 + \xi}{1+\xi}, \quad \tilde{x}_2 = \frac{\bar{x}_2}{1+\xi}, \\ \hat{k}_{1\perp} = \bar{k}_{1\perp} + \frac{1-\bar{x}_1}{1-\xi}\frac{\Delta_\perp}{2} &\quad \hat{k}_{2\perp} = \bar{k}_{2,\perp} - \frac{\bar{x}_2}{1-\xi}\frac{\Delta_\perp}{2} \\ \tilde{k}_{1\perp} = \bar{k}_{1\perp} - \frac{1-\bar{x}_1}{1+\xi}\frac{\Delta_\perp}{2}, &\quad \tilde{k}_{2\perp} = \bar{k}_{2,\perp} + \frac{\bar{x}_2}{1+\xi}\frac{\Delta_\perp}{2}, \end{aligned} \tag{27}$$

where $\bar{k}_i = (k'_i + k_i)/2$ and $\bar{x}_i = \bar{k}_i^+/P^+$. The variable $\bar{x}$ represents the momentum fraction carried by the partons, ξ is the skewedness parameter, and t is the squared momenta transfer.

$$x_1 = \bar{x}_1 + \xi, x'_1 = \bar{x}_1 - \xi; k_{1\perp} = \bar{k}_{1\perp} - \frac{\Delta}{2}, k'_{1\perp} = \bar{k}_{1\perp} + \frac{\Delta}{2}; k_2 = k'_2 = \bar{k}_2,$$

Pion and Photon LFWFS

The pion LFWF used are the ones presented at [1]

$$\psi^{\pi}_{\uparrow,\downarrow}(x,k_\perp) = 8\sqrt{15}\pi\frac{M^3}{(k_\perp^2+M^2)^2}x(1-x), \tag{29}$$

$$ik_\perp\psi^{\pi}_{\uparrow,\uparrow}(x,k_\perp) = 8\sqrt{15}\pi\frac{k_\perp M^2}{(k_\perp^2+M^2)^2}x(1-x), \tag{30}$$

For the photon we developed a LFWF model where the photon to photon GPD were consistent with [3]:

$$\psi_{\gamma\cdot n}(x,k_\perp) = -\mathcal{N}\frac{(1-2x)}{\left(k_\perp^2+M^2\right)}, \tag{31}$$

$$\psi_{\gamma\cdot n\gamma_5}(x,k_\perp) = \mathcal{N}\frac{1}{\left(k_\perp^2+M^2\right)}, \tag{32}$$

$$\psi_{\sigma^{n\perp}}(x,k_\perp) = \mathcal{N}\frac{M}{\left(k_\perp^2+M^2\right)}. \tag{33}$$

References

[1] N. Chouika et al. "A Nakanishi-based model illustrating the covariant extension of the pion GPD overlap representation and its ambiguities". In: *Physics Letters B* 780 (May 2018), pp. 287–293. ISSN: 0370-2693. DOI: 10.1016/j.physletb.2018.02.070. URL: http://dx.doi.org/10.1016/j.physletb.2018.02.070.

[2] M. Diehl et al. "The overlap representation of skewed quark and gluon distributions". In: *Nucl. Phys. B* 596 (2001). [Erratum: Nucl.Phys.B 605, 647–647 (2001)], pp. 33–65. DOI: 10.1016/S0550-3213(00)00684-2. arXiv: hep-ph/0009255.

[3] S. Friot, B. Pire, and L. Szymanowski. "Deeply virtual compton scattering on a photon and generalized parton distributions in the photon". In: *Phys. Lett. B* 645 (2007), pp. 153–160. DOI: 10.1016/j.physletb.2006.12.038. arXiv: hep-ph/0611176.