$\mathfrak{sl}(2,\mathbb{C})$ multipole decomposition of QCD matrix elements for any spin

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Towards improved hadron tomography with hard exclusive reactions

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Matrix elements for composite particles with arbitrary spin

- Decompose matrix element in independent non-perturbative objects while maintaining manifest Lorentz invariance

$$\left\langle d' \left| J^\mu \right| d \right\rangle = -\left(\left\{ G_1 \left[\epsilon'^* \cdot \epsilon \right] - G_3 \frac{\left(\epsilon'^* \cdot q \right) \left(\epsilon \cdot q \right)}{2m_d^2} \right\} 2P^\mu + G_M \left[\epsilon^\mu \left(\epsilon'^* \cdot q \right) - \epsilon'^{*\mu} (\epsilon \cdot q) \right] \right)$$

- Spin-j fields embedded in objects with $> 2j+1$ components
  $\rightarrow$ 4-vector, Rarita-Schwinger, Fierz-Pauli, ...
  - Need for constraints
  - Kinematical singularities

- Use $(2j+1)$-component (chiral) spinors [$(j,0)$ & $(0,j)$ irreps.]
  `[Joos; Barut-Muzinich-Williams; Weinberg 63+]`

Advantages of chiral spinor construction

- Leads to **systematic approach** for any spin j
- "Basic" algebraic construction su(2) $\rightarrow$ su(N) $\rightarrow$ sl(2,$\mathbb{C}$)
- Covariant "multipole" basis emerges $\rightarrow$ physical interpretation
- Parity conserving interactions $\rightarrow$ generalized Dirac algebra
- Easy to implement different types of spin
  $\rightarrow$ canonical, helicity, light-front
- Exact degrees of freedom, no need for constraints

Lorentz group basics

- Algebra for Generators of the Lorentz group

$$[\mathbb{J}_l, \mathbb{J}_m] = i\epsilon_{lmn}\mathbb{J}_n \;, \quad [\mathbb{J}_l, \mathbb{K}_m] = i\epsilon_{lmn}\mathbb{K}_n \;, \quad [\mathbb{K}_l, \mathbb{K}_m] = -i\epsilon_{lmn}\mathbb{J}_n$$

- Two independent su(2) subalgebras $\rightarrow$ irreps (j_A, j_B)

$$\mathbb{A}_m = \frac{1}{2}(\mathbb{J}_m + i\mathbb{K}_m) \quad , \quad \mathbb{B}_m = \frac{1}{2}(\mathbb{J}_m - i\mathbb{K}_m)$$

$$[\mathbb{A}_l, \mathbb{A}_m] = i\epsilon_{lmn}\mathbb{A}_n \;, \quad [\mathbb{B}_l, \mathbb{B}_m] = i\epsilon_{lmn}\mathbb{B}_n \;, \quad [\mathbb{A}_l, \mathbb{B}_m] = 0$$

- Simplest irreps that contain spin-j $\rightarrow$ ($2j + 1$ components)
  - Right-handed $(j, 0)$: $\mathbb{K}_m \rightarrow -i\mathbb{J}_m$
  - Left-handed $(0, j)$: $\mathbb{K}_m \rightarrow +i\mathbb{J}_m$

Weinberg's causal chiral fields (massive)

- Lorentz invariant S-matrix using a Hamiltonian density built up from causal fields

$$U_{[\Lambda,a]}\psi_\sigma(x)U^{-1}_{[\Lambda,a]} = \sum_{\sigma'} \left(D^{(j)}_{[\Lambda^{-1}]} \right)_{\sigma\sigma'} \psi_{\sigma'}(\Lambda x + a)$$

- No EoM for chiral fields (only obey KG eq.)

- Spinors appearing in the fields (not invariants, depend on choice boost)

Canonical $\rightarrow$

$$D^{(j)}_{[L(p)]} = e^{-\eta \hat{p}\cdot \boldsymbol{J}^{(j)}}$$

$$\overline{D}^{(j)}_{[L(p)]} = e^{+\eta \hat{p}\cdot \boldsymbol{J}^{(j)}}$$

Propagator and spinors: t–tensors

- Propagator numerator

$$\Pi^{(j)}_{\sigma\sigma'}(p) = m^{2j} D^{(j)}_{\sigma\sigma'}[L_p] \left(D^{(j)}_{\sigma'\sigma''}[L_p] \right)^{\dagger} = m^{2j} \left(e^{-2\eta\hat{p}\cdot \boldsymbol{J}^{(j)}} \right)_{\sigma\sigma'}$$

$$\overline{\Pi}^{(j)}_{\sigma\sigma'}(p) = m^{2j} \overline{D}^{(j)}_{\sigma\sigma'}[L_p] \left(\overline{D}^{(j)}_{\sigma'\sigma''}[L_p] \right)^{\dagger} = m^{2j} \left(e^{2\eta\hat{p}\cdot \boldsymbol{J}^{(j)}} \right)_{\sigma\sigma'}$$

- Introduction of 2j-rank t–tensors (symmetric/traceless)

$$\Pi^{(j)}_{\sigma\sigma'}(p) = t_{\sigma\sigma'}^{\mu_1\mu_2\ldots\mu_{2j}} p_{\mu_1} p_{\mu_2} \ldots p_{\mu_{2j}}$$

- Central role of t–tensors

  used to construct boosts/spinors

$$D^{(j)}_{[L(p)]} = t^{\mu_1\mu_2\ldots\mu_{2j}} \tilde{p}_{\mu_1} \tilde{p}_{\mu_2} \ldots \tilde{p}_{\mu_{2j}}$$

$$\overline{D}^{(j)}_{[L(p)]} = \bar{t}^{\mu_1\mu_2\ldots\mu_{2j}} \tilde{p}_{\mu_1} \tilde{p}_{\mu_2} \ldots \tilde{p}_{\mu_{2j}}$$

($\tilde{p}^{\mu}$ **not** 4-vectors) Canonical: $\tilde{p}^{\mu}_{C} = \sqrt{\frac{1}{2m(m+p^0)}}(p^0 + m, \boldsymbol{p})$

Same for any spin!

For helicity and LF spinors similar expression (but ℂ–numbers)

t–tensors

- Generalization of $\sigma^\mu = (1, \boldsymbol{\sigma}) \quad \bar{\sigma}^\mu = (1, -\boldsymbol{\sigma}^\mu)$ to arbitrary spin
- Intertwining map:

$$
\begin{aligned}
(j,0)\otimes(0,j) \quad & [\text{rank–2 in SL(2,}\mathbb{C})] \\
& \updownarrow \\
(j,j) \quad & [\text{rank–}2j \text{ symm. traceless in SO(3,1)}]
\end{aligned}
$$

- Recursion relation between different spins (CG)

$$
t^{\mu_1\mu_2\dots\mu_{2j}}_{\sigma\dot{\tau}} = \langle j\sigma|j-\tfrac{1}{2}\sigma_1 \tfrac{1}{2}\sigma_2\rangle \, \langle j\dot{\tau}|j-\tfrac{1}{2}\dot{\tau}_1 \tfrac{1}{2}\dot{\tau}_2\rangle \; t^{\mu_1\mu_2\dots\mu_{2j-1}}_{\sigma_1\dot{\tau}_1} \; t^{\mu_{2j}}_{\sigma_2\dot{\tau}_2}
$$

- Contain a basis of su(N=$2j+1$): use to expand $\langle\lambda'|\hat{O}|\lambda\rangle$.

Bi-spinors $(j,0) \oplus (0,j)$

- For Parity conserving interactions the direct sum of both chiral representations is used, **like the spin 1/2 case**
- The bispinor satisfy the Dirac eq.

$$\left(\gamma^{\mu_1\cdots\mu_{2j}} p_{\mu_1}\cdots p_{\mu_{2j}} - m^{2j}\right) u^{(j)}(p,s) = 0$$

$$\bar{u}^{(j)}(p,s)\left(\gamma^{\mu_1\cdots\mu_{2j}} p_{\mu_1}\cdots p_{\mu_{2j}} - m^{2j}\right) = 0$$

- Gamma matrices (chiral rep.)

$$\gamma^{\mu_1\cdots\mu_{2j}} = \begin{pmatrix} 0 & t^{\mu_1\cdots\mu_{2j}} \\ \bar{t}^{\mu_1\cdots\mu_{2j}} & 0 \end{pmatrix} \quad ; \quad \beta = \gamma^{0\cdots 0} = \begin{pmatrix} 0 & 1^{(j)} \\ 1^{(j)} & 0 \end{pmatrix} \quad ; \quad \gamma_5 = \begin{pmatrix} -1^{(j)} & 0 \\ 0 & 1^{(j)} \end{pmatrix}$$

Algorithm for construction of t-tensors

- Use recursion relation

$$t^{\mu_1\mu_2\cdots\mu_{2j}}_{\sigma\dot{\tau}} = \langle j\sigma|j-\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2\rangle\,\langle j\dot{\tau}|j-\frac{1}{2}\dot{\tau}_1\frac{1}{2}\dot{\tau}_2\rangle\; t^{\mu_1\mu_2\cdots\mu_{2j-1}}_{\sigma_1\dot{\tau}_1}\; t^{\mu_{2j}}_{\sigma_2\dot{\tau}_2}$$

- Efficient in +-RL Lorentz coordinates
  - Pauli matrices have only 1 non-zero element (=2)
  - $(t^{\mu_1\mu_2\cdots\mu_{2j}})_{\lambda'\lambda}$ elements in that basis have only 1 non-zero matrix element
    → position follows from +-RL counting
    → value from CG recursion
    → value depends only on j, λ', λ

- Appropriate for efficient numerical implementation

Reduction for Cubic Monomials

- Central role of the covariant t-tensors
  → spinors, boosts, propagators, gamma matrices
- Bilinear calculus involve products with alternating "barring" pattern: $t\bar{t}t\cdots$
- Matrices in t-tensors: $su(2j+1)$ basis → Products can be **linearized**
- Cubic products $(t\bar{t}t)_{\sigma\dot{\sigma}}$ are reduced with an Invariant Tensor

$$t^{\mu_1\cdots\mu_{2j}}\,\bar{t}^{\rho_1\cdots\rho_{2j}}\,t^{\sigma_1\cdots\sigma_{2j}} = \frac{1}{[(2j)!]^2}\underset{\{\rho_1\ldots\rho_{2j}\}}{\mathcal{S}}\,\underset{\{\sigma_1\ldots\sigma_{2j}\}}{\mathcal{S}}\left(\prod_{l=1}^{2j}\mathcal{C}^{\mu_l\rho_l\sigma_l\alpha_l}\right)t_{\alpha_1\cdots\alpha_{2j}}$$

$$\bar{t}^{\mu_1\cdots\mu_{2j}}\,t^{\rho_1\cdots\rho_{2j}}\,\bar{t}^{\sigma_1\cdots\sigma_{2j}} = \frac{1}{[(2j)!]^2}\underset{\{\rho_1\ldots\rho_{2j}\}}{\mathcal{S}}\,\underset{\{\sigma_1\ldots\sigma_{2j}\}}{\mathcal{S}}\left(\prod_{l=1}^{2j}\bar{\mathcal{C}}^{\mu_l\rho_l\sigma_l\alpha_l}\right)\bar{t}_{\alpha_1\cdots\alpha_{2j}}$$

$$\mathcal{C}^{\mu\rho\alpha\beta} = g^{\mu\rho}g^{\alpha\beta} - g^{\mu\alpha}g^{\rho\beta} + g^{\mu\beta}g^{\rho\alpha} + i\epsilon^{\mu\rho\alpha\beta} = \left(\bar{\mathcal{C}}^{\mu\rho\alpha\beta}\right)^* \quad \text{(Lorentz Invariants)}$$

- **Trade matrix multiplication by number multiplication**

Algebra of t-tensors: Quadratic reduction

Reduction for Quadratic Monomials

- Central role of the covariant t-tensors
  $\rightarrow$ spinors, boosts, propagators, gamma matrices

- Since, $t^{0\cdots 0} = \bar{t}^{\,0\cdots 0} = 1 \quad \longrightarrow \quad t^{\mu_1\cdots\mu_{2j}}\,\bar{t}^{\nu_1\cdots\nu_{2j}} = t^{\mu_1\cdots\mu_{2j}}\,\bar{t}^{\nu_1\cdots\nu_{2j}}\left(t^{\rho_1\cdots\rho_{2j}}\,\eta_{\rho_1}\cdots\eta_{\rho_{2j}}\right)$

$$t^{\mu_1\cdots\mu_{2j}}\,\bar{t}^{\rho_1\cdots\rho_{2j}} = \frac{1}{(2j)!}\underset{\{\rho_1\ldots\rho_{2j}\}}{\mathcal{S}}\left(\prod_{l=1}^{2j}\mathcal{C}^{\mu_l\rho_l\sigma_l\alpha_l}\,\eta_{\sigma_l}\right)t_{\alpha_1\cdots\alpha_{2j}}$$

$\eta^{\mu} = (1,0,0,0)$

$\mathcal{C}^{\mu\rho\sigma\alpha}\,\eta_{\sigma} = g^{\mu\rho}\,\eta^{\alpha} - g^{\rho\alpha}\,\eta^{\mu} + g^{\mu\alpha}\,\eta^{\rho} + i\,\epsilon^{\mu\rho\sigma\alpha}\,\eta_{\sigma}$ (Rotational Invariant)

- General result $\left(\mathcal{Q}_{\text{red}}^{\mu\rho\alpha} = -\mathcal{Q}_{\text{red}}^{\rho\mu\alpha} \equiv \mathcal{C}^{\mu\rho\sigma\alpha}\,\eta_{\sigma} - g^{\mu\rho}\,\eta^{\alpha}\right)$

$$t^{\mu_1\cdots\mu_{2j}}\,\bar{t}^{\rho_1\cdots\rho_{2j}} = \sum_{m=0}^{2j}\frac{1}{(2j)!}\underset{\{\rho_1\ldots\rho_{2j}\}}{\mathcal{S}}\left[\sum_{n=1}^{B_m^{2j}}\left(\prod_{l\in\pi_{m,n}}\mathcal{Q}_{\text{red}}^{\mu_l\rho_l\alpha_l}\ \prod_{k\in\pi_{m,n}^{\complement}} g^{\mu_k\rho_k}\,\eta^{\alpha_k}\right)\right]t_{\alpha_1\cdots\alpha_{2j}}$$

Terms can be linked to decomposition $(t\bar{t})_{\sigma}{}^{\tau} \sim (j,0)\otimes(j,0) \sim \oplus_{k=0}^{2j}\,(k,0)$.

Algebra of t-tensors: $\mathfrak{sl}(2,\mathbb{C})$ multipoles

- The $\mathfrak{sl}(2,\mathbb{C})$ multipole of order m is defined by

$$\mathcal{M}_m^{\mu_1\rho_1,\cdots,\mu_m\rho_m} = \frac{1}{m!}\underset{\{(\mu\rho)\}}{\mathcal{S}}\prod_{r=1}^{m}\mathbb{M}^{\mu_r\rho_r} - (\text{Traces})$$

- Relate terms of quadratic reduction to these multipoles:

$$\mathcal{M}_0 = 1^{(j)} = t_{\alpha_1\cdots\alpha_{2j}}\prod_{r=1}^{0}\prod_{s=1}^{2j}\eta^{\alpha_s} = t_{0\cdots 0}\,,$$

$$\mathcal{M}_1^{\mu\rho} = \mathbb{M}^{\mu\rho} = i\mathcal{Q}_{\text{red}}^{\mu\rho\alpha_1}\left(\prod_{s=2}^{2j}\eta^{\alpha_s}\right)(j)t_{\alpha_1\cdots\alpha_{2j}} = i\left(g^{\mu\alpha}\eta^{\rho} - g^{\rho\alpha}\eta^{\mu} + i\epsilon^{\mu\rho\sigma\alpha}\eta_{\sigma}\right)J_{\alpha} = \begin{pmatrix} 0 & -iJ_1 & -iJ_2 & -iJ_3 \\ iJ_1 & 0 & J_3 & -J_2 \\ iJ_2 & -J_3 & 0 & J_1 \\ iJ_3 & J_2 & -J_1 & 0 \end{pmatrix}$$

$$\begin{aligned}\mathcal{M}_2^{\mu_1\rho_1,\mu_2\rho_2} &= \frac{1}{2}\{\mathbb{M}^{\mu_1\rho_1},\mathbb{M}^{\mu_2\rho_2}\} + \frac{1}{3}j(j+1)\mathcal{C}_{\text{red}}^{\mu_1\rho_1\mu_2\rho_2}1^{(j)} \\ &= \frac{1}{2}j(2j-1)\left(-\mathcal{Q}_{\text{red}}^{\mu_1\rho_1\beta_1}\mathcal{Q}_{\text{red}}^{\mu_2\rho_2\beta_2}\,t_{\beta_1\beta_2 0\cdots 0} + \frac{1}{3}\mathcal{C}_{\text{red}}^{\mu_1\rho_1\mu_2\rho_2}1^{(j)}\right)\end{aligned}$$

- Decompose operators with **physical interpretation** for each term
  $\longrightarrow$ mono-, di-, quadrupole, ...

See also [Cotogno, Lorcé, Lowdon, Morales PRD 2020)]

Dirac Bilinear Calculus Generalization

Generalized Dirac basis (Weyl rep)

$$\gamma^{\mu_1\cdots\mu_{2j}}, \;\; \gamma^{\mu_1\cdots\mu_{2j}}\gamma_5, \;\; \gamma^{\mu_1\cdots\mu_{2j}}\gamma^{\nu_1\cdots\nu_{2j}}$$

Generalized Bilinears

- Chains of $t\bar{t}t\ldots$ contracted with $\tilde{p}^\mu$ and external 4-vectors (P, Δ, n)

$$\bar{u}^{(j)}_{(p_f,s_f)}\Gamma u^{(j)}_{(p_i,s_i)} = \overset{\circ}{u}{}^{(j)\dagger}_{s_f} \begin{pmatrix} 0 & t^{\beta_1\cdots}\tilde{p}^f_{\beta_1\cdots} \\ \bar{t}^{\beta_1\cdots}(\tilde{p}^f_{\beta_1\cdots})^* & 0 \end{pmatrix} \Gamma \begin{pmatrix} t^{\alpha_1\cdots}\tilde{p}^i_{\beta_1\cdots} & 0 \\ 0 & \bar{t}^{\bar{\alpha}_1\cdots}(\tilde{p}^i_{\alpha_1\cdots})^* \end{pmatrix} \overset{\circ}{u}{}^{(j)}_{s_i}$$

Canonical: $\tilde{p}^\mu_{\mathrm{C}} = \sqrt{\frac{1}{2m(m+p^0)}}(p^0 + m, \boldsymbol{p})$

- $2j$-rank Tensor bilinear $\widetilde{P} = \frac{1}{2}(\tilde{p}_f + \tilde{p}_i)$, $\widetilde{\Delta} = \tilde{p}_f - \tilde{p}_i$

$$\bar{u}_f\,\gamma^{\mu_1\cdots\mu_{2j}}\,u_f = m^{2j}\prod_{l=1}^{2j}\left[2\left(\widetilde{P}^{\mu_l}\widetilde{P}^{\tau_l} - \tfrac{1}{4}\widetilde{\Delta}^{\mu_l}\widetilde{\Delta}^{\tau_l}\right) - \left(\widetilde{P}^2 - \tfrac{1}{4}\tilde{\Delta}^2\right)g^{\mu_l\tau_l} + i\varepsilon^{\mu_l\tau_l\widetilde{P}\widetilde{\Delta}}\right]\langle\lambda_f|t_{\tau_1\cdots\tau_{2j}}|\lambda_i\rangle$$

$$+m^{2j}\prod_{l=1}^{2j}\left[2\left(\widetilde{P}^{\mu_l}\widetilde{P}^{\tau_l} - \tfrac{1}{4}\widetilde{\Delta}^{\mu_l}\widetilde{\Delta}^{\tau_l}\right) - \left(\widetilde{P}^2 - \tfrac{1}{4}\tilde{\Delta}^2\right)g^{\mu_l\tau_l} + i\varepsilon^{\mu_l\tau_l\widetilde{P}\widetilde{\Delta}}\right]^*\langle\lambda_f|\bar{t}_{\tau_1\cdots\tau_{2j}}|\lambda_i\rangle$$

- Generalized Gordon identities reduces number of independent bilinears

Generalized Gordon Identities

- Use Dirac Equation $\left(\gamma^{\mu_1\ldots\mu_{2j}} p_{\mu_1}\ldots p_{\mu_{2j}} - m^{2j}\right) u_p^s = 0$

$$\bar{u}_{p'}^{s'}\left(\Gamma\right) u_p^s = \frac{1}{2\bar{m}^{2j}} u_{p'}^{s'} \left(\left\{ \not{P}^{(j)}, \Gamma \right\} + \frac{1}{2}\left[\not{\Delta}^{(j)}, \Gamma \right] \right) u_p^s$$

$$0 = \bar{u}_{p'}^{s'} \left(\frac{1}{2}\left\{ \not{\Delta}^{(j)}, \Gamma \right\} + \left[\not{P}^{(j)}, \Gamma \right] \right) u_p^s$$

$$\Gamma = 1, \gamma_5, \gamma^{\mu_1\ldots\mu_{2j}}, \gamma^{\mu_1\ldots\mu_{2j}}\gamma_5, \gamma^{\mu_1\ldots\mu_{2j}}\gamma^{\nu_1\ldots\nu_{2j}}\ \text{[A,S]}$$

$$P_{\mu_1\ldots\mu_{2j}} = \frac{1}{2}\left(p'_{\mu_1}\ldots p'_{\mu_{2j}} + p_{\mu_1}\ldots p_{\mu_{2j}}\right)$$

$$\Delta_{\mu_1\ldots\mu_{2j}} = p'_{\mu_1}\ldots p'_{\mu_{2j}} - p_{\mu_1}\ldots p_{\mu_{2j}}$$

$$P^{\mu_1\ldots\mu_{2j}}\Delta_{\mu_1\ldots\mu_{2j}} = 0$$

$$P^{\mu_1\ldots\mu_{2j}}_{(p',p)} = +P^{\mu_1\ldots\mu_{2j}}_{(p,p')}$$

$$\Delta^{\mu_1\ldots\mu_{2j}}\left(p',p\right) = -\Delta^{\mu_1\ldots\mu_{2j}}\left(p,p'\right)$$

- Useful to reduce independent Dirac structures
- Rewrite independent terms to those with sl(2,C) multipoles appearing

Local EM current: spin-1 example

- Local current has the form

$$\langle p_f, \lambda_f | j^\mu(0) | p_i, \lambda_i \rangle = \bar{u}(p_f, \lambda_f)\, \Gamma^\mu(P, \Delta)\, u(p_i, \lambda_i)\,.$$

- Using all constraints

$$\Gamma^\mu = P^\mu \left(F_C(\Delta^2)\mathcal{M}_0 + F_Q(\Delta^2)\mathcal{M}_2^{\nu\rho,\xi\sigma} g_{\rho\sigma} \frac{\Delta_\nu \Delta_\xi}{M^2} \right) + \frac{i}{2M} F_D(\Delta^2)\, \mathcal{M}_1^{\mu\rho} \Delta_\rho$$

  - Monopole $\mathcal{M}_{\mathbf{0}} = \begin{pmatrix} 1^{(j)} & 0 \\ 0 & 1^{(j)} \end{pmatrix} = \begin{pmatrix} t_{\mathbf{00}} & 0 \\ 0 & \bar{t}_{\mathbf{00}} \end{pmatrix}$
  - Dipole $\mathcal{M}_{\mathbf{1}}^{\mu\rho} = \begin{pmatrix} \mathbb{M}^{\mu\rho} & 0 \\ 0 & \overline{\mathbb{M}}^{\mu\rho} \end{pmatrix} = \begin{pmatrix} \mathrm{i}(j)\mathcal{Q}_{\text{red}}^{\mu\rho\alpha}\, t_{\alpha\mathbf{0}} & 0 \\ 0 & -\mathrm{i}(j)\overline{\mathcal{Q}}_{\text{red}}^{\mu\rho\alpha}\, \bar{t}_{\alpha\mathbf{0}} \end{pmatrix}$
  - Quadrupole

$$\mathcal{M}_{\mathbf{2}}^{\mu_{\mathbf{1}}\rho_{\mathbf{1}},\mu_{\mathbf{2}}\rho_{\mathbf{2}}} = -\frac{j(2j-1)}{2} \begin{pmatrix} \mathcal{Q}_{\text{red}}^{\mu_{\mathbf{1}}\rho_{\mathbf{1}}\beta_{\mathbf{1}}} \mathcal{Q}_{\text{red}}^{\mu_{\mathbf{2}}\rho_{\mathbf{2}}\beta_{\mathbf{2}}}\, t_{\beta_{\mathbf{1}}\beta_{\mathbf{2}}} + \frac{1}{3}\mathcal{C}_{\text{red}}^{\mu_{\mathbf{1}}\rho_{\mathbf{1}}\mu_{\mathbf{2}}\rho_{\mathbf{2}}} 1^{(j)} & 0 \\ 0 & \overline{\mathcal{Q}}_{\text{red}}^{\mu_{\mathbf{1}}\rho_{\mathbf{1}}\beta_{\mathbf{1}}} \overline{\mathcal{Q}}_{\text{red}}^{\mu_{\mathbf{2}}\rho_{\mathbf{2}}\beta_{\mathbf{2}}}\, t_{\beta_{\mathbf{1}}\beta_{\mathbf{2}}} + \frac{1}{3}\overline{\mathcal{C}}_{\text{red}}^{\mu_{\mathbf{1}}\rho_{\mathbf{1}}\mu_{\mathbf{2}}\rho_{\mathbf{2}}} 1^{(j)} \end{pmatrix}$$

- Bilinear expressions can be evaluated using t-algebra relations.

Applications

- Using basis of bilinears and Gordon identities we can identify minimal set of independent bilinears
- These will form the basis in decompositions of matrix elements of QCD operators (currents/correlators)
- Has multipole interpretation, construction is **identical** for all spin cases
- **Unified framework** to discuss spin in hadronic physics
- Intuition from spin-1/2 carries over
- Extensions possible to transition matrix elements

Summary

- Construction allows for efficient and manifestly covariant calculations
- Central role of the covariant t-tensors
  $\rightarrow$ spinors, boosts, propagators, gamma matrices
- Very simple "basis ingredient"
  $\rightarrow$ reps of generators of rotations
- Covariant ***sl***(2,C)-multipole basis for operators
  $\rightarrow$ transparent interpretation
- Unique framework for **any spin**
  $\rightarrow$ intuition from spin 1/2
- Avoids calculations with (Dirac) matrices
  $\rightarrow$ everything reduces to number multiplication ($\mathcal{C}^{\mu\rho\sigma\alpha}$, $\mathcal{Q}^{\mu\rho\alpha}$)