QED Corrections for 3D Imaging of a Proton

ECT* Workshop

TOWARDS IMPROVED HADRON TOMOGRAPHY WITH HARD EXCLUSIVE REACTIONS

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SIDIS Calculations were performed by Stinson Lee (GWU), a graduate student
Coordination with Harut Avakian (JLab) is acknowledged

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- Motivation & Introduction
- Background
- Assumptions & Calculations
- Results
- Conclusions

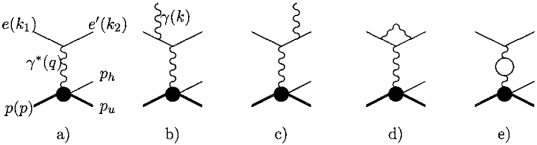
Radiative Corrections for Exclusive Processes

- Photon emission is a part of any electron scattering process: accelerated charges radiate;
- Exclusive electron scattering processes such as p(e,e'h_1)h_2 are actually inclusive p(e,e'h_1)h_2 nγ, where an infinite number of low-energy photons can be generated
- Low-energy photons do not affect polarization observables, thanks to Low theorem

QED Corrections for Electroproduction of Pions

Afanasev, Akushevich, Burkert, Joo, Phys.Rev.D66, 074004 (2002)

- Conventional RC, precise treatment of phase space, no peaking approximation, no dependence on hard/soft photon separation; extension to DVMP is straightforward;
- Can be used for any exclusive electroproduction of 2 hadrons, e.g., d(e,e'p)n (EXCLURAD code)



- Fortran code EXCLURAD is available at www.jlab.org/RC
- Used for data analysis at JLab, COMPASS, MAMI,...

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QED Corrections for Electroproduction of Pions

Sample results from EXCLURAD

- QED corrections to unpolarized cross sections reach tens of per cent
- Corrections are dependent on both polar and azimuthal angles of outgoing hadron (pion), which affects extraction of resonance parameters in the resonance region and GPDs in the deep-virtual region

AFANASEV *et al.* PHYSICAL REVIEW D **66**, 074004 (2002)

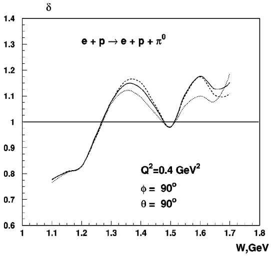

FIG. 3. *W* dependence of RC to the cross section of neutral pion

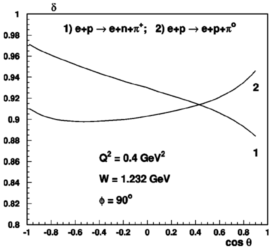

FIG. 5. RC to the cross section as a function of cos θ.

- QED corrections due to real-photon emission are smaller for polarization asymmetries

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Two-photon Exchange Corrections for Inclusive and Exclusive Processes

- Ge/Gm polarization vs Rosenbluth discrepancy is agreed to be partly due to two-photon exchange (resulting from about 5 per cent missing systematic correction at high momentum transfers (see for review A Afanasev, PG Blunden, D Hasell, BA Raue, Prog. Part. Nucl. Phys., 2017
- JLab experiment Katich et al., Phys.Rev.Lett. 113 (2014)022502 reveals about 5 per cent polarization asymmetries in DIS on 3He that are zero in one-photon exchange approximation
- Proposed positron beamline at JLab will provide a direct probe for two-photon effects via measurements of electron-positron asymmetries

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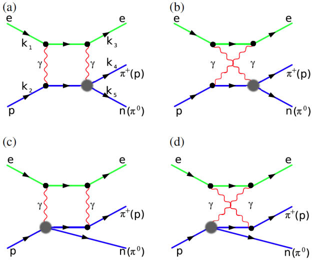
Two-Photon Exchange Corrections for Electroproduction of Pions

Afanasev, Aleksejevs, Barkanova, Phys.Rev. D88: 053008, 2013

- Calculated previously neglected QED corrections from two-photon exchange
- Used a soft-photon approximation, results expressed in terms of Passarino-Veltman integrals



- Computed corrections result in about 5 per cent variation of cross section from backward to forward scattering angles
- Conclusion: Important for the analysis of angular dependences, cos(ϕ) moments in particular

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Two-Photon Exchange Corrections for Electroproduction of Pions

Afanasev, Aleksejevs, Barkanova, Phys.Rev. D88: 053008, 2013

- Angular dependencies of two-photon corrections affect σ_L/σ_T extraction

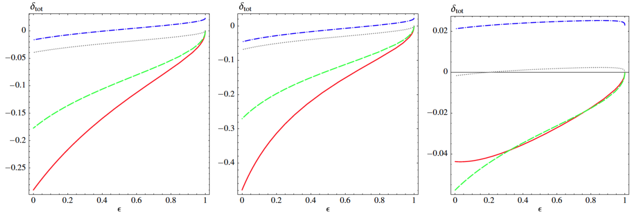

FIG. 5 (color online). π^0 electroproduction two-photon box correction (for detected proton) dependencies on virtual photon degree of polarization parameter ϵ for momentum transfers $Q^2 = 3.0\ \mathrm{GeV}^2$ (left plot), $Q^2 = 7.0\ \mathrm{GeV}^2$ (middle plot), and $Q^2 = 0.4\ \mathrm{GeV}^2$ (right plot). All plots are given for $\phi_4 = 90°$ and $\theta_4 = 90°$ and $W = 1.232$ GeV. Dot-dashed curve, SPT; dotted curve, SPT with $\alpha\pi$ subtracted; dashed curve, SPMT; solid curve, FM approach.

- These effects can be directly measured with proposed positron beamline at JLab
- Two-photon correction times two = electron-positron scattering asymmetry

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Theory Challenges

- Both soft and hard photons are present
- Soft photons do not resolve the quark/parton structure
- Soft/hard scale separation is necessary in the loop integral
- We used Grammer-Yennie procedure for soft/hard separation - as in AV Afanasev, SJ Brodsky, CE Carlson, YC Chen, M Vanderhaeghen, PRD72, 013008 (2005)
- The results become dependent on soft-hard separation scheme, QED and QCD have to be consistently combined
- Not all of the contributions are factorizable in terms of GPDs

Semi-Inclusive electroproduction and TMD studies

x-section for eN→e'hX assuming one-photon exchange

from Bacchetta et al, 1703.10157

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}$$

SIDIS phenomenology based on several assumptions[1], including:

- One-photon exchange dominates;
- Transverse photon cross section dominates, and F_{UU}^{L} can be ignored.

[1] Bacchetta et al. 1703.10157 [1]

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TMDs in SIDIS

Assuming the one-photon exchange and dominance of the transverse photon. SIDIS phenomenology for last decades was extracting the underlying transverse momentum dependent (TMD) distribution and fragmentation functions from multiplicities and single spin asymmetries in SIDIS.

Analysis of multiplicities was done based on factorization of the x-section from transverse part.

TMD Parton Distribution Functions   TMD Parton Fragmentation Functions

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = x\sum_q \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\,\delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

hard scattering

from Bacchetta et al, 1703.10157

Several JLab proposals focused on extraction of the longitudinal photon contributions.

No measurement so far available for evaluation of systematics from two-photon exchange.

SIDIS cross section: separating $F_{UU,L}$

Semi-Inclusive: from Avakian et al., Eur. Phys. J. A (2016) **52**: 150

ratio of longitudinal and transverse photon flux

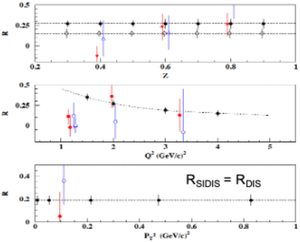
$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{ F_{UU,T} + \varepsilon F_{UU,L} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+ S_{\|}\left[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] + S_{\|}\lambda_e\sqrt{1-\varepsilon^2}\,F_{LL}\Big\}$$

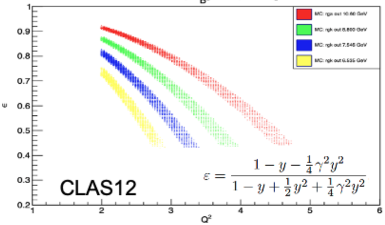
Hall-C E12-06-104
E12-23-014
Hall-B E12-16-010C

Separation of contributions from longitudinal and transverse photons critical for interpretation

Expected E12-06-104 assume $R=F_{UU,L}/F_{UU,T}$



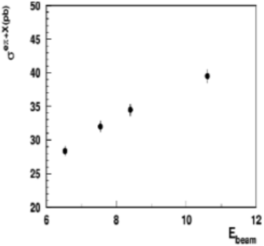
Wide ε-coverage needed!!!

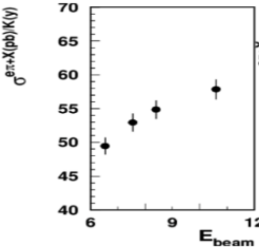


So far the P_T-dependence is neglected (DIS=SIDIS)!!

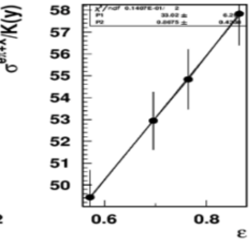
$F_{UU,L}$ studies at Hall-B

$$\frac{d\sigma}{dx dQ^2 dz dP_T} = GK(y)\left(F_{UU,T} + \epsilon F_{UU,L}\right)$$

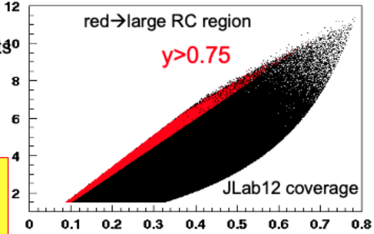




Measurements in a given small bin in x(0.3-0.32) and Q^2(2.4-2.6 GeV^2)



For evaluation of the longitudinal part, measurements at different beam energies are needed to fit the x-sections as a function of epsilon and extract the $F_{UU,L}$(or the ratio R)



Understanding the systematics from two-photon exchange, in particular at large P_T is critical for evaluation of systematics of TMD extractions

Considering the correction δ^{TPE},

$$\frac{d\sigma_{tot}}{dxdzdQ^2d^2P_T} \equiv d\sigma_{tot} = d\sigma_{exp}/(1+\delta^{TPE})$$
$$\sim (1-\delta^{TPE})\{K(y)[(1+\epsilon\frac{F_{UU,L}}{F_{UU,T}}) + \sqrt{2\epsilon(1+\epsilon)}\cos 2\phi\frac{F_{UU}^{\cos(2\phi)}}{F_{UU,T}}$$
$$+\epsilon\cos\phi\frac{F_{UU}^{\cos\phi}}{F_{UU,T}}]\} \tag{1}$$

with x is Bjorken-x, transverse momentum of the detected meson P_T, Q^2 relates to the momentum transfer of the virtual photon.

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$$K(y) = 1 - y + y^2/2 + \gamma^2 y^2/4 \quad (2)$$

$$\epsilon = \frac{1 - y - \gamma^2 y^2}{K(y)} \quad (3)$$

$$\gamma = 2Mx/Q \quad (4)$$

$$\nu = E_{lab} - E' \quad (5)$$

$$x = \frac{Q^2}{2M\nu} \quad (6)$$

$$y = \nu/E \quad (7)$$

$$z = E_h/\nu \quad (8)$$

$$P_T = P_h \sin(\theta_{h,\gamma}) \quad (9)$$

where E_{lab} and E' are the energies of the incoming electron beam and the scattered electron, respectively.

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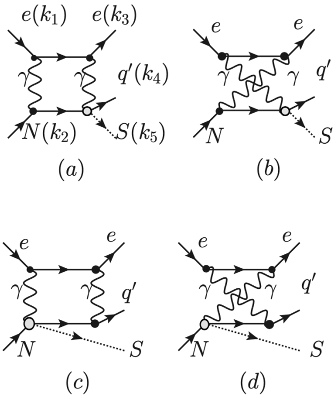
Assumptions & Calculations

$$e(k_1) + N(k_2) \to e(k_3) + q'(k_4) + S(k_5),$$



For quark-diquark model, q' represents quark and S represents diquark.

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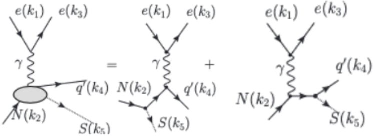
Assumptions & Calculations



Born-level one photon models, which equals to the sum of the "quark graph" and the "proton pole graph". q' and S stand for quark and diquark.[2]

[2] Afanasev and Carlson Phys. Rev. D 74.114027[2].

Using soft-photon approximation (SPT [3]) by neglecting the momentum for one of the photon while calculating the amplitude, such that

$$M^{2\gamma} = M^{1\gamma} \cdot \sum_{l} [\frac{-e^2}{2\pi} \cdot \sum_{i,j} (2k_i \cdot k_j) \cdot C_0(\{k_i, m_i\}, \{\mp k_j, m_j\}) \quad (10)$$

$$= \sum_{l=N,q',s} \sum_{i=a,b,c} M^{1\gamma} M_{l,i,box}, \quad (11)$$

where the Passarino-Veltman three-point scalar integral

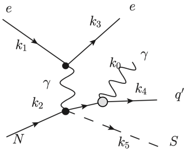
$$C_0(\{k_i, m_i\}, \{k_j, m_j\}) = \frac{1}{i\pi^2} \int d^4q \frac{1}{q^4} \cdot \frac{1}{(k_i - q)^2 - m_i^2} \cdot \frac{1}{(k_j - q)^2 - m_j^2}. \quad (12)$$

The correction

$$\delta_{box} = \frac{2Re[M^{2\gamma} M^{1\gamma\dagger}]}{|M^{1\gamma}|^2} = 2Re[\sum_{l;i} M_{l,i,box}]. \quad (13)$$

Assumptions & Calculations

Regularization of the infrared divergent integrals.



One of the possibilities for the Bremsstrahlung process [4].

[4] Afanasev et al. Phys. Rev. D 88, 053008 [3]

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The IR-divergence cancellation approximation of the photon in the numerator, the correction is

$$\delta_\gamma \sim \sum \frac{\alpha}{(2\pi)^2}(k_i \cdot k_j) I(k_i, k_j), \tag{14}$$

where $i, j = 1, 2, 3, 4$ correspond to the momenta from the Feynman diagram, and k_0 is the momentum of the virtual photon in Bremsstrahlung process.

$$I(k_i, k_j) = \int \frac{d^3 k_0}{\sqrt{\mathbf{k_0}^2 + \lambda^2}} \frac{1}{(k_i \cdot k_0)(k_j \cdot k_0)}. \tag{15}$$

Therefore,

$$\boxed{\delta^{TPE} = \delta_{box}^{TPE} + \delta_\gamma} \tag{16}$$

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$$d\sigma_{tot} \sim (1-\delta^{TPE})\{K(y)[(1+\epsilon\frac{F_{UU,L}}{F_{UU,T}})+\sqrt{2\epsilon(1+\epsilon)}\cos 2\phi\frac{F_{UU}^{\cos(2\phi)}}{F_{UU,T}}$$
$$+\epsilon\cos\phi\frac{F_{UU}^{\cos\phi}}{F_{UU,T}}]\} \quad (17)$$

The moments of $\cos(n\phi)$,

$$\langle\cos(n\phi)\rangle \sim \int d\sigma_{exp}d\phi(1-\delta^{TPE})\cos(n\phi) \quad (18)$$

The $\cos(n\phi)$ moments with the corrected terms only,

$$\langle\delta\cos(n\phi)\rangle \sim \int d\sigma_{exp}d\phi\delta^{TPE}\cos(n\phi) \quad (19)$$

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- $E_{lab} = 10.6$ GeV;
- $Q^2 \approx 2.5$ GeV^2;
- $y < 0.75$ to avoid the region most susceptible to radiative effects and lepton-pair symmetric background;
- $x = 0.31$ (the invariant mass $W \approx 2.7$ GeV);
- $z = 0.5$;
- The polar angle of the detected meson is $\cos\theta = 0.8$ ($P_T \approx 0.35$) for P_T independent figures;
- The azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent;
- $F_{UU,L}/F_{UU,T} \approx 0.2$;
- $F_{UU}^{\cos\phi}/F_{UU,T} \approx -0.05$;
- $F_{UU}^{\cos(2\phi)}/F_{UU,T} \approx 0.1$.

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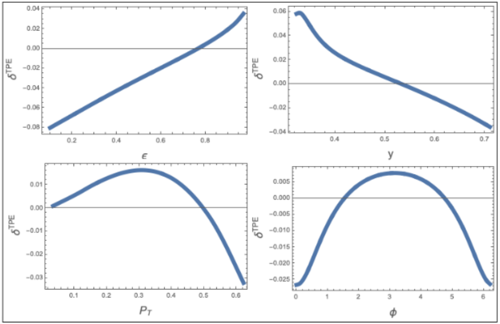
Results



$Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, $P_T \approx 0.35$ for P_T independent figures, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of spectator $m_x = 2$ GeV. Using data from JLab E12-06-104 [5].

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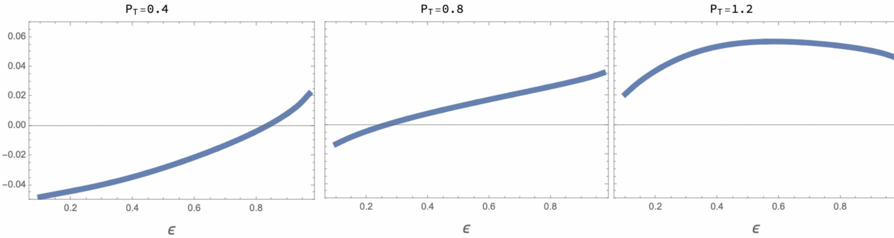
Results



$Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of spectator $m_x = 2$ GeV.

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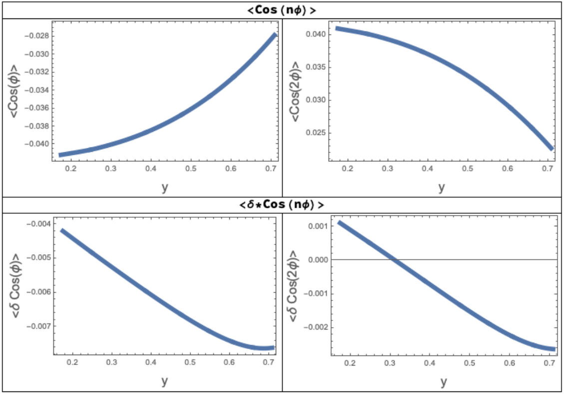
Results



The cosine moments in terms of y. $Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, $P_T \approx 0.35$, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of spectator $m_x = 2$ GeV. Using data from JLab E12-06-104 [5].

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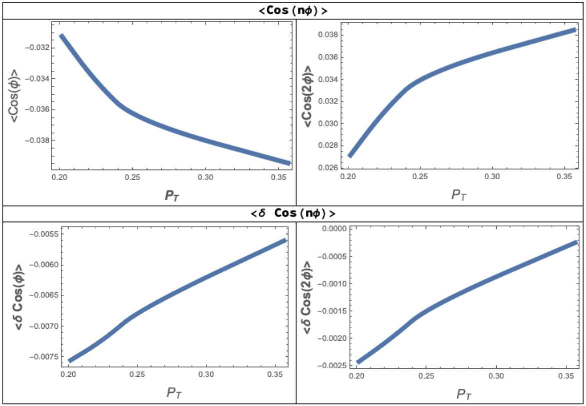
Results



The cosine moments in terms of transverse momentum of the detected meson. $Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of spectator $m_x = 2$ GeV. Using data from JLab E12-06-104 [5].

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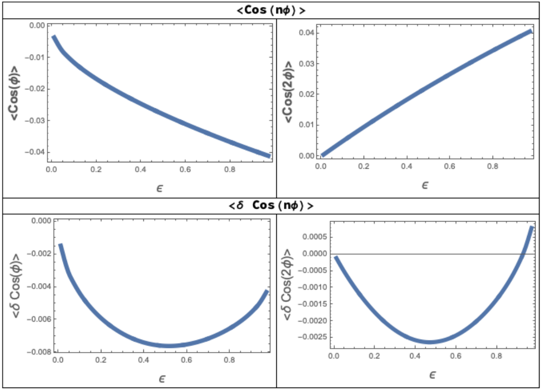
Results



The cosine moments in terms of polarization factor ϵ. $Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, $P_T \approx 0.35$, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of spectator $m_x = 2$ GeV. Using data from JLab E12-06-104 [5].

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Conclusion

- Two-photon effects alter angular dependence of cross sections in DVMP and SIDIS
- Their measurement is necessary for validation of the phenomenology to extract GPDs, 3D PDFs and FFs;
- For SIDIS, TPE corrections of the two-photon exchange are in the interval of $-4(-8) \sim 5\%$ for y, ϵ & P_T dependents;
- The corrections can affect the moments of $\cos(\phi)$ by nearly $0.5 \sim 1\%$ and 0.3% for $\langle\cos(2\phi)\rangle$;
- The experiments of multiparticle final-state observables in a multidimensional space in x, Q^2, z, P_T with the electron beam energies of $6.5, 7.5, 8.5, 10.5$ GeV have been measured at JLab;

The importance of calculating the cosine moments, $\langle\cos(n\phi)\rangle$:

- It is crucial for probing the transverse momentum distribution of partons;
- Constraining quark and gluon polarization;
- Testing quantum chromodynamics (QCD) factorization theorems;

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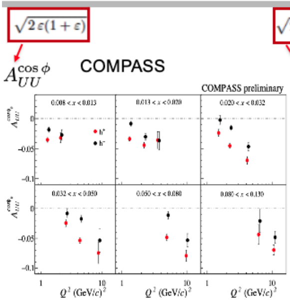
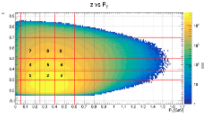
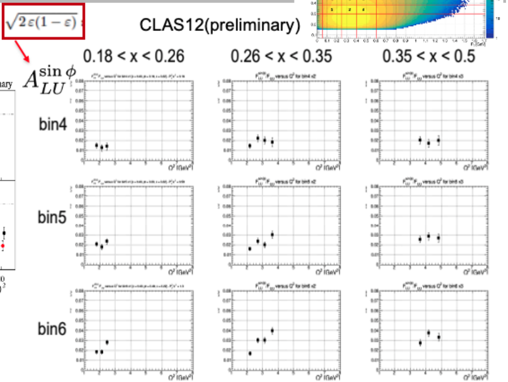
Supporting Slides

Supporting

Attempts to understand Q^2-dependence of HT







- We always measure ratio to $F_{UU,T} + \varepsilon F_{UU,L}$

- The moments defined as a ratio to ϕ-independent x-section(to $F_{UU,T}$), are not decreasing with Q!!!
- The HT observables, don't look much like HT observables, something missing in understanding
- **Understanding of these behavior can be a key to understanding of other inconsistencies**
- Checking the Q^2 and P_T-dependences of the $F_{UU,L}$ may provide crucial input for validation

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Longitudinal photon contributions in SIDIS

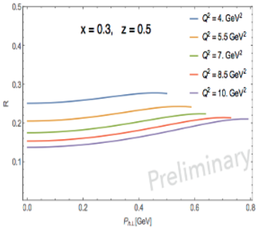
A. Bacchetta

	observable	twist (low P_T)	twist (high P_T)
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
"Cahn" - $f^\perp$	$F_{UU}^{\cos\phi_h}$	3	2
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^\perp$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	3
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

There are several possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF
- Twist 2 TMD matching twist 3 PDF
- Expected mismatch
- Twist 4 TMD matching twist 2 PDF?

→ Contributes everywhere,
→ we know nothing!!!



$$\frac{d\sigma}{dx\,dy\,dz\,dP_{h\perp}^2} = \frac{2\pi\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{2\pi F_{UU,T}(x,z,P_{h\perp}^2,Q^2) + \varepsilon\,2\pi F_{UU,L}(x,z,P_{h\perp}^2,Q^2)\right\}$$

$$\frac{d\sigma}{dx\,dy\,dz} = \frac{4\pi\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T}(x,z,Q^2) + \varepsilon\,F_{UU,L}(x,z,Q^2)\right\}$$

$$R = \frac{F_{UU,L}}{F_{UU,T}}$$

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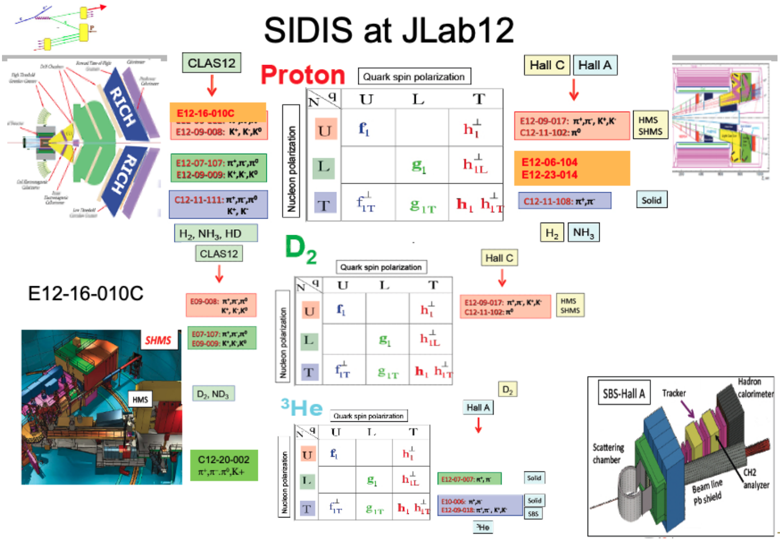
Supporting



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TMDs in Semi-Inclusive DIS

TMD Parton Distribution Functions

TMD Parton Fragmentation Functions

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = x\sum_q \mathcal{H}^q_{UU,T}(Q^2,\mu^2) \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

Major advance in theory in last years

$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} f_1^q(x,k_\perp^2;\mu_f,\zeta_f)$$

perturbative Sudakov form factor

$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = [C\otimes f_1](x,\mu_{b_*})\, e^{\int_{\mu_{b_*}}^{\mu_f}\frac{d\mu}{\mu}\left(\gamma_F-\gamma_K \ln\frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\rm resum}+g_K} f_{1\,NP}(x,b_T^2;\zeta_f,Q_0)$$

collinear PDF

matching coefficients (perturbative)

Collins-Soper kernel (perturbative and nonperturbative)

nonperturbative part of TMD

$$g_K(b_T^2) = -g_2^2\frac{b_T^2}{4}$$

CS kernel discribes the interaction of out-going parton with the confining potential
Provides nonperturbative part of evolution for TMDs

CS-kernel→independent on any other variables

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Possible sources of large P_T behavior

1. Perturbative contributions and P_T-dependence of unpolarized FFs (so far unlikely...);
2. Significantly wider in k_T distributions of u-quarks with spin opposite to proton spin (possible sign flips in asymmetries related to polarization of partons);
3. Significantly wider in k_T distributions of d-quarks (possible sign flips in asymmetries related to polarization of partons);
4. Significantly wider in k_T sea quark distributions (study contributions dominated by sea, K^-,..);
5. Increasing fraction of hadrons due to F^L_{UU} (needed for proper interpretation$\rightarrow$separation of F^L_{UU} from total);
6. Significant contributions from VMs to low P_T pion multiplicities, with direct pions showing up at large P_T (needed for proper interpretation$\rightarrow$much wider in k_T original parton distributions);
7. Radiative corrections (need the full x-section, typically applied to pions, while may be needed for underlying VMs,...).

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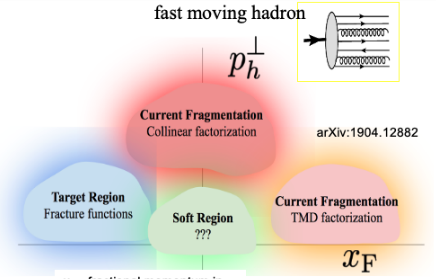
Kinematical regions in SIDIS



x_F – fractional momentum in the CM frame

1) Theory works well for $q_T/Q<0.25$,
2) Kinematic regions not trivial to separate, in particular for polarized measurements
3) Theoretical separation of kinematic region requires some assumptions (no decays,…)
4) Multi-dimensional measurements critical, requiring high lumi

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What we learned: missing parts of the mosaic

- SIDIS, with hadrons detected in the final state, from experimental point of view, is a measurement of observables in 5D space (x,Q^2,z,P_T,ϕ), 6D for transverse target, $+\phi_S$
  Collinear SIDIS, is just the proper integration, over P_T,ϕ,ϕ_S
- SIDIS observations relevant for interpretations of experimental results:
  1. Understanding the kinematic domain where non-perturbative effects of interest are significant (ex. x,P_T-range)
  2. Understanding of P_T-dependences of observables in the full range of P_T dominated by non-perturbative physics is important
  3. Understanding of phase space effects is important (additional correlations)
  4. Understanding the role of vector mesons is important
  5. Understanding of evolution properties and longitudinal photon contributions
  6. Understanding of radiative effects may be important for interpretation
  7. Overlap of modulations (acceptance, RC,…) is important in separation of SFs
  8. Multidimensional measurements with high statistics, critical for separation of different ingredients

- QCD calculations may be more applicable at lower energies when 1)-7) clarified

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The End

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