

# Probing quark & gluon Orbital Angular Momentum

**Shohini Bhattacharya**

Los Alamos National Laboratory

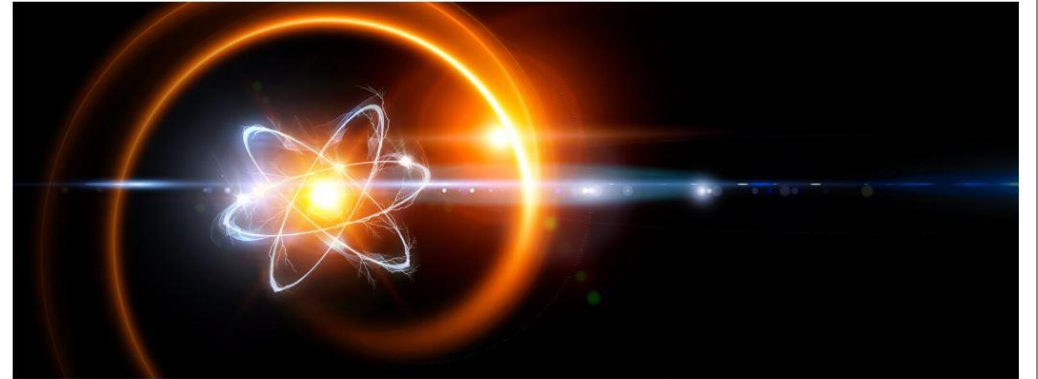
August 5, 2024

In Collaboration with:

**Duxin Zheng, Jian Zhou (PRL 133, 051901)**

**Renaud Boussarie, Yoshitaka Hatta (PRL 128, 182002 & arXiv: 2404.04209)**

TOWARDS IMPROVED HADRON  
TOMOGRAPHY WITH HARD  
EXCLUSIVE REACTIONS



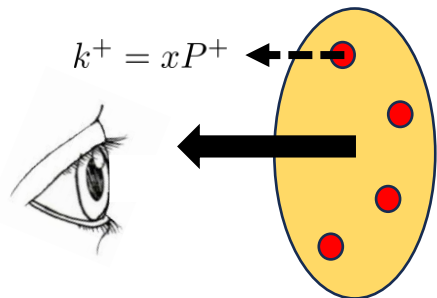


# Outline

- **Generalized TMDs & connection to spin physics**
- **Observable(s) for quark/gluon orbital angular momentum**
- **Summary**



# Wigner function - The “mother function”

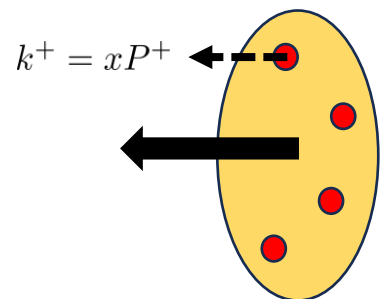


**Parton Distribution Functions**

**PDFs** ( $x$ )



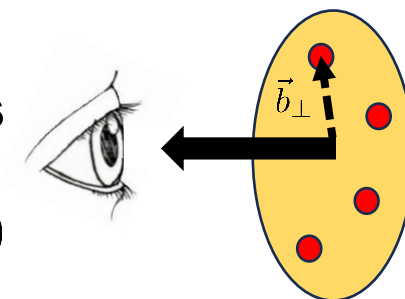
# Wigner function - The “mother function”



PDFs ( $x$ )

Form Factors

FFs ( $\Delta$ )



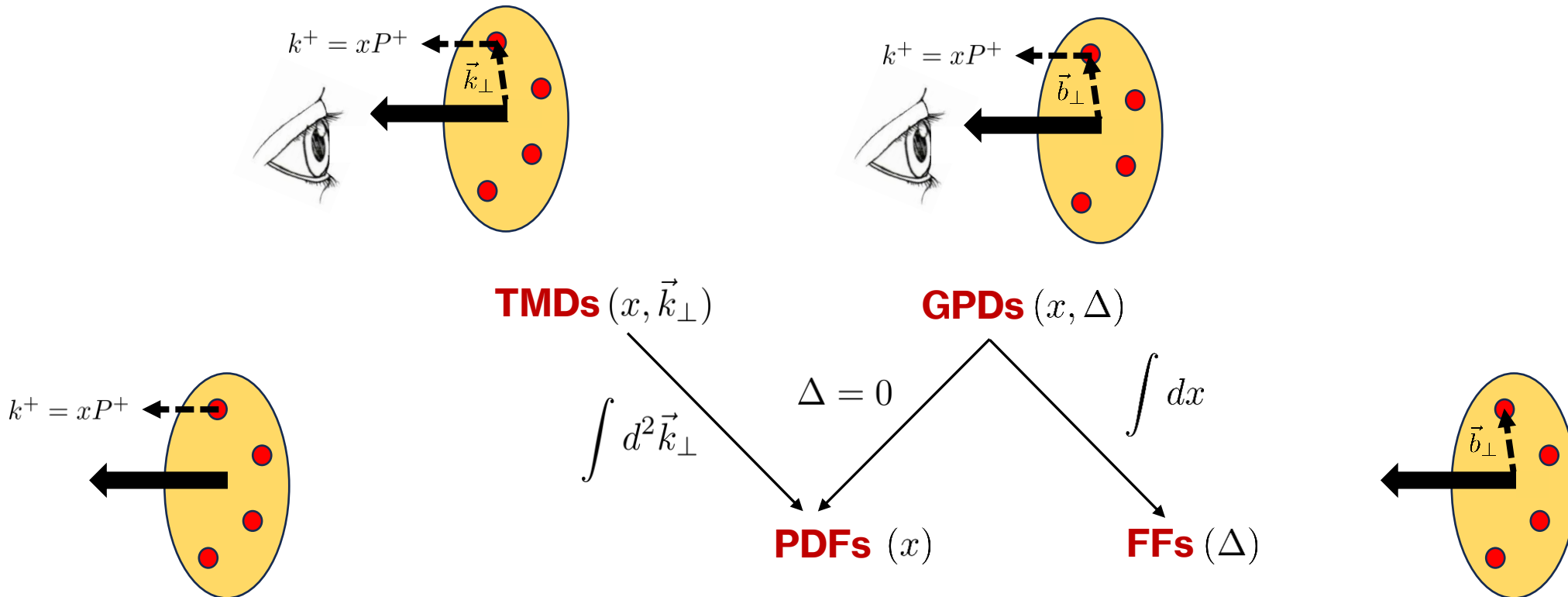


# Wigner function - The “mother function”



## Transverse Momentum-dependent Distributions

## Generalized Parton Distributions



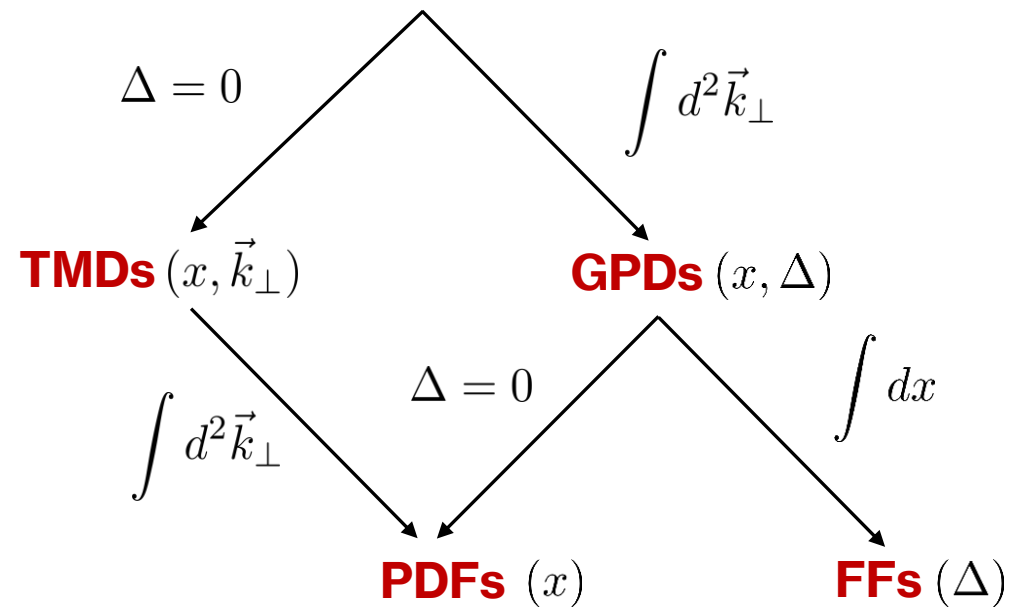


# Wigner function - The “mother function”

## Generalized **T**ransverse **M**omentum-dependent **D**istributions

(Meissner, Metz, Schlegel, 2009)

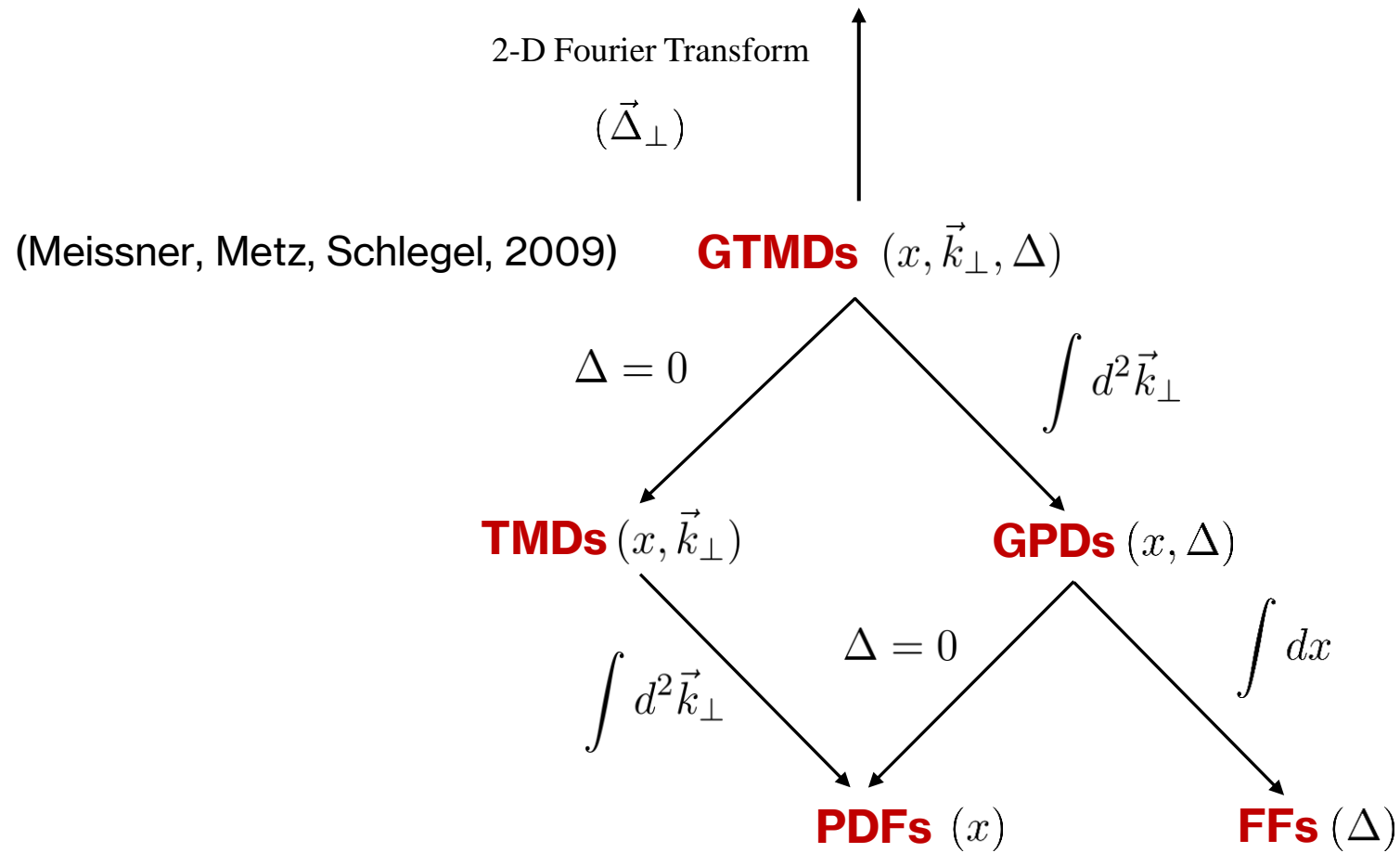
**GTMDs**  $(x, \vec{k}_\perp, \Delta)$





# Wigner function - The “mother function”

**Wigner functions**  $(x, \vec{k}_\perp, \vec{b}_\perp)$  (Belitsky, Ji, Yuan, 2003)

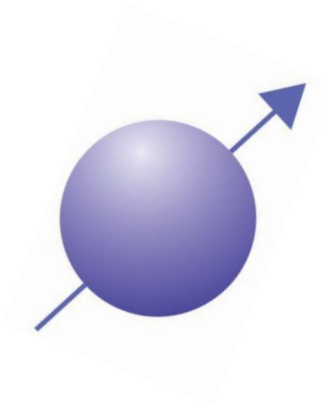


# Spin of proton



## Jaffe-Manohar spin decomposition

An incomplete story:



$$\frac{1}{2}$$

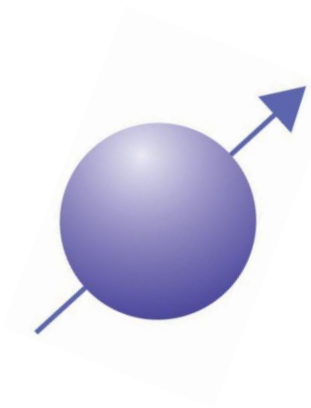




# Spin of proton

## Jaffe-Manohar spin decomposition

An incomplete story:



$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G$$

**Best known**

**How well do we know?**

Quark helicity  $\sim 30\%$

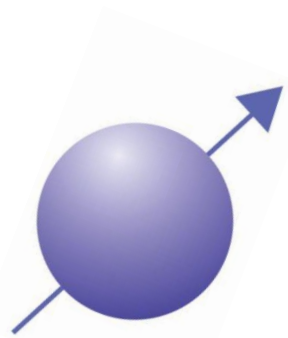
Gluon helicity  $\sim 40\%$



# Spin of proton

## Jaffe-Manohar spin decomposition

An incomplete story:



$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L^q + L^g$$

**Best known**

**How well do we know?**

**????????**

Quark helicity  $\sim 30\%$

Gluon helicity  $\sim 40\%$

OAM of quarks & gluons

# Wigner functions & Orbital Angular Momentum



## Wigner functions in Quantum Mechanics

(Wigner, 1932)

- Calculate from wave functions:

$$W(x, k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi\left(x + \frac{x'}{2}\right) \psi^*\left(x - \frac{x'}{2}\right)$$

- Expectation value of observables:

$$\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x, k) W(x, k)$$

# Wigner functions & Orbital Angular Momentum



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## Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

- Calculate from fourier transform of GTMD correlator:

$$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

- Application: **O**rbital **A**ngular **M**omentum (**OAM**)

$$L_z^{q,g} = \int dx \int d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

(Lorcé, Pasquini, 2011 / Hatta, 2011)

# Wigner functions & Orbital Angular Momentum



## Wigner functions in Quantum Mechanics

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## Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

- Calculate from fourier transform of GTMD correlator:

$$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

- Application: Relation between GTMD  $F_{1,4}^{q,g}$  & OAM

$$L_z^{q,g} = - \int dx \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, \vec{k}_\perp, \xi = 0, \Delta_\perp = 0)$$

(Lorcé, Pasquini, 2011 / Hatta, 2011)

# Wigner functions & Orbital Angular Momentum



Wigner functions in Quantum Mechanics

(Wigner, 1932)

Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

- Calculate from wave function

$$W(x, k) = \int \frac{dx'}{2\pi} \psi(x+x')$$

**Big question:**  
**Experimental observable?**

- Expectation value of observables:

$$\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x, k) W(x, k)$$

of GTMD correlator:

- Application: Relation between GTMD  $F_{1,4}^{q,g}$  & OAM

$$L_z^{q,g} = - \int dx \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, \vec{k}_\perp, \xi = 0, \Delta_\perp = 0)$$

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**arXiv: 1612.02438 (2016)**

**Hunting the Gluon Orbital Angular Momentum at the  
Electron-Ion Collider**

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>

# Developments



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GTMD distributions and the Odderons

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Signature of the gluon orbital angular momentum

Shohini Bhattacharya,<sup>1,\*</sup> Renaud Boussarie,<sup>2,†</sup> and Yoshitaka Hatta<sup>1,3,‡</sup>



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**arXiv: 2205.00045 (2022)**

Angular correlations in exclusive dijet photoproduction in ultra-peripheral PbPb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV

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arXiv: 1802.10550 (2018)

## Exclusive double Drell-Yan:

**Until now, this has been the sole known process sensitive to quark GTMDs**

arXiv: 2201.08709 (2022/2024)

Signature of the gluon orbital angular momentum

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# Probing quark OAM through double Drell-Yan

## Main findings

Physics Letters B 771 (2017) 396–400


Contents lists available at [ScienceDirect](#)

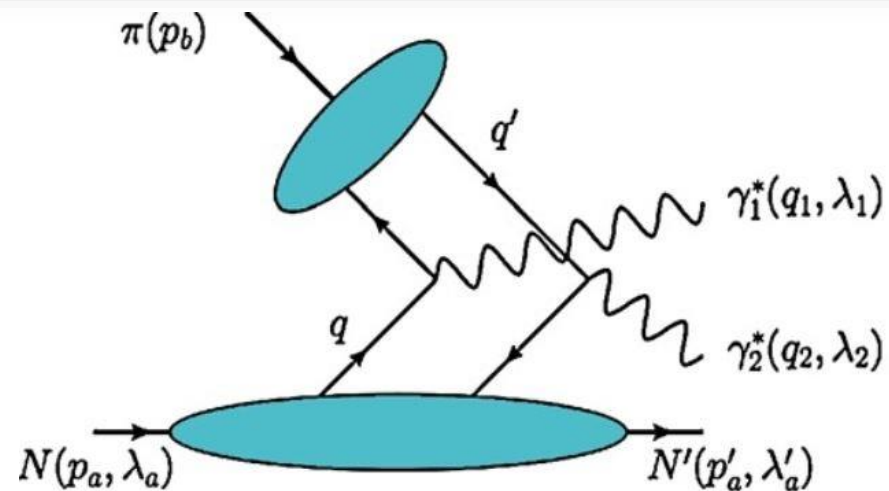
 **Physics Letters B** 

[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

Generalized TMDs and the exclusive double Drell–Yan process

Shohini Bhattacharya<sup>a</sup>, Andreas Metz<sup>a,\*</sup>, Jian Zhou<sup>b</sup>

 CrossMark



# Probing quark OAM through double Drell-Yan



## Main findings

Example of an observable sensitive to **OAM** & **spin-orbit correlation** :

$$\frac{1}{2}(\tau_{XY} - \tau_{YX}) = \frac{4}{M_a^2} (\varepsilon_{\perp}^{ij} \Delta q_{\perp}^i \Delta_{a\perp}^j) \text{Re.} \left\{ C^{(-)} [F_{1,1} \phi_{\pi}] C^{(+)} [\vec{\beta}_{\perp} \cdot \vec{k}_{a\perp} \mathbf{F}_{1,4}^* \phi_{\pi}^*] \right\}$$





# Probing quark OAM through double Drell-Yan

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Example of an observable sensitive to **OAM** & **spin-orbit correlation** :

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## Spin-orbit entanglement in the Color Glass Condensate

Shohini Bhattacharya,<sup>1,\*</sup> Renaud Boussarie,<sup>2,†</sup> and Yoshitaka Hatta<sup>3,4,‡</sup>

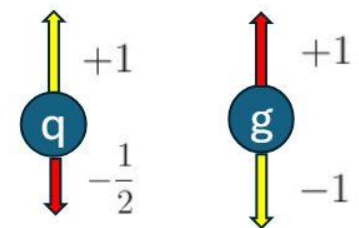
2404.04208

Recall Spin-Orbit coupling in H atom!



$$G_{1,1}^{q/g} \rightarrow L^{q/g} \cdot S^{q/g}$$

Perfect spin-orbit **anti**-correlation



# Probing quark OAM through double Drell-Yan



## Main findings

### Challenges:

- Low count rate (Amplitude  $\sim \alpha_{em}^2$ )

# Probing quark OAM through double Drell-Yan



## Main findings

### Challenges:

- Low count rate (Amplitude  $\sim \alpha_{em}^2$ )
- Sensitivity to GTMDs only in the ERBL region  $-\xi < x < \xi$

$$\text{OAM density: } L^{q/g}(x, \xi) = - \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, k_\perp, \xi, \Delta_\perp = 0)$$

$$\text{OAM: } L^{q/g} = \int dx L^{q/g}(x, \xi = 0)$$

The challenge lies in extrapolating the distribution to the forward limit, where the OAM equation is applicable

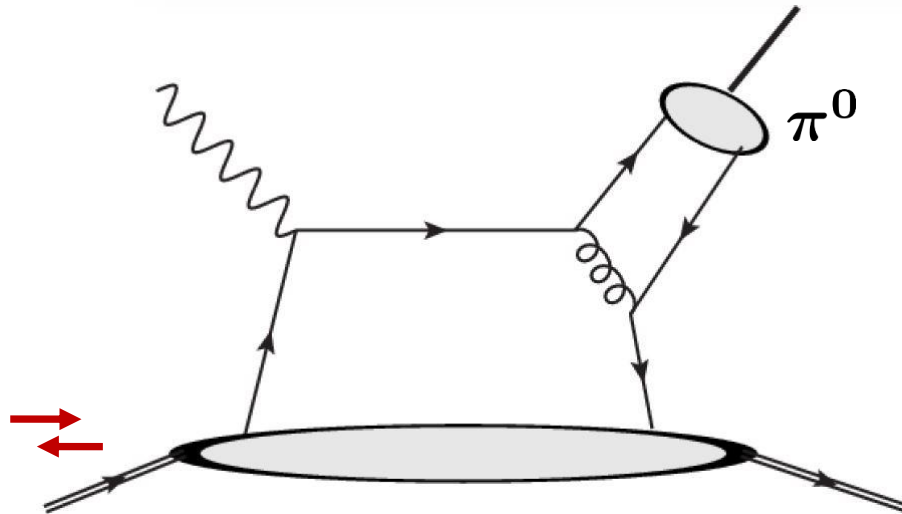


# Our work

PHYSICAL REVIEW LETTERS **133**, 051901 (2024)

## Probing the Quark Orbital Angular Momentum at Electron-Ion Colliders Using Exclusive $\pi^0$ Production

Shohini Bhattacharya<sup>1</sup>, Duxin Zheng<sup>2</sup>, and Jian Zhou<sup>3</sup>



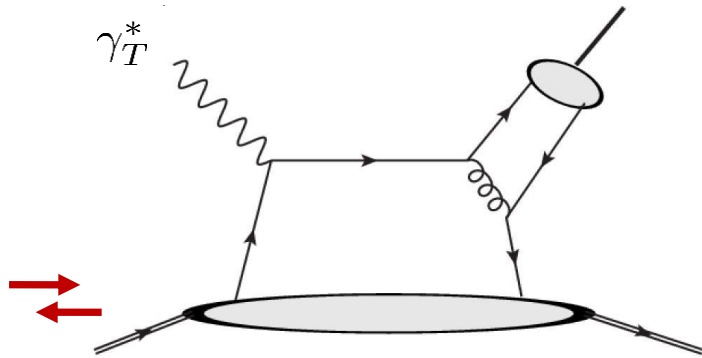
**Main Observable:**

**Longitudinal single-target spin asymmetry**

# Probing quark OAM through $\pi^0$ production in ep collisions



## Scattering amplitude



4 leading-order Feynman diagrams

# Probing quark OAM through $\pi^0$ production in ep collisions



## Scattering amplitude

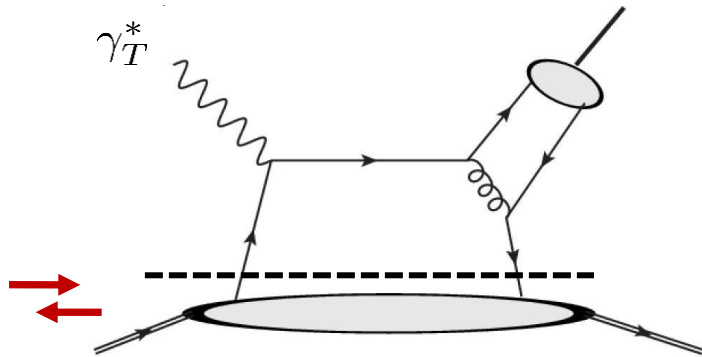
Scattering amplitude:

$$A \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^q(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_{\pi}(z)$$

Hard part

Soft part from  
proton

Pion Distribution  
Amplitude



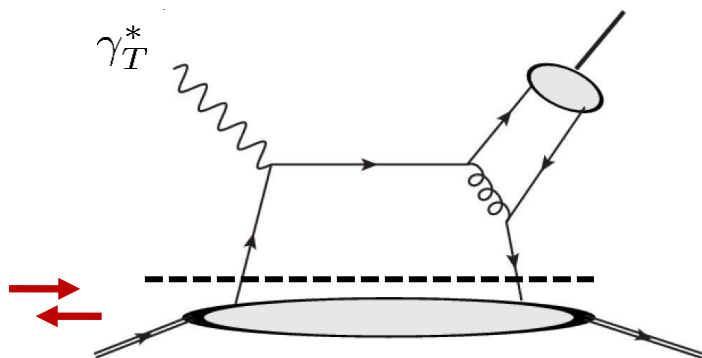
# Probing quark OAM through $\pi^0$ production in ep collisions



## Scattering amplitude

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**Collinear twist-expansion of hard part:**

$$H(k_{\perp}, \Delta_{\perp}) = H(k_{\perp} = 0, \Delta_{\perp} = 0) + \frac{\partial H(k_{\perp}, \Delta_{\perp} = 0)}{\partial k_{\perp}^{\mu}} \Big|_{k_{\perp}=0} k_{\perp}^{\mu} + \frac{\partial H(k_{\perp} = 0, \Delta_{\perp})}{\partial \Delta_{\perp}^{\mu}} \Big|_{\Delta_{\perp}=0} \Delta_{\perp}^{\mu} + \dots$$

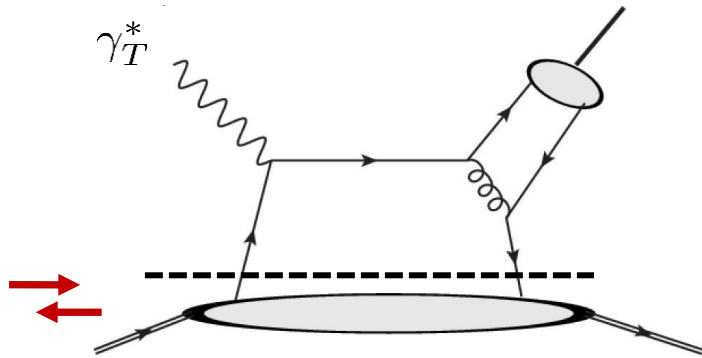
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Twist 2 term vanishes



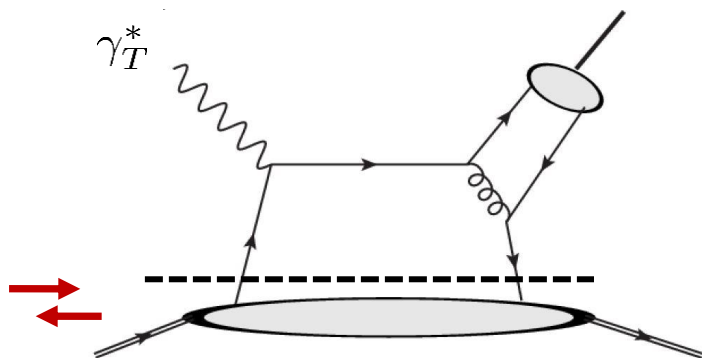
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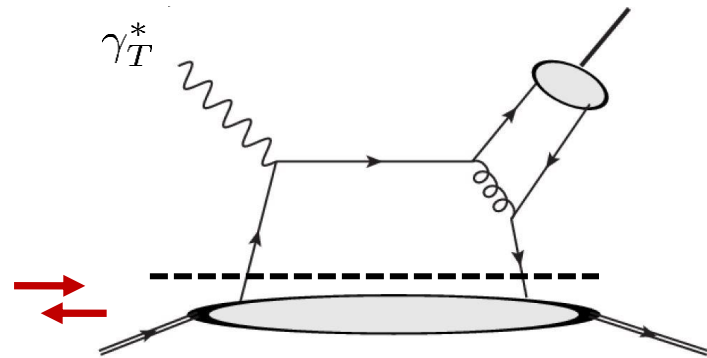
Use special-propagator technique to ensure electromagnetic gauge invariance

(J. W. Qiu, 1990)

# Probing quark OAM through $\pi^0$ production in ep collisions



## Scattering amplitude



Scattering amplitude:

$$A \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^q(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_{\pi}(z)$$

Collinear twist-expansion of hard part:

$$H(k_{\perp}, \Delta_{\perp}) = H(k_{\perp} = 0, \Delta_{\perp} = 0) + \frac{\partial H(k_{\perp}, \Delta_{\perp} = 0)}{\partial k_{\perp}^{\mu}} \Big|_{k_{\perp} = 0} k_{\perp}^{\mu} + \frac{\partial H(k_{\perp} = 0, \Delta_{\perp})}{\partial \Delta_{\perp}^{\mu}} \Big|_{\Delta_{\perp} = 0} \Delta_{\perp}^{\mu} + \dots$$

$$A \propto \int d^2 k_{\perp} k_{\perp}^2 \text{GTMD}$$

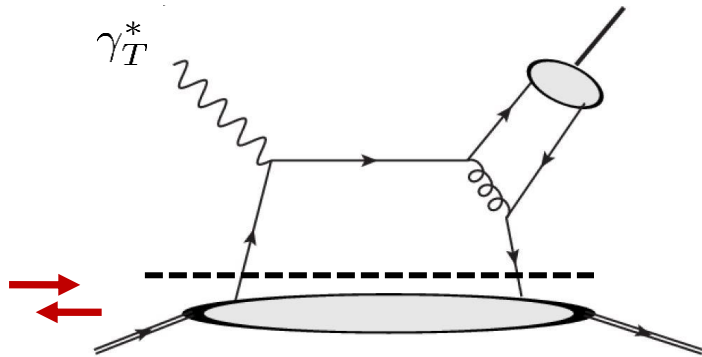
# Probing quark OAM through $\pi^0$ production in ep collisions



## Scattering amplitude

Scattering amplitude:

$$A \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^q(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_{\pi}(z)$$



Collinear twist-expansion of hard part:

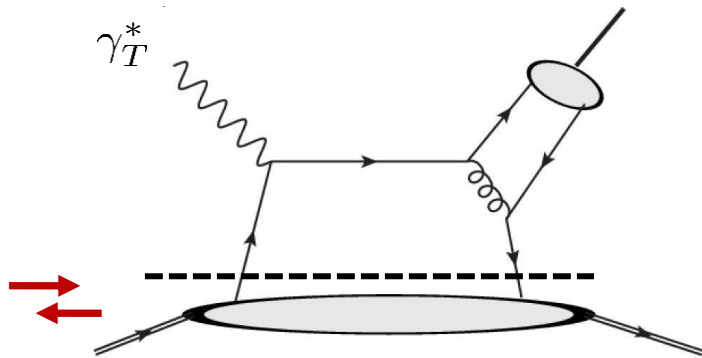
$$H(k_{\perp}, \Delta_{\perp}) = H(k_{\perp} = 0, \Delta_{\perp} = 0) + \frac{\partial H(k_{\perp}, \Delta_{\perp} = 0)}{\partial k_{\perp}^{\mu}} \Big|_{k_{\perp} = 0} k_{\perp}^{\mu} + \frac{\partial H(k_{\perp} = 0, \Delta_{\perp})}{\partial \Delta_{\perp}^{\mu}} \Big|_{\Delta_{\perp} = 0} \Delta_{\perp}^{\mu} + \dots$$

$$A \propto \text{GPD}$$

# Probing quark OAM through $\pi^0$ production in ep collisions



## Scattering amplitude



Scattering amplitude:

$$A \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^q(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_{\pi}(z)$$

## Collinear twist-expansion of hard part:

Consequently, the scattering amplitudes are a convolution of moments of GTMDs and GPDs and are of twist-3 nature

# Probing quark OAM through $\pi^0$ production in ep collisions



## Angular correlations

Scattering amplitudes depend on different angular correlations:

$$\mathcal{M}_1 = \frac{g_s^2 e f_\pi (N_c^2 - 1) 2\xi}{2\sqrt{2} N_c^2 \sqrt{1 - \xi^2}} \delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \times \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,1} + \mathcal{G}_{1,1}\}$$

$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi (N_c^2 - 1) 2\xi}{2\sqrt{2} N_c^2 \sqrt{1 - \xi^2}} \delta_{\lambda, -\lambda'} \frac{M \boldsymbol{\epsilon}_\perp \cdot \boldsymbol{S}_\perp}{Q^2} \{\mathcal{F}_{1,2} + \mathcal{G}_{1,2}\} \quad S_\perp^\mu = (0^+, 0^-, -i, \lambda)$$

$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi (N_c^2 - 1) 2\xi}{2\sqrt{2} N_c^2 \sqrt{1 - \xi^2}} \lambda \delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \cdot \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\}$$

# Probing quark OAM

through  $\pi^0$  production

## Compton Form Factors:



### Angular correlations

Scattering amplitudes depend on different angular correlations:

$$\mathcal{M}_1 = \frac{g_s^2 e f_\pi (N_c^2 - 1) 2\xi}{2\sqrt{2} N_c^2 \sqrt{1 - \xi^2}} \delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \times \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,1} + \mathcal{G}_{1,1}\}$$

$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi (N_c^2 - 1) 2\xi}{2\sqrt{2} N_c^2 \sqrt{1 - \xi^2}} \delta_{\lambda, -\lambda'} \frac{M \boldsymbol{\epsilon}_\perp \cdot \boldsymbol{S}_\perp}{Q^2} \{\mathcal{F}_{1,2} + \mathcal{G}_{1,2}\}$$

$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi (N_c^2 - 1) 2\xi}{2\sqrt{2} N_c^2 \sqrt{1 - \xi^2}} \lambda \delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \cdot \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\}$$

$$\mathcal{F}_{1,1} = \int_{-1}^1 dx \frac{x^2 \int d^2 k_\perp F_{1,1}^{u+d}(x, \xi, \Delta_\perp, k_\perp)}{(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1 + z^2 - z)}{z^2(1 - z)^2}, \quad (8)$$

$$\mathcal{G}_{1,1} = \int_{-1}^1 dx \int_0^1 dz \frac{\phi_\pi(z)(x^2 + 2x^2z + \xi^2)}{z^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int d^2 k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^{u+d}(x, \xi, \Delta_\perp, k_\perp), \quad (9)$$

$$\mathcal{F}_{1,2} = \int_{-1}^1 dx x \frac{\xi(1 - \xi^2) \int d^2 k_\perp k_\perp^2 F_{1,2}^{u+d}(x, \xi, \Delta_\perp, k_\perp)}{M^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1 + z^2 - z)}{z^2(1 - z)^2}, \quad (10)$$

$$\mathcal{G}_{1,2} = \int_{-1}^1 dx \int_0^1 dz \frac{\phi_\pi(z)(x^2 + 2x^2z + \xi^2)(1 - \xi^2)}{z^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int d^2 k_\perp \frac{k_\perp^2}{M^2} G_{1,2}^{u+d}(x, \xi, \Delta_\perp, k_\perp), \quad (11)$$

$$\mathcal{F}_{1,4} = \int_{-1}^1 dx \frac{x\xi \int d^2 k_\perp k_\perp^2 F_{1,4}^{u+d}(x, \xi, \Delta_\perp, k_\perp)}{M^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1 + z^2 - z)}{z^2(1 - z)^2}, \quad (12)$$

$$\mathcal{G}_{1,4} = \int_{-1}^1 dx \int_0^1 dz \frac{x(4\xi^2z + \xi^2 - 2x^2z + x^2)}{z^2\xi(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \phi_\pi(z) \times \int d^2 k_\perp G_{1,4}^{u+d}(x, \xi, \Delta_\perp, k_\perp). \quad (13)$$

# Probing quark OAM through $\pi^0$ production in ep collisions



## Angular correlations

Scattering amplitudes depend on different angular correlations:

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$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi (N_c^2 - 1) 2\xi}{2\sqrt{2} N_c^2 \sqrt{1 - \xi^2}} \delta_{\lambda, -\lambda'} \frac{M \boldsymbol{\epsilon}_\perp \cdot \boldsymbol{S}_\perp}{Q^2} \{\mathcal{F}_{1,2} + \mathcal{G}_{1,2}\}$$

$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi (N_c^2 - 1) 2\xi}{2\sqrt{2} N_c^2 \sqrt{1 - \xi^2}} \lambda \delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \cdot \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\}$$

## Sensitivity to quark OAM

$$\mathcal{F}_{1,1} = \int_{-1}^1 dx \frac{x^2 \int d^2 k_\perp F_{1,1}^{u+d}(x, \xi, \Delta_\perp, k_\perp)}{(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1 + z^2 - z)}{z^2(1 - z)^2}, \quad (8)$$

$$\mathcal{G}_{1,1} = \int_{-1}^1 dx \int_0^1 dz \frac{\phi_\pi(z)(x^2 + 2x^2z + \xi^2)}{z^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int d^2 k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^{u+d}(x, \xi, \Delta_\perp, k_\perp), \quad (9)$$

$$\mathcal{F}_{1,2} = \int_{-1}^1 dx x \frac{\xi(1 - \xi^2) \int d^2 k_\perp k_\perp^2 F_{1,2}^{u+d}(x, \xi, \Delta_\perp, k_\perp)}{M^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1 + z^2 - z)}{z^2(1 - z)^2}, \quad (10)$$

$$\mathcal{G}_{1,2} = \int_{-1}^1 dx \int_0^1 dz \frac{\phi_\pi(z)(x^2 + 2x^2z + \xi^2)(1 - \xi^2)}{z^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int d^2 k_\perp \frac{k_\perp^2}{M^2} G_{1,2}^{u+d}(x, \xi, \Delta_\perp, k_\perp), \quad (11)$$

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$$\mathcal{G}_{1,4} = \int_{-1}^1 dx \int_0^1 dz \frac{x(4\xi^2 z + \xi^2 - 2x^2 z + x^2)}{z^2 \xi (x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \phi_\pi(z) \times \int d^2 k_\perp G_{1,4}^{u+d}(x, \xi, \Delta_\perp, k_\perp). \quad (13)$$

# Probing quark OAM through $\pi^0$ production in ep collisions



## Cross section

$$\begin{aligned} \frac{d\sigma}{dtdQ^2dx_Bd\phi} &= \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2] \\ &\times \left\{ \left[ |\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[ -|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right. \\ &\quad \left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[ (i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\} \\ &\quad \uparrow \\ &\quad a = \frac{2(1-y)}{1+(1-y)^2} \end{aligned}$$



# Probing quark OAM through $\pi^0$ production in ep collisions



## Cross section

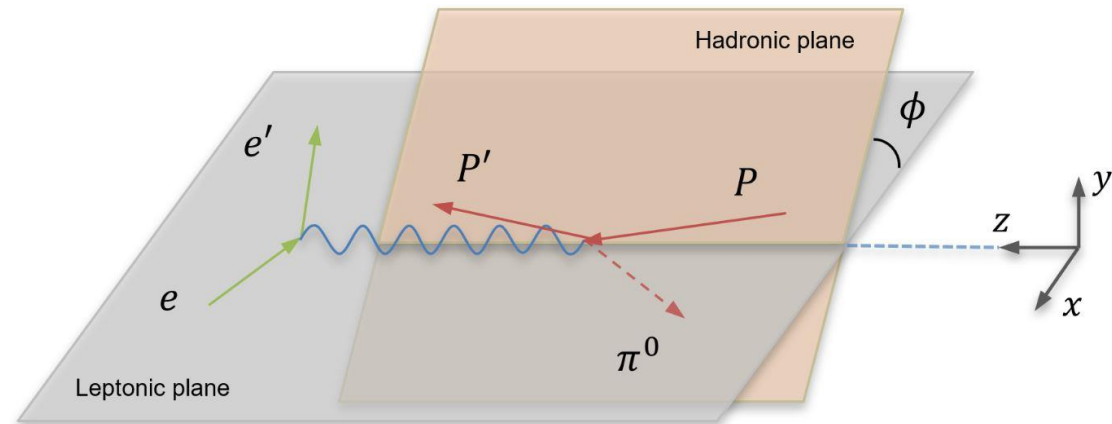
$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2]$$

$$\times \left\{ \left[ |\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[ -|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right.$$

$$\left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[ (i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\}$$

Distinguished experimental signature of quark OAM

$$\phi = \phi_{l_\perp} - \phi_{\Delta_\perp}$$



# Probing quark OAM through $\pi^0$ production in ep collisions



## Cross section

$$\frac{d\sigma}{dt dQ^2 dx_B d\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2]$$

$$\times \left\{ \left[ |\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[ -|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right.$$

$$\left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[ (i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\} \quad \uparrow \quad \text{Surprise!}$$

- Probe quark Sivers function through an unpolarized target

$$\operatorname{Im} [F_{1,2}]|_{\Delta=0} = -f_{1T}^\perp$$

(Similar to the gluon GTMD  $F_{1,2}$ , as discussed in Boussarie, Hatta, Szymanowski, Wallon, 2019)

# Probing quark OAM through $\pi^0$ production in ep collisions



## Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2]$$

$$\times \left\{ \left[ |\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[ -|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right.$$

$$\left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[ (i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\} \quad \uparrow \quad \text{Surprise!}$$

- Probe quark Sivers function through an unpolarized target

$$\operatorname{Im} [\mathbf{F}_{1,2}]|_{\Delta=0} = -\mathbf{f}_{1T}^\perp$$

- Probe quark worm gear function through an unpolarized target

$$\operatorname{Re} [\mathbf{G}_{1,2}]|_{\Delta=0} = \mathbf{g}_{1T}$$

# Probing quark OAM through $\pi^0$ production in ep collisions



## Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2]$$

$$\times \left\{ \left[ |\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[ -|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right.$$

$$\left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[ (i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\}$$



Helicity flip terms persist even when  $\Delta_\perp \rightarrow 0$

# Probing quark OAM through $\pi^0$ production in ep collisions



## Cross section

$$\begin{aligned} \frac{d\sigma}{dtdQ^2 dx_B d\phi} &= \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2] \\ &\times \left\{ \left[ |\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[ -|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right. \\ &\quad \left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[ (i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\} \end{aligned}$$

Since both unpolarized and polarized cross sections contribute at twist-3, the magnitudes of the asymmetries are not power-suppressed

# Probing quark OAM through $\pi^0$ production in ep collisions



## Theoretical complications

# Probing quark OAM through $\pi^0$ production in ep collisions



## Theoretical complications

End-point singularity & discontinuity:

$$\mathcal{F}_{1,4} = \int_{-1}^1 dx \frac{x\xi \int d^2k_{\perp} k_{\perp}^2 F_{1,4}^{u+d}(x, \xi, \Delta_{\perp}, k_{\perp})}{M^2(x + \xi - i\epsilon)^2(x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_{\pi}(z)(1 + z^2 - z)}{z^2(1 - z)^2}$$

Model-dependent method:

$$\int_{\langle p_{\perp}^2 \rangle / Q^2}^{1 - \langle p_{\perp}^2 \rangle / Q^2} dz$$

$\langle p_{\perp}^2 \rangle = 0.04 \text{ GeV}^2$  determined based on a fit to CLAS data

S. V. Goloskokov and P. Kroll, 2005

# Probing quark OAM through $\pi^0$ production in ep collisions



## Theoretical complications

End-point singularity & discontinuity:

$$\mathcal{F}_{1,4} = \int_{-1}^1 dx \frac{x\xi \int d^2k_{\perp} k_{\perp}^2 F_{1,4}^{u+d}(x, \xi, \Delta_{\perp}, k_{\perp})}{M^2 (x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_{\pi}(z)(1 + z^2 - z)}{z^2(1 - z)^2}$$

Model-dependent method:

$$\int_{\langle p_{\perp}^2 \rangle / Q^2}^{1 - \langle p_{\perp}^2 \rangle / Q^2} dz$$

S. V. Goloskokov and P. Kroll, 2005

$$\frac{1}{(x - \xi + i\epsilon)^2} \rightarrow \frac{1}{(x - \xi - \langle p_{\perp}^2 \rangle / Q^2 + i\epsilon)^2}$$

I. V. Anikin, O. V. Teryaev, 2003



# Probing quark OAM through $\pi^0$ production in ep collisions



## Numerical results

### Kinematics:

|             | $Q^2(\text{GeV}^2)$ | $\sqrt{s}_{ep}(\text{GeV})$ |
|-------------|---------------------|-----------------------------|
| <b>EIC</b>  | 10                  | 100                         |
| <b>EicC</b> | 3                   | 16                          |

# Probing quark OAM through $\pi^0$ production in ep collisions

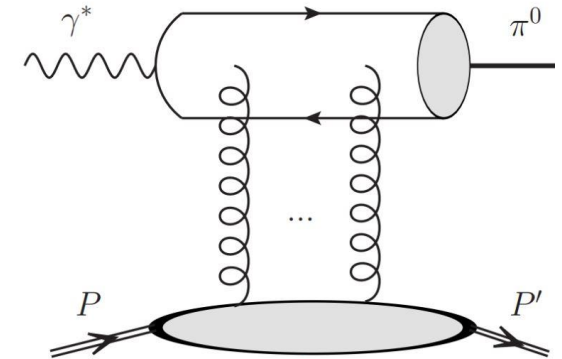


## Numerical results

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- We focus on large skewness ( $\xi$ ) region to suppress gluon contribution



# Probing quark OAM through $\pi^0$ production in ep collisions

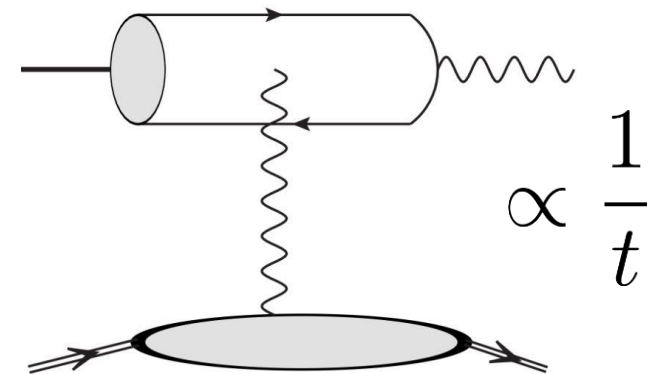


## Numerical results

### Kinematics:

|             | $Q^2(\text{GeV}^2)$ | $\sqrt{s}_{ep}(\text{GeV})$ |
|-------------|---------------------|-----------------------------|
| <b>EIC</b>  | 10                  | 100                         |
| <b>EicC</b> | 3                   | 16                          |

- We focus on large skewness ( $\xi$ ) region to suppress gluon contribution
- We focus on large momentum transfer ( $t$ ) region to suppress contribution from Primakoff process

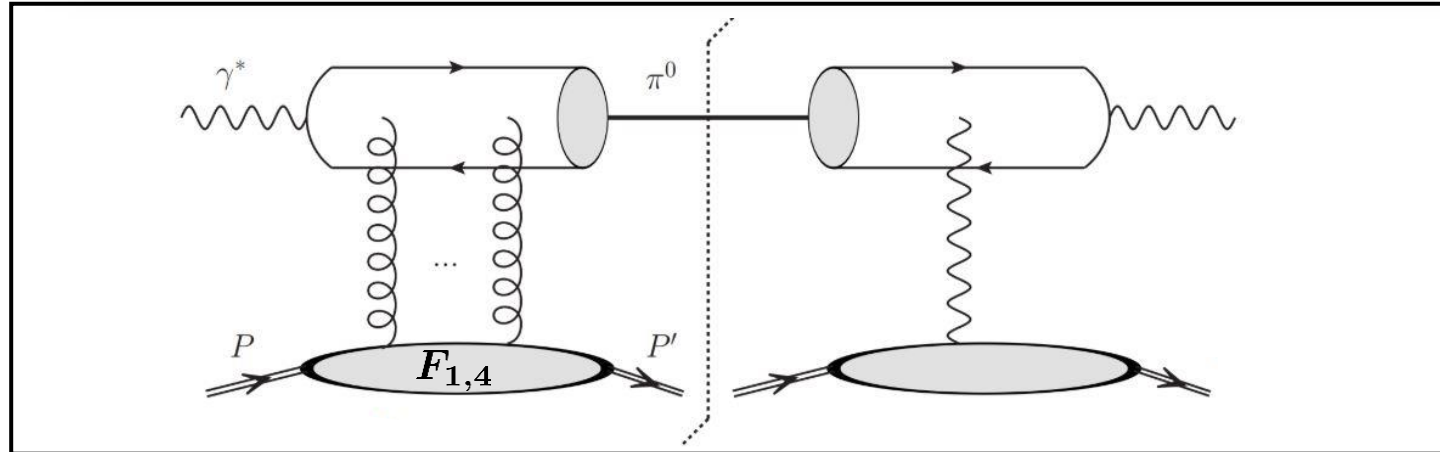




**Remark:**

Accessing the gluon GTMD  $F_{1,4}$  in exclusive  $\pi^0$  production in  $ep$  collisions

Shohini Bhattacharya<sup>1</sup>, Duxin Zheng<sup>2</sup>, and Jian Zhou<sup>3</sup>



$$\frac{d\Delta\sigma}{dt dQ^2 dx_B d\phi} = -\sin(2\phi) \frac{\alpha_{em}^3 \alpha_s f_\pi^2 (1-y) \xi x_B \mathcal{F}(t)}{3Q^8 N_c} \left[ \int_0^1 dz \frac{\phi_\pi(z)}{z(1-z)} \right]^2 \text{Im} \left[ \int_{-1}^1 dx \frac{F_{1,4}^{(1)}(x, \xi, \Delta_\perp) / M^2}{(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \right]$$

The same azimuthal asymmetry, precisely mirroring what we observe in this study, emerges from the interference between the Primakoff process and the contribution from the gluon GTMD

# Probing quark OAM through $\pi^0$ production in ep collisions



## Model input for numerical estimations

### Ingredients for non-perturbative functions:

- Model  $(H^q, \tilde{H}^q)$  according to the Double distribution approach (see Radyushkin, 9805342)

Example:

$$H^q(x, \xi, t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) \times \frac{3}{4} |\beta|^{-1.3 t} \frac{[(1 - |\beta|)^2 - \alpha^2]}{(1 - |\beta|)^3} q(|\beta|)$$



# Probing quark OAM through $\pi^0$ production in ep collisions



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The t-dependence is determined based on a fit to CLAS data

# Probing quark OAM through $\pi^0$ production in ep collisions



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- Model for OAM:
  1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^q(\boldsymbol{x}) = x \int_x^1 \frac{dx'}{x'} q(x') - x \int_x^1 \frac{dx'}{x'^2} \Delta q(x') + \text{genuine twist-three}$$

# Probing quark OAM through $\pi^0$ production in ep collisions



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$$L_{can}^q(\boldsymbol{x}) \stackrel{\text{WW approx}}{=} x \int_x^1 \frac{dx'}{x'} q(x') - x \int_x^1 \frac{dx'}{x'^2} \Delta q(x') + \text{genuine twist-three}$$



# Probing quark OAM through $\pi^0$ production in ep collisions



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- Model for OAM:
  1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^q(\boldsymbol{x}) \stackrel{\text{WW approx}}{=} x \int_x^1 \frac{dx'}{x'} q(x') - x \int_x^1 \frac{dx'}{x'^2} \Delta q(x') + \text{genuine twist-three}$$

2. Use the Double distribution approach to construct  $xL^q(x, \boldsymbol{\xi})e^{t/\Lambda}$  from  $xL^q(x)$

# Probing quark OAM through $\pi^0$ production in ep collisions



## Model input for numerical estimations

### Ingredients for non-perturbative functions:

- Pion distribution amplitude:

Asymptotic form

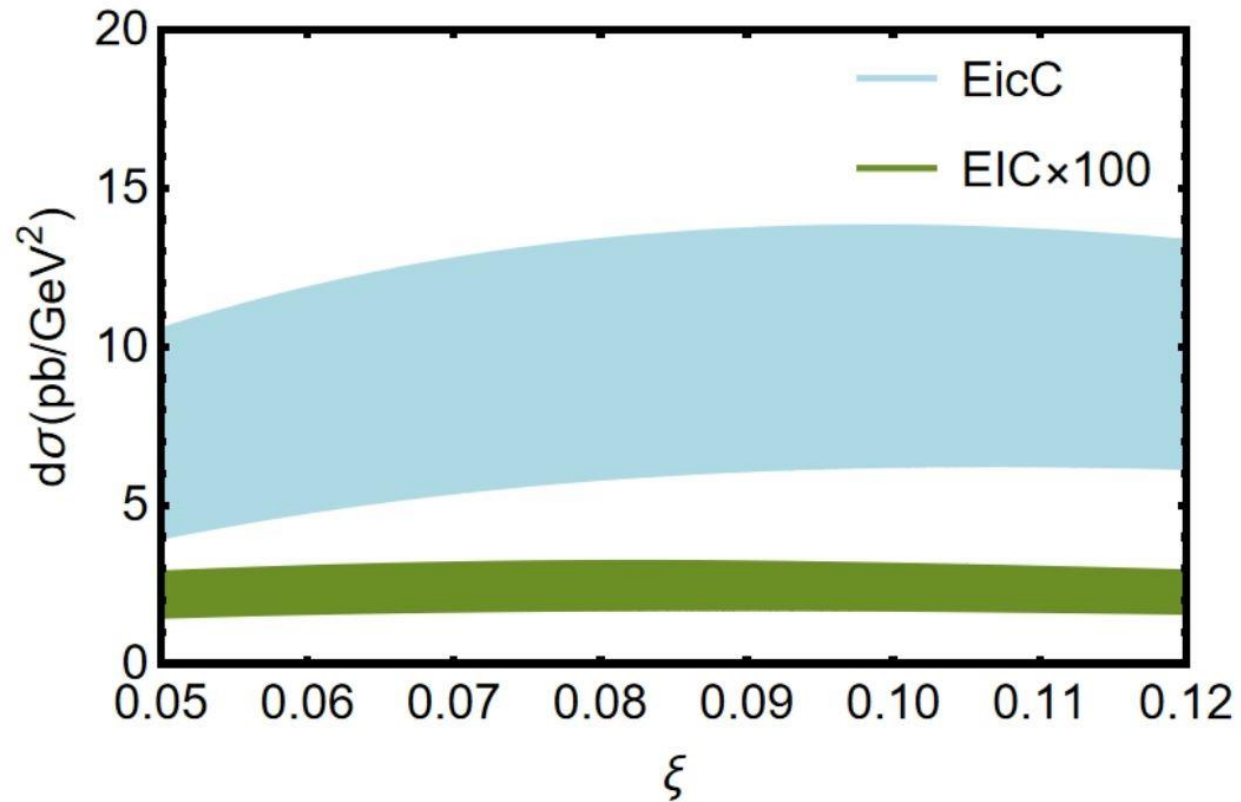
$$\phi_{\pi}(z) = 6z(1 - z)$$

# Probing quark OAM through $\pi^0$ production in ep collisions



## Numerical results

### Unpolarized cross section



#### Findings:

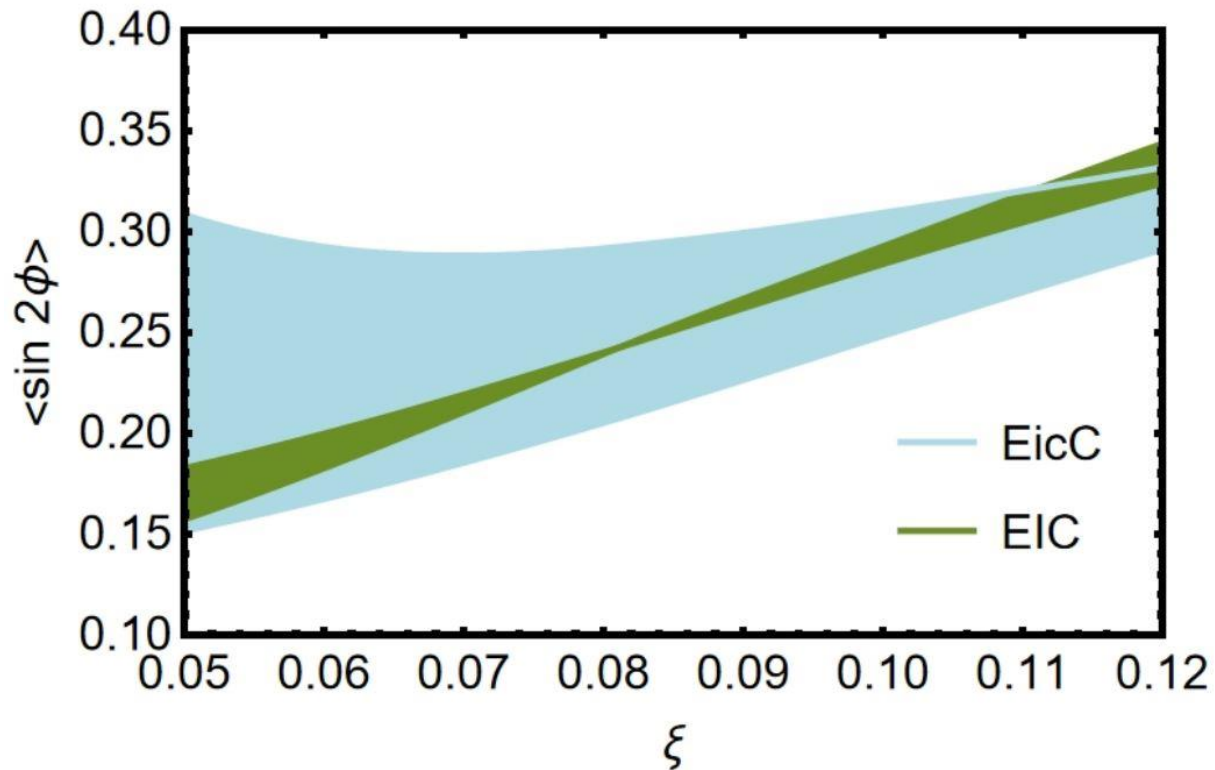
- The unpolarized cross section exhibits a notable magnitude at EicC energy
- Relatively small at EIC energy

# Probing quark OAM through $\pi^0$ production in ep collisions



## Numerical results

### Asymmetry



$$\langle \sin(2\phi) \rangle = \frac{\int \frac{d\Delta\sigma}{d\mathcal{P}.S.} \sin(2\phi) d\mathcal{P}.S.}{\int \frac{d\sigma}{d\mathcal{P}.S.} d\mathcal{P}.S.}$$

Findings:

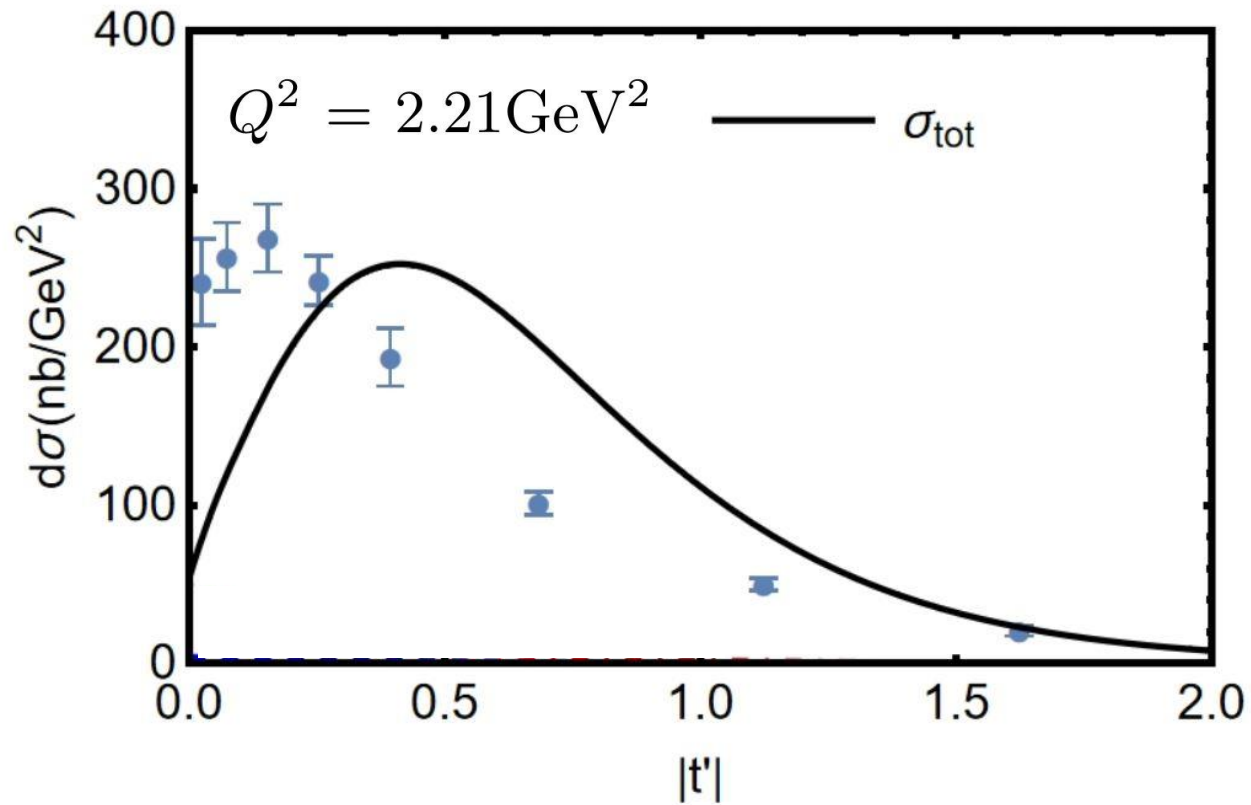
The asymmetries are substantial for both EIC & EicC kinematics

# Probing quark OAM through $\pi^0$ production in ep collisions



## Numerical results

### Comparison with CLAS data



Unpolarized cross section:

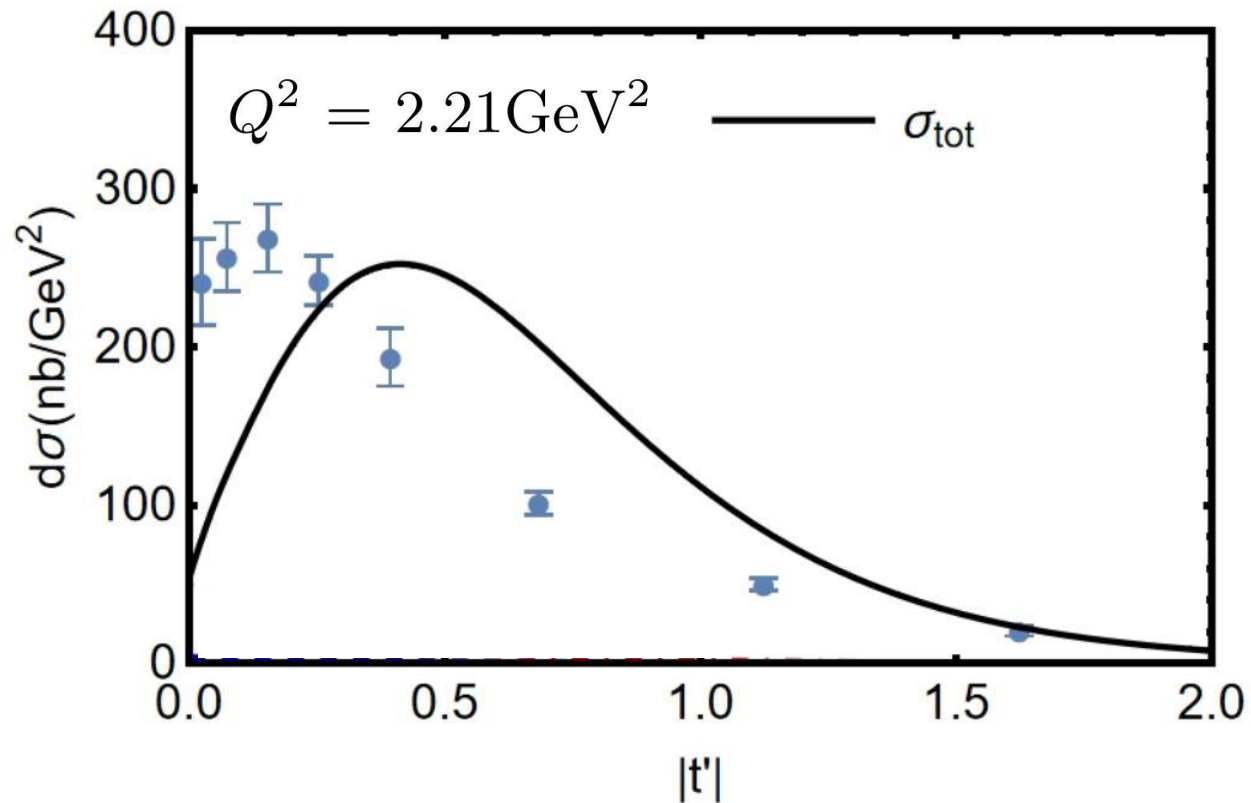
$$\frac{d\sigma_T}{dt} + a \frac{d\sigma_L}{dt}$$

# Probing quark OAM through $\pi^0$ production in ep collisions



## Numerical results

### Comparison with CLAS data



Findings:

- Our theoretical model is in reasonable agreement with experimental data

# Developments



arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>

arXiv: 1702.04387 (2017)

Generalized TMDs and the exclusive double Drell-Yan process

Shohini Bhattacharya,<sup>1</sup> Andreas Metz,<sup>1</sup> and Jian Zhou<sup>2</sup>

arXiv: 1802.10550 (2018)

Exclusive double quarkonium production and generalized TM

Shohini Bhattacharya,<sup>1</sup> Andreas Metz,<sup>1</sup> Vikash Kumar Ojha,<sup>2</sup> Jeng-Yuan Tsai,<sup>1</sup>

arXiv: 1807.08697 (2018)

Probing the Weizsäcker-Williams gluon Wigner distribution in  $pp$  collisions

Renaud Boussarie,<sup>1</sup> Yoshitaka Hatta,<sup>2</sup> Bo-Wen Xiao,<sup>3,4</sup> and Feng Yuan<sup>5</sup>

arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolarized GTMD distributions and the Odderons

Renaud Boussarie,<sup>1</sup> Yoshitaka Hatta,<sup>1</sup> Lech Szymanowski,<sup>2</sup> and S

arXiv: 2106.13466 (2021)

Probing the gluon tomography in photoproduction of di-pions

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

**arXiv: 2201.08709 (2022/2024)**

**Signature of the gluon orbital angular momentum**

Shohini Bhattacharya,<sup>1,\*</sup> Renaud Boussarie,<sup>2,†</sup> and Yoshitaka Hatta<sup>1,3,‡</sup>

arXiv: 2205.00045 (2022)

Angular correlations in exclusive dijet photoproduction in ultra-peripheral PbPb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV



# Selected works on gluon GTMDs

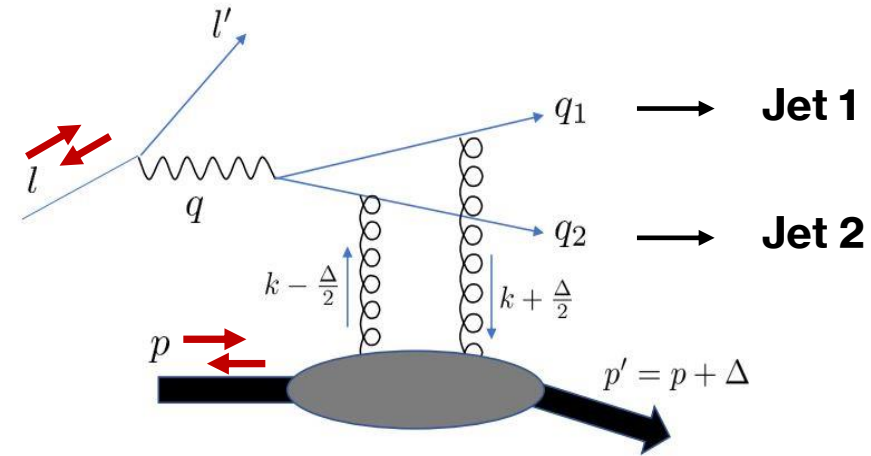
## Probing gluon OAM through exclusive di-jet production

PHYSICAL REVIEW LETTERS **128**, 182002 (2022)

**DOE Highlight**

**Signature of the Gluon Orbital Angular Momentum**

Shohini Bhattacharya<sup>1,\*</sup>, Renaud Boussarie<sup>2,†</sup> and Yoshitaka Hatta<sup>1,3,‡</sup>







# Selected works on gluon GTMDs

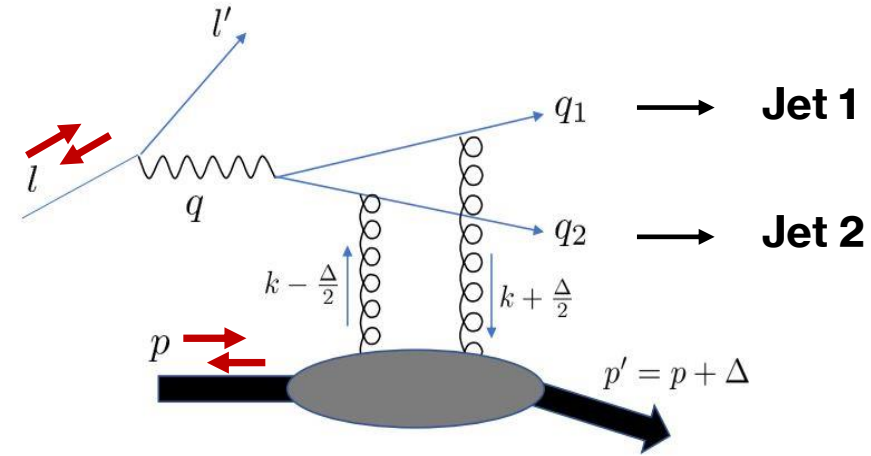
## Probing gluon OAM through exclusive di-jet production

PHYSICAL REVIEW LETTERS **128**, 182002 (2022)

**DOE Highlight**

**Signature of the Gluon Orbital Angular Momentum**

Shohini Bhattacharya<sup>1,\*</sup>, Renaud Boussarie<sup>2,†</sup> and Yoshitaka Hatta<sup>1,3,‡</sup>



**Main result (double spin asymmetry):**

**Signature of gluon OAM is cosine angular modulation**

$$d\sigma^{\text{asym}} \sim -\text{Re} \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \\ + \text{Re} \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$



# Selected works on gluon GTMDs

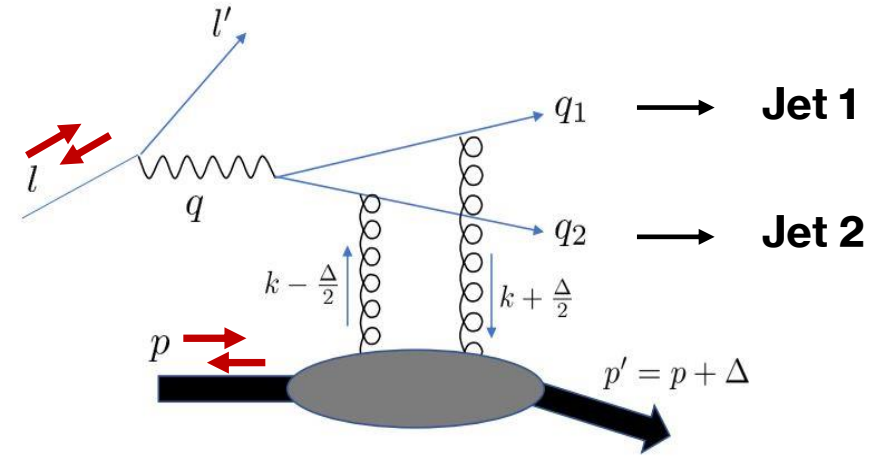
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## Gluon helicity contributes to the same angular modulation as that of OAM

$$d\sigma^{\text{asym}} \sim -\text{Re} \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

**Helicity GPD**  
(intrinsic spin)

$$+\text{Re} \left[ \mathcal{H}_g^{(1)}(\xi) + \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$



# Selected works on gluon GTMDs

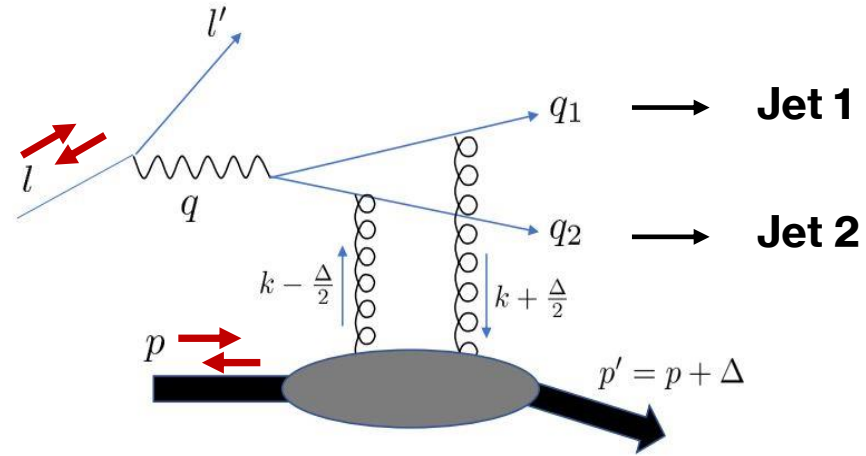
## Probing gluon OAM through exclusive di-jet production

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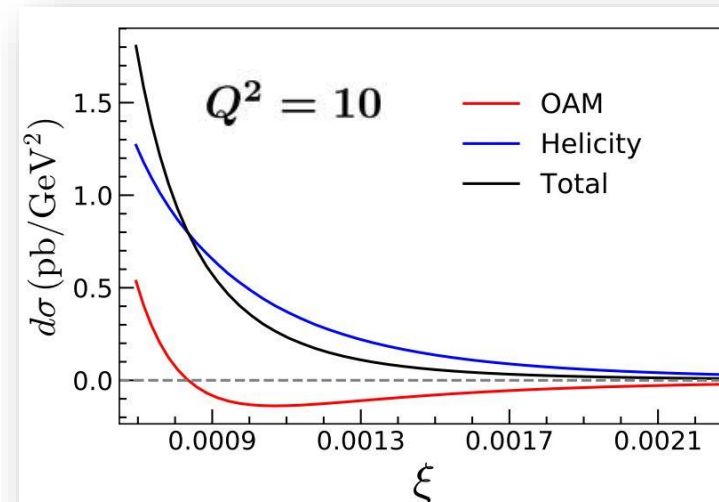
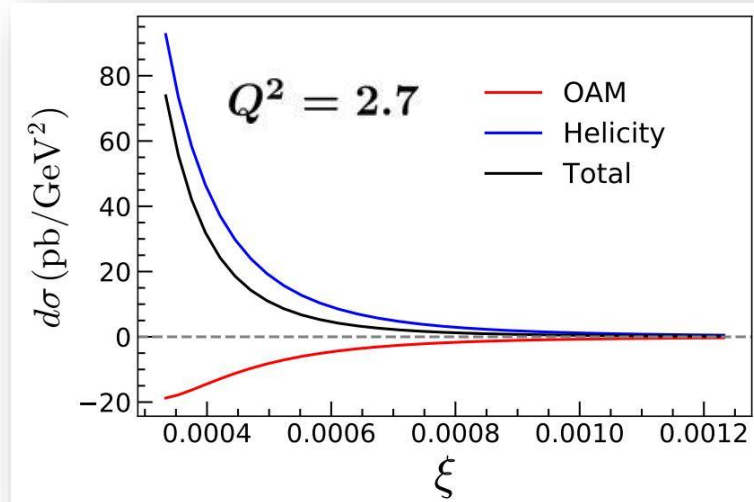
**Our observable is a simultaneous probe of gluon OAM & it's helicity**

$$d\sigma^{\text{asym}} \sim -\text{Re} \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$+ \text{Re} \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

# Selected works on gluon GTMDs



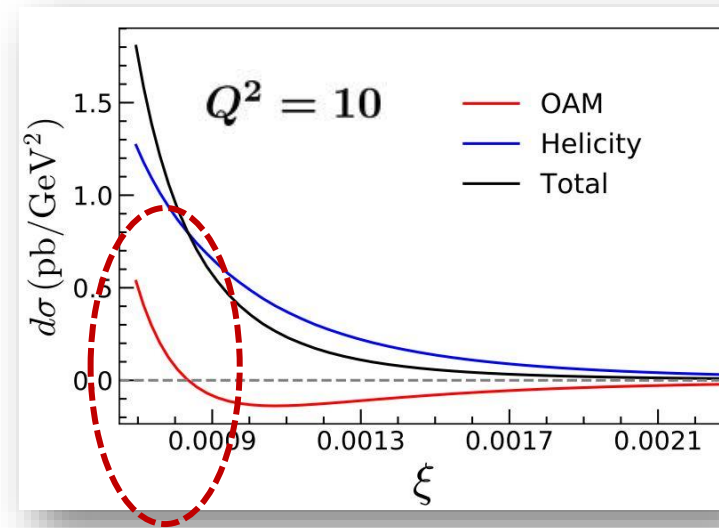
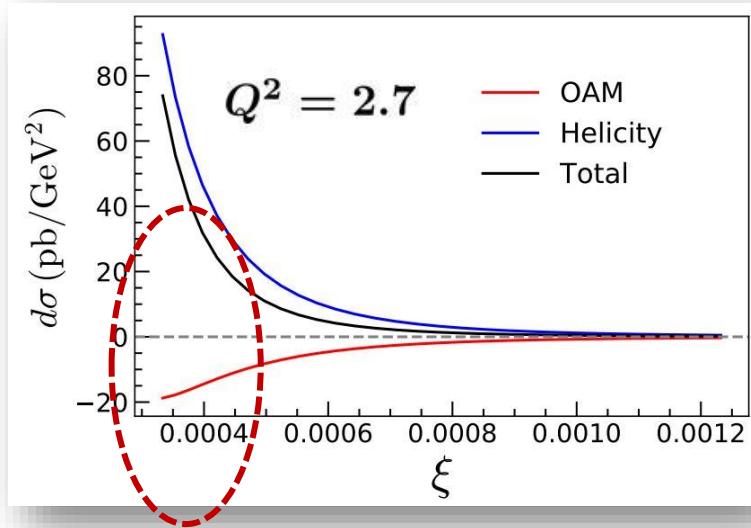
## Interplay between OAM and helicity at small $x$





# Selected works on gluon GTMDs

## Interplay between OAM and helicity at small $x$



### Schematic structure of our observable:

$$d\sigma^{\text{asym}} \sim \mathcal{H}_g^{(1)*}(\xi) \left( \tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_{\perp}^2 - Q^2/4}{q_{\perp}^2 + Q^2/4} \mathcal{L}_g(\xi) \right)$$

$\downarrow$   
 $\Delta G(x)$

$\downarrow$   
 $L_g(x)$

### Cancellation expected between helicity & OAM at small $x$

$$\Delta G(x) \approx -\frac{2}{1+c} L_g(x)$$

Boussarie, Hatta, Yuan (2019)  
Kovchegov, Manley (2023, 2024)

# Selected works on gluon GTMDs



## Contribution from spin-orbit correlation at small $x$ ?

Yet another contribution to the process:

$$d\sigma^{\text{asym}} \sim \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} C_g^{(2)}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi)$$

**Spin-orbit correlation:**

$$C^g(x) = \int d^2\vec{k}_{\perp} \frac{\vec{k}_{\perp}^2}{M^2} G_{1,1}^g(x, \vec{k}_{\perp}^2)$$

# Selected works on gluon GTMDs



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$$d\sigma^{\text{asym}} \sim \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} C_g^{(2)}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi)$$

**First insight into the small- $x$  behavior of spin-orbit correlation:**

$$C^g(x) \approx -2x \int_x^1 \frac{dx'}{x'^2} G(x') + \dots \propto -G(x)$$

(SB, Boussarie, Hatta,  
2404.04208 , 2404.04209)

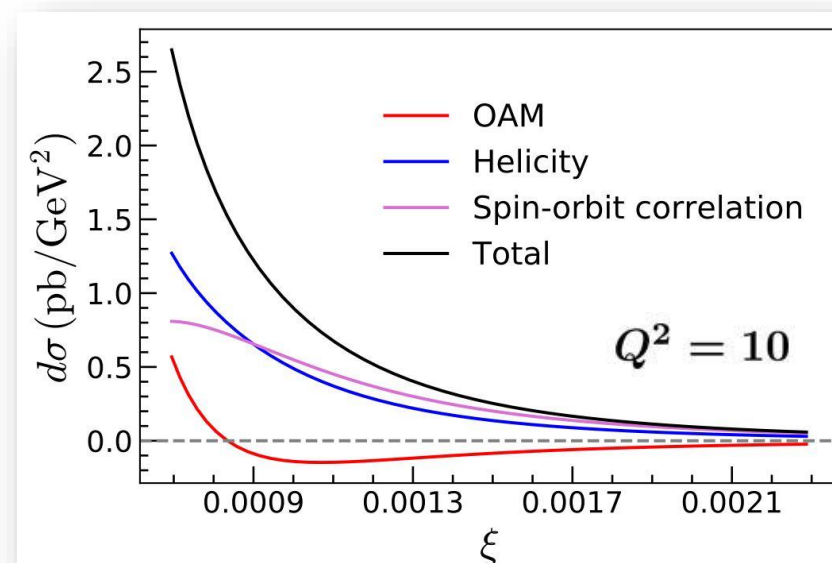
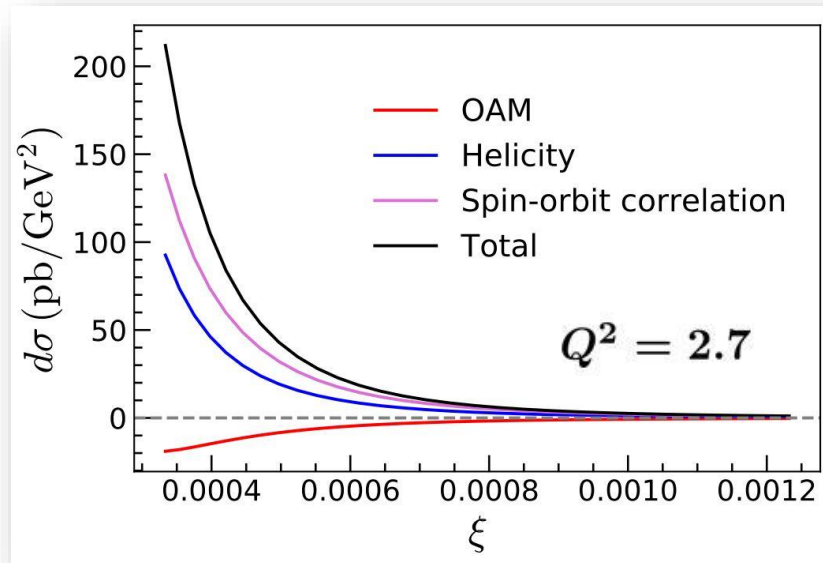
For a complete twist structure of spin-orbit correlation, see Hatta, Schoenleber, 2404.18872



# Selected works on gluon GTMDs

## Probing gluon OAM & spin-orbit correlation at small x

Updated numerical results (SB, Boussarie, Hatta, 2404.04209):



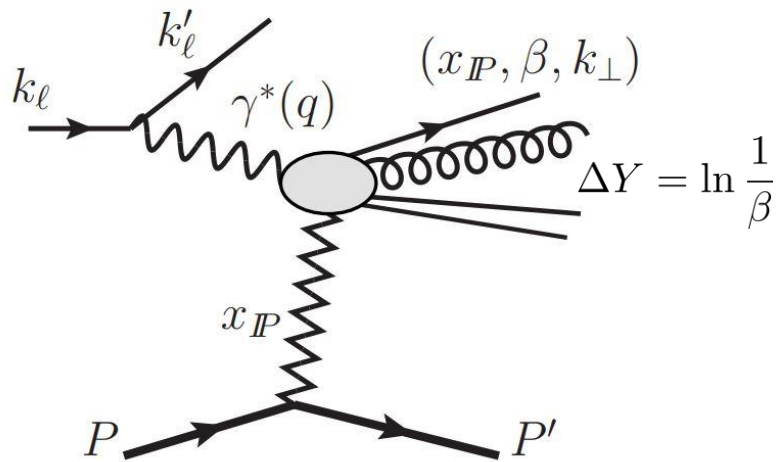
**Spin-orbit correlation** is more accurately constrained than **OAM** because the latter necessitates the precise determination of both unpolarized and polarized gluon distributions





# Selected works on gluon GTMDs

## Probing gluon OAM through Semi Inclusive Diffractive Deep Inelastic Scattering



- Measure invariant mass of diffractively produced system instead of reconstructing jets

$$M_X^2 = \frac{q_\perp^2}{z\bar{z}} = \frac{1-\beta}{\beta} Q^2$$

- Tag hadron species out of the diffractively produced system

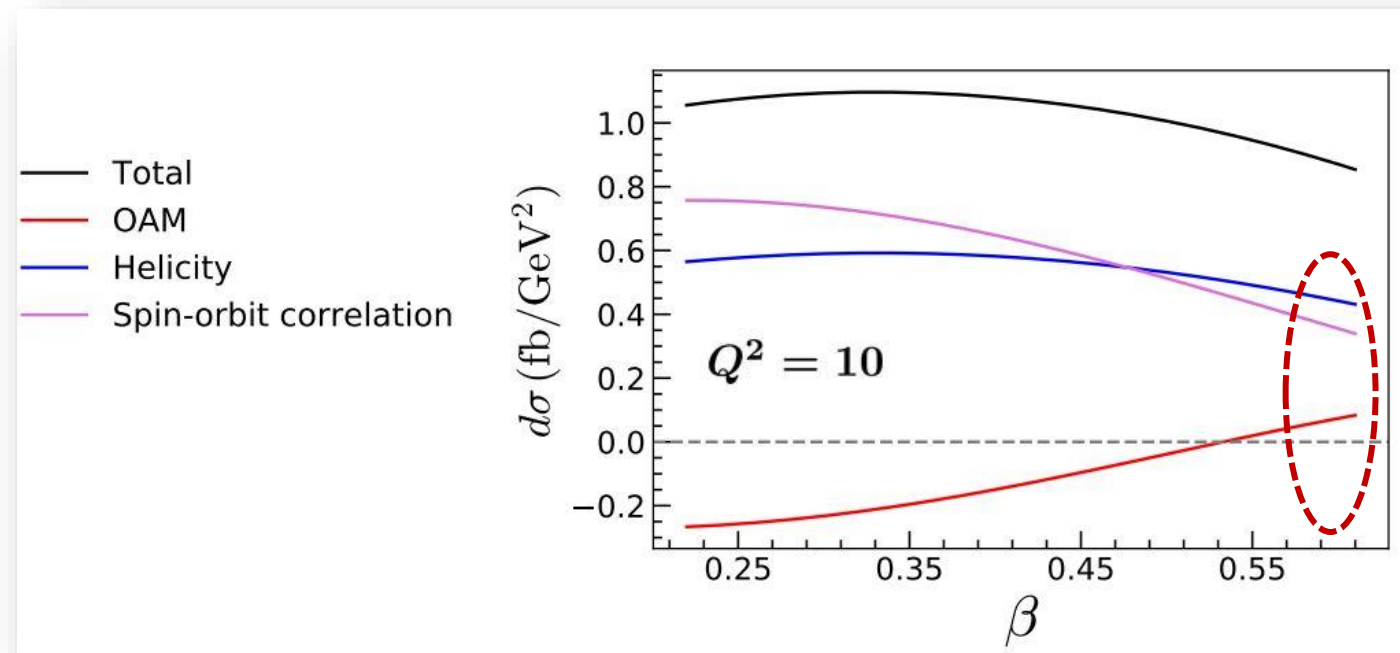
Hatta, Xiao, Yuan (2022)

# Selected works on gluon GTMDs



## Probing gluon OAM through Semi Inclusive Diffractive Deep Inelastic Scattering

Numerical results (SB, Boussarie, Hatta, 2404.04209):



Challenging, yet there is no requirement to reconstruct jets & we still maintain sensitivity to OAM



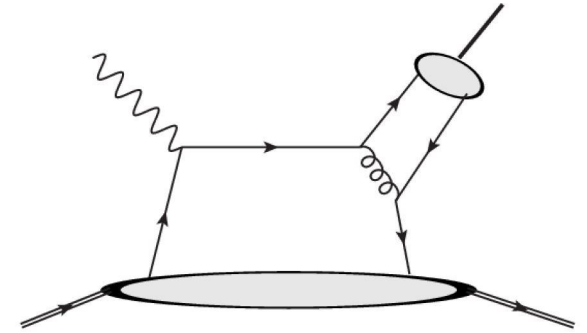
# Summary

- Generalized TMDs/Wigner functions are the holy grail of spin physics



# Summary

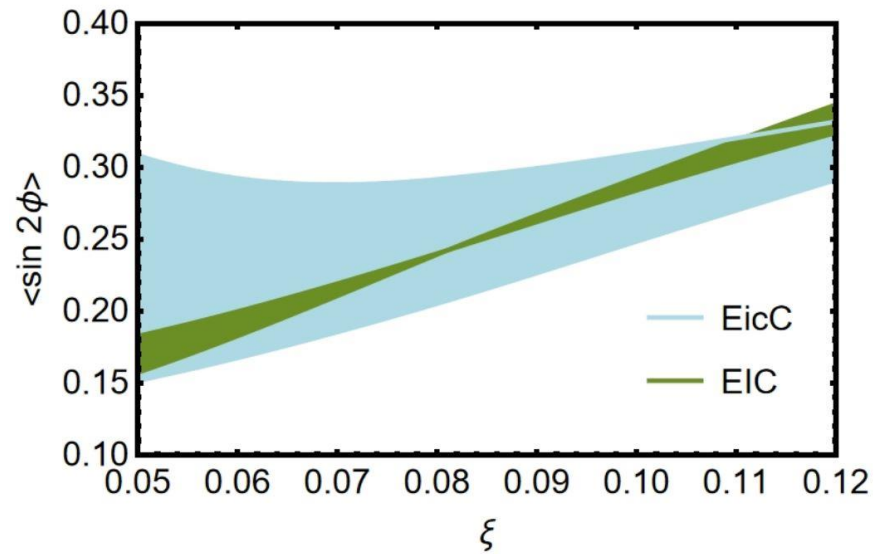
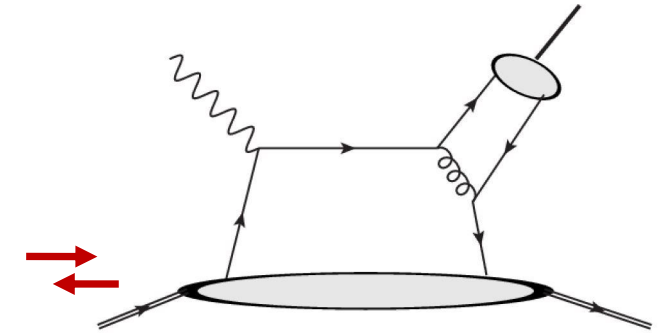
- Generalized TMDs/Wigner functions are the holy grail of spin physics
- Probe **quark OAM** via exclusive  $\pi^0$  production in ep collisions
- Circumvent challenges associated with double Drell-Yan process





# Summary

- Generalized TMDs/Wigner functions are the holy grail of spin physics
- Probe **quark OAM** via exclusive  $\pi^0$  production in ep collisions
- Circumvent challenges associated with double Drell-Yan process

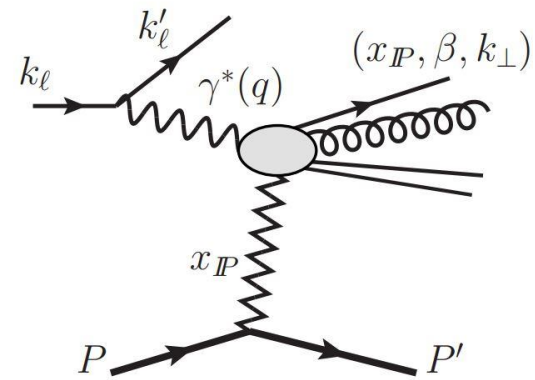
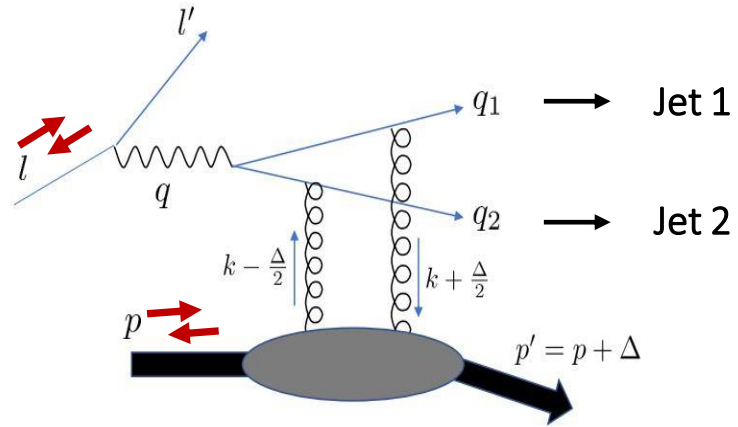


- Longitudinal single-target spin asymmetry is not power suppressed
- Asymmetry is substantial & thus exclusive  $\pi^0$  production in ep collisions maybe a promising route to constrain quark OAM



# Summary

- Probe **gluon OAM** via exclusive di-jet production/ SIDDIS in ep collisions





# Summary

- Probe **gluon OAM** via exclusive di-jet production/ SIDDIS in ep collisions

