Flying-focus beams as a tool for strong-field QED

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Outline

- Introduction on strong-field quantum electrodynamics (QED)
- Radiation reaction in a nutshell
- How I “stumbled into” flying-focus beams: On the transverse formation length of nonlinear Compton scattering (NCS)
- Applications of flying-focus beams to high-intensity physics
- Award to design NSF OPAL (more in the talk by C. Forrest)
- Conclusions

For more information see:

1. A. Di Piazza, Phys. Rev. A **103**, 012215 (2021) (at the MPIK)
2. M. Formanek et al., Phys. Rev. A **105**, L020203 (2022) (ADP at the MPIK)
3. M. Formanek et al., Phys. Rev. E **107**, 055213 (2023) (ADP at the MPIK)
4. M. Formanek et al., Phys. Rev. D **109**, 056009 (2024) (ADP at the UR/LLE and at the MPIK)

Collaborators:

Martin Formanek, John P. Palastro, Dillon Ramsey, Marija Vranic

Introduction on strong-field QED

	Classical Electrodynamics (CED)	Quantum Electrodynamics (QED)
Energy	Electron's rest energy: $\varepsilon_0 = mc^2 = 0.5\ \mathrm{MeV}$	
Momentum	$p_0 = \varepsilon_0/c = 0.5\ \mathrm{MeV}/c$	
Length	Classical electron's radius: $r_0 = e^2/4\pi mc^2 = 2.8\times10^{-13}\ \mathrm{cm}$ (from the Thomson cross section)	Compton wavelength: $\lambda_C = \hbar/p_0 = 3.9\times10^{-11}\ \mathrm{cm}$ (from Heisenberg uncertainty principle)
Time	$\tau_0 = r_0/c = 1.0\times10^{-23}\ \mathrm{s}$	$\tau_C = \lambda_C/c = 1.3\times10^{-21}\ \mathrm{s}$

$r_0 = \alpha\lambda_C$, where $\alpha = e^2/4\pi\hbar c \approx 1/137$ is the fine-structure constant

Field scales of QED (critical or Schwinger field)

$$E_{cr} = \frac{m^2c^3}{\hbar|e|} = 1.3\times10^{16}\ \mathrm{V/cm}$$

$$B_{cr} = \frac{m^2c^3}{\hbar|e|} = 4.4\times10^{13}\ \mathrm{G}$$

$\Rightarrow$

$$I_{cr} = cE_{cr}^2 = 4.6\times10^{29}\ \mathrm{W/cm^2}$$

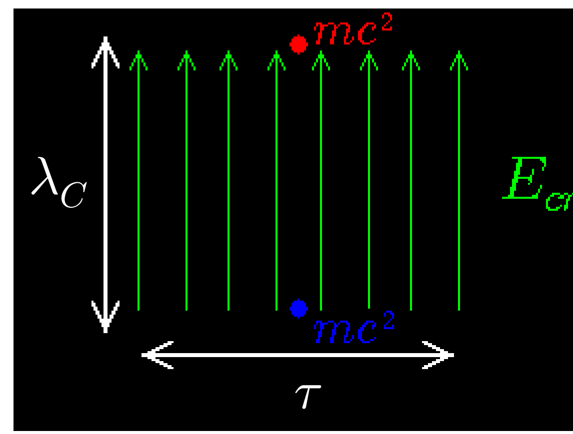
World-record intensity: $10^{23}\ \mathrm{W/cm^2}$ (Yoon et al. 2021)

The quantum vacuum

- The vacuum state is the lowest-energy state of the theory, where no particles are present
- In quantum field theory
  - "Fluctuations" of particles-antiparticles are present in the vacuum
  - They cover a short distance and annihilate again after a short time (for electrons and positrons $\lambda_C = \hbar / mc \sim 10^{-11}$ cm and $\tau = \lambda_C / c \sim 10^{-21}$ s, respectively)

- Physical meaning of the critical fields:

$$|e| E_{cr} \times \frac{\hbar}{mc} = mc^2$$



- Vacuum instability and electromagnetic cascades
- The interaction energy of a Bohr magneton with a magnetic field of the order of B_{cr} is of the order of the electron rest energy

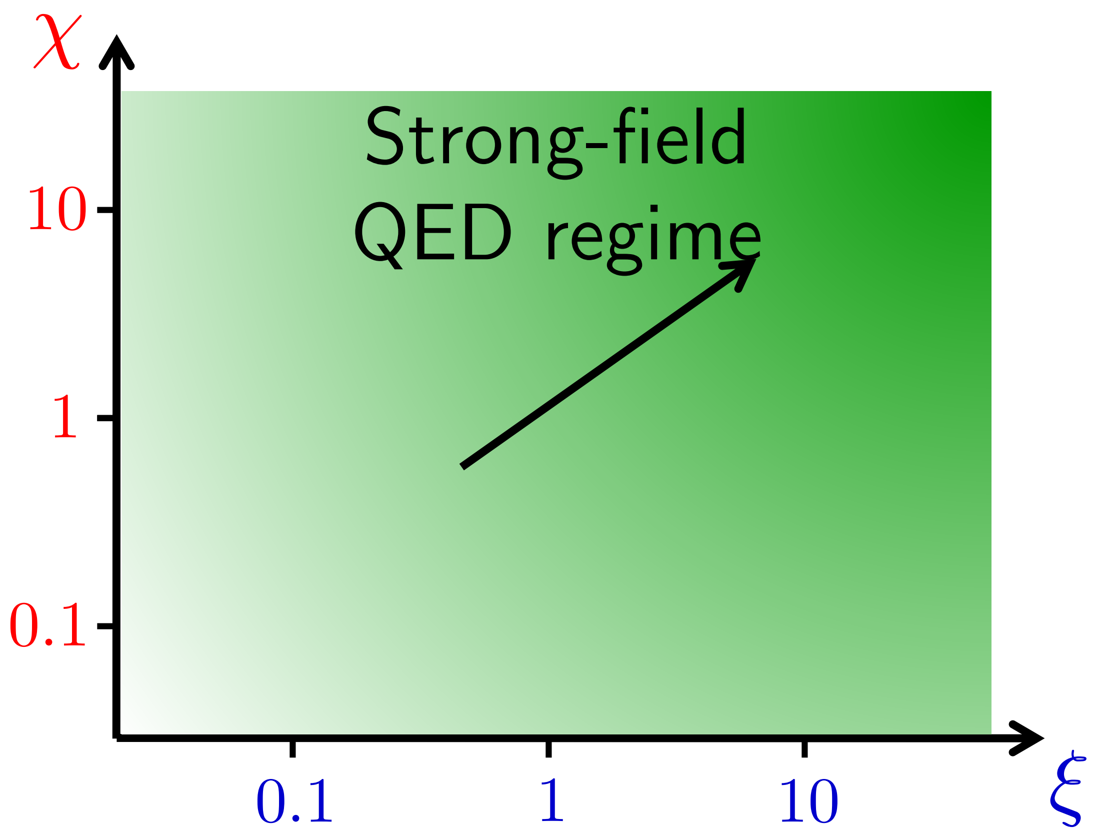
Strong-field QED in an intense laser field

- An electron with energy ε collides head-on with a plane wave with amplitude E_L and angular frequency ω_L (wavelength λ_L)

- The physical observables depend on the two Lorentz- and gauge-invariant parameters

$$\xi = \frac{1}{2\pi}\frac{|e|E_L\lambda_L}{mc^2} = \frac{|e|E_L\lambda_C}{\hbar\omega_L}$$

$$\chi = \left.\frac{E_L}{E_{cr}}\right|_{\text{rest frame}} \approx \frac{2\varepsilon}{mc^2}\frac{E_L}{E_{cr}}$$



Radiation reaction in a nutshell

Units: $\hbar = \epsilon_0 = c = 1$

- Accelerated electric charges emit electromagnetic radiation (relativistic Larmor formula, s is the electron's proper time)

$$\frac{d\mathcal{E}}{dt} = -\frac{2}{3}\frac{e^2}{4\pi}\frac{du^\mu}{ds}\frac{du_\mu}{ds}$$

- The exact dynamics of an electron in an external field includes the effects of this energy loss (and of the related momentum loss)
- By adding an extra force to the Lorentz equation due to the field produced by the electron itself, one can obtain the Lorentz-Abraham-Dirac (LAD) equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}\frac{e^2}{4\pi}\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

- The LAD equations features physical inconsistencies (runaway solutions, violation of causality) essentially due to the Schott term
- Landau and Lifshitz noticed that within CED a "reduction of order" can be carried out:

$$\frac{du^\mu}{ds} \approx \frac{e}{m}F^{\mu\nu} \text{ in } \frac{2}{3}\frac{e^2}{4\pi}\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

- One obtains the Landau-Lifshitz (LL) equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}\frac{e^2}{4\pi}\left[\frac{e}{m}(\partial_\alpha F^{\mu\nu})u^\alpha u_\nu - \frac{e^2}{m^2}F^{\mu\nu}F_{\alpha\nu}u^\alpha + \frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)(F_{\alpha\lambda}u^\lambda)u^\mu\right]$$

- The last term is responsible of the "radiation damping"

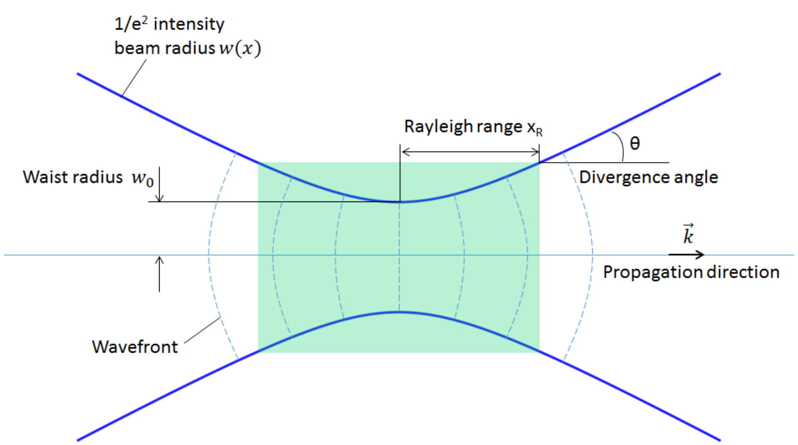
A limitation of Gaussian beams for strong-field QED

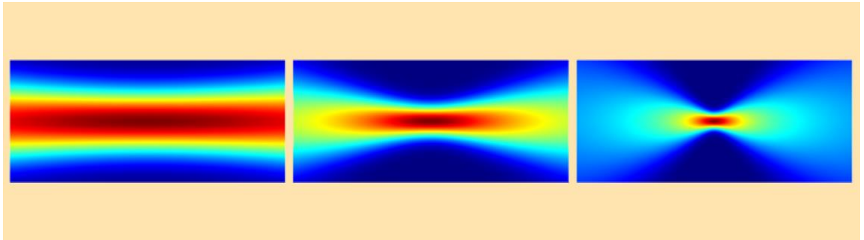
- The expressions of the classical and quantum nonlinearity parameters show that strong-field QED effects benefit from high laser intensities

$$\xi = \frac{1}{2\pi}\frac{|e|E_L\lambda_L}{mc^2} = \frac{|e|E_L\lambda_C}{\hbar\omega_L}$$

$$\chi = \left.\frac{E_L}{E_{cr}}\right|_{\text{rest frame}} \approx \frac{2\varepsilon}{mc^2}\frac{E_L}{E_{cr}}$$

- Interesting effects, like radiation reaction and vacuum birefringence, also depend on how long particles experience strong fields
- Gaussian laser beams feature an intrinsic limitation ultimately related with the properties of Maxwell's equations: at a given power, the more one focuses the beam to increase the intensity, the shorter becomes the longitudinal region where the field is strong





$$E_y(t,\boldsymbol{r}) = B_z(t,\boldsymbol{r})$$
$$= E_0\frac{e^{-r_\perp^2/w^2(x)}}{w(x)}\sin\left[\omega(t-x) - \frac{\omega x r_\perp^2}{x^2+x_R^2} + \psi_0\right]$$

$$r_\perp = \sqrt{y^2+z^2}, \quad w(x) = w_0\sqrt{1+\frac{x^2}{x_R^2}}, \quad x_R = \frac{\omega w_0^2}{2}$$

How I "stumbled into" flying-focus beams: On the transverse formation length of nonlinear Compton scattering (NCS)

- The classical spectrum emitted by an accelerated electron is given by

$$\frac{d\mathcal{E}}{d\omega d\Omega} = \frac{e^2}{16\pi^3}\left|\int dt \frac{\boldsymbol{n}\times\{[\boldsymbol{n}-\boldsymbol{v}(t)]\times\boldsymbol{a}(t)\}}{[1-\boldsymbol{n}\cdot\boldsymbol{v}(t)]^2} e^{-i\omega[t-\boldsymbol{n}\cdot\boldsymbol{r}(t)]}\right|^2 = \left|\int dt\, F(t) e^{-i\omega\int_0^t d\tau[1-\boldsymbol{n}\cdot\boldsymbol{v}(\tau)]}\right|^2$$

- The spectrum can also be written as

$$\frac{d\mathcal{E}}{d\omega d\Omega} = \int dt dt'\, F(t)F^*(t')e^{-i\omega\int_{t'}^t d\tau[1-\boldsymbol{n}\cdot\boldsymbol{v}(\tau)]} = \int dt_+ W(t_+)$$

where $t_\pm = (t \pm t')/2^{(1\pm1)/2}$ and

$$W(t_+) = \int dt_-\, F\left(t_+ + \frac{t_-}{2}\right) F^*\left(t_+ - \frac{t_-}{2}\right) e^{-i\omega\int_{t_+-t_-/2}^{t_++t_-/2} d\tau[1-\boldsymbol{n}\cdot\boldsymbol{v}(\tau)]}$$

- The emission of radiation at the time t_+ is formed on a time interval determined by the integral in t_-, which is called formation time/length
- The S-matrix element and the probability of NCS can be written as

$$S_{NCS} = \int d^4x A_{NCS}(x) \text{ and } P_{NCS} = |S_{NCS}|^2 = \int d^4x d^4x' A_{NCS}(x) A^*_{NCS}(x') = \int d^4x_+ W_{NCS}(x_+),$$

$$W_{NCS}(x_+) = \int d^4x_- A_{NCS}\left(x_+ + \frac{x_-}{2}\right) A^*_{NCS}\left(x_+ - \frac{x_-}{2}\right).$$

- The electron is described by a wave packet and the problem is four-dimensional (formation volume)

- I studied NCS in a focused laser field within a WKB-based approach valid for an ultrarelativistic electron barely deviated by the laser field
- The spectral probability can be written as (ADP 2014, 2015, 2021)

$$\frac{dP}{d\omega} = \int d^4x_+ \int dT_- d^2\boldsymbol{x}_{-,\perp} \exp\left\langle i\frac{T_-}{2}\left\{\text{[redacted]} - \frac{\varepsilon}{T_-^2}\left[\boldsymbol{x}_{-,\perp}\text{[redacted]}\right]^2 \text{[redacted]}\right\}\right\rangle \text{[redacted]}$$

- For the sake of illustration, the variable $\boldsymbol{x}_{-,\perp}$ can be used to introduce the transverse formation length (TFL) $l_\perp$ from the coefficient of $\boldsymbol{x}^2_{-,\perp}$:

$$l_\perp = \sqrt{\frac{2|T_-|}{\varepsilon}} = 2\lambda_C\sqrt{\omega_0|T_-|\frac{\xi_0}{\chi_0}}$$

- We look for a regime where $l_\perp \sim \sigma$, with σ being the laser focal spot
- The integral in T_- determines the longitudinal formation length (LFL) $l_\parallel$: $|T_-| \lesssim l_\parallel$
- For a conventional (realistic) Gaussian beam:

LFL	$\omega_0 l_\parallel \lesssim \omega_0 \min(2l_R,\tau)$ $l_R = \omega_0\sigma^2/2 =$ Rayleigh length, $\tau =$ pulse length
TFL	$l_\perp \lesssim 4\pi\lambda_C\frac{\sigma}{\lambda_0}\sqrt{\frac{\xi_0}{\chi_0}} \ll \sigma$

- One needs a laser pulse such that the ultrarelativistic electron can stay inside the focal region for a relatively long time
- In other words, one needs a laser beam with moving focus and I started searching in Google for that...

...and I found this paper: Esaray et al., Theory and group velocity of ultrashort, tightly focused laser pulses, JOSA B (1995), where the aim of the authors was to describe analytically an ultrashort and tightly-focused pulse with fixed focus

- In lightcone coordinates: $T=(t+z)/2$, $\phi = t - z$, and $\boldsymbol{x}_\perp = (x, y)$, the wave equation reads
$$2\frac{\partial^2 \boldsymbol{A}_\perp(x)}{\partial\phi\partial T} - \boldsymbol{\nabla}_\perp^2 \boldsymbol{A}_\perp(x) = \boldsymbol{0}$$

- By Fourier transforming in T (the pulse is assumed to propagate along the negative z direction) one obtains
$$-2ik\frac{\partial \tilde{\boldsymbol{A}}_\perp(k, \boldsymbol{x}_\perp, \phi)}{\partial\phi} - \boldsymbol{\nabla}_\perp^2 \tilde{\boldsymbol{A}}_\perp(k, \boldsymbol{x}_\perp, \phi) = \boldsymbol{0}$$

- This equation is in paraxial form but it is *exact* and then it admits the exact Gaussian-pulse form solution
$$\tilde{\boldsymbol{A}}_\perp(k, \boldsymbol{x}_\perp, \phi) = \tilde{\boldsymbol{A}}_0 f(k)\frac{e^{-\frac{\boldsymbol{x}_\perp^2}{2\sigma^2(1+i\phi/l_k)}}}{1+i\phi/l_k}$$

where $f(k)$ is an arbitrary function and l_k is the Rayleigh length of the mode k: $l_k = k\sigma^2$

- One can choose the shape function

$$f(k) = \frac{k}{k_0} e^{-(k-k_0)^2\tau^2/8}$$

describing a pulse with central angular frequency $\omega_0 = k_0/2$ and pulse length τ

- By going back to the "time" T, the exact analytical solution of Maxwell's equations is found ($\omega = k/2$)

$$\boldsymbol{A}_\perp(T, \boldsymbol{x}_\perp, \phi) = \boldsymbol{E}_0 \frac{\tau}{\sqrt{2\pi}} \mathrm{Re} \int \frac{d\omega}{\omega_0} \frac{\omega}{\omega_0} e^{-(\omega-\omega_0)^2 \frac{\tau^2}{2} - 2i\omega T} \frac{e^{-\frac{\boldsymbol{x}_\perp^2}{2\sigma^2(1+i\phi/l_\omega)}}}{1 + i\phi/l_\omega}$$

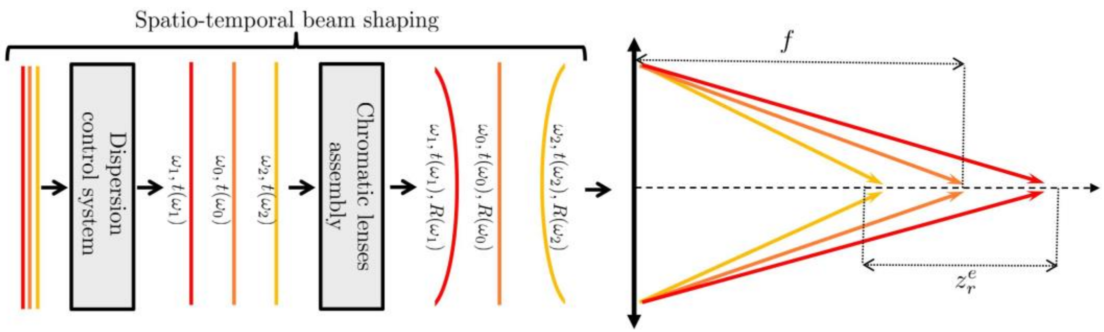
- Unlike Esarey et al., I looked to long pulses ($\omega_0\tau \gg 1$), where the integral can be taken approximately:

$$\boldsymbol{A}_\perp(T, \boldsymbol{x}_\perp, \phi) \approx \frac{\boldsymbol{E}_0}{\omega_0} e^{-2T^2/\tau^2} \frac{\sigma}{\sigma_\phi} e^{-\boldsymbol{x}_\perp^2/2\sigma_\phi^2} \cos\left[2\omega_0 T - \frac{\boldsymbol{x}_\perp^2}{2\sigma_\phi^2} \frac{\phi}{l_{\omega_0}} + \arctan\left(\frac{\phi}{l_{\omega_0}}\right)\right]$$

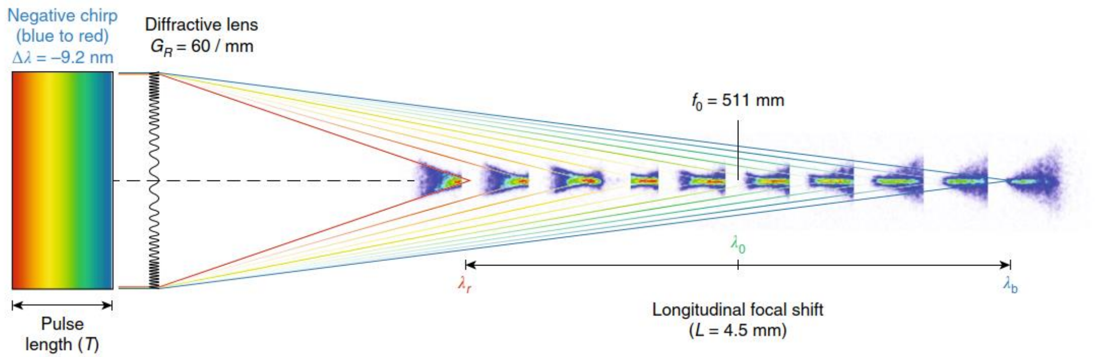
where $\sigma_\phi = \sigma[1+(\phi/l_{\omega_0})^2]^{1/2}$, with $l_{\omega_0} = \omega_0\sigma^2$

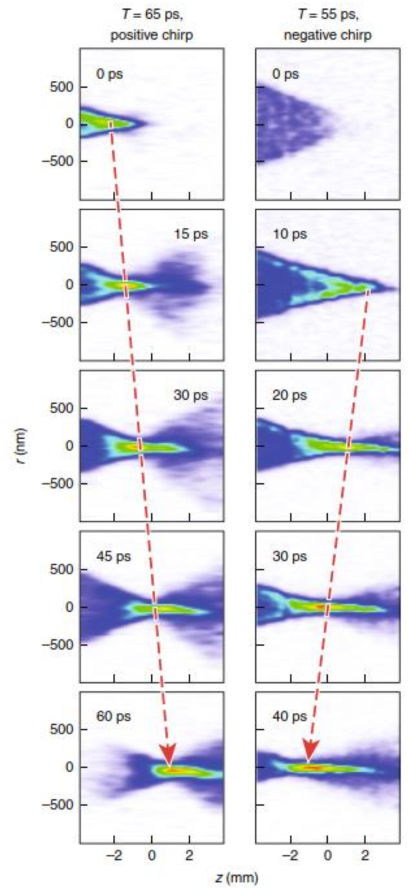
- This field looks exactly as a Gaussian beam but with $\phi = t - z$ instead of z: the focus is not fixed at $z = 0$ but it moves at the speed of light in the opposite direction of the phase velocity

- Such a beam can be realized experimentally by employing a chirped pulse (the frequency content of the pulse depends on time) and a chromatic lens (different frequencies are focused at different points):



Saint-Marie et al. Optica 2017 (theory)





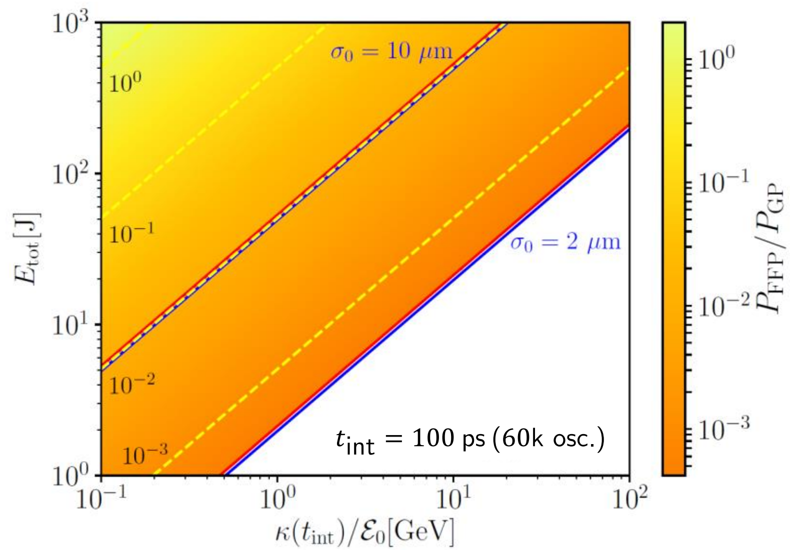
Froula et al. Nature Phys. 2018 (theory and experiment)

- The velocity of such a "flying-focus" pulse can be controlled, can be either positive or negative and the case $v_f \approx -1$ was demonstrated
- To sustain the pulse over a long length high energies are required

- Going back to the TFL, by considering an optical ($\lambda_0 = 1\ \mu$m) flying-focus beam with peak intensity 3×10^{18} W/cm^2 ($\xi_0 = 1$), spot radius $\sigma = 2\ \mu$m, and pulse duration $\tau = 100$ ps ($\Psi \sim 2\times10^5$) colliding head on with an electron with energy 8 GeV ($\chi_0 = 0.08$), substantial corrections due to a finite TFL are expected
- Such a pulse requires an energy of 40 J
- The idea of using a flying-focus pulse with focus moving at the speed of light in the opposite direction of the phase velocity can be exploited in different contexts
- Classical radiation reaction sizable at relatively low laser powers (Formanek et al. PRE 2022)
- Energy loss due to radiation reaction:

$$\varepsilon(t) \approx \frac{\varepsilon_0}{1 + \frac{4\varepsilon_0}{3m} r_e \omega_0^2 \int_0^t g^2(t')\xi^2(t')dt'} = \frac{\varepsilon_0}{1+\kappa(t)}$$

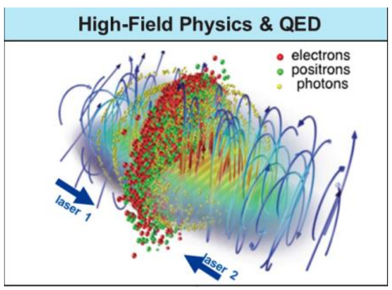
- The energy loss depends on the laser energy per unit surface ($\kappa(t) \sim \xi_0^2 t$)

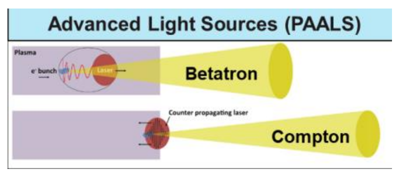


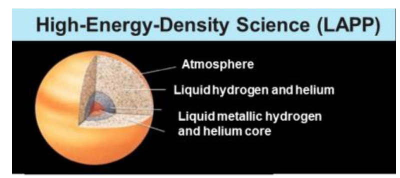
- A flying-focus allows for the same energy losses than a Gaussian beam but at much lower power

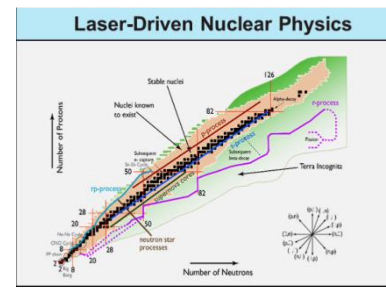
Recent award to design NSF OPAL

- The National Science Foundation (NSF) has funded an award ($18 million) to support the design of two 25-PW lasers at the LLE, as well as associated experimental and diagnostics systems (NSF OPAL)
- The design will be guided by the most pressing scientific questions that can be answered using such a laser system in four areas of frontier research:









- Join us by filling out the Working Group Interest Form at

https://app.smartsheet.com/b/form/99c146339a9a477690ffd711a86737bf

or by clicking on the QR code

Conclusions

- High-power lasers are becoming an important tool to test strong-field classical and quantum electrodynamics in still uncharted regimes
- Flying-focus beams are laser beams where the focus moves with controllable velocity
- They have been produced experimentally at relatively low intensities of the order of 10^{14} W/cm^2
- We have proposed to exploit the unique properties of flying-focus beams as a tool to test strong-field effects:
  1. Unveil the role of the transverse formation length of nonlinear Compton scattering
  2. Detect radiation-reaction effects under controlled conditions at relatively low laser powers/intensities
- NSF has awarded funding to design NSF OPAL: a two, 25-PW laser system at the LLE, where also nuclear-physics phenomena will be investigated

A post-doc position is available in my group at the University of Rochester starting at any time

If you are interested, please contact me at a.dipiazza@rochester.edu