Understanding Nature of Black Hole Holography

(A microscopic model)

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arXiv: 1712.02233 [hep-th]
1804.06154
1801.03918

with: M. Michel & S. Zell
arXiv: 1805.10292 [quant-ph]

Earlier work with
Cesar Gomez

✳ What is Holography?

✳ Is it exclusively gravitational?

Black Holes

Bekenstein: '73

't Hooft '93; Susskind '95

In Ads/CFT context

Witten '98;

Susskind, Witten '98

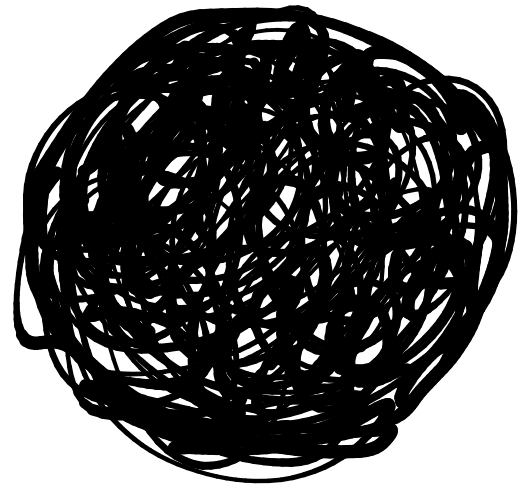
Black hole entropy (Bekenstein '73)

$$S_{BH} = \frac{R^2}{L_P^2}$$

Number of micro-states

$$N_{st} = e^{S_{BH}}$$

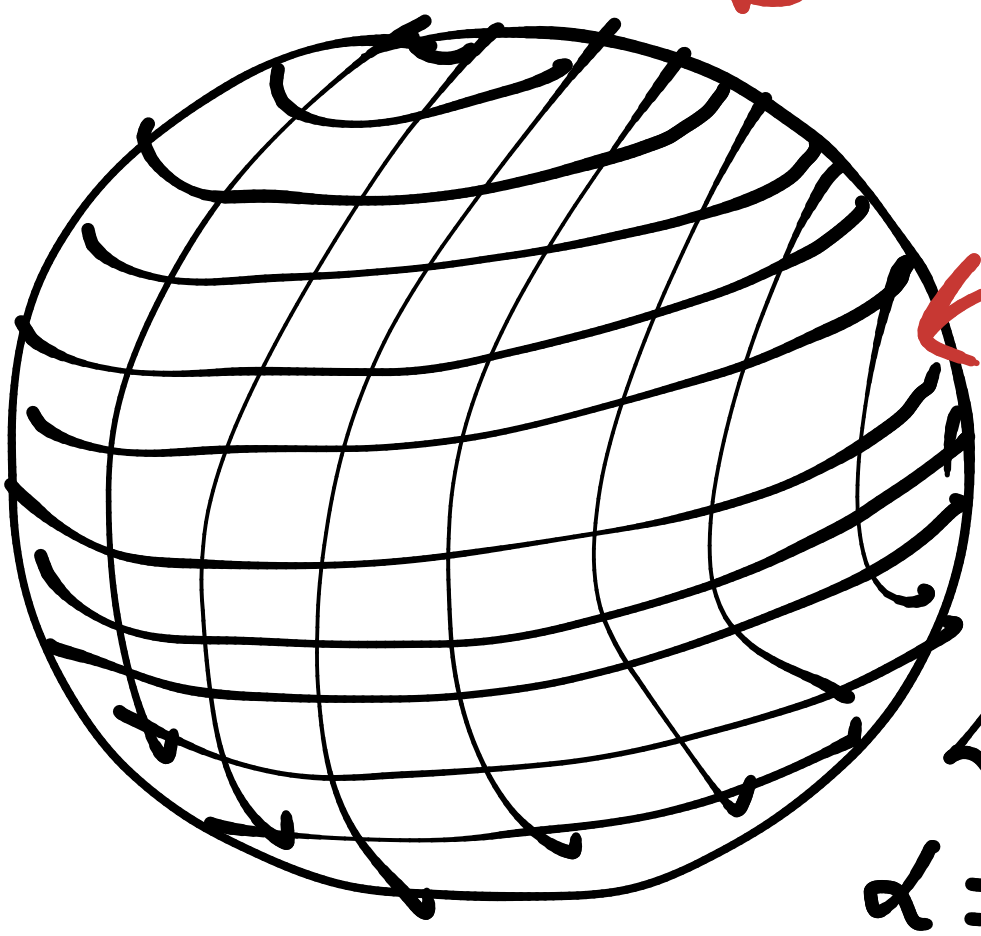
$$|BH\rangle_j =$$



Quantum Label j

Holographic degrees of freedom

2 Qubit



$\{|1\rangle$
$|0\rangle$

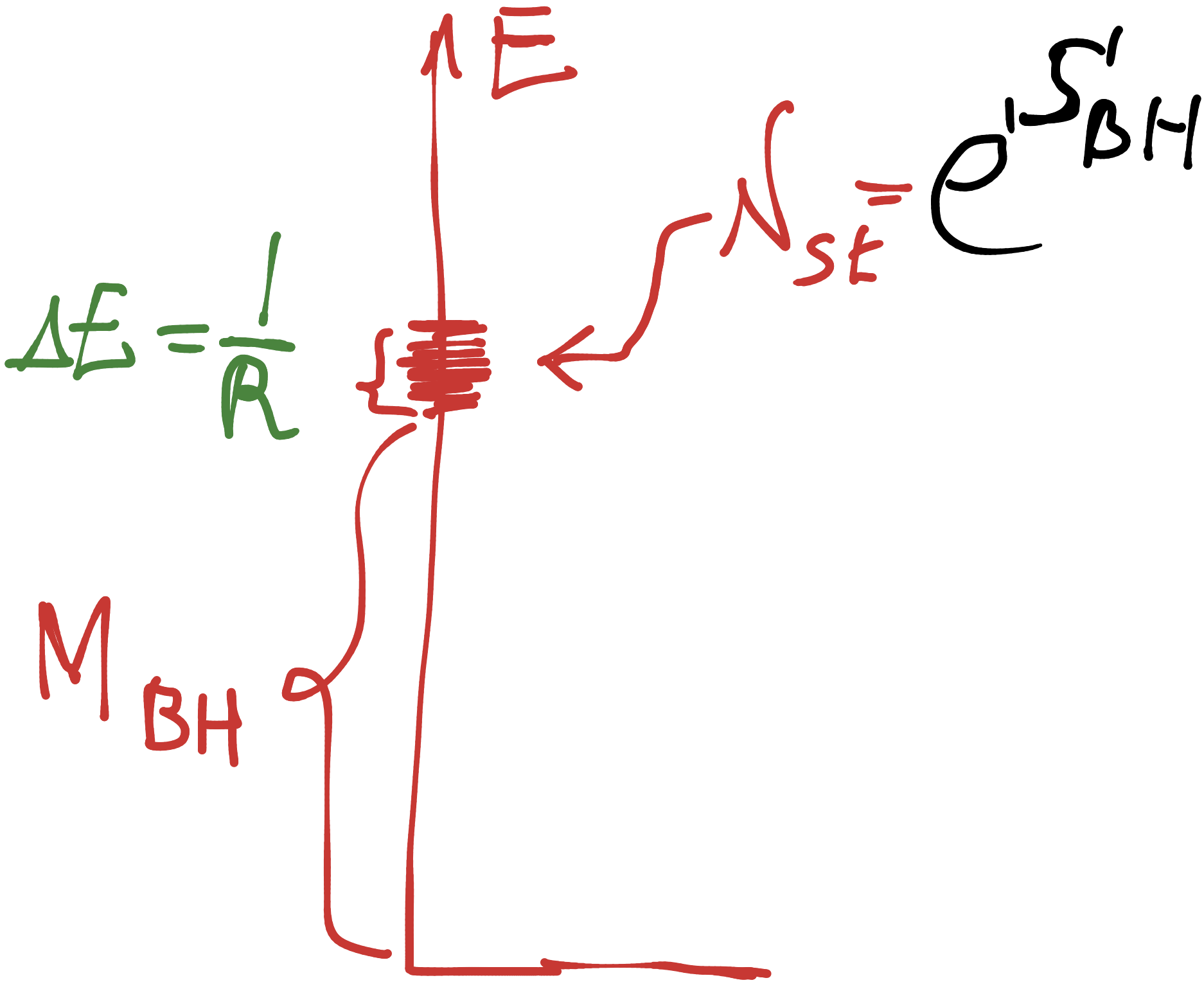
$\langle \hat{n}_\alpha \rangle = 0,1$

$\alpha = 1 \ldots S$

BH is S-long message:

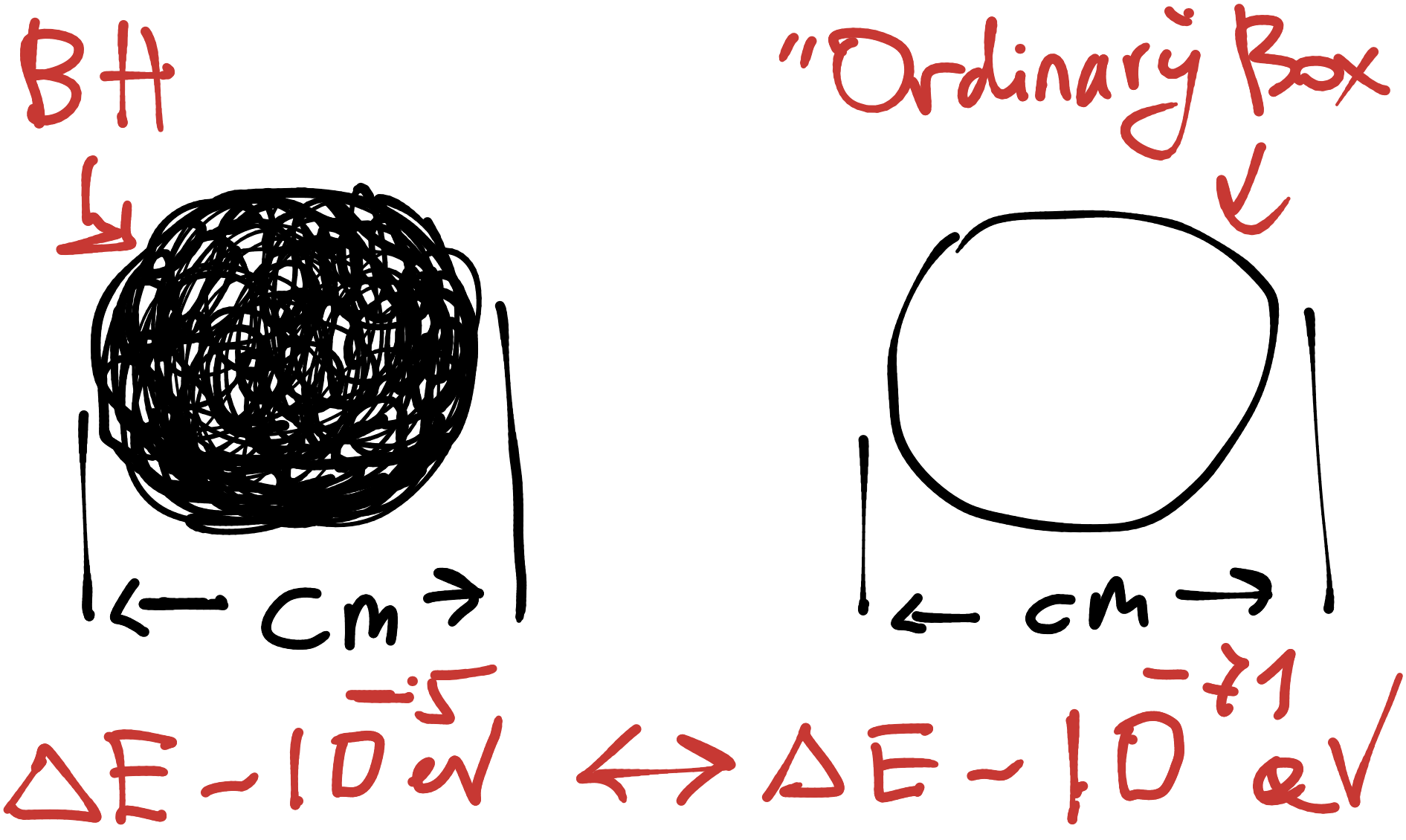
$$|BH\rangle = |n_1, n_2 \ldots\ldots n_S\rangle$$

But, unlike ordinary systems crowded in a tiny gap!



Thus, each holographic qubit must be almost gapless!

$$E_{|1\rangle} - E_{|0\rangle} = \frac{1}{S_{BH}} \frac{1}{R}$$



$\Delta E \sim 10^{-5}$ eV $\leftrightarrow$ $\Delta E \sim 10^{-71}$ eV

Two questions:

* What is the origin of gapless modes?

* What is the origin of Area-Scaling of their number?

storing pattern in a box

Number of momentum modes

$$\mathcal{N}_{mode} \sim (K_{MAX} R)^3 = \left(\frac{R}{L_P}\right)^3$$

$K_{max} = M_P$

Entropy $\sim (R M_P)^3$?

NO!

Energy cost of a pattern

$$|0,1,1,0,1,2 \ldots\ldots 1,0,1\rangle$$

$$n_k = 0,1 \qquad |k| \sim M_P$$

$$E_{Pattern} \sim \left(\frac{R}{L_P}\right)^3 M_P$$

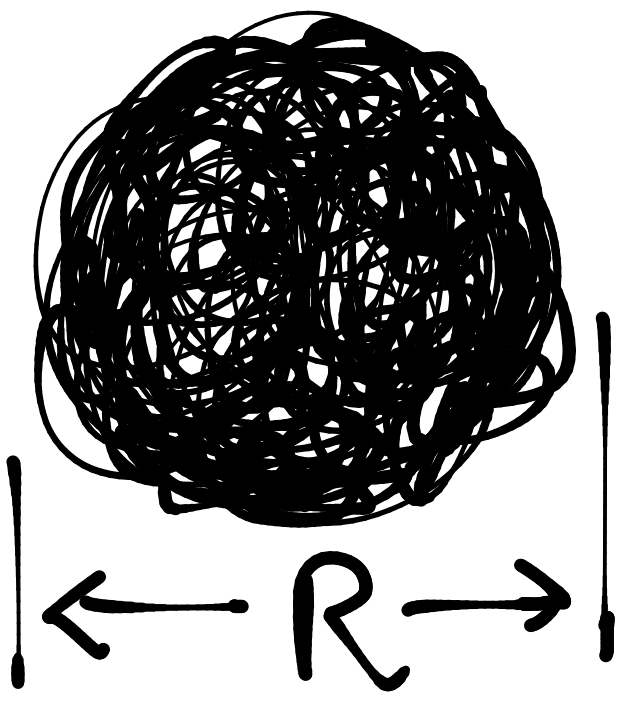
$$\left(\frac{R}{L_P}\right)^2 \cdot M_{BH}$$

NO WAY !

Black hole is achieving this differently:

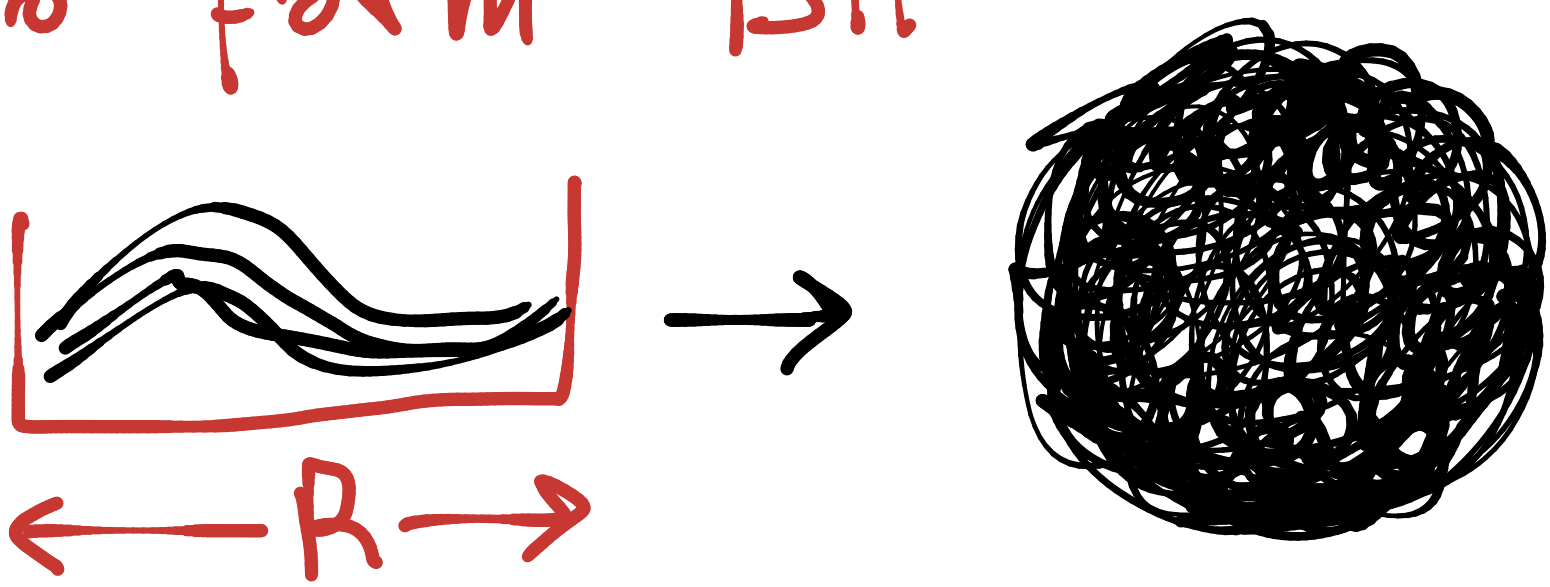
$$S_{BH} \sim \left(\frac{R}{L_P}\right)^2 = M_{BH} R$$

Bekenstein bound



$$S \leq MR$$

Alice has to populate box by $N \sim \alpha'^{-1} = \frac{R^2}{L_P^2}$ soft gravitons, in order to form BH



$$N = \frac{1}{\alpha}$$

Magic number

I shall present an effective model:

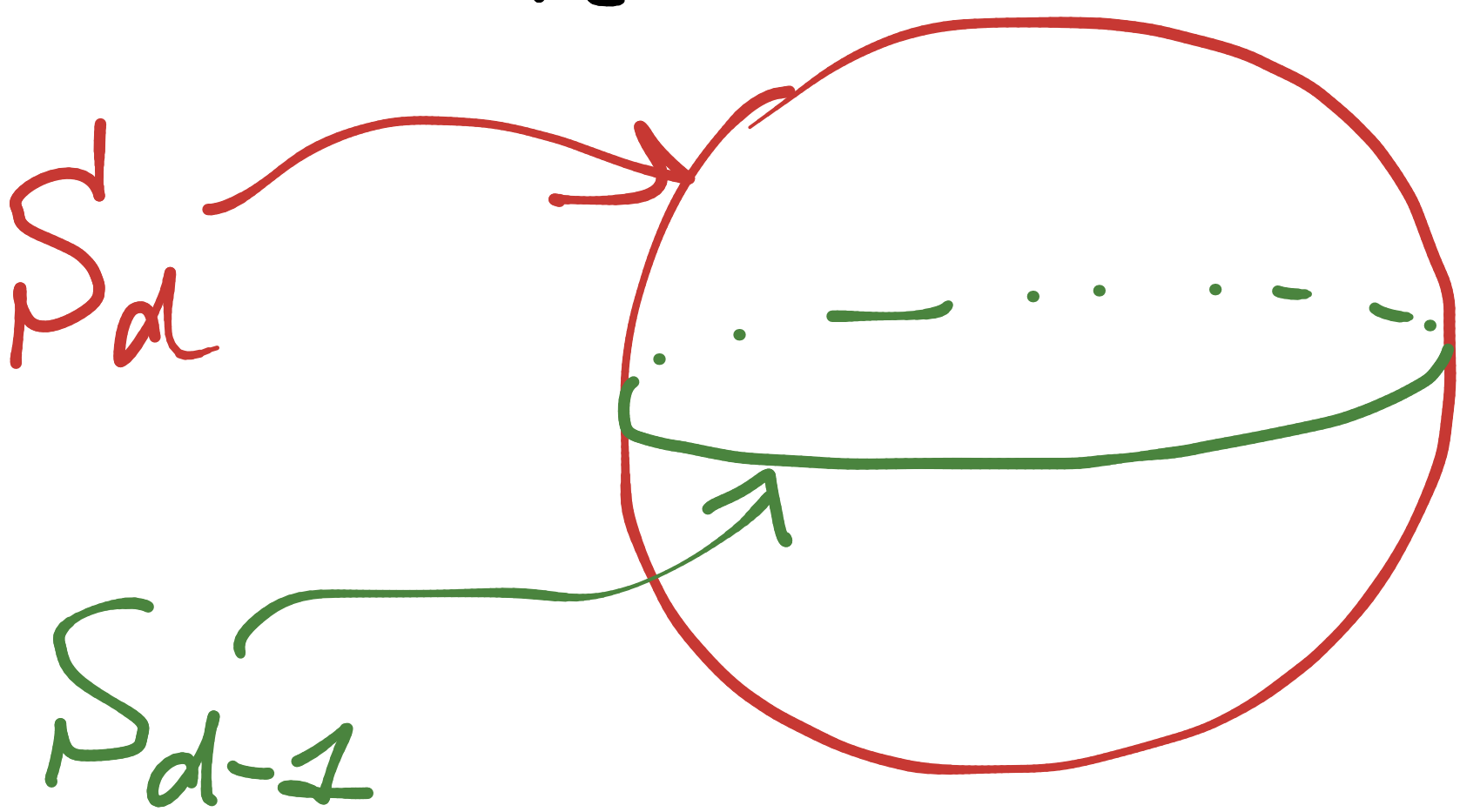
* Attractive momentum-dependent coupling.

* High occupation of soft modes to "magic" number $N \sim \frac{1}{\alpha}$.

See, what happens!

A Microscopic Model: Quantum field on S_d

$$\hat{\psi}(\theta_a) = \sum_K Y_K(\theta_a)\,\hat{a}_K$$



$$Y_k(\theta) \equiv Y_{k_1 \cdots k_d}(\theta)$$

$$k \equiv (k_1 \cdots k_d)$$

$$|k_1| \leq k_2 \leq \ldots \leq k_d = 0, 1, \ldots \infty$$

spherical harmonics on S_d.

$$[\hat{a}_k, \hat{a}^+_{k'}] = \delta_{kk'}$$

Hamiltonian:

$$\hat{H} = \int d^d x \; -\hat{\psi}^+ \Delta \psi - \frac{2}{\Lambda}\left(\hat{\psi}^+ \Delta \hat{\psi}^+\right)\left(\hat{\psi}\,\Delta\,\hat{\psi}\right) + \frac{1}{2\Lambda^2}\left(\hat{\psi}^+ \Delta^2 \hat{\psi}^+\right)\left(\hat{\psi}^2 \Delta \hat{\psi}^2\right) + h.c.$$

$$\Delta Y_k = -\epsilon_k Y_k$$

$$\epsilon_k \equiv \frac{k_d\left(k_d + d - 1\right)}{2mR^2}\, t^2$$

States with

$$\langle \hat{a}_0^\dagger \hat{a}_0 \rangle \equiv N \gg 1$$

Bogoliubov approximation:

$$\hat{a}_0 \to \sqrt{N}, \quad \hat{a}_0^\dagger \to \sqrt{N}.$$

Now, we take double-scaling limit:

$$N \to \infty, \quad \frac{N}{\Lambda} = \text{finite} \neq 0$$

Hamiltonian becomes:

$$\hat{H} = \sum_{k} \epsilon_k \left(1 - \frac{N}{\Lambda}\epsilon_k\right)^2 \hat{a}_k^\dagger \hat{a}_k$$

gapless modes:

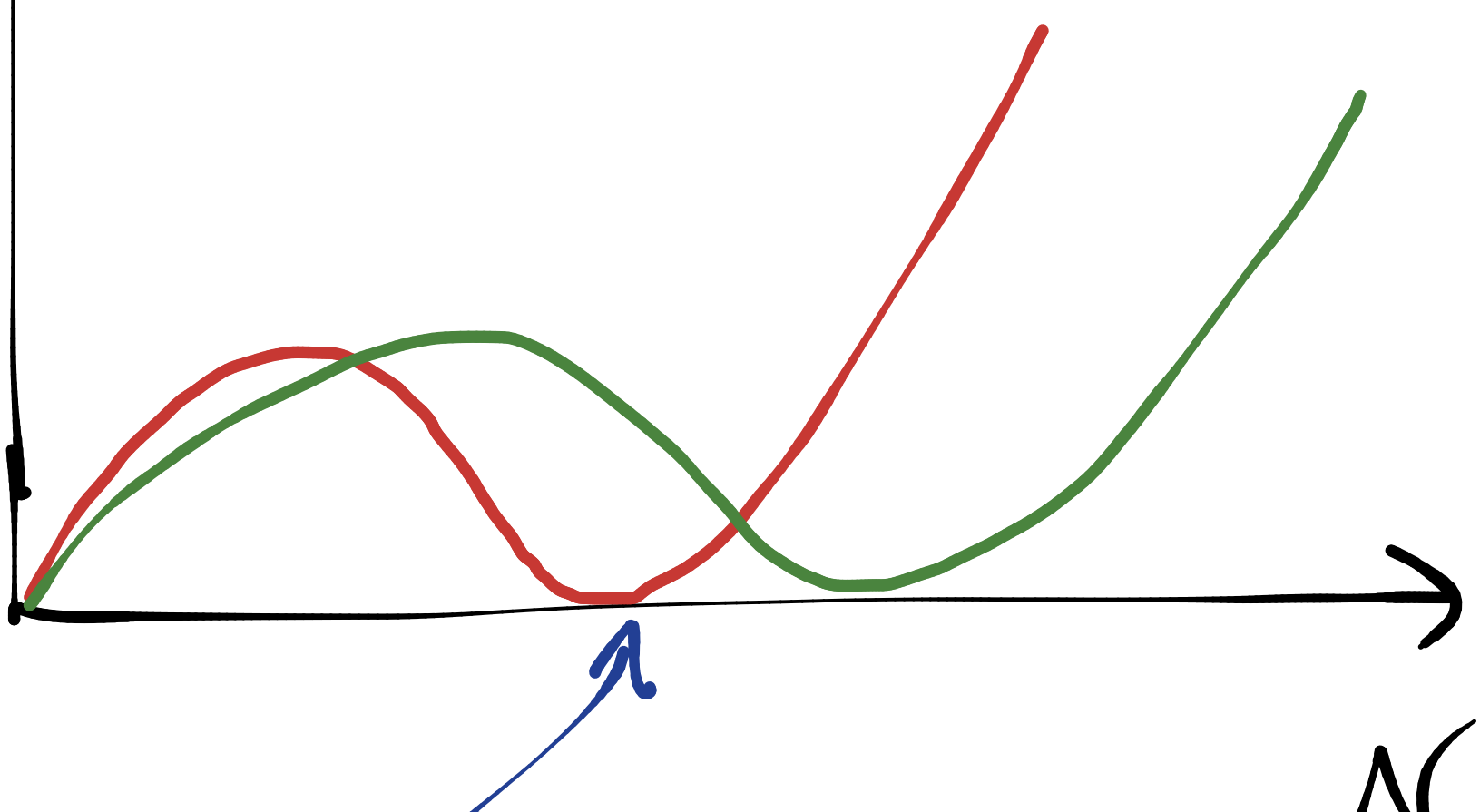
$$\epsilon_k = \frac{\Lambda}{N}$$

degeneracy:

$$S = \sum_{k_{d-1}}^{k_2} \cdots \sum_{k_1=-k_2}^{k_2} \simeq (k_2)^{d-1}$$

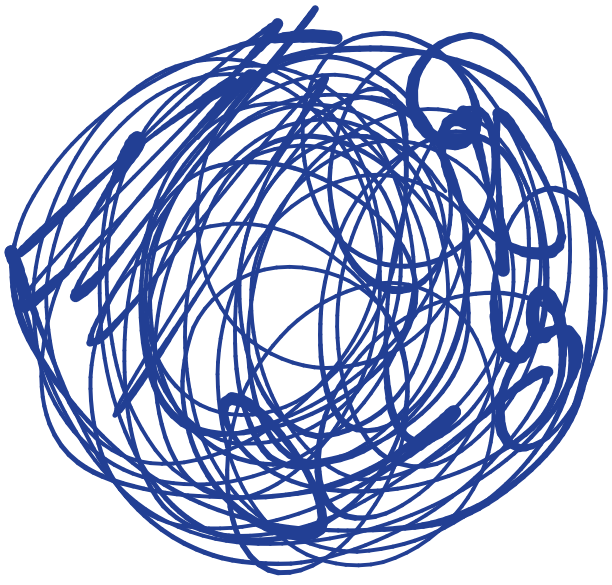
$\omega_K(N)$

$$\hat{H} = \sum_K \omega_K \hat{a}^\dagger_K \hat{a}_K$$



N

$S_{\lambda-1}$



$$\text{Entropy} = A_{S_{d-1}}$$

Entropy:

$$S = \sum_{K_{d-1}}^{K_1} \cdots \sum_{K_1 = -K_2}^{K_2} 2 \left(K_d\right)^{d-1}$$

$$\rho \equiv \left(\frac{R}{L_K}\right)^{d-1}$$

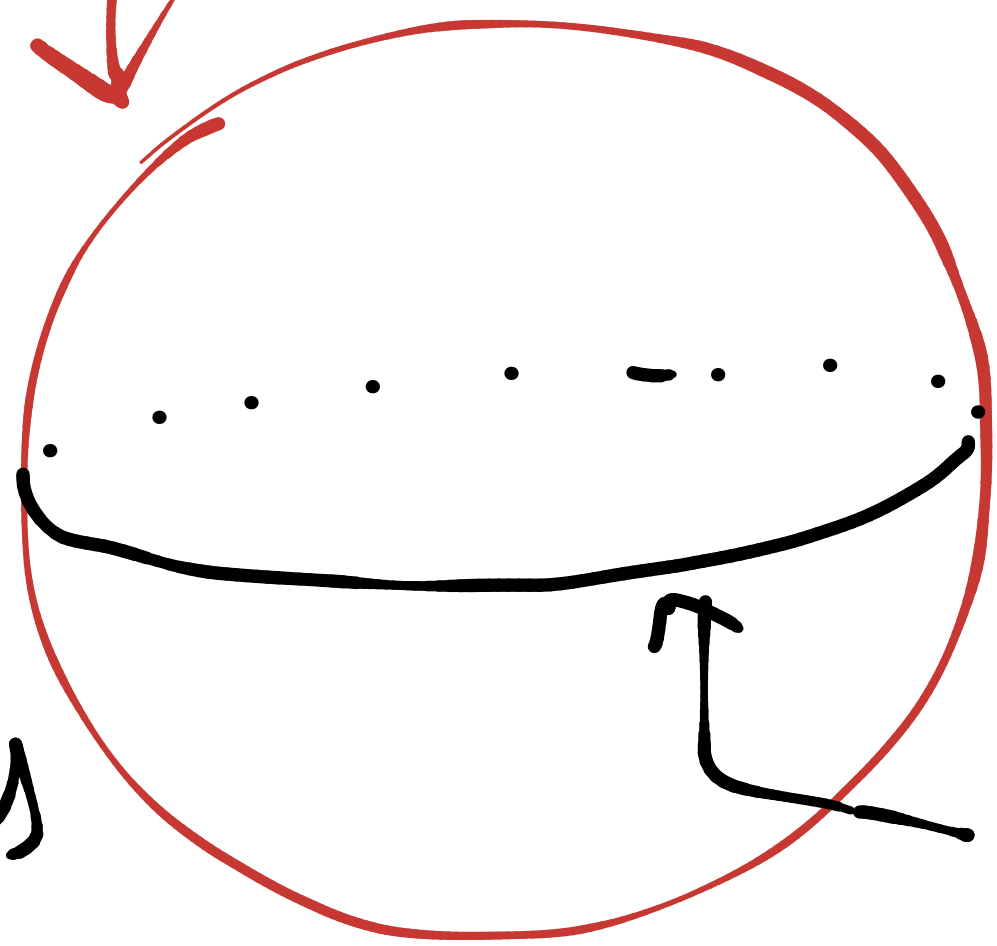
$$L_K \equiv \frac{\hbar}{\sqrt{2m\epsilon_K}}$$

Thus, excitation of a soft mode $\hat{a}_0$ to level N renders the modes $\hat{a}_k$, with

$$\epsilon_k = \frac{\Lambda}{N}$$, gaplen!

Entropy $= S = \left(\frac{R}{L_K}\right)^{d-1}$ Area of S_{d-1}!

$\vec{\mathcal{F}}(\theta)$



The modes $\epsilon_k = \frac{\Lambda}{N}$ are Holographic and populate S_{d-1}.

Number of poles modes scales as $\left(\frac{R}{L_K}\right)^{d-1}$!

A quantum field with attractive momentum-dependent interaction exhibits a holographic behaviour and Area-law entropy due to criticality..

(G.A., Gomez '11, 12 — criticality of attractive Bosons as origine of holography)

Our Model:

* BH = Many soft gravitons.

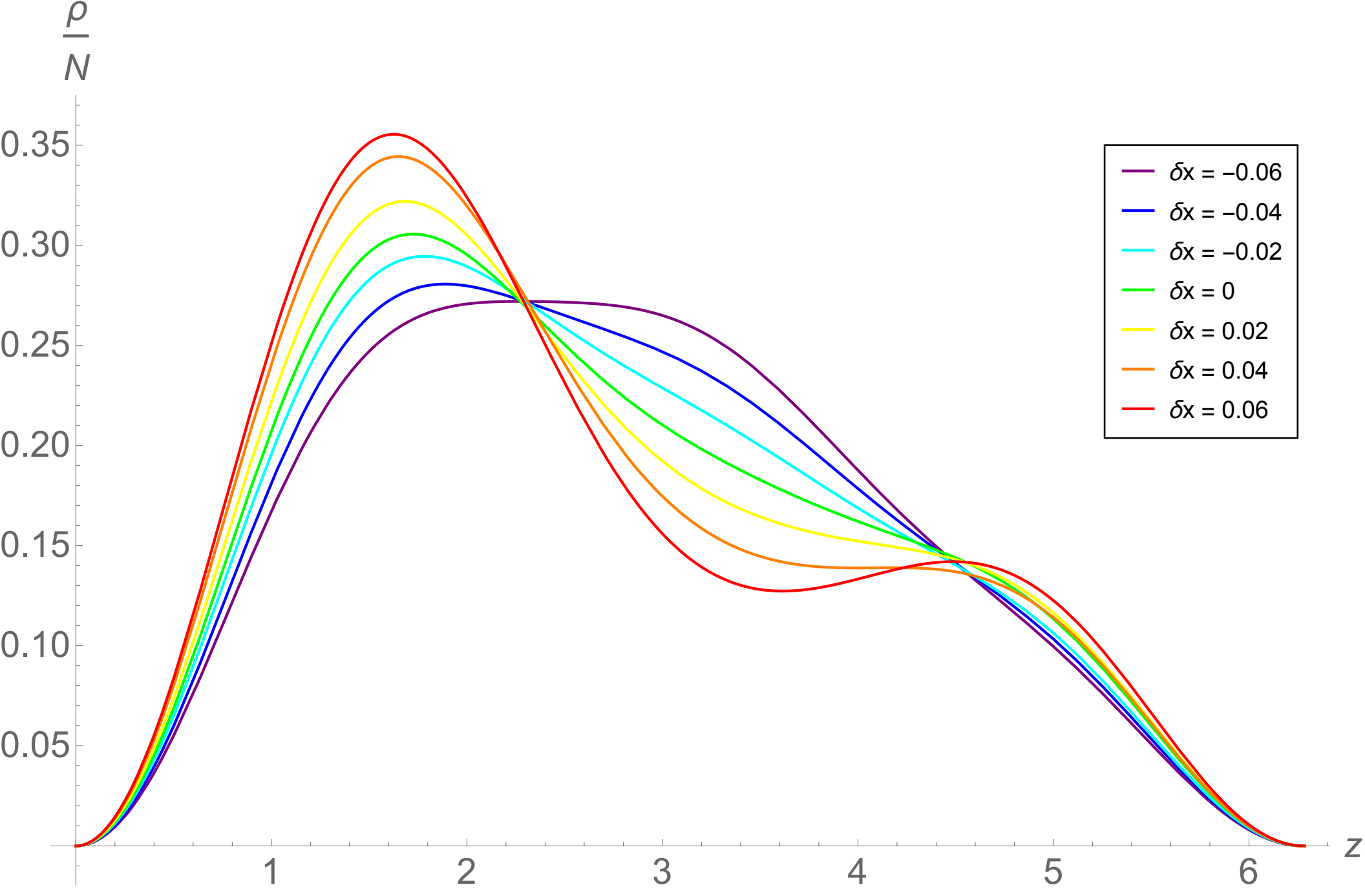
BH-macrostate:

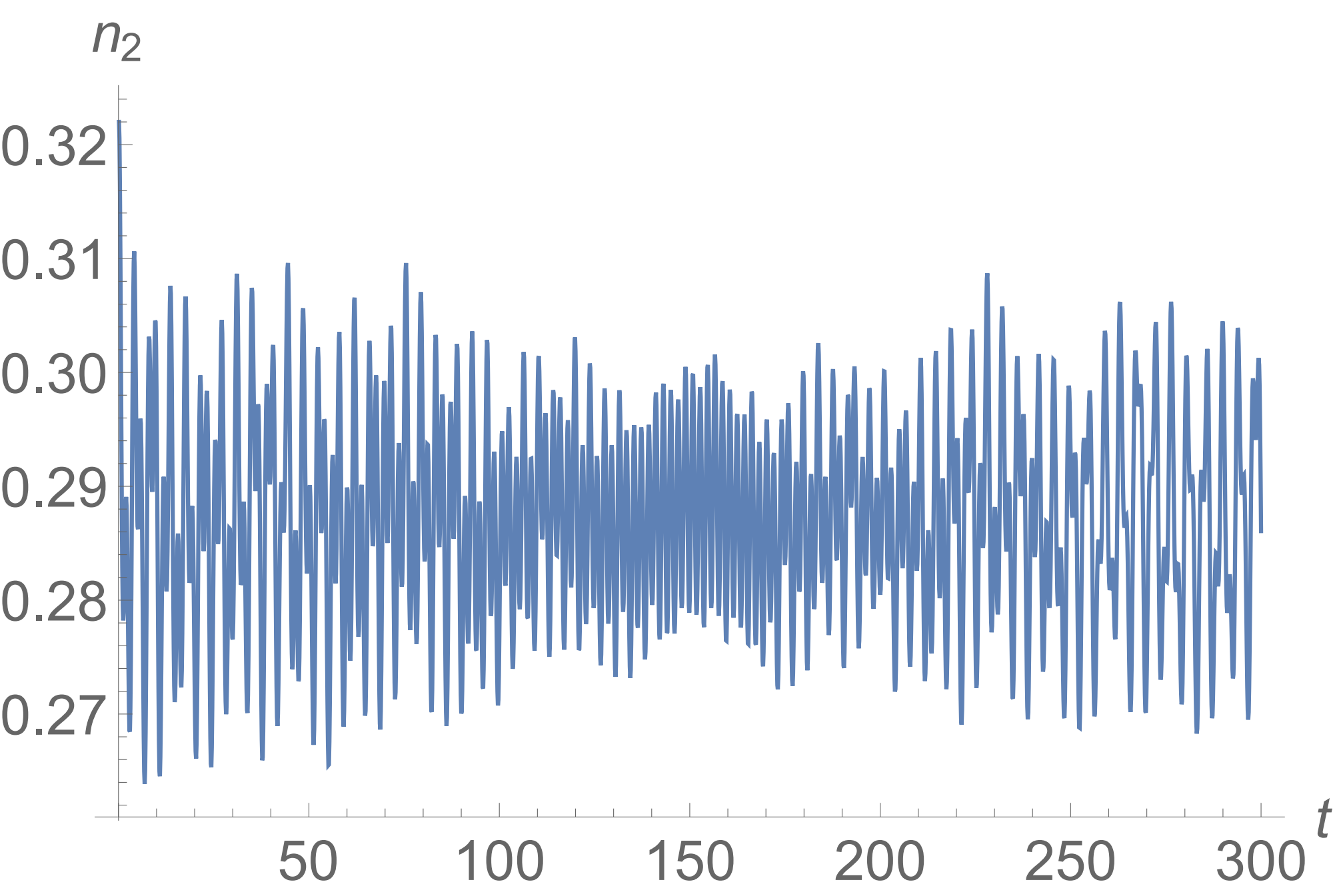
$$|BH\rangle = |N\rangle_{\lambda \sim R}$$

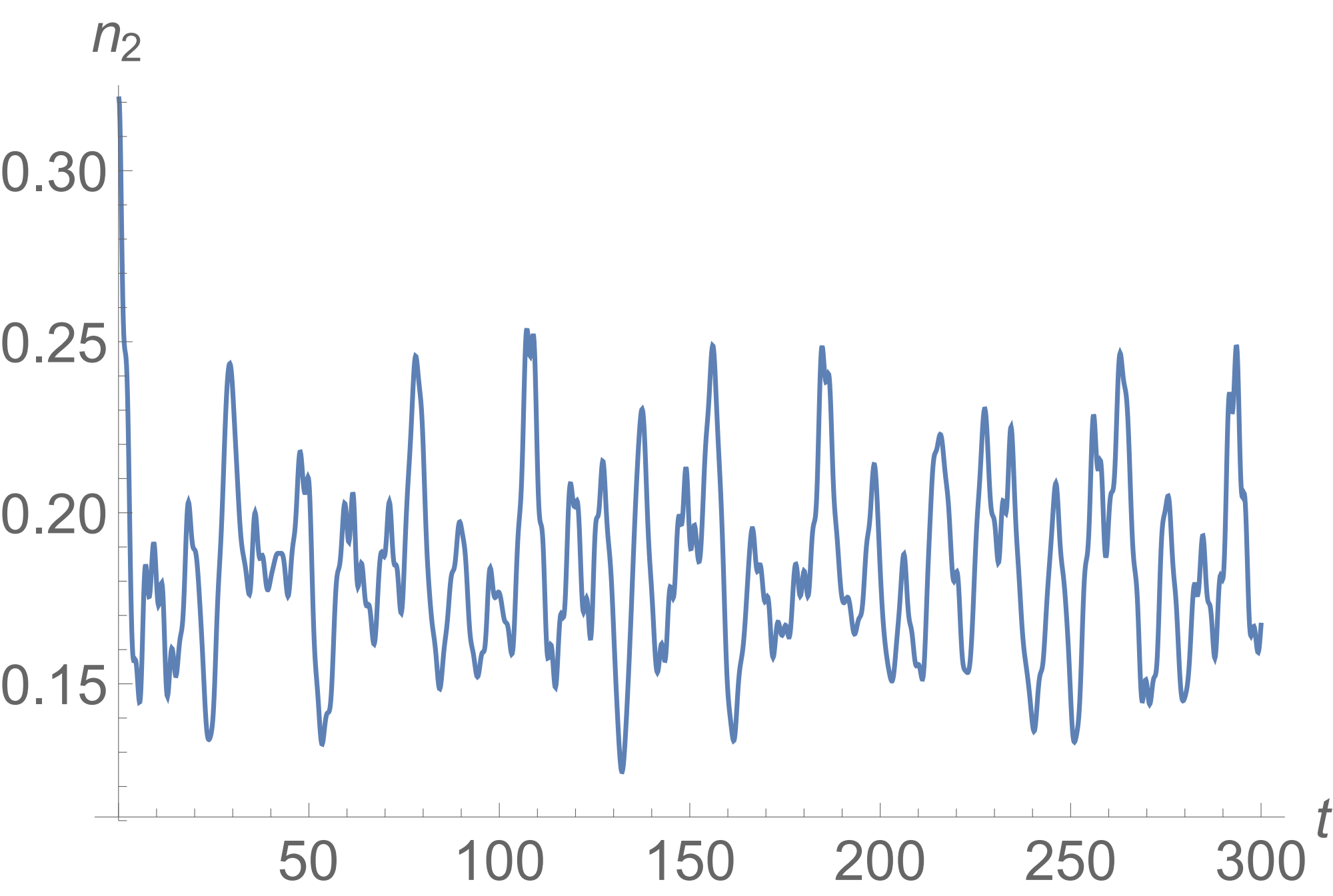
* Assisted gaplessness:

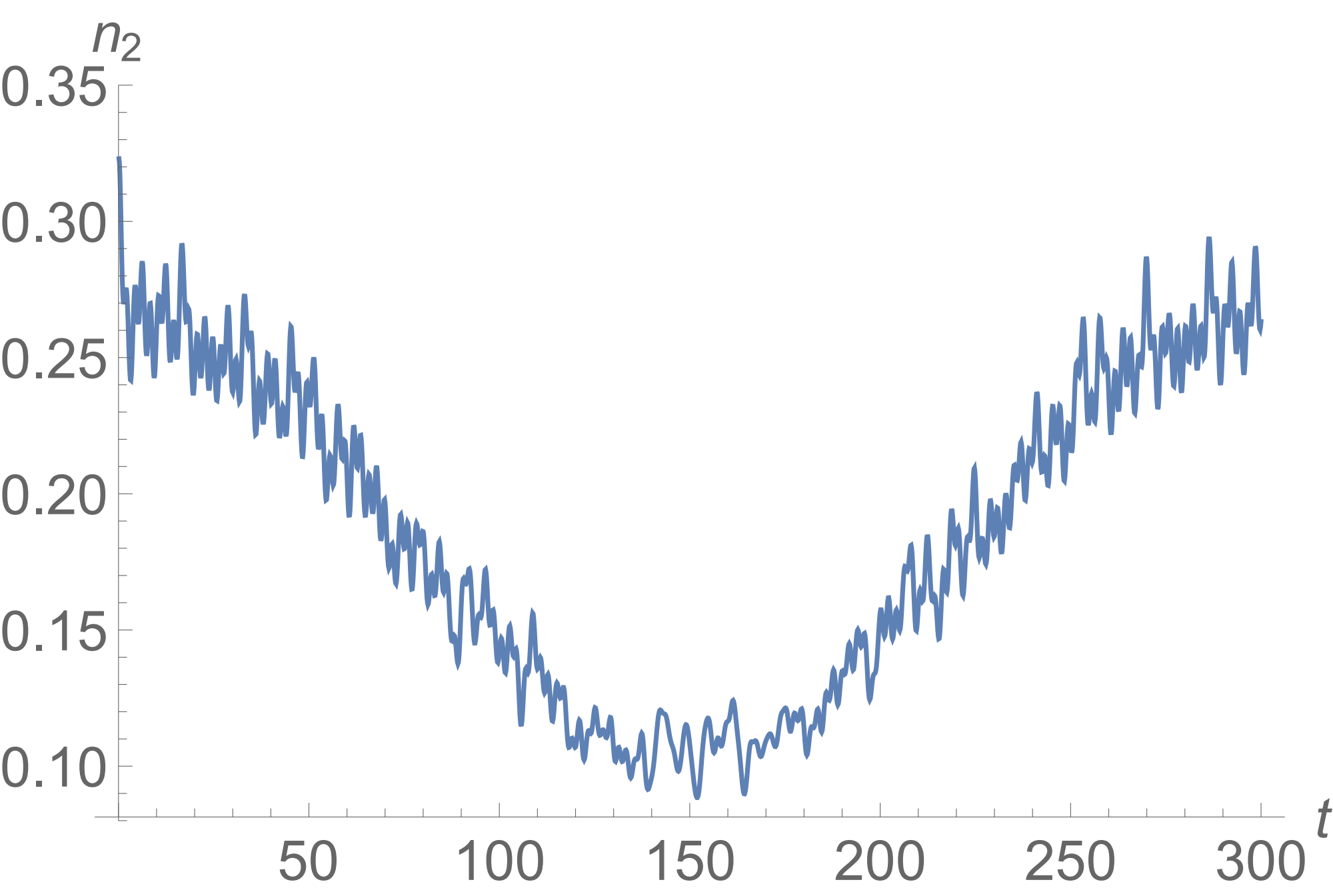
High occupation of soft modes, renders "hard" modes $|\vec{k}| \sim M_P$

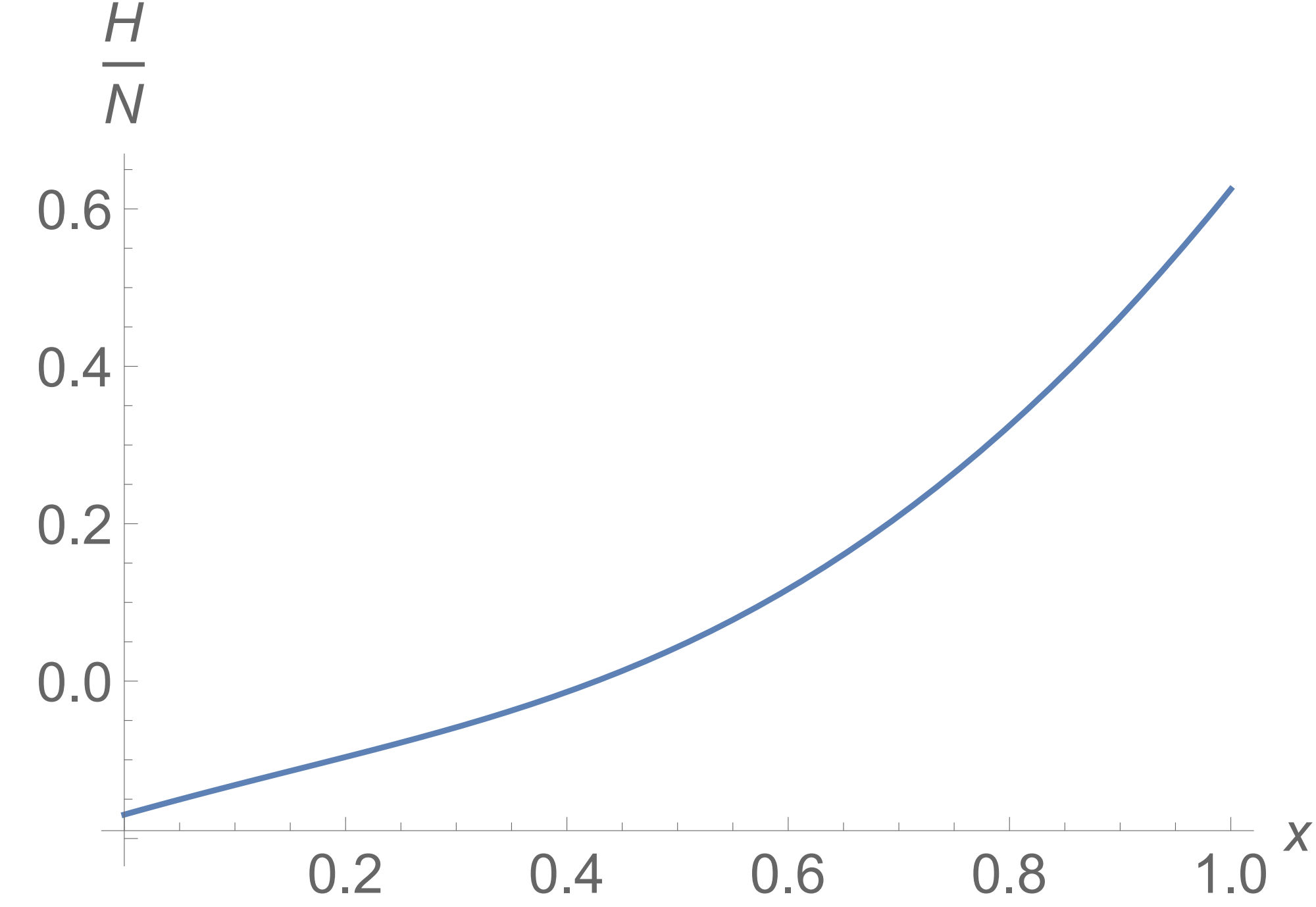
gapless!

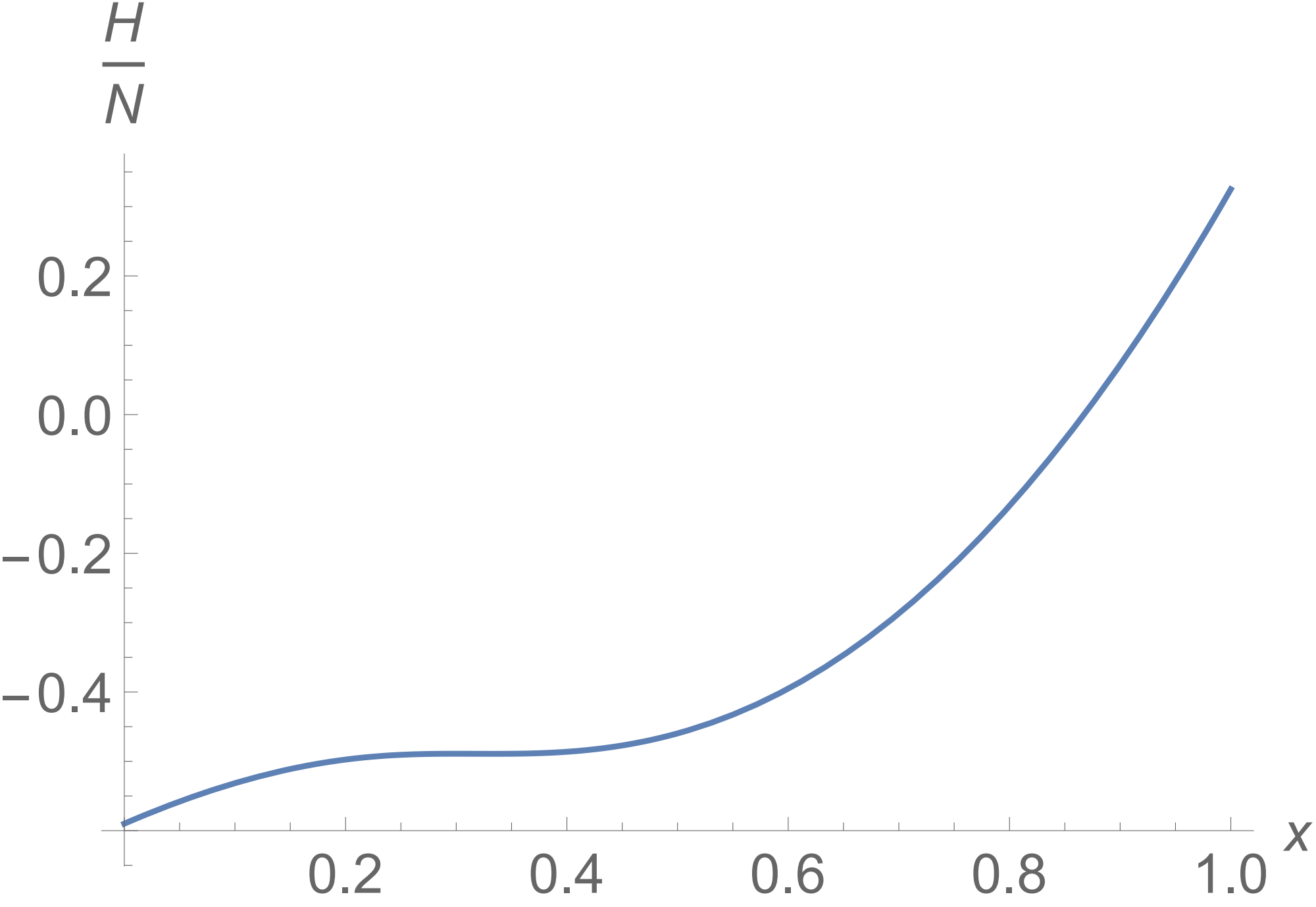

ρ/N
0.35
0.30
0.25
0.20
0.15
0.10
0.05
1
2
3
4
5
6
z
δx = −0.06
δx = −0.04
δx = −0.02
δx = 0
δx = 0.02
δx = 0.04
δx = 0.06

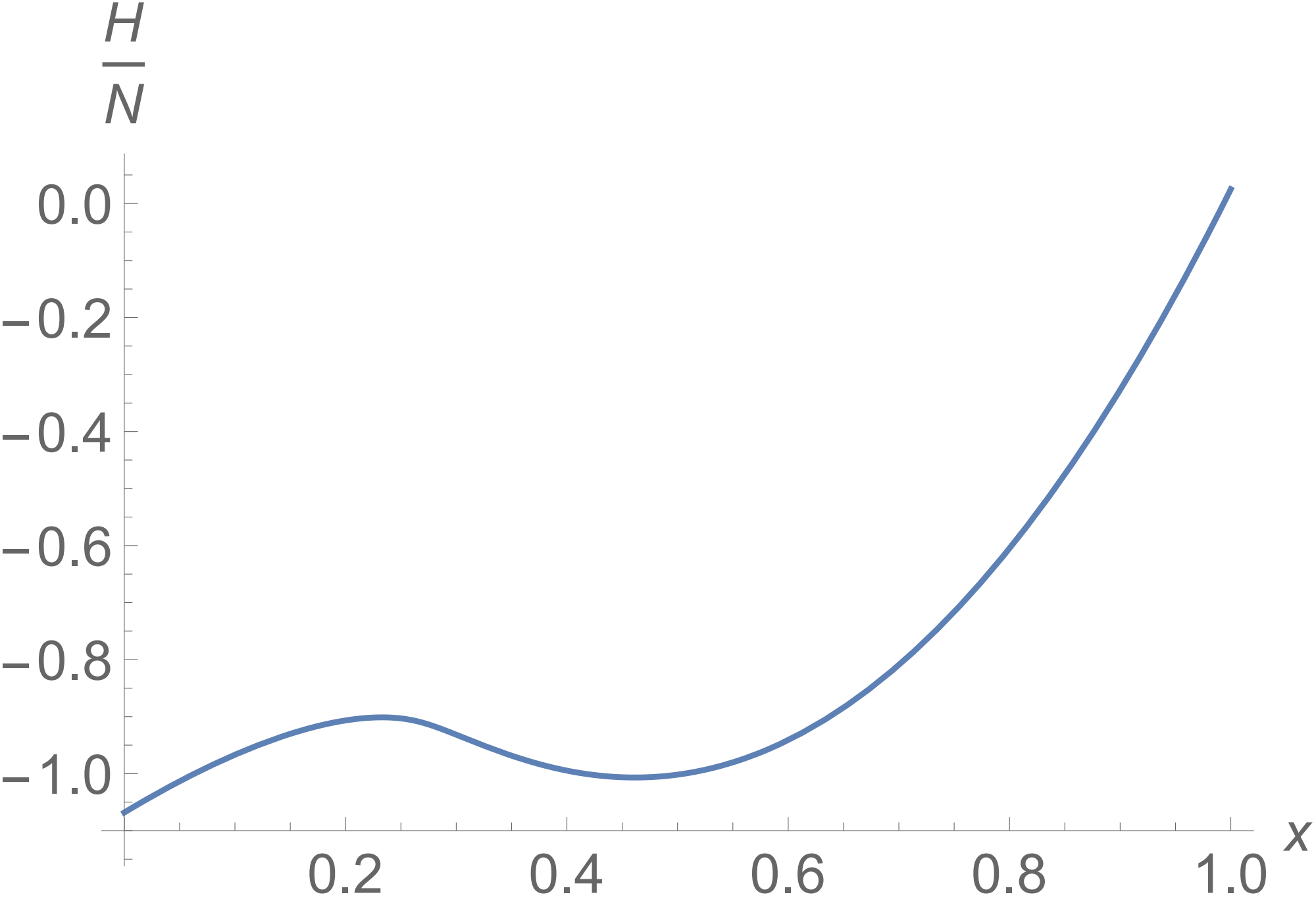

n_2
0.32
0.31
0.30
0.29
0.28
0.27
t
50
100
150
200
250
300

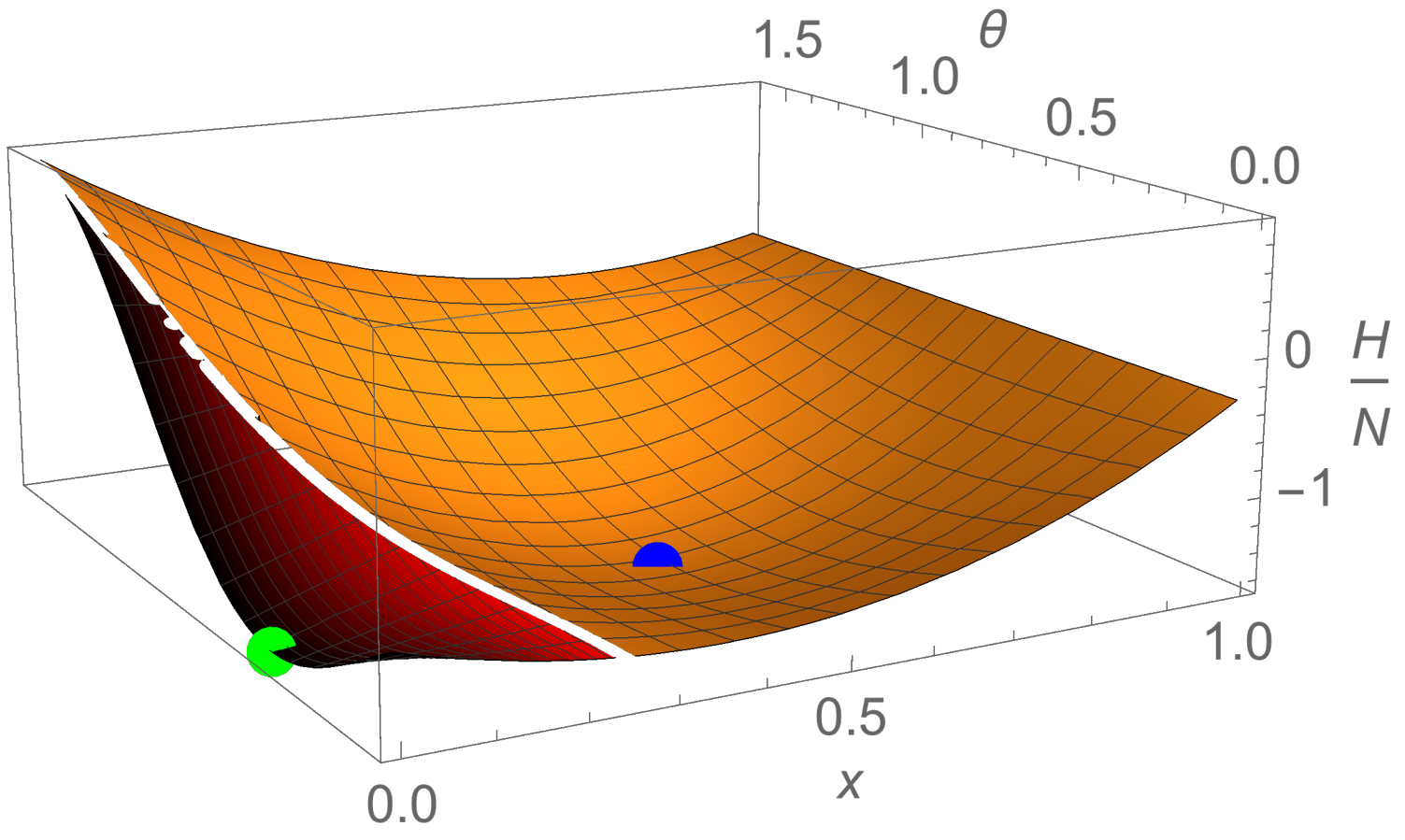

n_2
0.30
0.25
0.20
0.15
50
100
150
200
250
300
t


n_2
0.35
0.30
0.25
0.20
0.15
0.10
50
100
150
200
250
300
t


H/N
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
x


H/N
0.2
0.0
−0.2
−0.4
0.2
0.4
0.6
0.8
1.0
x


H/N
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0.2
0.4
0.6
0.8
1.0
x

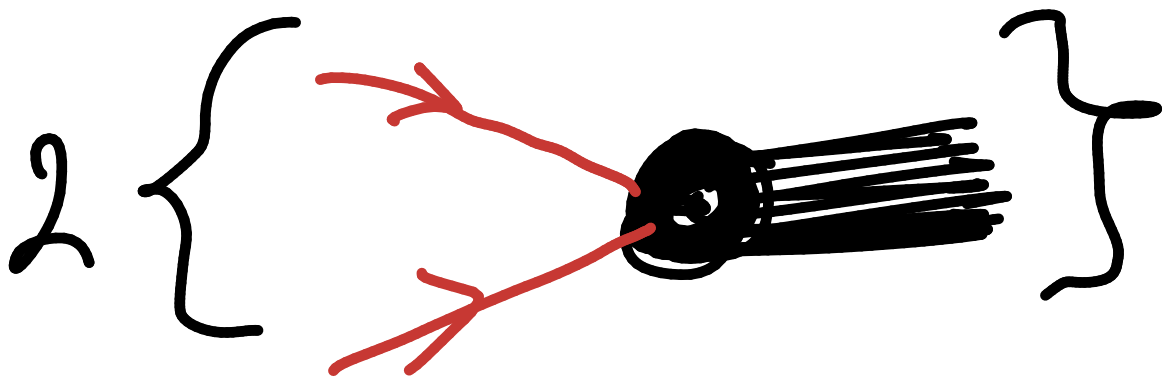

1.5
1.0
θ
0.5
0.0
0
H/N
−1
1.0
0.5
x
0.0

Open questions:

* Is this a model of BH holography?
* Does holography extend beyond gravity?
* A remarkable coincidence?
* Holography in neural networks ....

Classicalization: Quantum transition to Maximal memory storage capacity.

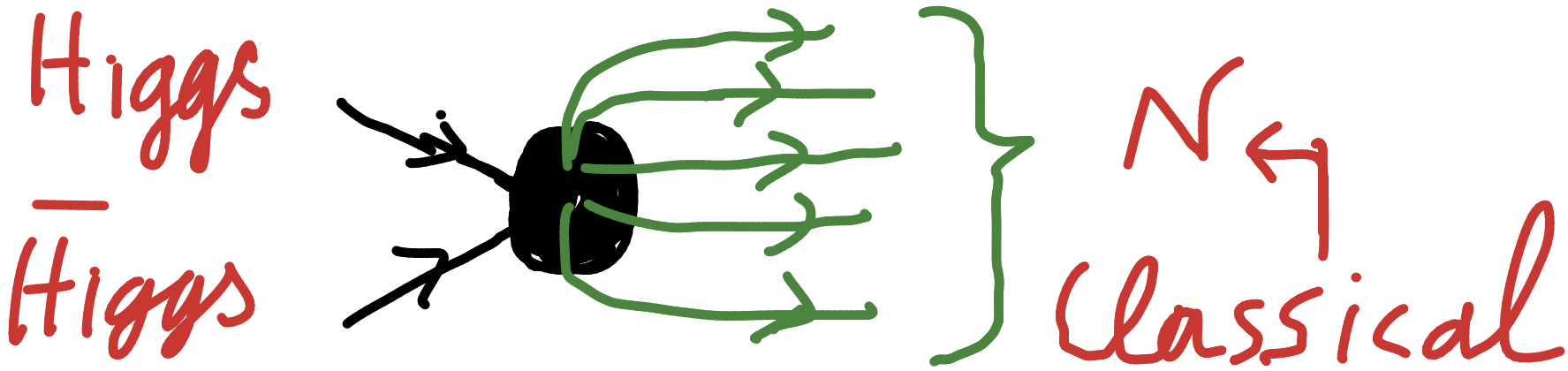
- Brain Networks;
- Solving Hierarchy Problem by classicalization;
- Understanding black hole production in particle collisions:



Non-Wilsonian UV-completion by Classicalization

G.D., Giudice, Gomez & Kehagias, '10

$\Lambda \leftarrow$ Strong coupling scale



Why not e^{-N} ?

$|N\rangle \leftarrow$ state with N emergent gapless modes!

Micro-state entropy $= N$

Maximal memory storage capacity

$$\hat{H}_0 = N\,\hat{b}^+\hat{b} + \hat{a}_0^+\hat{a}_0$$

$$\hat{H}_{tr} = g_N\,\hat{b}^+\hat{a}_0^N + h.c.$$

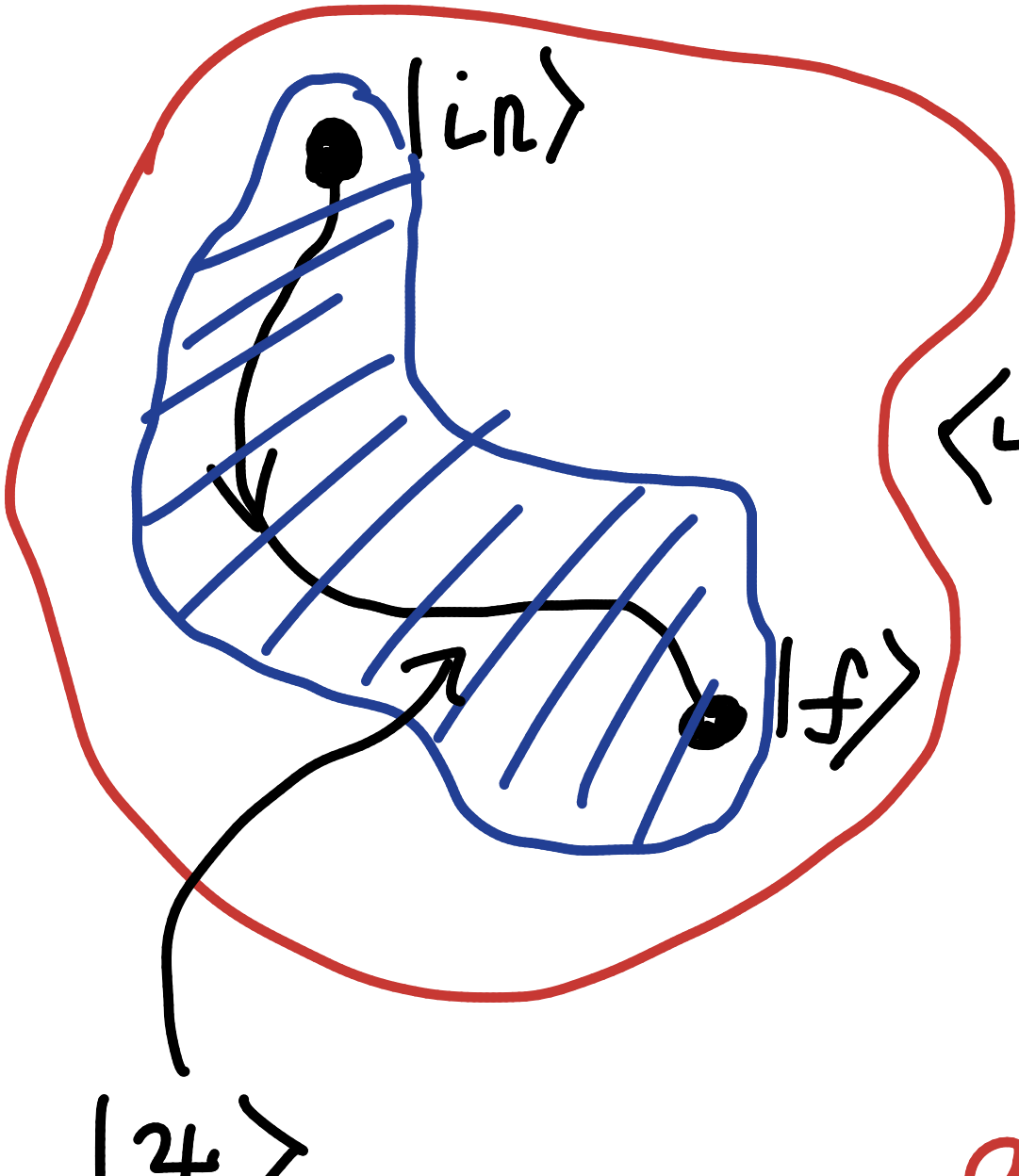
$$[\hat{a}_0, \hat{a}_0^+] = [\hat{b}_0, \hat{b}_0^+] = 1$$

$$|in\rangle_Q = |1\rangle_b \otimes |0\rangle_{a_0}$$

$$|f\rangle = |0\rangle_b \otimes |N\rangle_{a_0}$$

$$\hat{H}_{tr}|in\rangle = g_N\sqrt{N!}\,|f\rangle$$

$$|\langle in|\hat{H}_{tr}|f\rangle|^2 = g_N^2\,N!$$



Condition:

$$\langle \psi | \hat{H}_{tr} | \psi \rangle \lesssim \langle \psi | \hat{H}_0 | \psi \rangle$$

$$g^2 \lesssim N^{-N}$$

$$|\langle in | \hat{H}_{tr} | f \rangle|^2 \sim e^{-N} \quad !$$

However, N gapless modes give enhancement!

$$\hat{H} = N\hat{b}^{+}\hat{b} + \hat{a}_0^{+}\hat{a}_0 + \sum_{k=1}^{\tilde{N}} \epsilon_k \left(1 - \frac{\hat{a}_0^{+}\hat{a}_0}{N}\right) \hat{a}_k^{+}\hat{a}_k$$

$$\hat{H}_{tr} = \hat{b}^{+} \sum_{n_0 \dots n_N} g_{n_0 \dots n_N} \hat{a}_0^{n_0} \hat{a}_1^{n_1} \dots \hat{a}_N^{n_N} + \text{h.c.}$$

$$\epsilon_k^{eff} = \epsilon_k \left[1 - \frac{\hat{n}_0}{N}\right]$$

$\langle \hat{n}_0 \rangle = N \leftarrow$ gapless modes!

$$g_{n_0 \cdots n_N} \lesssim \bar{n}_0^{-n_0} \bar{n}_1^{-n_1} \cdots \bar{n}_N^{-n_N}$$

$$|N\rangle_{n_1 \cdots n_N} \equiv |N\rangle_{a_0} |n_1\rangle_{a_1} \cdots |n_N\rangle_{a_N}$$

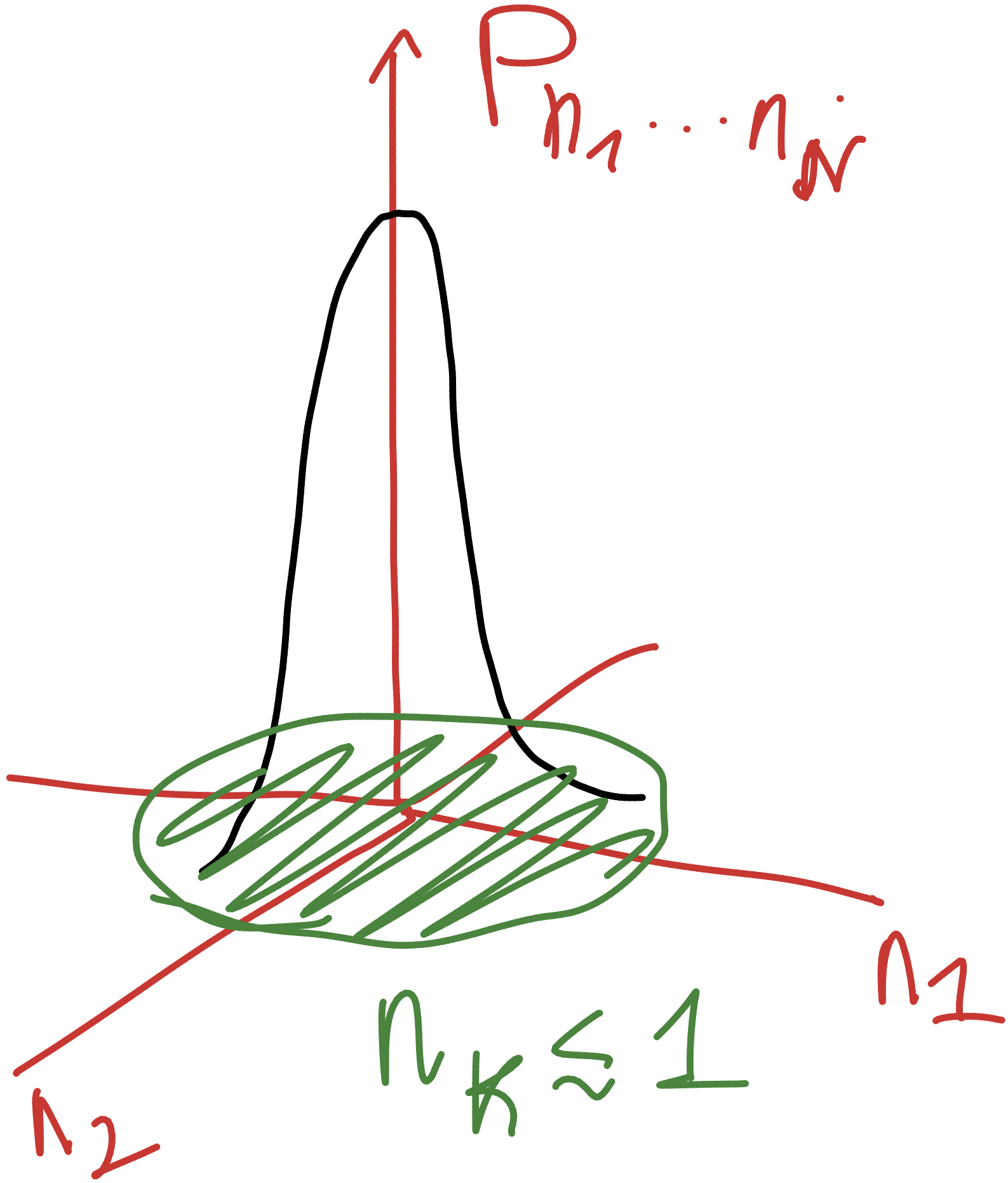
$$\downarrow$$

$$k_{in} |\hat{H}_{tr}| N\rangle_{n_1 \cdots n_N}|^2 \sim \bar{e}^{-[N+n_1+\cdots n_N]}$$

$$P_{1_b \to N_{a_0}} = \sum_{n_1 \cdots n_N} |\langle \quad \rangle|^2 = \bar{e}^{N} \cdot e^{N}$$

$$n_{k \neq 0} \lesssim 1 \leftarrow \text{contribute!}$$

$|1_b\rangle \rightarrow |N\rangle_{n_1 \dots n_N}$



Area-law entropy:

$$\hat{H} = \sum_N N\omega_N \hat{b}_N^\dagger \hat{b}_N + \omega_N \hat{a}_{0N}^\dagger \hat{a}_{0N}$$

$$+ \epsilon_N \left[1 - \frac{\hat{a}_{0N}^\dagger a_{0N}}{N}\right] \sum_{m,\ell} \hat{a}_{N\ell m}^\dagger \hat{a}_{N\ell m}$$

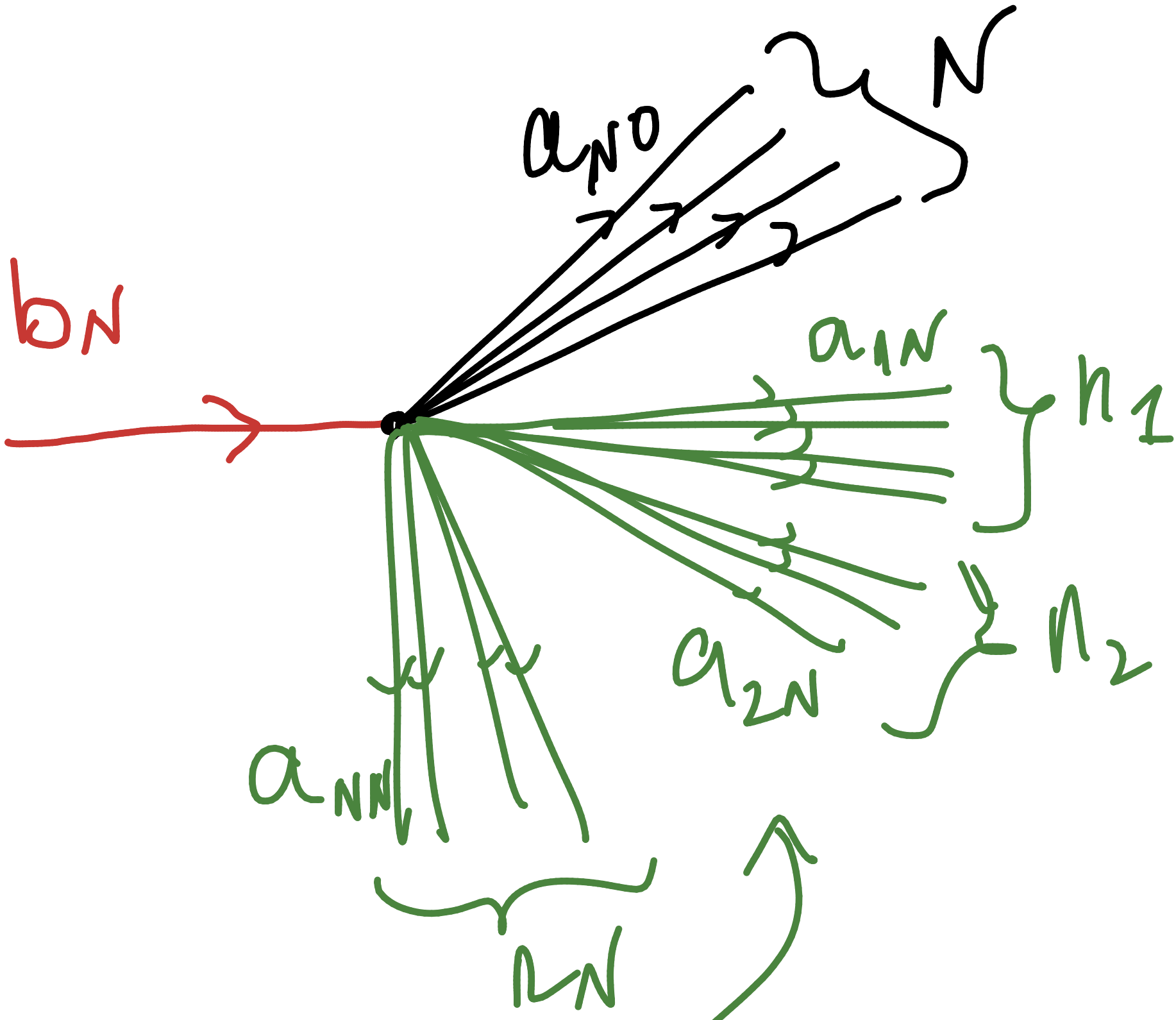
$$+ \sum \hat{b}_N^\dagger \hat{a}_{0N}^N \hat{a}_{N\ell_1 m_1}^{n_1} \cdots \hat{a}_{N\ell_N m_N}^{n_N} \times$$

$$N, n_1 \cdots n_N$$

$$\times g_{N n_1 \cdots n_N}.$$

$$N = S(S+2), \quad |m| \le \ell \le S$$

Entropy = $N \simeq S^2$.

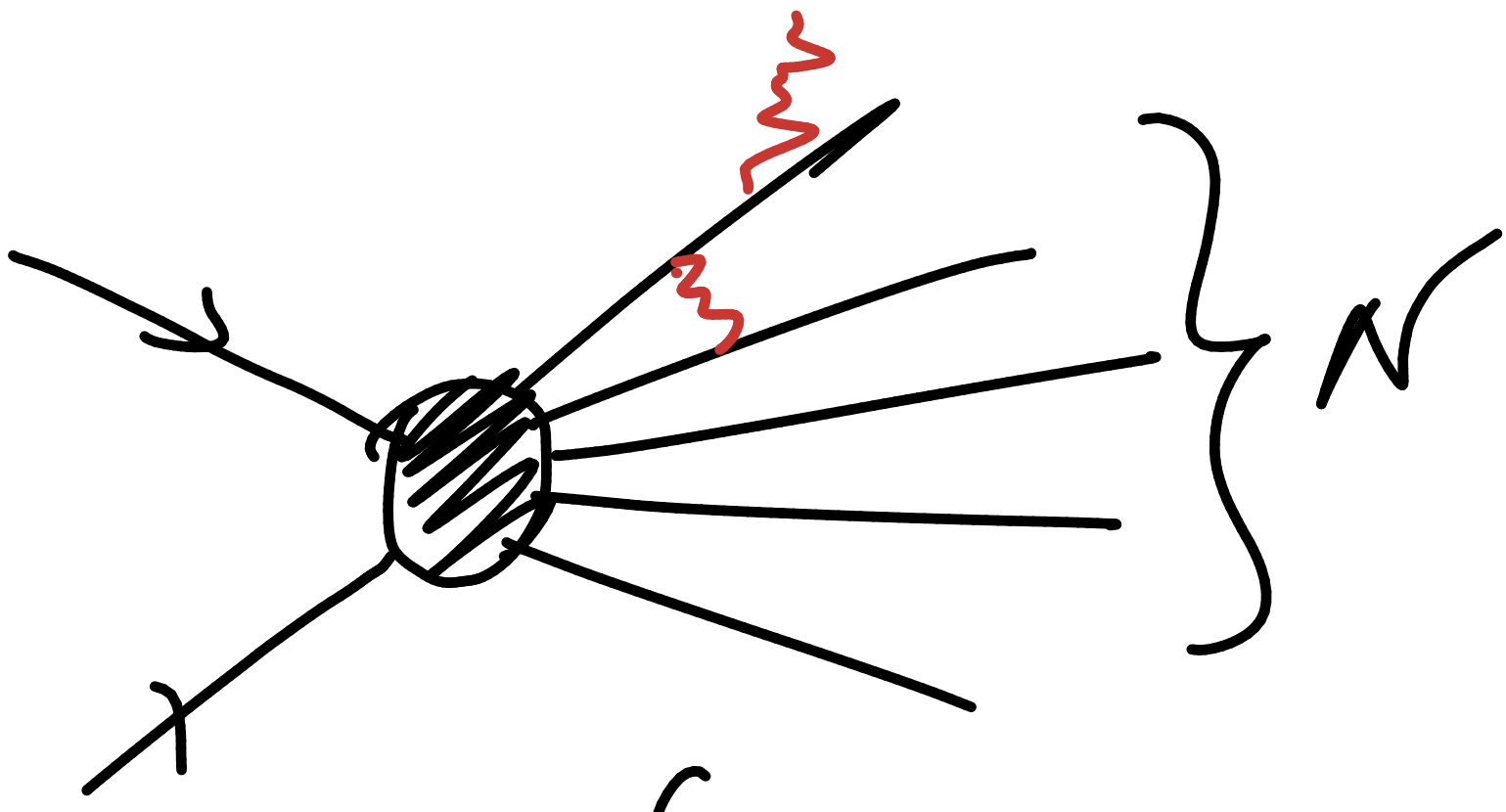


pressing by super-soft modes.

$|1\rangle_{\text{Quart}} \longrightarrow |N\rangle_{\text{class}}$

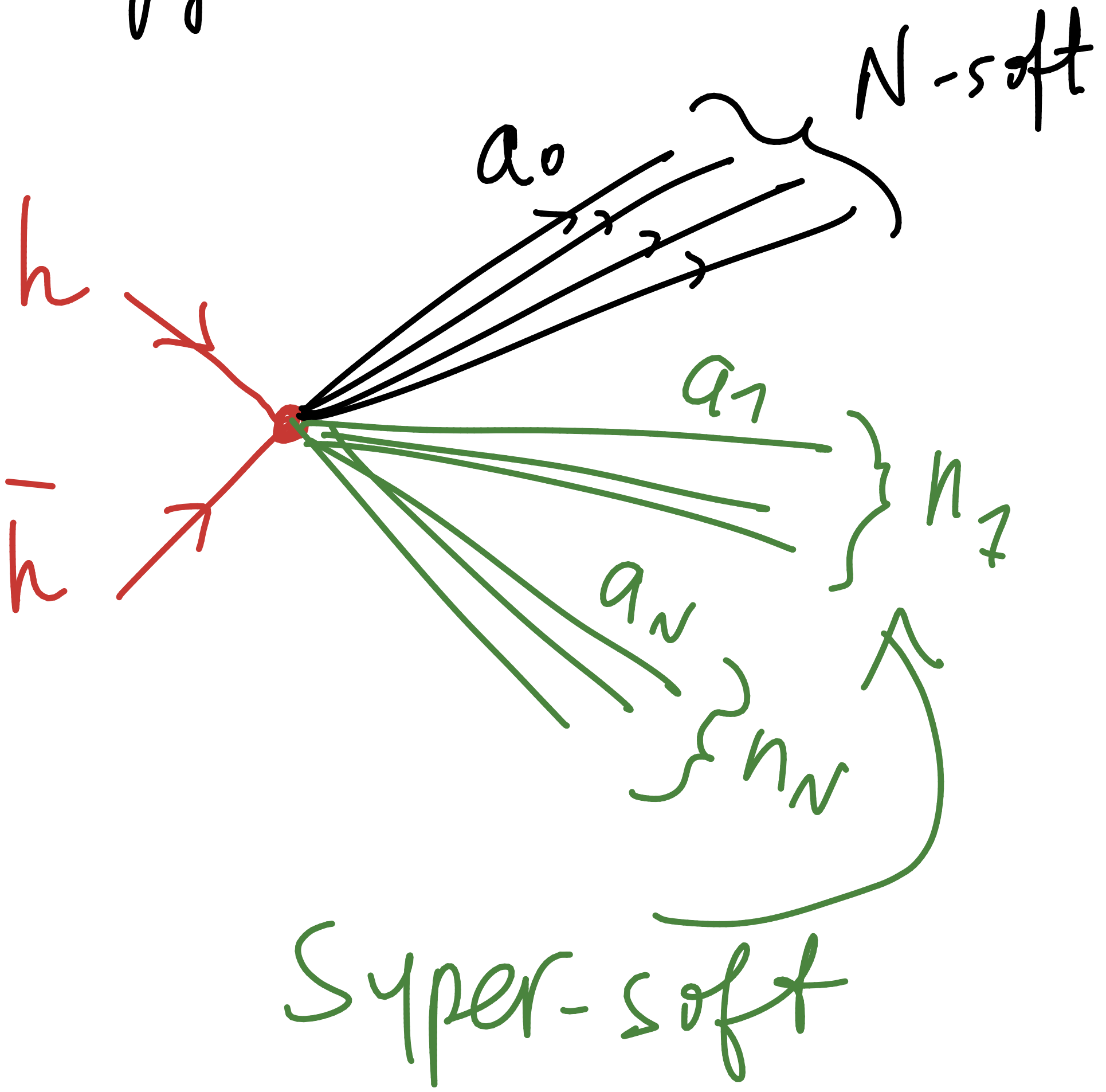
Very similar to $2 \to N$ graviton amplitudes

G.D., Gomez, Isermann, Lüst, Stieberger '15;
Addazi, Bianchi, Veneziano '16

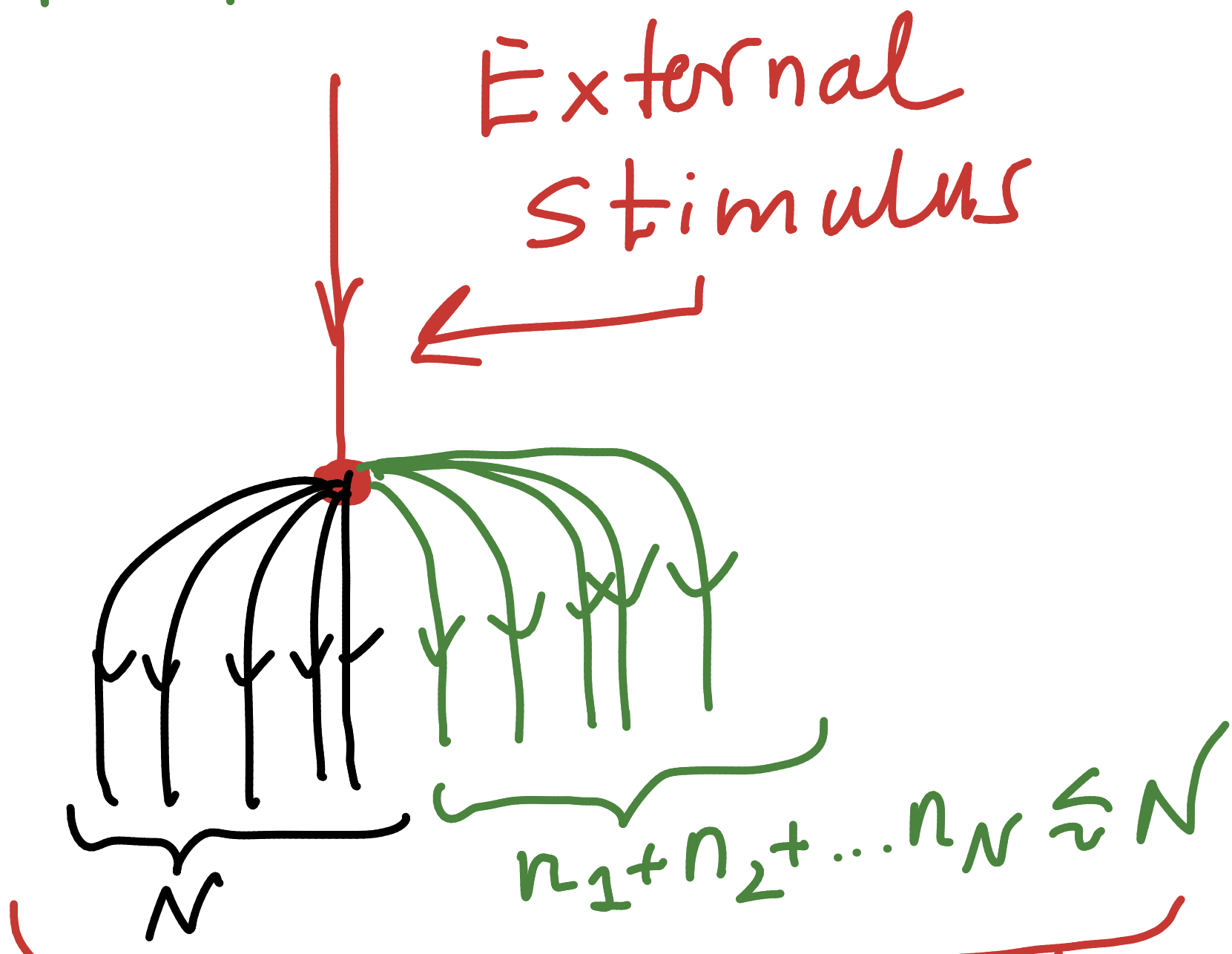


$P \sim e^N \times \text{dressing}.$

Higgs classicalization:



Gravity and SM-Higgs operate above Λ as Brain Network:



Maximal memory capacity state