Antiproton-nucleus annihilation in optical models

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Outline

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1. Introduction
2. The $N\bar{N}$ interaction
3. Formalism
4. Results
5. Conclusion

Antiproton physics at CERN

- The interest for **low-energy antiproton physics** has been revived with the development of dedicated facilities at CERN: **LEAR** (1983-1996), **AD**, **ELENA**.
- Opportunity to study the properties of antimatter, exotic particle-antiparticle systems, and standard matter.
- Some of the experiments: BASE ($p/\bar{p}$ properties), ALPHA ($H/\bar{H}$ properties), GBAR and AEGIS (gravitation), ASACUSA (antiprotonic helium spectroscopy), ...

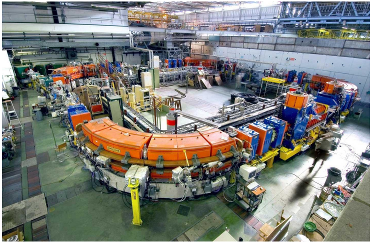


Figure: LEAR (Low Energy Antiproton Ring). Credits: CERN

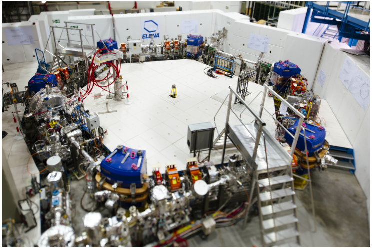


Figure: ELENA (Extra-Low Energy Antiproton Ring). Credits: CERN

AntiProton Unstable Matter Annihilation (PUMA) project

- Aims to study **nucleus skin densities** of short-lived nuclear isotopes, produced by ISOLDE, **using low-energy antiprotons** transported from ELENA[1].
- The antiproton-nucleus annihilation is expected to happen in the **periphery** of the nucleus $\longrightarrow$ study of the nuclear **density tail** by measuring the $p\bar{p}/n\bar{p}$ annihilation ratio.

- Remaining questions:
  1. How can we interpret the data from theoretical predictions ?
  2. Validity of the $N\bar{N}$ models ?
  3. **Model dependence** ?

$\longrightarrow$ **Microscopic** treatment of the antiproton-nucleus system.

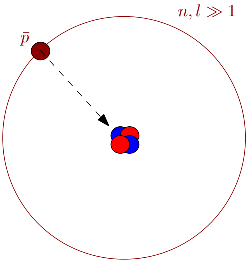


Figure: Antiproton-nucleus system

[1] T. Aumann, A. Obertelli, *et al.* *Eur. J. Phys. A* **58** (2022) 88

Antiproton-nucleus system

- Antiproton-nucleus system:
  1. **Capture** of the antiproton on a highly excited Coulomb orbital and formation of a quasi-bound state with

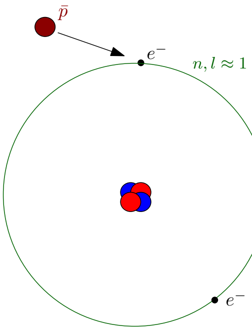
$$E = E_R - i\frac{\Gamma}{2}, \qquad E_R \approx E_B - \frac{\mathrm{Ryd}(\bar{\mathrm{p}}\mathrm{A})}{n^2}$$

  2. X-ray **cascade and annihilation** with a nucleon of the nucleus.

- Non-relativistic description by solving the **few-body Schrödinger equation**:

$$(\hat{H}_0 + \hat{V})\,\Psi = E\,\Psi$$

- *Ab initio* calculations for the simplest cases (3B, 4B).
- Difficult problem due to the $N\bar{N}$ interaction, the **annihilation dynamics**, and the presence of **different scales**.



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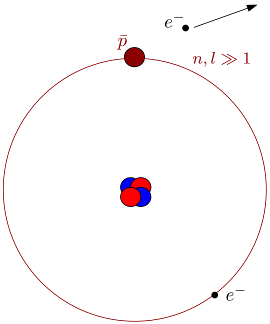

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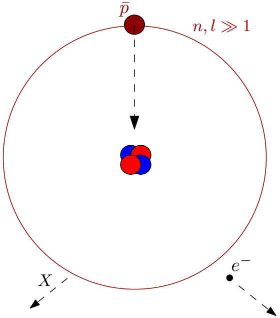

Figure: Antiproton-nucleus system

Outline

$N\bar{N}$ scattering

- Most of the $N\bar{N}$ **scattering data** come from the LEAR experiments.
- The $p\bar{p}$ scattering involves the **elastic scattering** ($p\bar{p} \to p\bar{p}$), the **charge-exchange** process ($p\bar{p} \to n\bar{n}$), and the **annihilation** ($p\bar{p} \to \pi\bar{\pi}, \pi\bar{\pi}\pi\bar{\pi}, \rho\bar{\rho}, ...$).

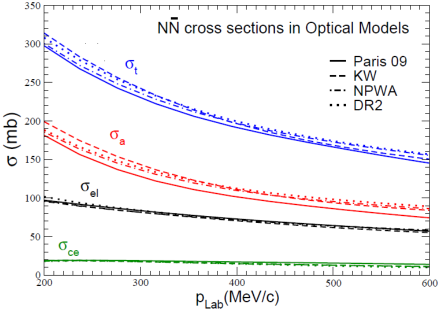

Figure: $N\bar{N}$ cross sections computed with different $N\bar{N}$ models[1]

[1] J. Carbonell, G. Hupin, and S. Wycech, *Eur. Phys. J. A* **59** (2023) 259

Protonium

- In the **absence of strong nuclear interaction**, $p\bar{p}$ would form an **hydrogenic state** with energy
$$\epsilon_n = -\frac{12.5}{n^2}\,\mathrm{keV}$$
and with a Bohr radius $B_{p\bar{p}} = 57\,\mathrm{fm}$.
- The nuclear interaction **shifts and broadens** the energy levels.
  1. The **level shift** $\Delta E_n = \Delta E_r - i\frac{\Gamma}{2}$ has been measured for low lying states[1,2].
  2. The level shift is related to the **scattering length**.
- Privileged system to test the $N\bar{N}$ interaction at low-energy.
- Hard to extract useful information from the study of antiprotonic atoms to construct $N\bar{N}$ models.

[1] M. Augsburger *et al.*, *Nucl. Phys. A* **658** (1999) 149

[2] K. Heitlinger *et al.*, *Z. Phys. A* **342** (1992) 359

Protonium: level shifts

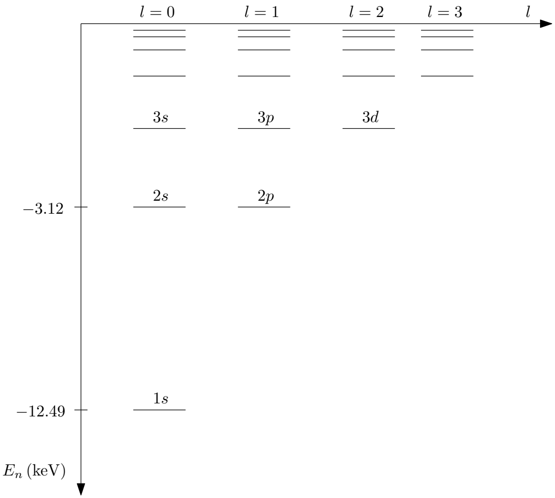

Figure: Protonium Rydberg states

Protonium: level shifts

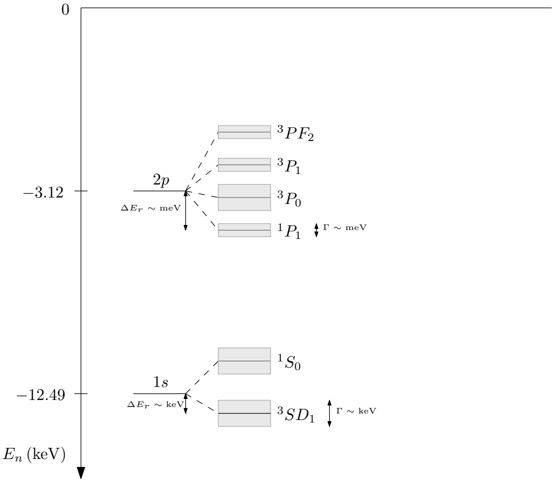

Figure: Protonium fine structure

$N\bar{N}$ interaction

- The attractive/repulsive features of the $N\bar{N}$ interaction is obtained from a G-**parity transform** of a NN **interaction** (meson exchange theory, χEFT).
- The $N\bar{N}$ **annihilation** is modelled with **phenomenological models**.

$$
\begin{aligned}
p\bar{p} &\to p\bar{p} \\
&\to n\bar{n} \\
&\to \pi^0\pi^0 \\
&\to \pi^0\pi^0\pi^0 \\
&\to \pi^0\pi^0\pi^0\pi^0 \\
&\to \pi^+\pi^- \\
&\to \pi^+\pi^-\pi^0 \\
&\vdots
\end{aligned}
\qquad
\begin{aligned}
n\bar{p} &\to n\bar{p} \\
&\to \pi^-\pi^0 \\
&\to \pi^-\pi^0 \\
&\to \pi^-\pi^-\pi^+ \\
&\to \pi^-\pi^-\pi^+\pi^0 \\
&\to \pi^-\pi^-\pi^+\pi^0 \\
&\to \pi^-\pi^-\pi^-\pi^+\pi^+ \\
&\vdots
\end{aligned}
$$

- Only $N\bar{N}$ channels are treated explicitly.

Optical model

- The annihilation is traditionally treated with **optical potentials**:

$$V_{N\bar{N}} \rightarrow V_{N\bar{N}} + W_R + iW_I$$

- All models **roughly reproduce low-energy observables and integrated cross sections** and provide a **reasonable agreement with experimental data**.
- Yet, "large differences have been observed in almost all the partial waves"[1].

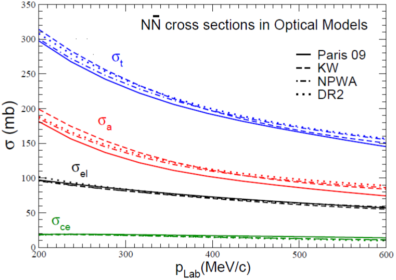

Figure: $N\bar{N}$ cross sections computed with different $N\bar{N}$ models[1]

[1] J. Carbonell, G. Hupin, and S. Wycech, *Eur. Phys. J. A* **59** (2023) 259

Coupled-channel model

- The annihilation is simulated by the addition of **effective meson-antimeson ($m\bar{m}$) channels** mimicking the real ones.
- Still **phenomenological** but provides a more realistic description of the annihilation process and involves quite **different dynamics** ($S^\dagger S = \mathbb{I}$).
- To investigate the model dependence, the parameters of the coupled-channel potential are here **adjusted to fit the (KW) optical model results**.

  - Annihilation channels:

$$\begin{aligned} p\bar{p} &\to p\bar{p} \\ &\to m_1\bar{m}_1 \\ &\to n\bar{n} \\ &\to m_2\bar{m}_2 \end{aligned}$$

$$\begin{aligned} n\bar{p} &\to n\bar{p} \\ &\to m_3\bar{m}_3 \end{aligned}$$

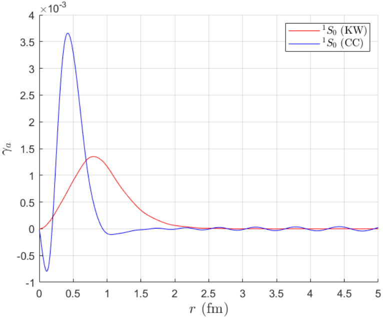

Figure: $p\bar{p}$ annihilation densities ($^1S_0$)

Outline

Three-body systems: Faddeev equations

- Decomposition of the wavefunction in **Faddeev components**[1]: $\Psi = \Psi_1 + \Psi_2 + \Psi_3$
- The Faddeev components are solutions of

$$(E - H_0 - V_i)\Psi_i(\boldsymbol{x}_i, \boldsymbol{y}_i) = V_i \left[\Psi_j(\boldsymbol{x}_j, \boldsymbol{y}_j) + \Psi_k(\boldsymbol{x}_k, \boldsymbol{y}_k)\right], \quad (ijk) = (123), (312), (231)$$

- **Independent boundary condition** for each Faddeev component $\longrightarrow$ adapted for scattering problems.
- However, **corrections required for long-range potentials**[2].

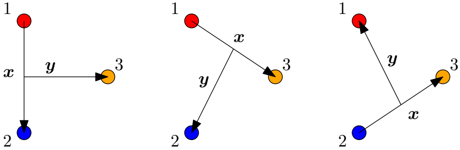

Figure: Jacobi coordinates for a three-body system

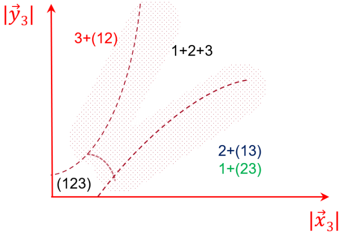

Figure: Partitions of the system

[1] L. D. Faddeev. *Sov. Phys. JETP* **39** (1960) 1459

[2] S. P. Merkuriev, *Ann. Phys.* **130** (1980) 395

Four-body systems: Faddeev-Yakubovksy equations

- **Generalisation of the Faddeev formalism** to four-body systems:

$$(E - H_0)\Psi = \sum_{i<j} V_{ij}\Psi, \qquad \Psi = \Phi_{12} + \Phi_{13} + \Phi_{14} + \Phi_{23} + \Phi_{24} + \Phi_{34}$$

- For each interacting pair, separation into $\mathcal{K}$ and $\mathcal{H}$ partitions:

$$\Phi_{ij} = \mathcal{K}_{ij,k}^{l} + \mathcal{K}_{ij,l}^{k} + \mathcal{H}_{ij,kl}$$

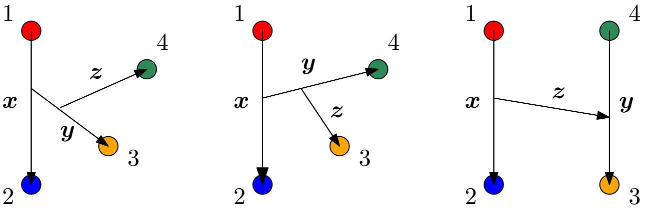

Figure: $\mathcal{K}$ and $\mathcal{H}$ partitions for a 4-body system.

Four-body systems: Faddeev-Yakubovksy equations

- The 18 FYCs are solution of the **Faddeev-Yakubovksy equations**[1]

$$(E - H_0 - V_{ij})\mathcal{K}^l_{ij,k} = V_{ij}\left(\mathcal{K}^l_{ik,j} + \mathcal{K}^l_{jk,i} + \mathcal{K}^j_{ik,l} + \mathcal{K}^i_{jk,l} + \mathcal{H}_{ik,jl} + \mathcal{H}_{jk,il}\right),$$
$$(E - H_0 - V_{ij})\mathcal{K}^k_{ij,l} = V_{ij}\left(\mathcal{K}^k_{il,j} + \mathcal{K}^k_{jl,i} + \mathcal{K}^j_{il,k} + \mathcal{K}^i_{jl,k} + \mathcal{H}_{il,jk} + \mathcal{H}_{jl,ik}\right),$$
$$(E - H_0 - V_{ij})\mathcal{H}_{ij,kl} = V_{ij}\left(\mathcal{K}^j_{kl,i} + \mathcal{K}^i_{kl,j} + \mathcal{H}_{kl,ij}\right), \qquad (i < j, k < l).$$

- **Efficient separation of binary channels asymptotes**.
- Such formulation does not provide stable numerical results for **long-range potentials** $\longrightarrow$ **corrections required**.
- Example: scheme proposed by Sasakawa and Sawada[2]:

$$\begin{aligned}(E - H_0 - V_{ij})\,\mathcal{K}^l_{ij,k} = V_{ij}&\left(\mathcal{K}^l_{ik,j} + \mathcal{K}^l_{jk,i} + \mathcal{K}^j_{ik,l} + \mathcal{K}^i_{jk,l} + \mathcal{H}_{ik,jl} + \mathcal{H}_{jk,il}\right),\\ &- q_i q_j\left[V^{(l)}_{z_{ikl,j}}\mathcal{K}^j_{ik,l} + V^{(l)}_{z_{jkl,i}}\mathcal{K}^i_{jk,l} + V^{(l)}_{z_{ik,lj}}\mathcal{H}_{ik,jl} + V^{(l)}_{z_{jk,il}}\mathcal{H}_{jk,il}\right]\\ &+ q_l(q_i + q_j + q_k)V^{(l)}_{z_{ijk,l}}K^l_{ij,k}\end{aligned}$$

[1] O.A. Yakubovsky, *Sov. J. Nucl. Phys.* **4** (1967) 937

[2] T. Sasakawa and T. Sawada, Phys. Rev. C **20** (1979) 1954.

Partial wave expansion

- Resolution in **configuration space** with a **partial wave expansion**:

$$\mathcal{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) = \sum_n \frac{F_n(x,y,z)}{xyz}\,\mathcal{Y}_n^{(F)}(\hat{x},\hat{y},\hat{z}),$$

with

$$\mathcal{Y}_n^{(K)} = \left[\left[\left[l_x(s_i s_j)_{s_x}\right]_{j_x}(l_y s_k)_{j_y}\right]_{j_{xy}}(l_z s_l)_{j_z}\right]_{J^\pi} \otimes \left[\left[(t_i t_j)_{t_x} t_k\right]_{t_3} t_l\right]_{T m_T}$$

$$\mathcal{Y}_n^{(H)} = \left[\left[\left[l_x(s_i s_j)_{s_x}\right]_{j_x}\left[l_y(s_k s_l)_{s_y}\right]_{j_y}\right]_{j_{xy}} l_z\right]_{J^\pi} \otimes \left[(t_i t_j)_{t_x}(t_k t_l)_{t_y}\right]_{T m_T}$$

- The radial functions are expressed as a linear combination of Lagrange functions:

$$F_n(x,y,z) = \sum_{i_x,i_y,i_z} c^{(F)}_{n i_x i_y i_z}\,\hat{f}_{i_x}\left(\frac{x}{h_x}\right)\hat{f}_{i_y}\left(\frac{y}{h_y}\right)\hat{f}_{i_z}\left(\frac{z}{h_z}\right)$$

- The coefficients $c^{(F)}_{n i_x i_y i_z}$ are computed by solving an **eigenvalues problem** for bound states and **linear systems** for scattering states ($N \sim 10^{7,8}$).

Outline

$\bar{p}d$ scattering lengths

- Study of the zero-energy $d\bar{p}$ collision $\longrightarrow$ **scattering length** a_l.
- **Complex scaling** to handle the three-body breakup in meson channels.

	MT+KW[1]	MT+CC[2]	AV18+KW[1]	AV18+CC[2]
	a_0 (fm)	a_0 (fm)	a_0 (fm)	a_0 (fm)
$S^+_{1/2}$	$1.34 - 0.72\,\mathrm{i}$	$1.32 - 0.71\,\mathrm{i}$	$1.34 - 0.72\,\mathrm{i}$	$1.31 - 0.68\,\mathrm{i}$
$S^+_{3/2}$	$1.39 - 0.72\,\mathrm{i}$	$1.40 - 0.73\,\mathrm{i}$	$1.39 - 0.72\,\mathrm{i}$	$1.39 - 0.74\,\mathrm{i}$
	a_1 (fm^3)	a_1 (fm^3)	a_1 (fm^3)	a_1 (fm^3)
$P^-_{5/2}$	$0.71 - 2.64\,\mathrm{i}$	$0.68 - 2.73\,\mathrm{i}$	$0.70 - 2.60\,\mathrm{i}$	$0.64 - 2.59\,\mathrm{i}$

- **Small dependence** on the NN and on the $N\bar{N}$ interactions.
- Quite **good agreement** between the KW and CC models within few percent **despite their very different dynamics**.

[1] P.-Y. Duerinck, R. Lazauskas, and J. Carbonell, *Phys. Lett. B* **841** (2023) 137936 (corrigendum)

[2] P.-Y. Duerinck, R. Lazauskas, and J. Dohet-Eraly, *Phys. Rev. C* **108** (2023) 054003

Comparison with other optical potentials

- The **level shifts** can be computed from the scattering lengths.
- **Low dependence** on both NN and $N\bar{N}$ interactions for S waves.
- **Larger dispersion**, yet comparable results for the $P^-_{5/2}$ state.

$S^+_{1/2}(n=1)$ (keV)	DR1	DR2	KW	Jülich
MT-I-III	$1.98-0.75$i	$2.02-0.74$i	$1.93-0.91$i	
AV18	$1.97-0.74$i	$2.01-0.74$i	$1.92-0.90$i	
I-N3LO			$1.92-0.89$i	$1.84-0.89$i
$S^+_{3/2}(n=1)$ (keV)	DR1	DR2	KW	Jülich
MT-I-III	$2.02-0.75$i	$2.06-0.76$i	$1.98-0.91$i	
AV18	$2.03-0.73$i	$2.08-0.75$i	$1.97-0.91$i	
I-N3LO			$1.99-0.89$i	$1.97-1.14$i
$P^-_{5/2}(n=2)$ (meV)	DR1	DR2	KW	Jülich
MT-I-III	$92.9-192$i	$95.1-209$i	$52.4-208$i	
AV18	$91.5-185$i	$93.6-193$i	$51.7-201$i	
I-N3LO				$33.1-219$i

Table: Level shifts for S (keV) and P waves (meV) computed with different $NN+N\bar{N}$ interactions.

Comparison with experiment

- Our results are the exact solution (in the numerical sense) of the three-body problem.
- **Strong discrepancy with experimental data**, especially for ΔE_R ($\sim 4\sigma$).

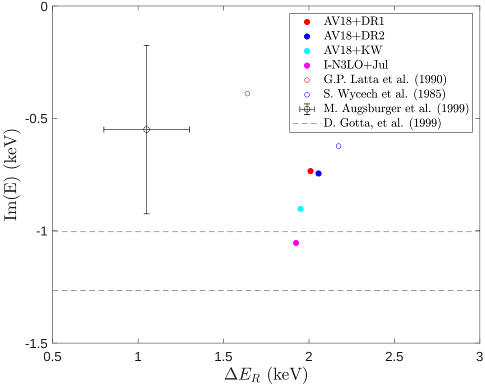

Figure: Comparison of the spin-average S wave level shifts and widths with previous results and experiments.

$N\bar{N}$ model-dependence

- While a nice agreement is observed for S waves, **sizeable differences** are found in some P waves, which is also observed in protonium.

	I-N3LO+KW	I-N3LO+Jülich
$^2P_{1/2}(n=2)$	$-25.9-229i$	$38.7-268i$
$^4P_{1/2}(n=2)$	$218-190i$	$192-220i$
$^2P_{1/2}(n=3)$	$-3.6-80.1i$	$25.7-95.9i$
$^4P_{1/2}(n=3)$	$60.7-46.0i$	$48.8-75.4i$
$^2P_{3/2}(n=2)$	$58.2-193i$	$54.0-163i$
$^4P_{3/2}(n=2)$	$-38.9-228i$	$-83.6-215i$
$^2P_{3/2}(n=3)$	$18.8-67.7i$	$17.5-57.1i$
$^4P_{3/2}(n=3)$	$-8.4-80.0i$	$-24.2-75.4i$

Table: Coupled P waves level shifts (meV) computed with different $N\bar{N}$ interactions.

Annihilation density

- The **annihilation density** $\gamma_a(r_{\bar{p}d})$ is related to the **probability of annihilation** of the antiproton with respect to the deuteron center of mass distance $(r_{\bar{p}d})$:

$$\Gamma = \int \gamma_a(r_{\bar{p}d})\mathrm{d}r_{\bar{p}d}$$

- **Peripheral absorption** for both S and P waves !

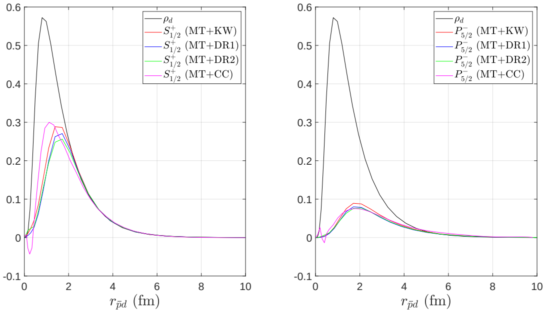

Figure: Annihilation density for S wave (left) and P wave (right).

$\bar{p}\,{}^3\mathrm{H}$ and $\bar{p}\,{}^3\mathrm{He}$ scattering lengths (preliminary)

- **Preliminary calculations** performed with the KW model.
- **Low dependence** on NN interaction.

$\bar{p} + {}^3\mathrm{H}$	a_0^+	a_1^+	a_0^-
MT+KW	$1.44 - 0.77\mathrm{i}$	$1.58 - 0.63\mathrm{i}$	$-0.3 - 4.2\mathrm{i}$
AV18+KW	$1.47 - 0.78\mathrm{i}$	$1.60 - 0.64\mathrm{i}$	

Table: $\bar{p}\,{}^3\mathrm{H}$ scattering lengths computed with different NN interactions

$\bar{p} + {}^3\mathrm{He}$	a_0^+	a_1^+	a_0^-
MT+KW	$1.48 - 0.54\mathrm{i}$	$1.38 - 0.62\mathrm{i}$	$3.8 - 1.97\mathrm{i}$
AV18+KW	$1.50 - 0.54\mathrm{i}$	$1.39 - 0.63\mathrm{i}$	$4.0 - 2.44\mathrm{i}$

Table: $\bar{p}\,{}^3\mathrm{He}$ scattering lengths computed with different NN interactions

Annihilation density (preliminary)

- **Annihilation density** $\gamma_a(r)$ is related to the **probability of annihilation** of the antiproton with respect to the its distance with the nucleus center of mass (r):

$$\Gamma = \int \gamma_a(r)\,\mathrm{d}r$$

- Preliminary calculation for $\bar{p}\,{}^3\mathrm{H}$:

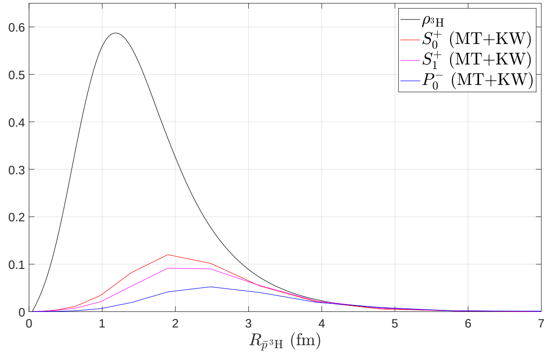

Figure: Annihilation density for S and P waves.

Outline

Conclusion

- **Three- and four-body calculations** for antiproton-nucleus systems.
- Investigation of the **model dependence** by comparing different approaches: **optical and coupled-channel models**.
- $\bar{p}d$:
  1. S states: low model-dependence, yet large discrepancies with experimental data.
  2. P waves: strong model dependence observed in some states $\longrightarrow$ our understanding of the $N\bar{N}$ interaction can still be improved.
  3. Annihilation densities: the annihilation is expected to be peripheral in all partial waves $\longrightarrow$ supports the major hypothesis of PUMA experiments.
- $\bar{p}\,^3\mathrm{H}$ and $\bar{p}\,^3\mathrm{He}$:
  1. Preliminary calculations for S and P states.
  2. $\bar{p}\,^3\mathrm{H}$ annihilation densities: similar conclusions as for $\bar{p}d$.
  3. Prospects: calculation of $\bar{p}\,^3\mathrm{He}$ annihilation densities and study of $N\bar{N}$ model dependence.