Microscopic optical potentials for antiproton-nucleus scattering

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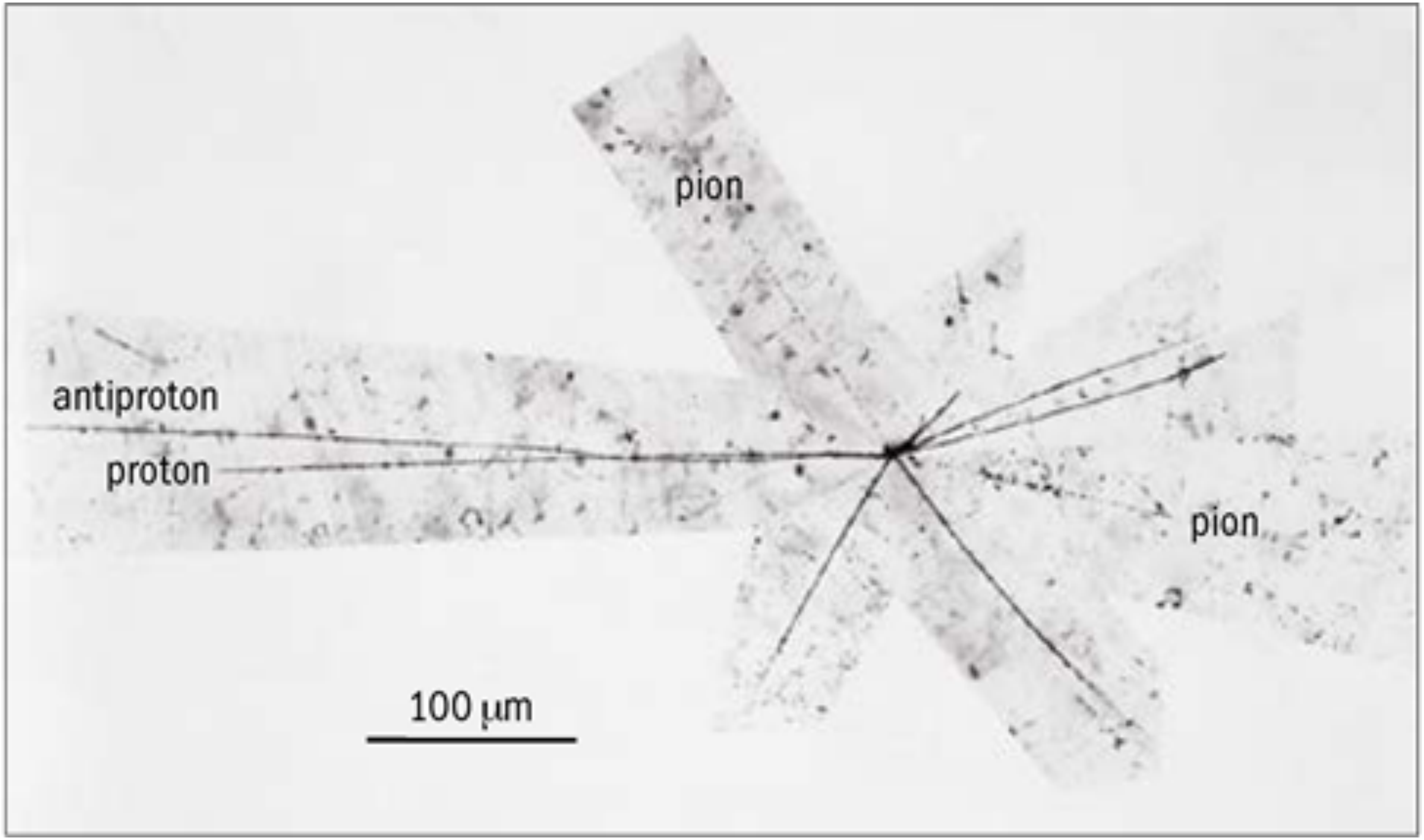
in collaboration
with M. Vorabbi (Surrey), C. Giusti (Pavia), M. Gennari (TRIUMF), P. Navratil (TRIUMF), C. Barbieri (MIlano) and V. Somà (Saclay)

A brief history



One of the first annihilations of an antiproton observed at the Bevatron with a photographic emulsion.

The antiproton enters from the left.

The fat tracks are from slow protons or nuclear fragments, the faint tracks from fast pions.

O Chamberlain et al., Nuovo Cimento 3 (1956) 447

- An antiproton is the antimatter counterpart of the proton.
- It carries a negative electric charge, opposite to the positively charged proton.
- Discovered in 1955 by E. Segrè and O. Chamberlain, the antiproton has the same mass as a proton but a negative charge and opposite magnetic moment.
- When antiprotons encounter protons, they annihilate each other, releasing energy.

Past experiments

EXPERIMENTS AT THE LOW-ENERGY ANTIPROTON RING (LEAR)

Th. Walcher

Ann. Rev. Nucl. Part. Sci. 1988. 38: 67–95

*"A pronounced diffractive pattern is visible that can be reproduced very well by an optical potential with a **strong absorptive imaginary part** and a **rather shallow attractive real part**.*

*Evidently, such a potential is well defined **only at the surface** of the nucleus because of the strong absorption. The inner part of the nucleus plays practically no role since the antiprotons are essentially absorbed at ranges where the nuclear density is 10% of the central value."*

many-body effects suppressed?

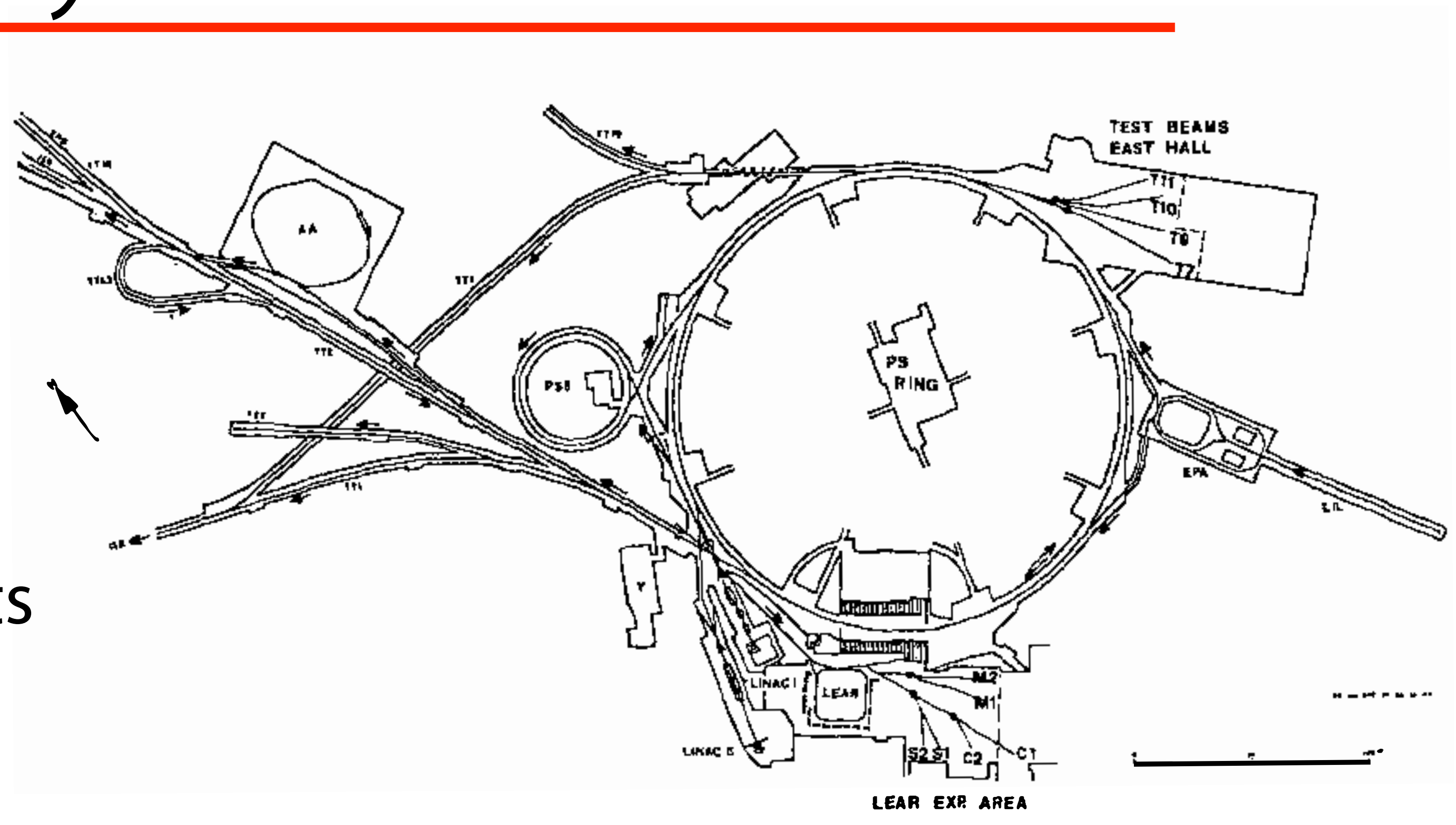
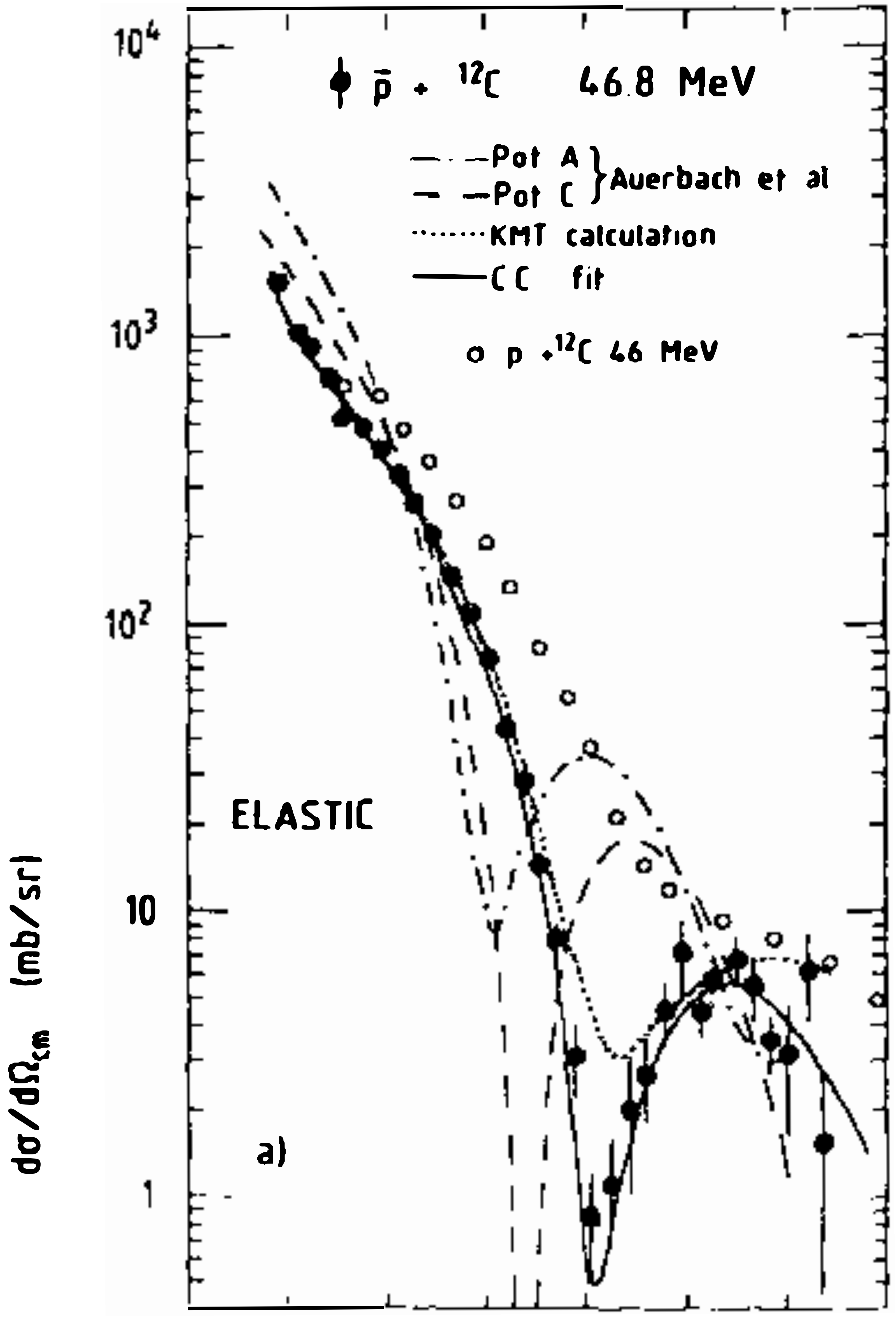


Figure 3 The PS, AA, and LEAR accelerator complex at CERN.



G-parity

- G-parity is a multiplicative quantum number that results from the generalization of C-parity to multiplets of particles.
- G-parity is a combination of charge conjugation and a **π** rad rotation around the 2nd axis of isospin space. Weak and electromagnetic interactions are not invariant under G-parity.

$$\mathcal{G} = \mathcal{C}\, e^{(i\pi I_2)}$$

As a consequence a dramatic change of the spin dependence of the interaction.

At very low energy, the **nucleon-nucleon interaction** is dominated by the spin-spin and tensor contributions of the one-pion exchange. However, when the energy increases, or, equivalently, when one explores shorter distances, the main pattern is a pronounced spin-orbit interaction. The tensor component of the *NN* interaction is known to play a crucial role.

In the case of the **antinucleon-nucleon** interaction, the most striking difference occurs in the tensor potential, especially in the case of isospin $I = 0$. A scenario with dominant tensor forces is somewhat unusual.

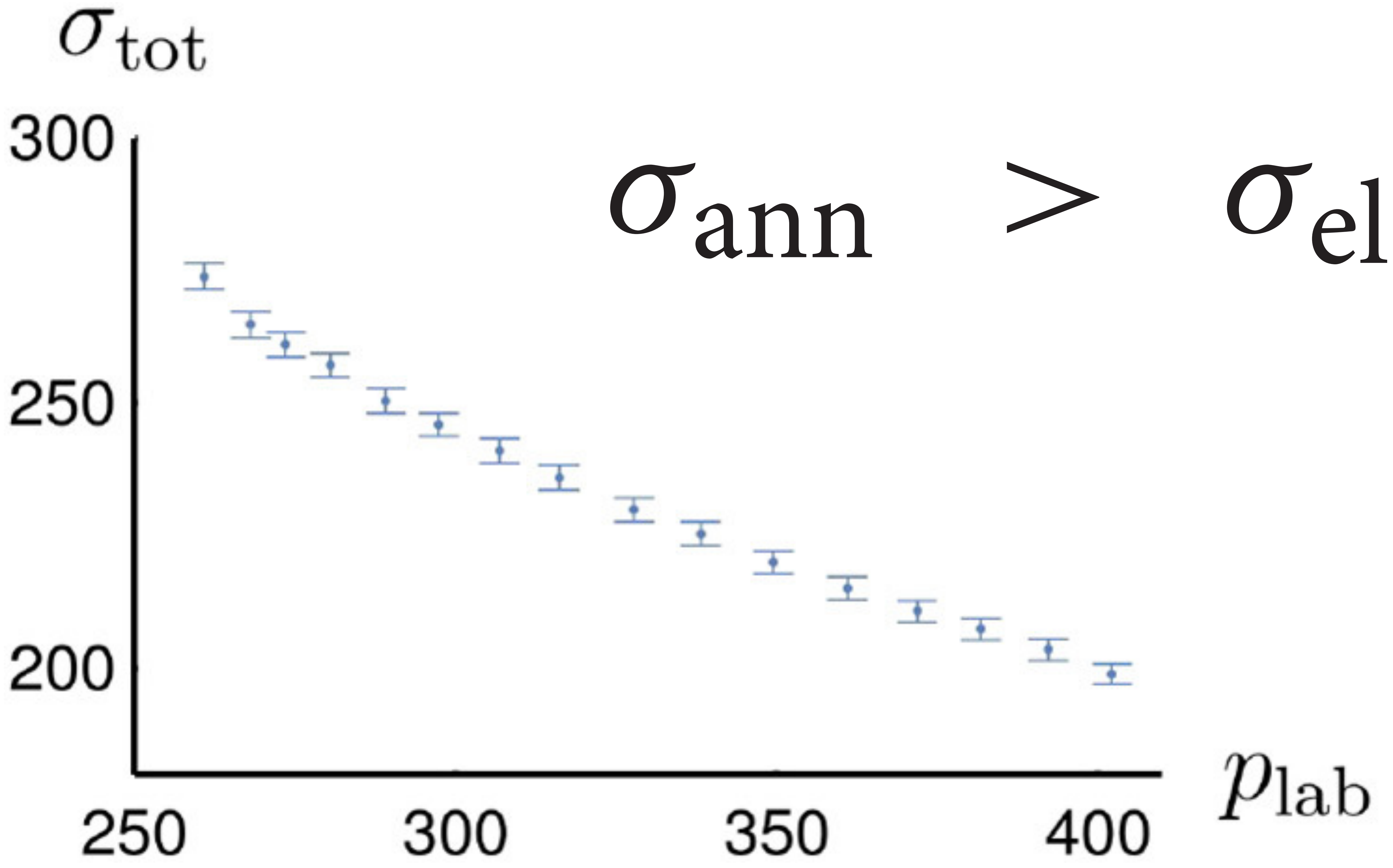
In a scattering process, there is no polarization if the tensor component is treated to first order, but polarization shows up at higher order.

Richard, Antiproton Physics, Frontiers in Physics, doi: 10.3389/fphy.2020.00006

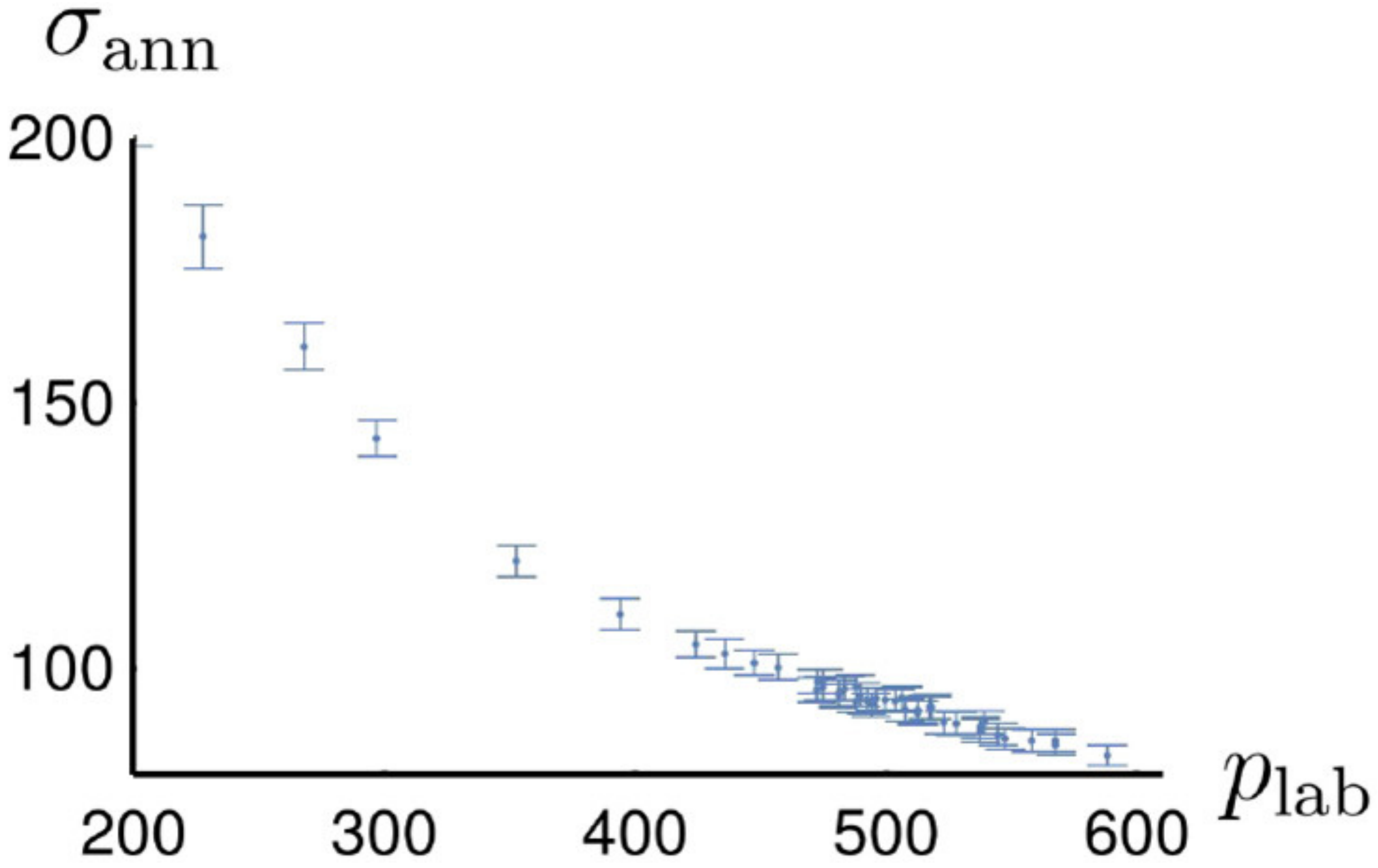
Integrated Cross Sections

When one compares the values at the same energy, one sees that annihilation is more than half the total cross section.

$$\sigma_{\rm ann} > \sigma_{\rm el}$$



Antiproton-proton cross section (in mb), as measured by the PS172 collaboration at LEAR

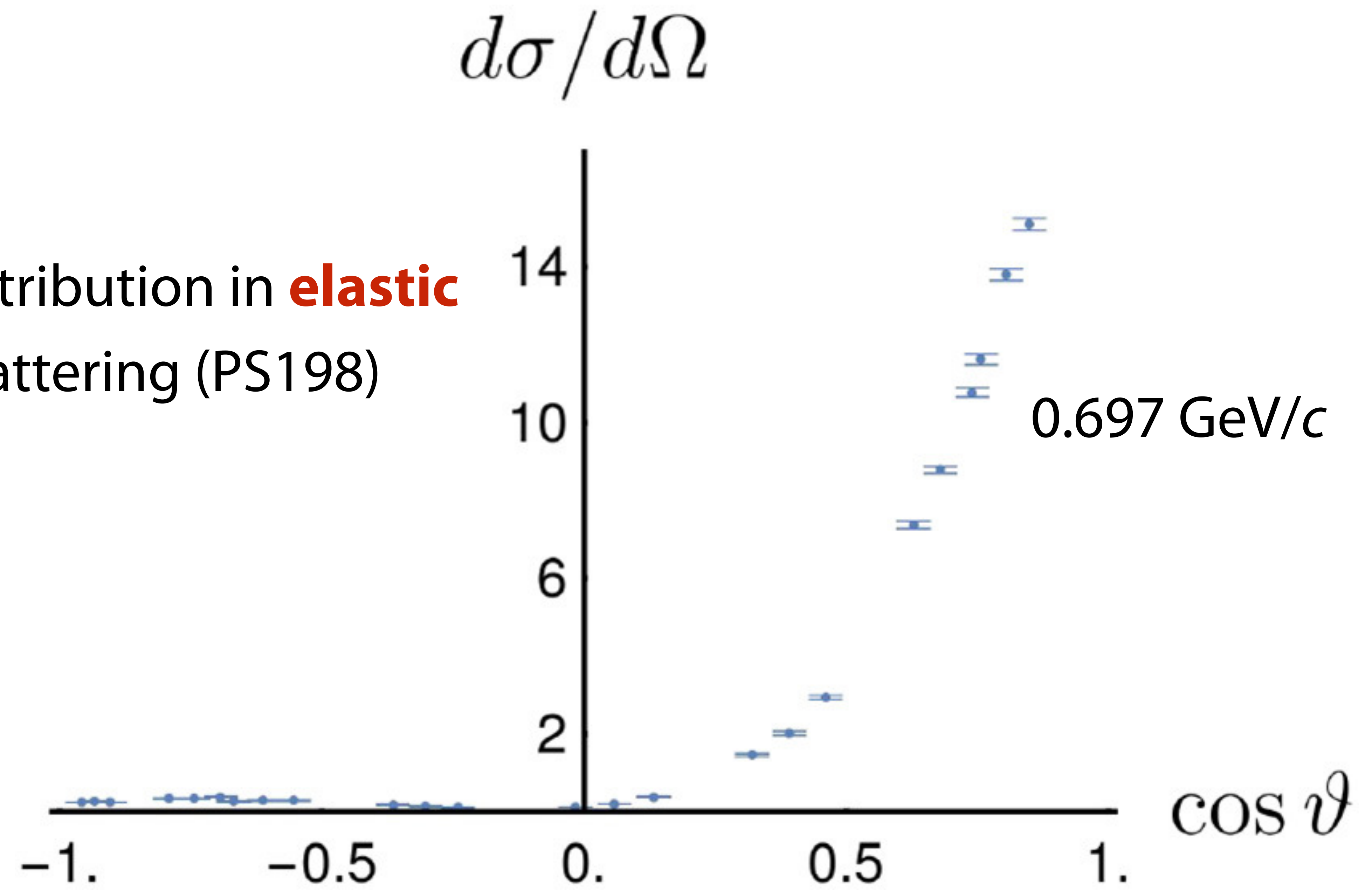


Antiproton-proton annihilation cross section (in mb), as measured by the PS173 collaboration.

Richard, Antiproton Physics, Frontiers in Physics, doi: 10.3389/fphy.2020.00006

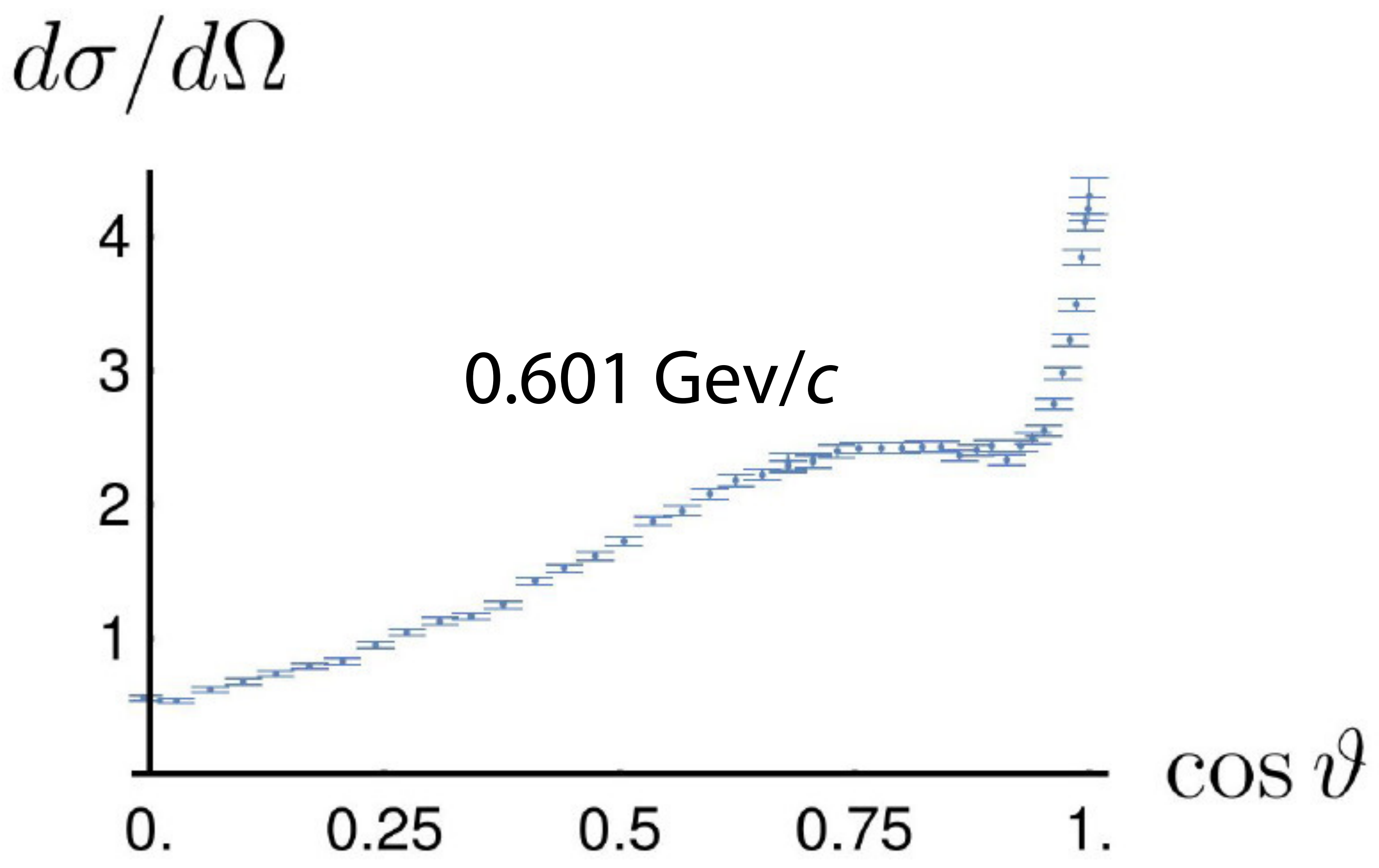
Angular Distribution for Elastic and Charge-Exchange Reactions

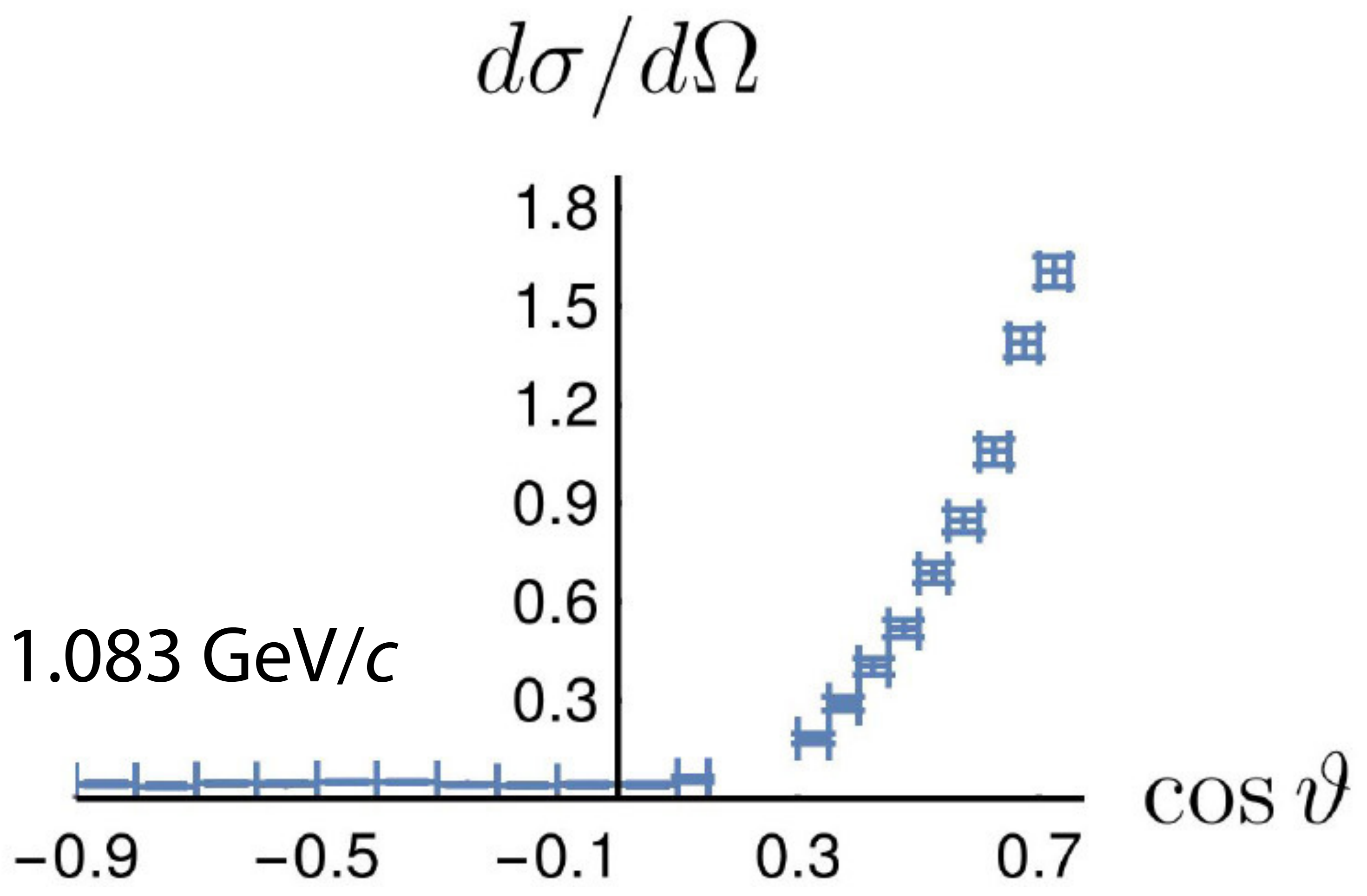
Angular distribution in **elastic** $\bar{p}p \to \bar{p}p$ scattering (PS198)



The distribution is far from flat.
High partial waves are believed to play a very important role.

Charge-exchange cross sections in **elastic** $\bar{p}p \to \bar{n}n$ scattering (PS198)



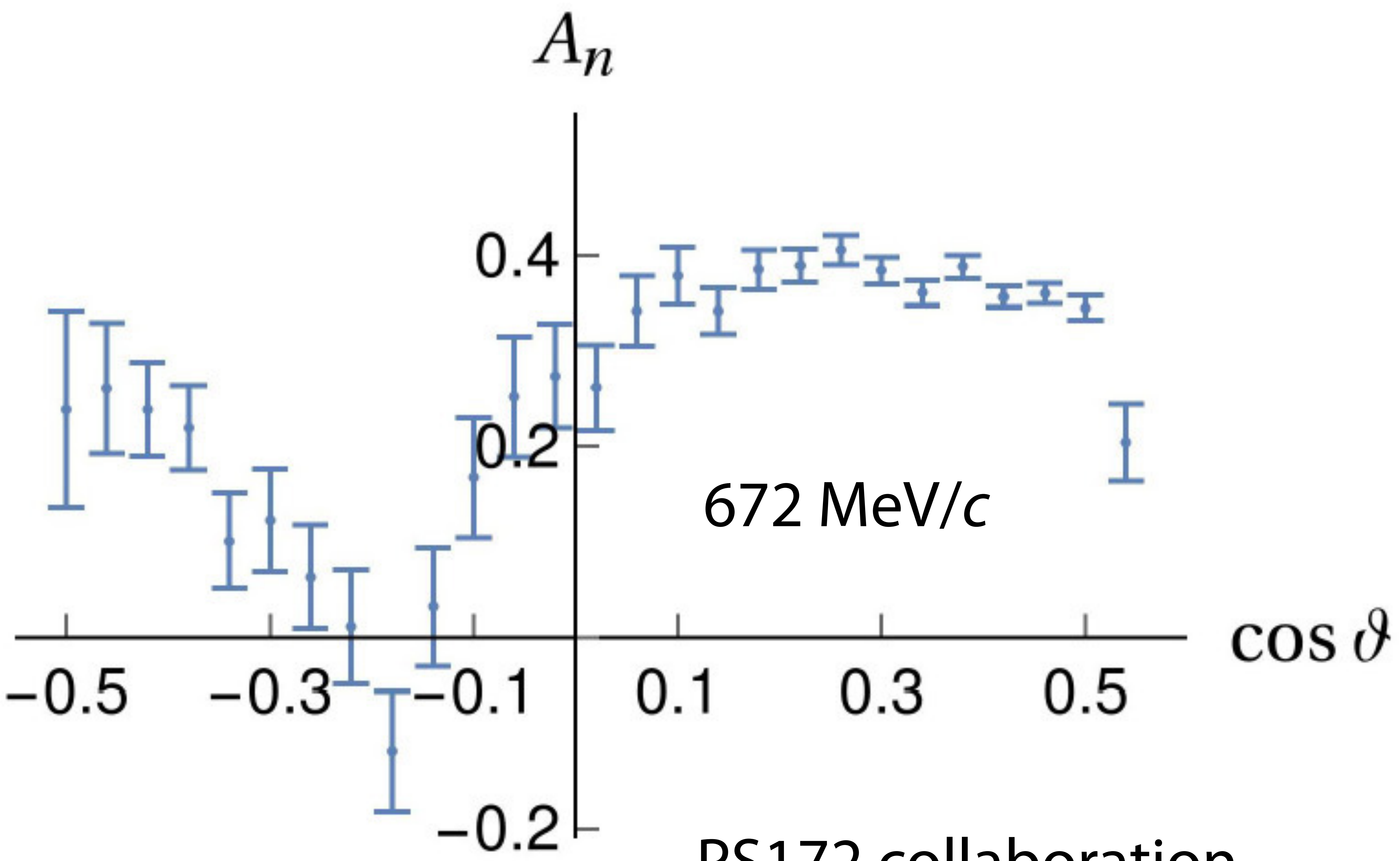


Richard, Antiproton Physics, Frontiers in Physics, doi: 10.3389/fphy.2020.00006

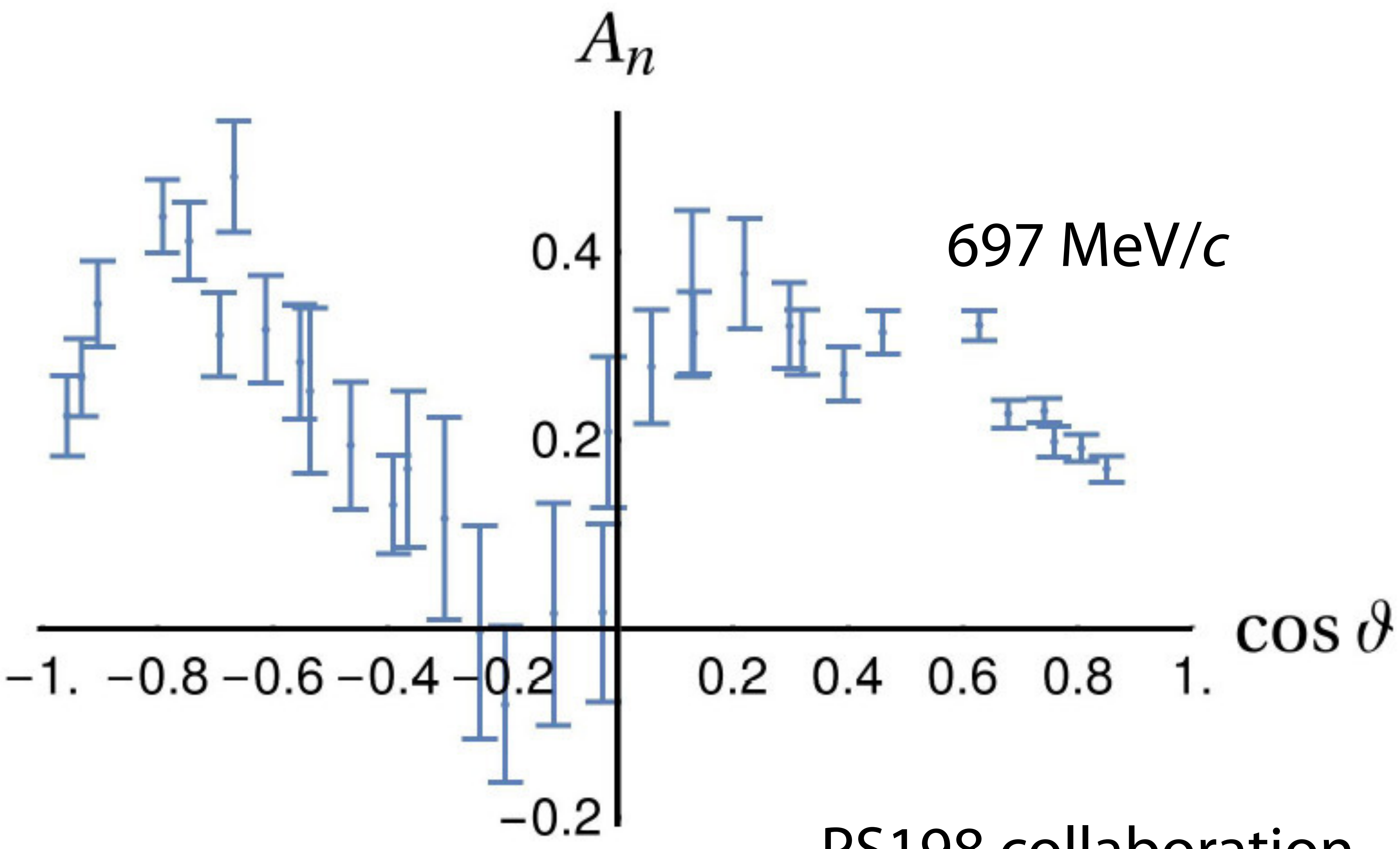
Few remarks about antiproton physics

The value of the analyzing power is sizable, but not very large.

It is compatible with either a moderate spin- orbit component of the interaction, or a rather strong tensor force acting at second order.



PS172 collaboration



PS198 collaboration

Richard, Antiproton Physics, Frontiers in Physics, doi: 10.3389/fphy.2020.00006

Potential Models

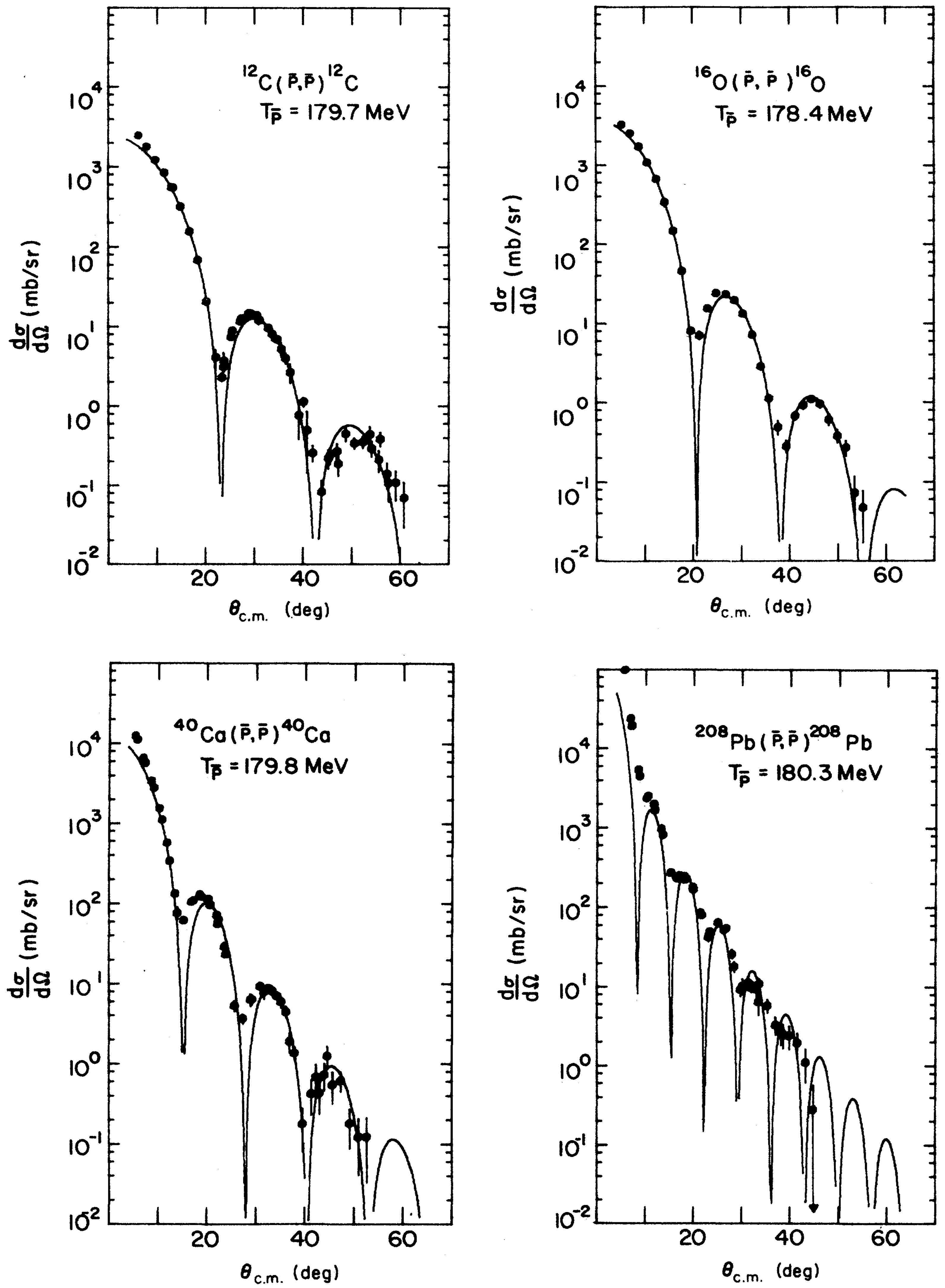
Historical way: the long range potential is the *G*-parity transformed of the Paris *NN* potential, regularized in a square-well manner, i.e., $V_{LR}(r < r_0) = V_{LR}(r_0)$ with $r_0 = 0.8$ fm, supplemented by a complex core to account for unknown short-range forces and for annihilation

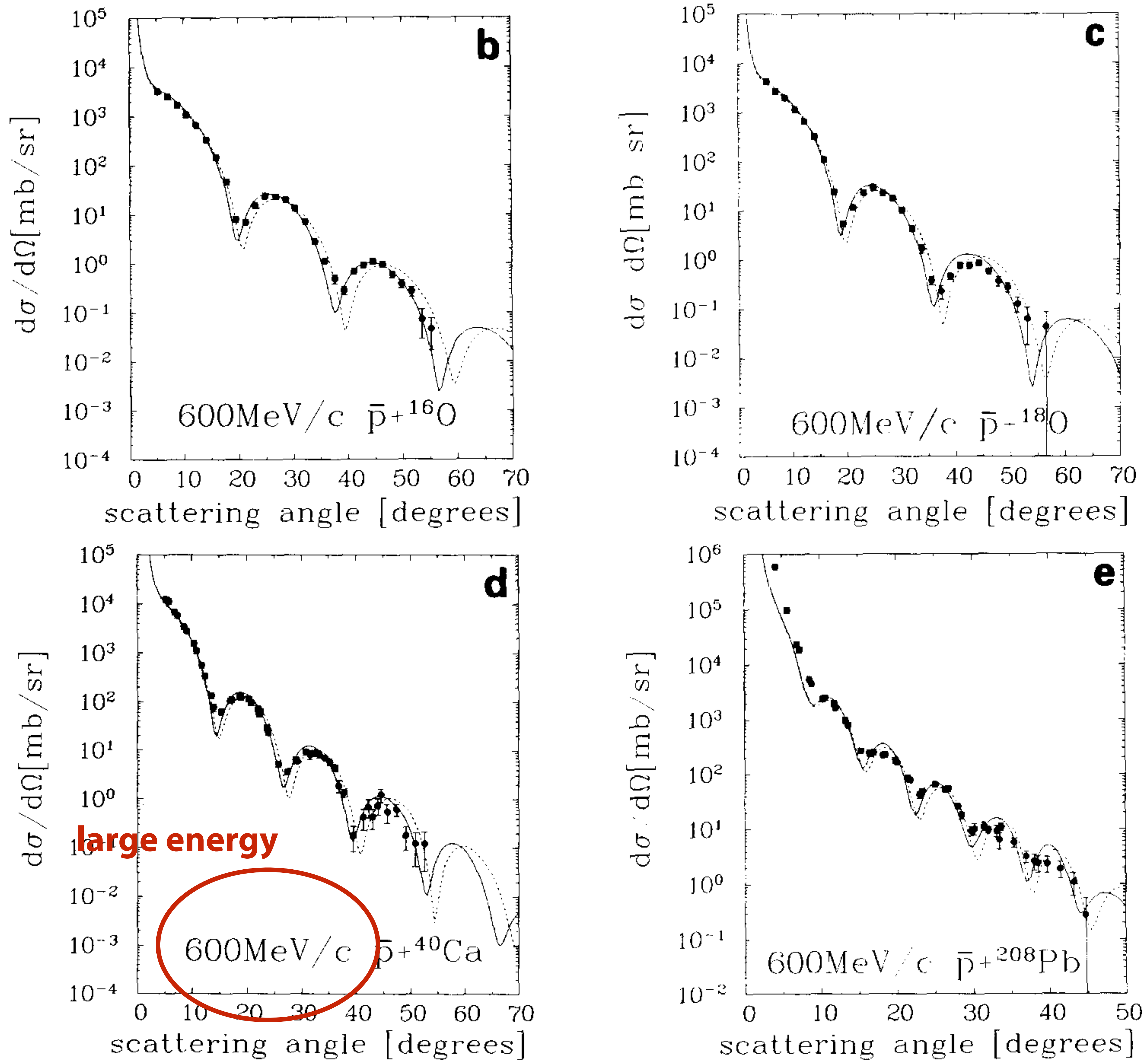
$$V_{\rm SR}(r) = -\frac{V_0 + i\,W_0}{1 + \exp(-(r - R)/a)}$$

The short-range interaction was taken as spin and isospin independent, for simplicity.

$$\begin{aligned}
&\text{model DR1} \quad R = 0\,\text{fm}, \quad a = 0.2\,\text{fm}, \quad V_0 = 21\,\text{GeV}, \\
&\qquad\qquad\quad W_0 = 20\,\text{GeV}, \\
&\text{model DR2} \quad R = 0.8\,\text{fm}, \quad a = 0.2\,\text{fm}, \quad V_0 = 0.5\,\text{GeV}, \\
&\qquad\qquad\quad W_0 = 0.5\,\text{GeV}.
\end{aligned}$$

Phenomenological interpretations





Phenomenological model analysis of elastic and inelastic scattering of ≈ 180 MeV antiprotons from various nuclei

D. C. Choudhury and T. Guo
Department of Physics, Polytechnic University, Brooklyn, New York 11201

PRC 39 (1987) 1883

MICROSCOPIC ANALYSIS OF ANTIPROTON-NUCLEUS ELASTIC SCATTERING

S. ADACHI[1] and H.V. VON GERAMB

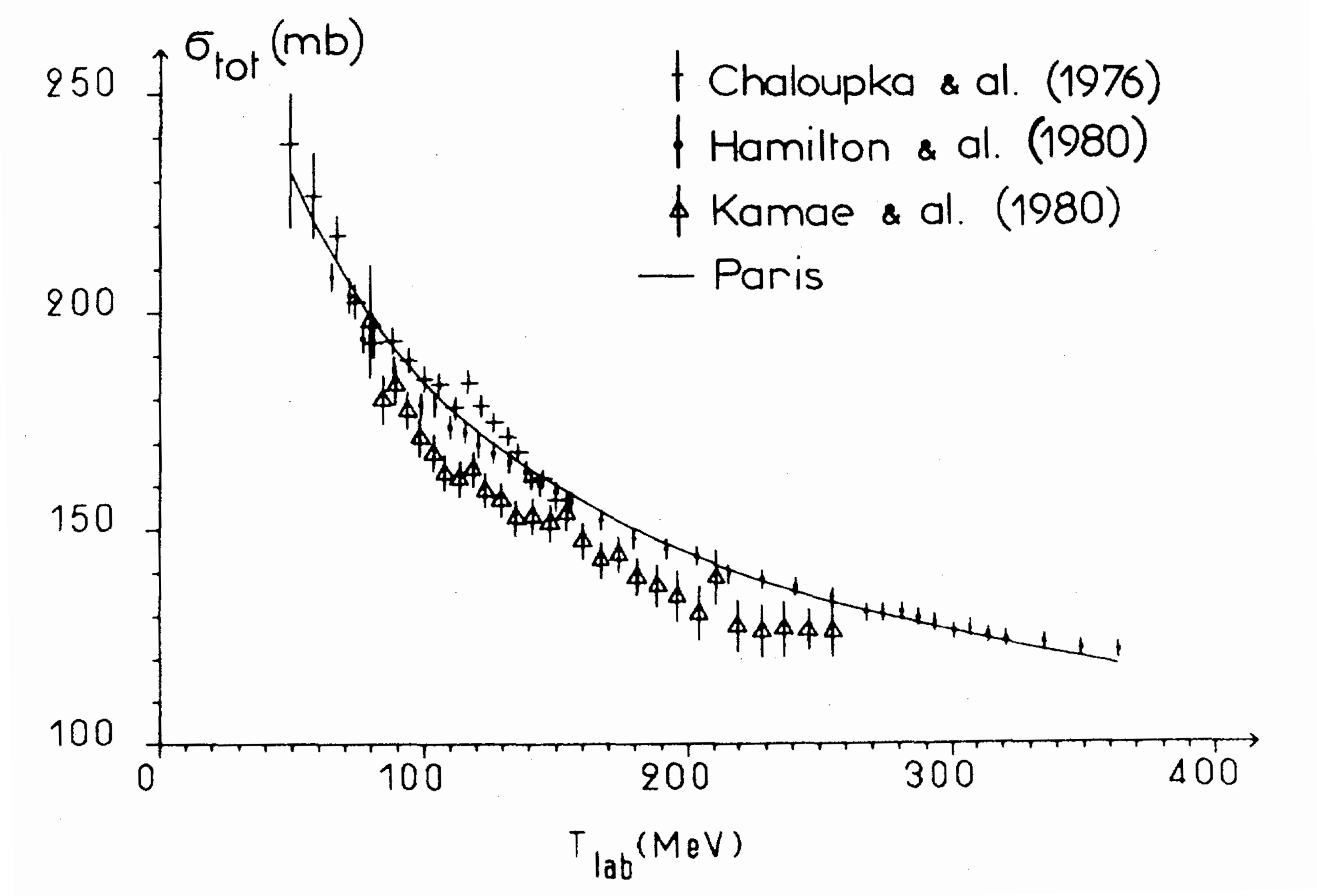
NPA 470 (1987) 461

Phenomenological interpretations

Nucleon-Antinucleon Optical Potential

J. Côté,[(a)] M. Lacombe, B. Loiseau, B. Moussallam, and R. Vinh Mau
Division de Physique Théorique, Insitut de Physique Nucléaire, F-91406 Orsay, France, and Laboratoire de Physique Théorique des Particules Elementaires, Université Pierre et Marie Curie, F-75230 Paris Cédex 05, France

PRL 48 (1982) 1320



The NN potential can be decomposed as follows in three contributions $\mathbf{V = U_S + U_L - i\,W}$:

- $\mathbf{U_S}$, long-range and medium range part generated by G-parity transformation of the Paris potential
- $\mathbf{U_L}$, the short-range component is described phenomenologically (quadratic function)
- **W** an absorptive part that is short-range and energy dependent

$$W_{N\bar{N}}(\vec{r},T_L)=\left(g_C(1+f_C T_L)+g_{SS}(1+f_{SS}T_L)\vec{\sigma}_1\cdot\vec{\sigma}_2+g_T S_{12}+\frac{g_{LS}}{4m^2}\vec{L}\cdot\vec{S}\frac{1}{r}\frac{d}{dr}\right)\frac{K_0(2mr)}{r}$$

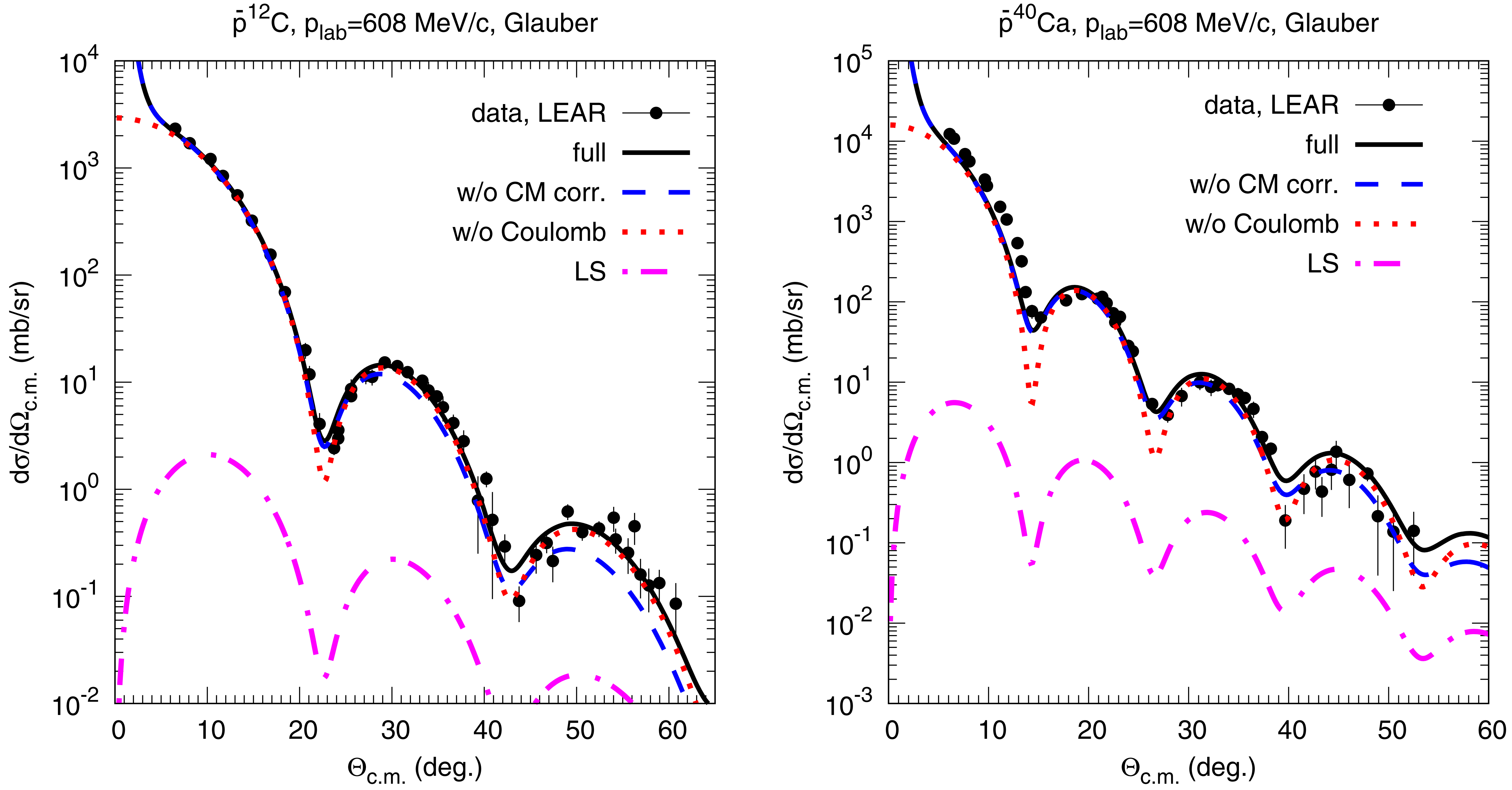


Fig. 2. Angular differential cross section of $\bar{p}$ elastic scattering at 608 MeV/c on ^{12}C, ^{40}Ca, and ^{208}Pb. Full GM calculation is shown by solid line. The dashed and dotted lines show, respectively, the results without recoil correction ($H_{cm}(q)=1$, Eq. (54)) and without Coulomb correction ($\xi=0$, Eq. (22)). The dot-dashed line shows the contribution of the spin–orbit amplitude G to the differential cross section, Eq. (44). Experimental data are from ref. [46].

Elastic scattering, polarization and absorption of relativistic antiprotons on nuclei

A.B. Larionov[a,b,*], H. Lenske[a]

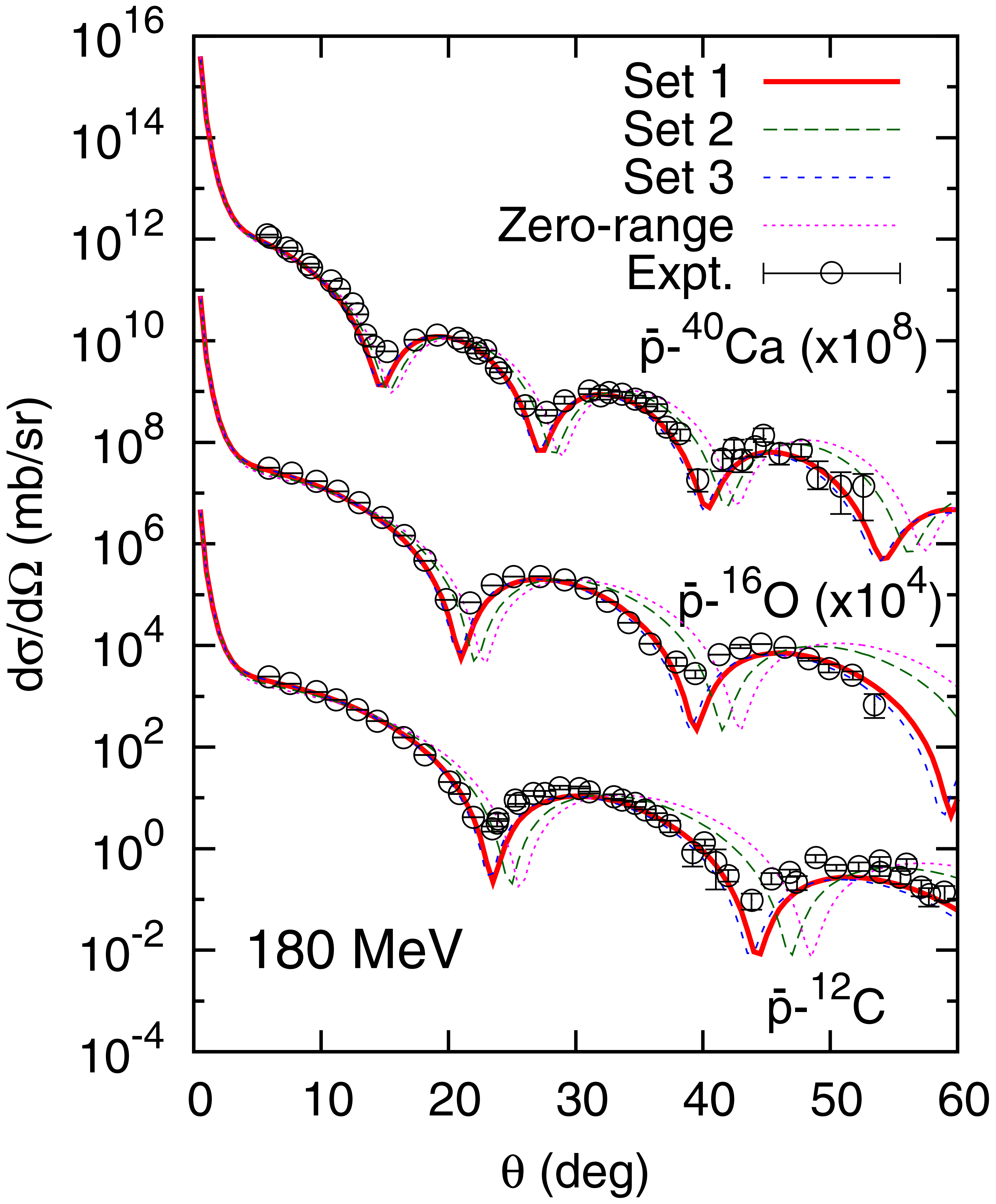
NPA 957 (2017) 450

Utility of antiproton-nucleus scattering for probing nuclear surface density distributions

K. Makiguchi,[1] W. Horiuchi,[1,*] and A. Kohama[2]

PHYSICAL REVIEW C **102**, 034614 (2020)

"In contrast to the proton-nucleus scattering, we find that the complete absorption occurs even beyond the nuclear radius due to the large p̄N elementary cross sections, which shows stronger sensitivity to the nuclear density distribution in the tail region."

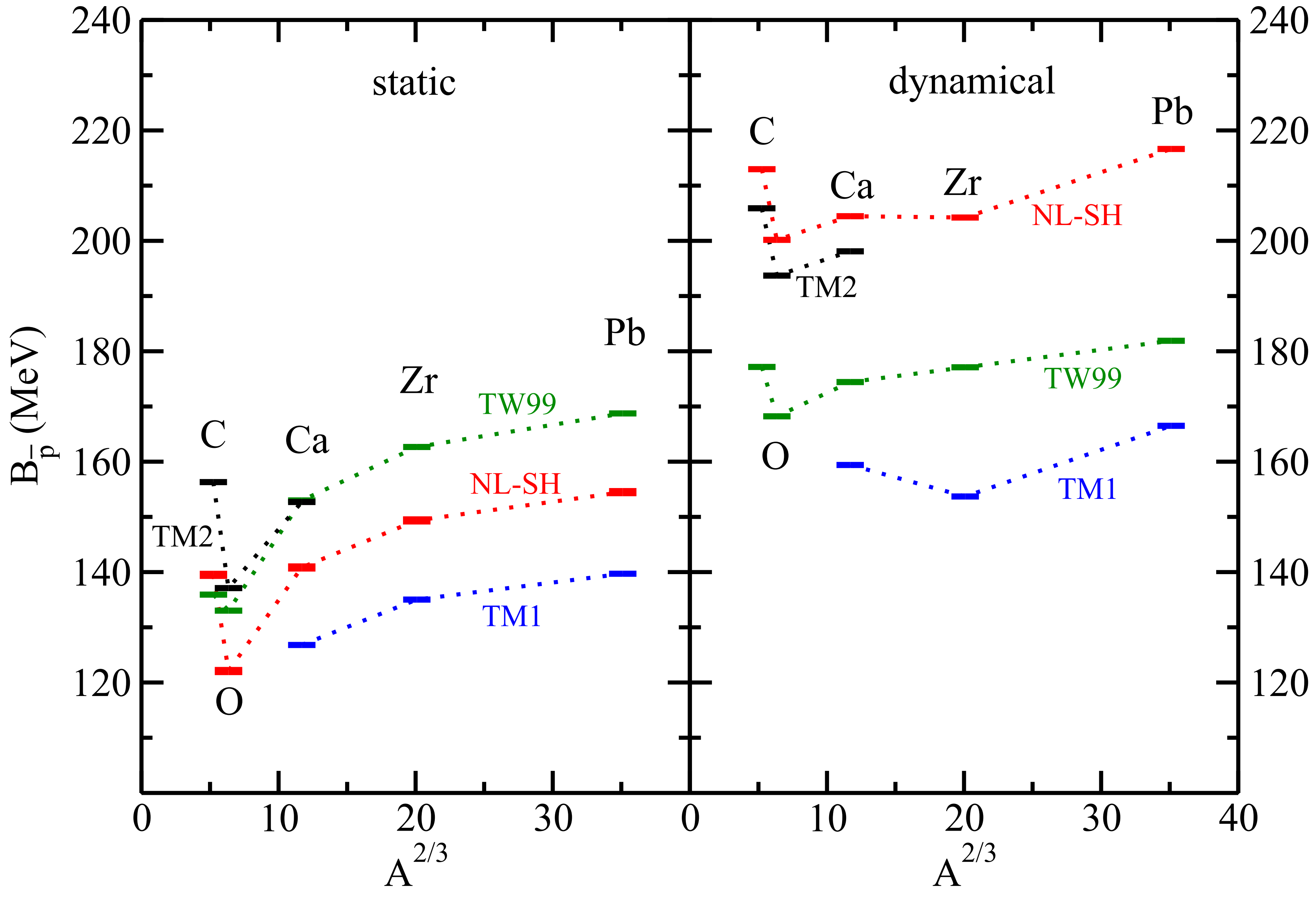


Interaction of antiprotons with nuclei

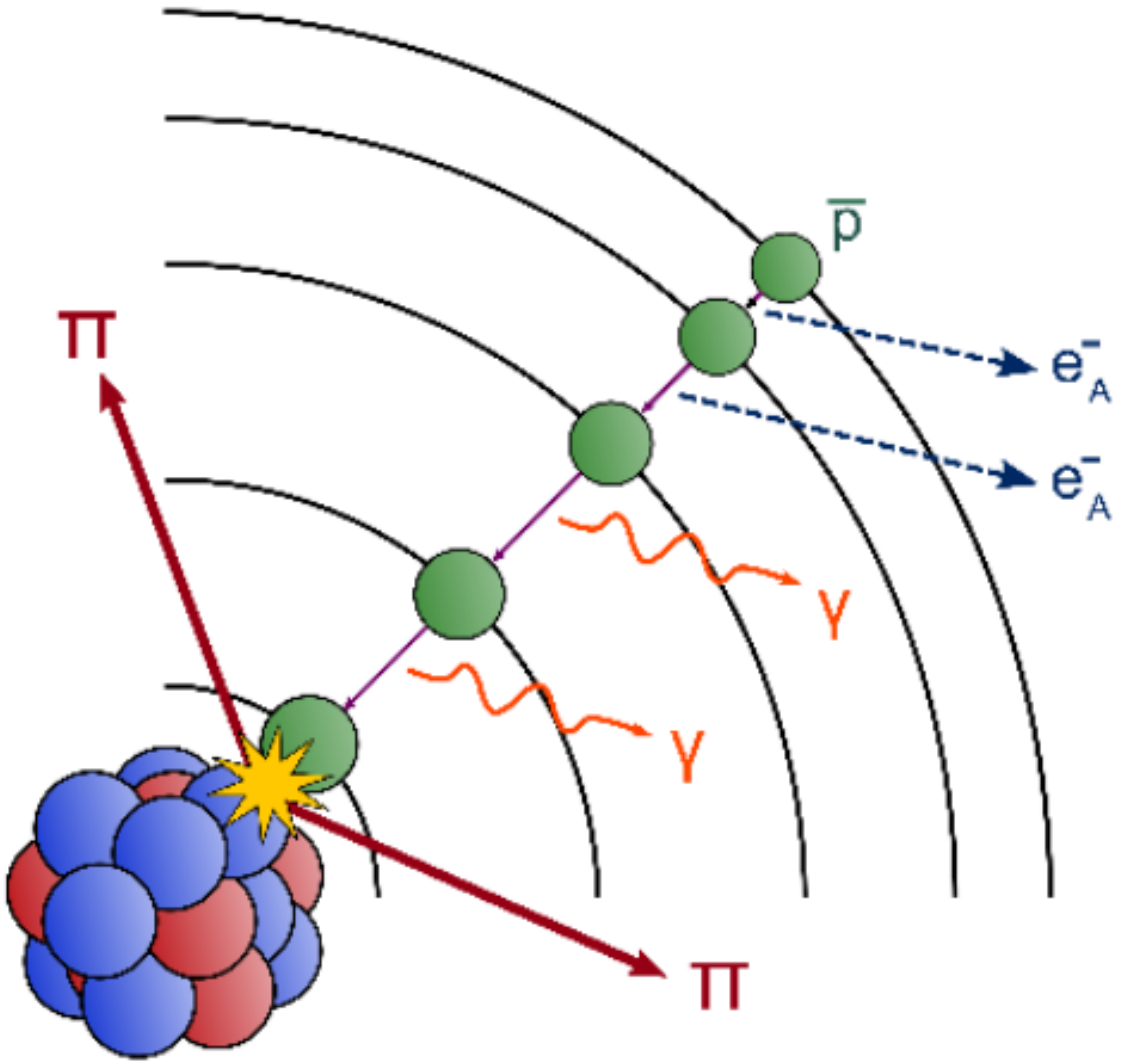
Jaroslava Hrtánková *, Jiří Mareš

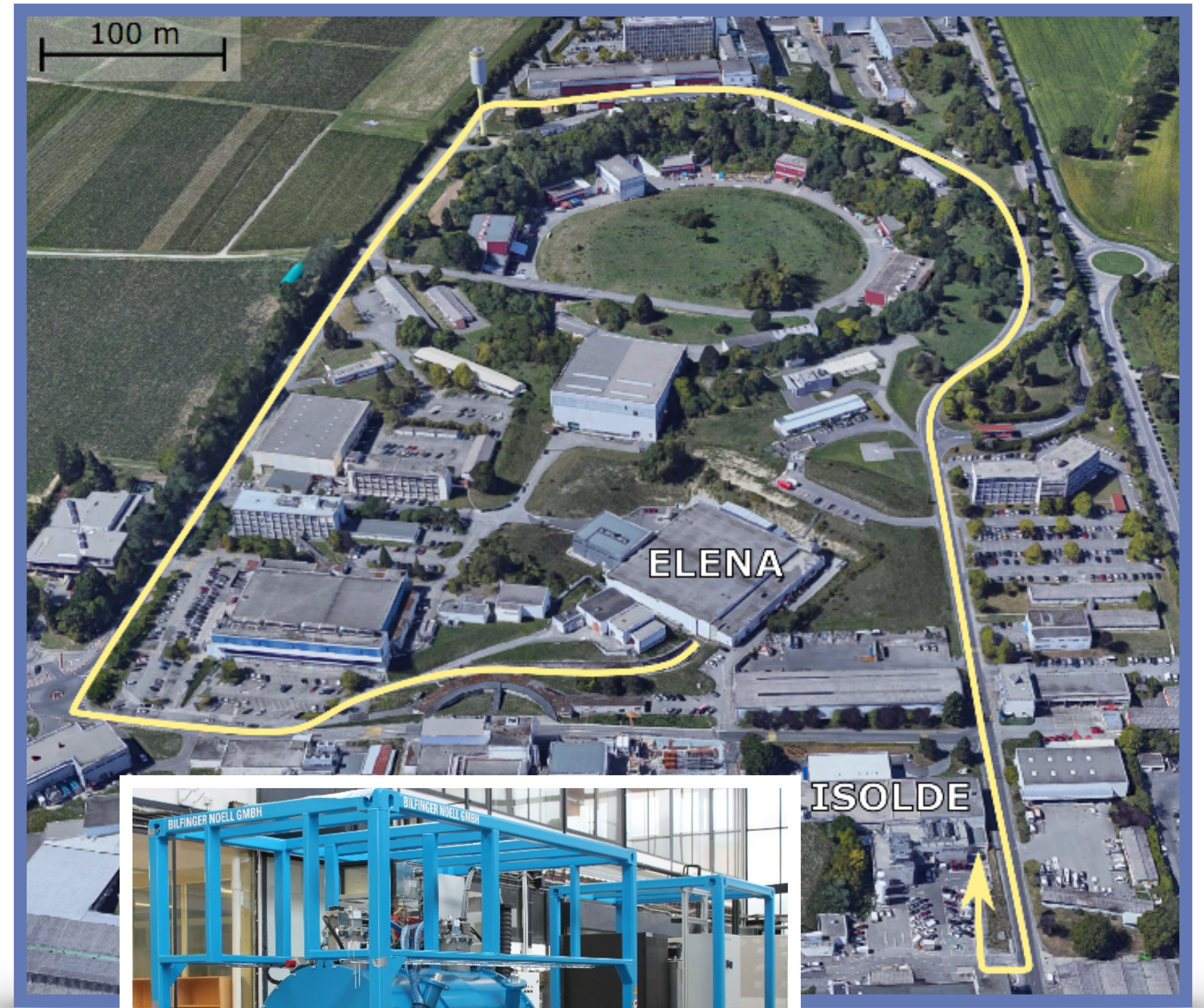
Nuclear Physics A 945 (2016) 197–215

"Fully self-consistent calculations of p-nuclear bound states using a complex p-nucleus potential accounting for p̄-atom data. While the real part of the potential is constructed within the relativistic mean-field (RMF) model, the p̄ annihilation in the nuclear medium is described by a phenomenological optical potential. We confirm large polarization effects of the nuclear core caused by the presence of the antiproton. The p annihilation is treated dynamically, taking into account explicitly the reduced phase space for annihilation from deeply bound states as well as the compressed nuclear density due to the antiproton."



PUMA





see A. Obertelli's talk @ ECT 2024 or Klink "THE PUMA EXPERIMENT AT ELENA AND ISOLDE" (2023)

why is interesting antiproton-nucleus scattering?

We can **constrain** and **test** the most recent chiral potentials

Predictive power

Accuracy

Convergence

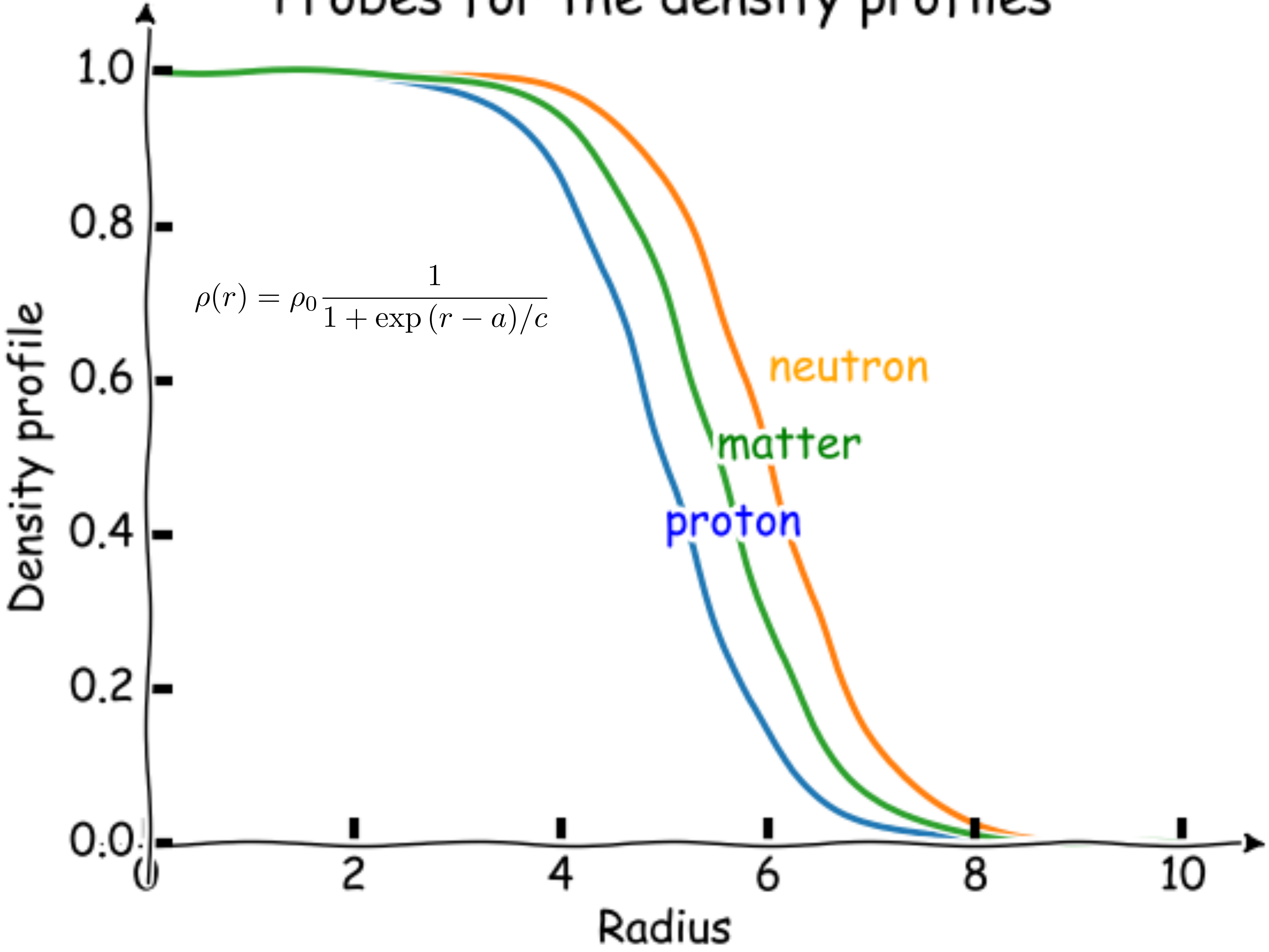
We can **test** multiple scattering theory (MST) outside the NN system

Many-body

We can **study** antiproton-nucleus systems to learn more about nuclei

Complementary approach to study nuclear systems

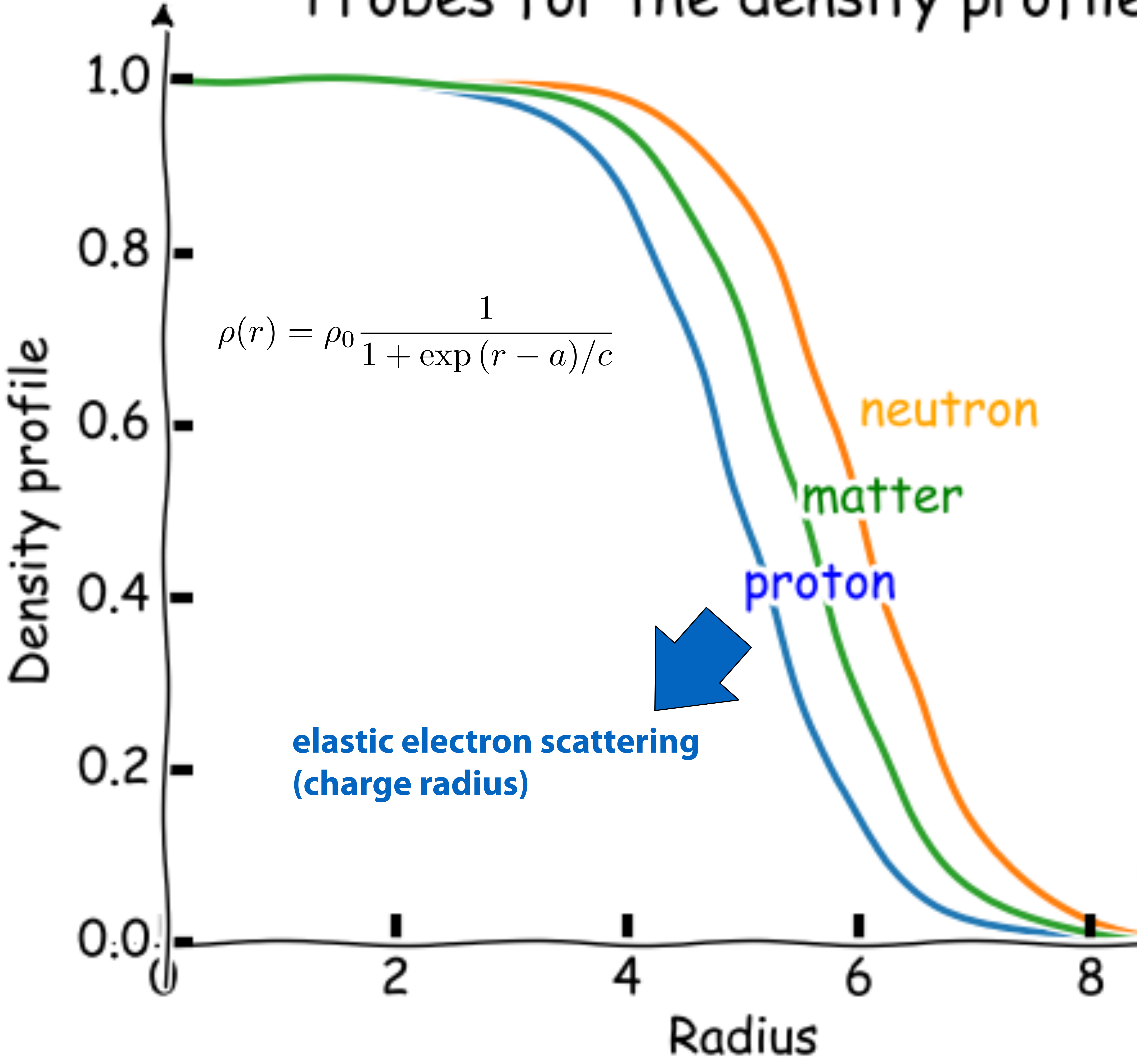
Probes for the density profiles



$$\rho(r) = \rho_0 \frac{1}{1 + \exp{(r-a)/c}}$$

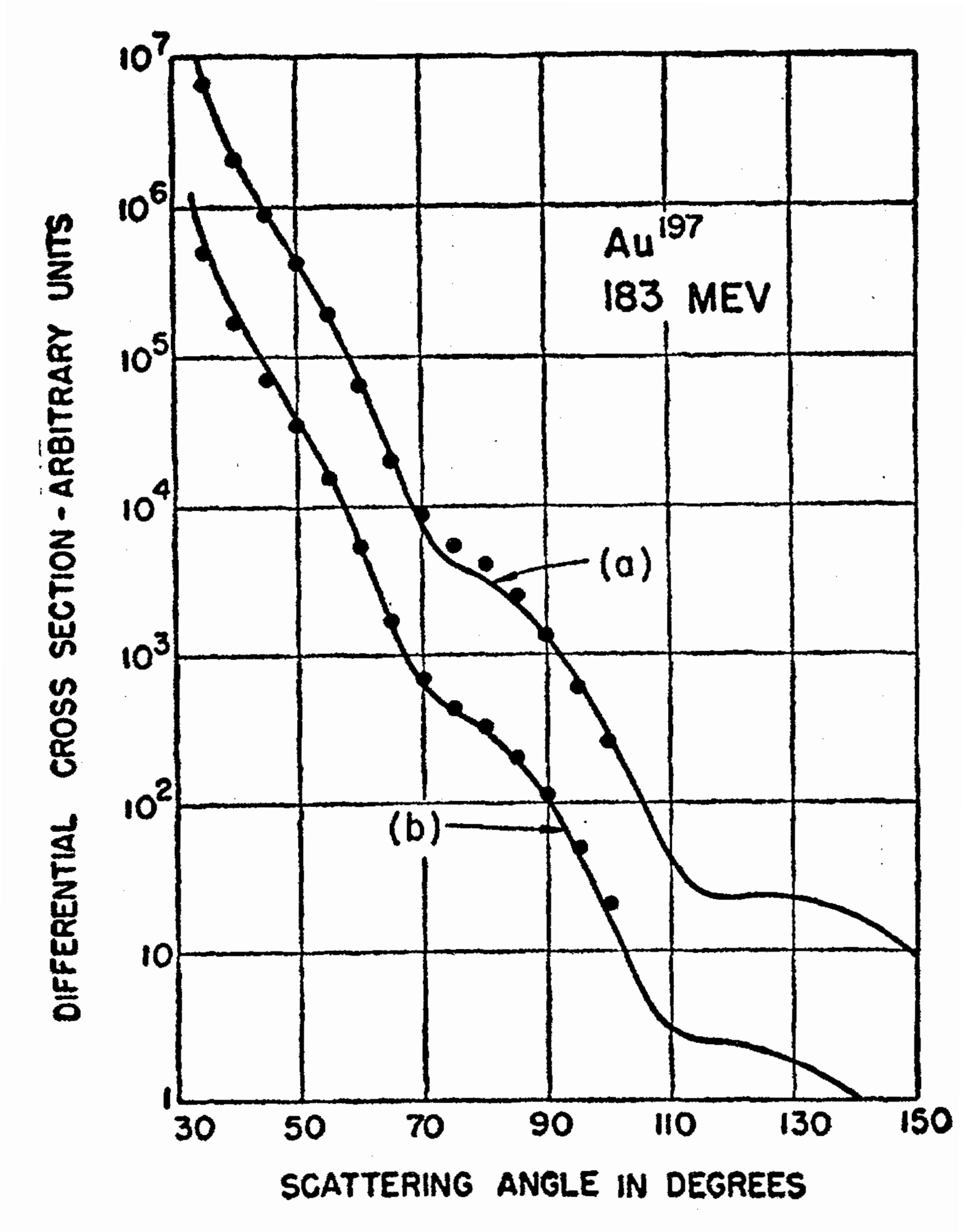
Hosftadter, Electron Scattering and Nuclear Structure

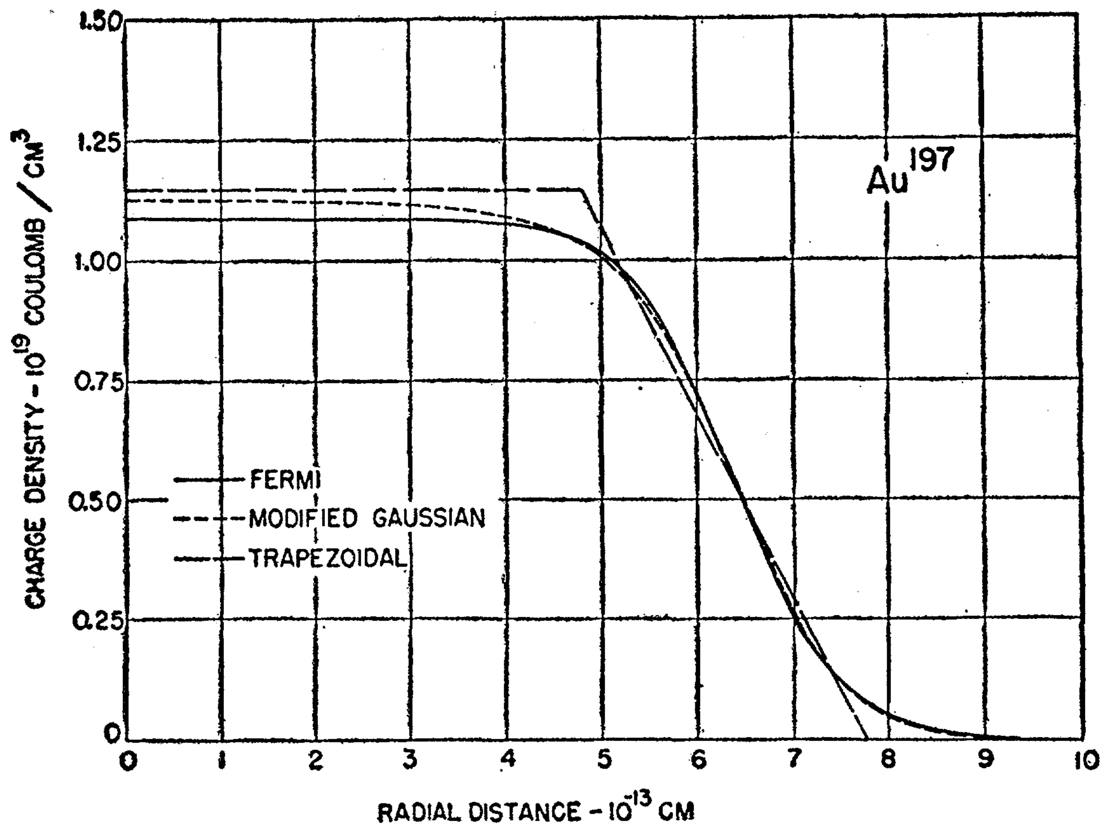
Probes for the density profiles



$$\rho(r) = \rho_0 \frac{1}{1 + \exp{(r-a)/c}}$$

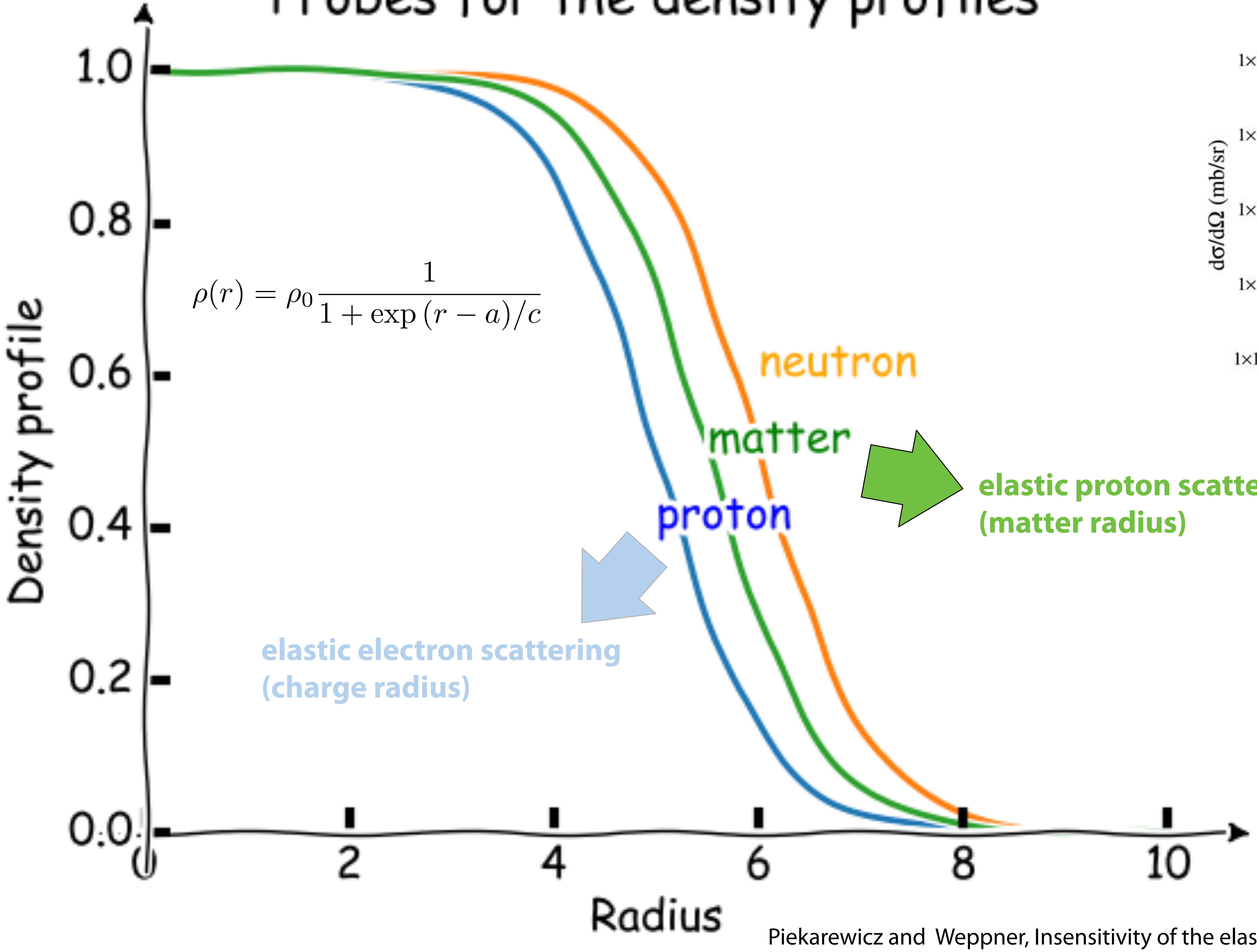
elastic electron scattering (charge radius)





Hosftadter, Electron Scattering and Nuclear Structure

Probes for the density profiles



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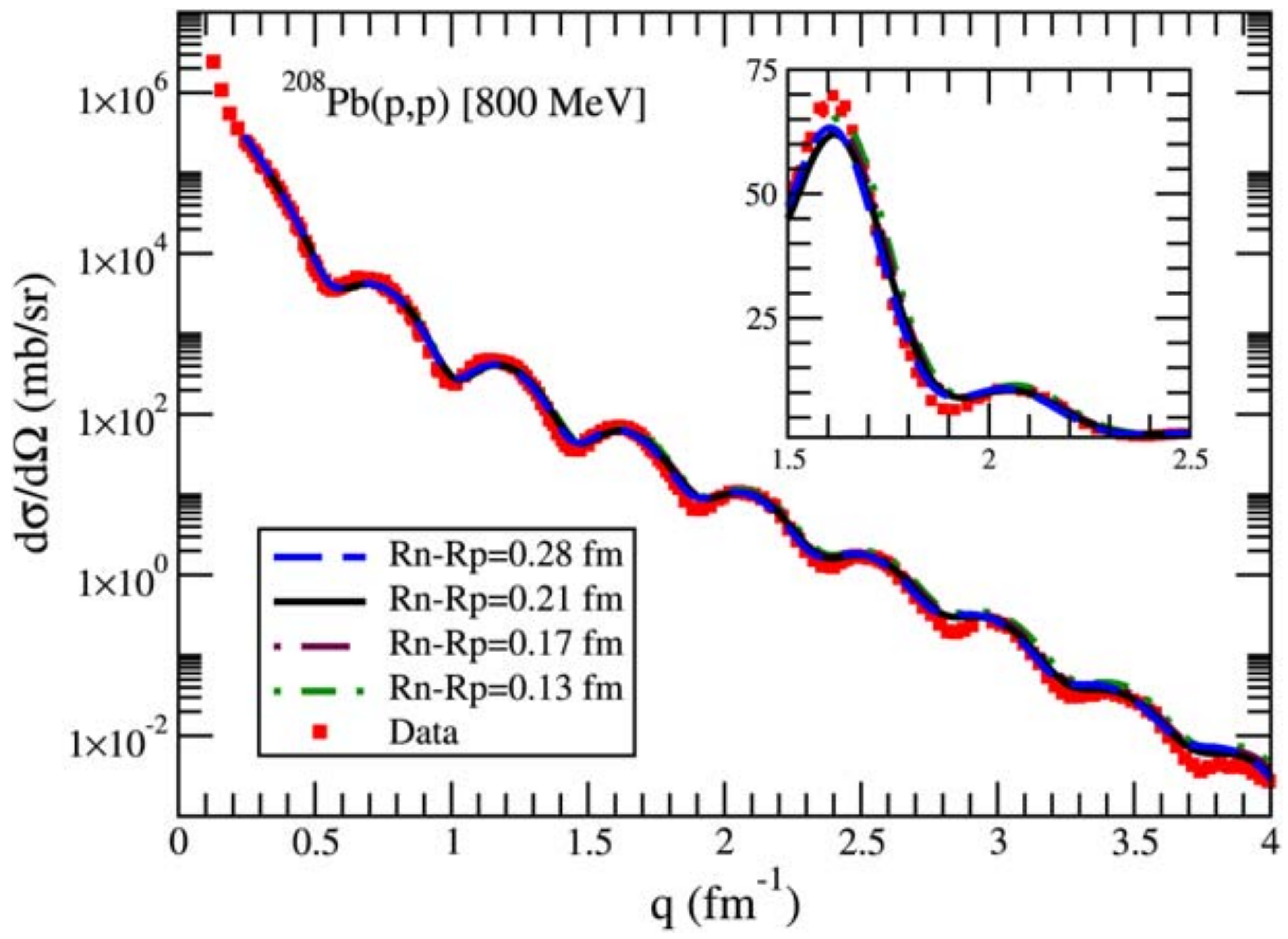
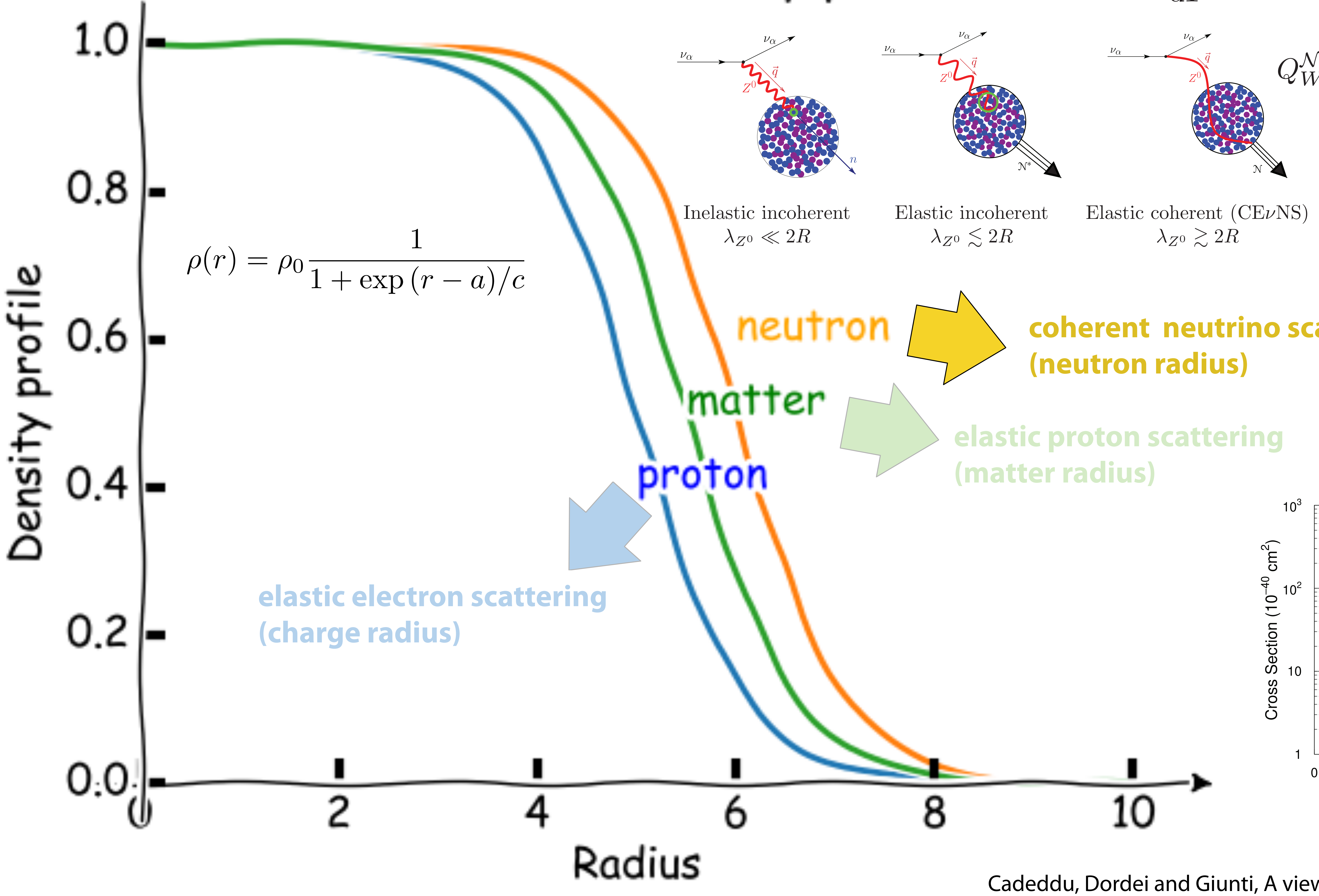


elastic proton scattering
(matter radius)

Piekarewicz and Weppner, Insensitivity of the elastic proton-nucleus reaction to the neutron radius of ^{208}Pb

Probes for the density profiles

$$\frac{d\sigma_{\nu\mathcal{N}}}{dT}(E_\nu, T) = \frac{G_F^2 M}{\pi}\left(1 - \frac{MT}{2E_\nu^2}\right)\left[Q_W^{\mathcal{N}}(|\vec{q}|)\right]^2$$

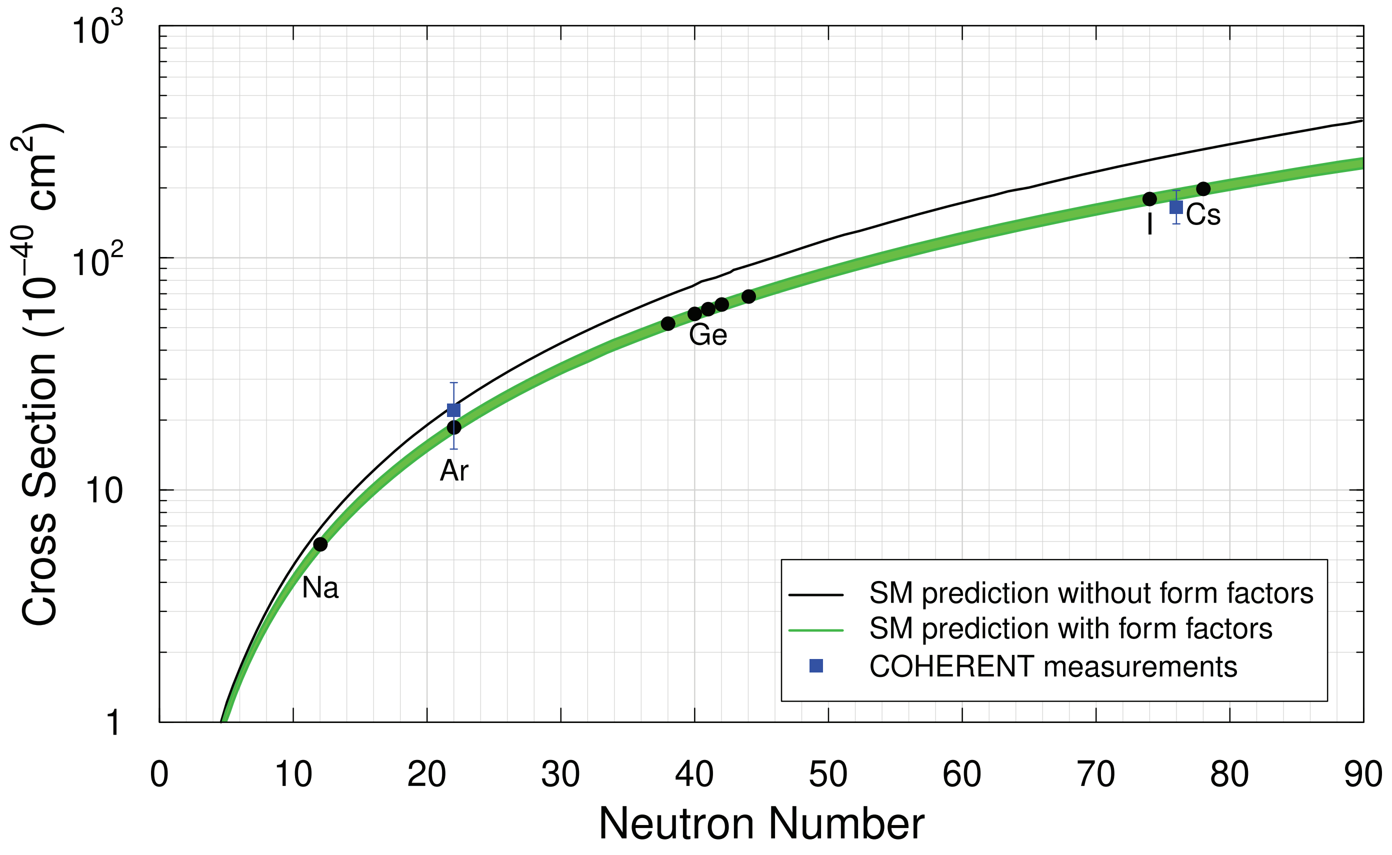


$$\rho(r) = \rho_0 \frac{1}{1 + \exp{(r-a)/c}}$$

Inelastic incoherent	Elastic incoherent	Elastic coherent (CEνNS)
$\lambda_{Z^0} \ll 2R$	$\lambda_{Z^0} \lesssim 2R$	$\lambda_{Z^0} \gtrsim 2R$

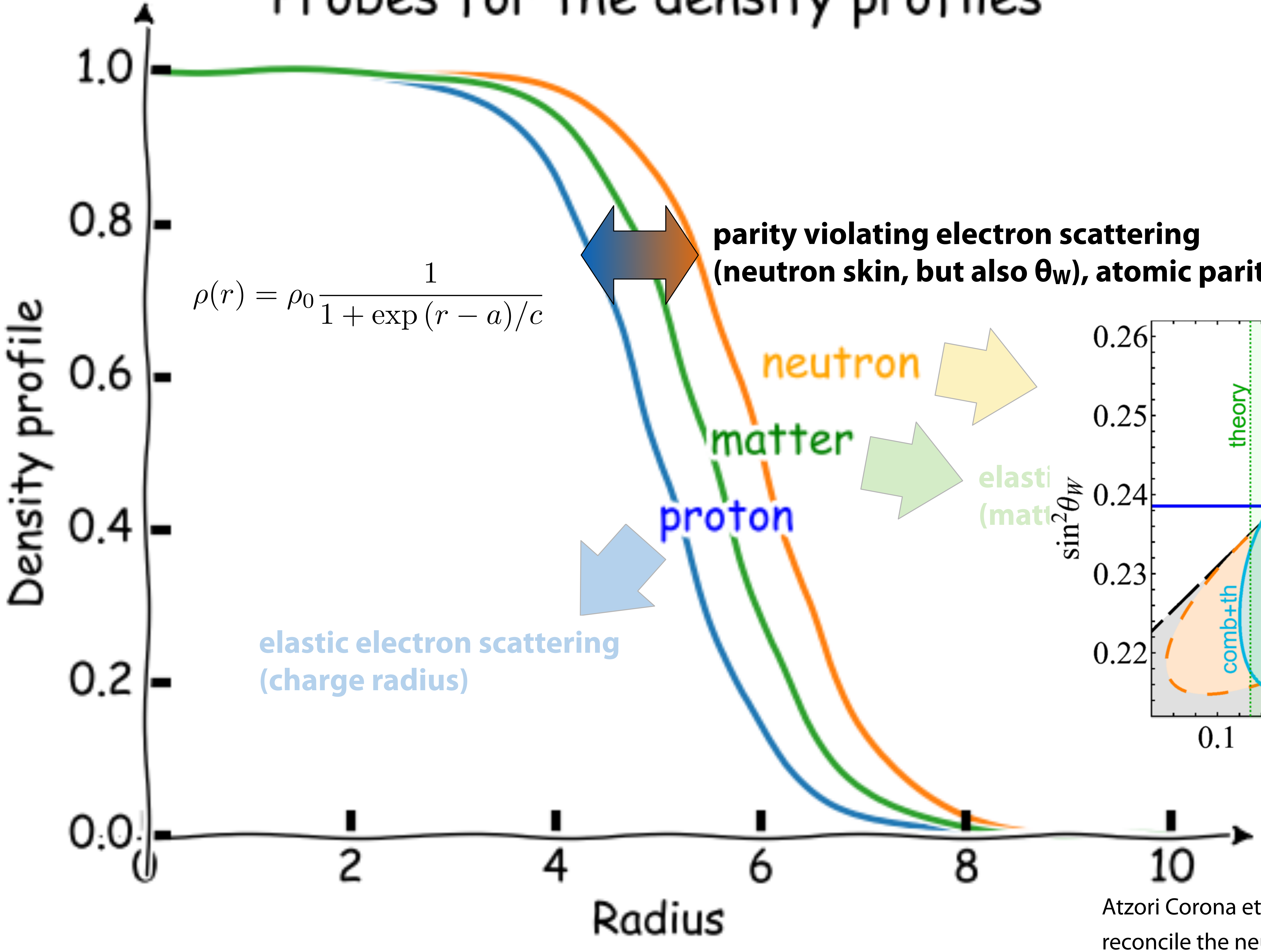
$$Q_W^{\mathcal{N}}(|\vec{q}|) = g_V^n\, N\, F_N^{\mathcal{N}}(|\vec{q}|) + g_V^p\, Z\, F_Z^{\mathcal{N}}(|\vec{q}|)$$

$$g_V^n \simeq -\frac{1}{2} \quad \text{and} \quad g_V^p \simeq \frac{1}{2} - 2\sin^2\vartheta_W \simeq 0.022$$



Cadeddu, Dordei and Giunti, A view of coherent elastic neutrino-nucleus scattering

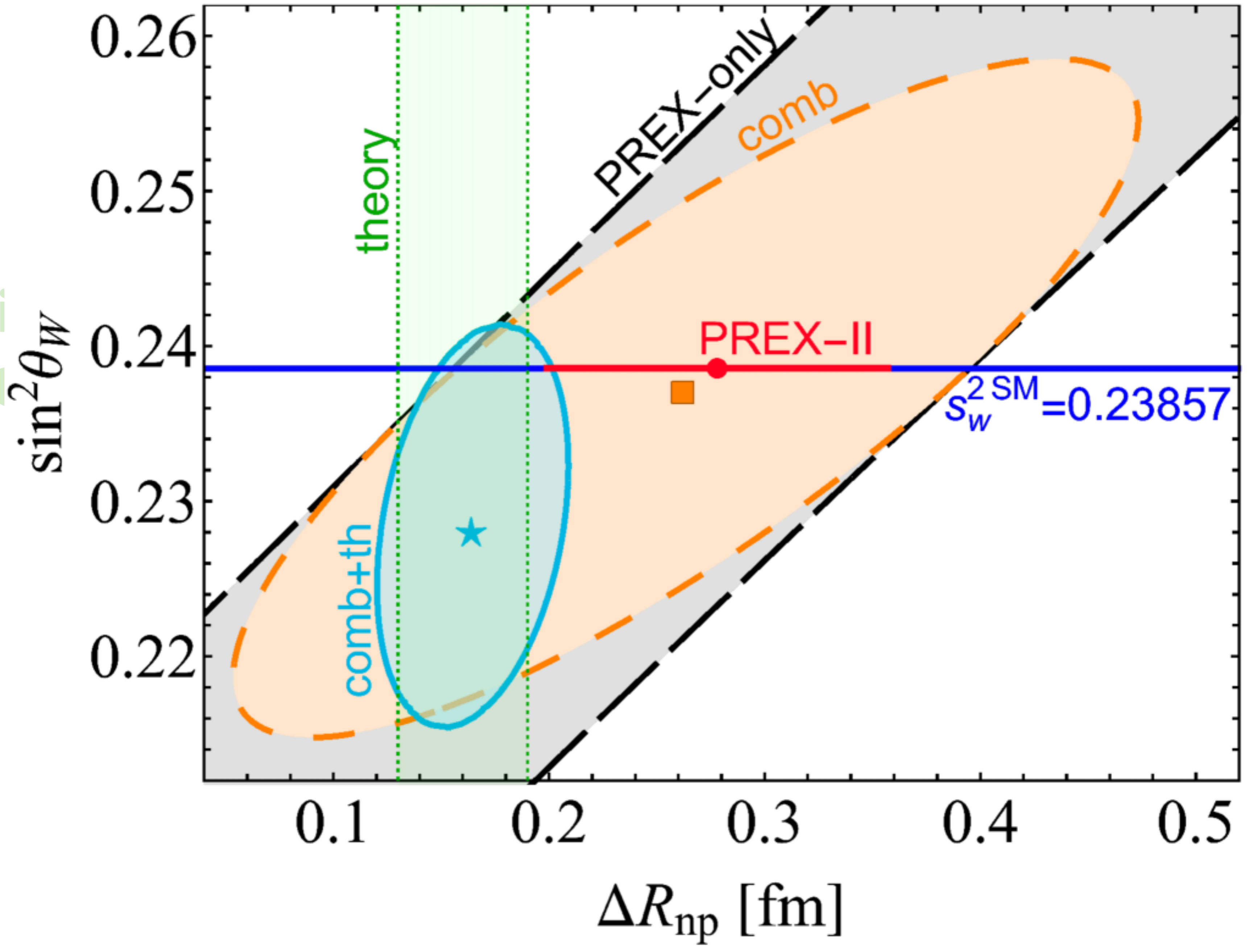
Probes for the density profiles



$$\rho(r) = \rho_0 \frac{1}{1+\exp{(r-a)/c}}$$

parity violating electron scattering (neutron skin, but also θ_W), atomic parity violation

elastic electron scattering (charge radius)



Atzori Corona et al., Incorporating the weak mixing angle dependence to reconcile the neutron skin measurement on ^{208}Pb by PREX-II

Probes for the density profiles

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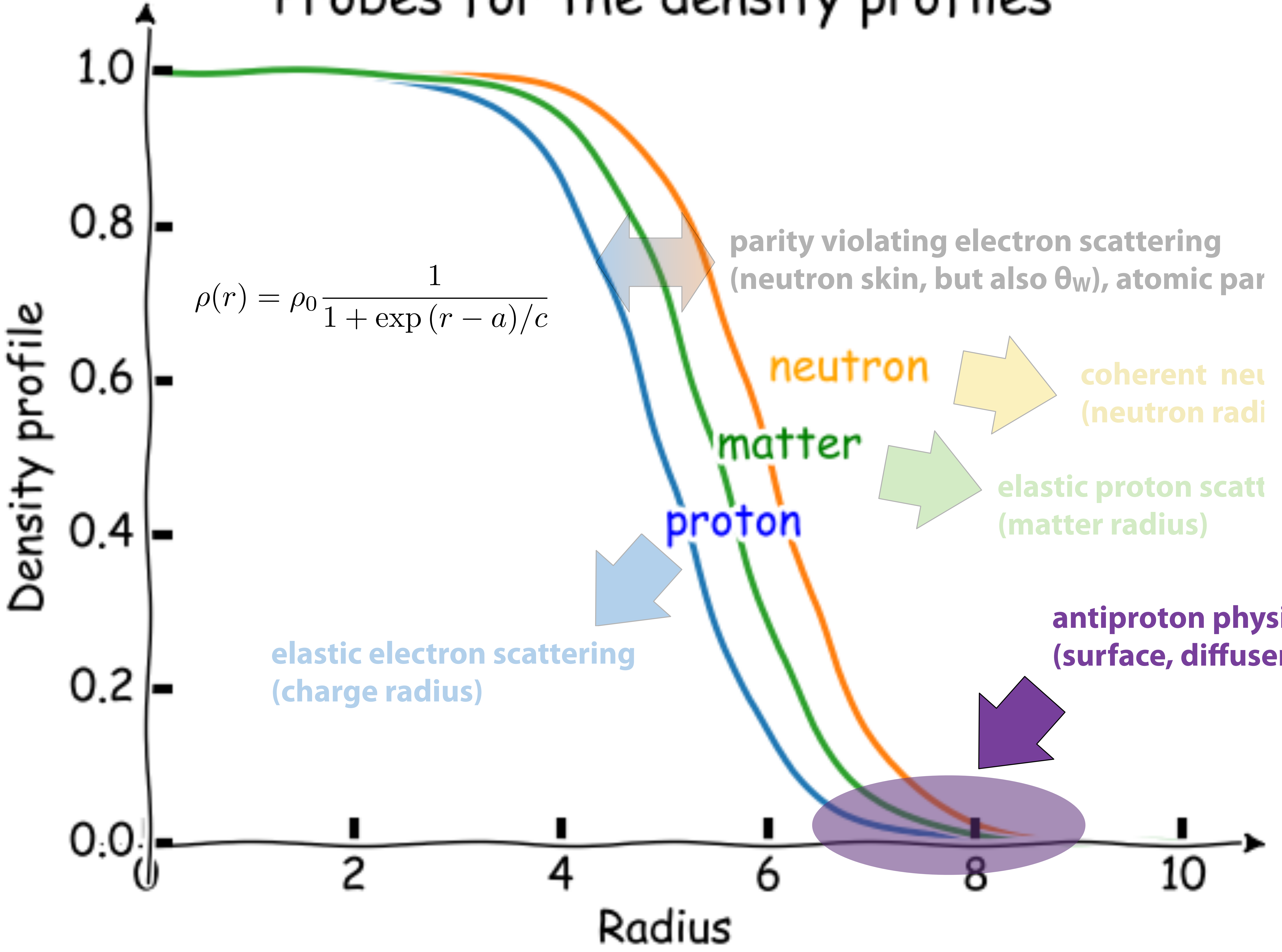
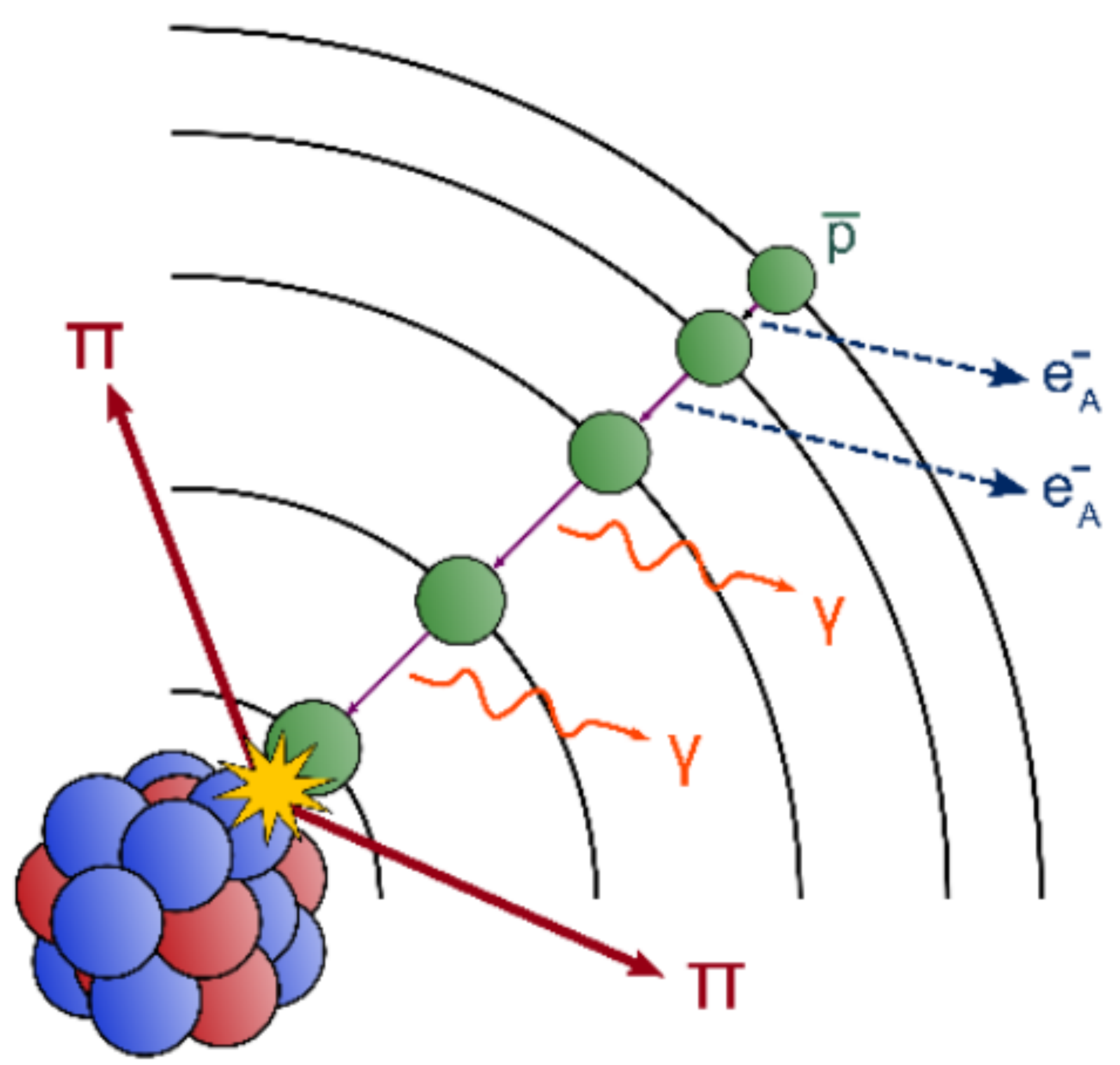
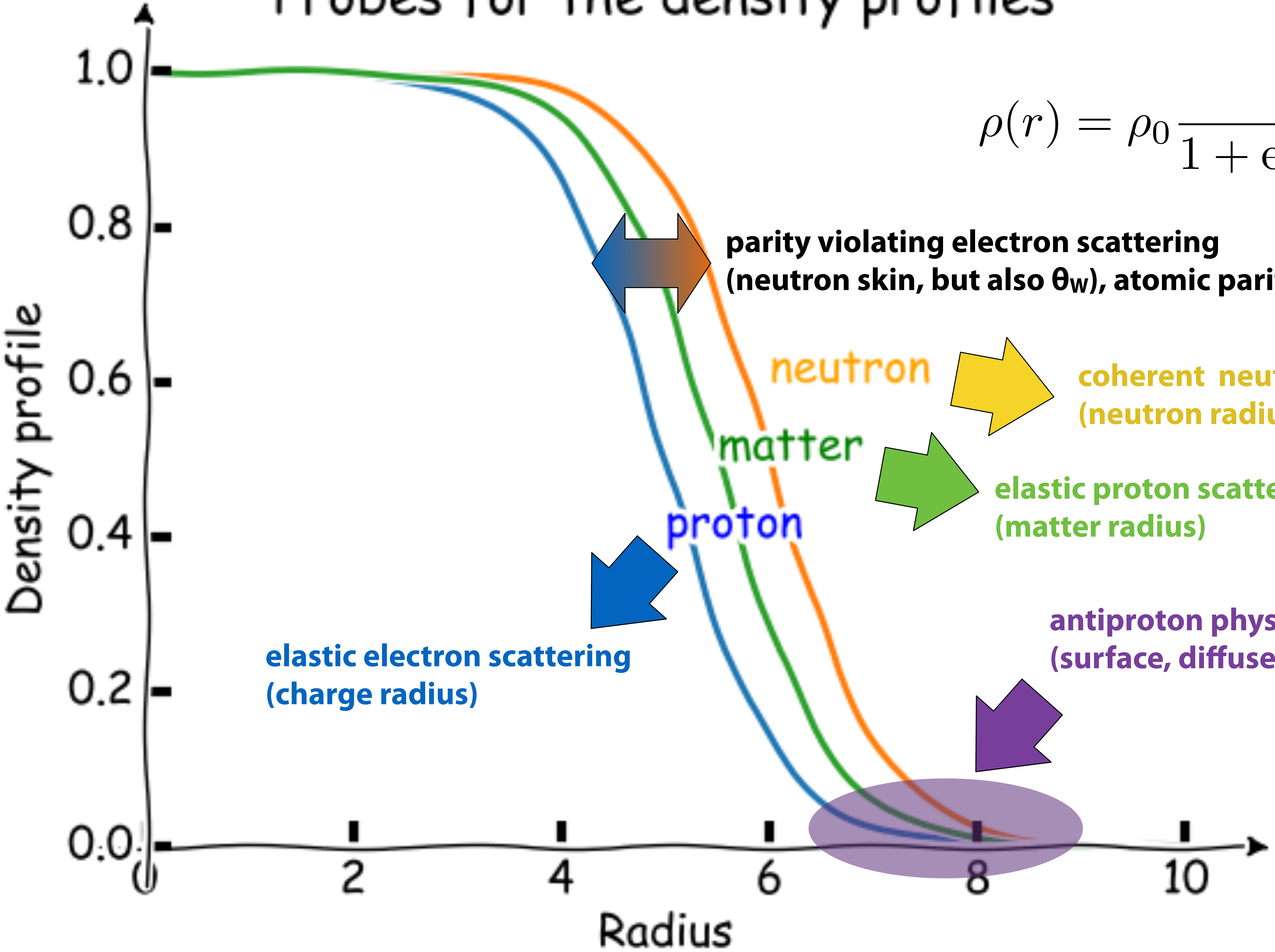


antiproton physics
(surface, diffuseness?)



Probes for the density profiles



$$\rho(r) = \rho_0 \frac{1}{1 + \exp{(r-a)/c}}$$

parity violating electron scattering (neutron skin, but also θ_W), atomic parity violation

coherent neutrino scattering (neutron radius)

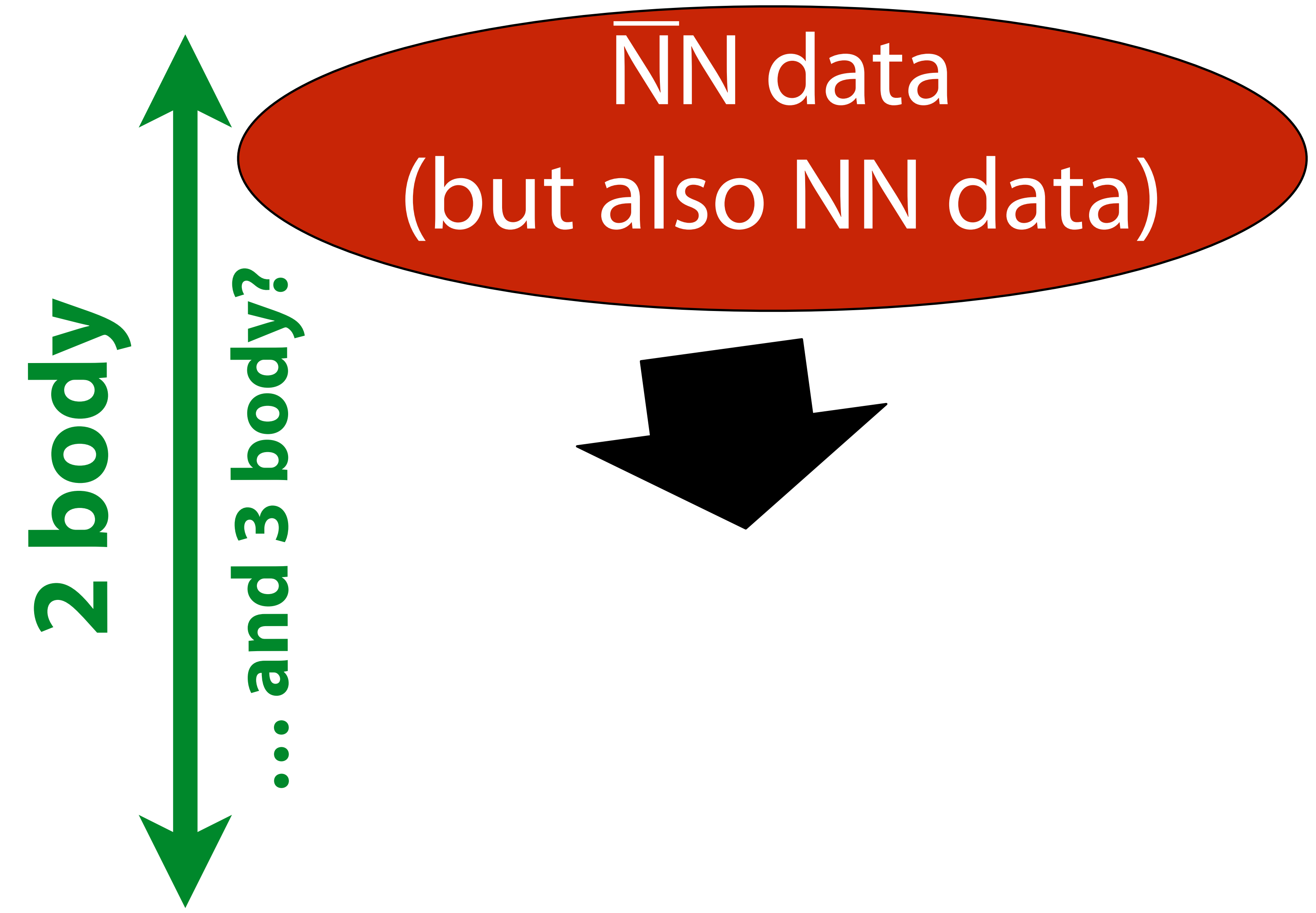
elastic proton scattering (matter radius)

elastic electron scattering (charge radius)

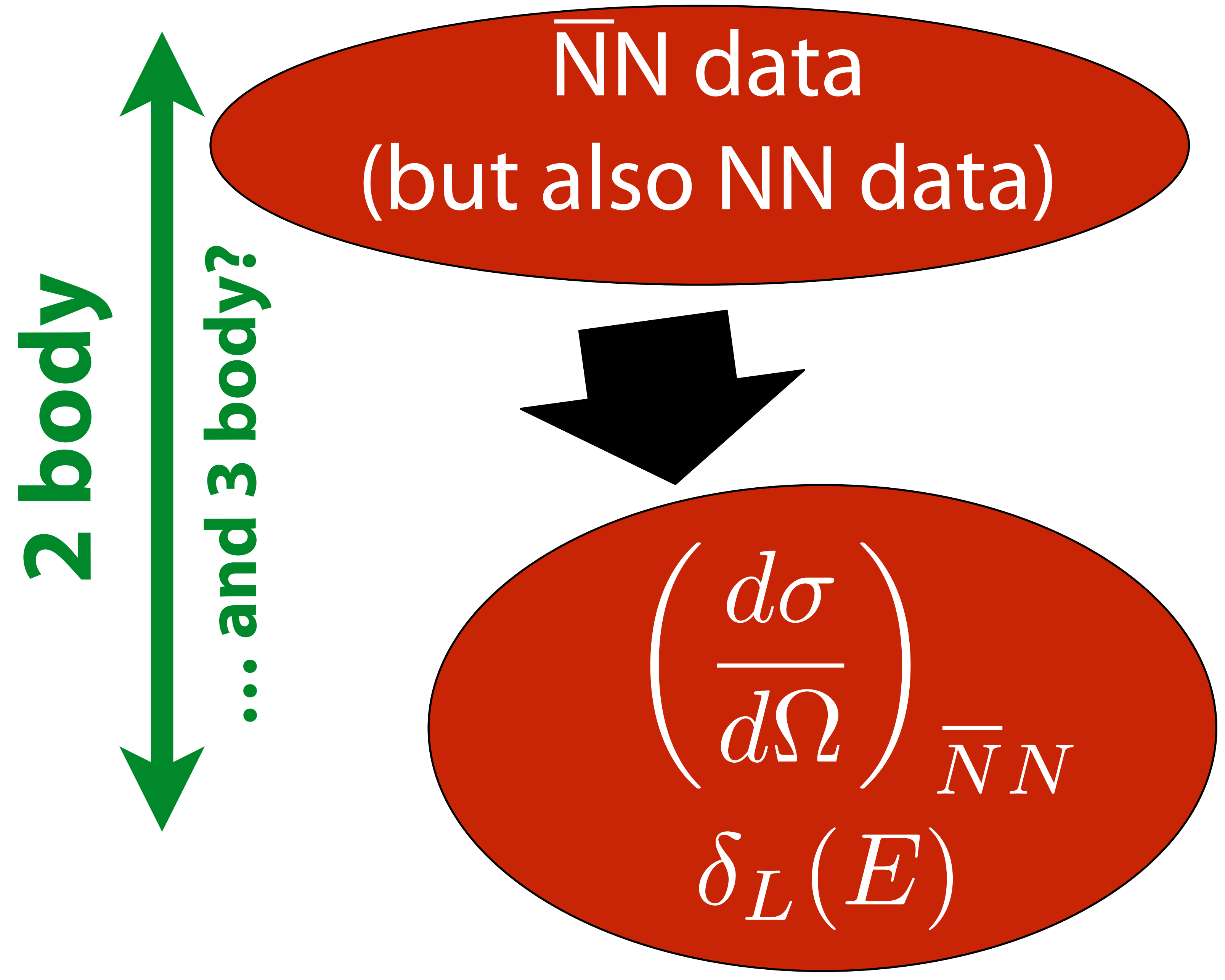
antiproton physics (surface, diffuseness?)

	Microscopic approach	Phenomenological approach
Symmetries	QCD symmetries are consistently respected	QCD symmetries are not respected
Truncation/expansion	Systematic expansion (order by order you know exactly the terms to be included)	Expansion determined by phenomenology (add whatever you need). A lot of freedom
Errors/uncertainties	Theoretical errors	Errors difficult to estimate (bayesian approach)
Many-body forces	Two- and three-body forces belong to the same framework	Two- and three-body forces are not related one to each other
Projectile/target	Both description belongs to the same framework	ad-hoc prescriptions

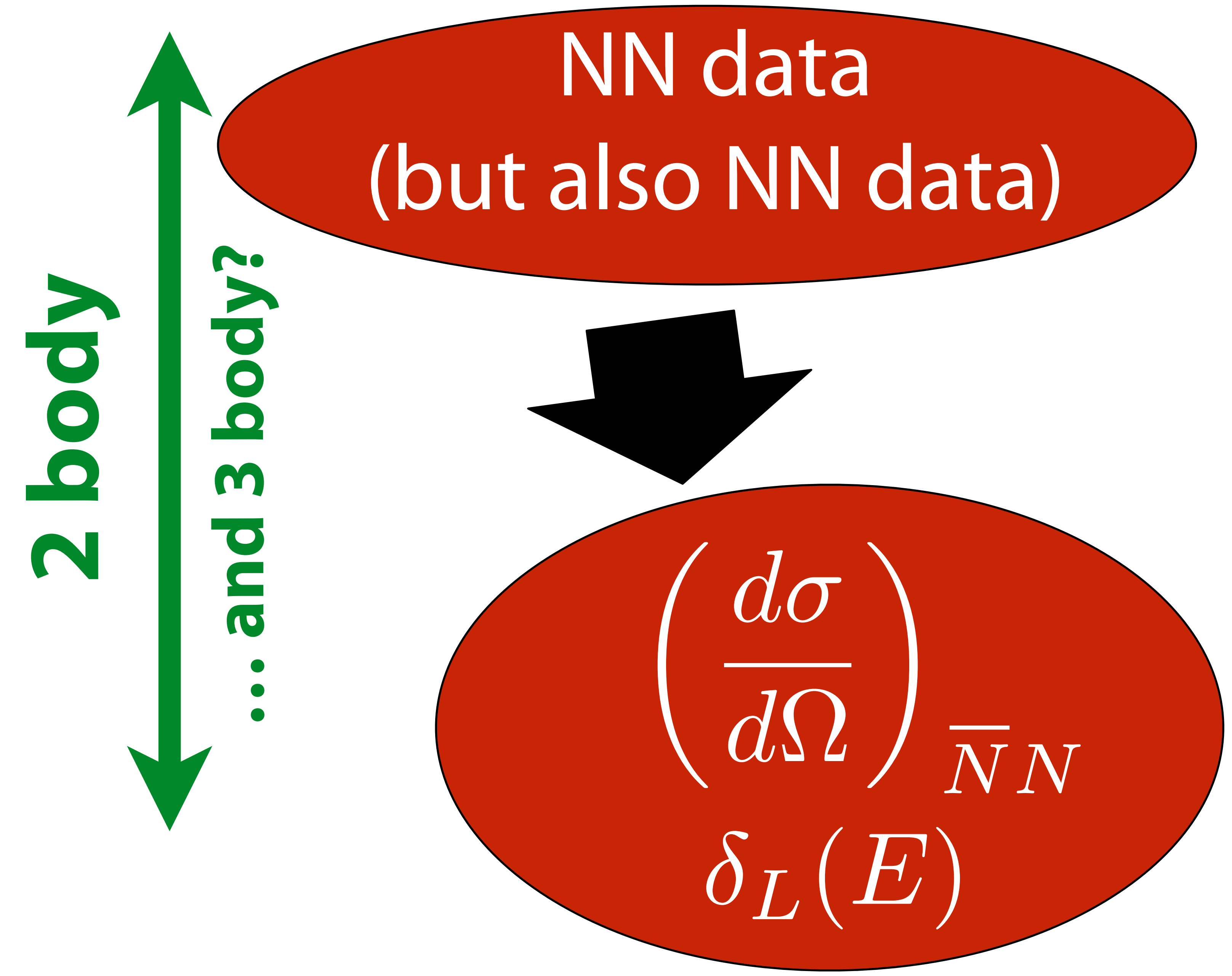
Method



Nuclear reaction theory relies on reducing the many-body problem to a problem with few degrees of freedom:
optical potentials.



Nuclear reaction theory relies on reducing the many-body problem to a problem with few degrees of freedom: **optical potentials**.

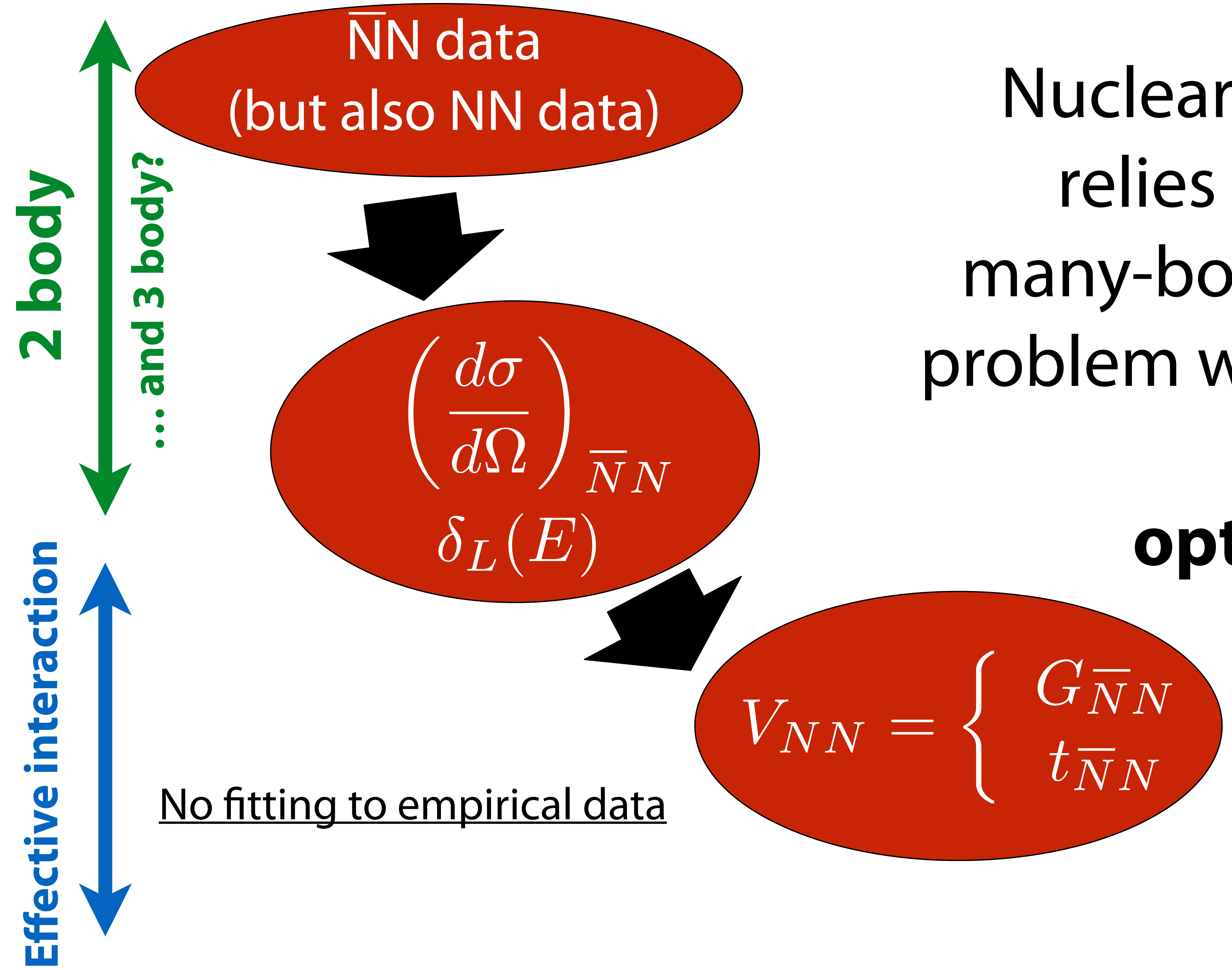


Nuclear reaction theory relies on reducing the many-body problem to a problem with few degrees of freedom: **optical potentials**.

The **optical potential** is used to describe the interaction between a projectile (such as a nucleon or a nucleus) and a target nucleus. This potential has both real and imaginary components.

Real Component It describes the average attractive or repulsive interaction between the projectile and the target nucleus. It is responsible for the coherent elastic scattering, where the projectile is deflected but neither absorbed nor inelastically scattered.

Imaginary Component It is crucial for describing absorption processes. It accounts for the loss of flux from the elastic channel due to various non-elastic processes



Nuclear reaction theory relies on reducing the many-body problem to a problem with few degrees of freedom: **optical potentials**.

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2 body

... and 3 body?

N̄N data (but also NN data)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\bar{N}N}$$

$$\delta_L(E)$$

Effective interaction

No fitting to empirical data

$$V_{NN} = \begin{cases} G_{\bar{N}N} \\ t_{\bar{N}N} \end{cases}$$

N̄ + A

More predictive power where no exp. data are available

$$U_{opt}(\boldsymbol{k}', \boldsymbol{k}, E)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\bar{N}A}$$

$$A_y$$

Model

The general goal when solving the scattering problem of a antinucleon (or a nucleon) from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

$$T = V + VG_0(E)T$$

all two-body interactions

$$V = \sum_{i=1}^{A} v_{0i}$$

Green Function propagator

$$G_0(E) = \frac{1}{E - H_0 + i\epsilon}$$

where

$$H_0 = h_0 + H_A$$

$$H_A \left|\Phi_A\right\rangle = E_A \left|\Phi_A\right\rangle$$ **target Hamiltonian**

$$h_0$$ **kinetic term of the projectile**

for the nucleon-nucleus case see
Vorabbi, Giusti and Finelli, Phys. Rev. C 93, 034619 (2016) and references therein

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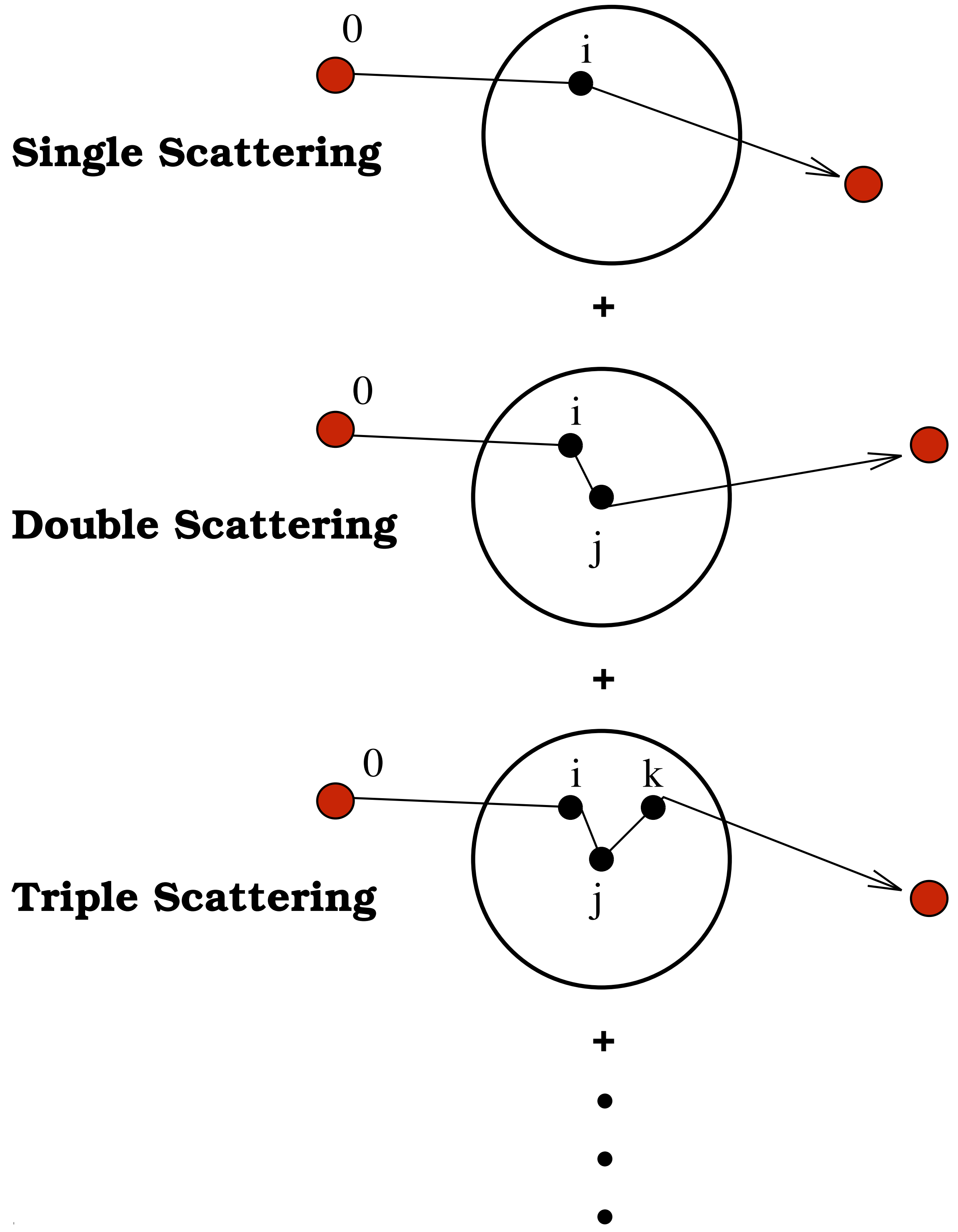

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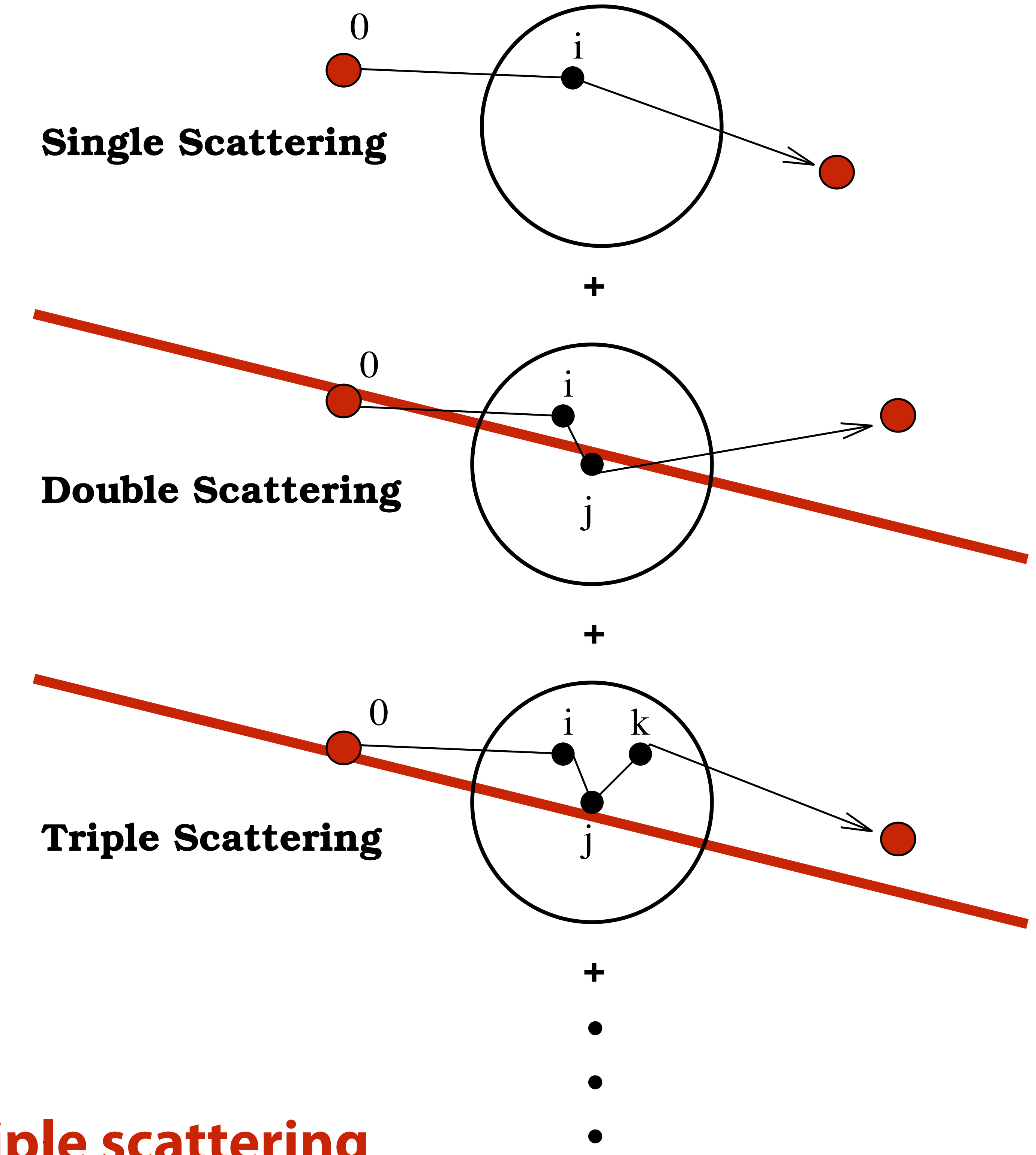
Spectator expansion

two particle interaction dominates the scattering process

$$T = \sum_{i=1} T_{0i}$$



$$T_{0i} = v_{0i} + v_{0i}G_0(E)T,$$

$$\begin{aligned} T_{0i} &= v_{0i} + v_{0i}G_0(E)\sum_j T_{0j} \\ &= v_{0i} + v_{0i}G_0(E)T_{0i} + v_{0i}G_0(E)\sum_{j\neq i} T_{0j} \\ (1 - v_{0i}G_0(E))T_{0i} &= v_{0i} + v_{0i}G_0(E)\sum_{j\neq i} T_{0j} \end{aligned}$$

$$T_{0i} = t_{0i} + t_{0i}G_0(E)\sum_{j\neq i} T_{0j}.$$

Watson multiple scattering

The general goal when solving the scattering problem of a antinucleon (or a nucleon) from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

$$T = V + VG_0(E)T$$

Let's introduce the **optical potential U**

$$T = U + UG_0(E)PT$$

$$U = V + VG_0(E)QU$$

$$P + Q = 1$$

$$[G_0, P] = 0$$

In the case of elastic scattering,

P projects onto the elastic channel

$$P = \frac{|\Phi_A\rangle \langle\Phi_A|}{\langle\Phi_A|\Phi_A\rangle}$$

The general goal when solving the scattering problem of a antinucleon (or a nucleon) from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

$$T = V + VG_0(E)T$$

transition amplitude T for elastic scattering

$$T_{\mathrm{el}} = PUP + PUPG_0(E)T_{\mathrm{el}}$$

we need to calculate *PUP*

expressions for U are derived such that PUP can be calculated accurately without having to solve the complete many-body problem

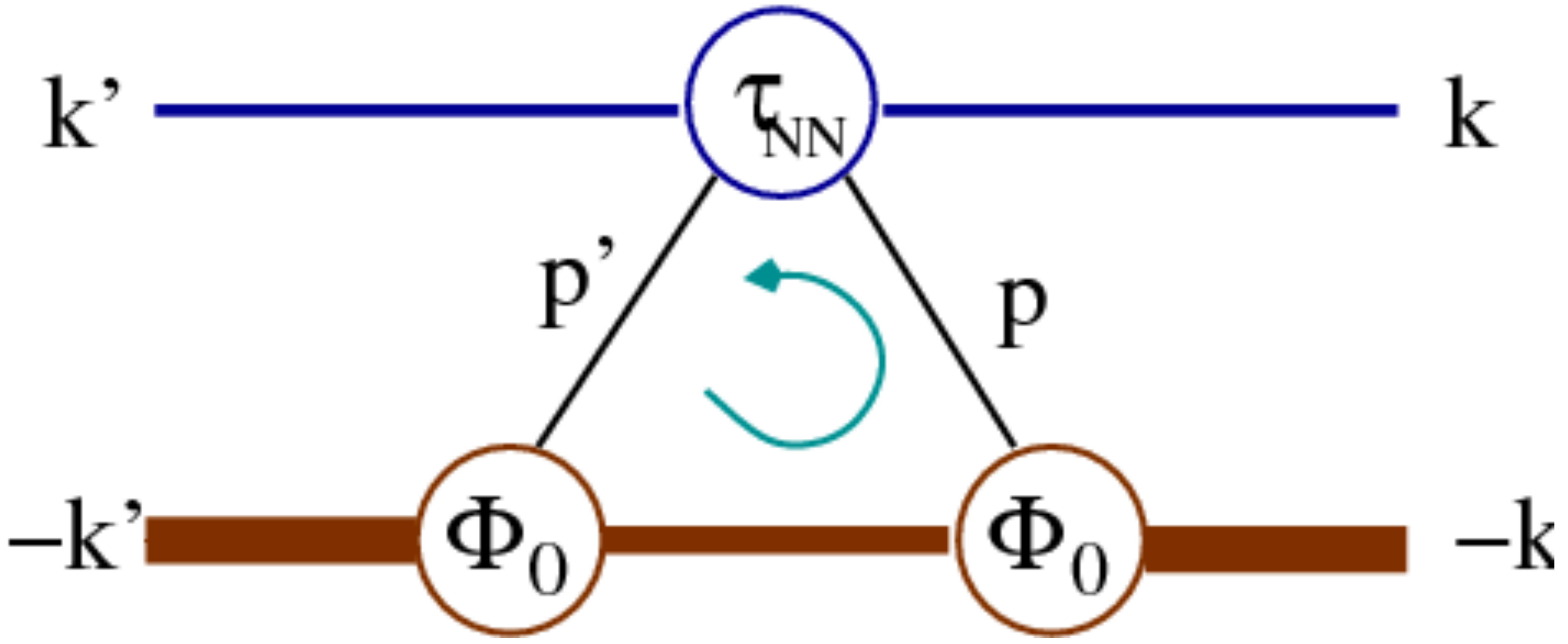
The general goal when solving the scattering problem of a antinucleon (or a nucleon) from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

$$T_{\rm el} = PUP + PUPG_0(E)T_{\rm el}$$

$$U = \sum_{i=1}^{A} \tau_i + \sum_{i,j\neq i}^{A} \tau_{ij} + \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{ijk} + \cdots$$

$$\hat{\tau}_i = v_{0i} + v_{0i}G_0(E)\hat{\tau}_i$$
$$= \tau_{0i} + \tau_{0i}G_0(E)P\hat{\tau}_{0i}.$$

$$\langle\Phi_A|\tau_i|\Phi_A\rangle = \langle\Phi_A|\hat{\tau}_i|\Phi_A\rangle - \langle\Phi_A|\hat{\tau}_i|\Phi_A\rangle \times \frac{1}{(E-E_A)-h_0+i\epsilon}\langle\Phi_A|\tau_i|\Phi_A\rangle$$



The general goal when solving the scattering problem of a antinucleon (or a nucleon) from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

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$$\hat{\tau}_i = v_{0i} + v_{0i}G_0(E)\hat{\tau}_i$$
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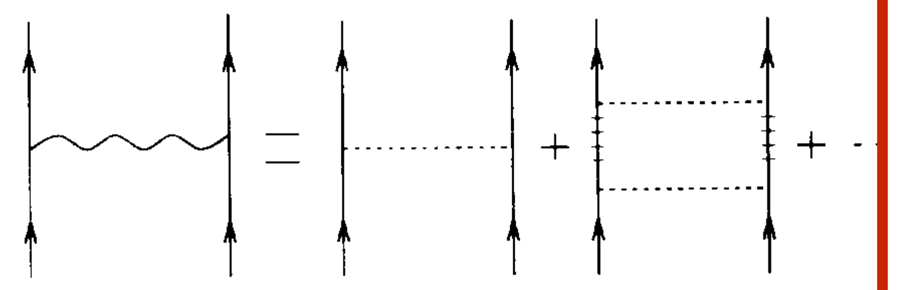
$$\langle\Phi_A|\tau_i|\Phi_A\rangle = \langle\Phi_A|\hat{\tau}_i|\Phi_A\rangle - \langle\Phi_A|\hat{\tau}_i|\Phi_A\rangle \times \frac{1}{(E-E_A)-h_0+i\epsilon}\langle\Phi_A|\tau_i|\Phi_A\rangle$$

Expanding the propagator $G_i(E) = \frac{1}{(E-E^i)-h_0-h_i-W_i+i\epsilon}$

$$\hat{\tau}_i = v_{0i} + v_{0i}G_i(E)\hat{\tau}_i = t_{0i} + t_{0i}g_iW_iG_i(E)\hat{\tau}_i$$

free t matrix

$$t_{0i} = v_{0i} + v_{0i}g_it_{0i}$$



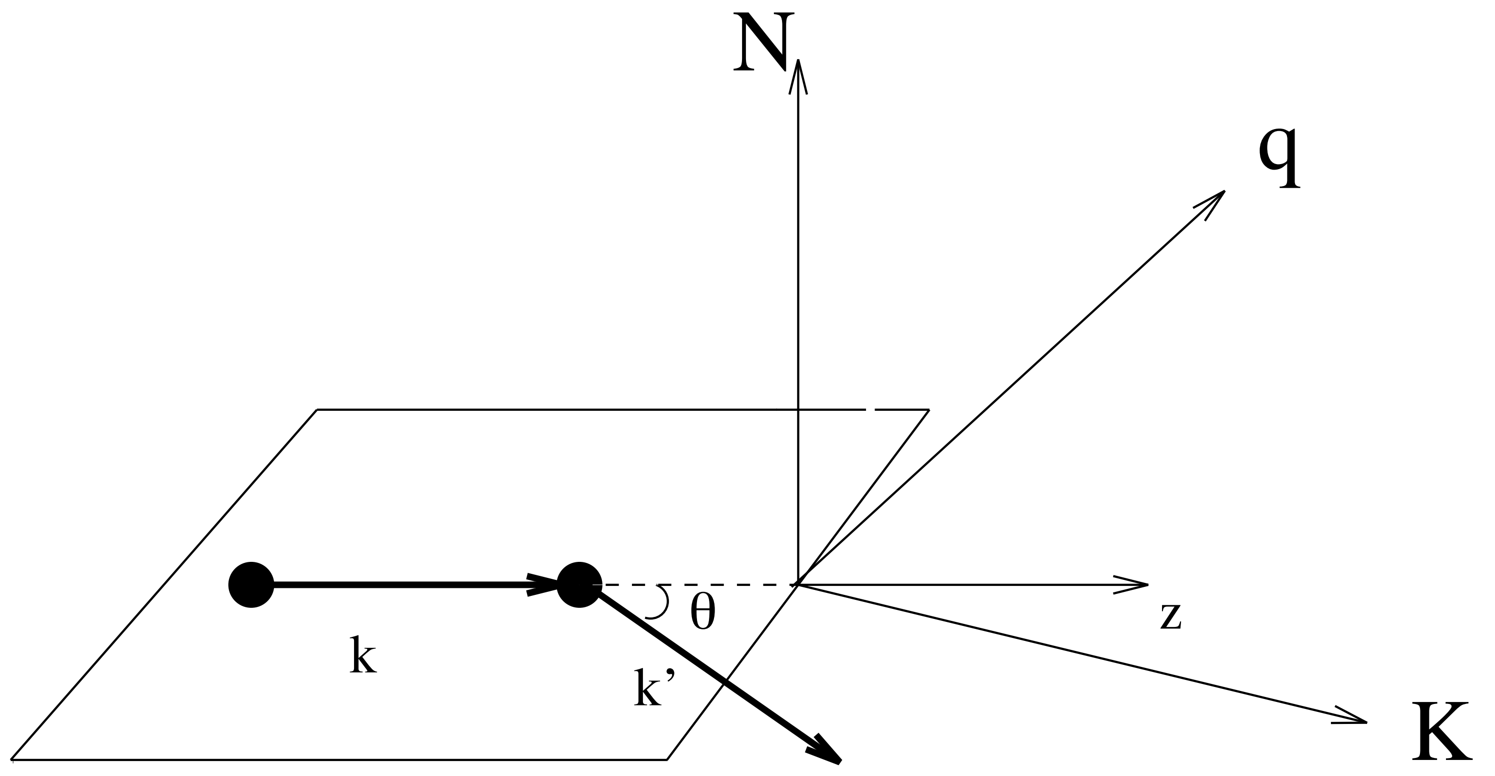
IMPULSE APPROXIMATION $\hat{\tau}_i \approx t_{0i}$

First-order optical potential

Kerman, McManus and Thaler, Ann. Phys. **8** (1959) 551 and many others

$$\hat{U}(\boldsymbol{k}', \boldsymbol{k}; \omega) = (A-1) \langle \boldsymbol{k}', \Phi_A | t(\omega) | \boldsymbol{k}, \Phi_A \rangle$$

$$\hat{U}(\boldsymbol{q}, \boldsymbol{K}; \omega) = \hat{U}^c(\boldsymbol{q}, \boldsymbol{K}; \omega) + \frac{i}{2} \boldsymbol{\sigma} \cdot \boldsymbol{q} \times \boldsymbol{K}\, \hat{U}^{ls}(\boldsymbol{q}, \boldsymbol{K}; \omega)$$



Central component

$$U_c^\alpha(\boldsymbol{q}, \boldsymbol{K}; E) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\alpha N}^c \left[\boldsymbol{q}, \frac{1}{2} \left(\frac{A+1}{A} \boldsymbol{K} + \sqrt{\frac{A-1}{A}} \boldsymbol{P} \right); E \right] \times \bar{\rho}_N \left(\boldsymbol{P} + \frac{1}{2} \sqrt{\frac{A-1}{A}} \boldsymbol{q}, \boldsymbol{P} - \frac{1}{2} \sqrt{\frac{A-1}{A}} \boldsymbol{q} \right),$$

$$\boldsymbol{q} \equiv \boldsymbol{k}' - \boldsymbol{k}\,, \qquad \boldsymbol{K} \equiv \frac{1}{2}(\boldsymbol{k}' + \boldsymbol{k})$$

Spin-orbit component

$$U_{ls}^\alpha(\boldsymbol{q}, \boldsymbol{K}; E) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\alpha N}^{ls} \left[\boldsymbol{q}, \frac{1}{2} \left(\frac{A+1}{A} \boldsymbol{K} + \sqrt{\frac{A-1}{A}} \boldsymbol{P} \right); E \right] \times \bar{\rho}_N \left(\boldsymbol{P} + \frac{1}{2} \sqrt{\frac{A-1}{A}} \boldsymbol{q}, \boldsymbol{P} - \frac{1}{2} \sqrt{\frac{A-1}{A}} \boldsymbol{q} \right).$$

Transition matrix

$$M(\boldsymbol{\kappa}', \boldsymbol{\kappa}, \omega) = \langle \boldsymbol{\kappa}' | M(\omega) | \boldsymbol{\kappa} \rangle = -4\pi^2 \mu \langle \boldsymbol{\kappa}' | t(\omega) | \boldsymbol{\kappa} \rangle$$

$$M = a + c(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{n}} + m(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{n}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{n}}) + (g+h)(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{l}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{l}}) + (g-h)(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{m}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{m}})$$

$$c_{\bar{p}N} = \frac{i}{f_{pN}\pi^2} \sum_{L=1}^{\infty} P_L^1(\cos\phi) \left[\left(\frac{2L+3}{L+1} \right) M_{LL}^{L+1,S=1} - \left(\frac{2L+1}{L(L+1)} \right) M_{LL}^{L,S=1} - \left(\frac{2L-1}{L} \right) M_{LL}^{L-1,S=1} \right]$$

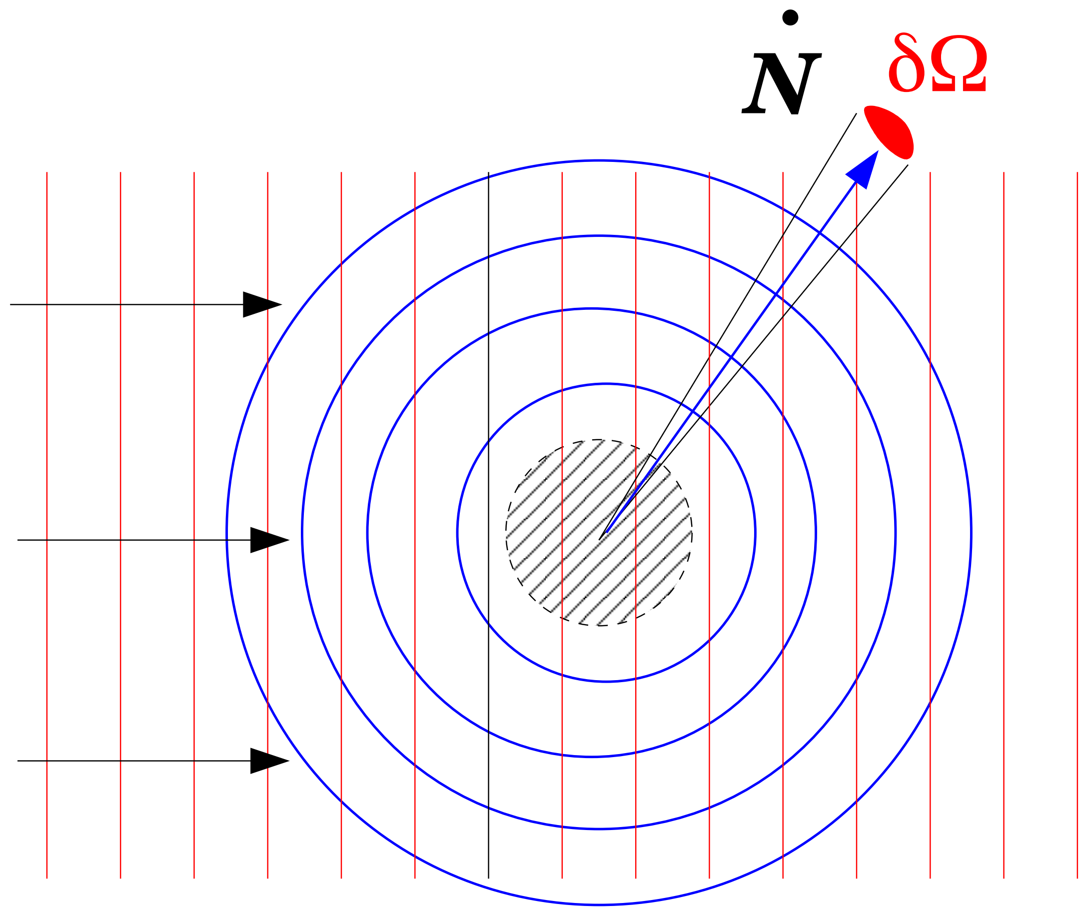
$$a_{\bar{p}N} = \frac{1}{f_{pN}\pi^2} \sum_{L=0}^{\infty} P_L(\cos\phi) \left[(2L+1) M_{LL}^{L,S=0} + (2L+1) M_{LL}^{L,S=1} + (2L+3) M_{LL}^{L+1,S=1} + (2L-1) M_{LL}^{L-1,S=1} \right]$$

The only relevant components for 0^+ nuclei, for $1/2^+$ more amplitudes must be included

Scattering observables

Spin-flip amplitude

$$M(k_0,\theta) = A(k_0,\theta) + \boldsymbol{\sigma}\cdot\hat{\mathbf{N}}\,C(k_0,\theta)$$



$$F_{LJ}(k_0) = -\frac{A}{A-1}4\pi^2\mu(k_0)\hat{T}_{LJ}(k_0,k_0;E)$$

$$\dot{N} \sim \left(\frac{d\sigma}{d\Omega}\right)\delta\Omega$$

$$A(\theta) = \frac{1}{2\pi^2}\sum_{L=0}^{\infty}\left[(L+1)F_L^{+}(k_0) + LF_L^{-}(k_0)\right]P_L(\cos\theta)$$

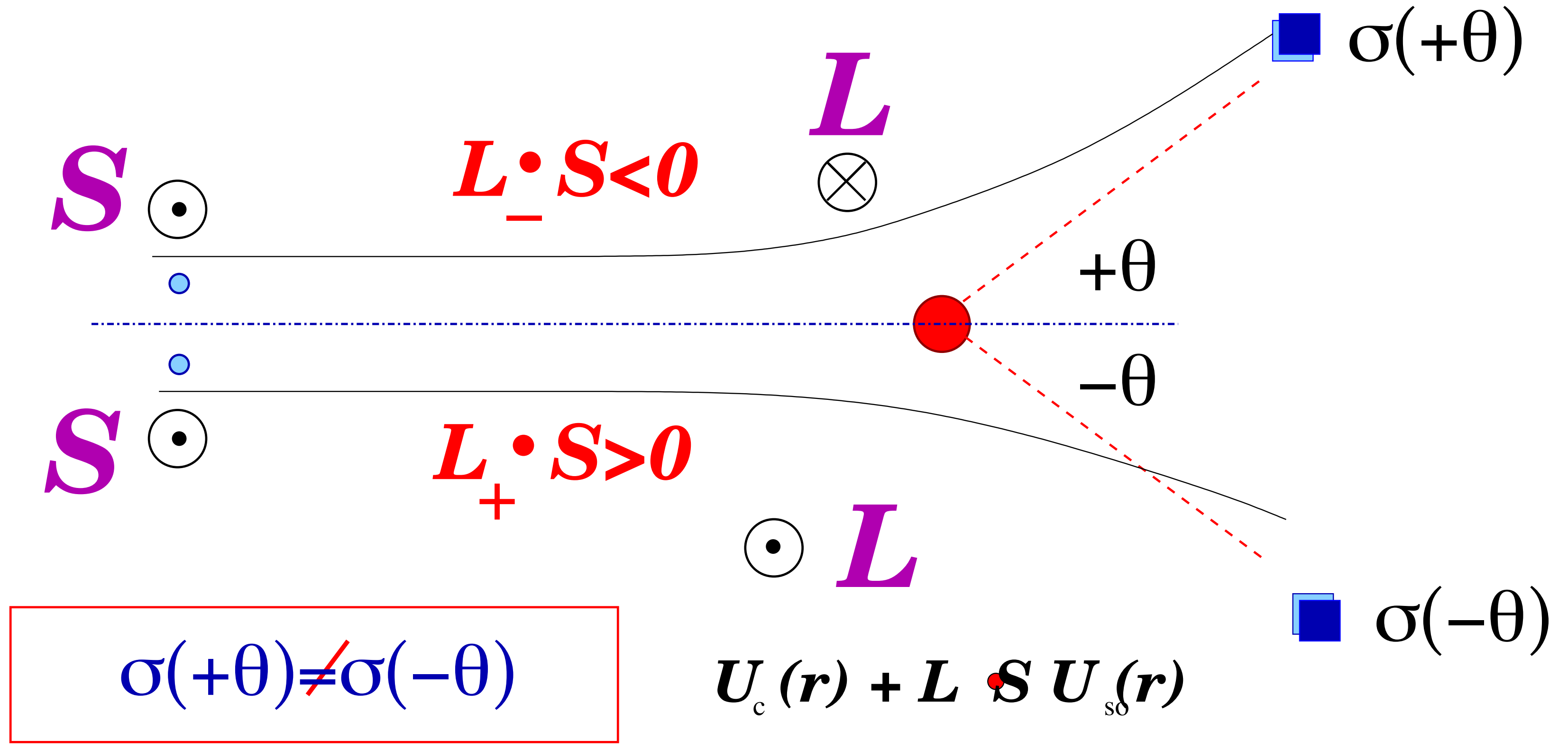
$$C(\theta) = \frac{i}{2\pi^2}\sum_{L=1}^{\infty}\left[F_L^{+}(k_0) - F_L^{-}(k_0)\right]P_L(\cos\theta)$$

Differential cross section

$$\frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |C(\theta)|^2$$

Analyzing power

$$A_y(\theta) = \frac{2\mathrm{Re}[A^*(\theta)\,C(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$



Spin rotation

$$Q(\theta) = \frac{2\mathrm{Im}[A(\theta)\,C^*(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

Rotation of the spin vector in the scattering plane, i.e. protons polarised along the **+*x*** axis have a finite probability of having the spin polarised along the **± *z*** axis after the collision

Densities

Ab initio no core shell model

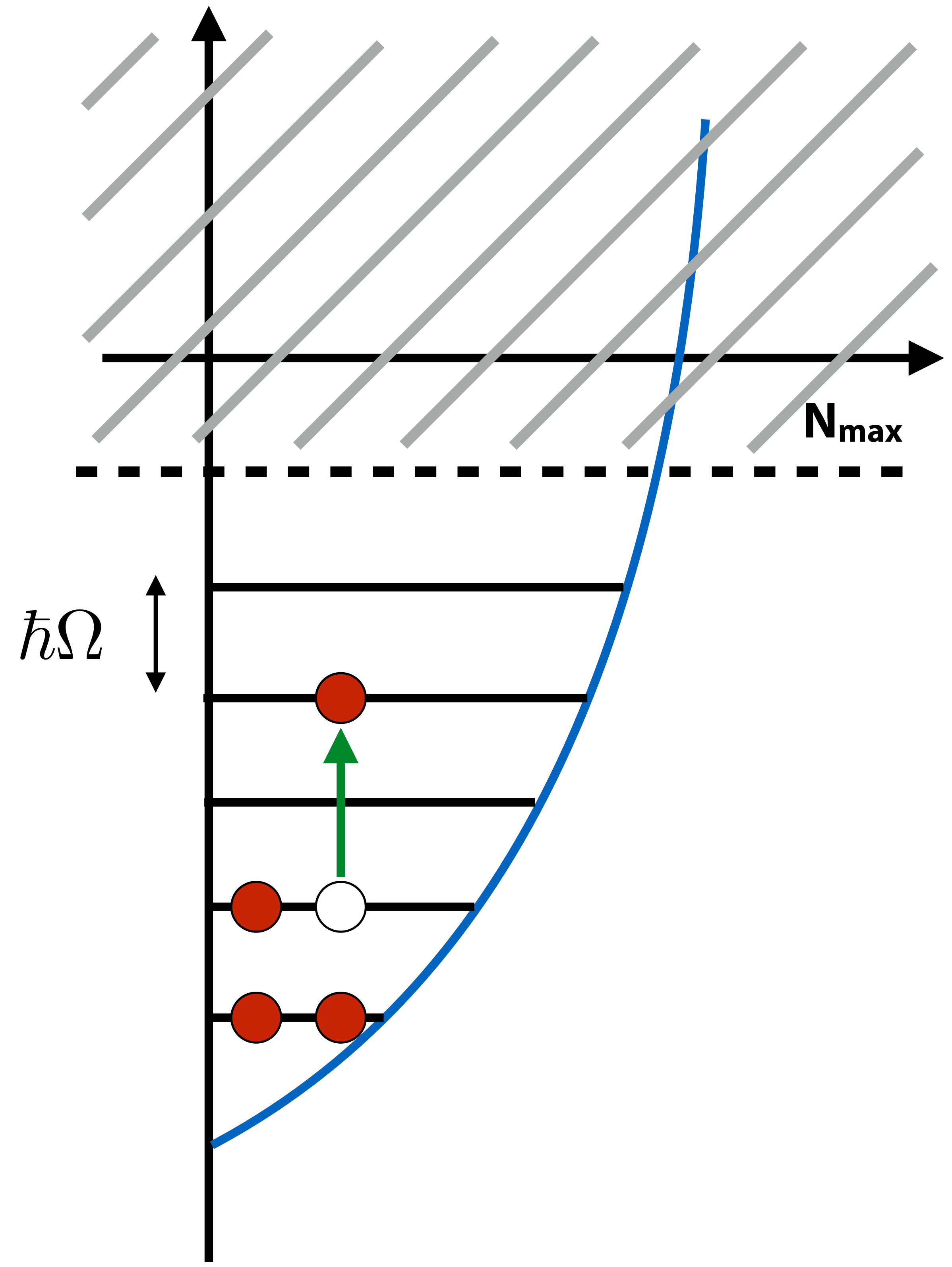
The basic idea of the NCSM is simply to treat **all A nucleons in a nucleus as active**: write down the Schrödinger equation for A nucleons and then solve it numerically.

Ab initio no core shell model

Bruce R. Barrett[a], Petr Navrátil[b], James P. Vary[c,*]

Progress in Particle and Nuclear Physics 69 (2013) 131–181

This approach avoids essentially all of the difficulties of the perturbative approaches (like problems related to excitations of nucleons from the core).



Being a non-perturbative approach, there are no difficulties related to convergence of such an expansion. It may also be formulated in terms of an **intrinsic Hamiltonian**, so as to **avoid spurious COM motion**.

Problems:

(1) need for **larger basis spaces**

(2) need for **effective many-body forces**, in order to treat all of the complexity of the excited-states.

Ab initio no core shell model

- In the ab initio no core shell model we consider a system of A point-like non-relativistic nucleons that interact by realistic two- or two- plus three-nucleon interactions.
- We employ NN potentials that fit nucleon–nucleon phase shifts with high precision up to a certain energy, typically up to 350 MeV.
- In the NCSM, **all the nucleons are considered active; there is no inert core like** in standard shell model calculations. Hence, the "no core" in the name of the approach.

$$H_A = T_{\rm rel} + \mathcal{V} = \frac{1}{A}\sum_{i<j}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{\mathrm{NN},ij} + \sum_{i<j<k}^{A} V_{\mathrm{NNN},ijk},$$

m is the nucleon mass

$V_{NN,ij}$ is the NN interaction

$V_{NNN,ijk}$ is the three-nucleon interaction

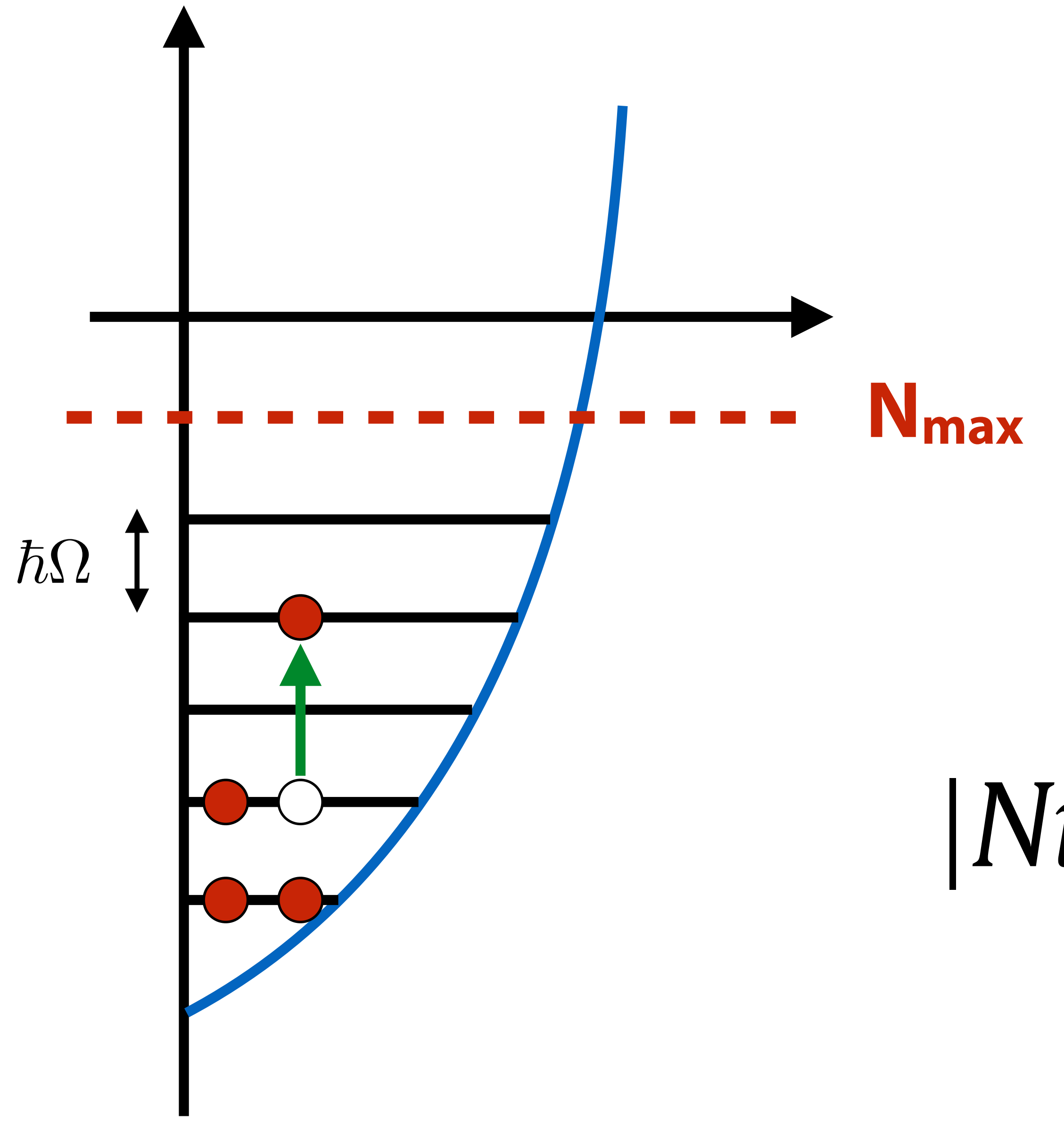
Progress in Particle and Nuclear Physics 69 (2013) 131–181

Ab initio no core shell model: the basis

NCSM uses the harmonic-oscillator (HO) basis, truncated by a chosen **maximal total HO energy** (N_{max}) of the A-nucleon system. The reason behind the choice of the HO basis is the fact that this is the only basis that allows for the use of single-nucleon coordinates without violating the translational invariance of the system.

$$\varphi_{nlm}(\vec{r}; b) = R_{nl}(r; b) Y_{lm}(\hat{r}),$$

As a downside, one has to face the consequences of the incorrect asymptotic behavior of the HO basis.



Jacobi's coordinate

$$|(nlsjt;\, \mathcal{N}\mathcal{L}\mathcal{J})JT\rangle.$$

antisymmetrized states

$$|NiJT\rangle = \sum \langle nlsjt;\, \mathcal{N}\mathcal{L}\mathcal{J}||NiJT\rangle|(nlsjt;\, \mathcal{N}\mathcal{L}\mathcal{J})JT\rangle$$

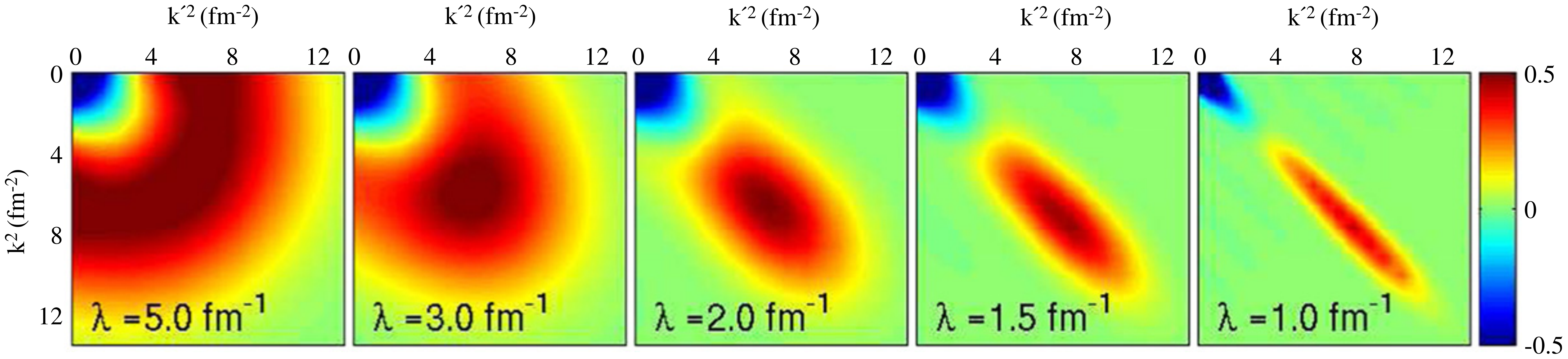
Slater determinants for A greater than 3

Progress in Particle and Nuclear Physics 69 (2013) 131–181

Ab initio no core shell model: the interaction

Ab initio NCSM calculations uses a truncated HO basis but the nuclear interactions act in the full space. As long as one uses soft potentials, such as the V_{lowk} and SRG NN, convergent NCSM results can be obtained (the similarity transformation softens the interactions and generates effective operators for all observables while preserving all experimental quantities in the low-energy domain).

The situation is different when standard NN potentials that generate strong short-range correlations are used.



$$H_\lambda = U_\lambda H_{\lambda=\infty} U_\lambda^\dagger$$

$$\frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[[T_{\rm rel}, H_\lambda], H_\lambda]$$

The derived "effective" interactions still act among all A nucleons and preserve all the symmetries of the initial or "bare" NN +NNN interactions.

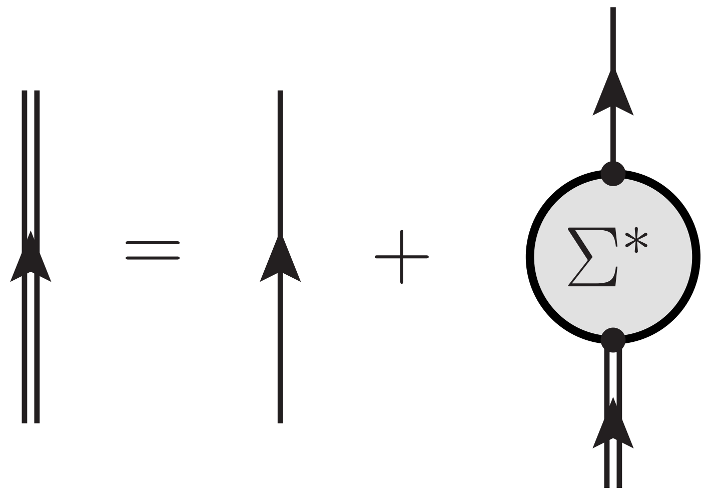
Progress in Particle and Nuclear Physics 69 (2013) 131–181

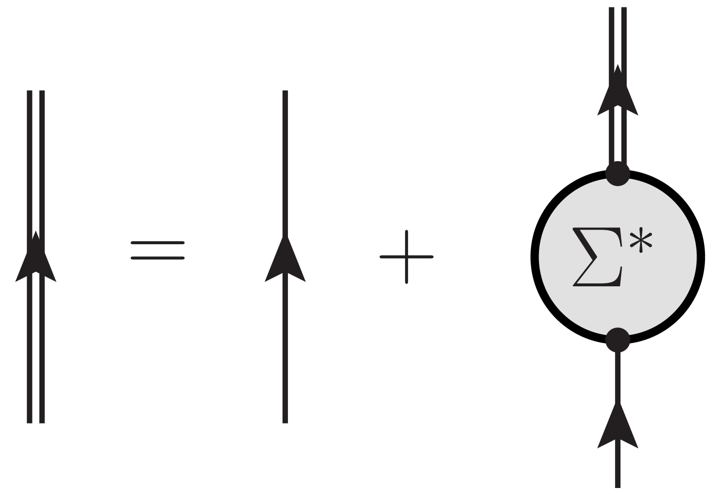
An alternative approach: *Self-consistent* Green's functions

$$G = G^{(0)} + G^{(0)}\,\Sigma\,G$$ Dyson equation

unperturbed Green's function

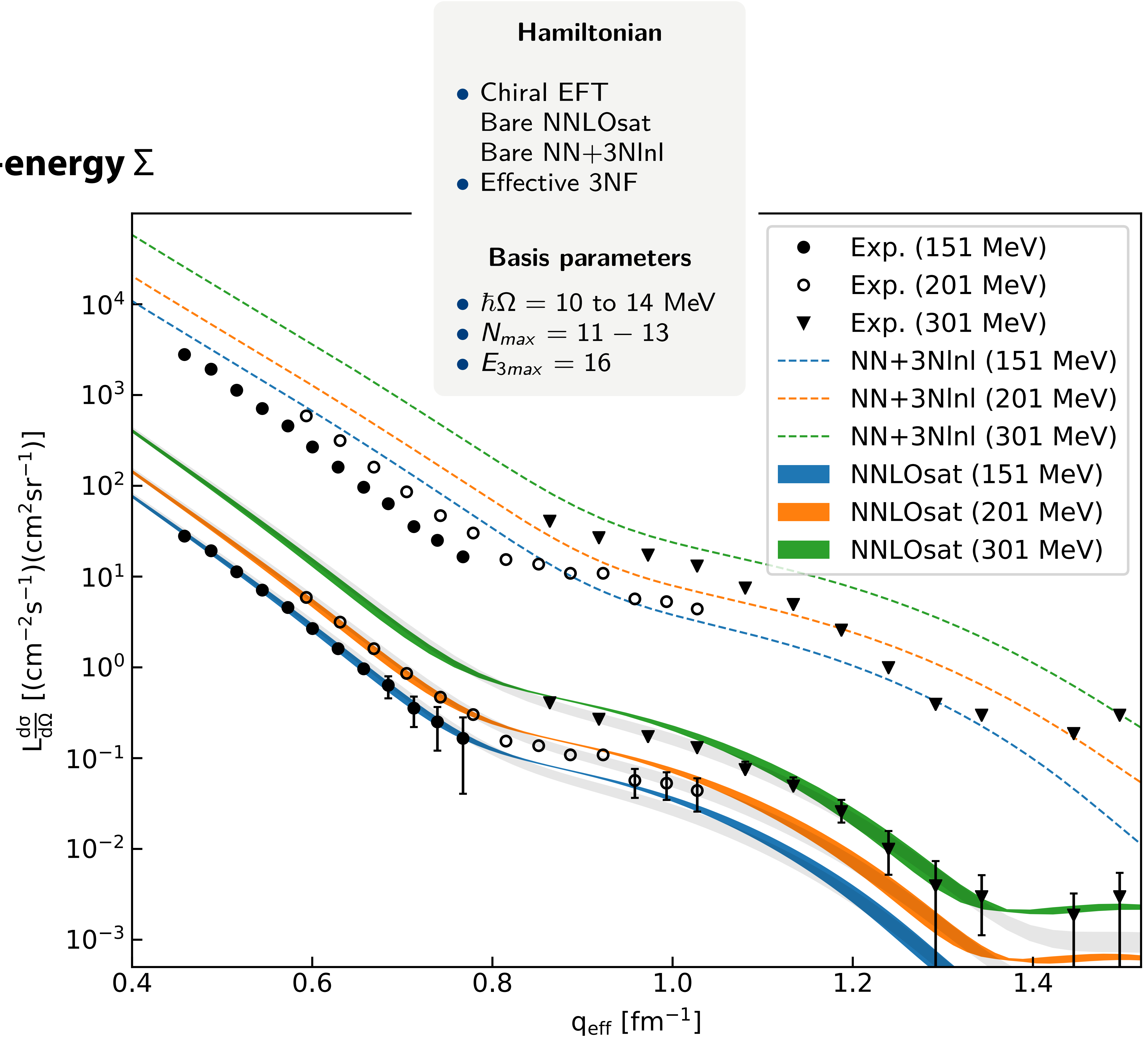
many-body effects contained in the **self-energy** Σ





Through the one-body Green's function:

- Ground-state energy
- One-body observables (radii, densities…)
- Spectroscopy of the $A \pm 1$-body systems
- Elastic nucleon-nucleus scattering

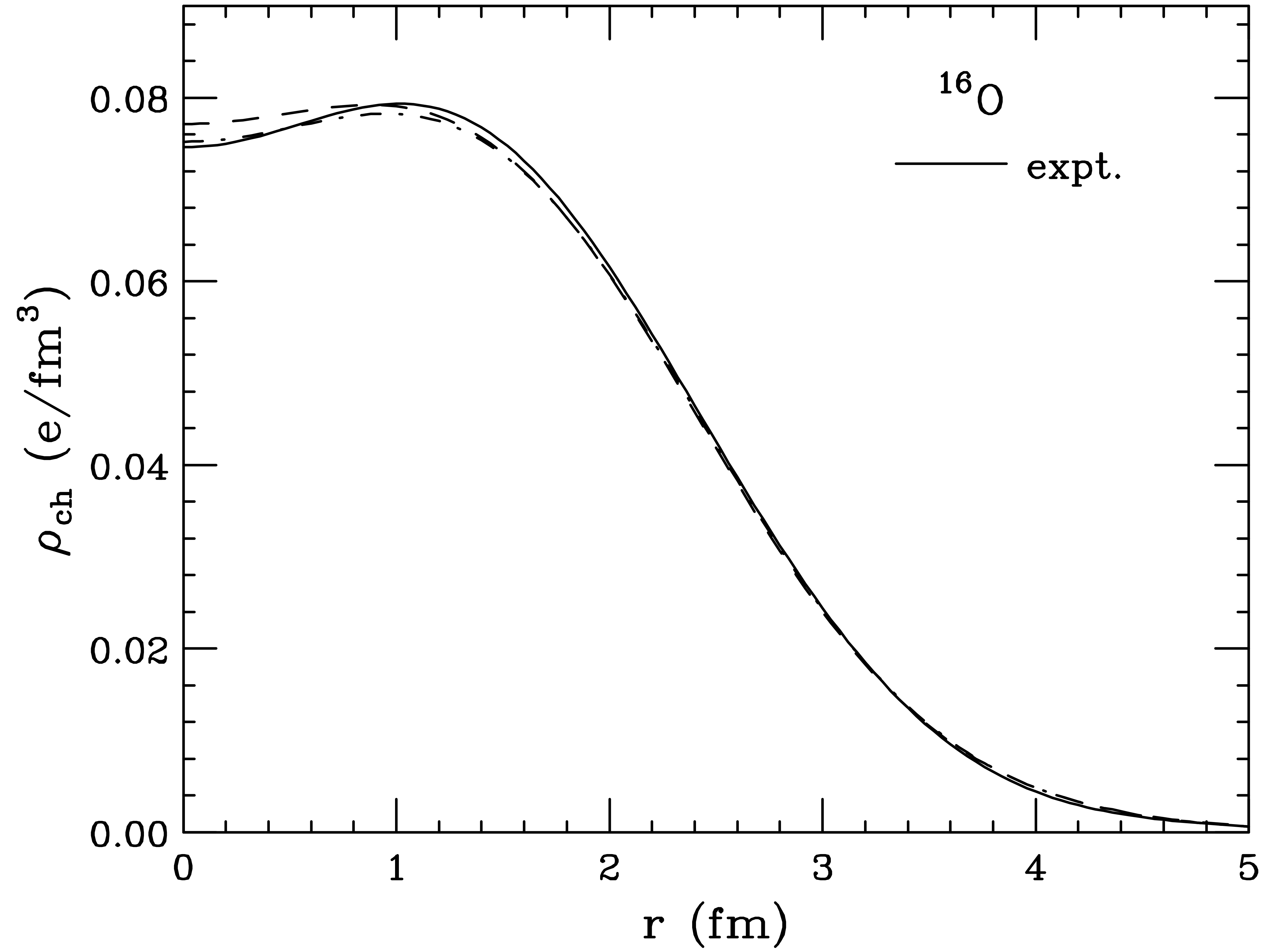


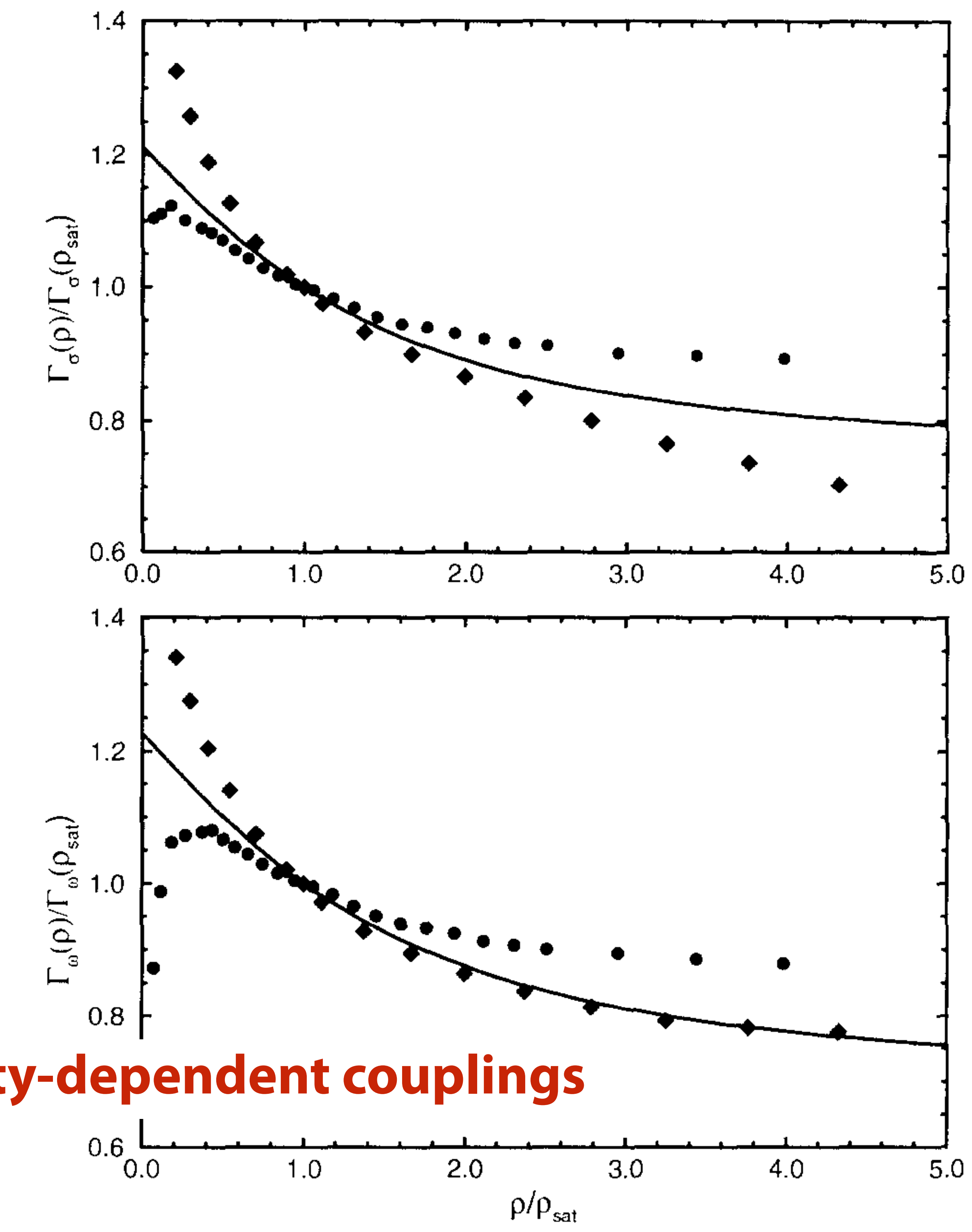
Arthuis, Barbieri Vorabbi, Somà and Finelli,

Ab Initio Computation of Charge Densities for Sn and Xe Isotopes

For heavy systems, a simpler approach: matter densities from DFT

Typel and Wolter , Nuc. Phys. A **656** (1999) 331





Density-dependent couplings

$$\mathcal{L} = \bar{\Psi}\left[\gamma^\mu\left(i\partial_\mu - \Gamma_\omega A_\mu^{(\omega)} - \Gamma_\varrho \frac{\boldsymbol{\tau}}{2}\cdot \boldsymbol{A}_\mu^{(\varrho)} - e\frac{1+\tau_3}{2}A_\mu^{(\gamma)}\right) - (m - \Gamma_\sigma \phi)\right]\Psi$$
$$+\frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2 - \frac{1}{2}F_{\mu\nu}^{(\omega)}F^{(\omega)\mu\nu} + m_\omega^2 A_\mu^{(\omega)}A^{(\omega)\mu}\right.$$
$$\left.-\frac{1}{2}\boldsymbol{F}_{\mu\nu}^{(\varrho)}\cdot\boldsymbol{F}^{(\varrho)\mu\nu} + m_\varrho^2 \boldsymbol{A}_\mu^{(\varrho)}\cdot\boldsymbol{A}^{(\varrho)\mu} - \frac{1}{2}F_{\mu\nu}^{(\gamma)}F^{(\gamma)\mu\nu}\right]$$

DDME1 parametrization

T. Nikšić, D. Vretenar, P. Finelli and P. Ring
Phys. Rev. C **66** (2002) 024306

$\bar{N}N$ potentials

basic features of $\bar{\mathrm{N}}$N potentials

In the limit where the neutron-to-proton mass difference can be neglected, as well as Coulomb corrections, the $\bar{\mathrm{N}}$N system obeys isospin symmetry:

- antiproton–neutron is pure isospin I =1
- while $\bar{\mathrm{p}}$p and $\bar{\mathrm{n}}$n, with $I_3 = 0$, are combinations of I =1 and I =0

Antinucleon–nucleon interaction at low energy: scattering and protonium

Eberhard Klempt[a], Franco Bradamante[b], Anna Martin[b], Jean-Marc Richard[c,d,*]

Physics Reports 368 (2002) 119

$$|\bar{\mathrm{p}}\mathrm{p}\rangle = \frac{|I=1\rangle + |I=0\rangle}{\sqrt{2}}, \quad |\bar{\mathrm{n}}\mathrm{n}\rangle = \frac{|I=1\rangle - |I=0\rangle}{\sqrt{2}}$$

so that the elastic and charge-exchange amplitudes are given by

$$\mathscr{T}(\bar{\mathrm{p}}\mathrm{p} \to \bar{\mathrm{p}}\mathrm{p}) = \tfrac{1}{2}(\mathscr{T}^{1}_{\bar{\mathrm{N}}\mathrm{N}} + \mathscr{T}^{0}_{\bar{\mathrm{N}}\mathrm{N}}), \quad \mathscr{T}(\bar{\mathrm{p}}\mathrm{p} \to \bar{\mathrm{n}}\mathrm{n}) = \tfrac{1}{4}(\mathscr{T}^{1}_{\bar{\mathrm{N}}\mathrm{N}} - \mathscr{T}^{0}_{\bar{\mathrm{N}}\mathrm{N}})$$

G-parity

G-parity is a multiplicative quantum number that results from the generalization of C-parity to multiplets of particles.

G-parity is a combination of charge conjugation and a π rad rotation around the 2nd axis of isospin space. Weak and electromagnetic interactions are not invariant under G-parity.

$$\mathcal{G} = \mathcal{C}\, e^{(i\pi I_2)}$$

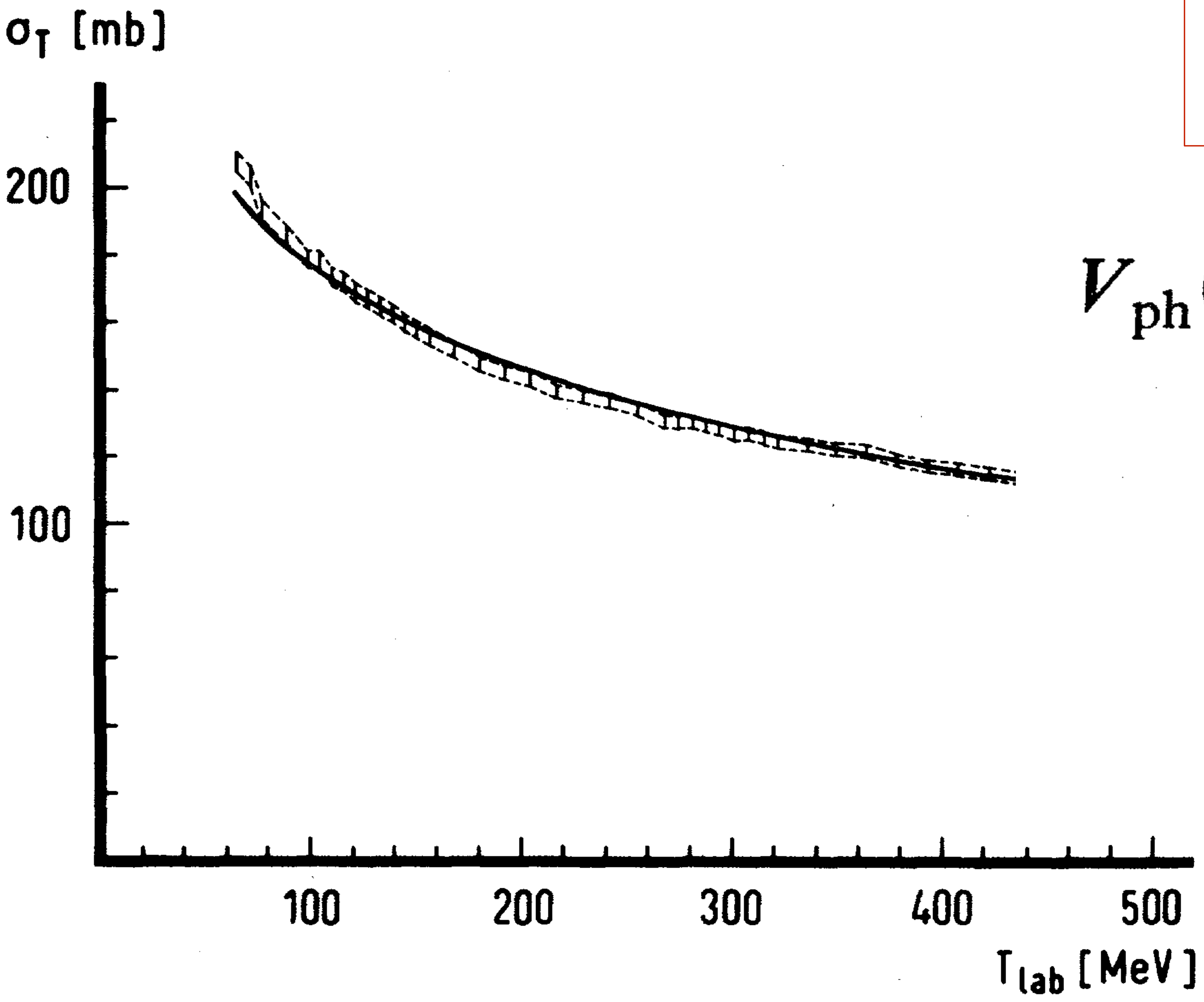
a little bit of history

The $\bar{N}N$ potential is generated by a G-parity-transformed Nijmegen model-D potential plus a phenomenological short-range potential:

Antinucleon-nucleon potential

P. H. Timmers, W. A. van der Sanden, and J. J. de Swart
nstitute for Theoretical Physics, University of Nijmegen, Nijmegen, The Netherlands

PRD 29 (1984) 1928



$$V_{\mathrm{ph}}(r) = \Big[V_C + V_{SS}\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T S_{12} m_e r + V_{SO}\vec{\mathbf{L}}\cdot\vec{\mathbf{S}}\frac{1}{m_e{}^2 r}\frac{d}{dr}\Big] V_{WS}(r)$$

$$V_{WS}(r) = \frac{1}{1+\exp(m_e r)}$$

Measurement	Incoming $\bar{p}$ momentum (MeV/c)	Experiment
integrated cross sections		
$\sigma_{tot}(\bar{p}p)$	222-599 (74 momenta)	PS172
	181,219,239,261,287,505,590	PS173
$\sigma_{ann}(\bar{p}p)$	177-588 (53 momenta)	PS173
	38-174 (14 momenta)	PS201
$\bar{p}p$ elastic scattering		
ρ = Re $f(0)$/Im $f(0)$	233,272,550,757,1077	PS172
	181,219,239,261,287,505,590	PS173
$d\sigma/d\Omega$	679-1550 (13 momenta)	PS172
	181,287,505,590	PS173
	439,544,697	PS198
A_{0n}	497-1550 (15 momenta)	PS172
	439,544,697	PS173
D_{0n0n}	679-1501 (10 momenta)	PS172
$\bar{p}p$ charge exchange		
$d\sigma/d\Omega$	181-595 (several momenta)	PS173
	546,656,693,767,875,1083,1186,1287	PS199
	601.5,1202	PS206
A_{0n}	546,656,767,875,979,1083,1186,1287	PS199
D_{0n0n}	546,875	PS199
K_{n00n}	875	PS199

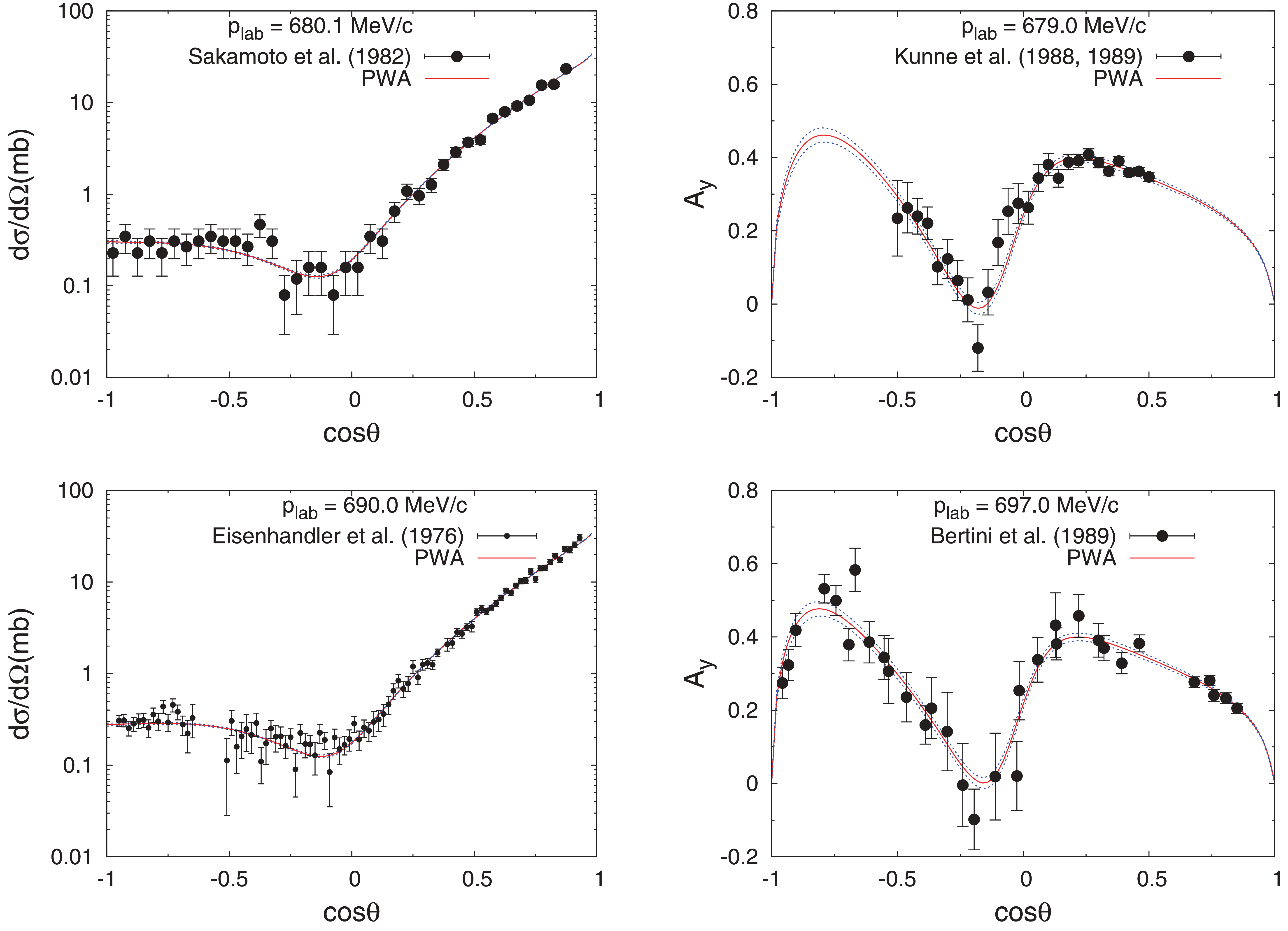
Haidenbauer, Antiproton-proton interaction from chiral effective field theory

the most recent Plane Wave Analysis

Energy-dependent partial-wave analysis of all antiproton-proton scattering data below 925 MeV/*c*

Daren Zhou and Rob G. E. Timmermans
KVI, Theory Group, University of Groningen, Zernikelaan 25, NL-9747 AA Groningen, The Netherlands
PHYSICAL REVIEW C **86**, 044003 (2012)

- Energy-dependent partial-wave analysis of all antiproton-proton elastic ($\bar{p}p \rightarrow \bar{p}p$) and charge-exchange ($\bar{p}p \rightarrow \bar{n}n$) scattering data below 925 MeV/c antiproton laboratory momentum.
- The relevant long-range parts of the electromagnetic and the one- and two-pion exchange interactions are included exactly, where the short-range interactions, including the coupling to the mesonic annihilation channels, are parametrized by a complex boundary condition at a radius of r = 1.2 fm.
- They concluded that chiral effective field theory provides an excellent long-range antinucleon-nucleon interaction.



the most recent Plane Wave Analysis

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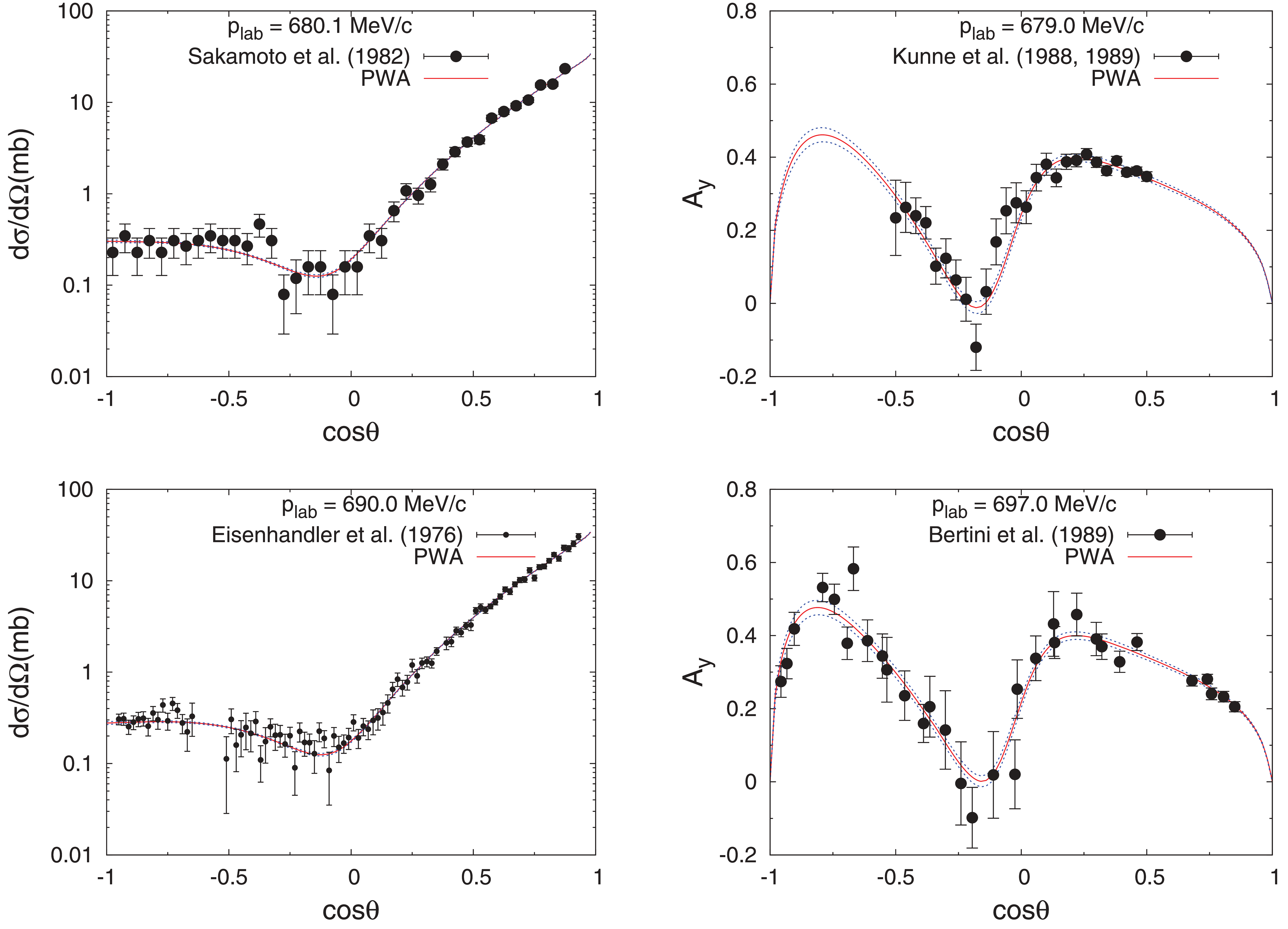
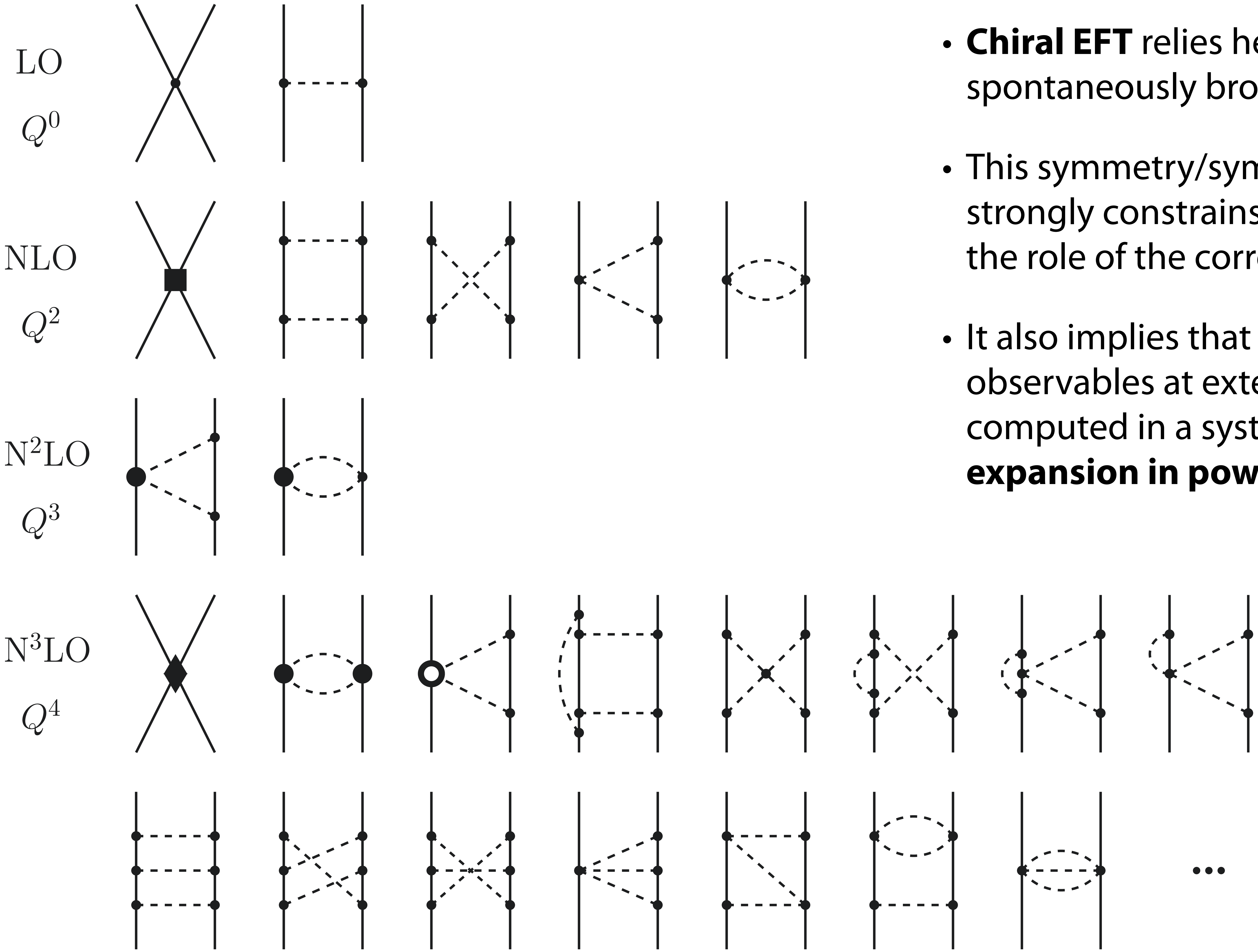
PHYSICAL REVIEW C **86**, 044003 (2012)

criticism

- Some $\bar{N}N$ scattering data are incompatible
- Prejudice in favor of pre-LEAR (pre 1980) data
- Uniqueness of solution
- no Pauli principle, phase shifts are complex: 4 × more PW's than in NN! few polarization data



$\bar{\text{N}}$N potentials from ChPT



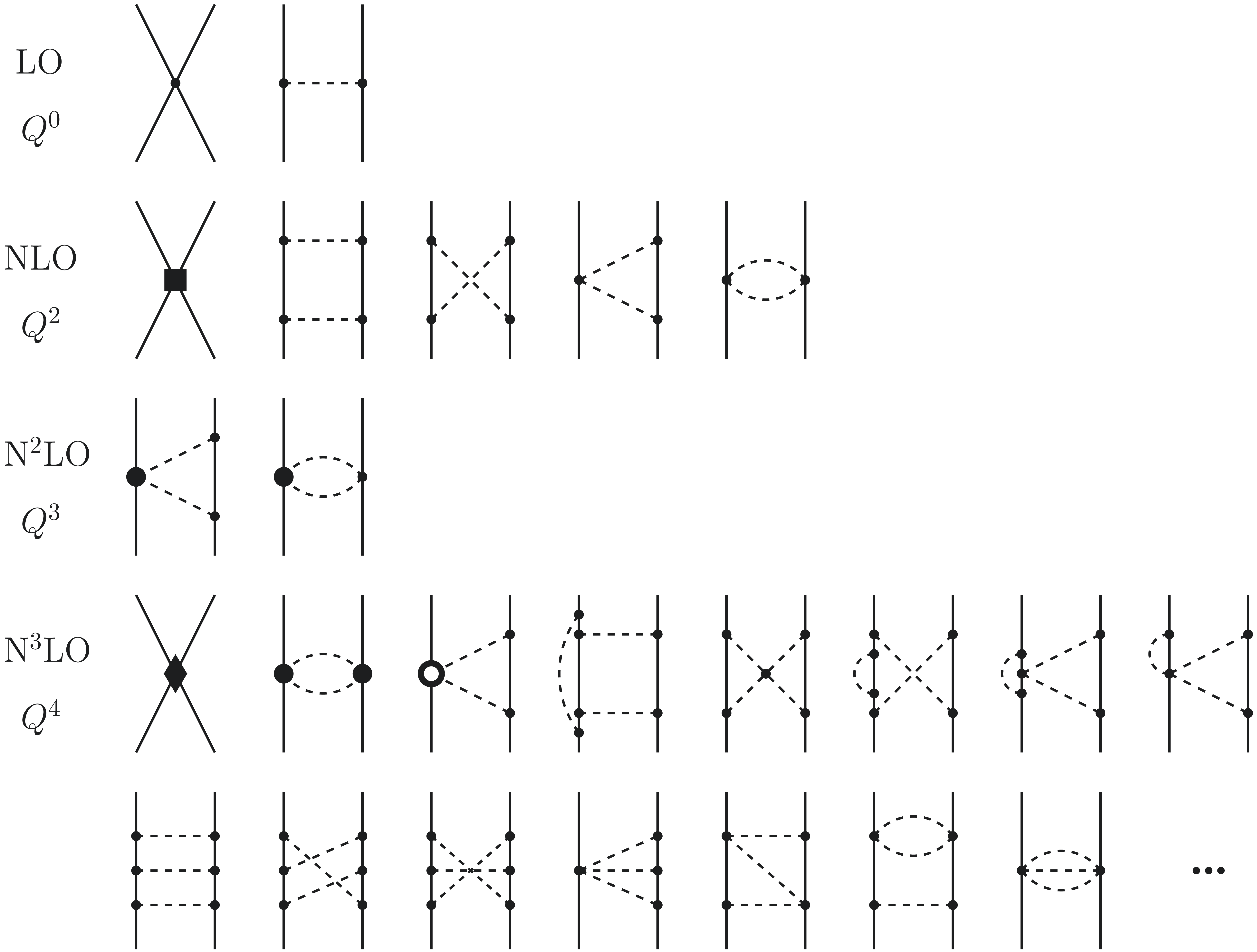
- **Chiral EFT** relies heavily on the approximate spontaneously broken chiral symmetry of QCD.
- This symmetry/symmetry-breaking pattern of QCD strongly constrains the interaction of pions which play the role of the corresponding Goldstone bosons.
- It also implies that pion- and pion-nucleon low-energy observables at external momenta Q~M_π can be computed in a systematic way via a **perturbative expansion in powers of Q/Λ_χ**

Antinucleon-nucleon interaction at next-to-next-to-next-to-leading order in chiral effective field theory

Ling-Yun Dai,[a] Johann Haidenbauer[a] and Ulf-G. Meißner[a,b]

JHEP 07 (2017) 078

$\bar{N}N$ potentials from ChPT



A new local regularization scheme has been used for the pion-exchange contributions

$$f(r) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n$$

The contact interactions are non-local anyway. In this case they use again the standard nonlocal regulator of Gaussian type.

$$f(p', p) = \exp\left(-\frac{p'^m + p^m}{\Lambda^m}\right)$$

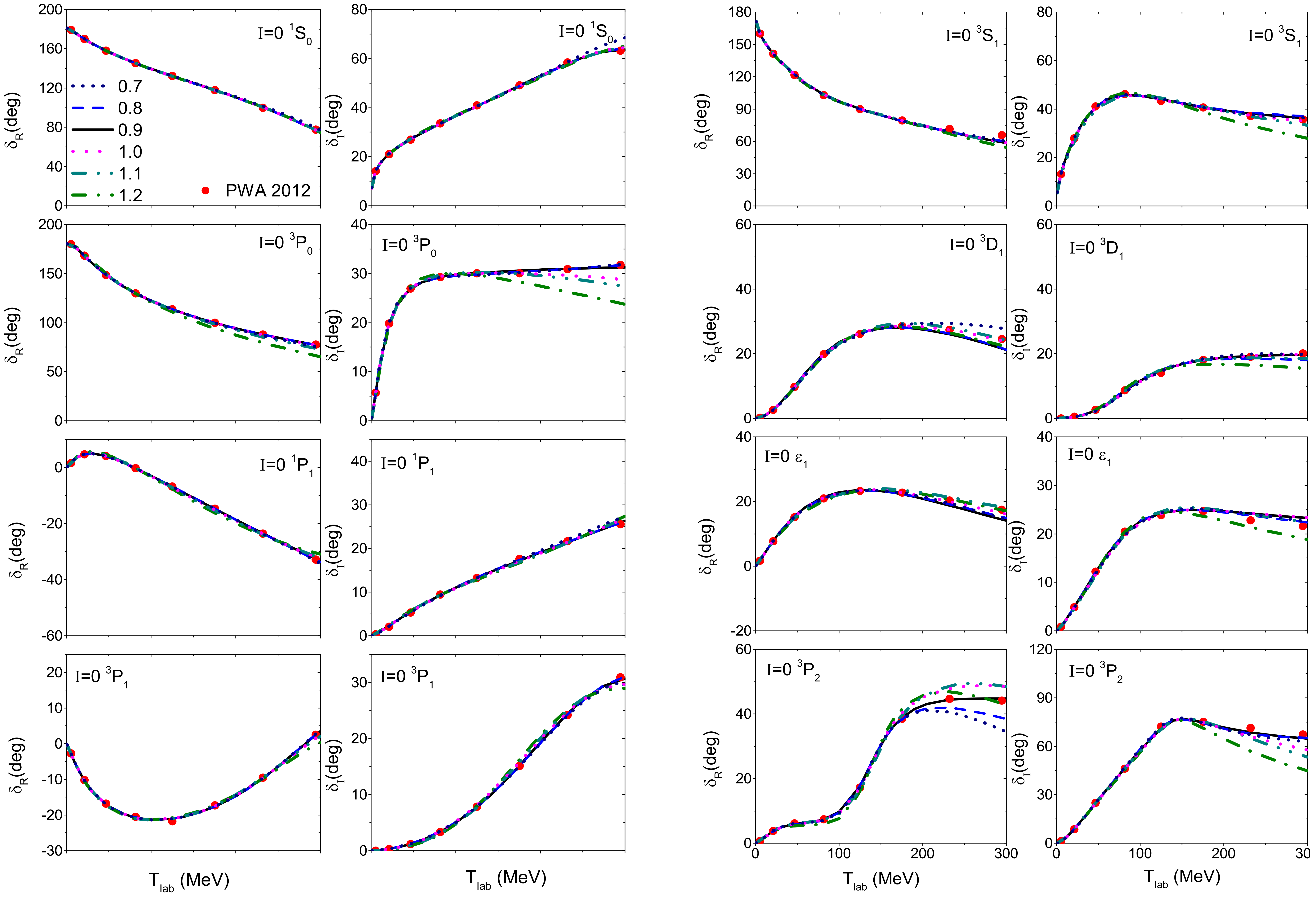
Antinucleon-nucleon interaction at next-to-next-to-next-to-leading order in chiral effective field theory

Ling-Yun Dai,[a] Johann Haidenbauer[a] and Ulf-G. Meißner[a,b]

JHEP 07 (2017) 078

N̄N potentials from ChPT

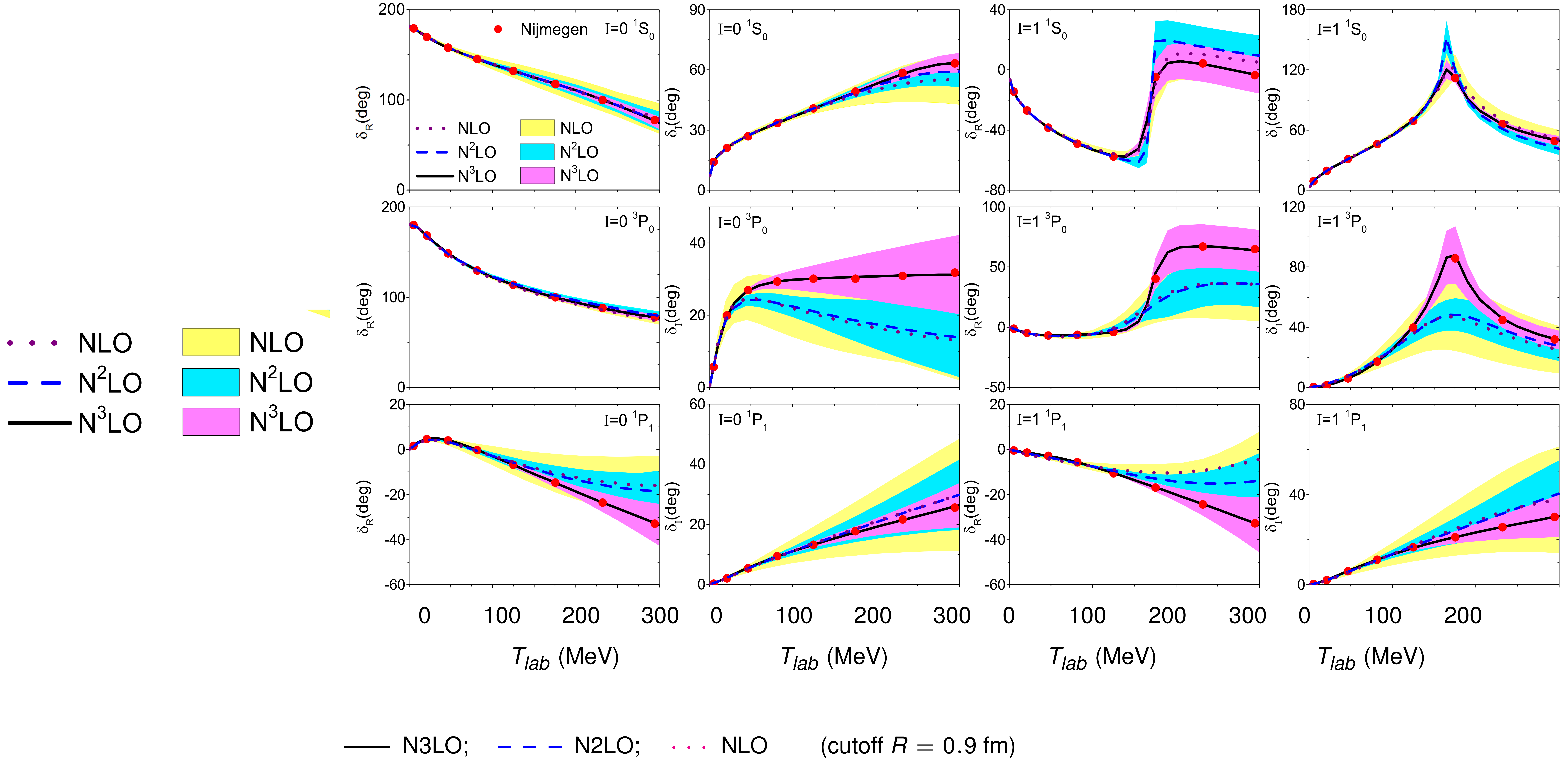
JHEP 07 (2017) 078



Real and imaginary parts of various N̄N phase shifts at N^{3}LO for cutoffs R = 0.7 – 1.2 fm

$\bar{N}N$ potentials from ChPT

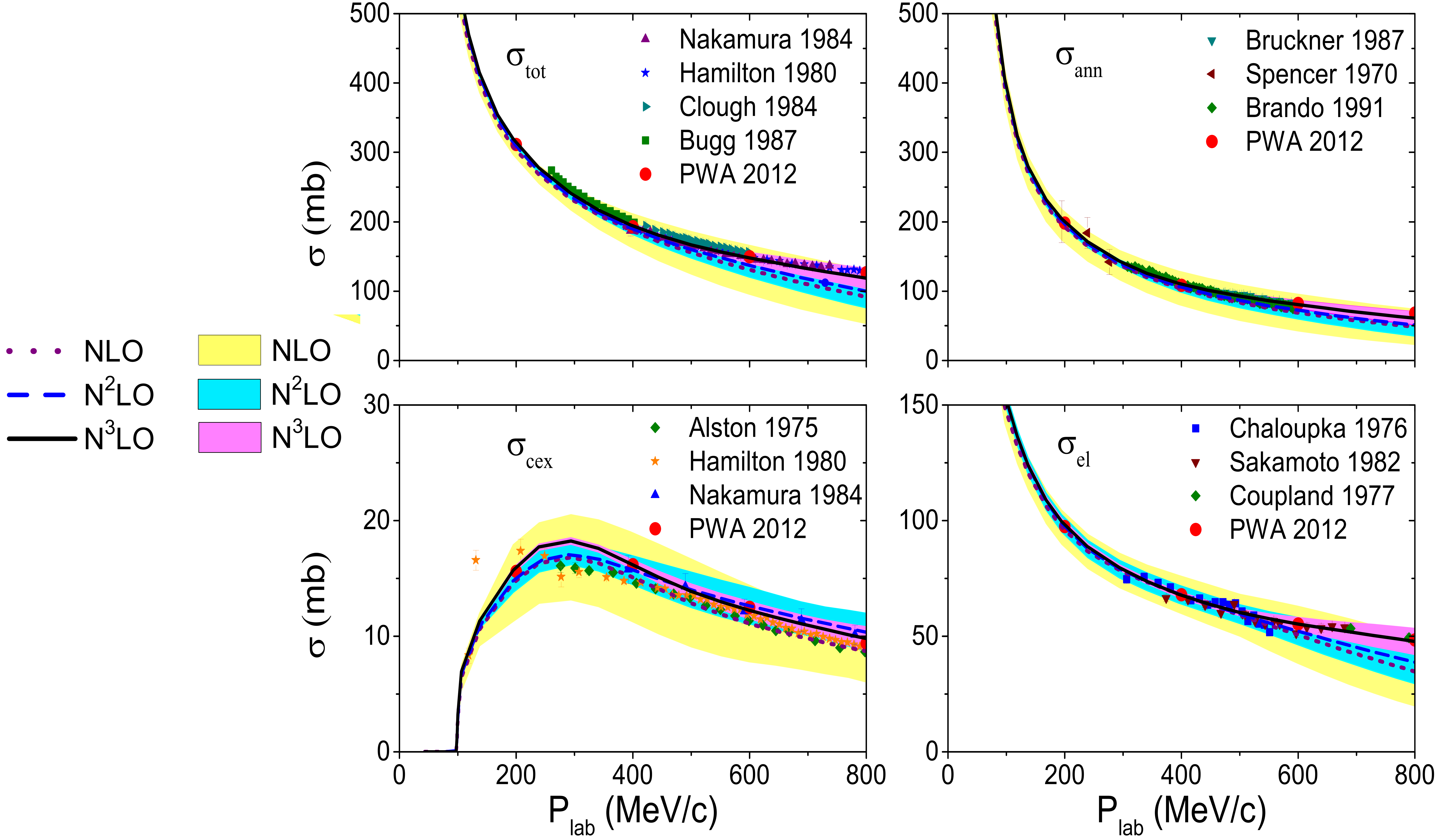
JHEP 07 (2017) 078



p̄p integrated cross sections

N̄N potentials from ChPT

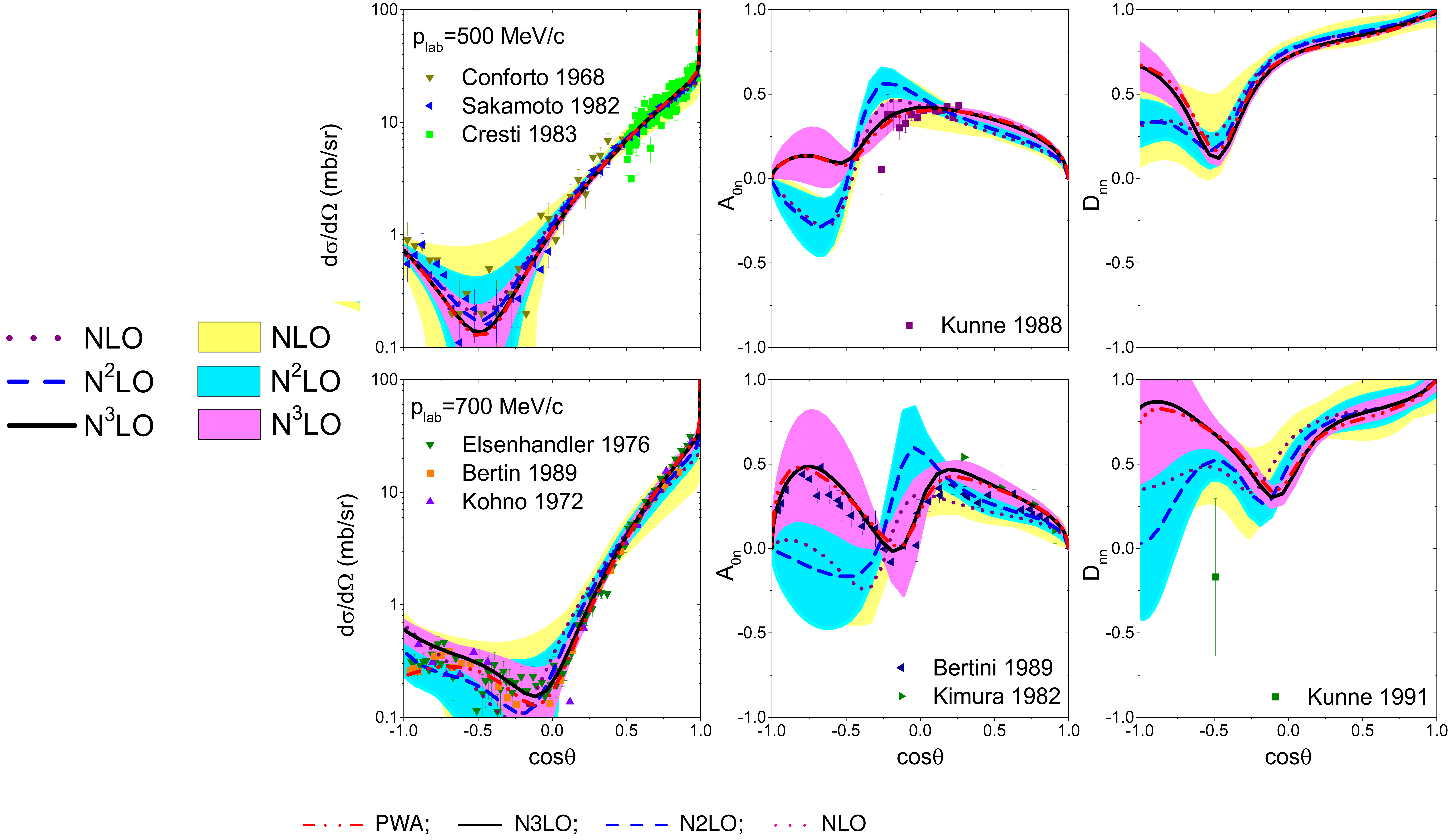
JHEP 07 (2017) 078



$\bar{p}p$ elastic cross sections

$\bar{N}N$ potentials from ChPT

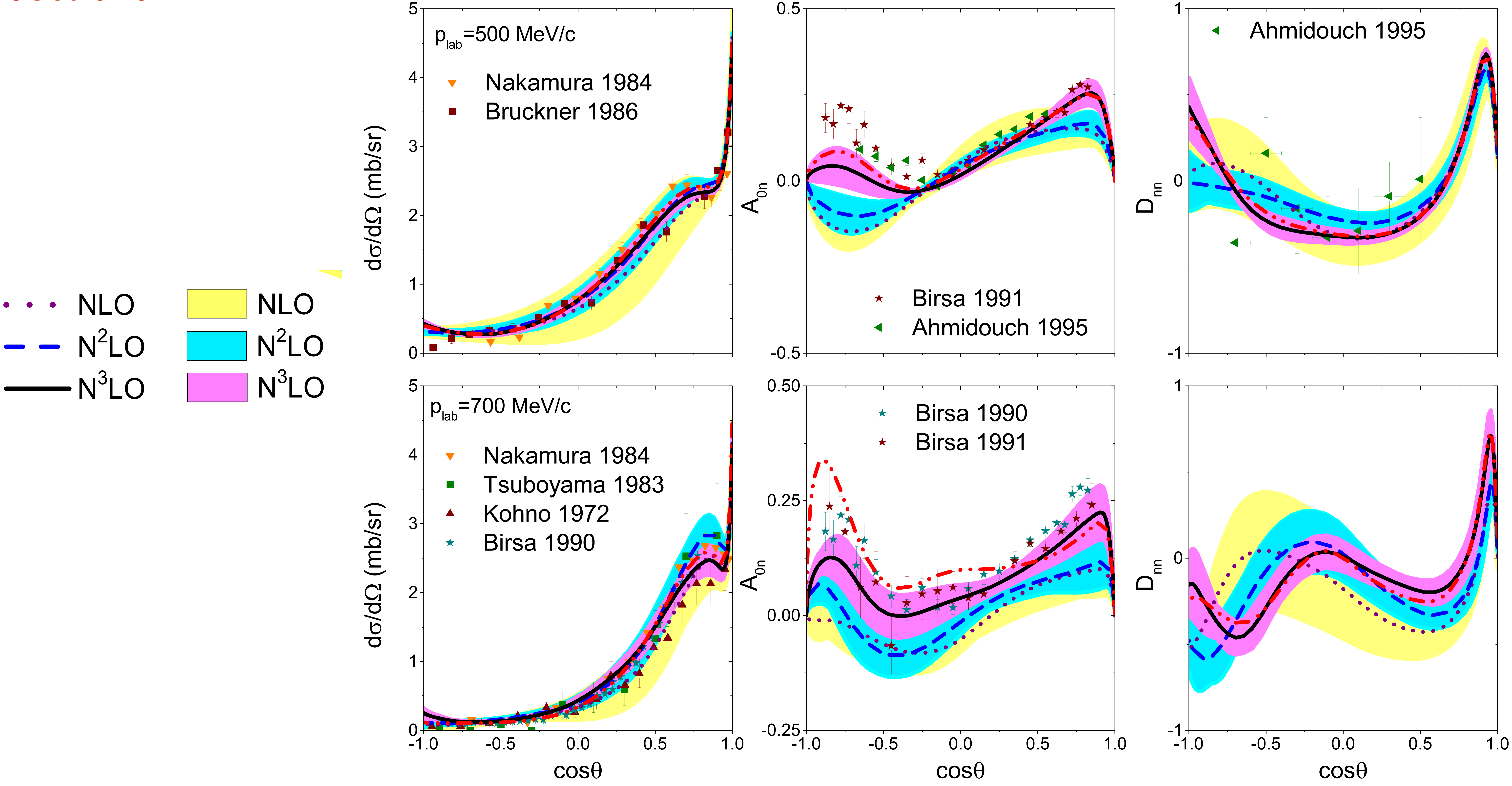
JHEP 07 (2017) 078



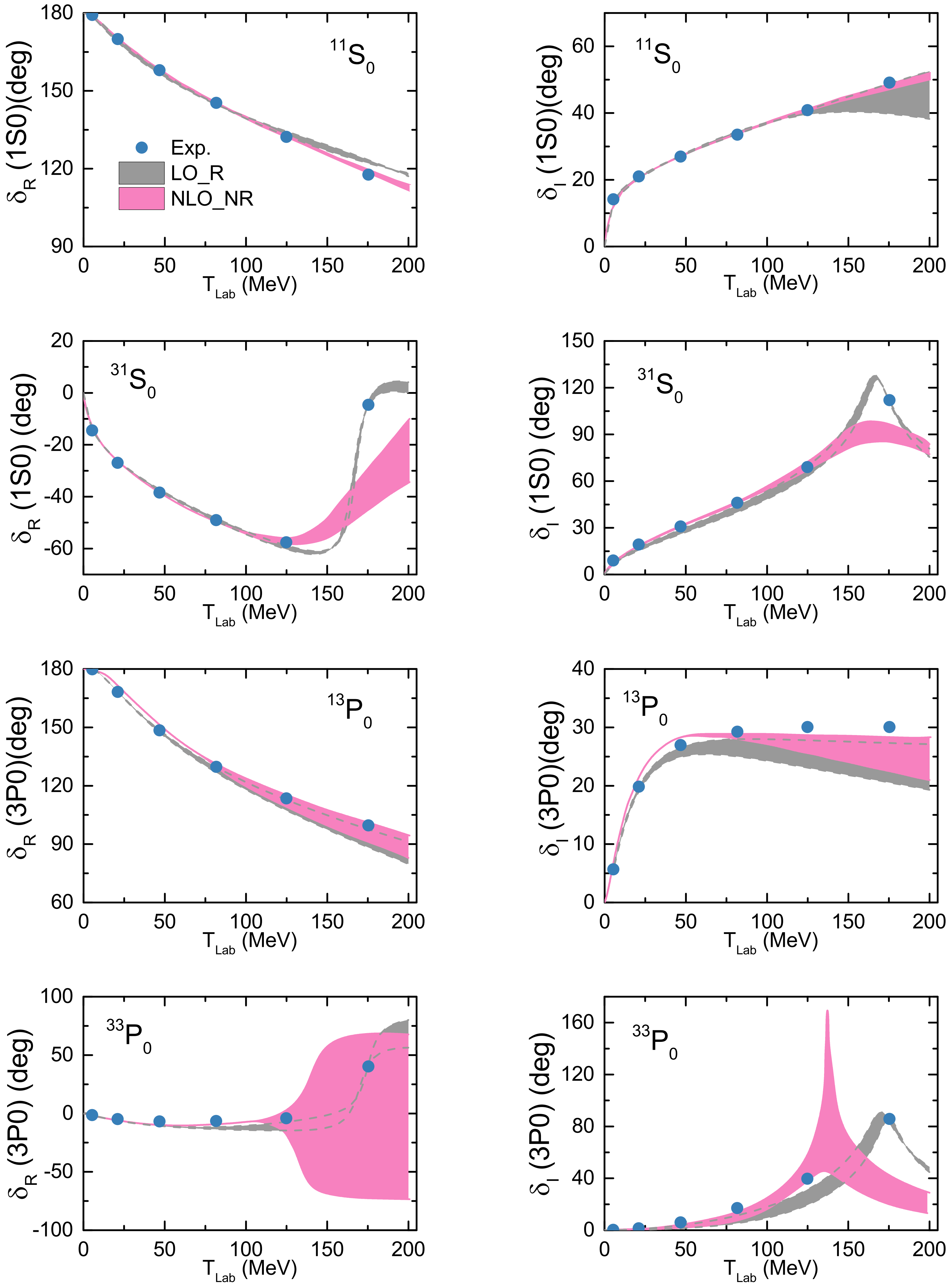
N̄N potentials from ChPT

JHEP 07 (2017) 078

p̄p charge-exchange cross sections



$\bar{N}N$ potentials from ChPT: a recent alternative



Antinucleon-nucleon interactions in covariant chiral effective field theory

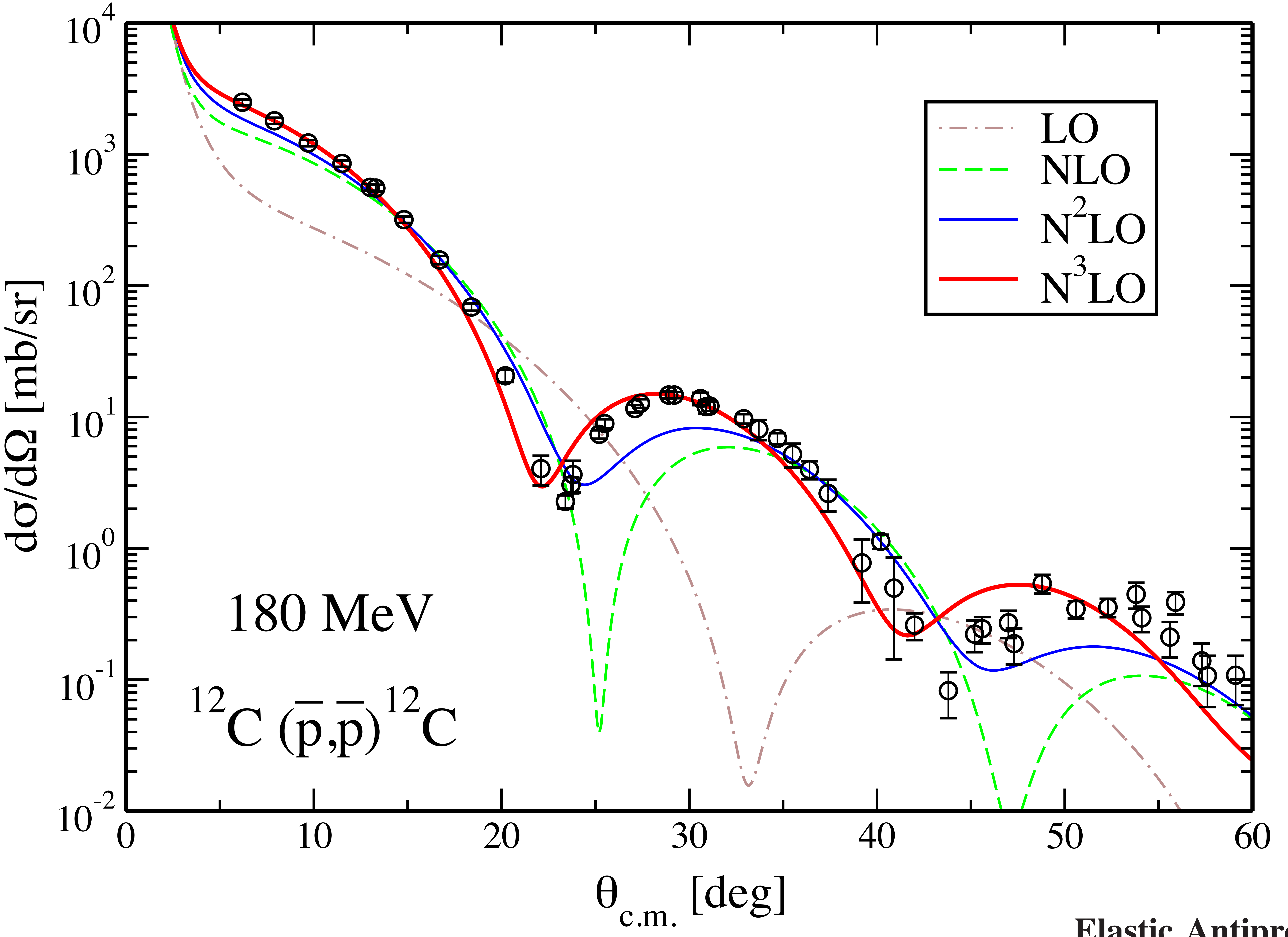
Yang Xiao,[1,2] Jun-Xu Lu,[2] and Li-Sheng Geng[2,3,4,5,*]

arXiv:2406.01292v1 [nucl-th]

- Real and imaginary parts of the phase shift for the $^1S_0$ and $^3P_0$ partial waves.
- The gray bands show the LO relativistic chiral EFT results with the cutoff in the range Λ = 450–600 MeV.
- The pink bands show the NLO non-relativistic chiral EFT results.

Results

Results with NCSM densities



Differential cross section as a function of the center-of-mass scattering angle for elastic antiproton scattering off ^{12}C at 180 MeV, computed at different chiral orders.

Elastic Antiproton-Nucleus Scattering from Chiral Forces
PHYSICAL REVIEW LETTERS **124,** 162501 (2020)

Results with NCSM densities

Elastic Antiproton-Nucleus Scattering from Chiral Forces
PHYSICAL REVIEW LETTERS **124,** 162501 (2020)

Results with NCSM densities

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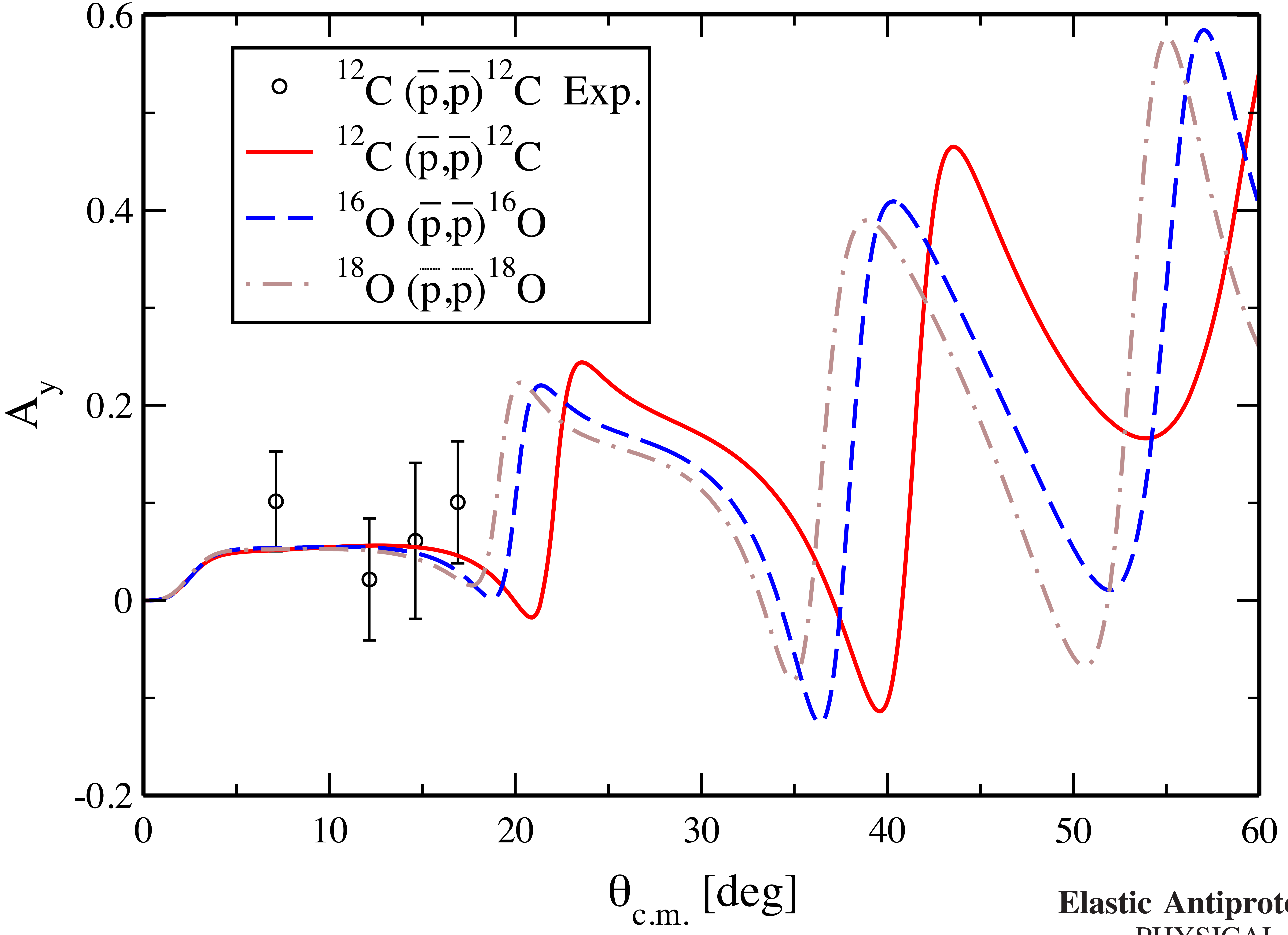
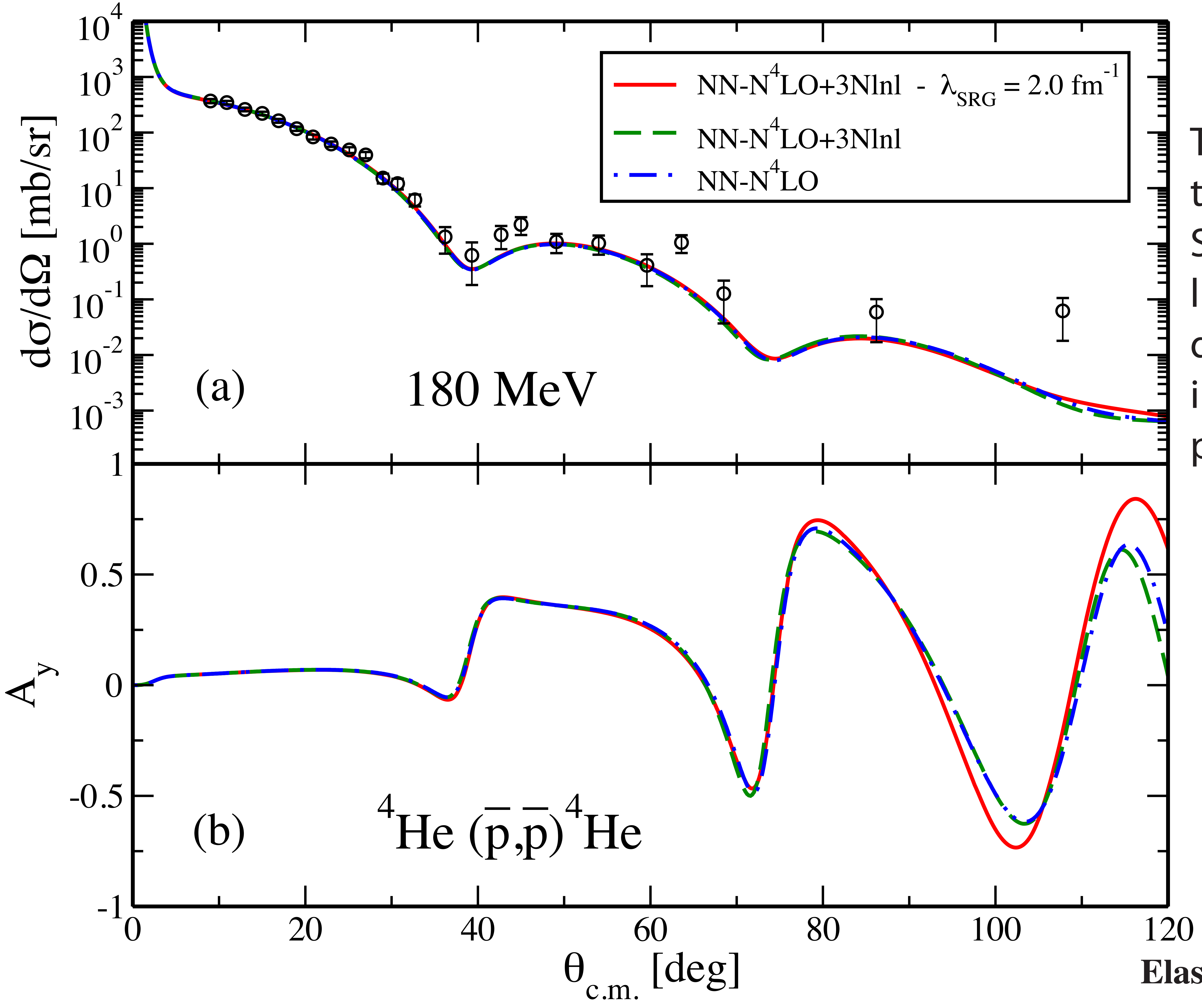
Results with NCSM densities



Analyzing power as function of the center-of-mass scattering angle for elastic antiproton scattering off ^{12}C and $^{16;18}O$

Elastic Antiproton-Nucleus Scattering from Chiral Forces
PHYSICAL REVIEW LETTERS **124,** 162501 (2020)

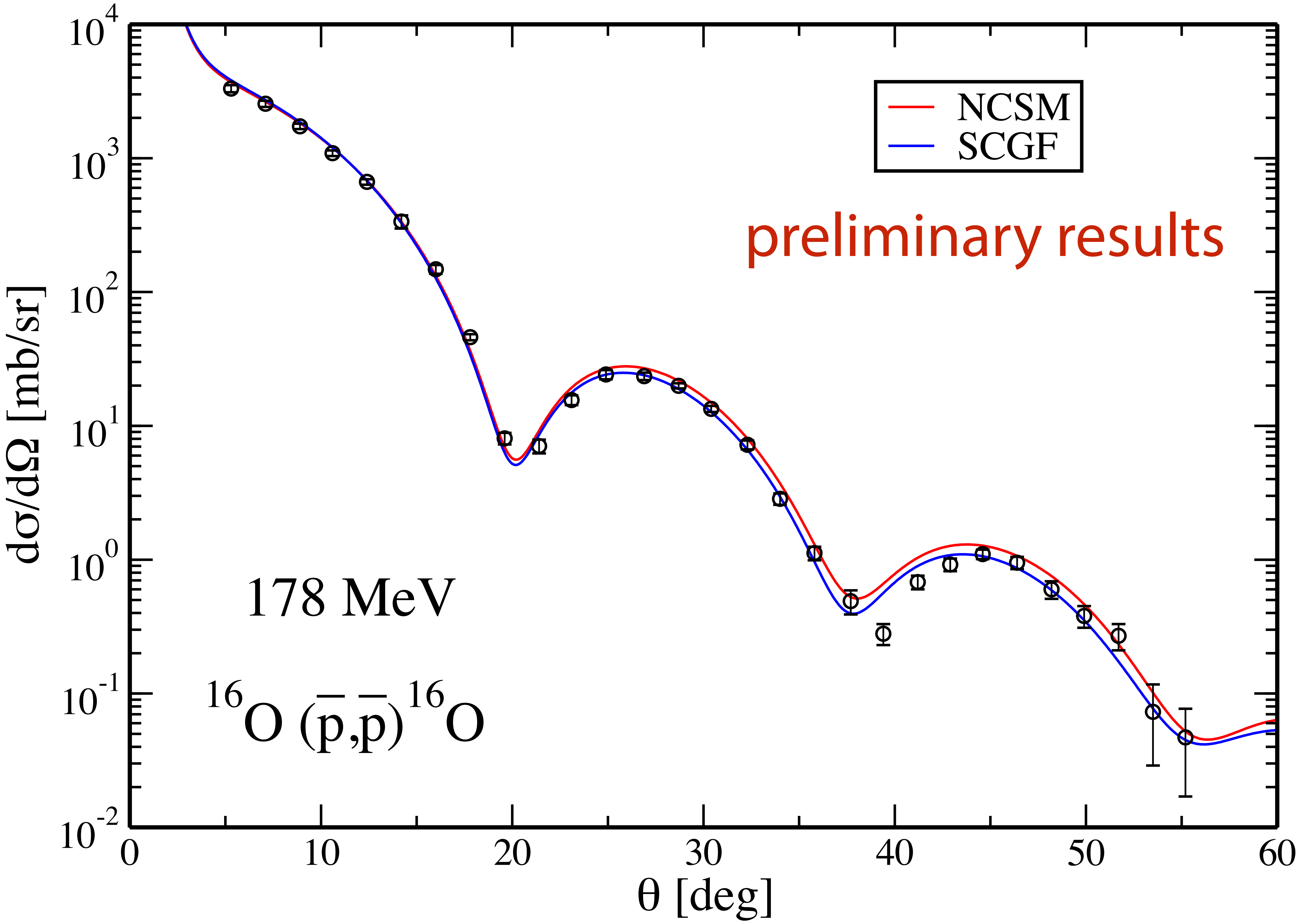
Results with NCSM densities

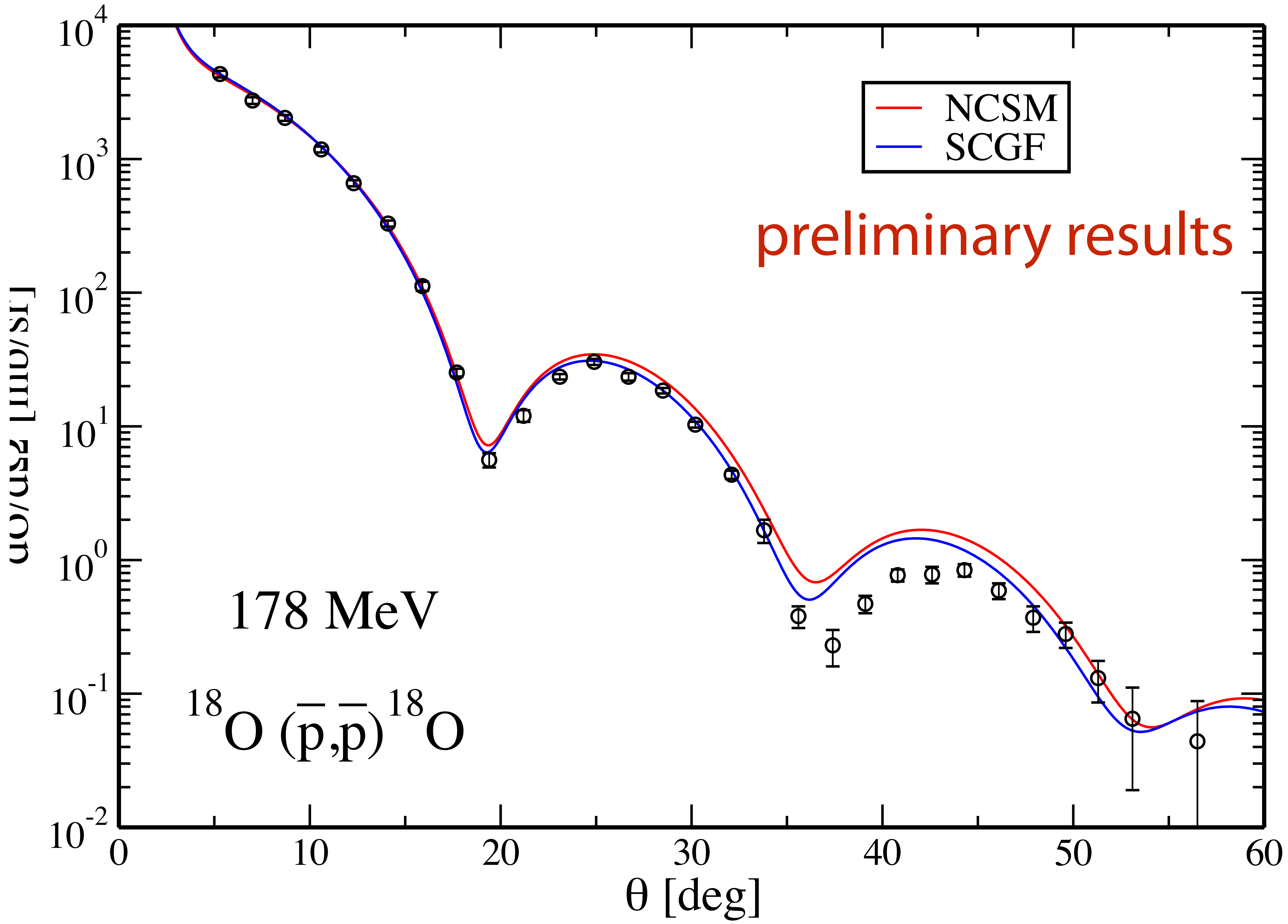


The dashed line has been obtained with the target density computed without the SRG procedure, while the dash-dotted line has been obtained with the target density computed with only the NN interaction and without the SRG procedure.

Elastic Antiproton-Nucleus Scattering from Chiral Forces
PHYSICAL REVIEW LETTERS **124,** 162501 (2020)

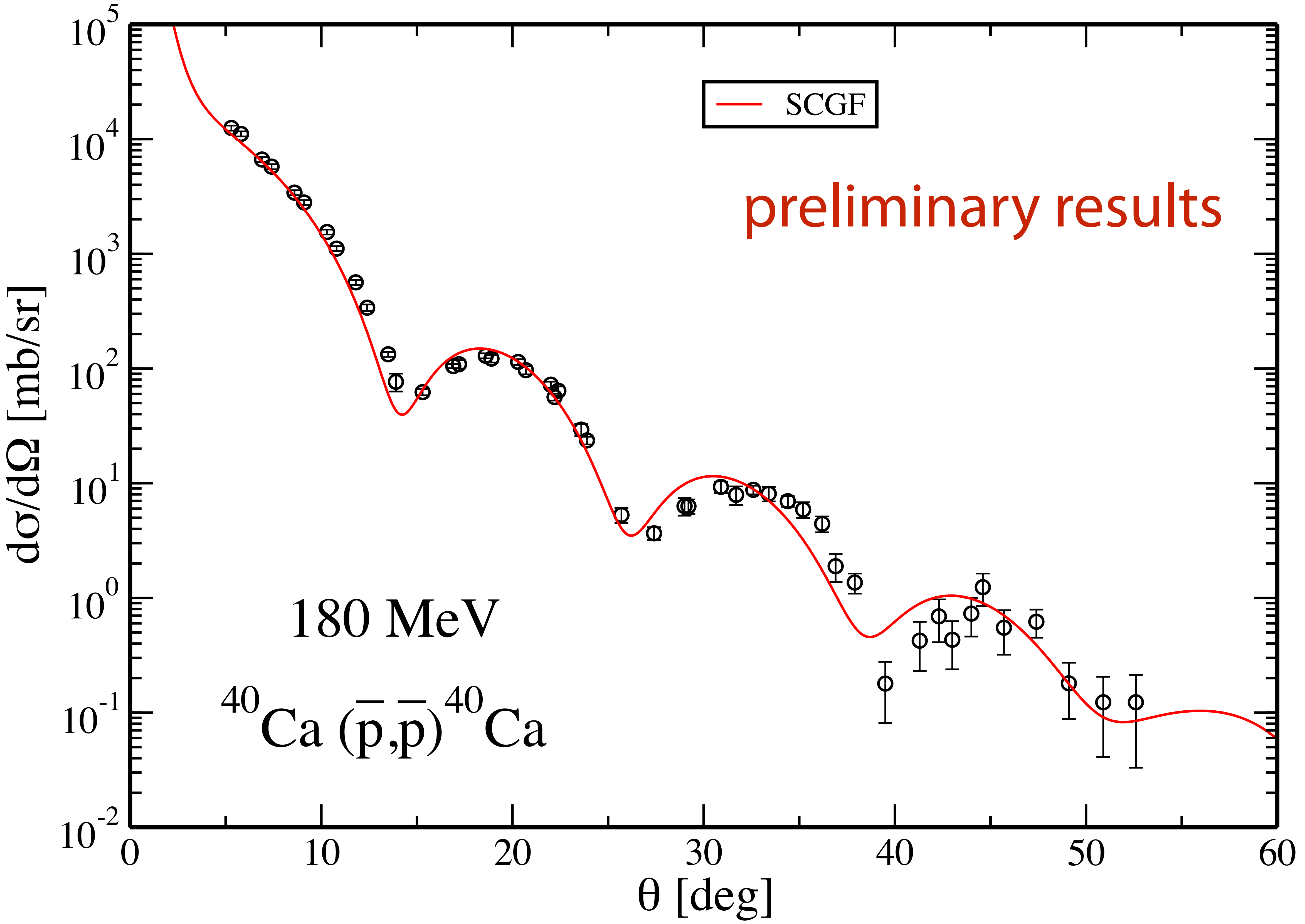
What about SCGF densities?

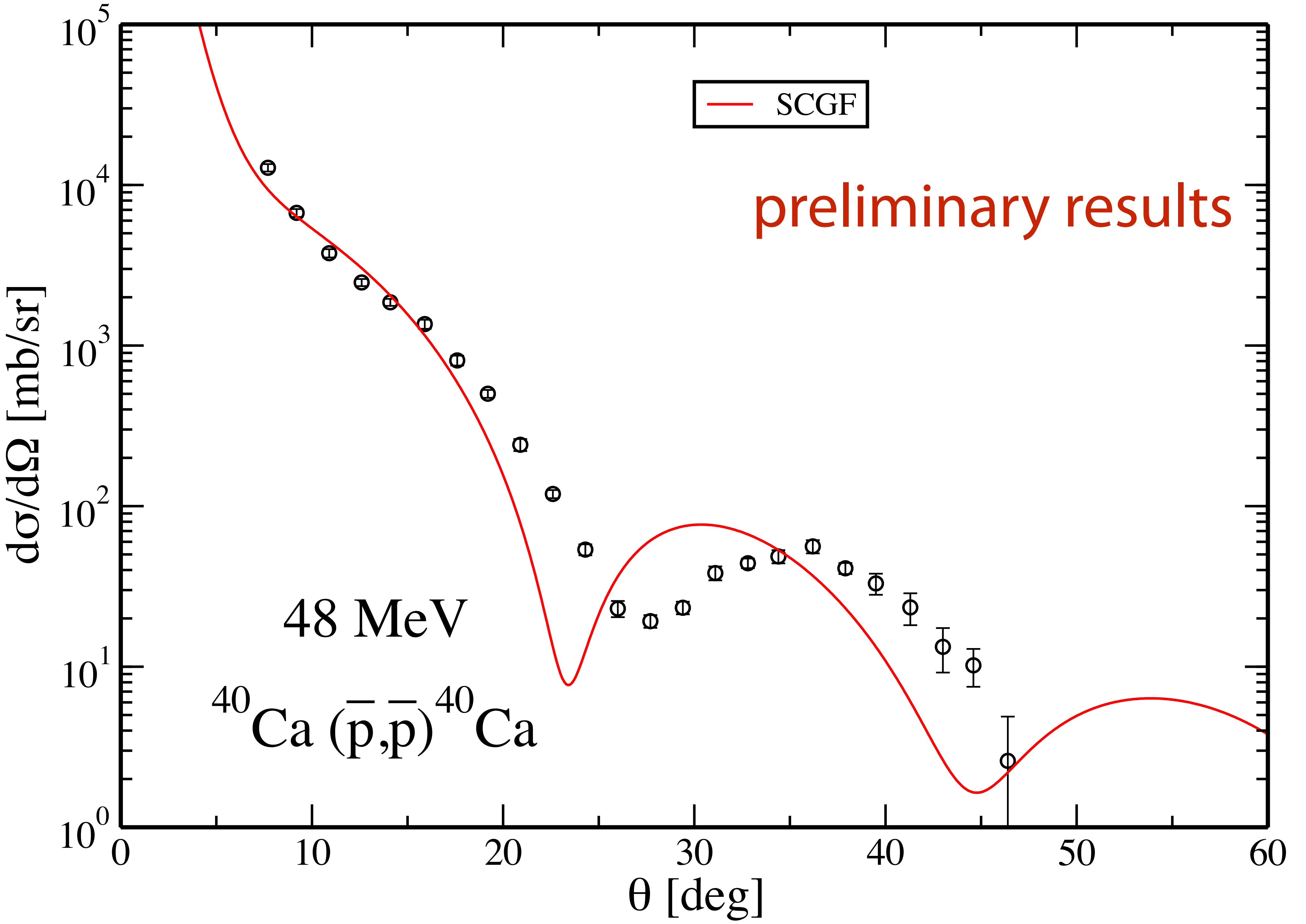




SCGF and NCSM densities both reproduce well the experimental data

What about SCGF densities?

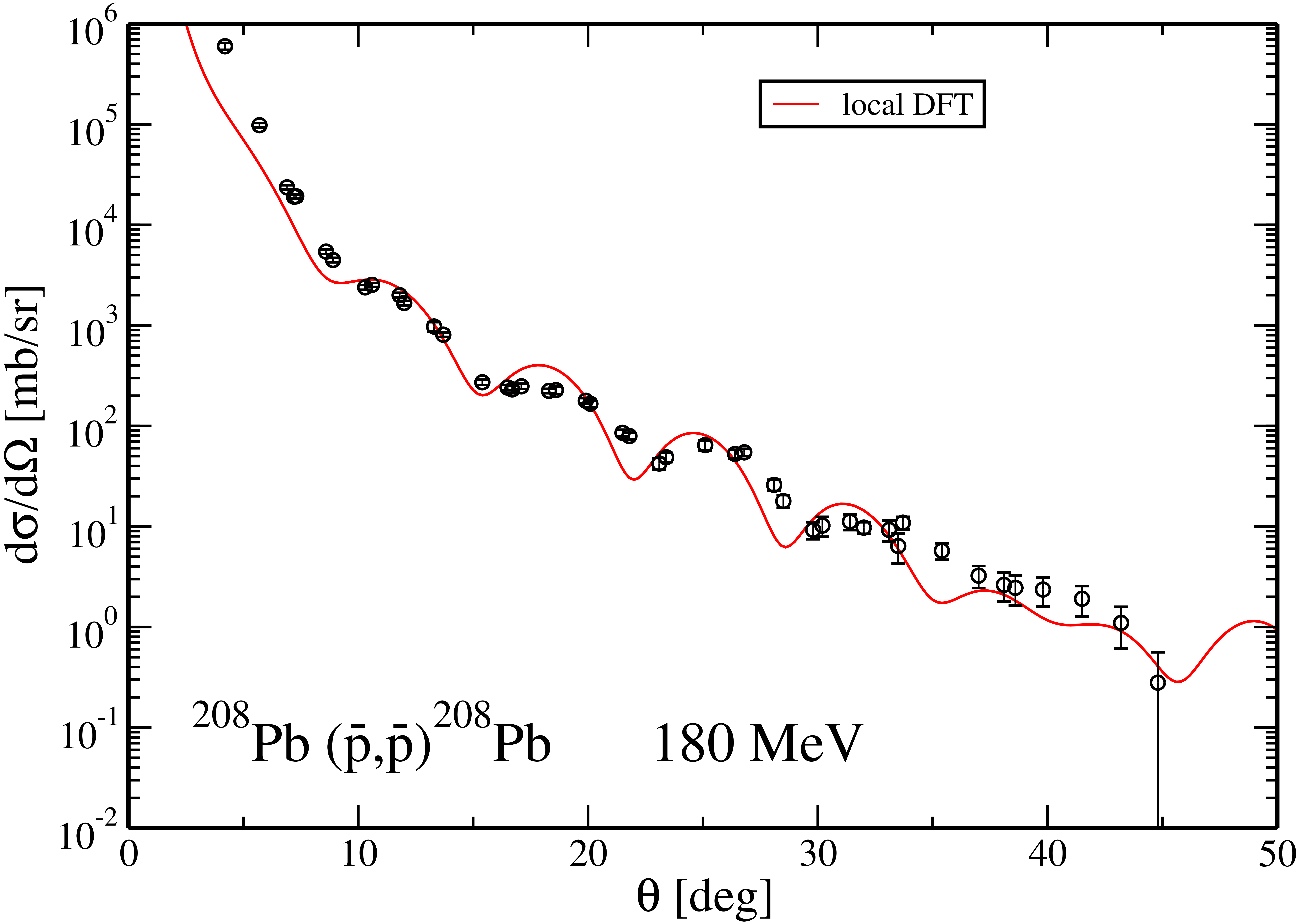




SCGF approach allows to study heavier nuclei (medium mass range)

At relatively low energy experimental data are better reproduced respect to the NN case

Heavy nuclei (only DFT)



- For the first time, a **full microscopic** description of elastic antiproton scattering off light nuclei (but not light anymore…)
- In-medium contributions and many-body forces appear to be rather small

- To describe low-energy observables, the impulse approximation has to improved
- Annihilation processes in the near future

Thanks!