Nucleus-Informed NN Interaction from a Dispersive Optical Model

Mack C. Atkinson

Lawrence Livermore National Laboratory



LLNL-PRESS-865739-DRAFT



This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract 24-LW-062. Lawrence Livermore National Security, LLC

Motivation for nucleus-informed NN interaction

- The DOM provides consistent ingredients for knockout reactions
- Discrepancy between ${}^{40}\text{Ca}(e,e'p){}^{39}\text{K}$ and ${}^{40}\text{Ca}(p,2p){}^{39}\text{K}$





Motivation for nucleus-informed NN interaction

- The DOM provides consistent ingredients for knockout reactions
- Discrepancy between ${}^{40}Ca(e, e'p){}^{39}K$ and ${}^{40}Ca(p, 2p){}^{39}K$

• Through Green's function formalism, the DOM can also describe how a proton propagates through the nucleus (propagator G_{DOM})





GDON

Why not just use (e, e'p) for all single-knockout experiments?



$$G_{\ell j}(r, r'; E) = \sum_{m} \frac{\langle \Psi_{0}^{A} | a_{r \ell j} | \Psi_{m}^{A+1} \rangle \langle \Psi_{m}^{A+1} | a_{r' \ell j}^{\dagger} | \Psi_{0}^{A} \rangle}{E - (E_{m}^{A+1} - E_{0}^{A}) + i\eta} + \sum_{n} \frac{\langle \Psi_{0}^{A} | a_{r' \ell j}^{\dagger} | \Psi_{n}^{A-1} \rangle \langle \Psi_{n}^{A-1} | a_{r \ell j} | \Psi_{0}^{A} \rangle}{E - (E_{0}^{A} - E_{n}^{A-1}) - i\eta}$$

$$G_{\ell j}(r, r'; E) = \sum_{m} \frac{\langle \Psi_{0}^{A} | a_{r\ell j} | \Psi_{m}^{A+1} \rangle \langle \Psi_{m}^{A+1} | a_{r'\ell j}^{\dagger} | \Psi_{0}^{A} \rangle}{E - (E_{m}^{A+1} - E_{0}^{A}) + i\eta} + \sum_{n} \frac{\langle \Psi_{0}^{A} | a_{r'\ell j}^{\dagger} | \Psi_{n}^{A-1} \rangle \langle \Psi_{n}^{A-1} | a_{r\ell j} | \Psi_{0}^{A} \rangle}{E - (E_{0}^{A} - E_{n}^{A-1}) - i\eta}$$

• Poles correspond to excitation energies of (A + 1) or (A - 1) nucleus

$$G_{\ell j}(r,r';E) = \sum_{m} \frac{\langle \Psi_{0}^{A} | a_{r\ell j} | \Psi_{m}^{A+1} \rangle \langle \Psi_{m}^{A+1} | a_{r'\ell j}^{\dagger} | \Psi_{0}^{A} \rangle}{E - (E_{m}^{A+1} - E_{0}^{A}) + i\eta} + \sum_{n} \frac{\langle \Psi_{0}^{A} | a_{r'\ell j}^{\dagger} | \Psi_{n}^{A-1} \rangle \langle \Psi_{n}^{A-1} | a_{r\ell j} | \Psi_{0}^{A} \rangle}{E - (E_{0}^{A} - E_{n}^{A-1}) - i\eta}$$

- Poles correspond to excitation energies of (A + 1) or (A 1) nucleus
- Numerator like a transition probability to given excitation

$$G_{\ell j}(r,r';E) = \sum_{m} \frac{\langle \Psi_{0}^{A} | a_{r\ell j} | \Psi_{m}^{A+1} \rangle \langle \Psi_{m}^{A+1} | a_{r'\ell j}^{\dagger} | \Psi_{0}^{A} \rangle}{E - (E_{m}^{A+1} - E_{0}^{A}) + i\eta} + \sum_{n} \frac{\langle \Psi_{0}^{A} | a_{r'\ell j}^{\dagger} | \Psi_{n}^{A-1} \rangle \langle \Psi_{n}^{A-1} | a_{r\ell j} | \Psi_{0}^{A} \rangle}{E - (E_{0}^{A} - E_{n}^{A-1}) - i\eta}$$

- Poles correspond to excitation energies of (A + 1) or (A 1) nucleus
- Numerator like a transition probability to given excitation

$$S^{h}_{\ell j}(r; E) = rac{1}{\pi} \operatorname{Im} G_{\ell j}(r, r; E) \theta(E - (E^{\mathcal{A}}_{0} - E^{\mathcal{A}-1}_{0}))$$

$$G_{\ell j}(r, r'; E) = \sum_{m} \frac{\langle \Psi_{0}^{A} | a_{r\ell j} | \Psi_{m}^{A+1} \rangle \langle \Psi_{m}^{A+1} | a_{r'\ell j}^{\dagger} | \Psi_{0}^{A} \rangle}{E - (E_{m}^{A+1} - E_{0}^{A}) + i\eta} + \sum_{n} \frac{\langle \Psi_{0}^{A} | a_{r'\ell j}^{\dagger} | \Psi_{n}^{A-1} \rangle \langle \Psi_{n}^{A-1} | a_{r\ell j} | \Psi_{0}^{A} \rangle}{E - (E_{0}^{A} - E_{n}^{A-1}) - i\eta}$$

- Poles correspond to excitation energies of (A + 1) or (A 1) nucleus
- Numerator like a transition probability to given excitation

$$S_{\ell j}^{h}(r; E) = \frac{1}{\pi} \operatorname{Im} G_{\ell j}(r, r; E) \theta(E - (E_{0}^{A} - E_{0}^{A-1}))$$

$$\underbrace{=}_{S_{0}}^{10^{-1}} \underbrace{=}_{S_{0}}^{10^{-1}} \underbrace{=$$

experiment

 ${}^{40}\mathbf{Ca}(e,e'p){}^{39}\mathbf{K}$

$$G_{\ell j}(r, r'; E) = \sum_{m} \frac{\langle \Psi_{0}^{A} | a_{r\ell j} | \Psi_{m}^{A+1} \rangle \langle \Psi_{m}^{A+1} | a_{r'\ell j}^{\dagger} | \Psi_{0}^{A} \rangle}{E - (E_{m}^{A+1} - E_{0}^{A}) + i\eta} + \sum_{n} \frac{\langle \Psi_{0}^{A} | a_{r'\ell j}^{\dagger} | \Psi_{n}^{A-1} \rangle \langle \Psi_{n}^{A-1} | a_{r\ell j} | \Psi_{0}^{A} \rangle}{E - (E_{0}^{A} - E_{n}^{A-1}) - i\eta}$$

- Poles correspond to excitation energies of (A + 1) or (A 1) nucleus
- Numerator like a transition probability to given excitation

$$S^{h}_{\ell j}(r; E) = rac{1}{\pi} \operatorname{Im} G_{\ell j}(r, r; E) \theta(E - (E^{A}_{0} - E^{A-1}_{0}))$$

• Close connection with experimental observables



• Perturbative expansion of G leads to the Dyson equation



- Perturbative expansion of G leads to the Dyson equation
- $\bullet~\Sigma^*$ corresponds to an optical potential



- Perturbative expansion of G leads to the Dyson equation
- $\bullet~\Sigma^*$ corresponds to an optical potential
- $\Sigma^*(\mathbf{r}, \mathbf{r'}; E)$ is nonlocal



- Perturbative expansion of G leads to the Dyson equation
- $\bullet~\Sigma^*$ corresponds to an optical potential
- $\Sigma^*(\mathbf{r}, \mathbf{r'}; E)$ is nonlocal

$$rac{\hat{oldsymbol{p}}^2}{2\mu}\psi(oldsymbol{r})+\int doldsymbol{r'}\Sigma^*(oldsymbol{r},oldsymbol{r'};E)\psi(oldsymbol{r'})=E\psi(oldsymbol{r})$$



- \bullet Perturbative expansion of G leads to the Dyson equation
- $\bullet~\Sigma^*$ corresponds to an optical potential
- $\Sigma^*(\mathbf{r}, \mathbf{r'}; E)$ is nonlocal

$$rac{\hat{oldsymbol{p}}^2}{2\mu}\psi(oldsymbol{r})+\int doldsymbol{r}'\Sigma^*(oldsymbol{r},oldsymbol{r}';E)\psi(oldsymbol{r}')=E\psi(oldsymbol{r})$$



- \bullet Perturbative expansion of G leads to the Dyson equation
- $\bullet~\Sigma^*$ corresponds to an optical potential
- $\Sigma^*(\mathbf{r}, \mathbf{r'}; E)$ is nonlocal

$$rac{\hat{oldsymbol{p}}^2}{2\mu}\psi(oldsymbol{r})+\int doldsymbol{r}'\Sigma^*(oldsymbol{r},oldsymbol{r}';E)\psi(oldsymbol{r}')=E\psi(oldsymbol{r})$$

• Use the same functional form as standard optical potentials for the self-energy

$$\Sigma^*(\mathbf{r},\mathbf{r'};E) \rightarrow V_{vol}(\mathbf{r},\mathbf{r'};E) + V_{sur}(\mathbf{r},\mathbf{r'};E) + V_{so}(\mathbf{r},\mathbf{r'};E)$$



- \bullet Perturbative expansion of G leads to the Dyson equation
- $\bullet~\Sigma^*$ corresponds to an optical potential
- $\Sigma^*(\mathbf{r}, \mathbf{r'}; E)$ is nonlocal

$$rac{\hat{oldsymbol{
ho}}^2}{2\mu}\psi(oldsymbol{r})+\int doldsymbol{r}'\Sigma^*(oldsymbol{r},oldsymbol{r}';E)\psi(oldsymbol{r}')=E\psi(oldsymbol{r})$$

• Use the same functional form as standard optical potentials for the self-energy

$$\Sigma^*(\boldsymbol{r}, \boldsymbol{r}'; E) \rightarrow V_{vol}(\boldsymbol{r}, \boldsymbol{r}'; E) + V_{sur}(\boldsymbol{r}, \boldsymbol{r}'; E) + V_{so}(\boldsymbol{r}, \boldsymbol{r}'; E)$$

• Nonlocality is parametrized with β





- \bullet Perturbative expansion of G leads to the Dyson equation
- $\bullet~\Sigma^*$ corresponds to an optical potential
- $\Sigma^*(\mathbf{r}, \mathbf{r'}; E)$ is nonlocal

$$rac{\hat{oldsymbol{
ho}}^2}{2\mu}\psi(oldsymbol{r})+\int doldsymbol{r}'\Sigma^*(oldsymbol{r},oldsymbol{r}';E)\psi(oldsymbol{r}')=E\psi(oldsymbol{r})$$

• Use the same functional form as standard optical potentials for the self-energy

$$\Sigma^*(\mathbf{r},\mathbf{r}';E) \rightarrow V_{vol}(\mathbf{r},\mathbf{r}';E) + V_{sur}(\mathbf{r},\mathbf{r}';E) + V_{so}(\mathbf{r},\mathbf{r}';E)$$

• Nonlocality is parametrized with β

$$\Sigma^*(\mathbf{r},\mathbf{r'};E) = \Sigma^*\left(rac{r+r'}{2};E
ight)e^{rac{-(r-r')^2}{eta^2}}\pi^{-rac{3}{2}}eta^{-3}$$

- Perturbative expansion of G leads to the Dyson equation
- $\bullet~\Sigma^*$ corresponds to an optical potential
- $\Sigma^*(\mathbf{r}, \mathbf{r'}; E)$ is nonlocal

$$rac{\hat{oldsymbol{
ho}}^2}{2\mu}\psi(oldsymbol{r})+\int doldsymbol{r}'\Sigma^*(oldsymbol{r},oldsymbol{r}';E)\psi(oldsymbol{r}')=E\psi(oldsymbol{r})$$

• Use the same functional form as standard optical potentials for the self-energy

$$\Sigma^*(\mathbf{r},\mathbf{r'};E) \rightarrow V_{vol}(\mathbf{r},\mathbf{r'};E) + V_{sur}(\mathbf{r},\mathbf{r'};E) + V_{so}(\mathbf{r},\mathbf{r'};E)$$

• Nonlocality is parametrized with β

Can this also describe negative energy observables?

$$\Sigma^{*}(\mathbf{r},\mathbf{r}';E) = \Sigma^{*}\left(\frac{r+r'}{2};E\right)e^{\frac{-(r-r')^{2}}{\beta^{2}}}\pi^{-\frac{3}{2}}\beta^{-3}$$

• The DOM makes use of complex analysis to formulate a consistent self-energy

Dispersive Correction

$$Re\Sigma_{\ell j}(r, r'; E) = Re\Sigma_{\ell j}(r, r'; \epsilon_F) - \frac{1}{\pi}(\epsilon_F - E)\mathcal{P}\int_{\epsilon_T^+}^{\infty} dE' Im\Sigma_{\ell j}(r, r'; E') \left[\frac{1}{E - E'} - \frac{1}{\epsilon_F - E'}\right] \\
+ \frac{1}{\pi}(\epsilon_F - E)\mathcal{P}\int_{-\infty}^{\epsilon_T^-} dE' Im\Sigma_{\ell j}(r, r'; E') \left[\frac{1}{E - E'} - \frac{1}{\epsilon_F - E'}\right]$$

Mack C. Atkinson LLNL

¹ C. Mahaux, R. Sartor, Adv. Nucl. Phys., 20, 96 (1991)

• The DOM makes use of complex analysis to formulate a consistent self-energy

Dispersive Correction

$$Re\Sigma_{\ell j}(r, r'; E) = Re\Sigma_{\ell j}(r, r'; \epsilon_F) - \frac{1}{\pi}(\epsilon_F - E)\mathcal{P}\int_{\epsilon_T^+}^{\infty} dE' Im\Sigma_{\ell j}(r, r'; E') \left[\frac{1}{E - E'} - \frac{1}{\epsilon_F - E'}\right] \\
+ \frac{1}{\pi}(\epsilon_F - E)\mathcal{P}\int_{-\infty}^{\epsilon_T^-} dE' Im\Sigma_{\ell j}(r, r'; E') \left[\frac{1}{E - E'} - \frac{1}{\epsilon_F - E'}\right]$$

• (subtracted) Dispersion relation constrains self-energy at all energies

¹ C. Mahaux, R. Sartor, Adv. Nucl. Phys., 20, 96 (1991)

• The DOM makes use of complex analysis to formulate a consistent self-energy

Dispersive Correction

$$Re\Sigma_{\ell j}(r, r'; E) = Re\Sigma_{\ell j}(r, r'; \epsilon_F) - \frac{1}{\pi}(\epsilon_F - E)\mathcal{P}\int_{\epsilon_T^+}^{\infty} dE' Im\Sigma_{\ell j}(r, r'; E') \left[\frac{1}{E - E'} - \frac{1}{\epsilon_F - E'}\right] \\
+ \frac{1}{\pi}(\epsilon_F - E)\mathcal{P}\int_{-\infty}^{\epsilon_T^-} dE' Im\Sigma_{\ell j}(r, r'; E') \left[\frac{1}{E - E'} - \frac{1}{\epsilon_F - E'}\right]$$

- (subtracted) Dispersion relation constrains self-energy at all energies
- This constraint ensures bound and scattering quantities are simultaneously described

¹ C. Mahaux, R. Sartor, Adv. Nucl. Phys., 20, 96 (1991)

 \bullet Parameters of self-energy varied to minimize χ^2

• Parameters of self-energy varied to minimize χ^2



M.C. Atkinson et al., PRC 98, 044627 (2018)

• Parameters of self-energy varied to minimize χ^2



M.C. Atkinson et al., PRC 98, 044627 (2018)

Mack C. Atkinson LLNL

• Parameters of self-energy varied to minimize χ^2



M.C. Atkinson et al., PRC 98, 044627 (2018)

Mack C. Atkinson LLNL

• Parameters of self-energy varied to minimize χ^2



Mack C. Atkinson LLNL

- Parameters of self-energy varied to minimize χ^2
- Reproducing the data means self-energy is found



0.09

0.08

0.07

Mack C. Atkinson LLNL

Experiment DOM



• Excitation spectrum provides evidence of many-body correlations beyond mean-field



• Excitation spectrum provides evidence of many-body correlations beyond mean-field



M.C. Atkinson et al., PRC 98, 044627 (2018)

Mack C. Atkinson LLNL

- Excitation spectrum provides evidence of many-body correlations beyond mean-field
- Momentum distribution is closely tied to the boundstate wavefunction



M.C. Atkinson et al., PRC 98, 044627 (2018)

- Excitation spectrum provides evidence of many-body correlations beyond mean-field
- Momentum distribution is closely tied to the boundstate wavefunction
- Spectroscopic factor



M.C. Atkinson et al., PRC 98, 044627 (2018)

- Excitation spectrum provides evidence of many-body correlations beyond mean-field
- Momentum distribution is closely tied to the boundstate wavefunction
- Spectroscopic factor





Mack C. Atkinson LLNL

- Excitation spectrum provides evidence of many-body correlations beyond mean-field
- Momentum distribution is closely tied to the boundstate wavefunction
- Spectroscopic factor
- Electron interaction means clean knockout reactions





Mack C. Atkinson LLNL

Spectroscopic factor is **not** the same as Occupation



Mack C. Atkinson LLNL




• No imaginary component of Σ^* around ϵ_F

$$J^{\ell}_{W}(E) = (4\pi)^2 \int_0^\infty dr r^2 \int_0^\infty dr' r'^2 \operatorname{Im} \{ \Sigma^*_{\ell}(r,r';E) \}$$

 Spectroscopic factor for states near ε_F is well defined from Σ*

$$S_{F} = \left(1 - \frac{\partial \Sigma^{*}(\alpha_{qh}, \alpha_{qh}; E)}{\partial E}\Big|_{\epsilon}\right)^{-1}$$



• No imaginary component of Σ^* around ϵ_F

$$J_W^\ell(E) = (4\pi)^2 \int_0^\infty dr r^2 \int_0^\infty dr' r'^2 \operatorname{Im}\{\Sigma_\ell^*(r,r';E)\}$$

 Spectroscopic factor for states near ε_F is well defined from Σ*

$$S_{F} = \left(1 - \frac{\partial \Sigma^{*}(\alpha_{qh}, \alpha_{qh}; E)}{\partial E}\Big|_{\epsilon}\right)^{-1}$$

$$n_{n\ell j} = \int_{-\infty}^{\epsilon_f} dE S^h_{n\ell j}(E)$$



• No imaginary component of Σ^* around ϵ_F

$$J^{\ell}_{W}(E) = (4\pi)^2 \int_0^\infty dr r^2 \int_0^\infty dr' r'^2 \operatorname{Im} \{ \Sigma^*_{\ell}(r,r';E) \}$$

 Spectroscopic factor for states near ε_F is well defined from Σ*

$$S_{F} = \left(1 - \frac{\partial \Sigma^{*}(\alpha_{qh}, \alpha_{qh}; E)}{\partial E}\Big|_{\epsilon}\right)^{-1}$$

$$n_{n\ell j} = \int_{-\infty}^{\epsilon_f} dES^h_{n\ell j}(E) \qquad d_{n\ell j} = \int_{\epsilon_f}^{\infty} dES^p_{n\ell j}(E)$$



Mack C. Atkinson LLNL

• No imaginary component of Σ^* around ϵ_F

$$J^{\ell}_{W}(E) = (4\pi)^2 \int_0^\infty dr r^2 \int_0^\infty dr' r'^2 \operatorname{Im}\{\Sigma^*_{\ell}(r,r';E)\}$$

 Spectroscopic factor for states near ε_F is well defined from Σ*

$$S_{F} = \left(1 - \frac{\partial \Sigma^{*}(\alpha_{qh}, \alpha_{qh}; E)}{\partial E}\Big|_{\epsilon}\right)^{-1}$$

$$n_{n\ell j} = \int_{-\infty}^{\epsilon_f} dES^h_{n\ell j}(E) \qquad d_{n\ell j} = \int_{\epsilon_f}^{\infty} dES^p_{n\ell j}(E)$$

Orbital	S _F	n _{nℓj}	d _{nℓj}
$0d\frac{3}{2}$	0.71	0.80	0.17
$1s\frac{\overline{1}}{2}$	0.60	0.82	0.15



atkinson27@llnl.gov

Mack C. Atkinson LLNL

DOM calculation of ${}^{40}Ca(e, e'p){}^{39}K$

• DWIA for exclusive reaction (C. Giusti's DWEEPY code)

$$J^{\mu}(\mathbf{q}) = \int \chi^{(-)*}_{E\alpha}(\mathbf{r}) j^{\mu}(\mathbf{r}) \phi_{E\alpha}(\mathbf{r}) [S_{F\alpha}(E)]^{1/2} e^{i\mathbf{q}\cdot\mathbf{r}} d^3r$$



DOM calculation of ${}^{40}Ca(e, e'p){}^{39}K$

• DWIA for exclusive reaction (C. Giusti's DWEEPY code)

$$J^{\mu}(\mathbf{q}) = \int \chi^{(-)*}_{E\alpha}(\mathbf{r}) j^{\mu}(\mathbf{r}) \phi_{E\alpha}(\mathbf{r}) [S_{F\alpha}(E)]^{1/2} e^{i\mathbf{q}\cdot\mathbf{r}} d^3r$$

• DOM provides all ingredients



DOM calculation of ${}^{40}Ca(e, e'p){}^{39}K$

• DWIA for exclusive reaction (C. Giusti's DWEEPY code)

$$J^{\mu}(\mathbf{q}) = \int \chi_{E\alpha}^{(-)*}(\mathbf{r}) j^{\mu}(\mathbf{r}) \phi_{E\alpha}(\mathbf{r}) [S_{F\alpha}(E)]^{1/2} e^{i\mathbf{q}\cdot\mathbf{r}} d^3r$$

• DOM provides all ingredients



p /

 (\mathbf{q},ω)

A-1

e



Data: G. J. Kramer et. al, Nucl. Phys. A, 679, 267 (2001)

M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, 798, 135027 (2019)



Data: G. J. Kramer et. al, Nucl. Phys. A, 679, 267 (2001)

$S_{F_{\ell j}}^n$	$0d\frac{3}{2}$	$1s\frac{1}{2}$
⁴⁰ Ca	0.71 ± 0.04	0.60 ± 0.03
⁴⁸ Ca	0.58 ± 0.03	0.55 ± 0.03

M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, 798, 135027 (2019)



Data: G. J. Kramer et. al, Nucl. Phys. A, 679, 267 (2001)

$S_{F_{\ell j}}^n$	$0d\frac{3}{2}$	$1s\frac{1}{2}$
⁴⁰ Ca	0.71 ± 0.04	0.60 ± 0.03
⁴⁸ Ca	0.58 ± 0.03	0.55 ± 0.03

M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, 798, 135027 (2019)





Protons are more correlated in ⁴⁸Ca!

$S_{F_{\ell j}^n}$	$0d\frac{3}{2}$	$1s\frac{1}{2}$
⁴⁰ Ca	0.71 ± 0.04	0.60 ± 0.03
⁴⁸ Ca	0.58 ± 0.03	0.55 ± 0.03

M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, 798, 135027 (2019)

 \bullet Dispersion relation "pulls" strength of Σ^* around



- \bullet Dispersion relation "pulls" strength of Σ^* around
- More strength above ϵ_f depletes the strength below



- \bullet Dispersion relation "pulls" strength of Σ^* around
- More strength above ϵ_f depletes the strength below
- High-energy σ_{react} has strong effect on S_F



- \bullet Dispersion relation "pulls" strength of Σ^* around
- More strength above ϵ_f depletes the strength below
- High-energy σ_{react} has strong effect on S_F





- \bullet Dispersion relation "pulls" strength of Σ^* around
- More strength above ϵ_f depletes the strength below
- High-energy σ_{react} has strong effect on S_F





Mack C. Atkinson LLNL

- \bullet Dispersion relation "pulls" strength of Σ^* around
- More strength above ϵ_f depletes the strength below
- High-energy σ_{react} has strong effect on S_F





Mack C. Atkinson LLNL

Quenching of "Spectroscopic Factors"



atkinson27@llnl.gov

Mack C. Atkinson LLNL

13

Quenching of "Spectroscopic Factors"



atkinson27@llnl.gov

Mack C. Atkinson LLNL

13



$$T \approx \int d\boldsymbol{R} t_{NN} \chi_1^{(-)*}(\boldsymbol{R}) \chi_2^{(-)*}(\boldsymbol{R}) \chi_0^{(+)}(\boldsymbol{R}) e^{-i\alpha_R \boldsymbol{K}_0 \cdot \boldsymbol{R}} \phi_{ljm}^n(\boldsymbol{R}).$$

• Same DOM ingredients used



$$T \approx \int d\boldsymbol{R} t_{NN} \chi_1^{(-)*}(\boldsymbol{R}) \chi_2^{(-)*}(\boldsymbol{R}) \chi_0^{(+)}(\boldsymbol{R}) e^{-i\alpha_R \boldsymbol{K}_0 \cdot \boldsymbol{R}} \phi_{ljm}^n(\boldsymbol{R}).$$

• Same DOM ingredients used



$$T \approx \int d\boldsymbol{R} t_{NN} \chi_1^{(-)*}(\boldsymbol{R}) \chi_2^{(-)*}(\boldsymbol{R}) \chi_0^{(+)}(\boldsymbol{R}) e^{-i\alpha_R \boldsymbol{K}_0 \cdot \boldsymbol{R}} \phi_{ljm}^n(\boldsymbol{R})$$

• Same DOM ingredients used

"S _F "	(p,2p)	(e, e'p)
DOM	0.560	0.71 ± 0.04



$$T \approx \int d\boldsymbol{R} t_{NN} \chi_1^{(-)*}(\boldsymbol{R}) \chi_2^{(-)*}(\boldsymbol{R}) \chi_0^{(+)}(\boldsymbol{R}) e^{-i\alpha_R \boldsymbol{K}_0 \cdot \boldsymbol{R}} \phi_{ljm}^n(\boldsymbol{R})$$

• Same DOM ingredients used

"S _F "	(p,2p)	(e, e'p)
DOM	0.560	0.71 ± 0.04

$$\mathcal{S}_{\mathcal{F}} = \left(1 - rac{\partial \Sigma^*(lpha_{qh}, lpha_{qh}; E)}{\partial E}\Big|_{\epsilon}
ight)^{-1}$$

• Remember that S_F comes directly from Σ^*_{DOM}





$$T \approx \int d\boldsymbol{R} t_{NN} \chi_1^{(-)*}(\boldsymbol{R}) \chi_2^{(-)*}(\boldsymbol{R}) \chi_0^{(+)}(\boldsymbol{R}) e^{-i\alpha_R \boldsymbol{K}_0 \cdot \boldsymbol{R}} \phi_{ljm}^n(\boldsymbol{R})$$

• Same DOM ingredients used

"S _F "	(p,2p)	(e, e'p)
DOM	0.560	0.71 ± 0.04

$$\mathcal{S}_{\mathcal{F}} = \left(1 - \frac{\partial \Sigma^*(\alpha_{qh}, \alpha_{qh}; E)}{\partial E}\Big|_{\epsilon}\right)^{-1}$$

- Remember that S_F comes directly from Σ^*_{DOM}
- Main difference is the probe \implies problem is likely V_{pp}

K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff PRC 105, 014622 (2022)

atkinson27@llnl.gov



• Try varying V_{NN} to see effect on S_F



• Try varying V_{NN} to see effect on S_F



S _F	V_{NN}	(<i>p</i> , 2 <i>p</i>)
DOM	FL	0.560 ± 0.05
DOM	Mel	0.489 ± 0.05
DOM	Mel (free)	0.515 ± 0.05

- Try varying V_{NN} to see effect on S_F
- Dependence of S_F on choice of V_{NN} is another sign the problem lies in V_{NN}



S _F	V_{NN}	(<i>p</i> , 2 <i>p</i>)
DOM	FL	0.560 ± 0.05
DOM	Mel	0.489 ± 0.05
DOM	Mel (free)	0.515 ± 0.05

- Try varying V_{NN} to see effect on S_F
- Dependence of S_F on choice of V_{NN} is another sign the problem lies in V_{NN}
- Interactions need information about the nucleus



S _F	V_{NN}	(<i>p</i> , 2 <i>p</i>)
DOM	FL	0.560 ± 0.05
DOM	Mel	0.489 ± 0.05
DOM	Mel (free)	0.515 ± 0.05

- Try varying V_{NN} to see effect on S_F
- Dependence of S_F on choice of V_{NN} is another sign the problem lies in V_{NN}
- Interactions need information about the nucleus





S _F	V_{NN}	(p, 2p)
DOM	FL	0.560 ± 0.05
DOM	Mel	0.489 ± 0.05
DOM	Mel (free)	0.515 ± 0.05

K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff PRC 105, 014622 (2022)

- Try varying V_{NN} to see effect on S_F
- Dependence of S_F on choice of V_{NN} is another sign the problem lies in V_{NN}
- Interactions need information about the nucleus



• $G_{pp} \approx \int G_{\rm DOM} \times G_{\rm DOM}$



S _F	V_{NN}	(p, 2p)
DOM	FL	0.560 ± 0.05
DOM	Mel	0.489 ± 0.05
DOM	Mel (free)	0.515 ± 0.05

K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff PRC 105, 014622 (2022)

- Try varying V_{NN} to see effect on S_F
- Dependence of S_F on choice of V_{NN} is another sign the problem lies in V_{NN}
- Interactions need information about the nucleus



- $G_{pp} \approx \int G_{\rm DOM} \times G_{\rm DOM}$
- Similar to G-matrix, except this is calculated in finite nuclei



S_F	V_{NN}	(p, 2p)
DOM	FL	0.560 ± 0.05
DOM	Mel	0.489 ± 0.05
DOM	Mel (free)	0.515 ± 0.05

K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff PRC 105, 014622 (2022)

- Try varying V_{NN} to see effect on S_F
- Dependence of S_F on choice of V_{NN} is another sign the problem lies in V_{NN}
- Interactions need information about the nucleus



- $G_{pp} \approx \int G_{\rm DOM} \times G_{\rm DOM}$
- Similar to *G*-matrix, except this is calculated in finite nuclei
- Good approximation for typical (p, 2p) energies



S_F	V_{NN}	(<i>p</i> , 2 <i>p</i>)
DOM	FL	0.560 ± 0.05
DOM	Mel	0.489 ± 0.05
DOM	Mel (free)	0.515 ± 0.05

K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff PRC 105, 014622 (2022)

Looking (far) ahead: microscopic (ab initio) OMPs?

 \bullet Currently, the goal is to improve reaction models by providing nucleus-informed Γ_{NN}
- \bullet Currently, the goal is to improve reaction models by providing nucleus-informed Γ_{NN}
 - In any way possible (phenomenologically or microscopically)

- \bullet Currently, the goal is to improve reaction models by providing nucleus-informed Γ_{NN}
 - In any way possible (phenomenologically or microscopically)
- Γ can also be used to calculate the nucleon self-energy (OMP)

- Currently, the goal is to improve reaction models by providing nucleus-informed Γ_{NN}
 - In any way possible (phenomenologically or microscopically)
- Γ can also be used to calculate the nucleon self-energy (OMP)



- Currently, the goal is to improve reaction models by providing nucleus-informed Γ_{NN}
 - In any way possible (phenomenologically or microscopically)
- Γ can also be used to calculate the nucleon self-energy (OMP)



- Currently, the goal is to improve reaction models by providing nucleus-informed Γ_{NN}
 - In any way possible (phenomenologically or microscopically)
- Γ can also be used to calculate the nucleon self-energy (OMP)



- \bullet Currently, the goal is to improve reaction models by providing nucleus-informed Γ_{NN}
 - In any way possible (phenomenologically or microscopically)
- Γ can also be used to calculate the nucleon self-energy (OMP)



Summary

- The DOM provides consistent ingredients for knockout reactions
- Discrepancy between ${
 m ^{40}Ca}(e,e'p)^{39}{
 m K}$ and ${
 m ^{40}Ca}(p,2p)^{39}{
 m K}$
- DOM provides a path toward improvement through the nucleus-informed NN interaction Γ_{NN}





Thanks

- Willem Dickhoff
- Robert Charity
- Hossein Mahzoon
- Lee Sobotka
- Natalie Calleya



Cole Pruitt Gregory Potel Sofia Quaglioni



• Louk Lapikás

• Henk Blok





Backup Slides

 Monte-Carlo results borrowed from Bob Wiringa's website_[1]





https://www.phy.anl.gov/theory/research/QMCresults.html
 R.B. Wiringa *et al.*, PRC **89**, 024305 (2014)
 D. Lonardoni *et al.*, PRC **96**, 024326 (2017)

atkinson27@llnl.gov

Backup

- "Smearing" of self-energy poles inflates S_F
- Renormalize with experimental excitation energy spectrum

$$\frac{\mathcal{Z}_{F}^{\text{DOM}}}{\int dE \ S^{\text{DOM}}(E)} = \frac{\mathcal{Z}_{F}^{\text{exp}}}{\int dE \ S^{\text{exp}}(E)}$$



M. Atkinson et al., PRC 98, 044627 (2018)

$$n(\mathbf{k}) = \int d^3r \int d^3r' e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\rho(\mathbf{r},\mathbf{r}')$$

 $^{40}\mathrm{Ca}$ DOM Single-Particle Momentum Distribution



Mack C. Atkinson LLNL

$$n(\mathbf{k}) = \int d^3r \int d^3r' e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\rho(\mathbf{r},\mathbf{r}')$$

 $^{40}\mathrm{Ca}$ DOM Single-Particle Momentum Distribution



Mack C. Atkinson LLNL

 Short-range correlations (SRC) responsible for this high-momentum content

$$n(\mathbf{k}) = \int d^3r \int d^3r' e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\rho(\mathbf{r},\mathbf{r}')$$

 $\rm ^{40}Ca$ DOM Single-Particle Momentum Distribution



Mack C. Atkinson LLNL

• Short-range correlations (SRC) responsible for this high-momentum content

$$n(\mathbf{k}) = \int d^3r \int d^3r' e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \rho(\mathbf{r},\mathbf{r}')$$

⁴⁰Ca DOM Single-Particle Momentum Distribution Proton Spectral Functions in ⁴⁰Ca 10^{4} 10^{0} 10^{3} HF $k_F \approx 1.4 \text{ fm}^{-1}$ 10^{-1} 10^{2} n(k) [fm³] $n_{
ho}(k_{
m high})=14\%$ $n_n(k_{
m high})=14.7\%$ 10^{1} $\underbrace{\textcircled{H}}_{0}$ 10⁻² 10^{0} 10^{-1} 10^{-3} 10^{-2} 10^{-4} 10^{-3} -100 - 90 - 80 - 70 - 60 - 50 - 40 - 30 - 20-100.51.52 2.53 3.5 ε_F E_{cm} [MeV] $k \, [{\rm fm}^{-1}]$

Mack C. Atkinson LLNL

$$S^{h}(\alpha,\beta;E) = rac{1}{\pi} \mathrm{Im}\{G(\alpha,\beta;E)\} \qquad S^{h}(E) = \sum_{\alpha} S(\alpha,\alpha;E)$$





$$S^{h}(\alpha,\beta;E) = \frac{1}{\pi} \operatorname{Im} \{G(\alpha,\beta;E)\} \qquad S^{h}(E) = \sum_{\alpha} S(\alpha,\alpha;E)$$

1.00

$$\begin{array}{c} 10^{-1} & 0.81/2 & & \\ 0.91/2 & -... & \\ 0.93/2 & -.. & \\ 0.03/2 & & \\ 0.05/2 & -..$$

$$\rho_{\alpha,\beta} = \int_{-\infty}^{\varepsilon_F} dES(\alpha,\beta;E)$$

$$S^{h}(\alpha,\beta;E) = \frac{1}{\pi} \operatorname{Im} \{G(\alpha,\beta;E)\} \qquad S^{h}(E) = \sum_{\alpha} S(\alpha,\alpha;E)$$



$$\rho_{\alpha,\beta} = \int_{-\infty}^{\varepsilon_F} dES(\alpha,\beta;E) \qquad N, Z = \sum_{\alpha} \rho_{\alpha,\alpha}^{N,Z}$$



. .

$$S^{h}(\alpha,\beta;E) = \frac{1}{\pi} \operatorname{Im} \{G(\alpha,\beta;E)\} \qquad S^{h}(E) = \sum_{\alpha} S(\alpha,\alpha;E)$$



$$\rho_{\alpha,\beta} = \int_{-\infty}^{\varepsilon_F} dES(\alpha,\beta;E) \qquad N, Z = \sum_{\alpha} \rho_{\alpha,\alpha}^{N,Z}$$

$$E_0^{A} = \frac{1}{2} \sum_{\alpha\beta} \left[T_{\beta\alpha} \rho_{\alpha\beta} + \delta_{\alpha\beta} \int_{-\infty}^{\epsilon_f^{-}} dEES_h(\alpha; E) \right]$$



$$S^{h}(\alpha,\beta;E) = \frac{1}{\pi} \operatorname{Im} \{G(\alpha,\beta;E)\} \qquad S^{h}(E) = \sum_{\alpha} S(\alpha,\alpha;E)$$



$$\rho_{\alpha,\beta} = \int_{-\infty}^{\varepsilon_F} dES(\alpha,\beta;E) \qquad N, Z = \sum_{\alpha} \rho_{\alpha,\alpha}^{N,Z}$$

$$E_{0}^{A} = \frac{1}{2} \sum_{\alpha\beta} \left[T_{\beta\alpha} \rho_{\alpha\beta} + \delta_{\alpha\beta} \int_{-\infty}^{\epsilon_{f}^{-}} dEES_{h}(\alpha; E) \right]$$

	Ν	Z	DOM E_0^A/A	Exp. E_0^A/A
⁴⁰ Ca	19.9	19.8	-8.50	-8.55
⁴⁸ Ca	27.9	19.9	-8.59	-8.66
²⁰⁸ Pb	126.2	82.1	-7.81	-7.87







• *n*-*p* interaction stronger than *n*-*n* or *p*-*p*



- *n*-*p* interaction stronger than *n*-*n* or *p*-*p*
- More neutrons means more *n*-*p* SRC pairs (*n*-*p* dominance)



- *n*-*p* interaction stronger than *n*-*n* or *p*-*p*
- More neutrons means more *n*-*p* SRC pairs (*n*-*p* dominance)
- \implies Protons more correlated (more high-momentum content)









atkinson27@llnl.gov